Astrophysical Turbulence:

Observational Testing

"Turbulence is the last unsolved problem
in classical statistical mechanics" R. Feinman

Alex Lazarian

Astronomy & CMSO

UW Collaboration:
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Kowal G.

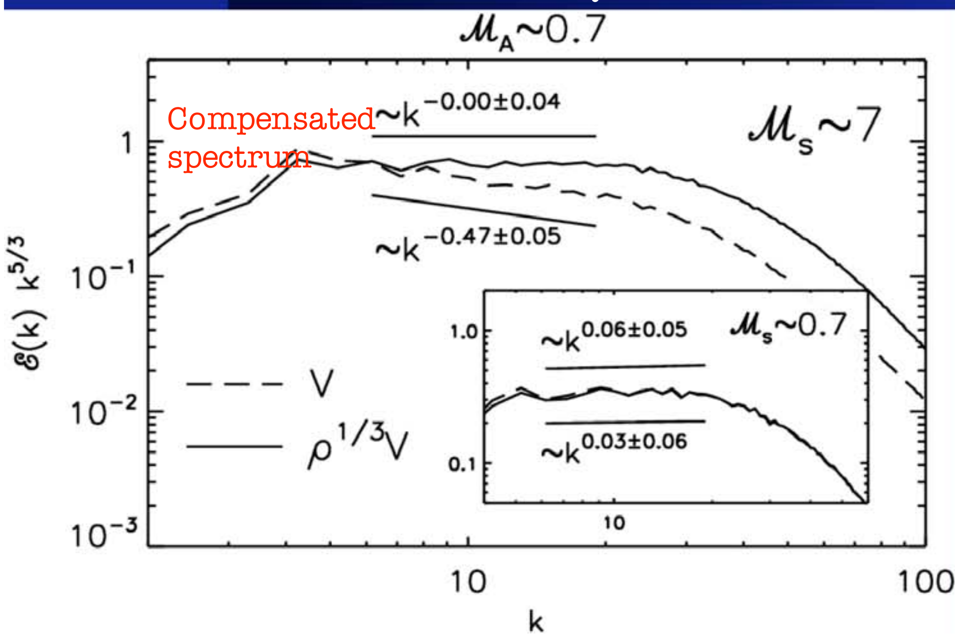
THE UNIVERSITY of WISCONSIN MADISON

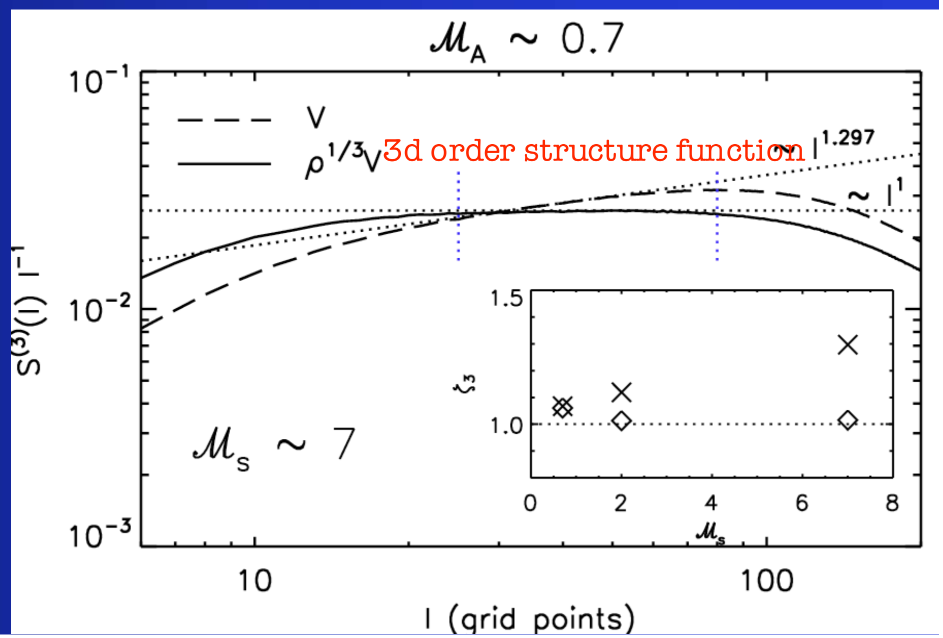
Spectrum of Compressible MHD Turbulence

Fleck's predictions for hydro: $V_l \sim l^{1/3+\alpha}$ $\rho_l \sim l^{-3\alpha}$

Kritsuk et al. 07 used spectra of $u = \rho^{1/3}V$ *and got Kolmogorov scaling for "U" in hydro*

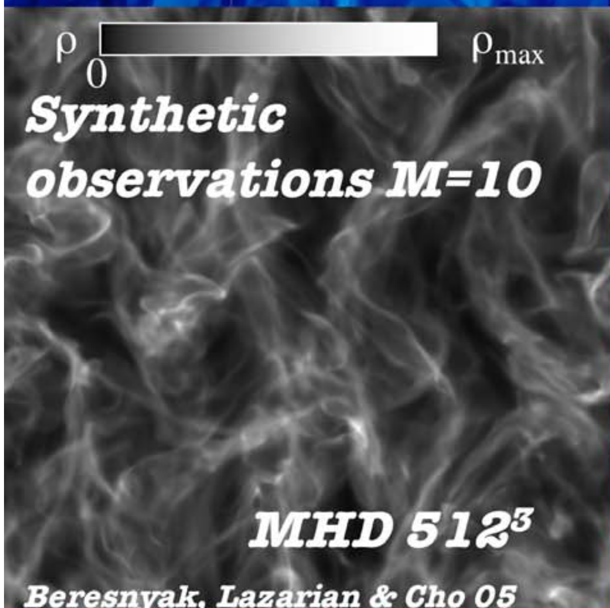
Kolmogorov scaling is valid in compressible MHD turbulence for modified velocity (Kowal & Lazarian 07)

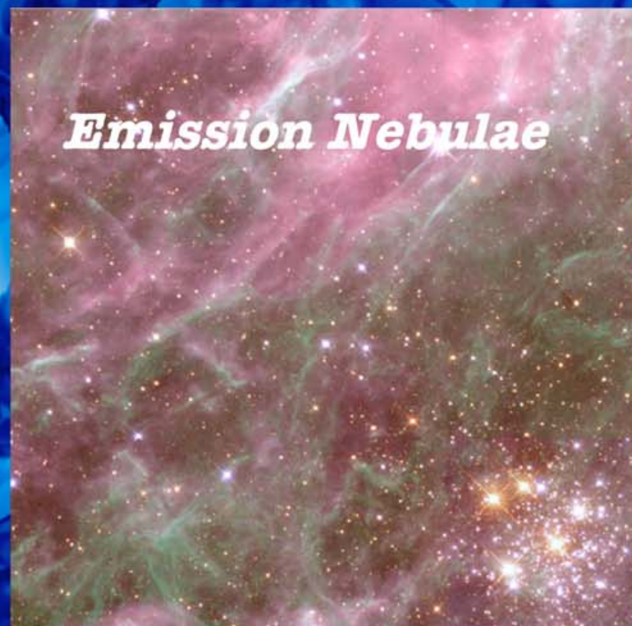




Is Visual Correspondence Enough?

Astrophysical flows: $Re \sim VL/\nu \sim 10^{10} >> 1$

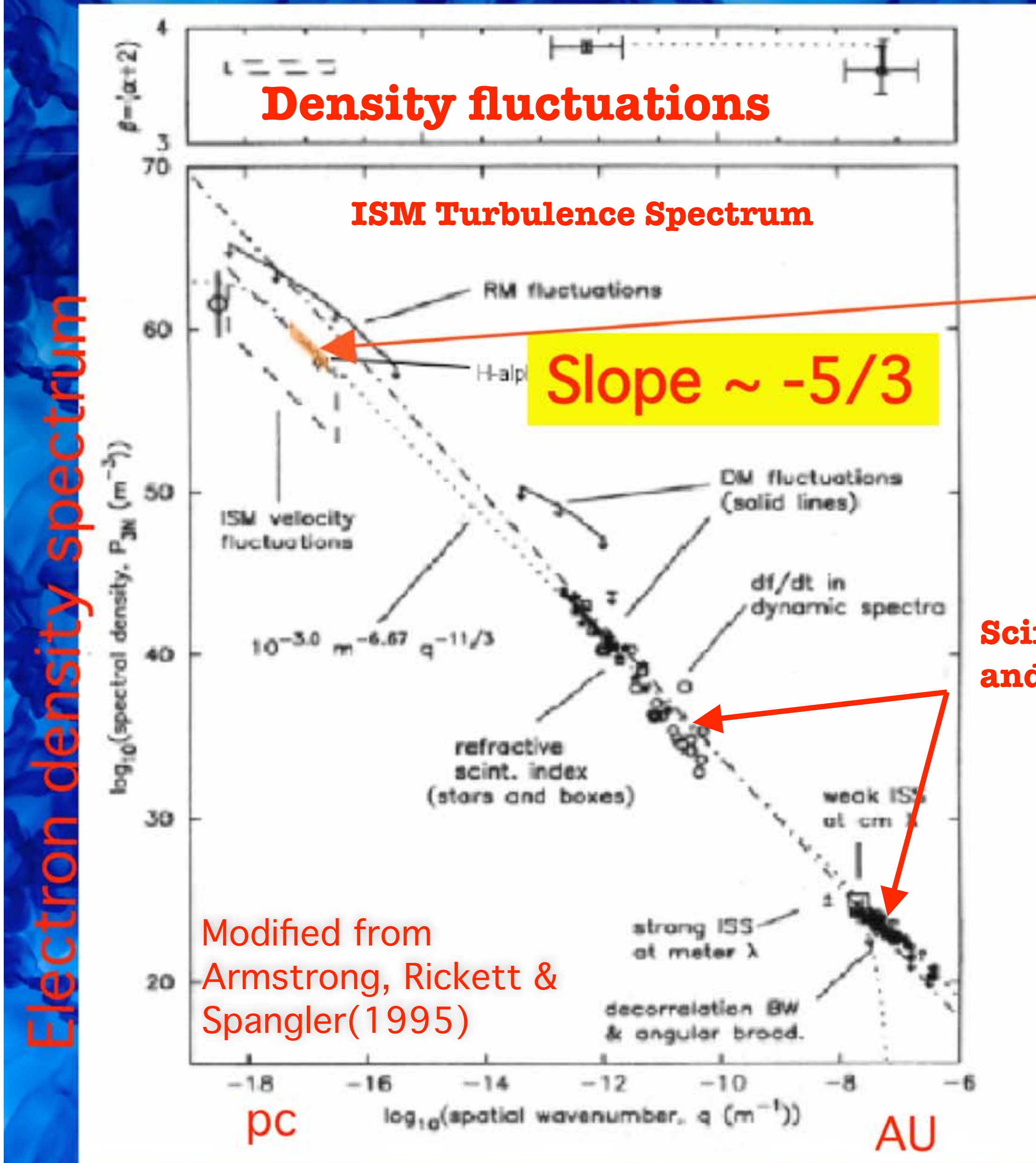


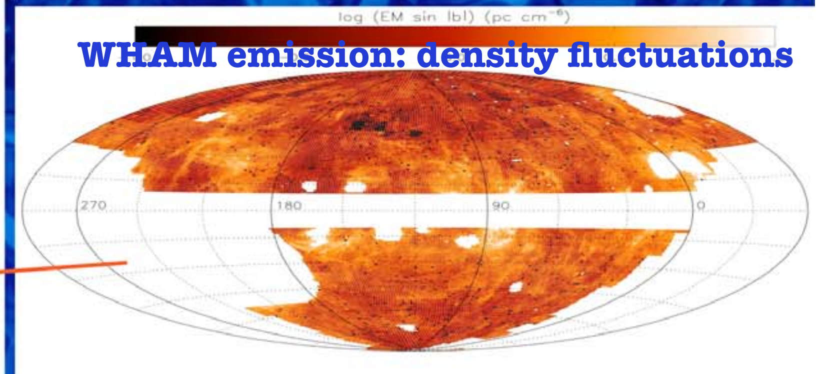


Computational efforts scale as Re^4!!!
Currently max Re of order $<10^4$

Numerics will not get to astro Re in foreseeable future. Flows in ISM and computers are and will be different!

