Supersonic Hydrodynamic Turbulence

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Outline

- Some History
- Numerical Experiments
- Statistics of supersonic turbulence
  - Density
  - Velocity
  - Mixed
- Phenomenology of compressible cascade
- Summary
- References:

Kritsuk, Norman, & Padoan, ApJL **638**, L25, 2006

Kritsuk, Wagner, Norman, & Padoan, ASP Conf. Ser. **359**, 84, 2006

Kritsuk, Norman, Padoan, & Wagner, ApJ **665**, 416, 2007

Kritsuk, Padoan, Wagner, & Norman, AIP Conf. Proc., **932**, 393, 2007

Some History: Theory

- ☞ 1759 Euler (“Euler equations”)
- ☞ 1822 Navier (“Navier-Stokes equations”)
- ☞ 1845 Stokes (friction of fluids in motion)
- ☞ 1895 **Reynolds** (“Reynolds decomposition”, “Reynolds equation”)
- ☞ 1922 Richardson (“Richardson cascade”)
- ☞ 1935 Taylor (isotropic turbulence)
- ☞ 1941 Kolmogorov (“K41 phenomenology” $\Rightarrow$ 46 yrs since Reynolds)
- ☞ 1946 99% of papers on density fluctuations were published since this yr.
- ☞ 1958 **Favre** (density-weighted average)
- ☞ 1962 Kolmogorov (K62 refined similarity hypothesis)

. . .

- ☞ 2000 **$1M bounty** from Clay Mathematics Institute “to unlock the secrets hidden in the Navier-Stokes equations”

Spitzer: “Sculpting the South Pillar”, Carina Nebula

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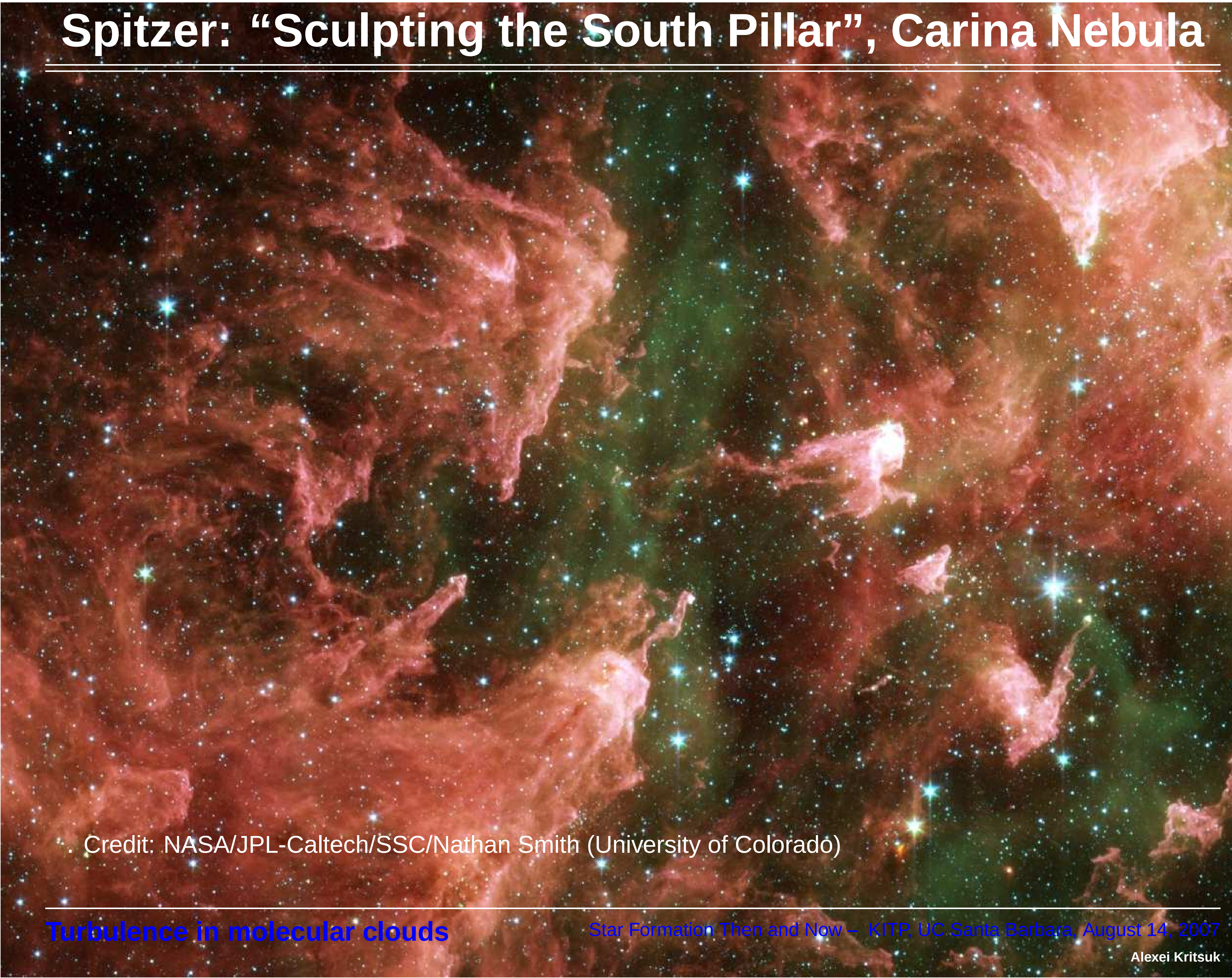


. Credit: NASA/JPL-Caltech/SSC/Nathan Smith (University of Colorado)

Some History: Astrophysics

Kaplan & Pikelner (1970): *"Unfortunately, the question of the nature of turbulence in a magnetic field remains far from solved. ... we must stress that interstellar gas turbulence is known not to have an isotropic or homogeneous nature. Therefore, we can draw no further conclusions by comparing theoretical assumptions with observational data."*

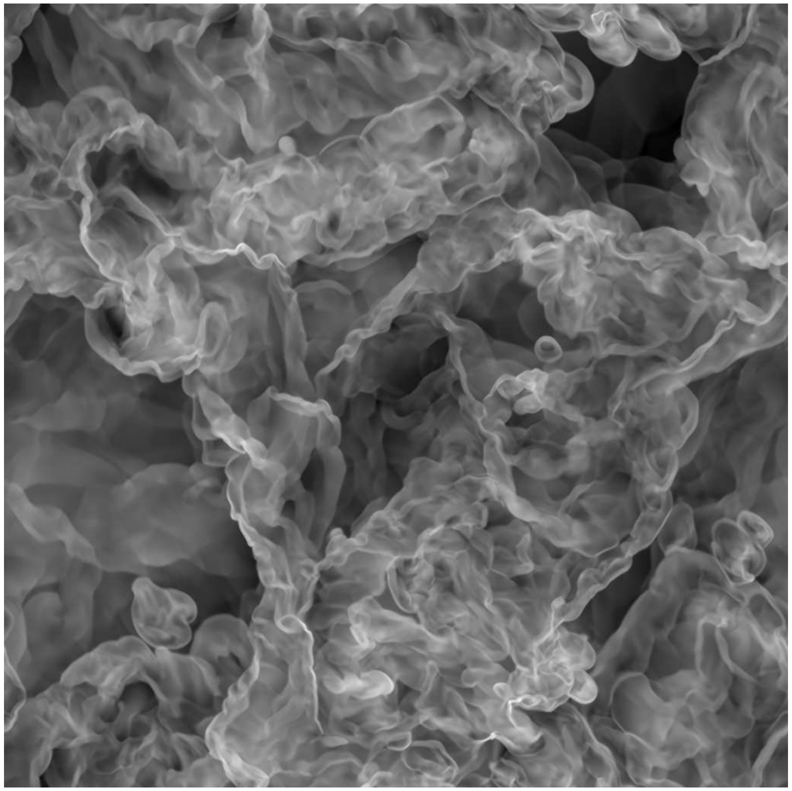
Pouquet, Passot & Léorat (1991): *"Although interstellar cloud turbulence certainly includes magnetic fields, stellar energy sources, radiative cooling and gravitation,* ***nonlinear advection is a major common feature to take into account****. Homogeneous compressible turbulence has not been extensively studied, partly due to the fact that the incompressible case remains unsolved."*

McKee & Ostriker (2007): *"Unfortunately, for the case of strong compressibility and moderate or strong magnetic fields, which generally applies within molecular clouds, there is as yet no simple conceptual theory to characterize the energy transfer between scales and to describe the spatial correlations in the velocity and the magnetic fields."*

