Teleportation of electronic many-qubit states via single photons

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Teleportation of single photon states via EPR pair

Theory: Bennett et al., PRL **70**, 1895 (1993)

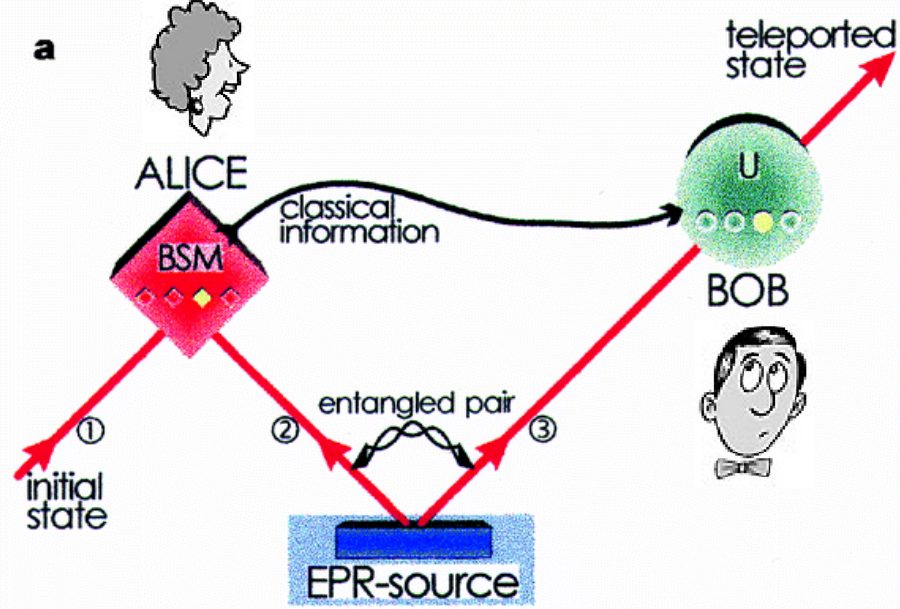
Experiment:
Bouwmeester et al., Nature **390**, 575 (1997)

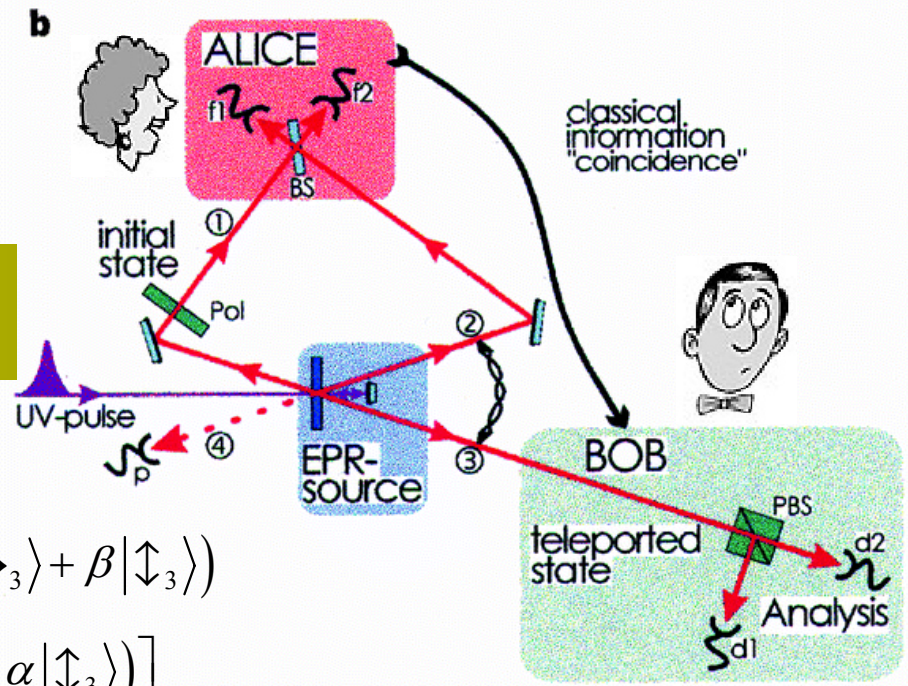
Initial state: $|\psi_1\rangle = \alpha|\leftrightarrow_1\rangle + \beta|\updownarrow_1\rangle$