What do Observations Tell Us?

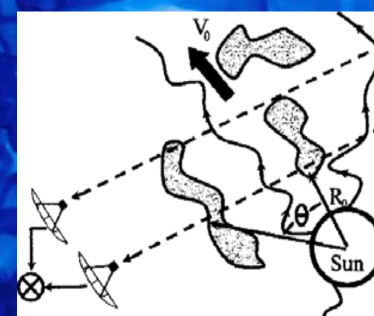




Chepurnov et al. 2007

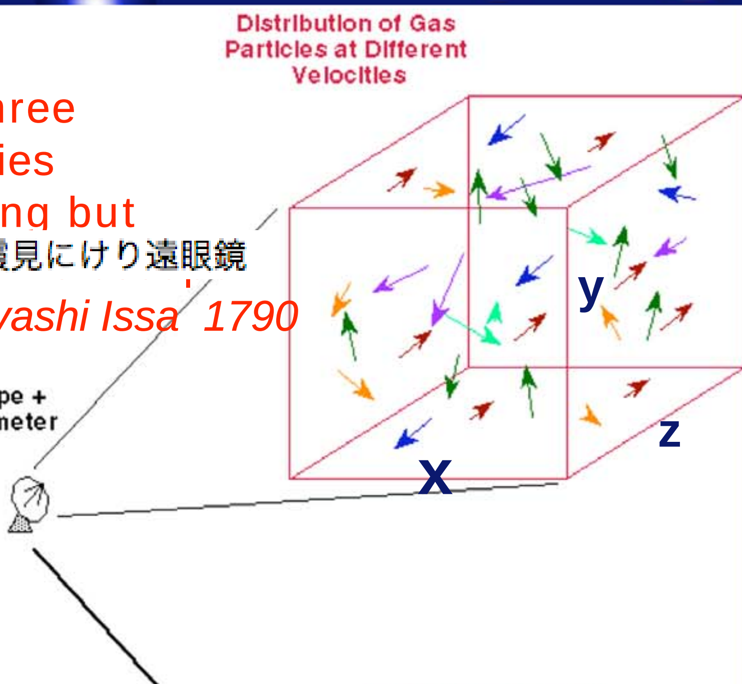
Extension of the range

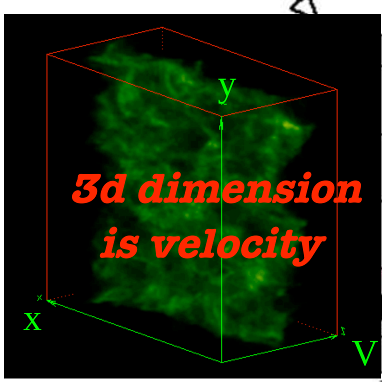
Scincillations and scattering

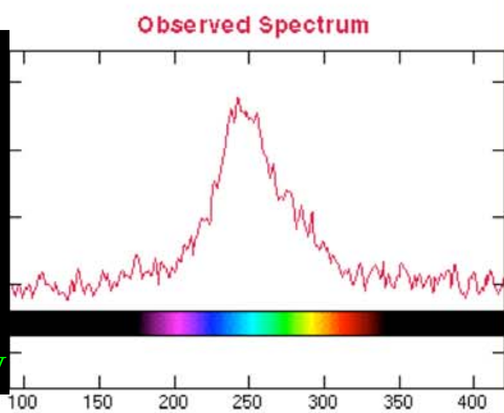


Velocity Statistics VCA and VCS: Keeping Theorists Honest

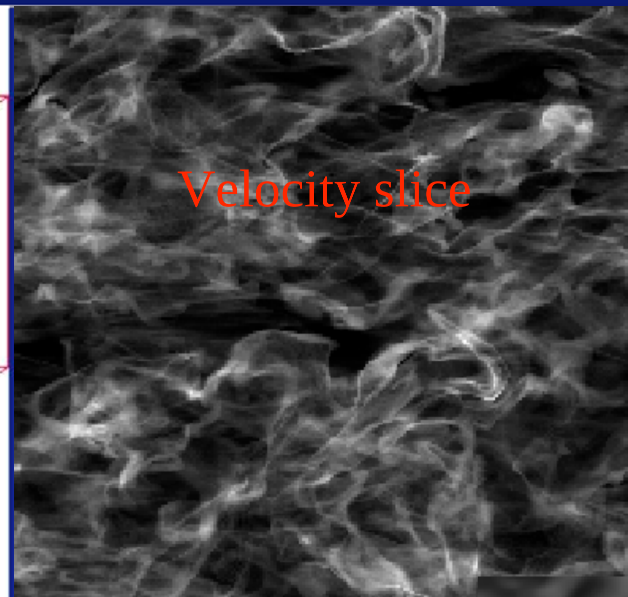
for three
pennies
nothing but
三文が霞見にけり遠眼鏡
Kobayashi Issa 1790





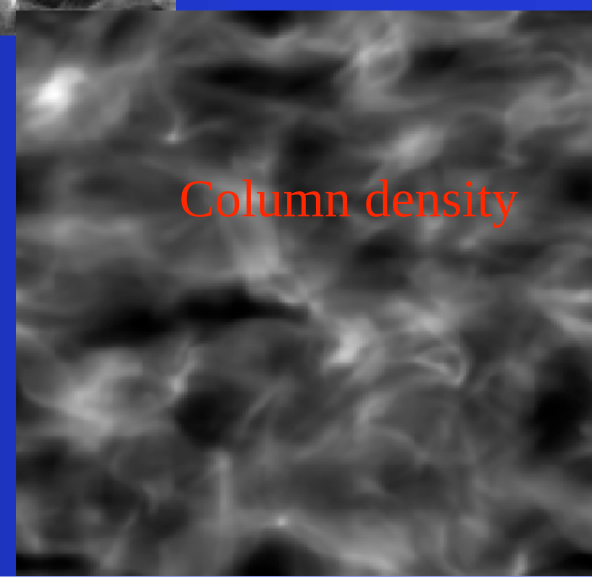


Modified from A Goodman



Structures are present even for incompressible simulations

What is the relation to the underlying velocity statistics?



Mathematical Setting

Brace yourself, marshal all your strength

Nahum 2:1

$$\rho_s(\mathbf{X},v)\mathrm{d}\mathbf{X}dv = \left[\int_0^S \mathrm{d}z\ \rho(\mathbf{x})\phi_v(\mathbf{x})\right]\mathrm{d}\mathbf{X}dv$$

Density in PPV

$$\phi_v(\mathbf{x})\mathrm{d}v = \frac{1}{(2\pi\beta)^{1/2}}\exp\left[-\frac{(v-v_{gal}(\mathbf{x})-u(\mathbf{x}))^2}{2\beta}\right]\mathrm{d}v$$

Velocity distribution

$$\xi_s(R,v_1,v_2)\equiv\langle\rho_s(\mathbf{X}_1,v_1)\rho_s(\mathbf{X}_2,v_2)\rangle$$

Correlation function in PPV

$$\xi_s(\mathbf{R},v)\sim\int dz\,\frac{\xi(\mathbf{r})}{(D_z(\mathbf{r})+2\beta)^{1/2}}\exp\left[\frac{v^2}{2(D_z(\mathbf{r})+2\beta)}\right]$$

where

$$\xi(\mathbf{r}) = \langle\rho(\mathbf{x})\rho(\mathbf{x}+\mathbf{r})\rangle \propto 1+(r/r_0)^\gamma$$

(xyz) density correlation

$$D_z(\mathbf{r})\propto r^m$$

Velocity correlation, m=2/3 for Kolmogorov

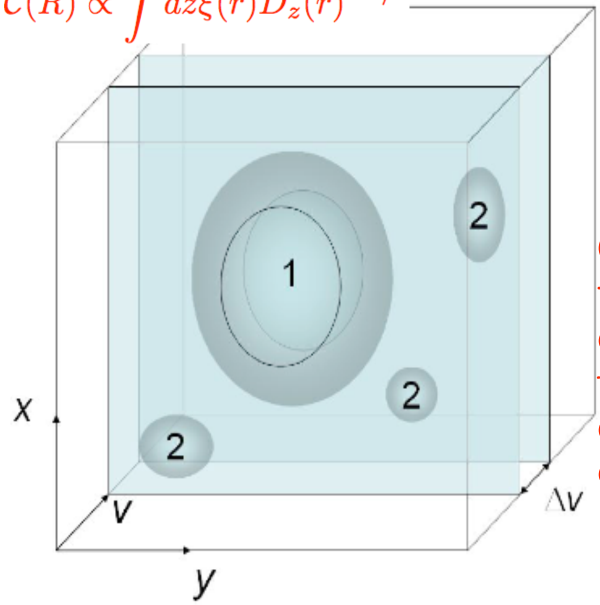
Lazarian & Pogosyan 00

VCA: Spectrum of "Thin" Slice

For "thin" slice: $\mathcal{C}(R) \propto \int dz \xi(r) D_z(r)^{-1/2}$

PPV Cube

VCA



Changing slice thickness one can separate velocity and density contributions

Successful testing in Lazarian et al. 01, Esquivel et al. 03, Padoan et al. 06, Chepurnov et al. 06

Theory without absorption is in Lazarian & Pogosyan 00, and with is in Lazarian & Pogosyan 04.

Most important is "thin slice":

As $D_z(\mathbf{r}) \propto r^m$ **and** $\xi(\mathbf{r}) \propto 1 + (r/r_0)^\gamma$

$$\mathcal{C}(R) \propto R^{1-m/2} + R^{1+\gamma-m/2}$$

2D Channel Map Spectrum:

$$P_2(K) \propto \int dR R \exp(-RK) \mathcal{C}(R)$$

thus

$$P_2(K) \propto K^{-3+m/2} + K^{-3-\gamma+m/2}$$

$\gamma < 0$ "Shallow"

$$P_2(K) \propto K^{-3-\gamma+m/2}$$

$\gamma > 0$ "Steep"

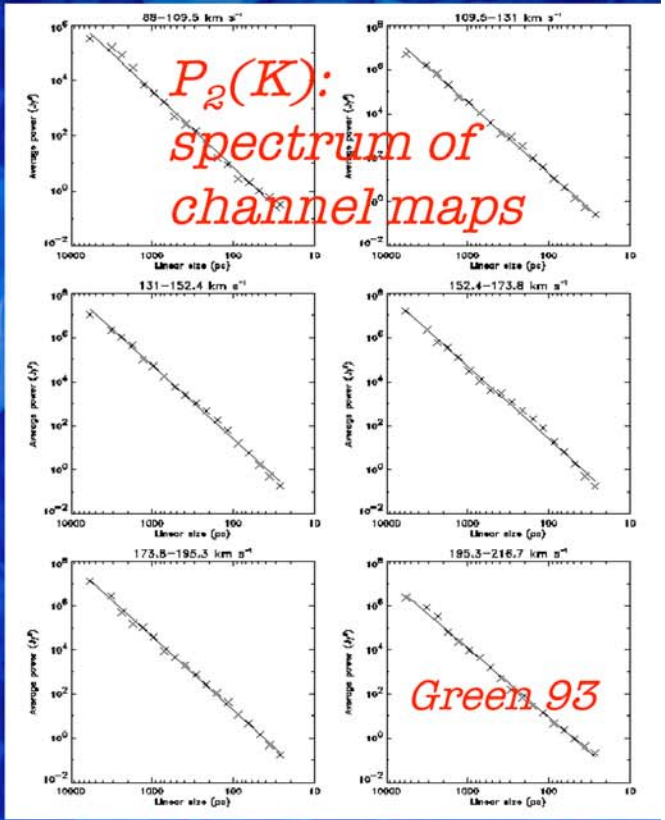
$$P_2(K) \propto K^{-3+m/2}$$

Velocity scaling can be obtained from "thin" slices

VCA: Successful Application to HI data

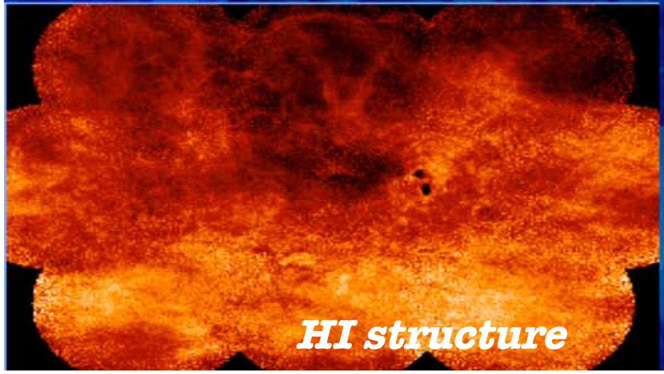
Predictions of theory:

1. $P_2(K)$ *is not* turbulence spectrum.
2. $P_2(K)$ is a function of channel map width.
3. $P_2(K)$ can be used to find the V-spectrum.
4. $P_2(K) \sim K^{-3}$ may be due to absorption.