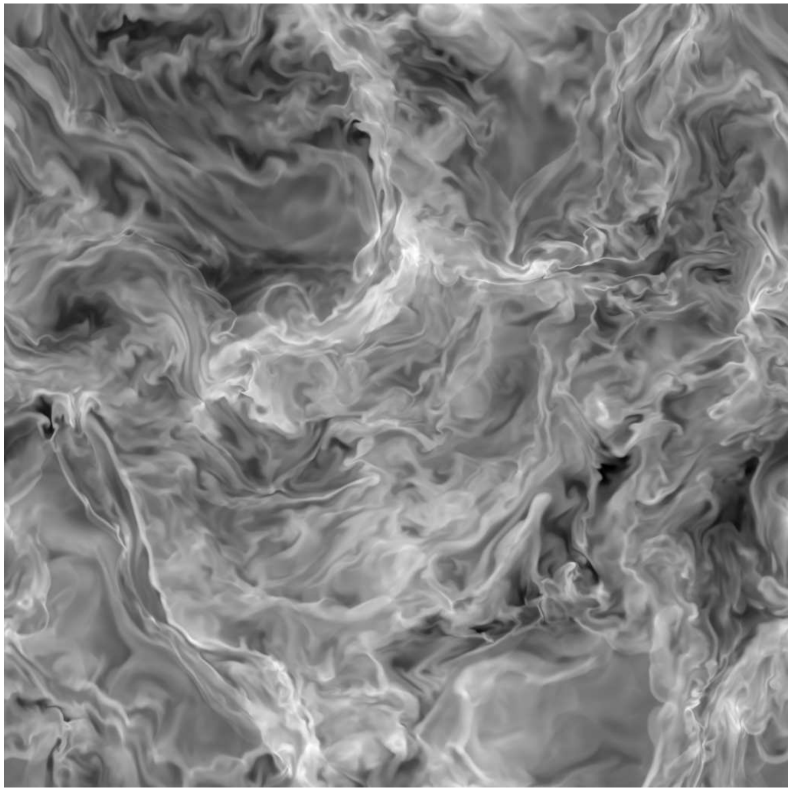
Turbulent structures: HD vs. MHD

Density slices from two simulations with resolution 1024^3 points

Zeus HD



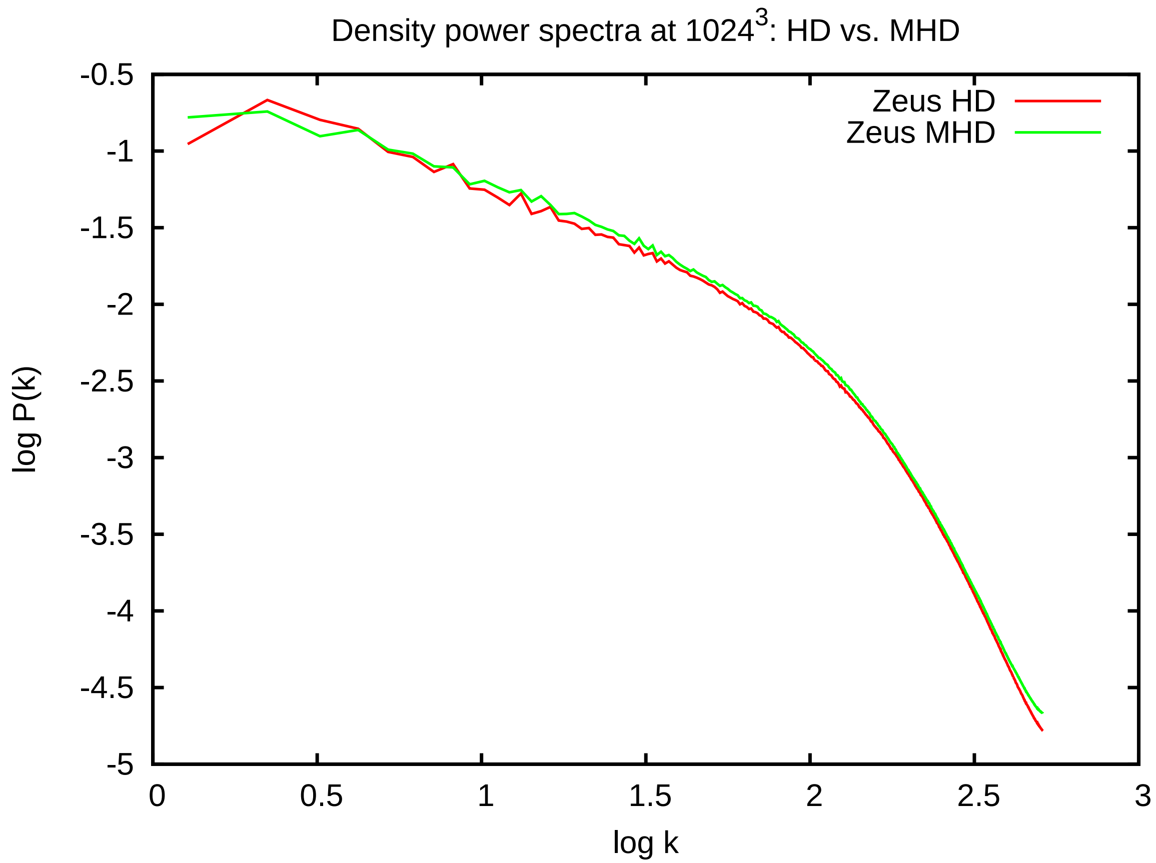
Zeus MHD



Structures are different due to suppression of K-H instability by B-fields

Turbulent structures: HD vs. MHD

Density power spectra for two snapshots with resolution 1024^3 points



While structures are different, power spectra appear identical.

See also Padoan et al. (2007) and 512^3 MHD by Kowal & Lazarian (2007)

Some History: Selected Simulations

Grid	Mach	Force	Authors	Year	Milestone
64^3	–	No	Orszag & Patterson	1972	First DNS
$??^3$	$\ll 1$	shear	Feireisen, Reynolds & Ferziger	1981	First compressible
256^2	0.03-1.7	No	Passot, Pouquet	1987	
512^2	1, 4	No	Passot, Pouquet & Woodward	1988	First PPM
64^3	0.4-0.8	No	Kida & Orszag	1990	
64^3	1	Yes	Kida & Orszag	1990	
2048^2	≤ 1	No	Porter, Pouquet & Woodward	1992	
256^3	≤ 1	No	Porter, Pouquet & Woodward	1992	
512^3	≤ 1	No	Porter, Pouquet & Woodward	1994	
1024^3	≤ 1	No	Porter, Woodward & Pouquet	1998	First 1K Euler
1024^3	≤ 0.5	No	Sytine et al.	2000	
512^3	1	Yes	Porter, Pouquet & Woodward	2002	
1024^3	6	Yes	This work	2006	

Numerical Experiments

☞ **In**

➠ Euler equations; 3D periodic box; Cartesian mesh

➠ Isothermal EOS

➠ **Mach 6**

➠ Random driving force (with a stationary pattern)

➠ Uniform grids $64^3, \ldots, 1024^3$ with **PPM**
[Kritsuk et al. 2007, ApJ 665, 416]

➠ Structured **AMR** with refi nement on shocks & shear up to 2048^3
[Kritsuk, Norman & Padoan 2006, ApJL 638, L25]

➠ **ENZO** code for cosmology and astrophysics [http://lca.ucsd.edu]