Bell states: $|\Psi_{23}^{(\pm)}\rangle = \left(|\leftrightarrow\updownarrow\rangle \pm |\updownarrow\leftrightarrow\rangle\right)/\sqrt{2}$

$|\Phi_{23}^{(\pm)}\rangle = \left(|\leftrightarrow\leftrightarrow\rangle \pm |\updownarrow\updownarrow\rangle\right)/\sqrt{2}$

Shared EPR state: $|\Psi_{23}^{(-)}\rangle = \left(|\leftrightarrow\updownarrow\rangle - |\updownarrow\leftrightarrow\rangle\right)/\sqrt{2}$





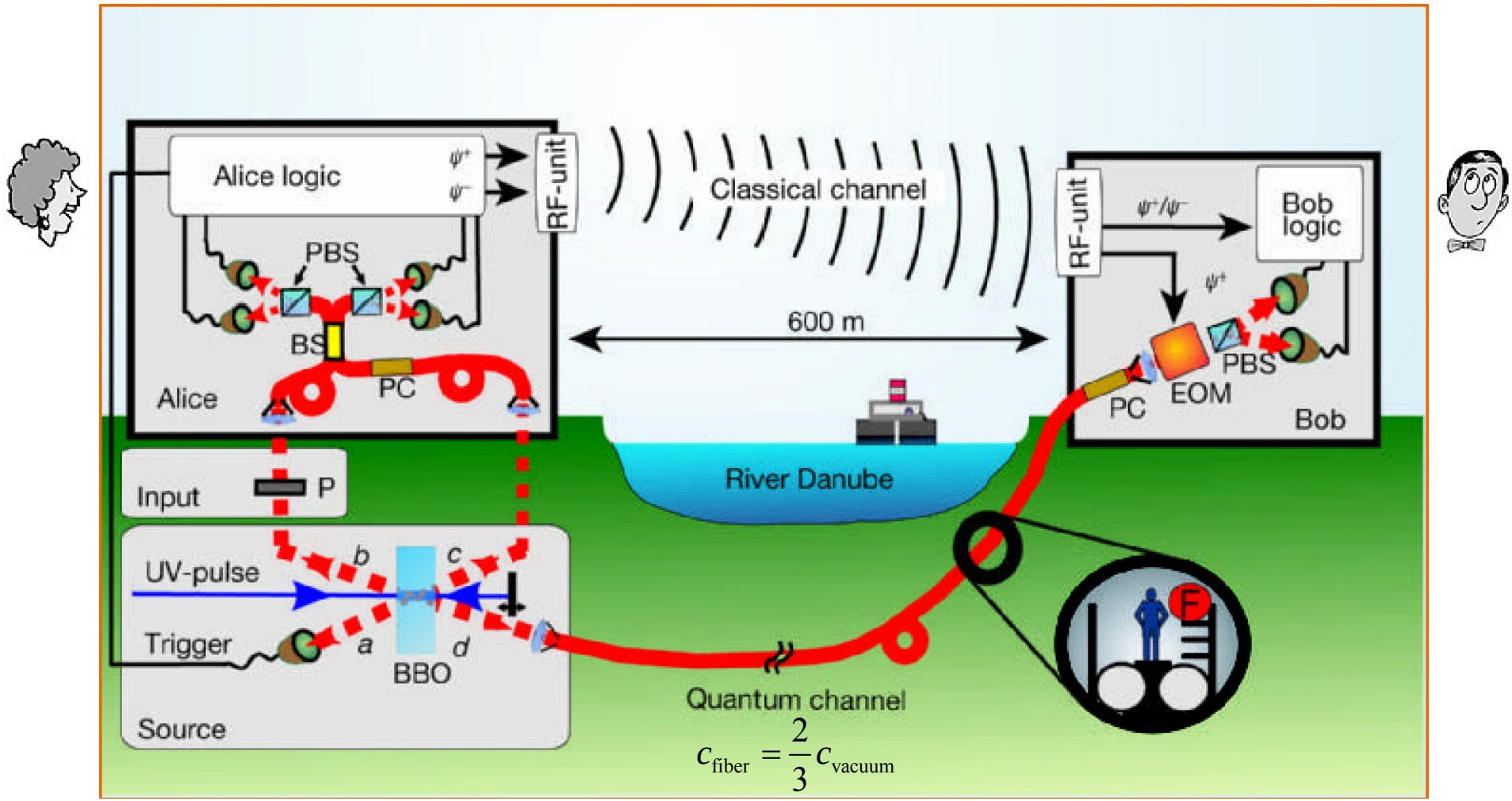
Teleportation by means of Bell measurement (correlation) of photon 1 and 2.