Results:

Kolmogorov type scaling for V was obtained for Milky Way HI in Lazarian & Pogosyan 00, for SMC in Stanimirovic & Lazarian 01, for south Magellanic Bridge in Muller et al. 04.

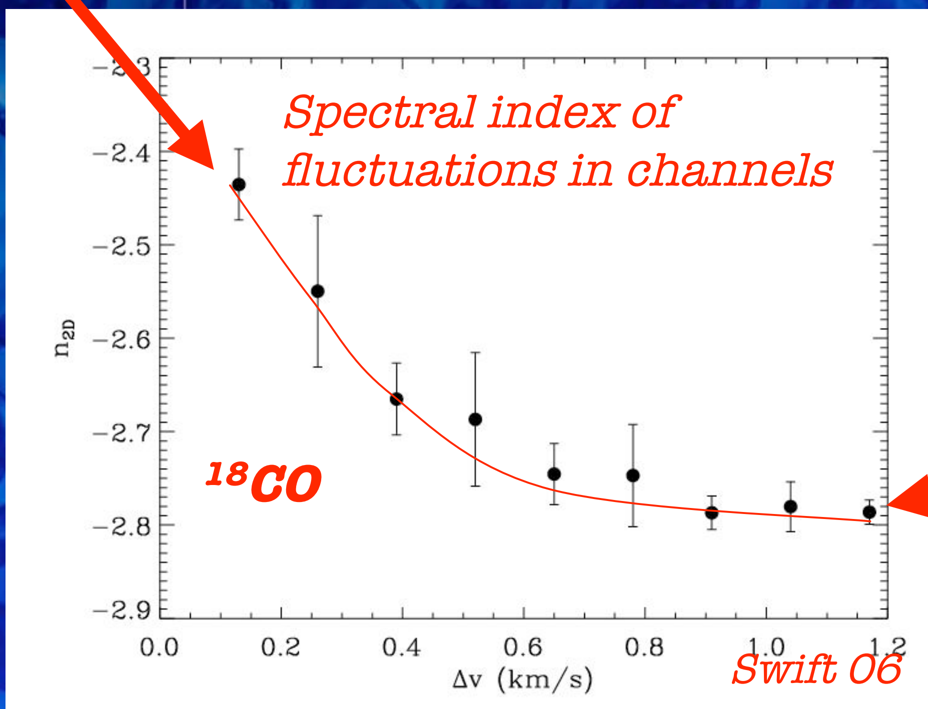


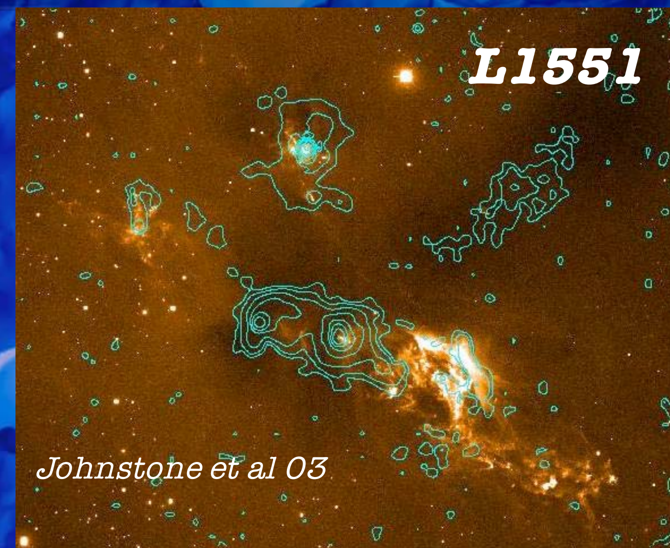
More Results:

Universal K^{-3} spectrum was observed in Dickey et al. 01 for inner part of Milky Way, Khalil et al. 06, Begum et al. 06 for DDO 210 galaxy. Deshpande et al. 01 found evidence of shallow density $E_k \sim k^{-0.8}$ at subpc scales.

VCA for ^{18}CO data (L1551): Optically Thin Case

$$-3-\gamma+m/2 \approx -2.44 \Longrightarrow m \approx 0.72$$





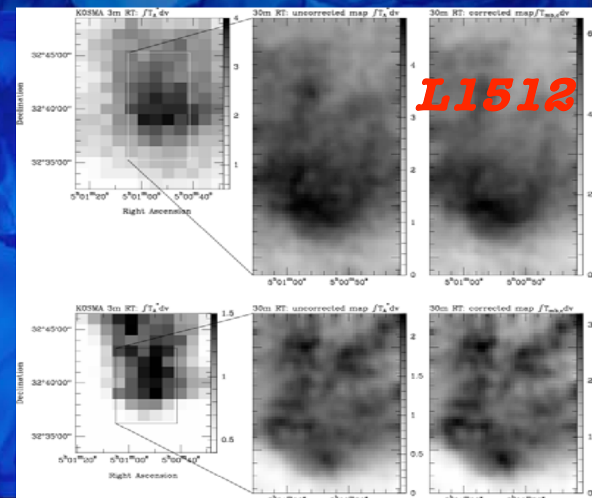
$$-3-\gamma \approx -2.8 \Longrightarrow \gamma \approx -0.2$$

***Swift 06* applied VCA to ^{18}CO and obtained density spectrum $E_n(k) \sim k^{-0.8}$ and velocity spectrum $E_v \sim k^{-1.7}$.**

Current work:

We test numerical fitting of the power index change with channel map width

VCA for ^{12}CO and ^{13}CO: Effects of Radiation Transfer



Integrated spectral lines:

Stutzki et al. 98 *got spectrum $P_2(K)$ of integrated lines ^{12}CO and ^{13}CO ~ $K^{-2.8}$ for L1512.* ***Theory*** *suggests $E_n(k) \sim k^{-0.8}$ spectrum of density. Similar to ^{18}CO results by Swift 06.*

$$P_2(K) \sim K^{-3+\gamma} \to E_n(k) \sim k^{-1+\gamma}$$

Optically thick
LP04

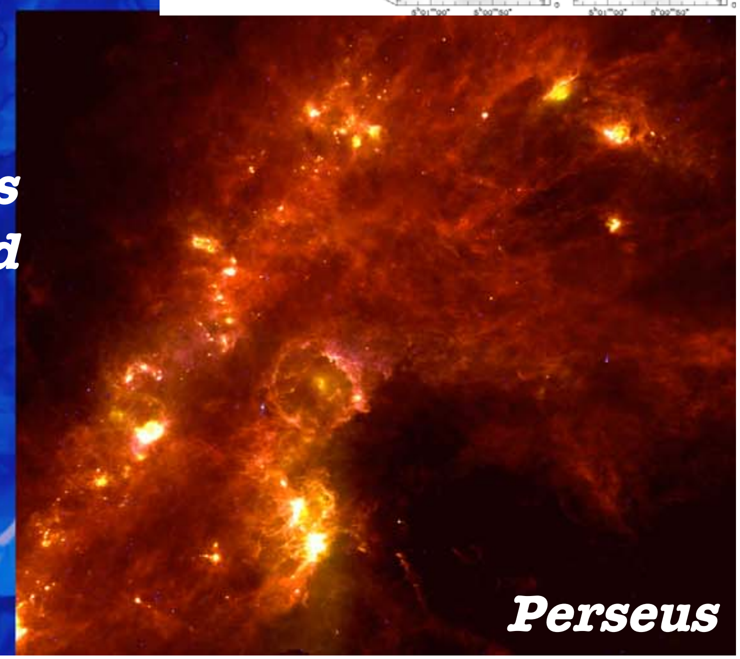
Channel Maps:

Padoan et al. 06 *proved the VCA predictions for synthetic ^{13}CO channel maps and applied VCA to Perseus molecular cloud. They obtained $E_v \sim k^{-1.8}$ for velocity.*

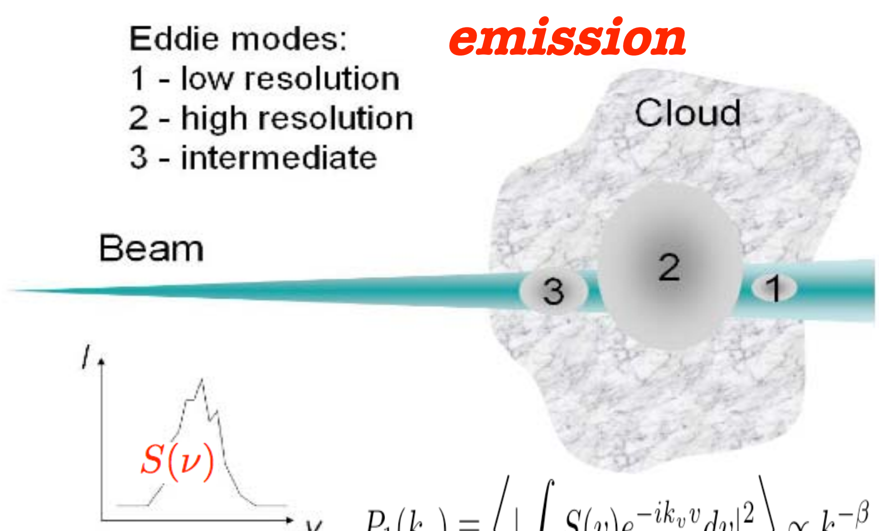
$\alpha_\rho \approx 0.07$ $\alpha_v \approx 0.06$

Kritsuk et al. 07

Consistent with Fleck 96 model predictions!