☞ **Out**

➠ Hydro fi elds, visualizations & statistical properties of turbulent structures

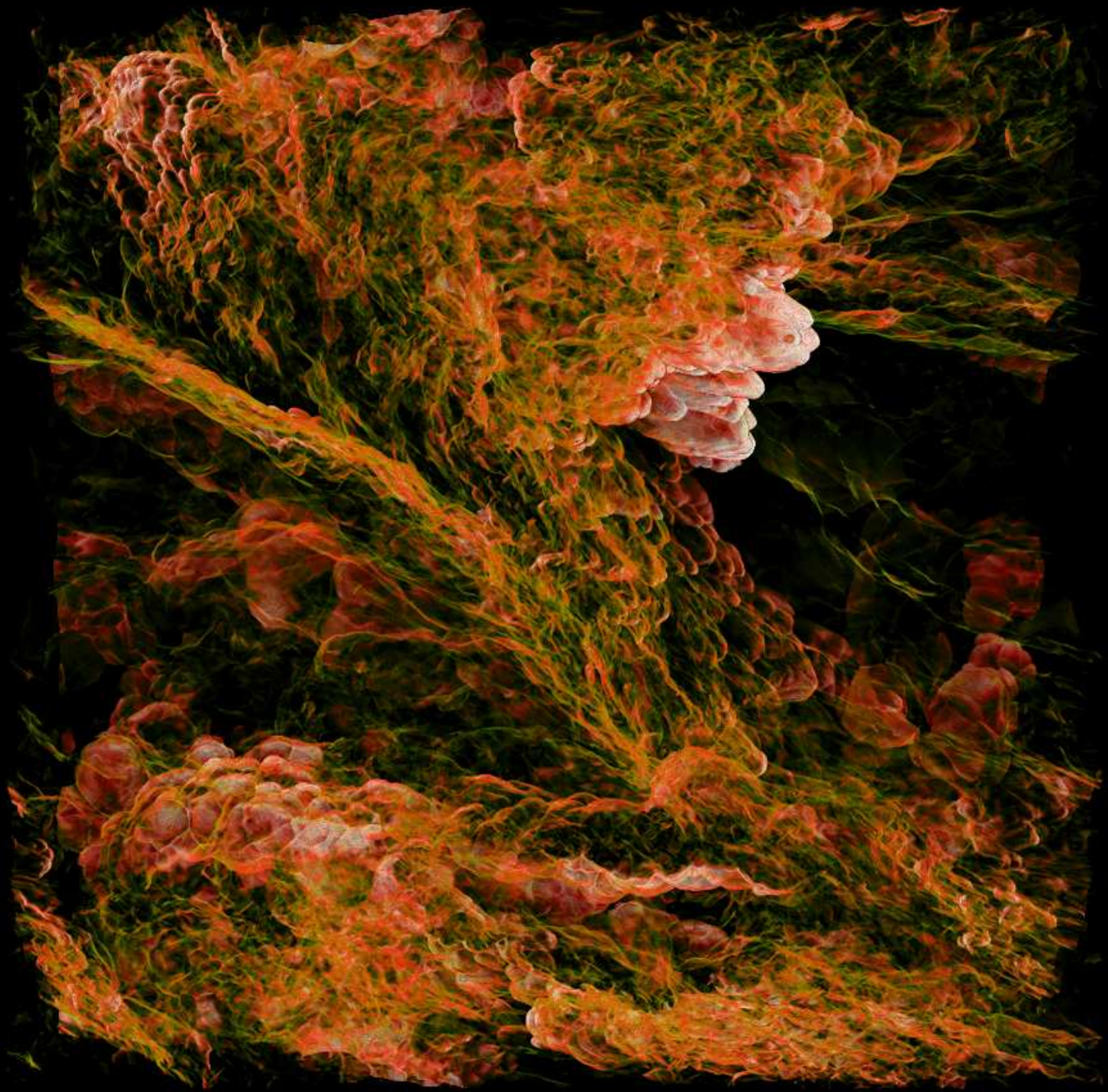
Structures in Physical Space

Turbulent Structures: density

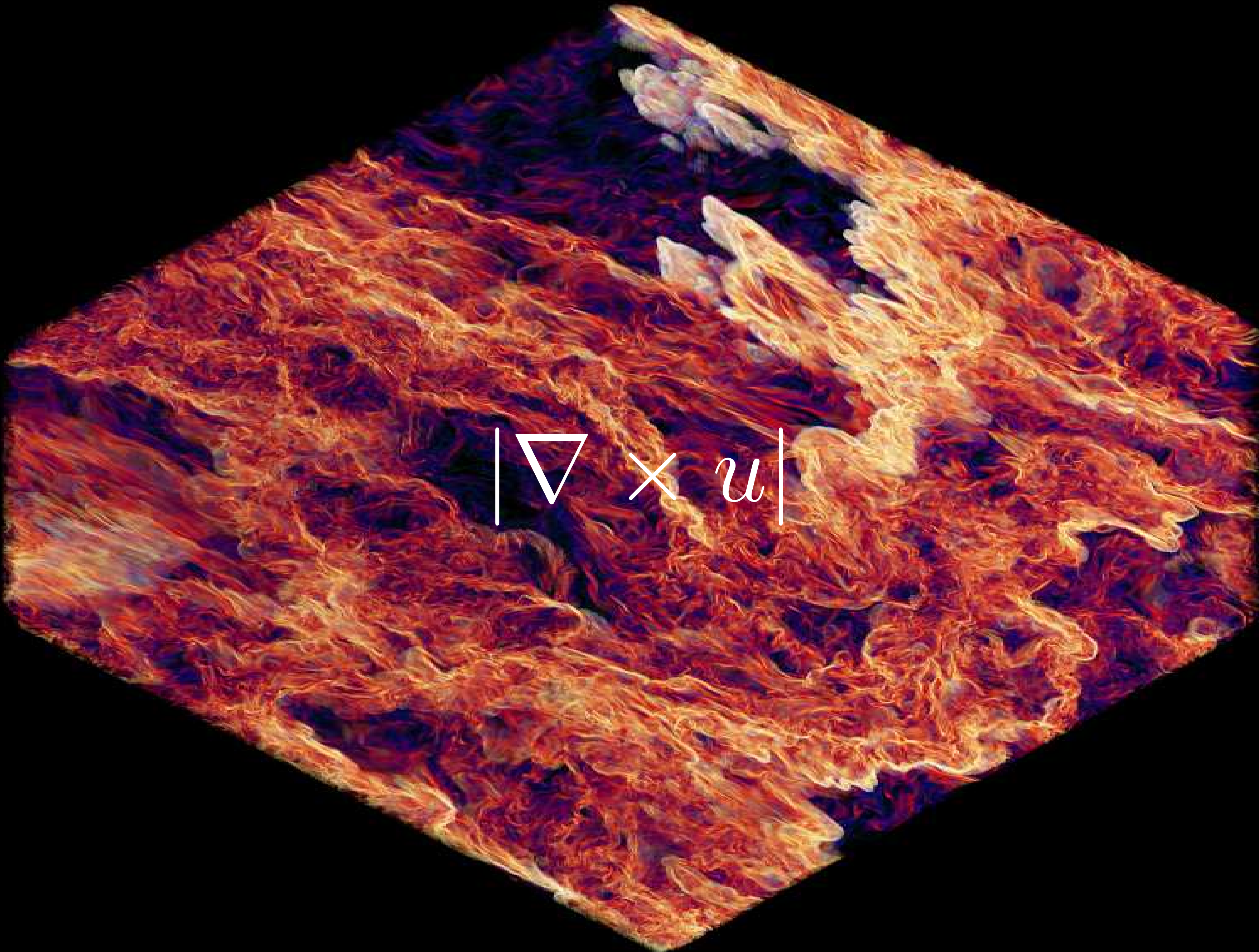
Turbulent Structures: dilatation

Dilatation

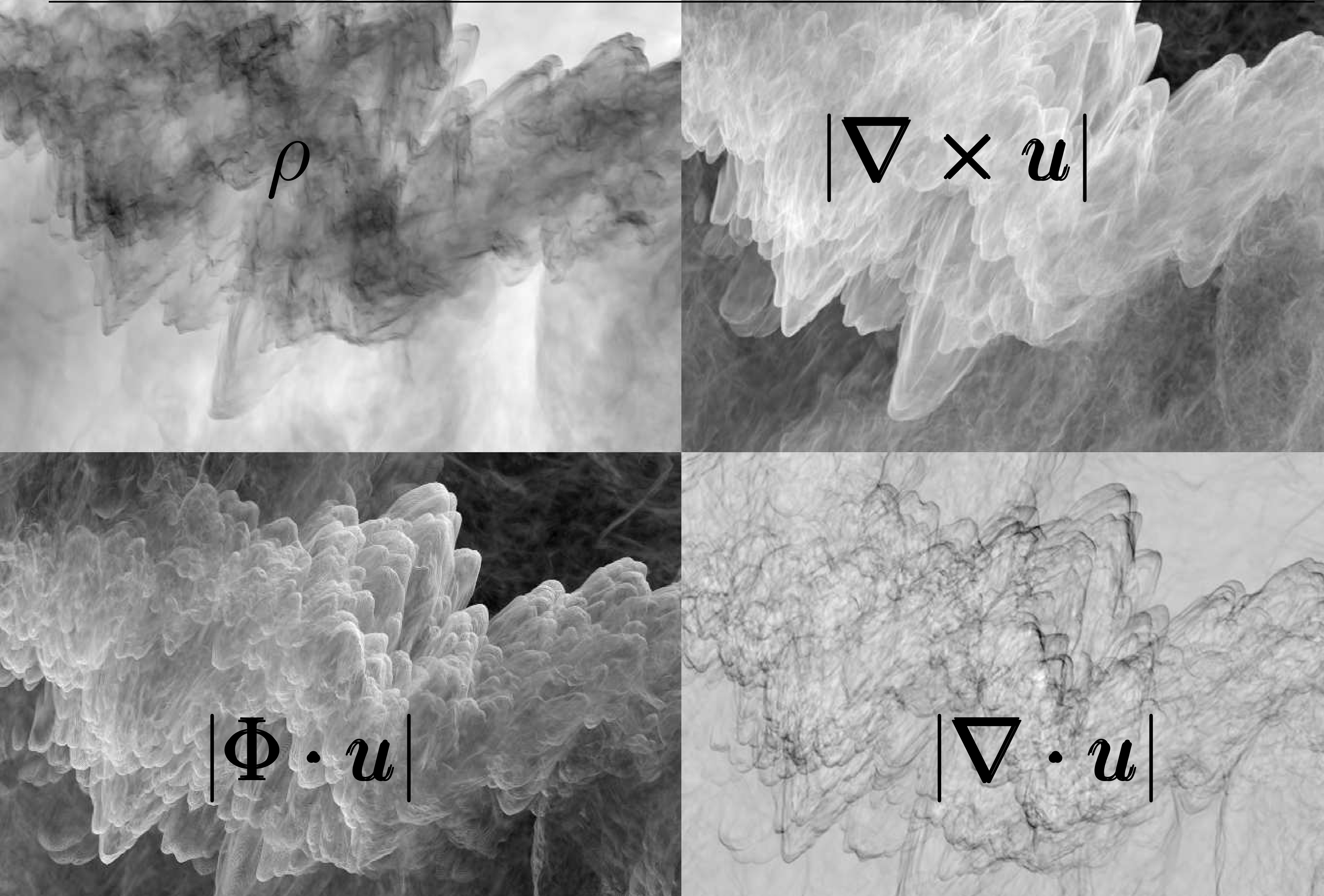
(MPEG animation)



Turbulent Structures: vorticity



Dissipative Structures in Mach 6 Turbulence



Results II

Statistics of Turbulence

Lognormal PDF of Density

In **isothermal turbulence** the density PDF is lognormal (this follows from an invariance property of the equations, see Vazquez-Semadeni 1994; Padoan, Nordlund & Jones 1997; Passot & Vázquez-Semadeni 1998; Nordlund & Padoan 1999; Biskamp 2003)

$$p(\ln\rho)d\ln\rho = \frac{1}{\sqrt{2\pi\sigma^2}} \times \exp\left[-\frac{1}{2}\left(\frac{\ln\rho - \overline{\ln\rho}}{\sigma}\right)^2\right] d\ln\rho, \tag{1}$$

where the mean of the logarithm of the density, $\overline{\ln\rho}$, is determined by

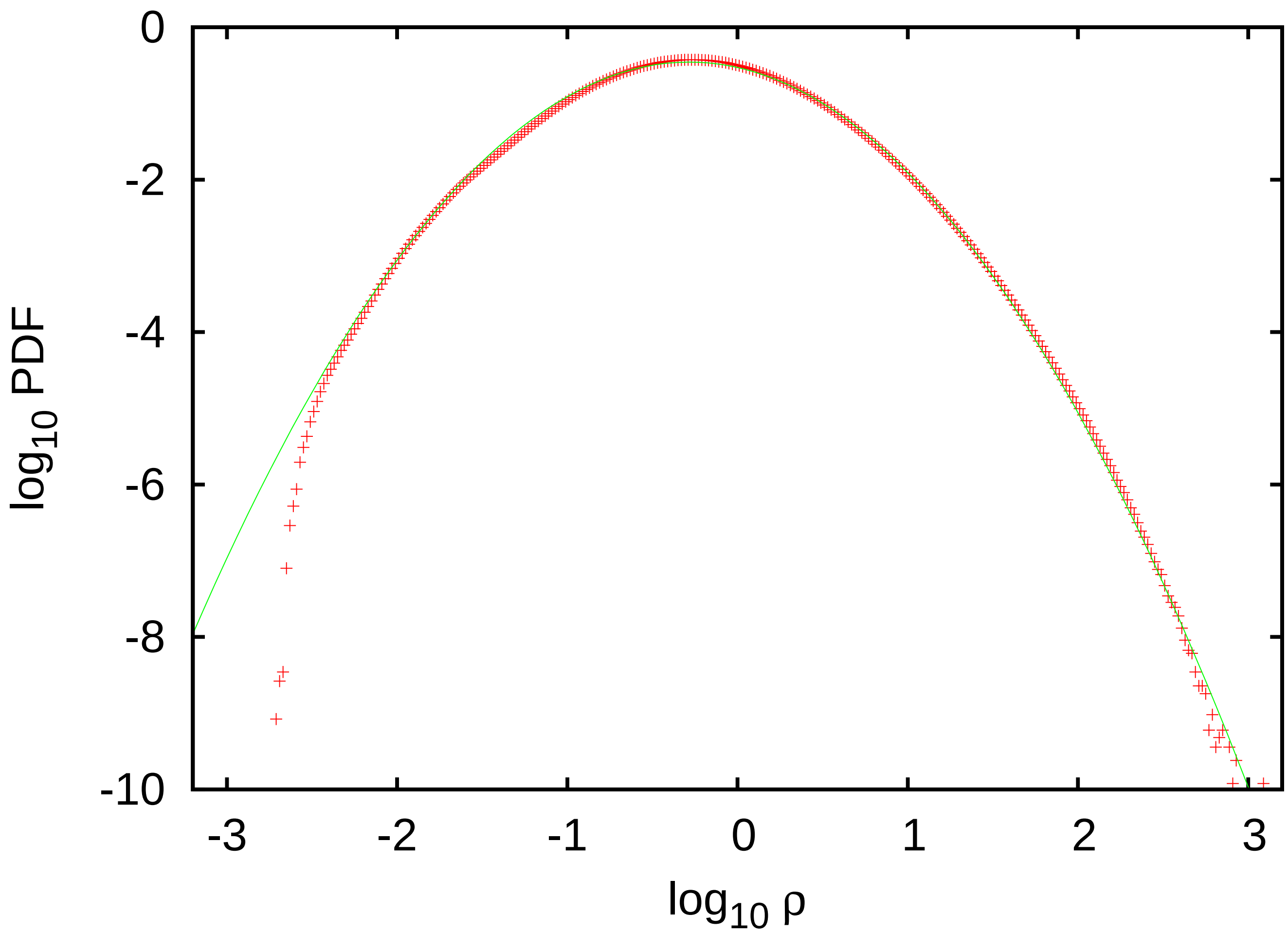
$$\overline{\ln\rho} = -\sigma^2/2. \tag{2}$$

The standard deviation σ is a function of Mach number $\mathcal{M}$

$$\sigma^2 = \ln\left(1 + b^2\mathcal{M}^2\right). \tag{3}$$

Lognormal PDF of Density

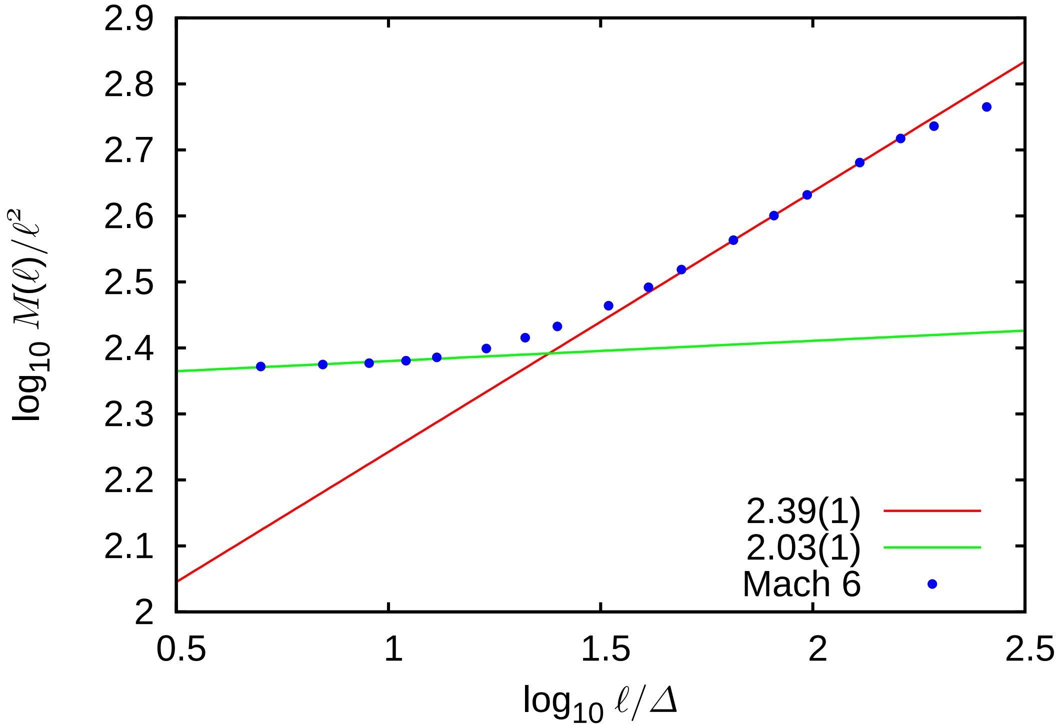
- Excellent fit quality over 8 decades in probability!
- Sample size 2×10^{11}
- The best-fit value of $b \approx 0.260 \pm 0.001$ for $\log_{10} \rho \in [-2, 2]$