3-photon state: $|\Psi_{123}\rangle = |\psi_1\rangle|\Psi_{23}^{(-)}\rangle$

$$|\Psi_{123}\rangle = \frac{1}{2}\Big[|\Psi_{12}^{(-)}\rangle\left(-\alpha|\leftrightarrow_3\rangle - \beta|\updownarrow_3\rangle\right) + |\Psi_{12}^{(+)}\rangle\left(-\alpha|\leftrightarrow_3\rangle + \beta|\updownarrow_3\rangle\right)$$
$$+|\Phi_{12}^{(-)}\rangle\left(\beta|\leftrightarrow_3\rangle + \alpha|\updownarrow_3\rangle\right) + |\Phi_{12}^{(+)}\rangle\left(-\beta|\leftrightarrow_3\rangle + \alpha|\updownarrow_3\rangle\right)\Big]$$

Quantum teleportation across the river Danube

Ursin et al., Nature **430**, 849 (2004)



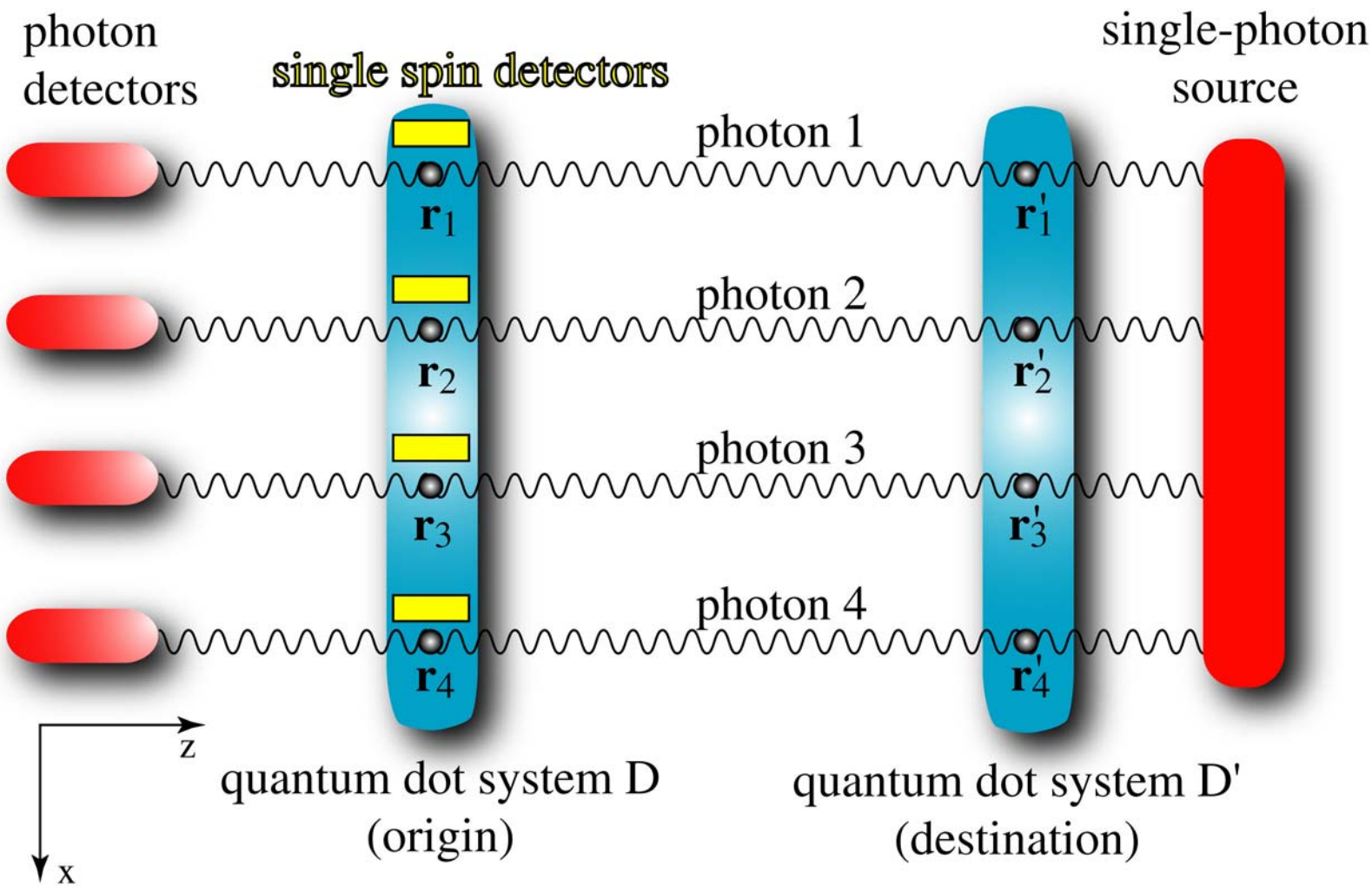
$$c_{\text{fiber}} = \frac{2}{3} c_{\text{vacuum}}$$

