Nongeometric String Backgrounds

Albion Lawrence
Brandeis University

Outline

I. Introduction

II. D-branes on "nongeometric" backgrounds

A. Lawrence,
M. Schulz,
and B. Wecht
hep-th/0602025

III. Nongeometric backgrounds and spacetime supersymmetry

A. Lawrence, R. Minasian, T. Sander, M. Schulz, and B. Wecht, in progress

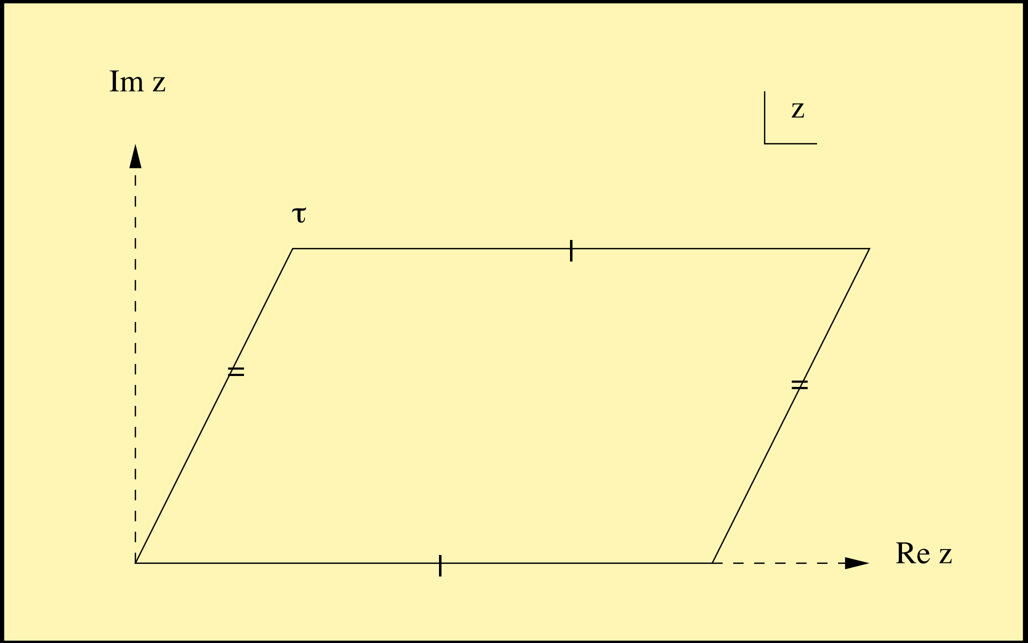
I. Introduction

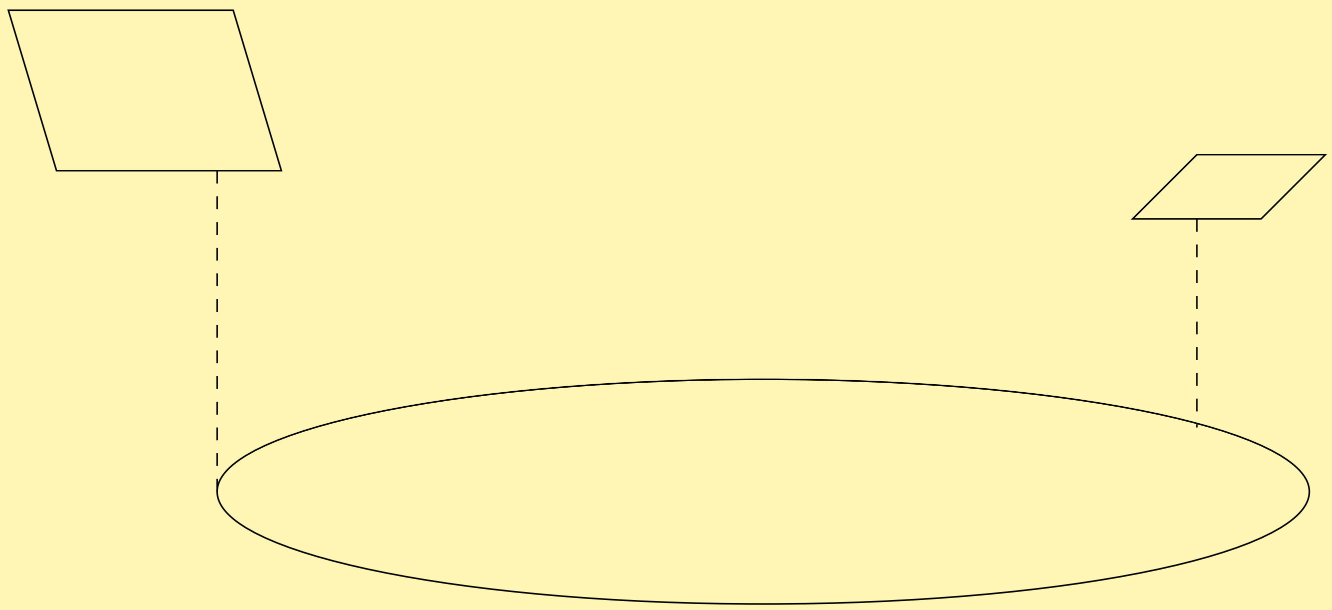
A. Example of non-geometric "T-fold" backgrounds