Fractal Dimension of Mass Distribution

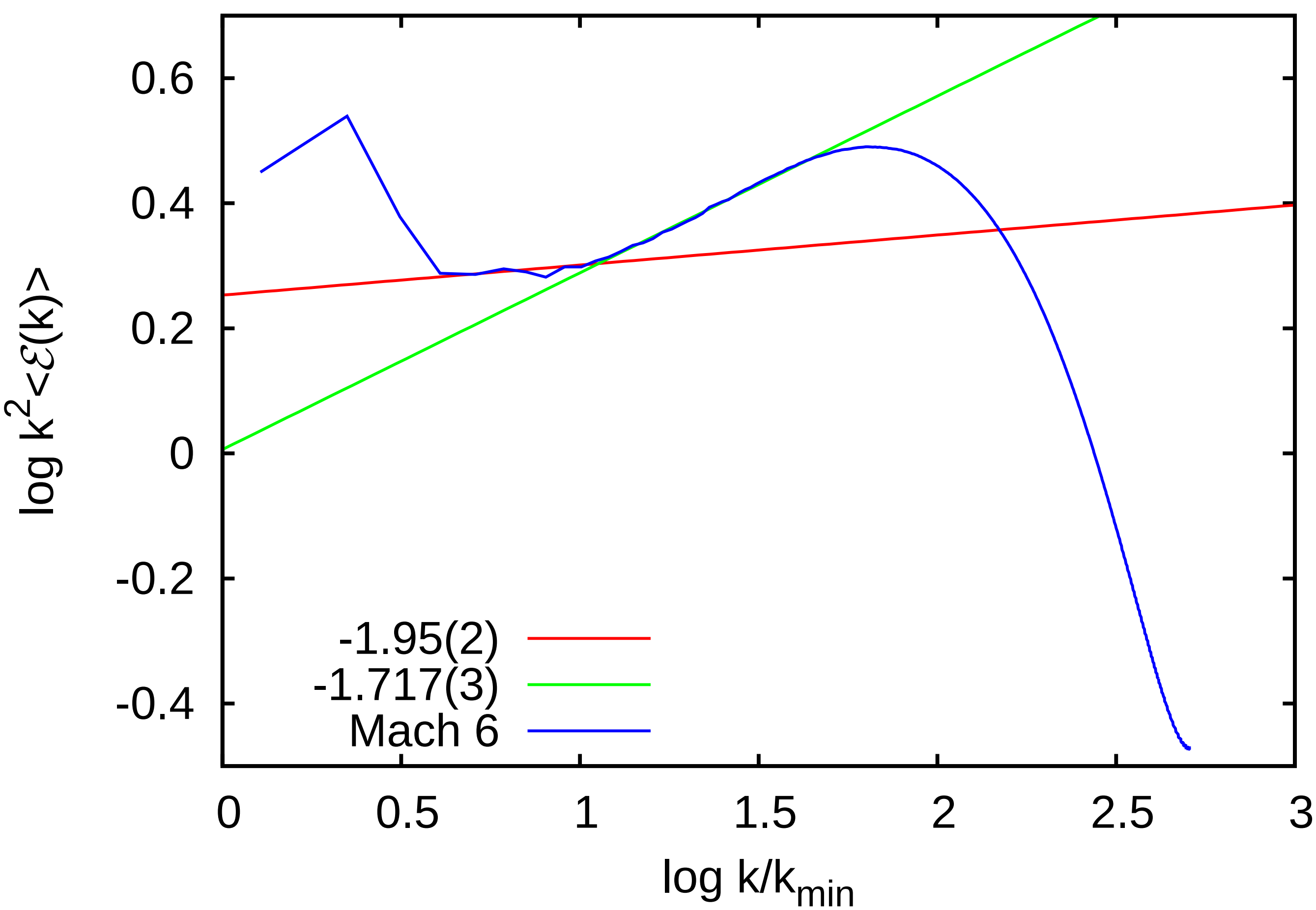
- Mass dimension, D_m, is defi ned via $M(\ell) \propto \ell^{D_m}$
- On small scales, where dissipation dominates, $D_m \approx 2 \;\Rightarrow$ shocks
- At $\ell \in [40,\ 160]\Delta$ dissipation is negligible and $D_m = 2.4 \;\Rightarrow$ inertial range
- D_m is consistent with observations of molecular clouds

  [e.g., Elmegreen & Falgarone 1996; Chappell & Scalo 2001]



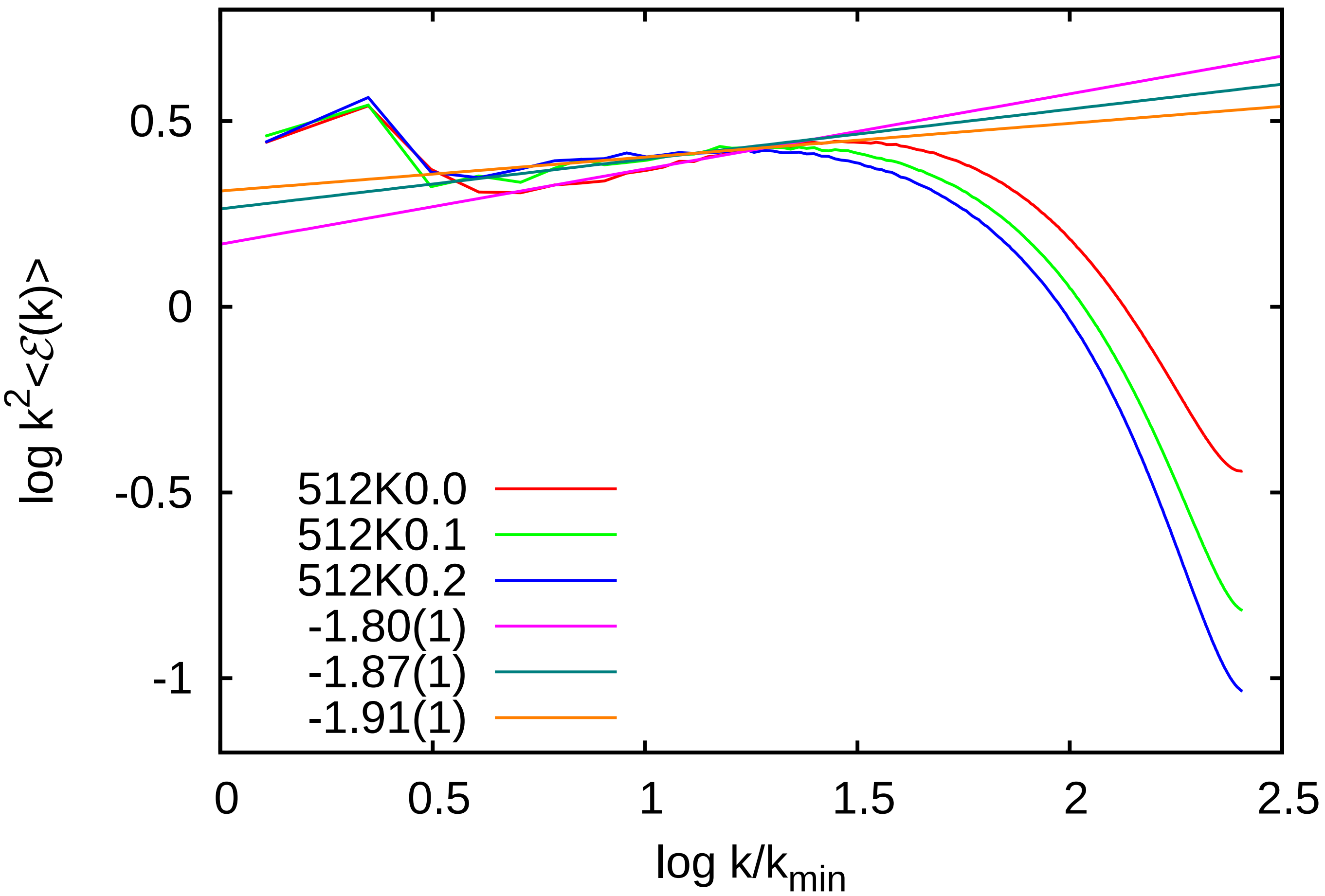
Velocity Power Spectrum at 1024^3

- Large-scale excess of power at $\ell \in [256, 1024]\Delta$ due to external forcing
- Short straight section in the inertial subrange $\ell \in [40, 256]\Delta$, slope $\beta = 1.95 \pm 0.02$
- Small-scale excess at $\ell < 40\Delta$ due to the bottleneck phenomenon



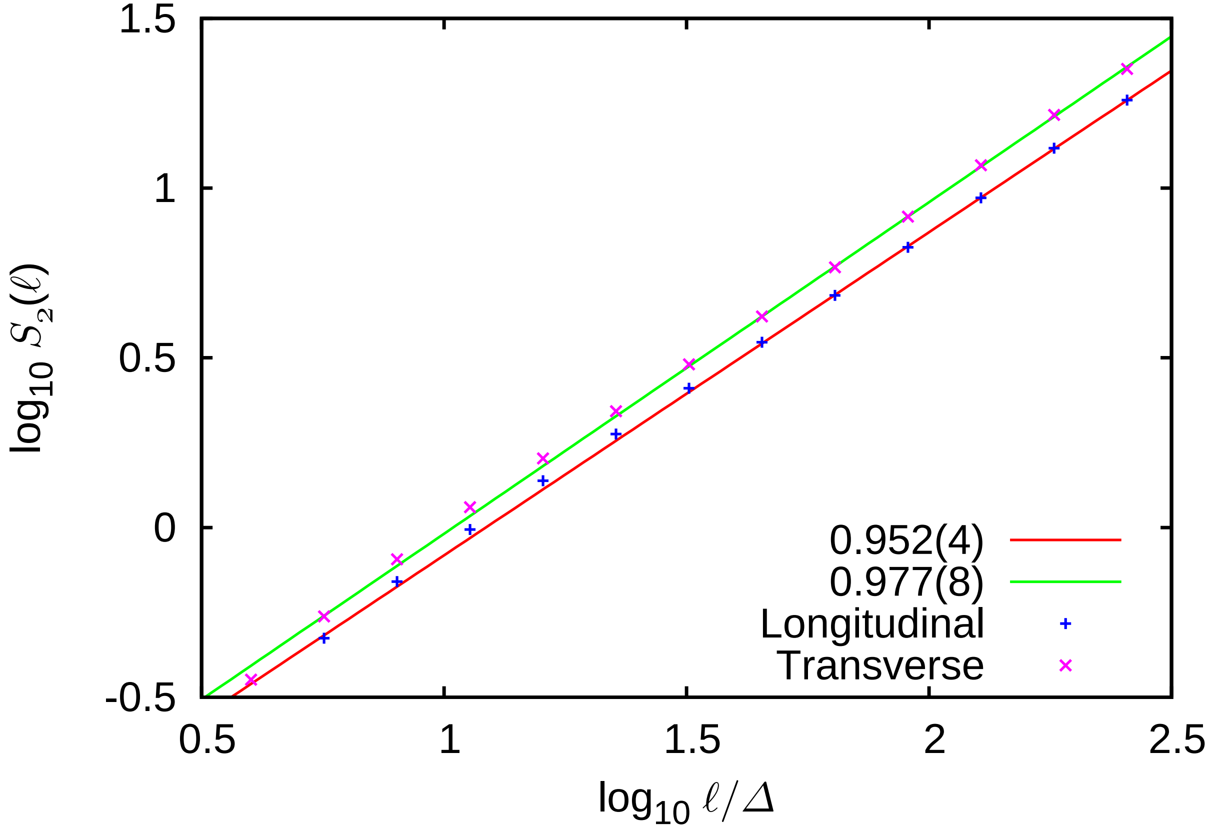
Numerical Dissipation and Bottleneck Phenomenon

- At 512^3, the slope of the "flat" part of the spectrum is primarily controlled by numerical diffusion
- The resulting uncertainty is $\sim 30\%$ of the difference between K41 and Burgers slopes!