The quantum channel (optical fiber F) rests in a sewage-pipe tunnel below the Danube in Vienna with a total length of 800 m. Fidelity>80%.

Teleportation of electronic many-qubit states via single photons

GHZ teleportation method:

- GHZ state of spin-photon-spin system
- No Bell measurements required!
- fully scalable to arbitrarily many qubits



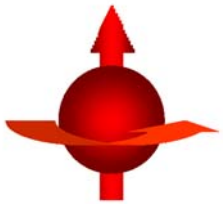
Greenberger-Horne-Zeilinger states (GHZ)

$$\left|\Psi_{\mathrm{GHZ}}\right\rangle = \left(\left|\uparrow\uparrow\uparrow\right\rangle + e^{i\varphi}\left|\downarrow\downarrow\downarrow\right\rangle\right)/\sqrt{2}$$

Preparation of GHZ states by means of interaction

Let us assume Alice measures in z direction (50% probability):

Then Bob measures in z direction (100% probability):



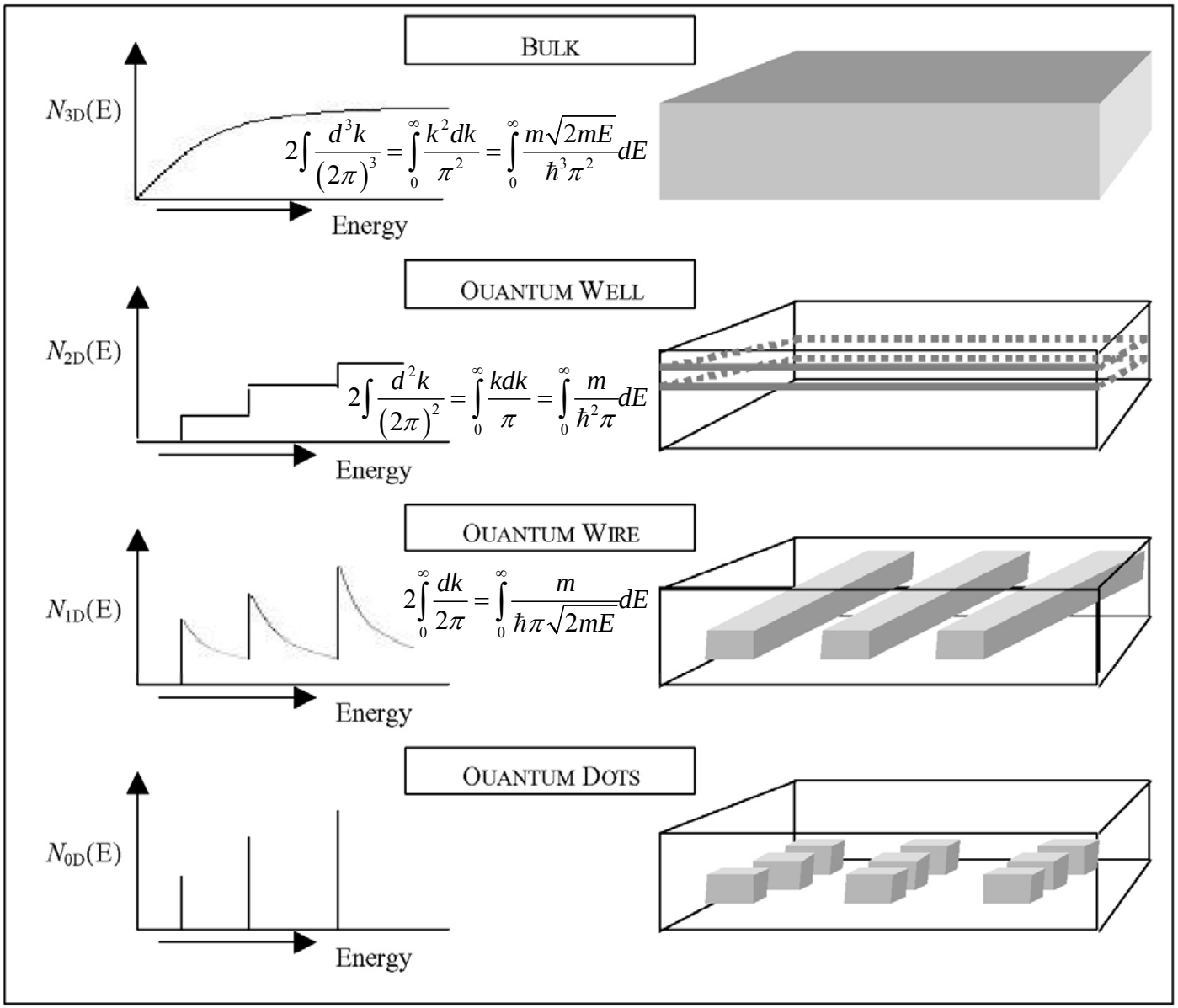
← quantum correlation →
(entanglement)

↑
quantum correlation
(entanglement)
↓

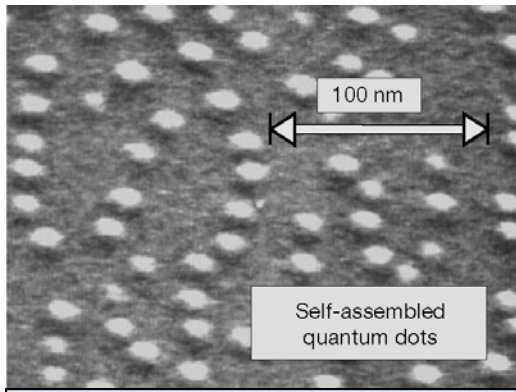
Then Carol measures in z direction (100% probability):