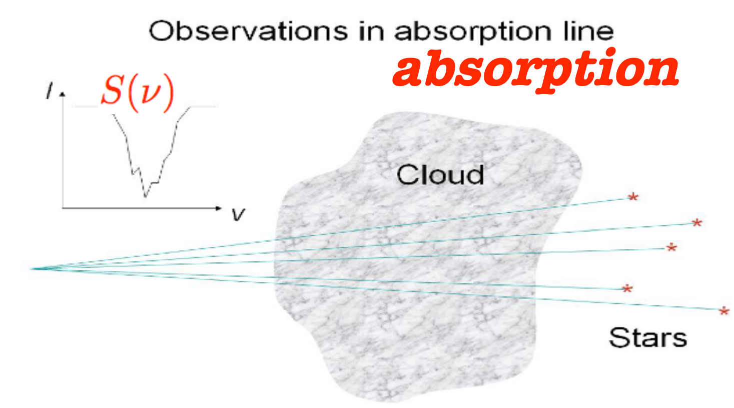


Velocity Coordinate Spectrum (VCS): Spectrum Along V-axis



$$P_1(k_v) \equiv \left\langle \left| \int S(v) e^{-ik_v v} dv \right|^2 \right\rangle \propto k_v^{-\beta}$$

New technique proposed in Lazarian & Pogosyan 06. Can work for resolved and unresolved objects.



Weak absorption case is simple. Saturated absorption lines are in Lazarian & Pogosyan 07

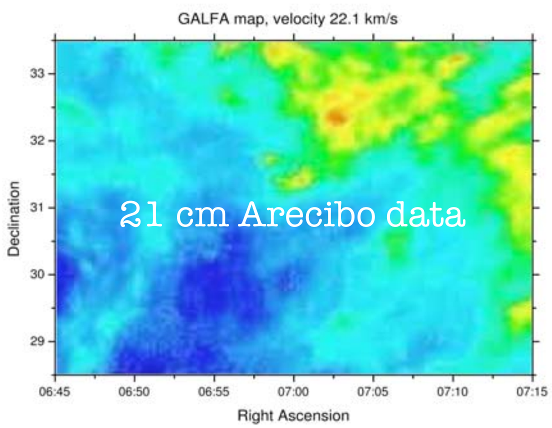
For shallow $\gamma < 0$ ***density***

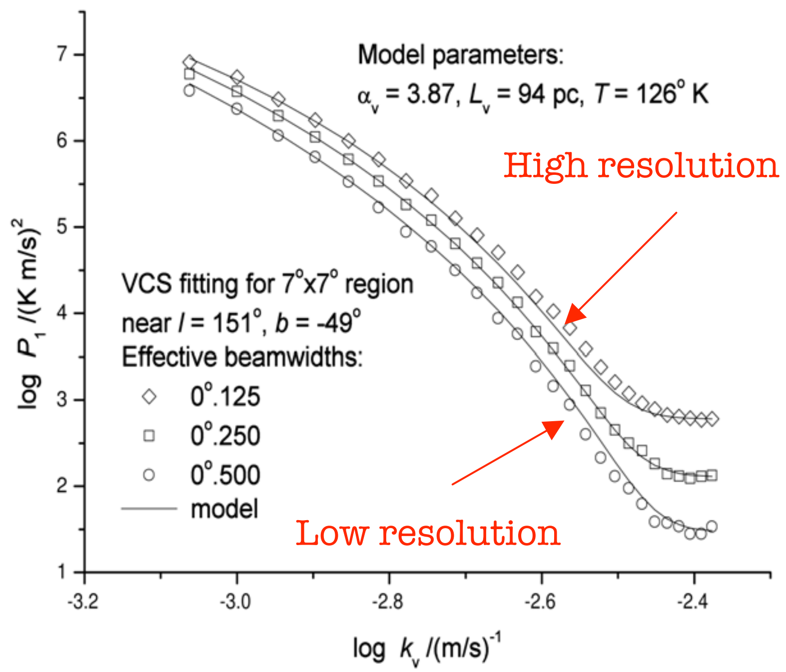
LOS geometry	high resolution		low resolution
	pencil beam	flat beam	
parallel	2(1+γ)/*m*	2(2+γ)/*m*	2(3+γ)/*m*
crossing	2(1+γ)/*m*	(not a power law)	2(2+γ)/*m*

$\gamma = 0$ *for steep density*

VCS: Application to GALFA HI

This is the first analysis of the spectrum of fluctuations along the V-coordinate.



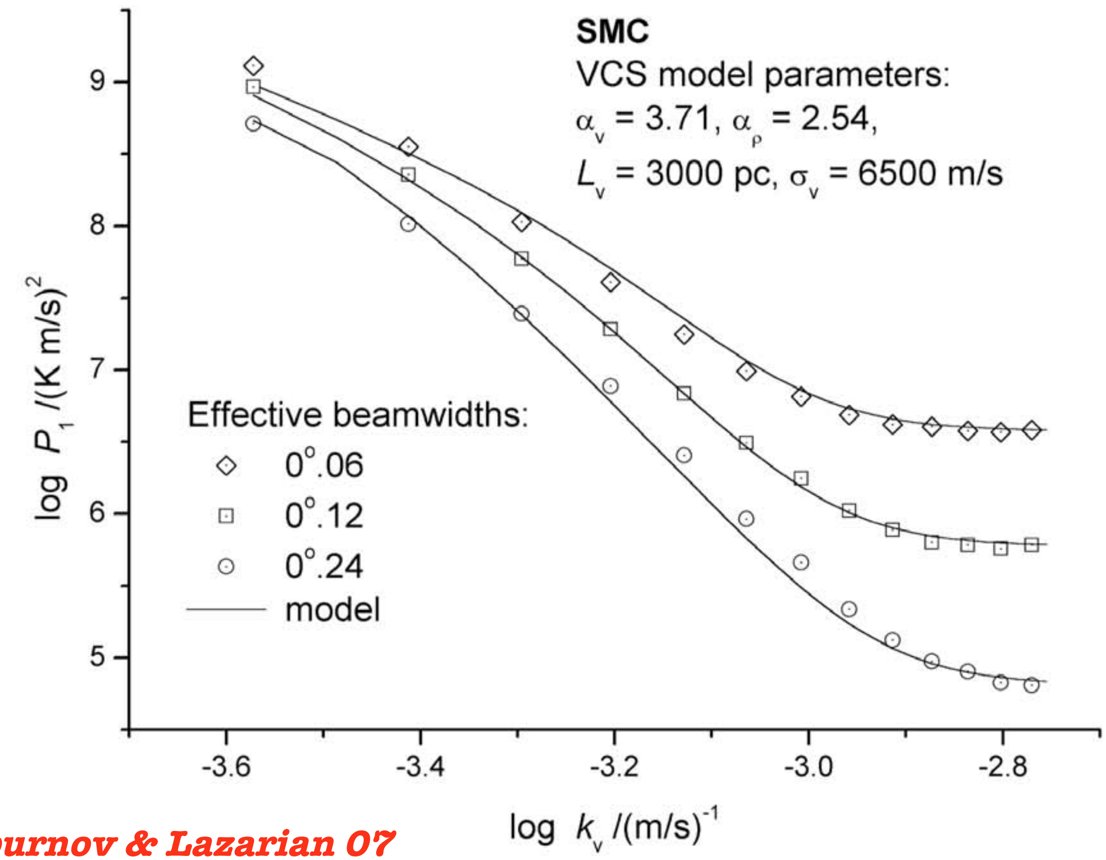


VCS uses info along v-axis. It does not need good spatial resolution for emission line

Data handling by Chepunov & Lazarian 06

Model with $T=10^2$K fits the data for different resolutions. Spectral index is steep (-3.9)

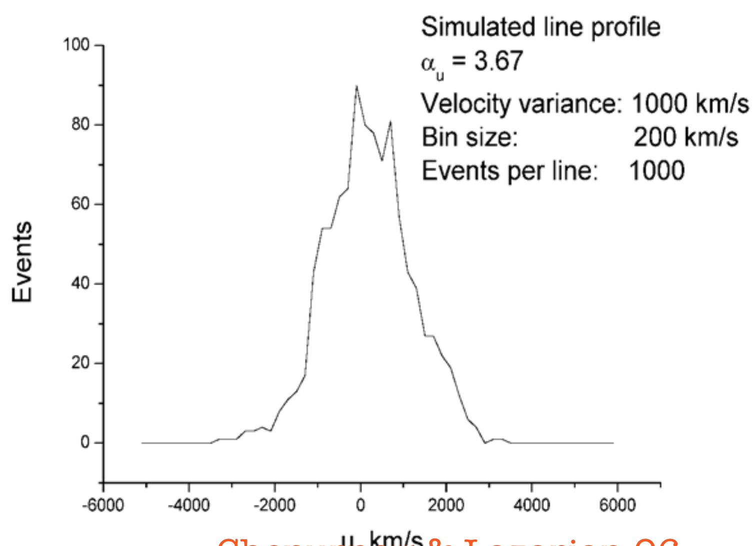
Extragalactic Studies: SMC



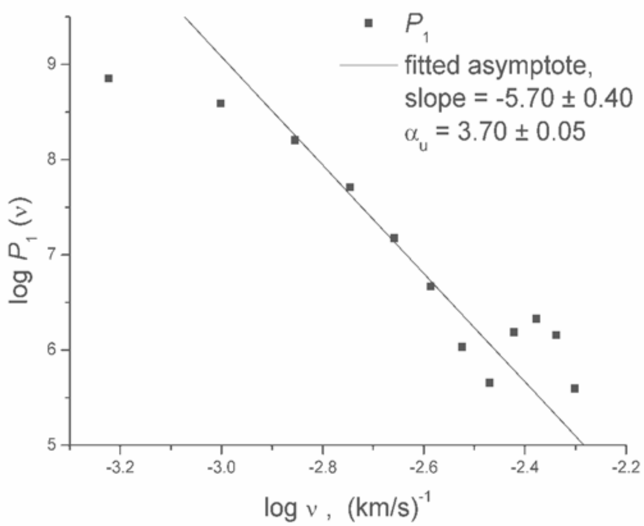
Chepurnov & Lazarian 07

Low curve correspods to unresolved galaxy

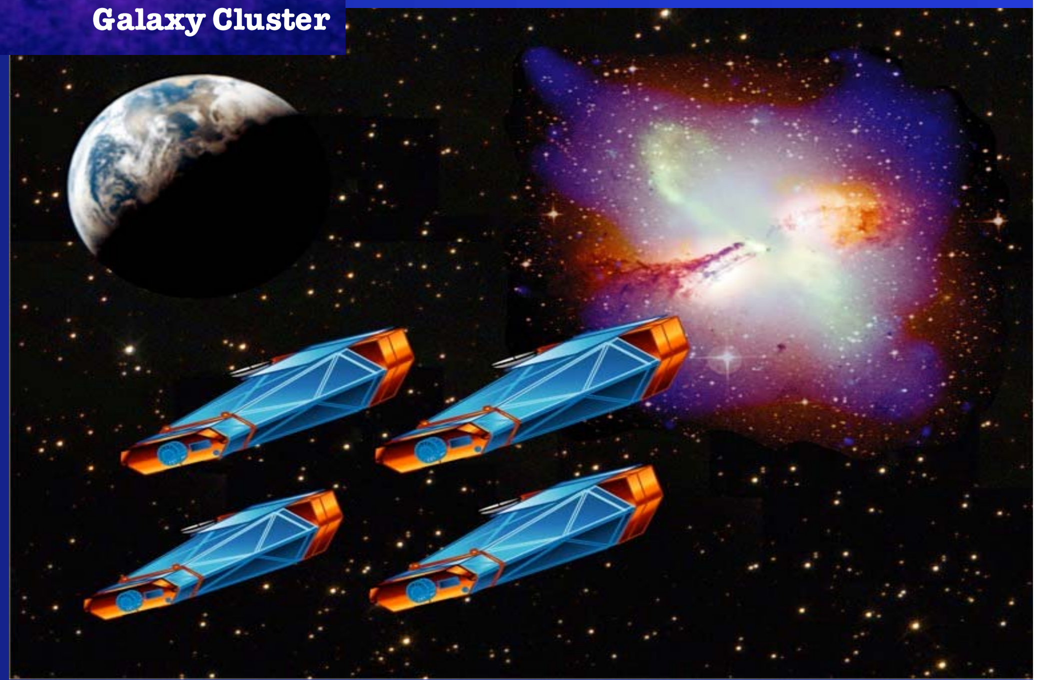
Future Missions: Spectrum of Turbulence with Constellation X



