F- and D-terms from D7-branes

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in collaboration with

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Type IIB Orientifolds:

We consider type IIB orientifold compactifications with background spaces $\mathcal{M}_{10} = R^{3,1} \otimes Y/\mathcal{O}$,

Y : compact CY, $\mathcal{O}$: O3/O7 orientifold projection.

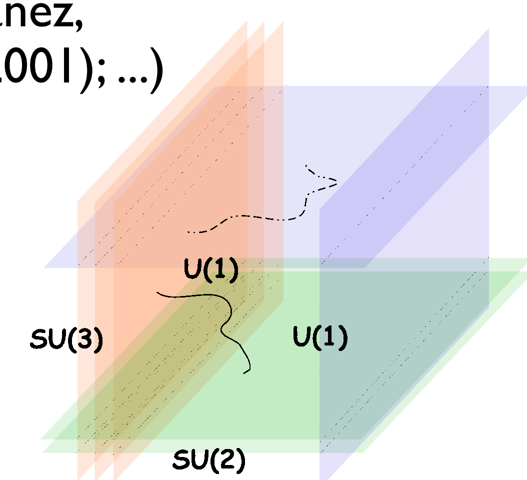
They offer several phenomenologically attractive features:

- Semi-realistic models for particle physics: MSSM from open strings on D7-branes.
- Moduli stabilization from 3-form G-fluxes and non-perturbative effects on D7-branes.
  KKLT: possibilty of dS-vacua with positive cosmological constant!
- (Possibly, cosmological models with D3/D7-brane inflation.)

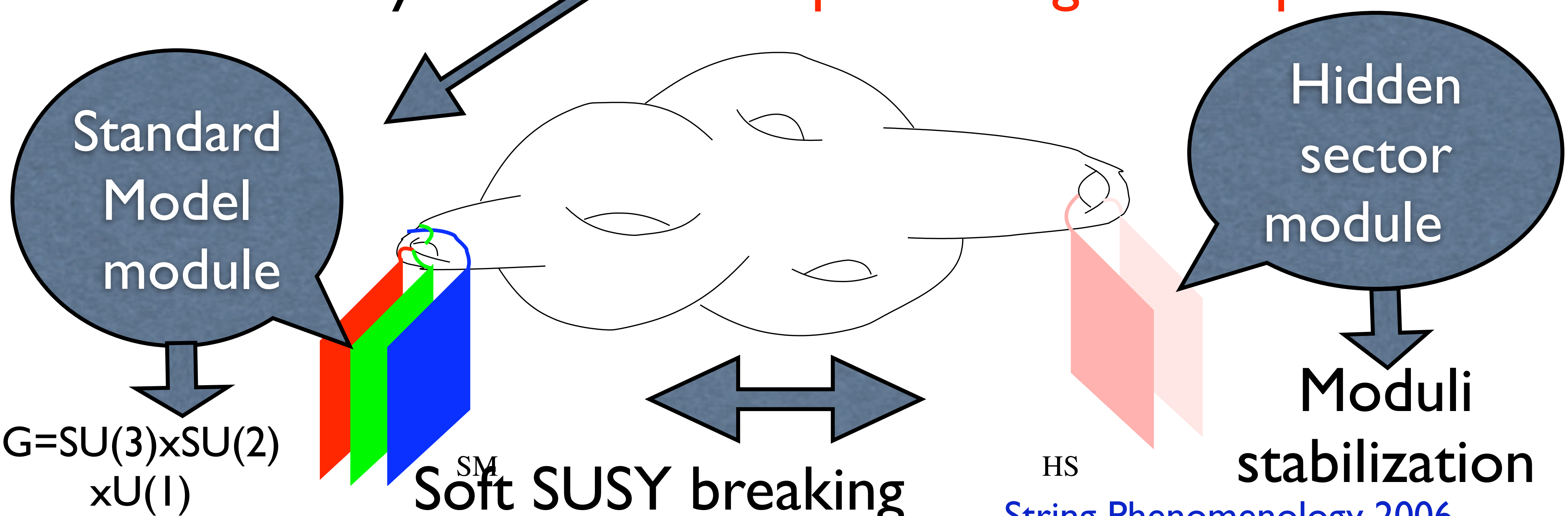
(Intersecting) D-brane model building

(Bachas (1995); Blumenhagen, Görlich, Körs, Lüst (2000); Angelantonj, Antoniadis, Dudas Sagnotti (2000); Ibanez, Marchesano, Rabadan (2001); Cvetic, Shiu, Uranga (2001); ...)