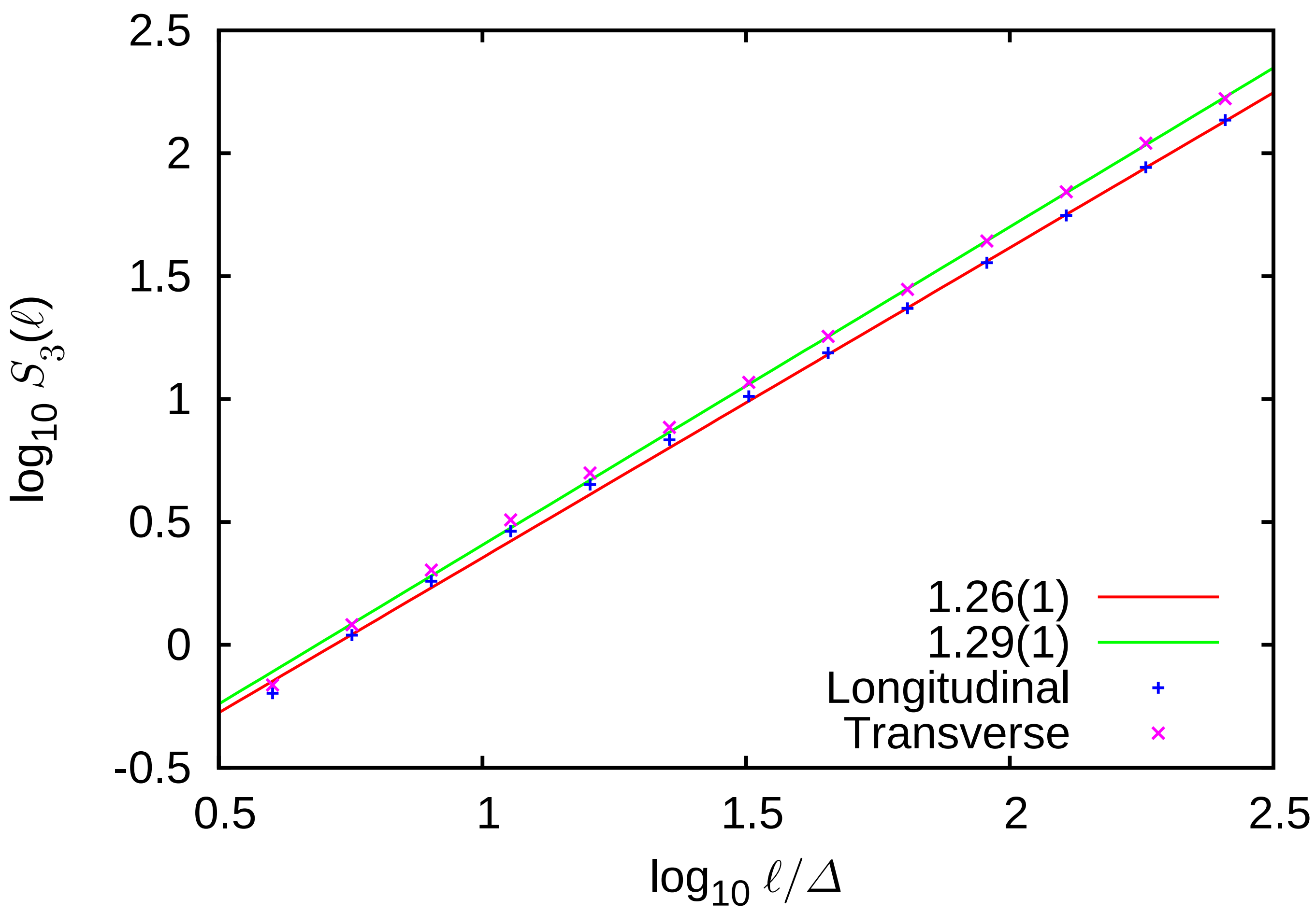
Velocity Structure Functions: 2nd order

- **Non-Kolmogorov exponents:** $\zeta_2^{\parallel} = 0.952 \pm 0.004$ and $\zeta_2^{\perp} = 0.977 \pm 0.008$; $\zeta_2^{K41} \equiv \frac{2}{3}$;
- Very good agreement with the velocity power spectrum index $\beta = 1 + \zeta_2 = 1.95 \pm 0.02$
- The PS and SF applications are completely independent and even rely on different parallelization paradigms



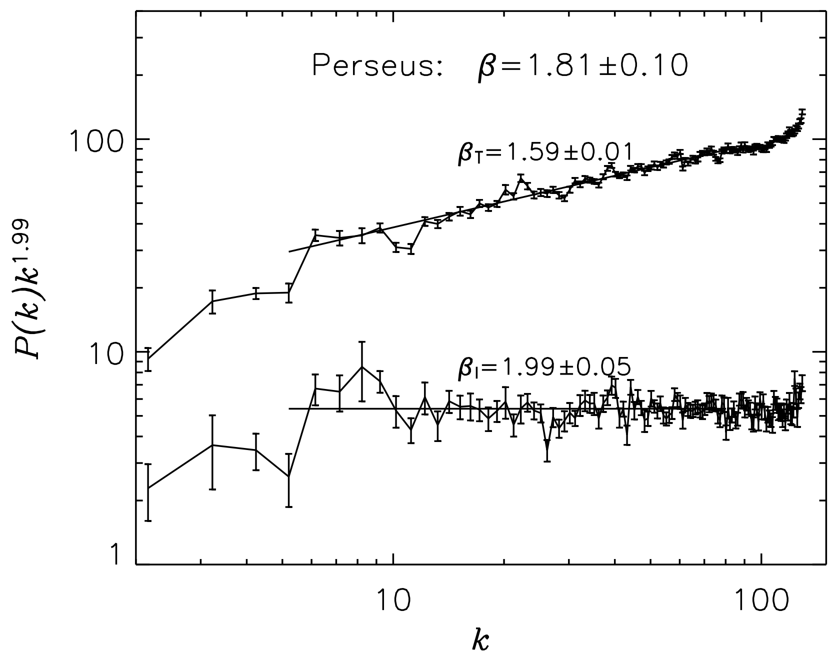
Velocity Structure Functions: 3rd order

- **Non-Kolmogorov exponents:** $\zeta_3^{\parallel} = 1.26 \pm 0.01$ and $\zeta_3^{\perp} = 1.29 \pm 0.01$
- **Four-fifths law** for incompressibler turbulence requires $\zeta_3^{K41} \equiv 1$



The power spectrum of supersonic turbulence in Perseus

- Power index $\beta = 1.81 \pm 0.10$ (compare with $\beta = 1.95 \pm 0.02$)
- Obtained via comparison of power spectra of integrated intensity maps and single-velocity-channel maps [Lazarian & Pogosyan 2000]
- Modifi cations of β due to magnetic effects appear to be small, while turbulence remains super-Alfvénic [Padoan et al., ApJ 661, 972, 2007]



[Padoan et al. 2006, ApJL 653, L125]

Cascade Phenomenology

A Simple Compressible Cascade Model

The kinetic energy is transferred through a hierarchy of scales by nonlinear interactions. In a compressible fluid, the mean *volume* energy transfer rate $\rho u^2 u/\ell$ is constant in a statistical steady state [e.g., Lighthill 1955], therefore

$$u \sim (\ell/\rho)^{1/3}. \tag{4}$$

Let's consider scaling relations for $\boldsymbol{v} \equiv \rho^{1/3}\boldsymbol{u}$

$$v^p = (\rho^{1/3}u)^p \sim \ell^{\,p/3}. \tag{5}$$

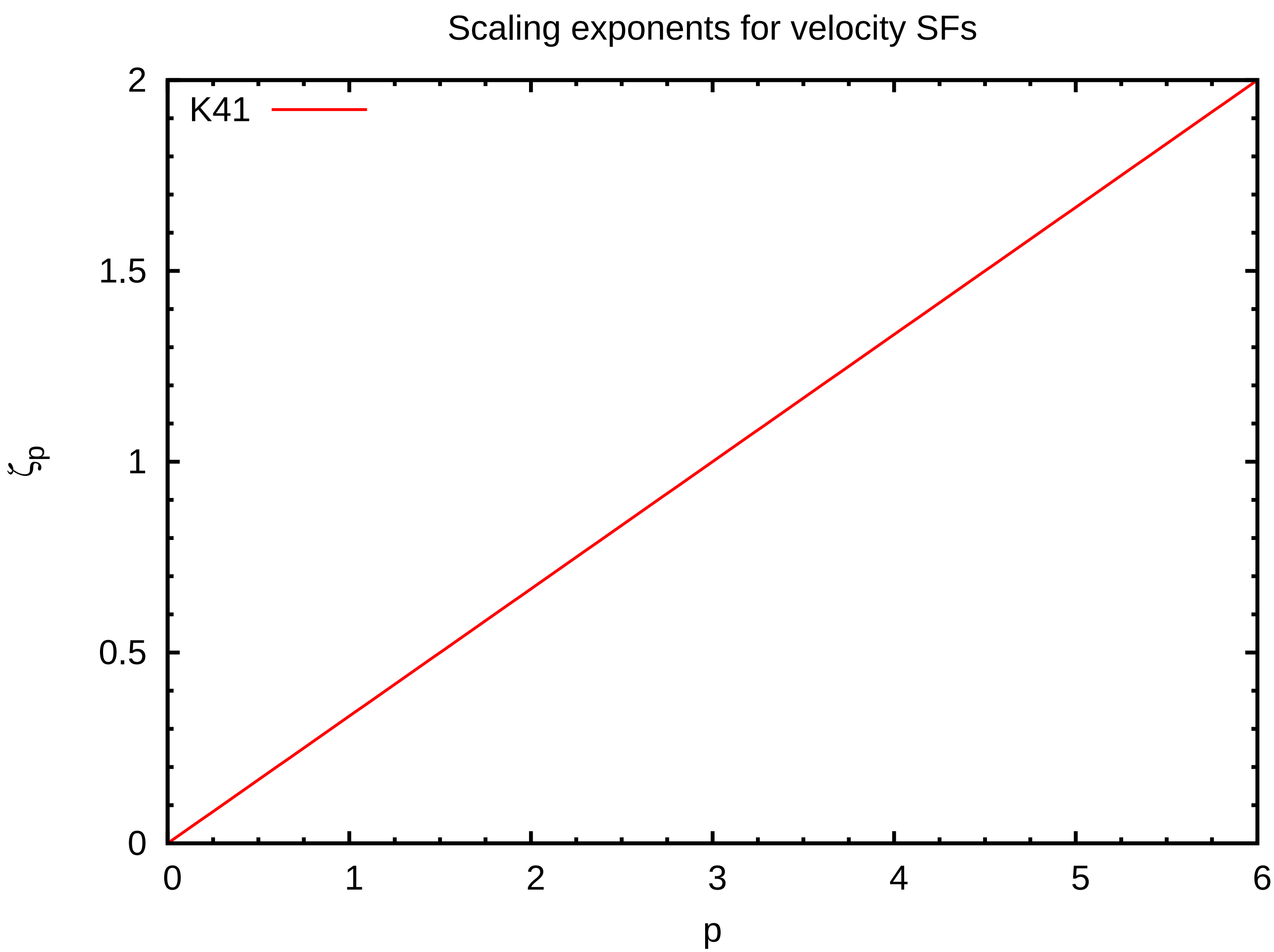
For compressible flows, structure functions of v should be used instead of u

$$\mathcal{S}_p(\ell) \equiv \langle|\boldsymbol{v}(\boldsymbol{r}+\boldsymbol{\ell})-\boldsymbol{v}(\boldsymbol{r})|^p\rangle \sim \ell^{\,p/3}, \tag{6}$$

with $\mathcal{S}_3 \sim \ell$. The scaling laws expressed by equation (6) are not necessarily exact and, as the incompressible K41 scaling, may require intermittency corrections. Using v instead of u, one properly accounts for the important density–velocity correlations in compressible flows.