Chepurnov & Lazarian 06



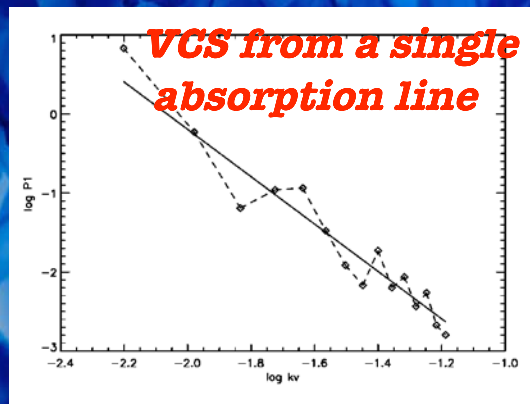
Studies of turbulence with new X-ray missions

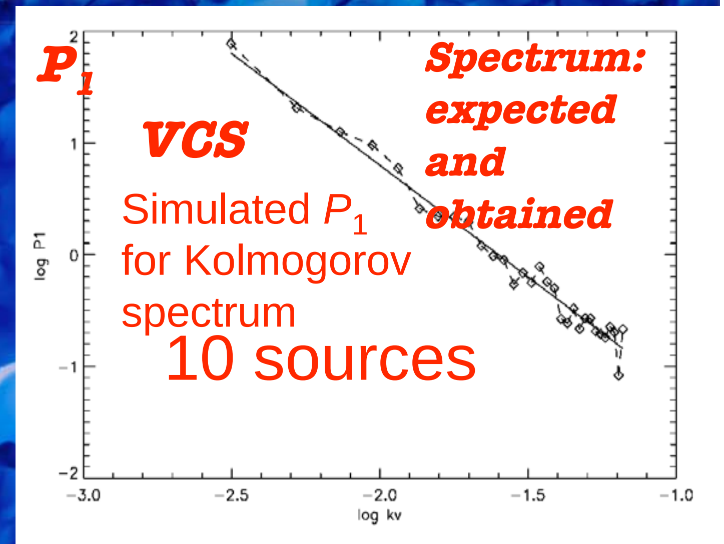


Constellation X will get turbulent spectra with VCS technique in 1 hour

VCA and VCS: Prospects for Turbulence Studies

Absorption lines can be used to study turbulence (extragalactic objects, Lyman alpha, supernovae remnants).





In addition:

To increase velocity coverage use heavy species.

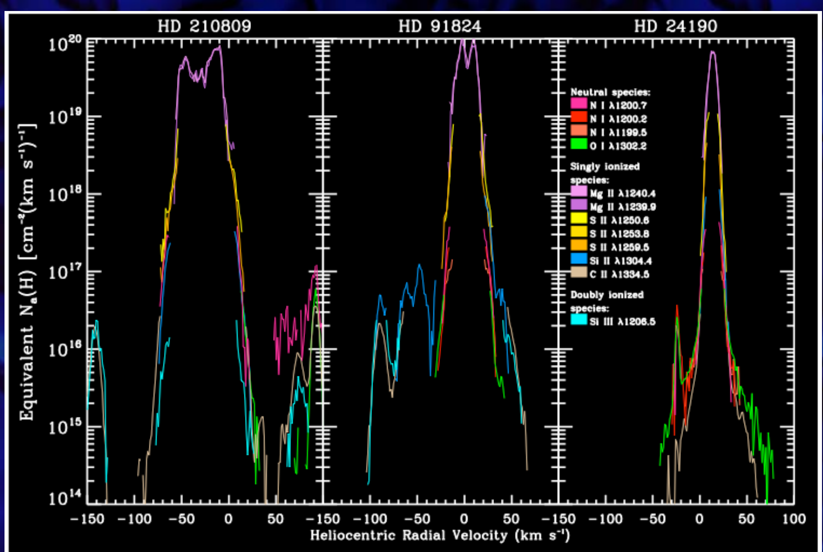
Measure cold gas temperature.

Combine VCS and VCA to extract info on compressibility

Use of entire 3D PPV cubes is promising!

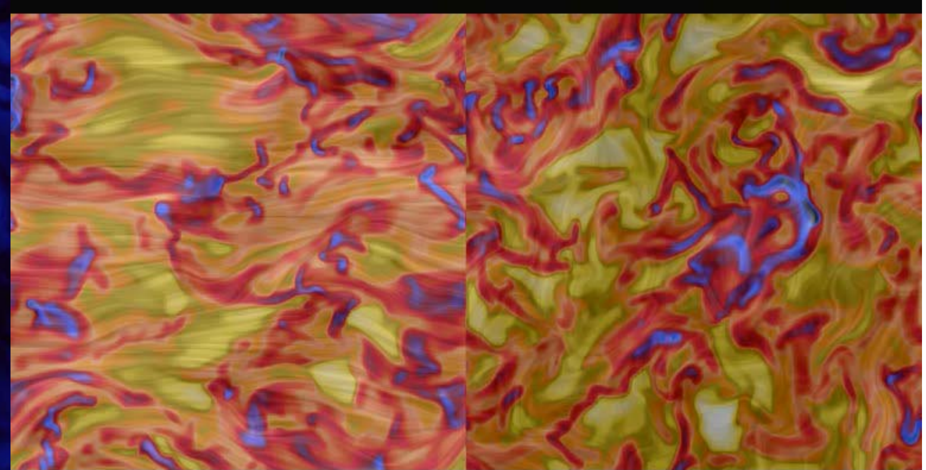
Summary

- VCA and VCS are new techniques to study turbulence with observational data
- First applications deliver spectra consistent with theoretical expectations
- More tests are expected as VCA and VCS mature



Jenkins 07

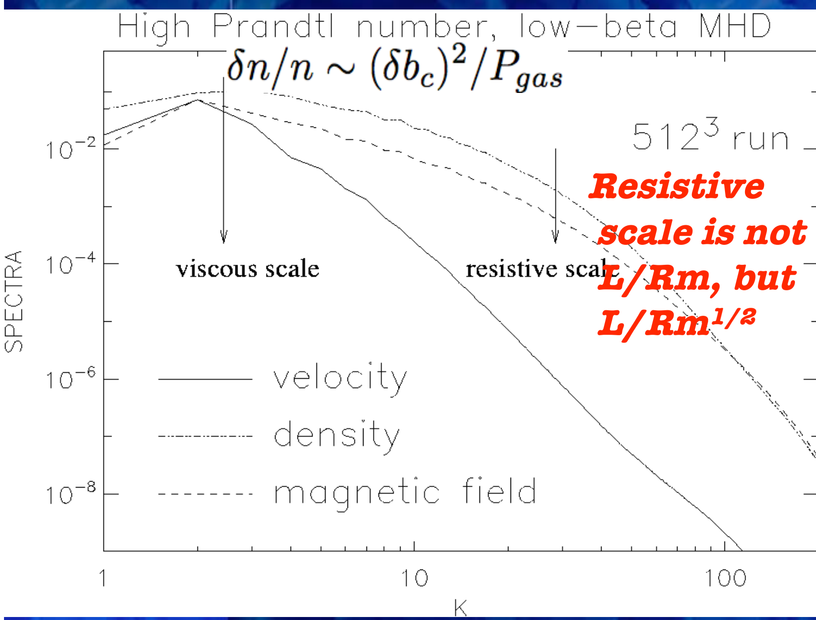
512^3 Compressible MHD



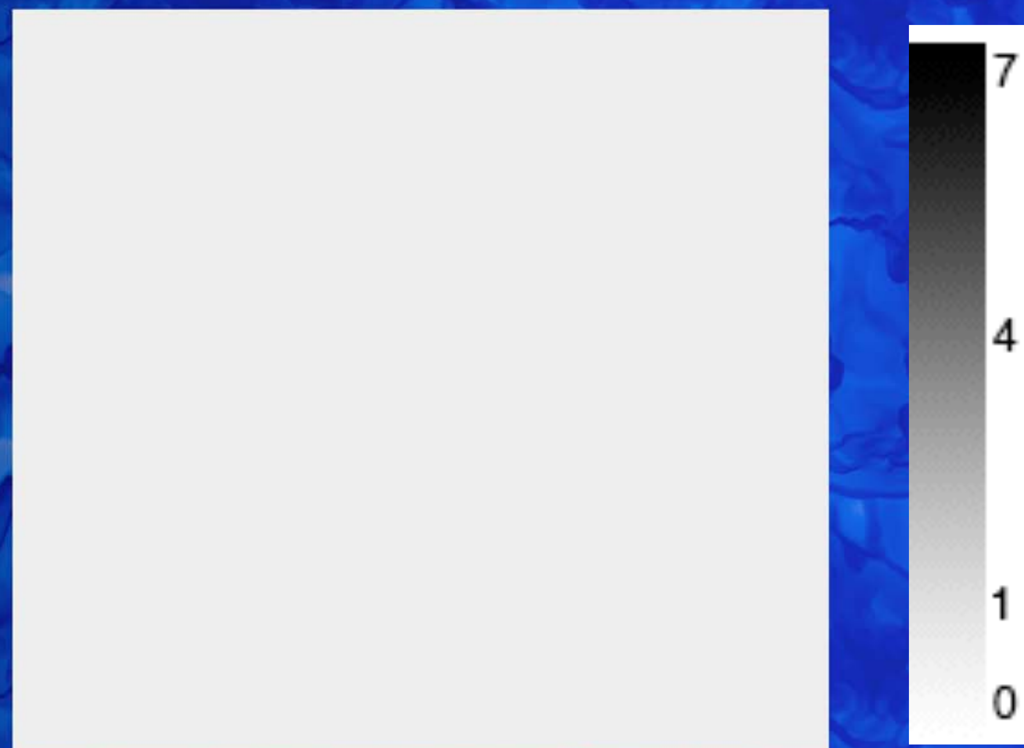
Kowal & Lazarian 07

Formation of Density Structures in Viscous Turbulent Flow

Magnetic field in viscous fluid compresses density



Beresnyak & Lazarian 07



Possible cause of SINS.