Local D-brane module of the Standard Model by 4 stacks of intersecting D-branes:



D-brane compactifications: Wrapping the D-brane modules around cycles of the compact background space:



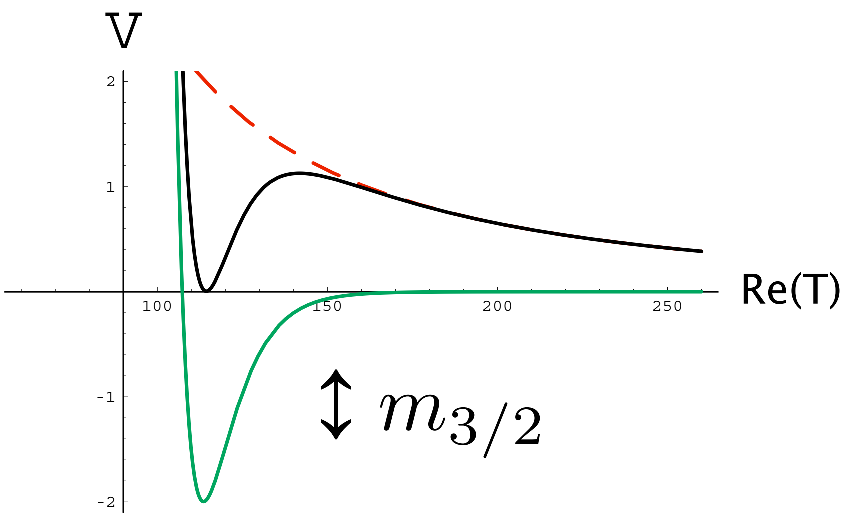
Moduli Stabilization - KKLT

(Kachru Kallosh, Linde, Trivedi (2002))

Step 1: F-terms: Fix all moduli (preserving SUSY)
Dilaton S and complex structure moduli U are stabilized with 3-form fluxes, Kähler moduli T are fixed by non-perturbative effects on D3/D7-branes → SUSY AdS vacuum.

$$W_{np} = \gamma(z) e^{-\alpha T}$$

Step 2: Lift the minimum of the potential to a metastable dS vacuum



(i) introducing $\overline{D3}$ branes

$$V_{\overline{D3}} \sim \frac{\mu_3}{[\mathrm{Re}(T)]^2}$$

(ii) D7-branes with F-flux → D-term potential:

$$V_D \sim \frac{q^2}{[\mathrm{Re}(T)]^2}$$

(Burgess, Kallosh, Quevedo (2003))

Outline:

The talk will adress two questions:

- Can all moduli be fixed in AdS vacuum by F-terms from D3/D7 branes in blown-up orbifolds models?

(Lüst, Reffert, Scheidegger, Schulgin, Stieberger)

Related work by: Choi, Falowski, Nilles, Olechowski, Pokorski, hep-th/0411066;
Denef, Douglas, Florea, Grassi, Kachru: hep-th/0503125;
Lüst, Reffert, Schulgin, Stieberger, hep-th/0506090)

- Can D-terms be obtained from D7-branes, i.e is the D-term uplift possible using D7-branes, or is there are clash between F- and D-terms?

(Haack, Krefl, Lüst, Van Proeyen, Zagermann)

Related work by: Jockers, Louis, hep-th/0502059; Villadoro, Zwirner, hep-th/0508167,0602120;
Acucarro, de Carlos, Casas, Doplicher, hep-th/0601190; Lebedev, Nilles, Ratz, hep-th/0603047.