Compactify string theory to $d = 7$ via NS-NS fields as follows:

Compactification to $d = 8$ on a T^2. Data:

- Complex structure τ
- Complexified volume $\rho = b + i\sqrt{G}$, $b = \int_{T^2} B_{12}$. b has period 1.





Compactify to $d = 7$ on additional S^1_R, coordinate $x \equiv x + 2\pi R$.

$$\begin{aligned} \rho &\equiv \rho(x) \\ \tau &\equiv \tau(x) \end{aligned}$$

As $x \to x + 2\pi R$, (τ, ρ) must return to selves up to symmetry of $d = 8$ compactification. The symmetry action is called a "monodromy".

Symmetries of string compactification on T^2:

- Isometries: $SL(2,\mathbb{Z})_\tau$:

$$\tau \to \frac{a\tau + b}{c\tau + d} \; ; \qquad a, b, c, d \in \mathbb{Z} \; ; \qquad ad - bc = 1$$

- Stringy symmetries: $SL(2,\mathbb{Z})_\rho$. Includes $b \to b+1$, T-duality $\rho \to -\frac{1}{\rho}$.

- Mirror symmetry $\tau \leftrightarrow \rho$. This is a T-duality transformation $R \to 1/R$ on one cycle of torus.

- $(\tau, \rho) \to (-\tau^*, -\rho^*)$

Classification by monodromy

1. Shifts $b \to b+n$, $n \in \mathbb{Z}$ lead to "magnetic" NS-NS flux: $H = dB$, $\int_{T^3} H = n$.

2. Shifts in $SL(2,\mathbb{Z})_\tau$ isometries lead to manifolds. "Geometric flux".

Kachru, Schulz, Tripathy, Trivedi;
Tomasiello; Shelton,Taylor,Wecht

3. T-duality shifts such as $\rho \to \frac{-1}{\rho}$, $\rho \leftrightarrow \tau$ lead to *nongeometric* compactifications. "Nongeometric flux".

KSTT; Hellerman, McGreevy, Williams;
Hull; STW.

Fiberwise T-duality (with rectangular T^2)

1. $\rho \leftrightarrow \tau$ at every z: magnetic flux $\rightarrow$ geometric flux.
2. $\rho \rightarrow \frac{-1}{\rho}$ at every z: magnetic flux $\rightarrow$ nongeometric flux

KSTT,STW,LSW

More general story:

1. T^n fibres: GL(n,Z) monodromies lead to geometric models. O(n,n;Z) monodromies lead to non-geometric compactifications
2. More general manifolds: T^n fibration over more general base manifold B.
   a. Geometric: GL(n,Z) transition functions
   b. Non-geometric: O(n,n;Z)

Motivation

1. Magnetic fluxes useful for model building GKP,KKLT
2. More general class includes “nongeometric fluxes” STW
3. Will argue: important for understanding SUSY breaking
4. Nongeometric compactifications intrinsically interesting!

II. D-branes on "T-folds"

A. Motivation

1. Add open strings to nongeometric flux models
2. Wrapped D-branes: nonperturbative objects (solitons, instantons)
3. D-branes are probes of $L \sim g_s l_s$: potentially useful for understanding exotic compactifications

Shenker; Kabat&Pouliot; Douglas, Kabat, Pouliot & Shenker

B. Questions

1. What are allowed D-brane configurations?
2. What is geometry or topology of moduli space/ low-energy configuration space of D-branes?

C. D-branes and monodromy

D-branes transform nontrivially under O(n,n;Z)
Example: T^2

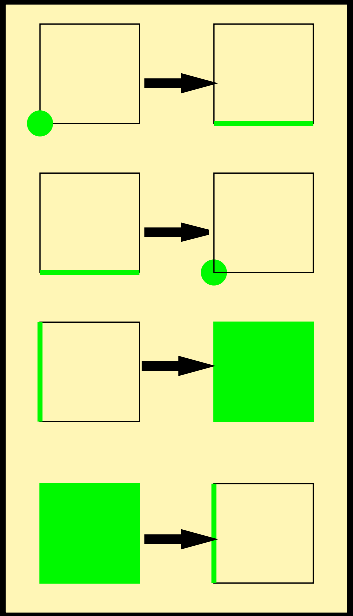
1. T-duality along a cycle exchanges Dirichlet and Neumann