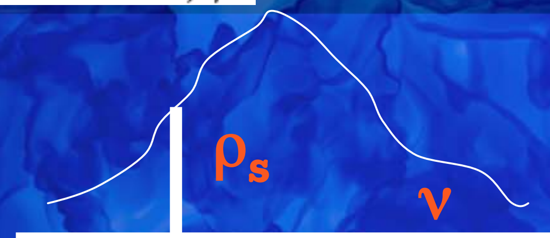
Centroids: Problematic Tool

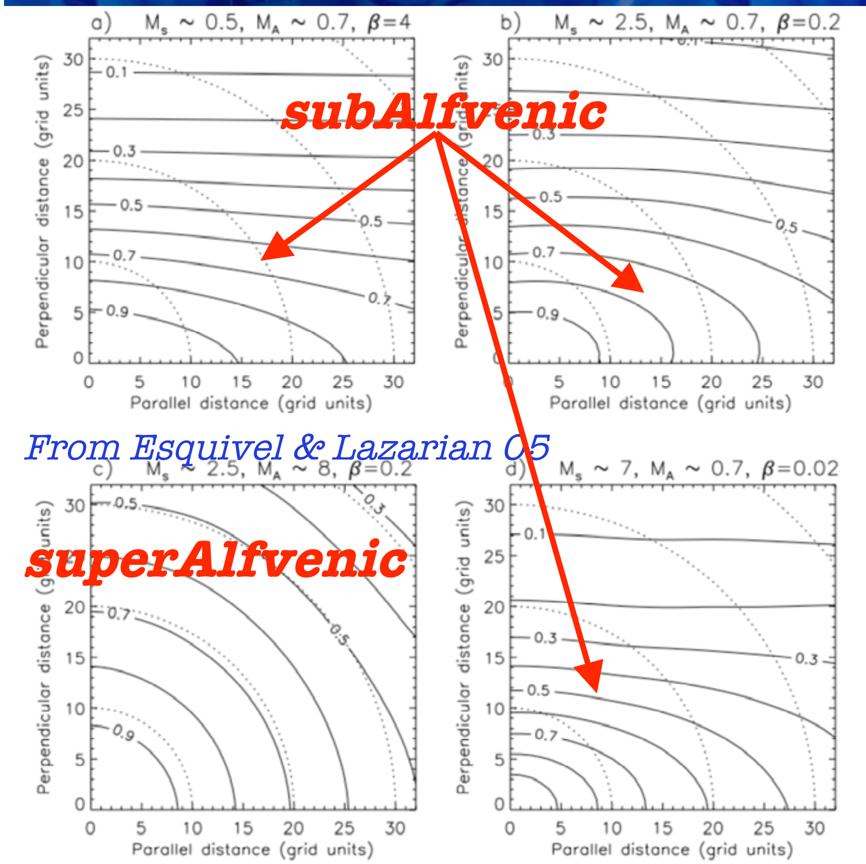
$$\mathcal{COR} \equiv \langle \delta S(\mathbf{X}) \delta S(\mathbf{X}+\mathbf{R}) \rangle$$

Definition: $$S(x,y) = \int v_z \, \rho_s(x,y,v_z) dv_z$$

ρ_s = antennae temperature at frequency ν

(depends on both velocity and density)



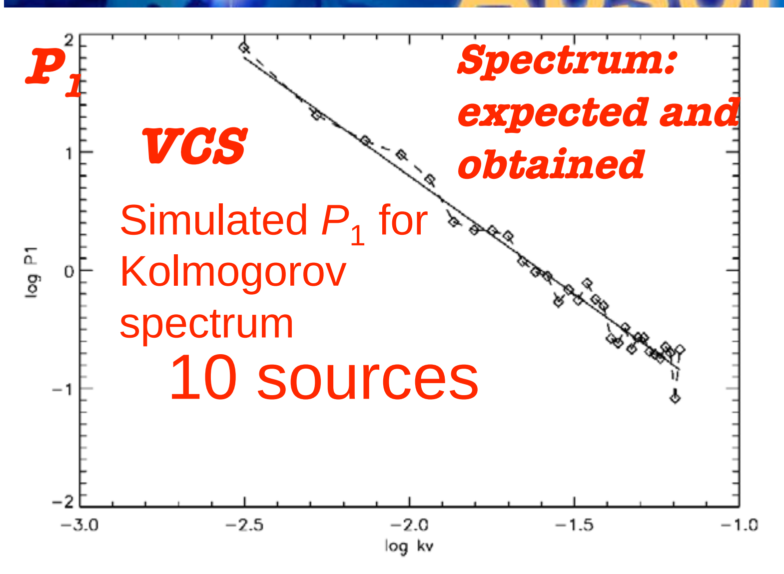


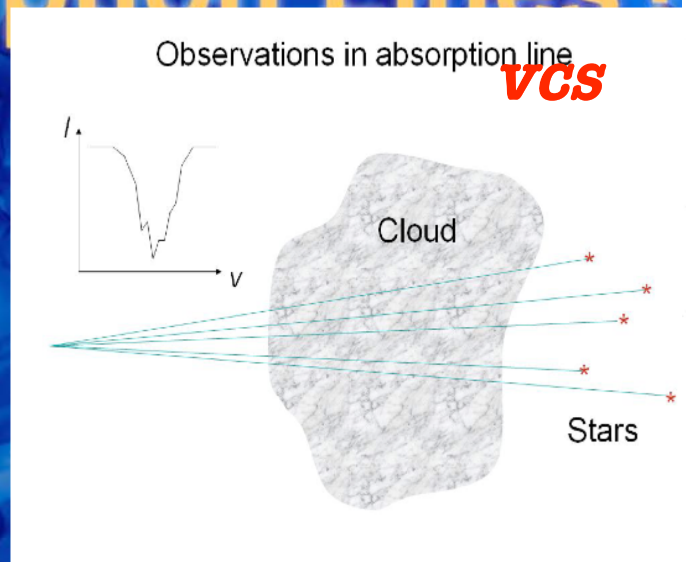
1. Centroids are OK to reveal anisotropy due to B (Lazarian et al.01), for subAlfvenic turbulence.

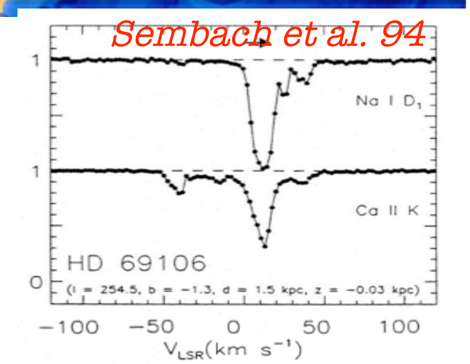
2. Centroids are not OK to study M_s>1 turbulence (Esquivel & Lazarian 05).

3. Necessary observational criterion obtained in Lazarian & Esquivel 03 is not valid for most of molecular line data.

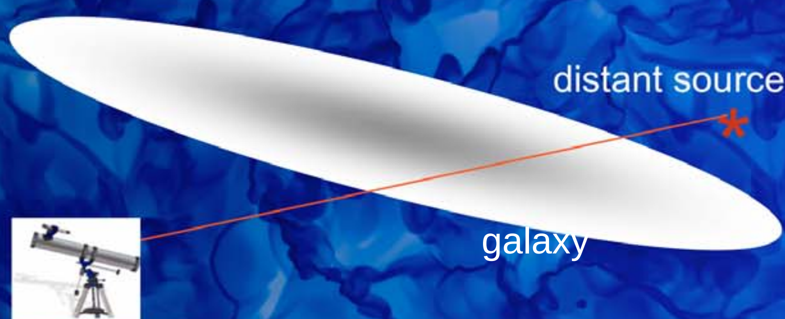
How Many Sources is Necessary to Study with Absorption Lines?

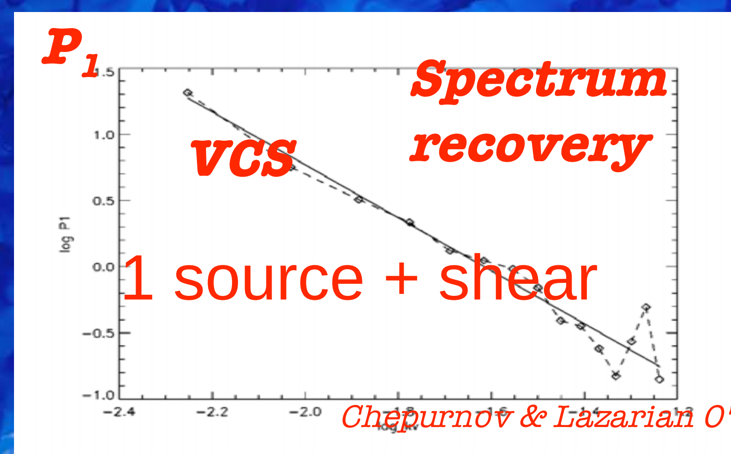






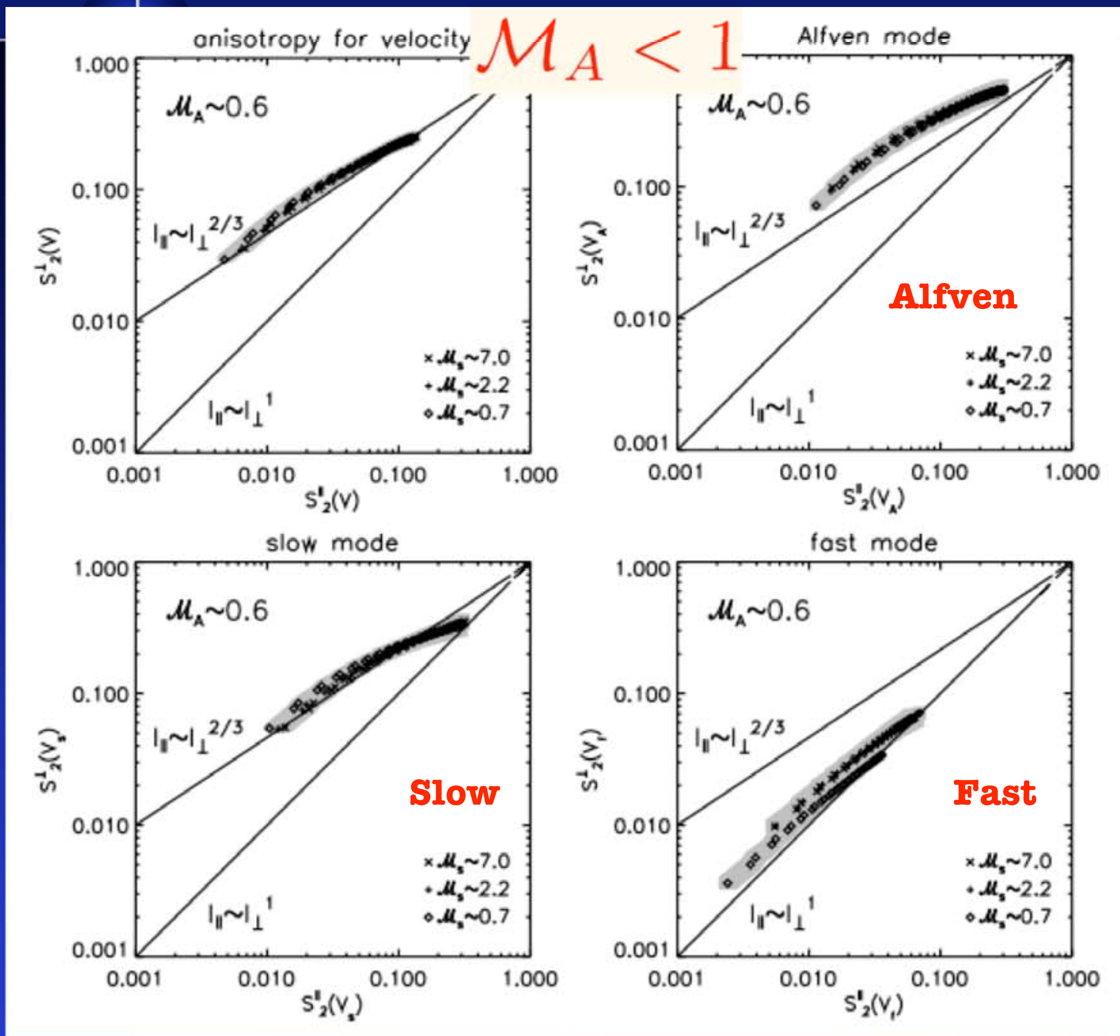
One line of sight + galactic shear



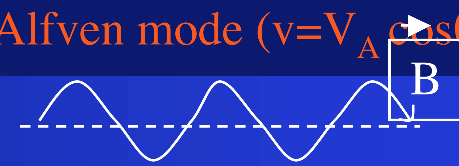


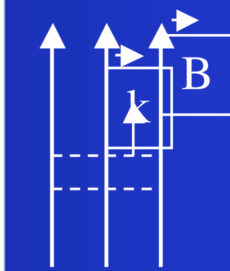
One realization. Δ*v* due to galaxy rotation is 8 times the turbulent line width

Scaling of MHD Turbulence Modes



Alfven mode ($v=V_A \cos\theta$)