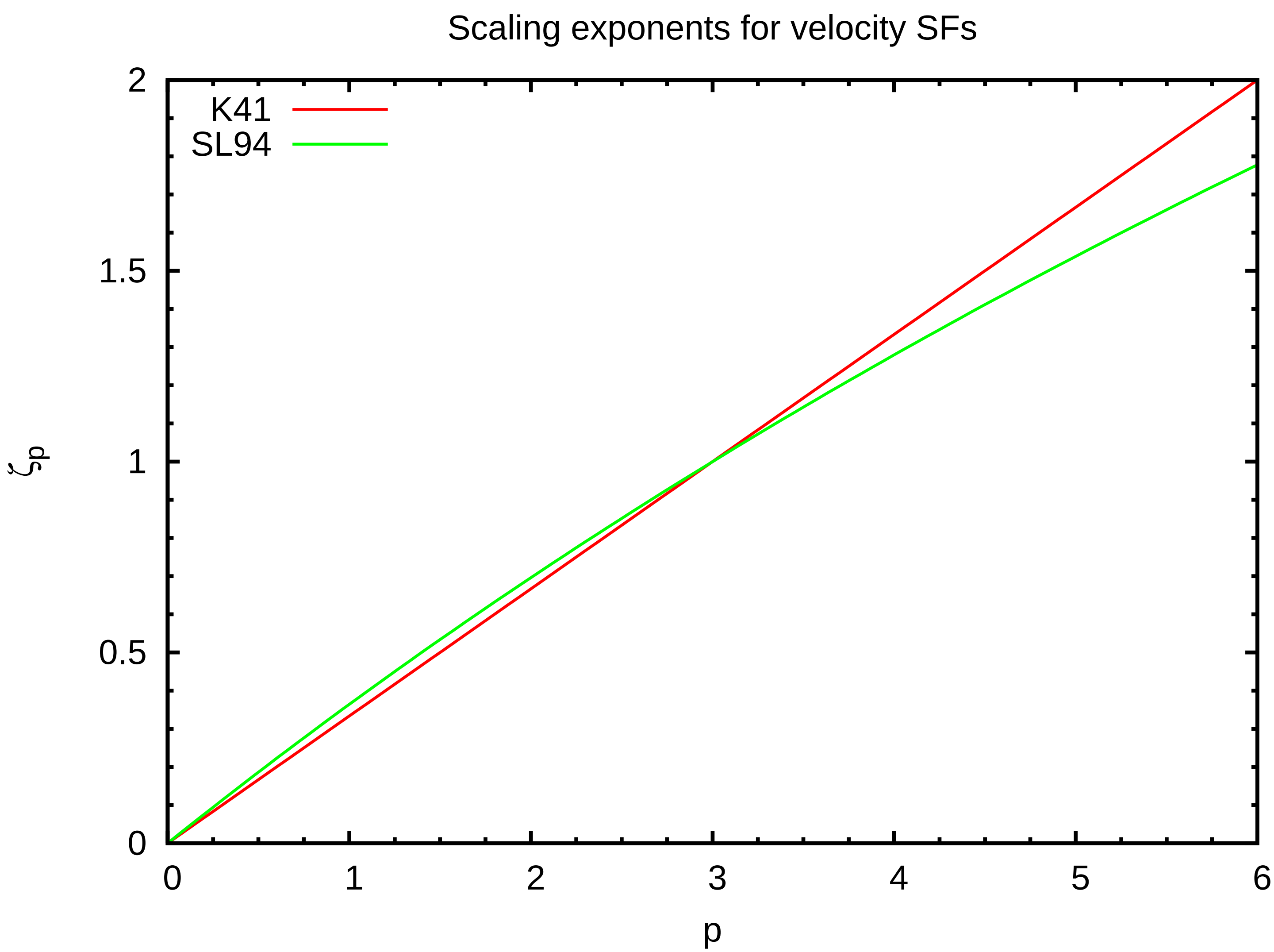
Turbulence Statistics: Models vs. Data

Kolmogorov (1941) incompressible scaling, $\zeta_p = p/3$



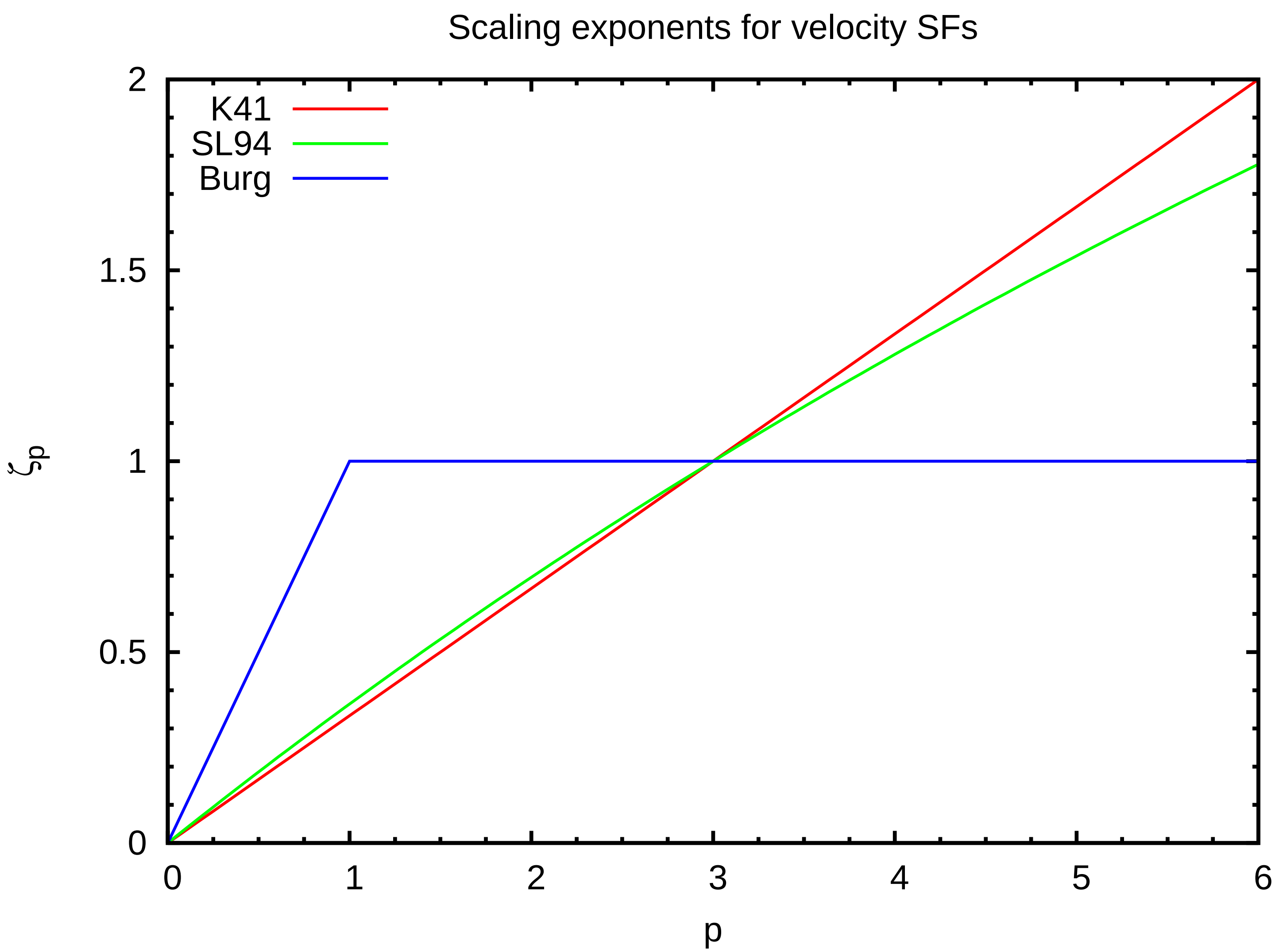
Turbulence Statistics: Models vs. Data

She & Lévêque (1994) incompressible intermittency corrections



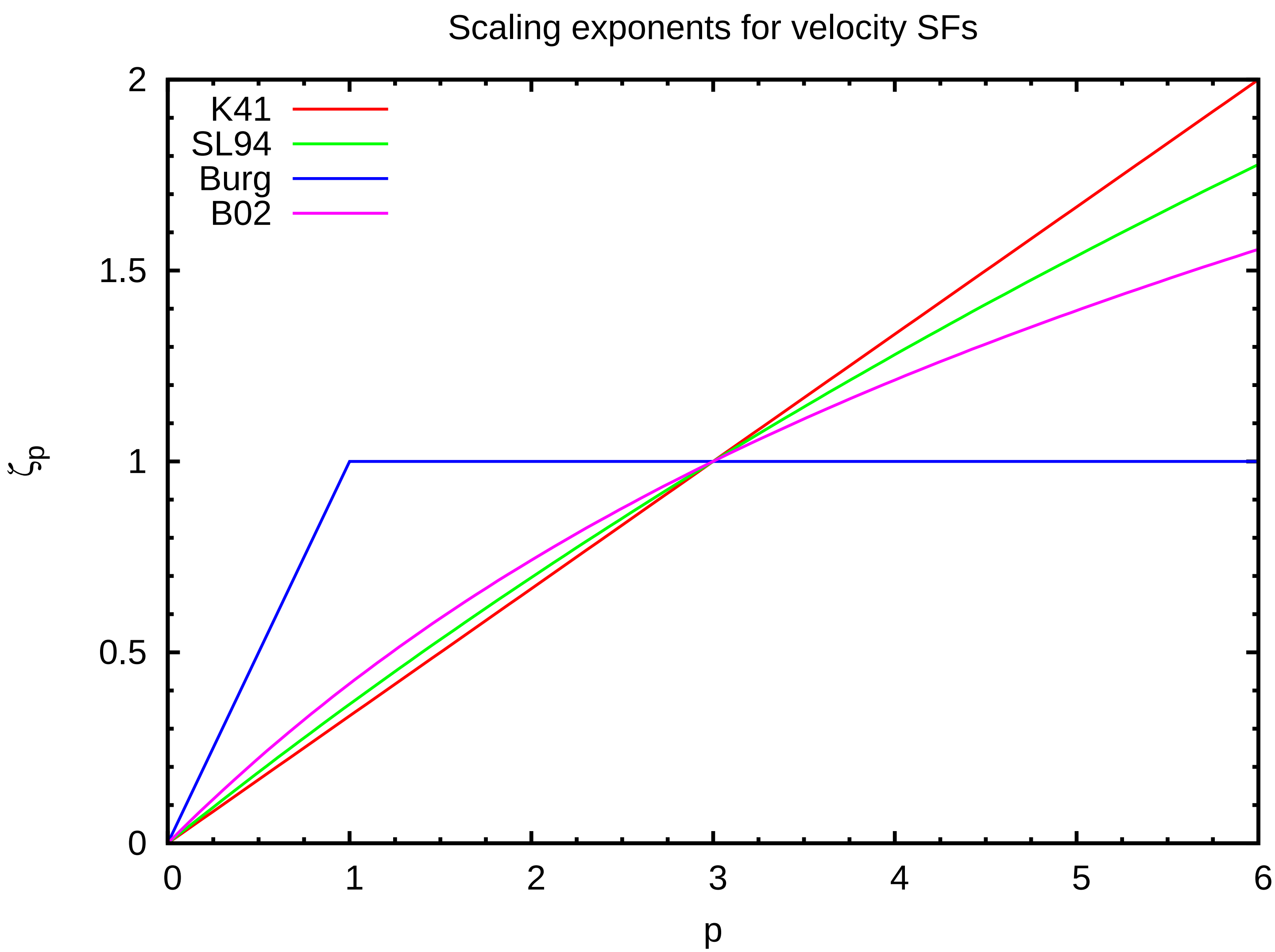
Turbulence Statistics: Models vs. Data

Burgulence. The phase transition at $p = 1$ is due to the isolated nature of shocks