F-terms:

Non-perturbative superpotential:

- Euclidean D3-brane wrapped on 4-cycle $C_4^j \subset Y$ with volume T^j: $W \sim \gamma_j(z) e^{-2\pi T^j}$

Condition for non-vanishing superpotential: (Witten; Tripathy, Trivedi; Kallosh, Kashani-Poor, Tomasiello; Saulina; E. Bergshoeff, R. Kallosh, A. K. Kashani-Poor, D. Sorokin, A. Tomasiello; J.Park)

$$\frac{N_F}{2} = \chi(C_4) = h^{0,0}(C_4) - h^{1,0}(C_4) + h^{2,0}(C_4) = 1$$

(Arithmetic genus χ also depends on O7-plane and 3-form flux!)

- Gaugino condensation in gauge theory on D7-branes wrapped on 4-cycle $C_4^j \subset Y$:

$$W \sim \gamma_j(z) e^{-T^j/b_j}$$

Condition for non-vanishing superpotential: no massless adjoint chiral multiplets (open string moduli of D7!) (Gomis, Mateos, Marchesano)

Prefactor in gaugino condensate superpotential:

- Gauge kinetic function for D7-brane wrapped on divisor C_4^j :

$$f_j = T^j + \Delta(S,U) \quad \Longrightarrow \quad \gamma_j \sim e^{\Delta(S,U)} \sim \eta(U)^2$$

tree level — loop corrections (Lüst, Stieberger)

- additional instantons in the D7-gauge theory:

$$\gamma_j \sim e^{-S\int_{C_4} F\wedge F}$$

- Charged matter fields: see next section.

F-terms ➠ AdS vacua

All together:

Total effective N=1 superpotential:

$$W = W_{\rm flux}(S,U) + W_{\rm np}(T) = \kappa_{10}^{-2}\int G_3\wedge\Omega + \sum_{j=1}^{h^{(+)}_{(1,1)}(X_6)} \gamma_j(S,U)\; e^{a_j\; T^j}$$

Kähler potential: $K = -\ln(S-\overline{S}) - \ln\int\Omega\wedge\overline{\Omega} + 2\,\ln V$

$$V = \frac{1}{6}\int J\wedge J\wedge J$$

F-term scalar potential:

$$V = e^{\kappa_4^2 K}\left(|D_S W|^2 + \sum_i |D_{T^i}W|^2 + \sum_j |D_{U^j}W|^2 - 3|W|^2\right)$$

Supersymmetric vacua ➠ impose F-term SUSY-conditions:

$$D_i W = \partial_i W + \kappa_4^2\; W\;\partial_i K = 0\,, \quad i = S, U^i, T^i$$

All known SUSY vacua with all moduli fixed are AdS.

Fixing all moduli in resolved orbifolds

We consider IIB orientifolds with D3/D7-branes, compactified on resolved orbifolds $\mathcal{M}_6 = T^6/Z_N\,, T^6/(Z_N \times Z_M)$ (smooth Calabi-Yau!) using the methods of toric geometry: (hep-th/0609013,0609014)

- Describe the local patches near the singularities with toric methods. Resolve the singularities via blow-ups.
- Glue the patches to get a smooth Calabi-Yau
- Perform an orientifold projection on the smooth CY $\Rightarrow$ O-planes and D-branes
- Determine divisor topologies to decide whether a n.p.-superpotential is generated
- Determine the CY triple intersection no. ➡ Kähler potential:
- Fix the dilaton and the complex structure moduli with 3-form flux
- Look for critical points of the scalar potential
- Stable dS-uplift: $(\mathrm{mass}_{\mathrm{moduli}})^2 > 0$

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Results for F-term moduli stabilization to AdS on blown-up orbifolds:

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- Not in every model all the divisors have the correct topology to contribute the n.p. superpotential.

- "No-go Theorem" for models with no complex structure moduli: a stable uplift to a dS vacuum is not possible!

- In models with odd cohomology under the orientifold projection, $(h^{(-)}_{(1,1)} \neq 0)$, the F-terms cannot fix all moduli. However these can be stabilized by additional D-terms.