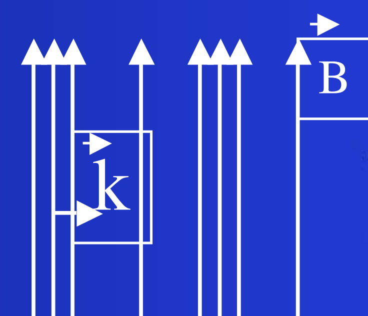




Alfven and slow modes are anisotropic

$l_{\parallel} \sim l_{\perp}^{2/3}$ GS95

slow mode ($v=c_s \cos\theta$)



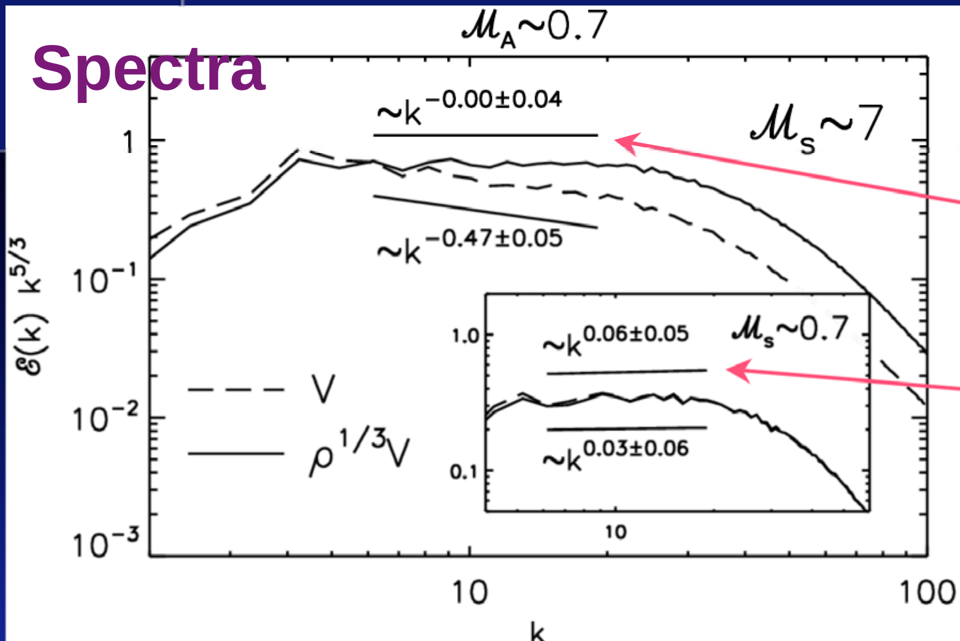
fast mode are isotropic

fast mode ($v=V_A$)

-anisotropy is independent on the sonic Mach number

Kowal & Lazarian 07 and Cho & Lazarian 03

Can we restore K41 using “u”?



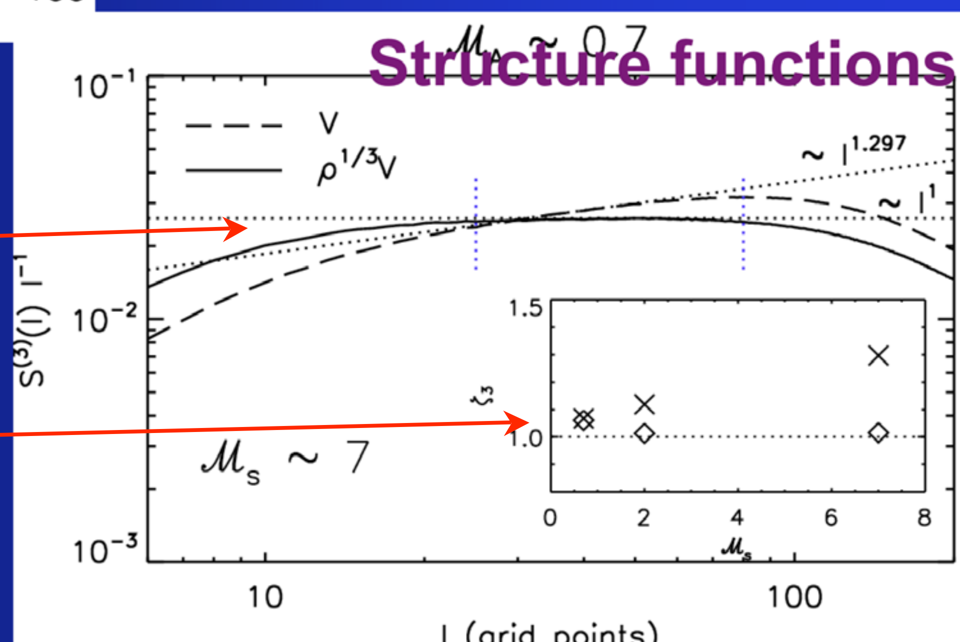
$u?\ ?^{1/3}\ v$

For high Mach numbers **u** has Kolmogorov scaling

For low Mach numbers both **u** and **v** have Kolmogorov scalings

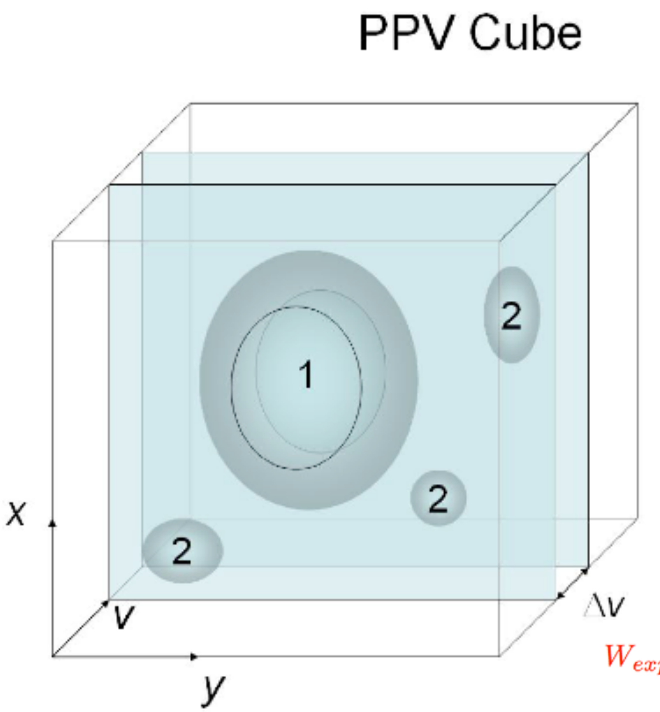
For high Mach number third moment for **u** scales as l

For low Mach number the third moment for **u** and **v** scale as l



Kowal & Lazarian 07

Velocity Channel Analysis (VCA): Analysis of Channel Maps



VCA: slice in velocity

1 - thin slice
2 - thick slice

Window Function

$W_{exp} = \Delta V^{-1} \exp[-\frac{v^2}{2\Delta V}]$

We correlate $I_C(\mathbf{X}) = \int dv \rho_s(\mathbf{X}, v) W(v)$

"Channel Correlation Function":

$$\mathcal{C}(R) \equiv \langle I_C(\mathbf{X}) I_C(\mathbf{X}+\mathbf{R}) \rangle$$

is equal to

$$\mathcal{C}(R) \propto \int dv_1 W(v_1) \int dv_2 W(v_2) \xi_s(\mathbf{R}, v_1, v_2)$$

thus

$$\mathcal{C}(R) \propto \int dz \xi(r) D_z(r)^{-1/2} \int dv W_{exp} \exp\left[-\frac{v^2}{2D_z(r)}\right]$$

$\Delta V \gg D_z(R)^{1/2}$ *"Thick slice"*

$$\mathcal{C}(R) \propto \int dz \xi(r)$$

Only density

$\Delta V \ll D_z(R)^{1/2}$ *"Thin slice"*

$$\mathcal{C}(R) \propto \int dz \xi(r) D_z(r)^{-1/2}$$

Density + velocity

Channel maps were used but not understood:

e.g. Crovisier & Dickey 83, Green 93, Stanimirovic et al. 99

Difference in slopes, thickness was arbitrarily chosen

VCA and VCS: Absorption

"It was absorption that induced us to stop our work"

G. Munch 98

Emission lines:

In the presence of absorption the effective window function changes

$$\tilde{W}_{exp} = W_{exp} \exp[-\kappa_{abs}(\xi_s(0,v) - \xi_s(0,0))]$$

$$\kappa_{abs}(\xi_s(0,v) - \xi(0,0)) > 1$$ **is absorption-dominated regime**

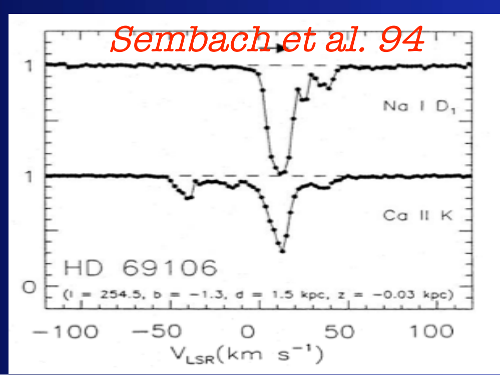
VCA: $P_2(K) \propto K^{-3-\gamma}$ **for shallow density** $P_2(K) \propto K^{-3}$ **for steep one**

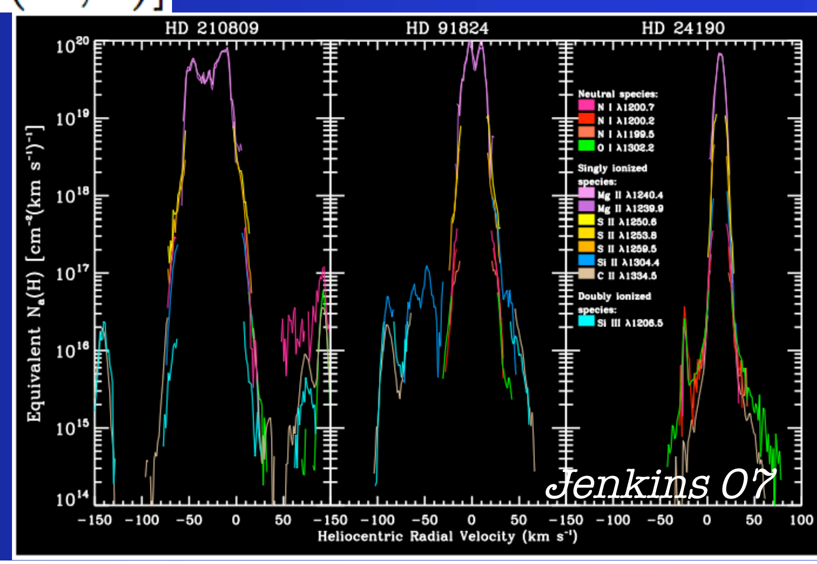
For absorption lines: $I_{\mathbf{X}}(v) = I_0 \exp[-\tau(\mathbf{X}, v)]$ **a new measure**