   a. D0 --> D1 along cycle

   b. D1 along cycle --> D0

   c. D1 along dual cycle --> D2

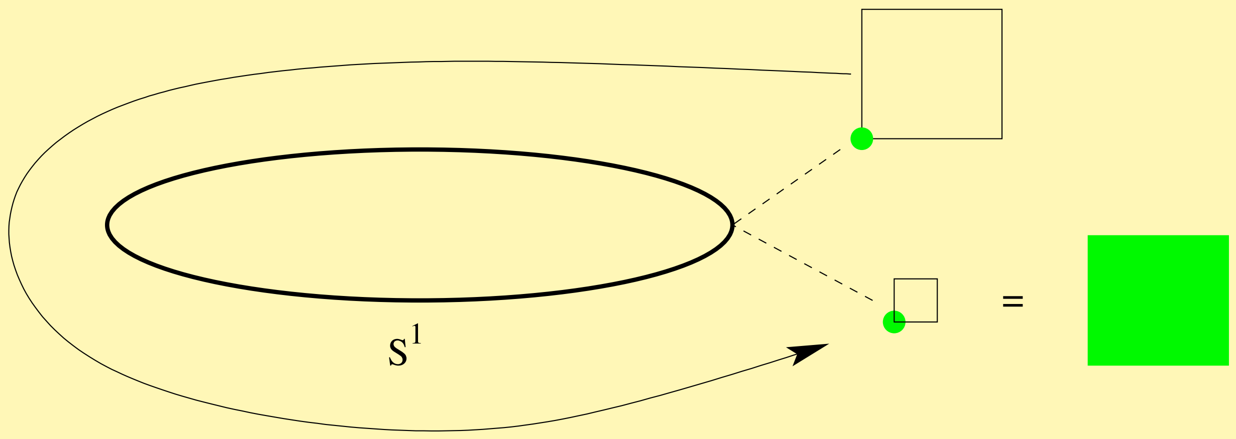
   d. D2 --> D1 along dual cycle



2. b --> b+1 takes D2 --> D2 + D0

D. Allowed D-brane configurations

1. D-brane wrapping base circle: fibre directions must be invariant under monodromy g^n: else it does not close on itself.



A T^2 fibration with $\rho \to \frac{-1}{\rho}$ monodromy

This example is not allowed

If fibre directions (and brane orientation) are invariant under g^n, it may be wrapped n times around the base.

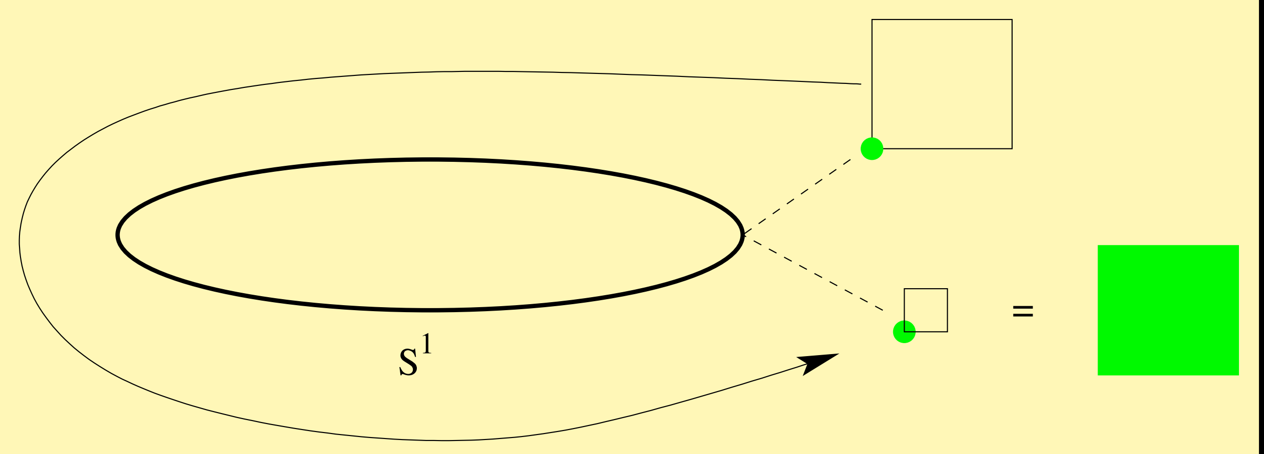
2. D-branes at points on the base are all allowed

Other examples

1. T with H-flux: D3 brane not allowed
2. Non-geometric flux: D1-branes or D3-branes wrapping base not allowed.

Conditions can be stated elegantly using Hull's "doubled torus" formalism

E. “D-topology” of T-folds



A T^2 fibration with $\rho \longrightarrow \frac{-1}{\rho}$ monodromy

Transport D0-brane around S^1

g: D0 --> D2
g^2: D0 --> D0

Configuration space of D0 is
geometric double cover of T-fold

Conjectures:

1. Configuration space of D-branes at point on base is always a geometric n-fold cover of “T-fold”
2. If g does not preserve D-brane for any n, there is a potential on the configuration space (else an infinite degeneracy of D-brane states), or brane is otherwise unstable.