- Conclusion: Moduli stabilization is model dependent! However in particular examples like Z4, Z6, Z2xZ2, Z2xZ4, all moduli can be fixed!

D-terms:

A D-term potential arises from the existence of a gauged shift symmetry:

(Anomalous) $U(1)_f$ gauge transformation:

$$V_f \to V_f + i(\Lambda - \bar{\Lambda})\,. \quad (V_f:\ U(1)_f \text{ vector superfield})$$

V_f mixes in the Kähler potential with a chiral superfield T_c :

$$K = -\log(T_c + \bar{T}_c + \delta_{GS} V_f)\,.$$

Hence to keep K invariant, T_c has to shift under $U(1)_f$ as:

$$T_c \to T_c + i\delta_{GS}\Lambda\,,\ \ a_c \to a_c + \delta_{GS}\Lambda\,,\ (a_c = Im(T_c))$$

Hence a_c appears with a gauge covariant derivative:

$$\mathcal{L} \sim (\partial_\mu a^c + qA_\mu^{(f)})^2\,,\ \ q = \delta_{GS}$$

D-terms:

The $(a_c \leftrightarrow U(1)_f)$ mixing leads to the following FI D-term potential:

$$V_D \sim \frac{1}{g_f^2}\, \xi_{FI}^2 \,, \qquad \xi_{FI} = g_f^2 \left(\frac{\partial K}{\partial V_f} \right)_{V_f=0} .$$

Hence: $$V_D \sim \delta_{GS}^2 \, \frac{g_f^2}{(ReT_c)^2} \, .$$

V_D arises from the standard expression:

$$V_D \sim D^2 \, , \;\; D \sim \eta^{T_c} \partial_{T_c} K$$

with constant Killing vector $\eta^{T_c} = i\delta_{GS}$ $\leftrightarrow$ gauged shift symmetry.

There can be two non $U(1)_f$ invariant terms in the action:

(i) If there is a gauge group $G_c = U(1)_c \times SU(N_c)$

with gauge coupling $1/g_c^2 \sim ReT_c$

then the term $\mathrm{Im}(T_c)\mathrm{tr}[F_c \wedge F_c]$

is not $U(1)_f$ invariant.

(ii) If the gauge group $SU(N_c)$ induces a gaugino condensate

then the n.p. superpotential $W \sim e^{-T_c/b_c}$

is not $U(1)_f$ invariant.

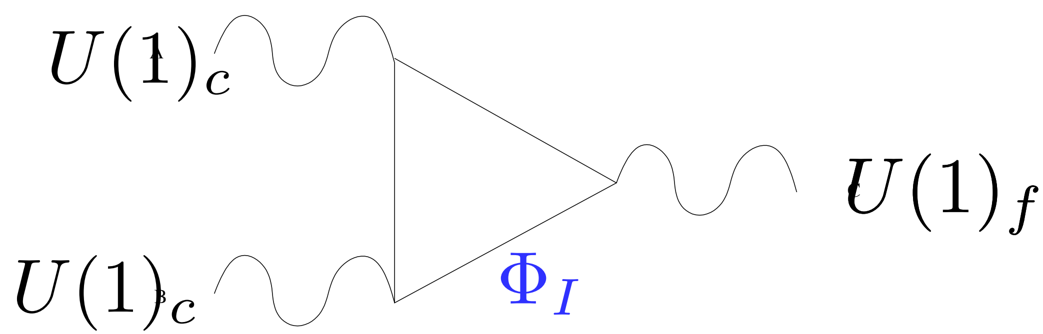
D-terms:

Both non-invariances can be cancelled by the same mechanism if N_f matterfields Φ_I are present:

$U(1)_f$ charge: $Q_f (= 1)$; $U(1)_c$ charge: Q_c

$N_c \oplus \bar{N}_c$ representation of $SU(N_c)$

(i) Mixed triangle graph: $U(1)_f$ is anomaleous



$$A_{(i)} \sim tr((Q_c^2 Q_f) = tr(Q_f)$$

GS-mechanism: Together with the $T_c/U(1)_f$ FI mixing term, the $U(1)_f$ non-invariance (i) is cancelled.