$$\langle [log(I(v_1) - log I(v_2)]^2 \rangle$$

is appropriate

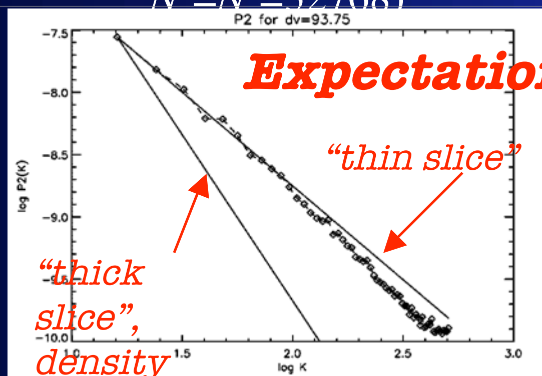
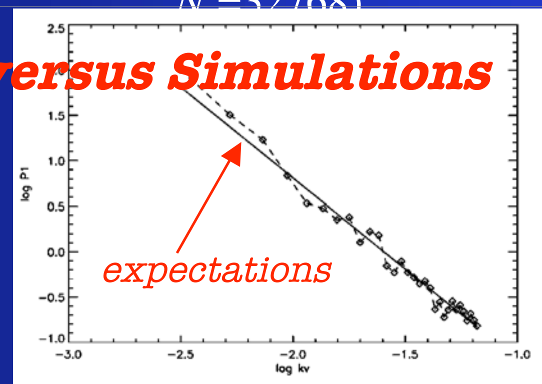
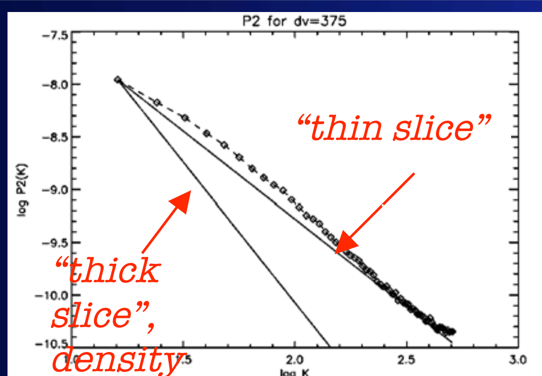
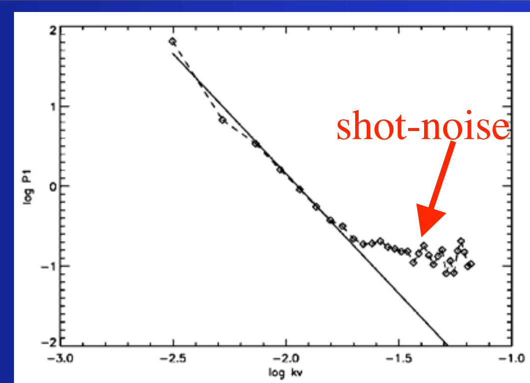
Many absorption lines are available. Testing stage





Example of VCA/VCS Testing with Simulations

Expectations versus Simulations

	VCA (spatial spectrum, N =N =32768)	**VCS** (spectrum over *v*, N =32768)
α_u= 4.0 need N_z: 20000		
α_u=3.67 need N_z: 420000		

Other tests of VCA are in Lazarian et al. 01, Esquivel et al. 02, Padoan et al. 07.

Tests by Chepurnov & Lazarian 06

Scales corresponding to resolved k_v (or channel thickness) should contain a sufficient number of emitting points

$$N_z \approx 15 \frac{L_s}{L_{inj}} N_{ch}^{\frac{2}{\alpha_u - 3}}$$

VCS: Predictions and Testing

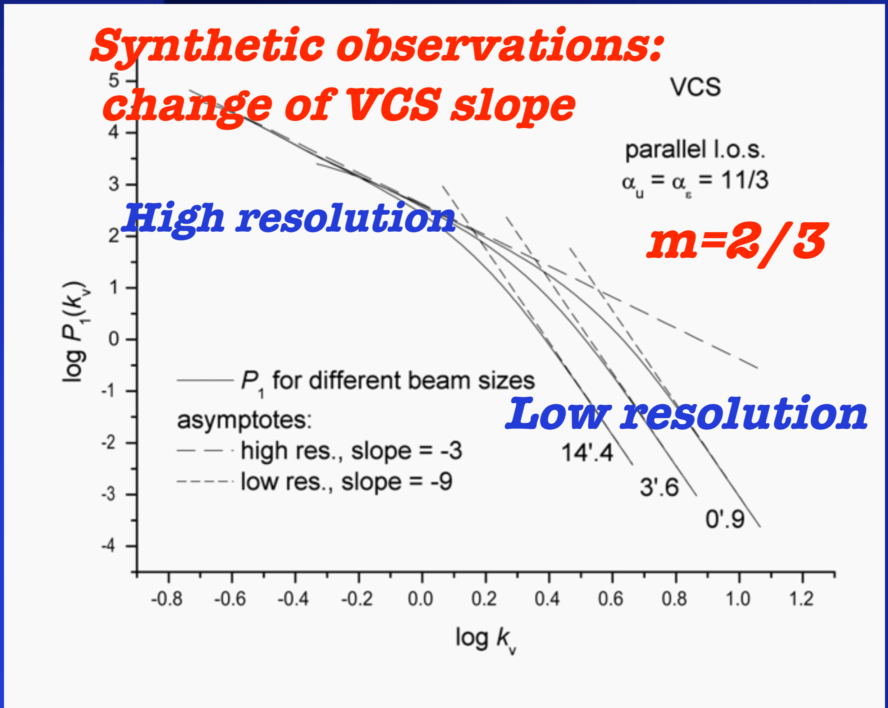
VCS expression:

$$P_1(k_v) \equiv \left\langle \left| \int S(v) e^{-ik_v v} dv \right|^2 \right\rangle \propto k_v^{-\beta}$$

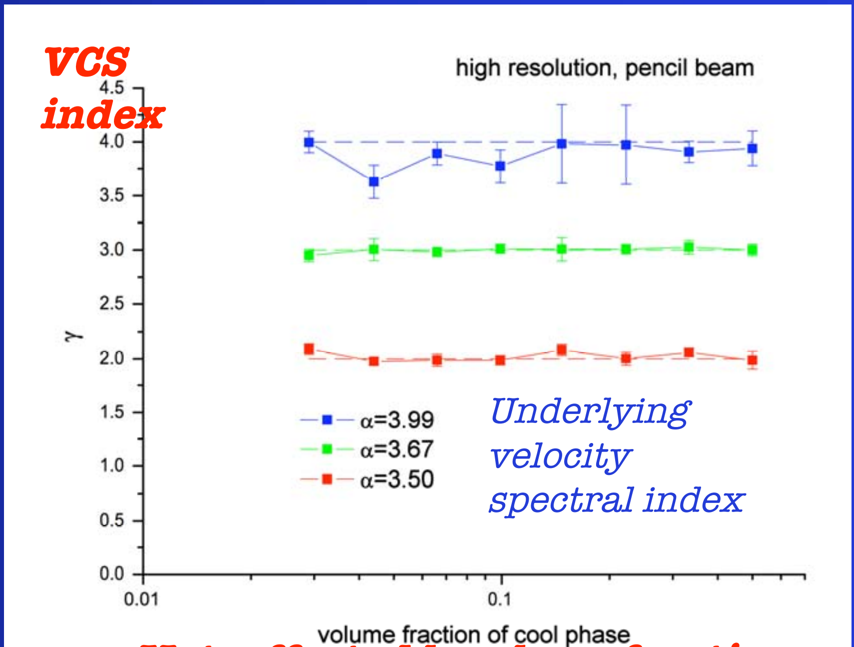
S(v) is observed line

Prediction for VCS in Lazarian & Pogosyan 06 and Chepurnov & Lazarian 06 are tested in Chepurnov & Lazarian 07

Synthetic observations: change of VCS slope



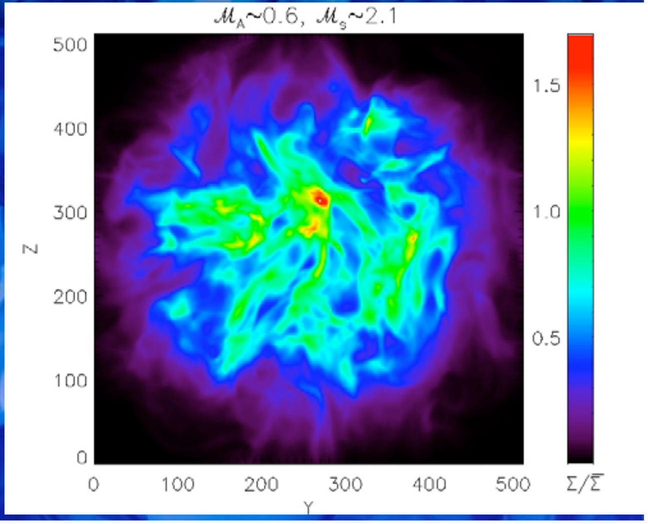
VCS index



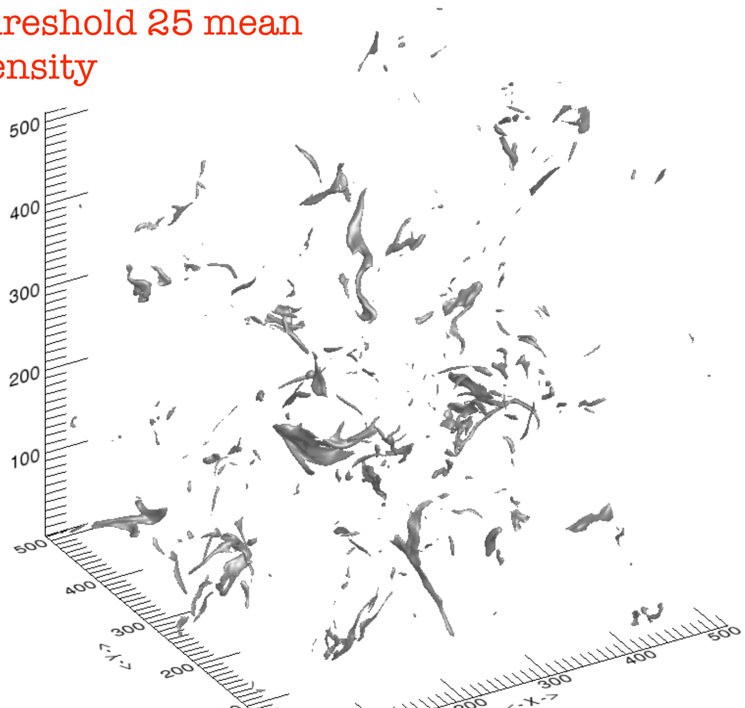
Not affected by phase fraction

Spectrum of Density

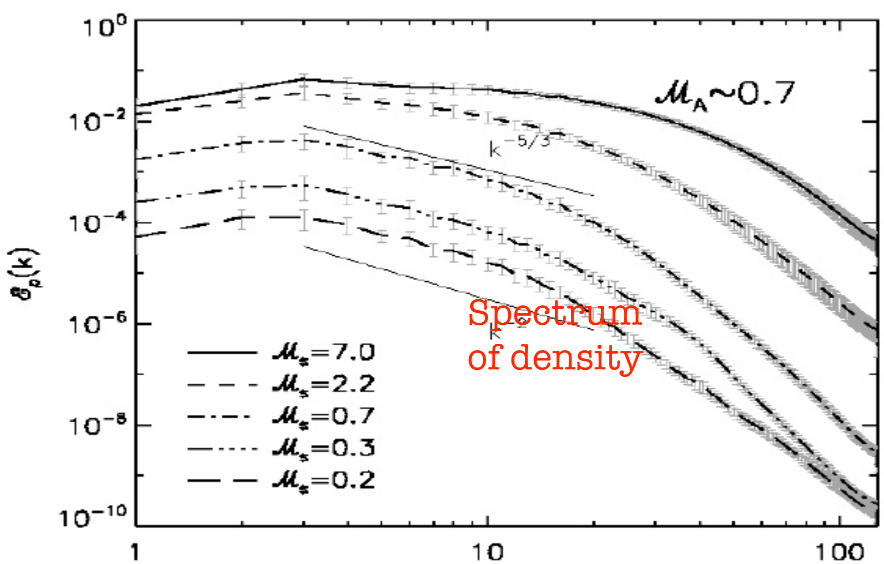
Density gets shallow (more energy at small scales) for high Mach numbers.



Peaks of density at threshold 25 mean density



Mach number is 7



Kowal, Lazarian & Beresnyak 07