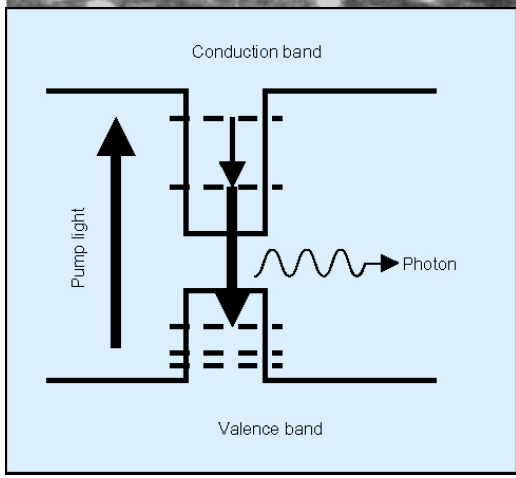
Quantum dots

$$k = \frac{\sqrt{2mE}}{\hbar} \Rightarrow dk = \frac{m}{\hbar\sqrt{2mE}}dE \qquad n = 2\int \frac{d^d k}{(2\pi)^d} f(E(k)) \qquad f(E) = \frac{1}{e^{(E-\mu)/k_B T} + 1}$$



Bayer et al., Nature **405**, 923 (2000):



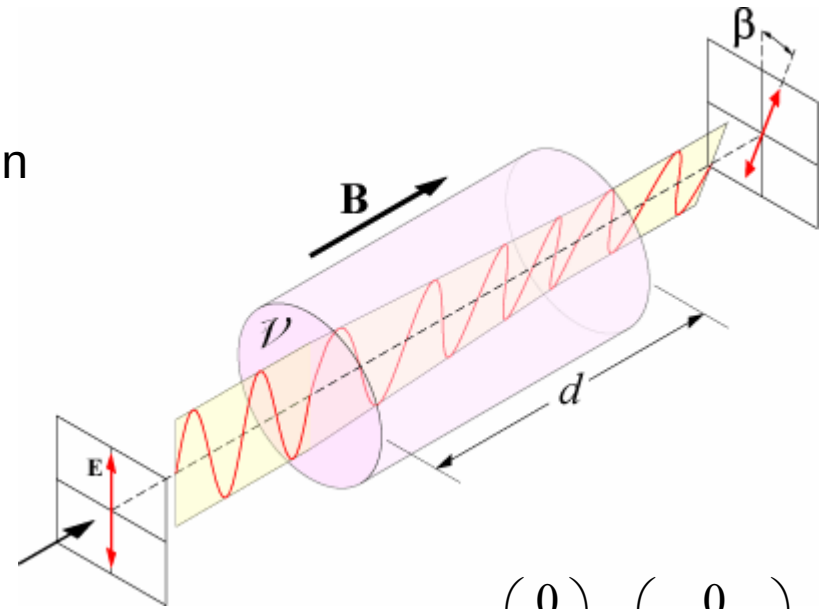


Classical Faraday effect (circular dichroism, circular birefringence)

Faraday rotation angle: $\beta = \nu B d$

Verdet constant is proportional to the difference in index of refraction for right (σ^+) and left (σ^-) circularly polarized light, which is proportional to the magnetization of the material:

$$\nu \propto \eta_{\sigma^+} - \eta_{\sigma^-} \propto M$$



Horizontal linear polarization (↔): $\mathbf{e} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

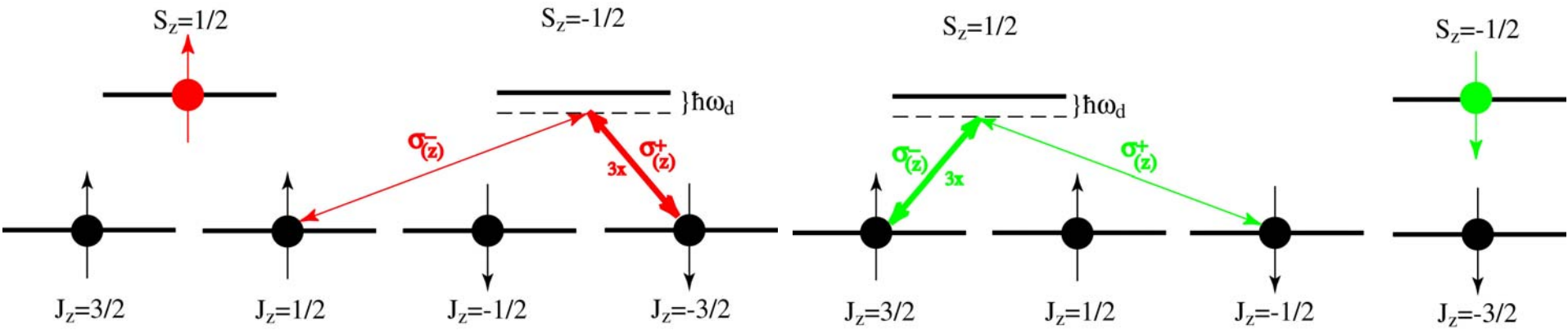
Wavevector of light: $\mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ k_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2\pi / \lambda \end{pmatrix}$