D-terms:

(ii) The matter fields contribute to the effective superpotential:

$$W_{n.p.} \sim \gamma_{1-loop}(U)(\det M)^{-\frac{1}{N_c-N_f}} e^{-\frac{8\pi^2 T_c}{N_c-N_f}},$$

(Assume $N_f < N_c$) $\qquad \det M \equiv \det(\Phi\bar{\Phi})$

Now the n.p. superpotential from gauge condensation is also $U(1)_f$ invariant, if:

$$\delta_{GS} = \frac{1}{8\pi^2} N_f \quad (\Phi \to e^{i\Lambda}\Phi)\,.$$

D-term potential: $V_D \sim \frac{1}{\mathrm{Re}(T^f)} \left(\frac{\delta_{GS}}{\mathrm{Re}(T^c)} + \sum |\Phi_i|^2 \right)^2$

D-terms from D7-branes:

Now we want to derive the D-term potential microscopically in type II orientifolds with D7-branes with F-fluxes:

Set-up: Consider two kinds of (stacks of) D7-branes:

(i) $(n_f)\, D7_f$:
- carry world volume 2-form flux F
- are wrapped around a divisor Σ
- gauge group: $G_f = U(1)_f (\times SU(n_f))$

↑ anomalous

Intersect each other ⇒ Bi-fundamental matter fields:

$$\Phi_I \, : \, Q_{\alpha\Sigma} \times (n_f, N_c \oplus \bar{N}_c)_{Q_f=1}$$

↑ Intersection no.

(ii) $N_c^\alpha \; D7_c$:
- are wrapped around divisors $C_\alpha, C_\beta, \ldots$
- gauge group: $G_c = G_\alpha \times G_\beta \times \ldots$

$G_\alpha = U(1)_c \times SU(N_c^\alpha)$ ⇒ Gaugino condensation

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DBI and WZ actions:

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D-terms correspond to unbalanced tensions (NS-tadpoles) of the D7-branes.

Consider the DBI actions for $D7_f$ with F-flux:

$$S_{DBI} = -\mu_7 \int_{\mathcal{W}} d^8\xi e^{-\phi} \sqrt{-\det(\iota^* g + \mathcal{F})}$$

$$\mathcal{W} = \mathcal{M}_4 \times \Sigma, \quad \mu_7 = \frac{1}{\sqrt{2}} (2\pi)^{-7} \alpha'^{-4}, \; \mathcal{F} = (\iota^* B +) 2\pi\alpha' F.$$

WZ-action: $$S_{WZ} = -\mu_7 \int_{\mathcal{W}} \sum_p \iota^* C_p \wedge e^{\mathcal{F}}$$

BPS calibration conditions on Σ :

$$\frac{1}{2} (\iota^* J + i\mathcal{F}) \wedge (\iota^* J + i\mathcal{F}) = e^{i\theta} \sqrt{\frac{\det(g_\Sigma + \mathcal{F})}{\det(g_\Sigma)}} \mathrm{Vol}_\Sigma \,, \quad \mathcal{F}^{2,0} = \mathcal{F}^{0,2} = 0 \,.$$

Scalar potential and gauge coupling:

Using the space-time ansatz $\mathcal{M}_{10} = \mathcal{M}_4 \times Y$

the DBI action can be rewritten as:

$$S_{DBI} = -\mu_7 \int_{\mathcal{M}_4} d^4x e^{-\phi} \sqrt{-\det(g_{(4)})} \sqrt{\det\left(1 + 2\pi\alpha' g_{(4)}^{-1} F_{(4)}\right) \Gamma_\Sigma}$$

with $\Gamma_\Sigma = \int_\Sigma d^4z \sqrt{\det(g_\Sigma + \mathcal{F})}$

A low energy expansion provides:

- the 4D scalar potential

$$V_{D7_f} = \mu_7 e^{3\phi} \mathcal{V}^{-2} \Gamma_\Sigma$$

- the 4D gauge coupling

$$g_{D7_f}^{-2} = \mu_7 (2\pi\alpha')^2 e^{-\phi} \Gamma_\Sigma$$

Some elements of CY geometry and topology:

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- Kähler form J for ambient Calabi-Yau Y:

$$J = v^\alpha \omega_\alpha \ (\alpha = 1, \cdots H^{(1,1)}(\Sigma))$$ v^α: geometrical 4-cycle volumes

- CY volume and triple intersection numbers:

$$\mathcal{V} = \frac{1}{6}(2\pi\sqrt{\alpha'})^{-6} \int_Y J \wedge J \wedge J = \frac{1}{6}\mathcal{K}_{\alpha\beta\gamma} v^\alpha v^\beta v^\gamma$$ $\mathcal{K}_{\alpha\beta\gamma}$: triple intersection #'s

- Cohomology of divisor $\Sigma \subset Y$:

2 type of (1,1)-forms: $\iota^* w_\alpha$: pullbacks of (1,1)-forms from Y

$\tilde{w}_a$: harmonic only locally on Σ , lie in the cokernel of ι^*

Hence: $$H^{(1,1)}(\Sigma) = \iota^* H^{(1,1)}(Y) \oplus \tilde{H}^{(1,1)}(\Sigma)$$

$$\Longrightarrow \quad \mathcal{F} = f^\alpha \iota^* \omega_\alpha + \tilde{f}^a \tilde{\omega}_a$$

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Relate D-term potential to CY geometry:

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Use calibration condition:

$$\Gamma_\Sigma = \tilde{\Gamma}_\Sigma e^{-i\theta}\,,\ \tilde{\Gamma}_\Sigma \equiv \frac{1}{2}\int_\Sigma (\iota^* J \wedge \iota^* J - \mathcal{F}\wedge\mathcal{F}) + i\int_\Sigma (\iota^* J \wedge \mathcal{F})$$

$$\mathrm{Re}\tilde{\Gamma}_\Sigma = \frac{1}{2}\int_\Sigma (\iota^* J \wedge \iota^* J - \mathcal{F}\wedge\mathcal{F}) = \left(\frac{1}{2}v^\alpha v^\beta \mathcal{K}_{\alpha\beta\Sigma} - f_\Sigma\right)(2\pi\sqrt{\alpha'})^4\,,$$

$$\mathrm{Im}\tilde{\Gamma}_\Sigma = \int_\Sigma \iota^* J \wedge \mathcal{F} = v^\alpha Q_{\alpha\Sigma}\,(2\pi\sqrt{\alpha'})^4$$

with $$Q_{\alpha\Sigma} = (2\pi\sqrt{\alpha'})^{-4}\int_\Sigma \iota^*\omega_\alpha \wedge \mathcal{F} = f^\beta \mathcal{K}_{\alpha\beta\Sigma}\,,$$

$$\Rightarrow\ N_f = (n_f)Q_{\alpha\Sigma}$$

$$f_\Sigma = \frac{1}{2}\left(f^\alpha f^\beta \mathcal{K}_{\alpha\beta\Sigma} + \tilde{f}^a \tilde{f}^b \mathcal{K}^{(\Sigma)}_{ab}\right)\,,$$

$$\mathcal{K}^{(\Sigma)}_{ab} = (2\pi\sqrt{\alpha'})^{-4}\int_\Sigma \tilde{\omega}_a \wedge \tilde{\omega}_b\,.$$

Relate D-term potential to CY geometry:

SUSY condition:

$$\theta = 0 \quad \Leftrightarrow \quad \mathrm{Im}\tilde{\Gamma}_\Sigma = \int_\Sigma \iota^* J \wedge \mathcal{F} = 0 \ .$$