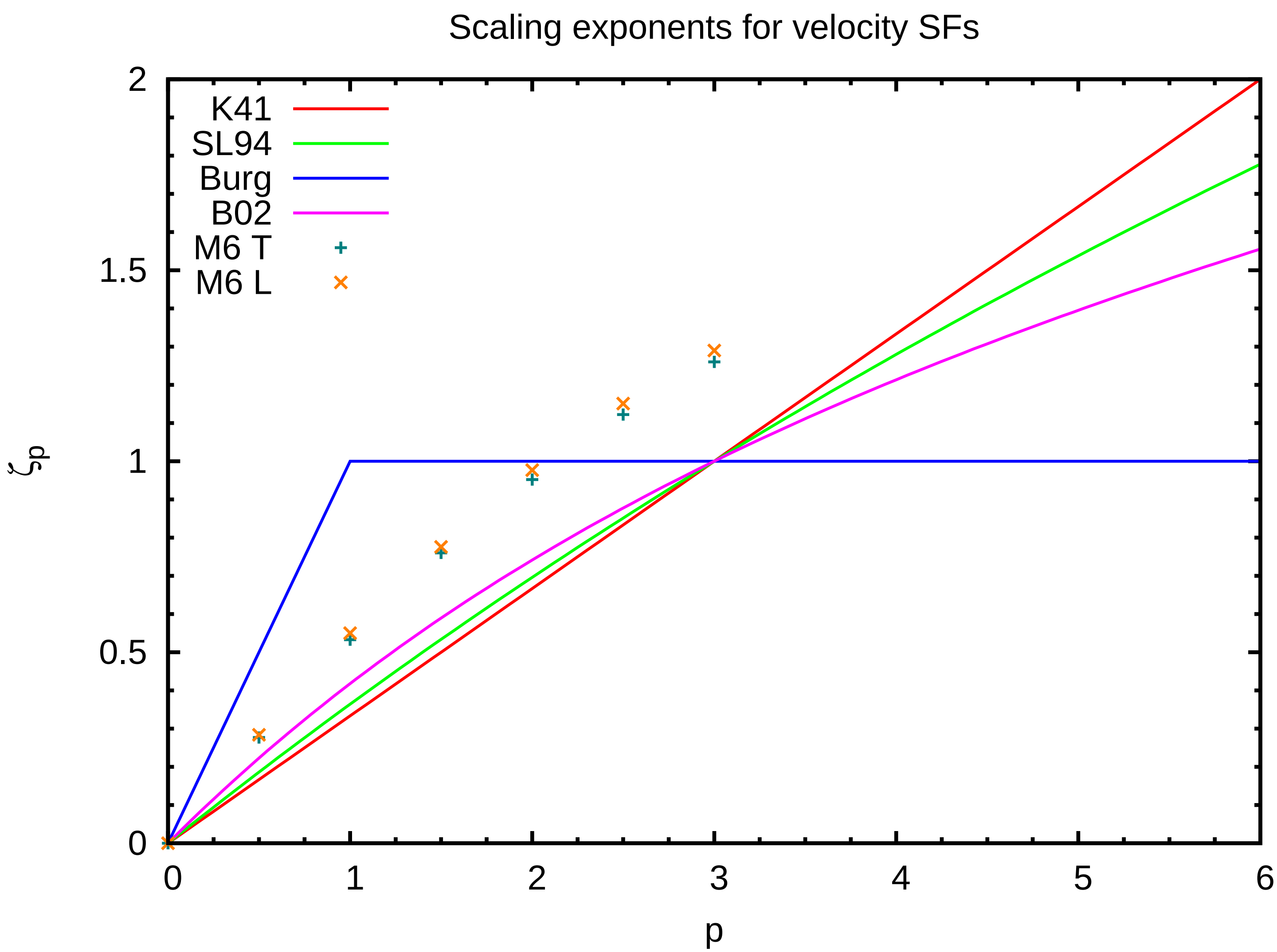
Turbulence Statistics: Models vs. Data

Boldyrev (2002) Kolmogorov-Burgers intermittency model



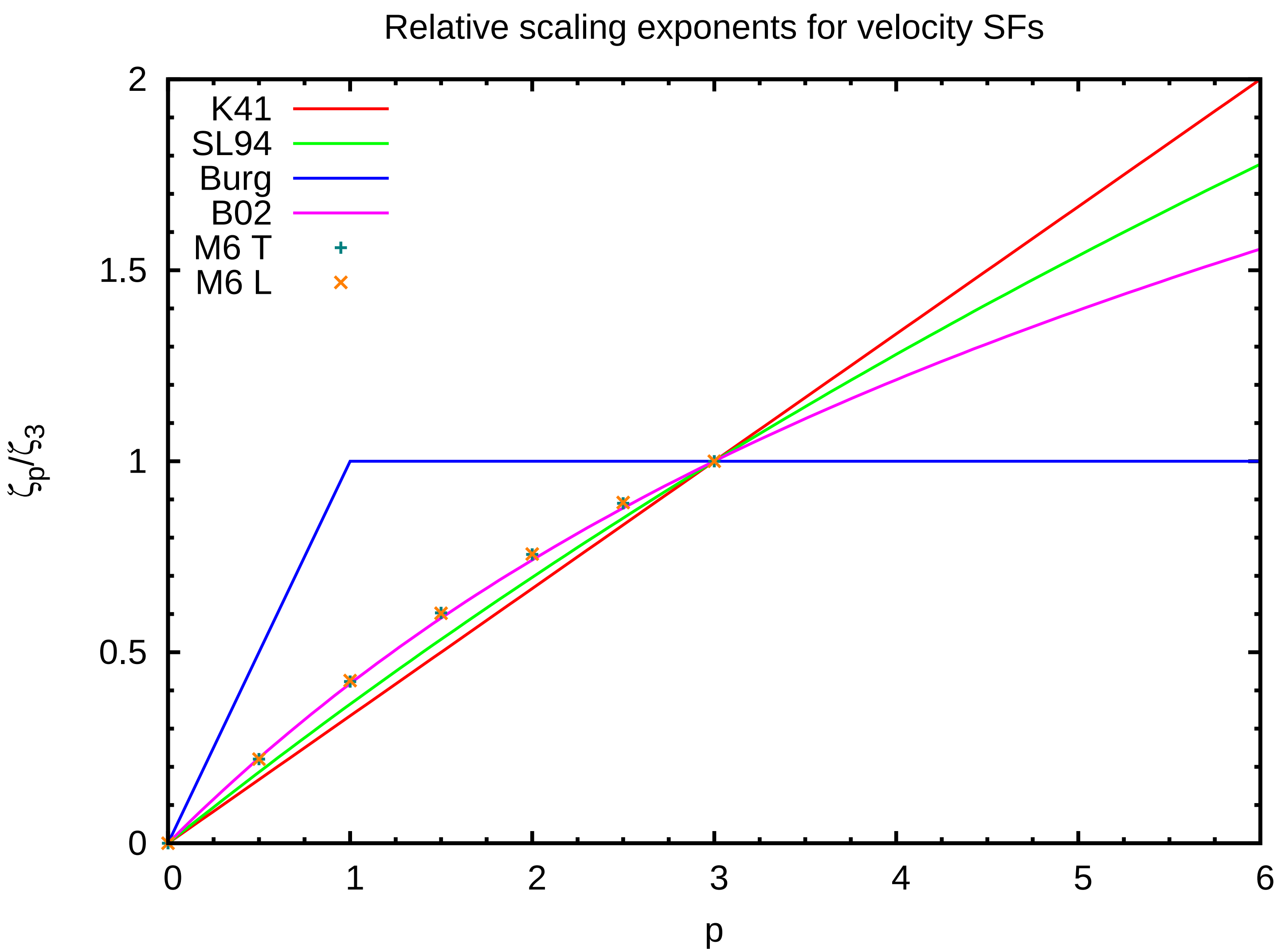
Turbulence Statistics: Models vs. Data

Absolute exponents for supersonic turbulence at Mach 6



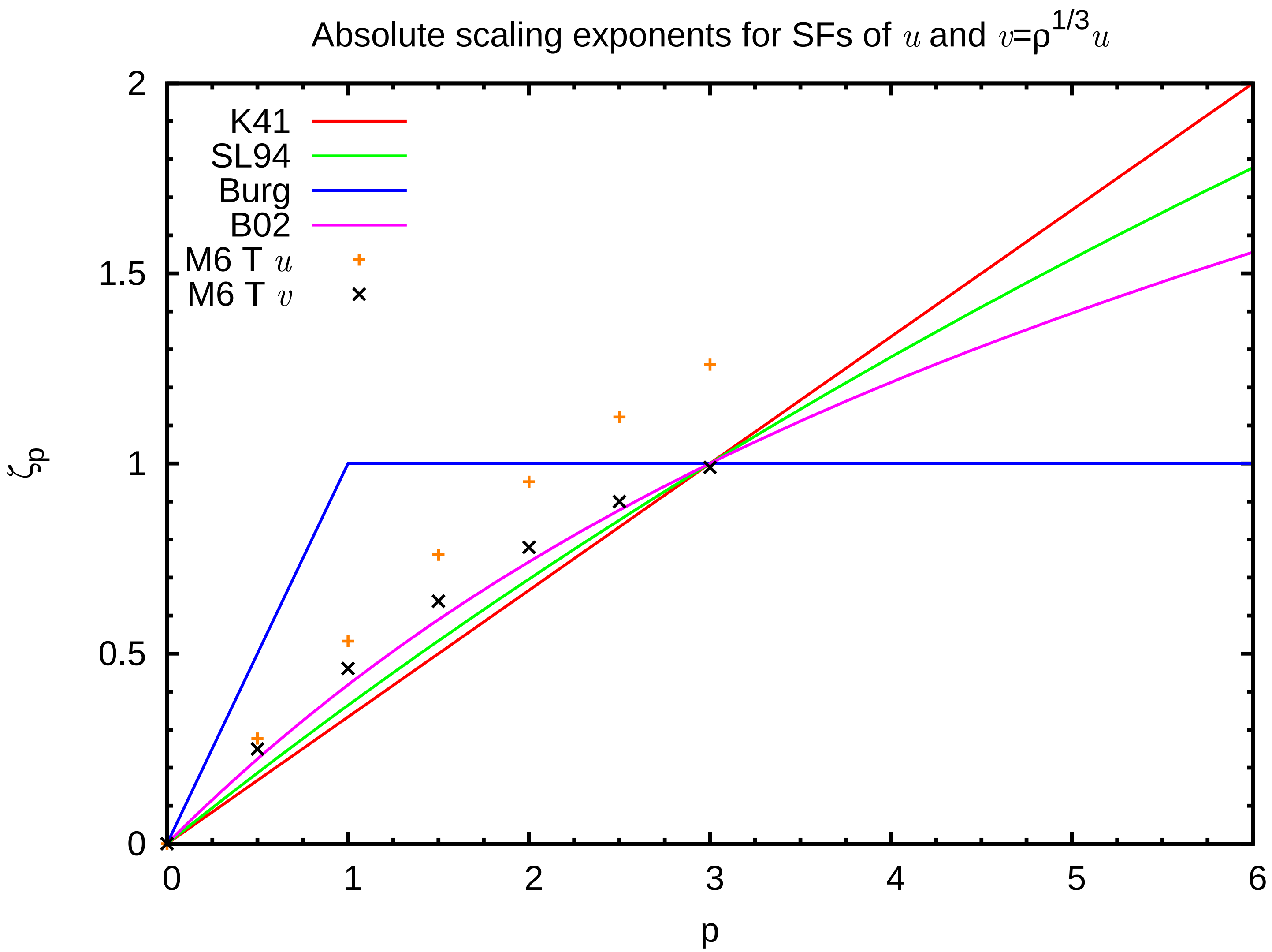
Turbulence Statistics: Models vs. Data

Relative exponents for supersonic turbulence at Mach 6



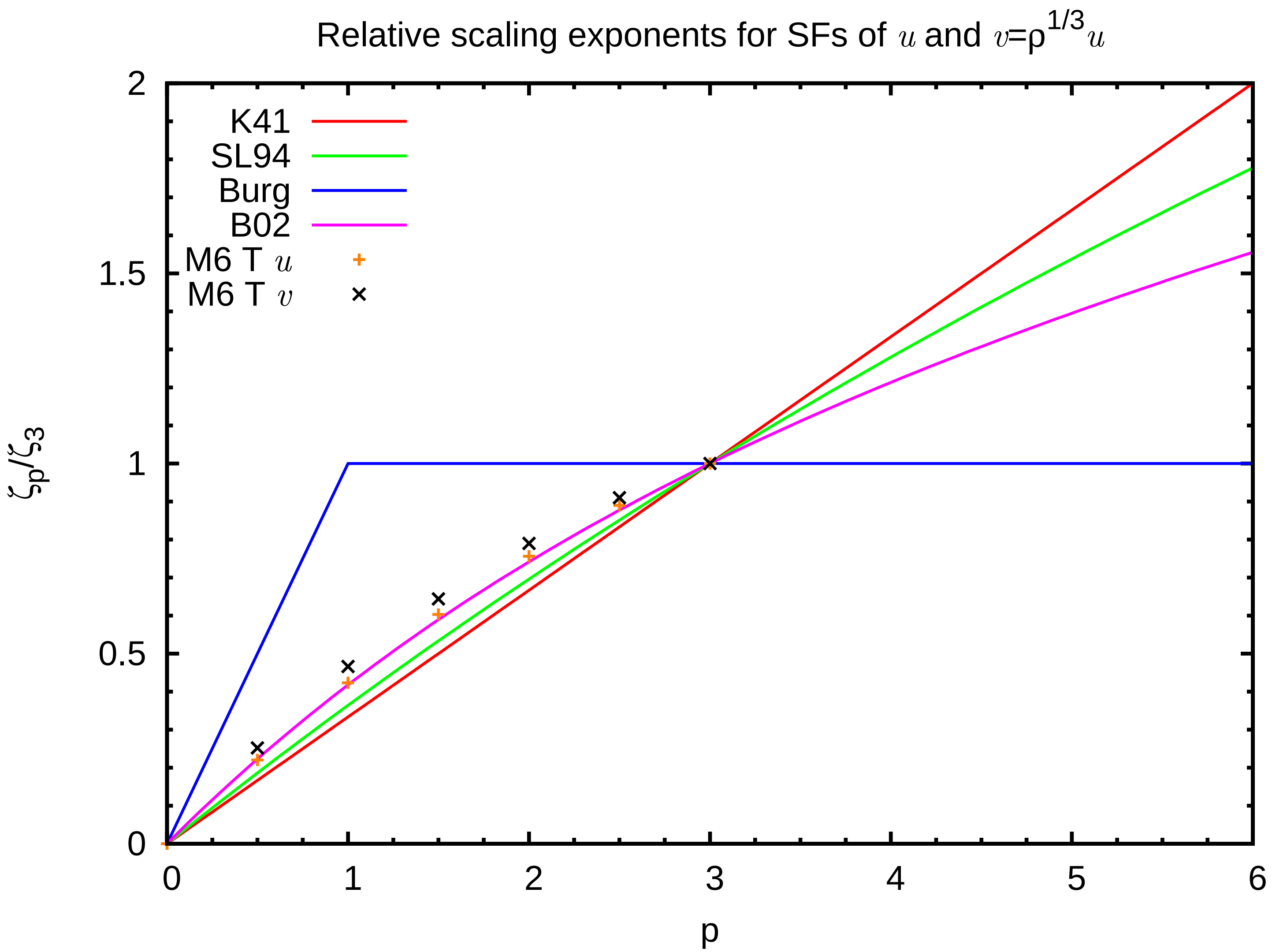
Turbulence Statistics: Models vs. Data

Absolute mixed exponents for supersonic turbulence at Mach 6



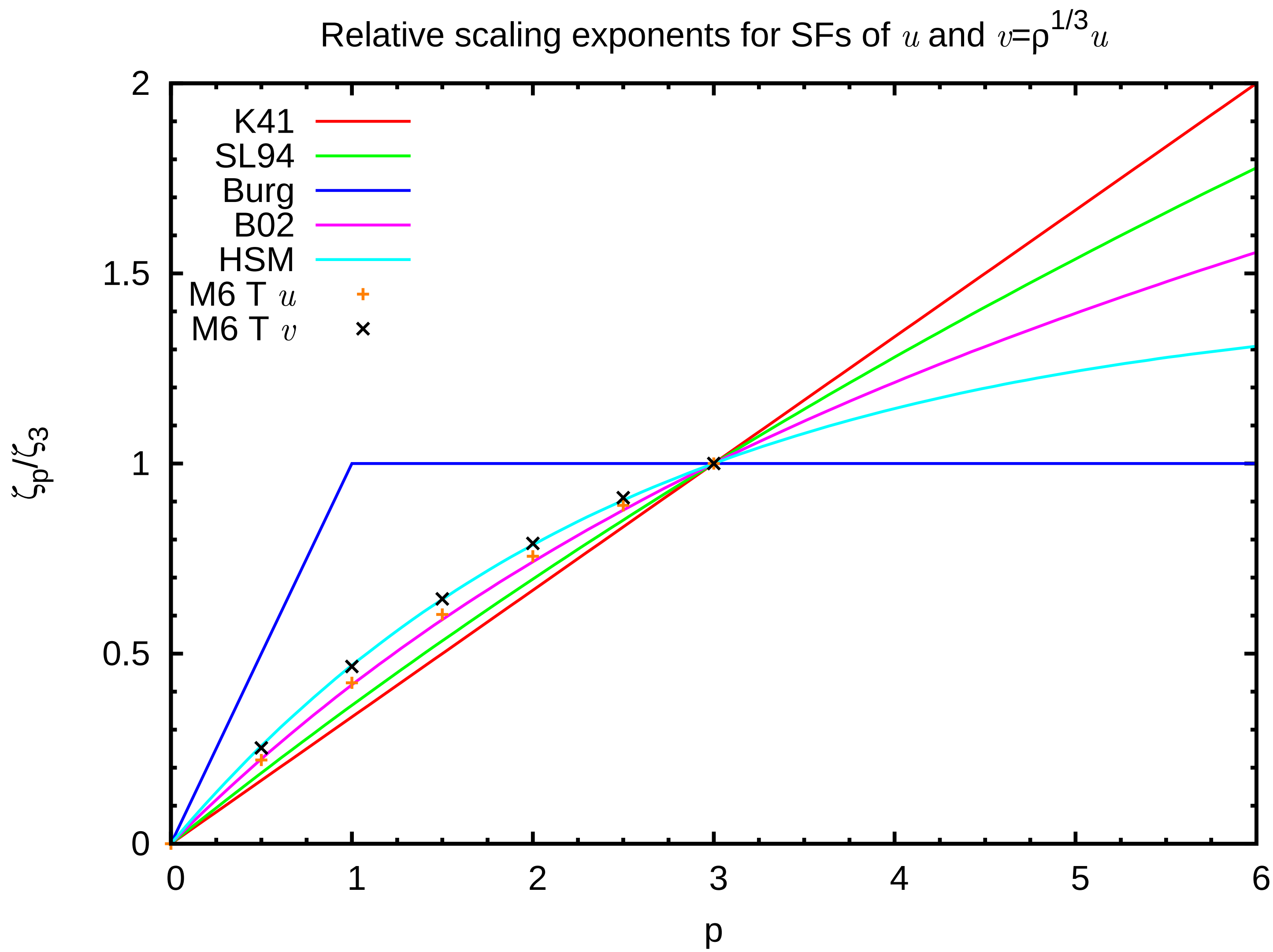
Turbulence Statistics: Models vs. Data

Relative mixed exponents for supersonic turbulence at Mach 6



Turbulence Statistics: Models vs. Data

A model for the mixed exponents at Mach 6



Intermittency Models: Summary

The Hierarchical Structure (HS) model [She & Lévêque 1994] predicts

$$\frac{\zeta_p}{\zeta_3} = \gamma p + C(1 - \eta^p). \quad (7)$$

The **codimension of the support** of the most singular dissipative structures

$$C = (1 - 3\gamma)/(1 - \eta^3). \quad (8)$$