(Hellerman, private correspondence)

III. Nongeometric fluxes and SUSY

A. Fluxes and soft SUSY breaking

Consider type IIB on a Calabi-Yau with D-branes.
Lagrangian for open strings (gauge bosons,
charged matter) depends on closed strings

1. Perturbative superpotential for open string chiral scalar superfields: couple to complex structure moduli

2. FI D-terms, tree-level gauge couplings: couple to Kahler moduli

3. Kahler potential for open string scalars: couple to all moduli

Brunner, Douglas, Lawrence & Romelsberger;
Douglas; Lawrence & McGreevy

Auxiliary components of closed string fields: soft SUSY-breaking terms in open string Lagrangian

Closed string modes descend from N=2 multiplets.

1. Expand $\mathcal{N}=2$ superfield in $SU(2)_R$ doublet $(\theta, \hat{\theta})$ of superspace variables.

2. Vector multiplets are chiral in $(\theta, \hat{\theta})$. $SU(2)_R$ triplet of auxiliary fields:

$$V = w + \theta\lambda + \hat{\theta}\hat{\lambda} + \theta^2 D_{++} + \hat{\theta}^2 D_{--} + \theta^\alpha \hat{\theta}^\beta \epsilon_{\alpha\beta}\left(D_{+-} + \sigma^{\mu\nu}_{\alpha\beta} F_{\mu\nu}\right) + \ldots$$

Here D_{ab} are auxiliary fields.

Grimm, Sohnius & West; de Wit & van Holten; de Roo, van Holten, de Wit & van Proeyen

3. Hypermultiplets are "twisted chiral" :

$$H = t + \theta\psi + \bar{\hat{\theta}}\bar{\hat{\psi}} + \theta^2 y + \bar{\hat{\theta}}^2 \bar{y} + \theta^\alpha \bar{\hat{\theta}}^{\dot{\beta}} \sigma^\mu_{\alpha\dot{\beta}} F_\mu + \ldots$$

where $t, y, \bar{y}, F_\mu = \partial_\mu \phi$ are complex. Here $y, \bar{y}$ are auxiliary fields.

Berkovits & Siegel

Only certain auxiliary fields are understood

Type IIB vector multiplets

- w: complex structure deformations.
- $J^{\mu}{}_{\nu}$: almost complex structure; $\omega = g_{\mu\lambda} J^{\lambda}{}_{\nu} dx^{\mu} \wedge dx^{\nu}$.
- $D_{\pm\pm}$: built from $d\omega$ and NS-NS 3-form field strength H, both $\in H^{(2,1)} \oplus H^{(1,2)}$.
- D_{+-} are built from RR 3-form F in $H^{(2,1)} \oplus H^{(1,2)}$.

Vafa; Lawrence &McGreevy

Type IIA hypermultiplets

- t are complex structure deformations.
- $y, \bar{y}$ built from $d\omega$, $H \in H^{(2,1)} \oplus H^{(1,2)}$.

Lawrence &McGreevy

What about IIB hypermultiplets, IIA vectormultiplets?

Mirror symmetry for NS-NS flux?

1. Mirror symmetry exchanges IIA and IIB, Kahler and complex structure moduli.
2. y, $\bar{y}$ in IIB should be “mirrors of NS flux”
3. Mirror symmetry is a form of T-duality for most Calabi-Yau compactifications Strominger, Yau & Zaslow
4. T-duality applied to H-flux: geometric, non-geometric fluxes

Worldsheet calculation in sigma model limit

- $\Omega \in H^{(3,0)}$ is holomorphic 3-form which (together with ω determines metric.
- $y, \bar{y}$ built from H with all holomorphic indices and $d\Omega$ with 2 holomorphic indices. (= Particular class of "intrinsic torsion".)

Vafa; Gurrieri. Louis, Micu & Waldram; Gurrieri & Micu; Fidanza, Minasian & Tomasiello; LMSSW

Puzzles

1. How does this relate to nongeometric flux? Does that emerge globally?
2. y, $\bar{y}$ in IIA allow for spacetime SUSY in supergravity approximation. y, y in IIB do not.

Answer: worldsheet instantons correct IIB SUSY conditions in the presence of y, $\bar{y}$

LMSSW

Conclusions

1. Nongeometric compactifications lead to interesting modification of stringy topology
2. Nongeometric “flux” is generic and important in type II models with reduced/broken SUSY
3. Worldsheet instantons always crucial for mirror symmetry