Small SUSY breaking: $\theta \sim \mathrm{Im}\tilde{\Gamma}_\Sigma \neq 0\,,\ |\mathrm{Im}\tilde{\Gamma}_\Sigma| \ll |\mathrm{Re}\tilde{\Gamma}_\Sigma|$

$$\begin{aligned} V_{D7_f} &= \mu_7 e^{3\phi}\mathcal{V}^{-2}\mathrm{Re}\tilde{\Gamma}_\Sigma\sqrt{1+\left(\frac{\mathrm{Im}\tilde{\Gamma}_\Sigma}{\mathrm{Re}\tilde{\Gamma}_\Sigma}\right)^2} \\ &\approx \mu_7 e^{3\phi}\mathcal{V}^{-2}\mathrm{Re}\tilde{\Gamma}_\Sigma + \frac{1}{2}\mu_7 e^{3\phi}\mathcal{V}^{-2}\frac{1}{\mathrm{Re}\tilde{\Gamma}_\Sigma}(\mathrm{Im}\tilde{\Gamma}_\Sigma)^2 \ . \end{aligned}$$

The first term vanished due to the RR-tadpole condition.

$$\Longrightarrow \quad V_{D7_f} \equiv V_D = \frac{1}{2}\mu_7 e^{3\phi}\mathcal{V}^{-2}\frac{1}{\mathrm{Re}\tilde{\Gamma}_\Sigma}(\mathrm{Im}\tilde{\Gamma}_\Sigma)^2 \, .$$

Likewise for the gauge coupling constant:

$$g_\Sigma^{-2} = \mu_7(2\pi\alpha')^2 e^{-\phi}\mathrm{Re}\tilde{\Gamma}_\Sigma = \mu_7(2\pi)^6\alpha'^4\left(e^{-\phi}\frac{1}{2}v^\alpha v^\beta \mathcal{K}_{\alpha\beta\Sigma} - e^{-\phi} f_\Sigma\right)$$

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D-term potential in supergravity variables:

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The v^α are the 4-cycle volumes in the geometrical string basis!

Define the following Kähler moduli in the SUGRA basis:

$$T^\alpha = \hat{\mathcal{V}}^\alpha + i a^\alpha\,, \qquad S = e^{-\phi} - iC_0\,.$$

with $$\hat{\mathcal{V}}^\alpha = e^{-\phi}\frac{1}{2} v^\beta v^\gamma \mathcal{K}_{\beta\gamma\alpha}\,.$$

Gauge coupling: $$g_\Sigma^{-2} = \mu_7 (2\pi)^6 \alpha'^4 \left(\mathrm{Re} T^\Sigma - f_\Sigma \mathrm{Re} S\right)$$

D-term potential:

$$V_D = \frac{1}{\mathrm{Re}(T^f)}\left(\mu_7 (2\pi)^5 \alpha'^3 (\partial_{T^\alpha} K) n_f Q_{\alpha\Sigma}\right)^2 \Rightarrow \delta_{GS} \sim n_f Q_{\alpha\Sigma}$$

This is of the familiar form from supergravity with a gauged U(1) symmetry.

Summary and remarks:

- Wrapped D7-branes branes can produce coexisting F- and D-terms: Anomaleous $U(1)_f$ gauge symmetry.

$$D7_c: \quad W_{n.p.} \sim \gamma_{1-loop}(U)(\det M)^{-\frac{1}{N_c-N_f}} e^{-\frac{8\pi^2 T_c}{N_c-N_f}},$$

$$D7_f: \quad V_D \sim \frac{1}{\mathrm{Re}(T^f)}\left(\frac{\delta_{GS}}{\mathrm{Re}(T^c)} + \sum |\Phi_i|^2\right)^2$$

- $Q_{\alpha\Sigma} \neq 0$: Only axions on 4-cycles that intersect Σ can be charged under the gauged U(1).

If Σ has self-intersections, then T^{Σ} will be charged.

- One can construct a Z2xZ2 orientifold with F-flux as toy model.

- The Φ_I must be taken into account when minimizing the potential! $(\langle\Phi_I\rangle \neq 0?)$ (see also Achucarro et al.)

- Generalizations:

  ➪ $G_c = SU(N_c)$ with $N_f = N_c$

  ➪ Orientifolds with (anti)-symmetric repr. of $SU(N_c)$

  ➪ Orientifolds with $G_c = SO(N_c)/Sp(N_c)$

Thank You!