Two parameters: η — a measure of intermittency; γ — a measure of singularity of structures

Model	η^3	γ	C
Kolmogorov (1941)	1	1/3	0^*
She & Lévêque (1994)	2/3	1/9	2
Boldyrev (2002)	1/3	1/9	1
HS1 model, $\boldsymbol{v = \rho u^{1/3}}$	1/3	0	1.5
HS2 model, $\boldsymbol{v = \rho u^{1/3}}$	1/6	1/9	0.8
Burgulence	0	0	1

Summary

- Absolute scaling exponents of supersonic turbulence are measured for the fi rst time.
- Low-order velocity statistics of supersonic turbulence deviate substantially from Kolmogorov's laws for incompressible turbulence. In particular, exponents of the 3rd order velocity structure functions $\zeta_3 > 1$ at Mach 6.
- The fractal dimension of the mass distribution in the inertial range $D_m \approx 2.4$.
- **The mean volume energy transfer rate in compressible turbulent flows, $\rho u^2 u/\ell$, is very close to a constant. Therefore, $\boldsymbol{v} \equiv \rho^{1/3}\boldsymbol{u}$ is the primary variable of interest for such flows.**
- **The statistics of density-weighted velocity $\boldsymbol{v}$ seem to obey the K41 laws in incompressible, nearly incompressible, weakly compressible, compressible and strongly compressible regimes.**