Vertical linear polarization (↕): $\mathbf{e} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

Right circular polarization (σ^+): $\mathbf{e} = \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix}$

Left circular polarization (σ^-): $\mathbf{e} = \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix}$

Optical selection rules
⇒ single-photon Faraday rotation
⇒ entanglement of photon and spin



$\sigma^+_{(z)}$ **virtual process involving a heavy hole**

$\sigma^-_{(z)}$ virtual process involving a light hole

$\sigma^-_{(z)}$ **virtual process involving a heavy hole**

$\sigma^+_{(z)}$ virtual process involving a light hole

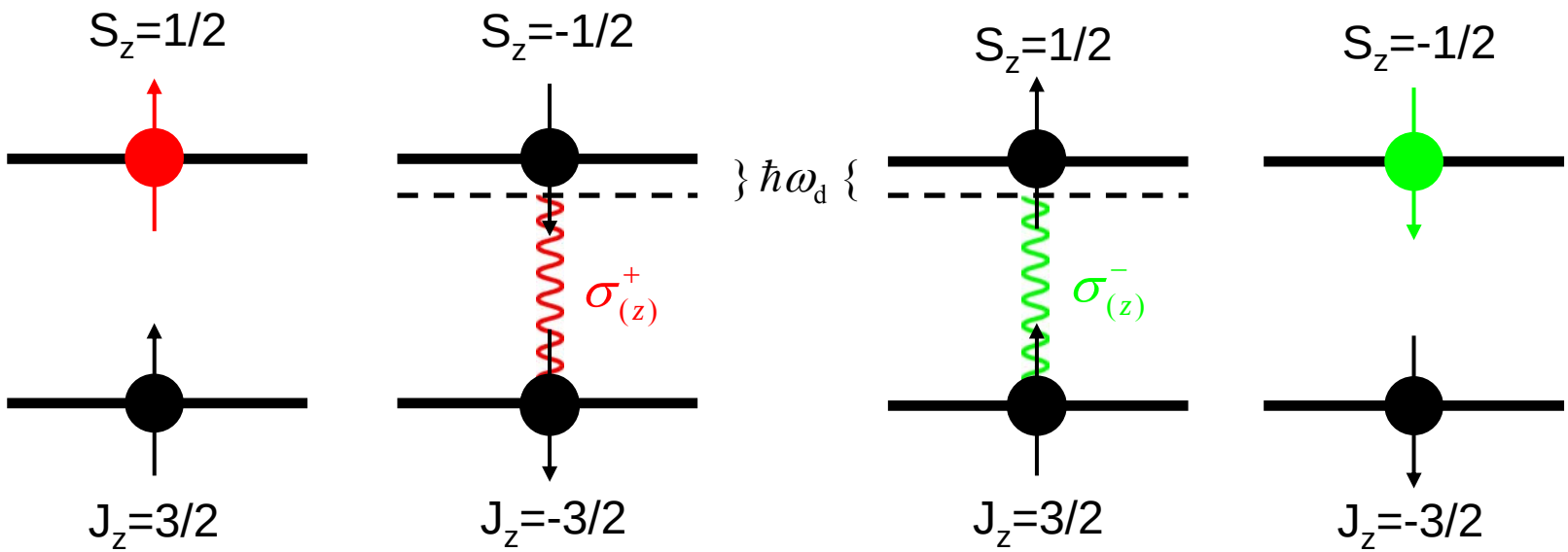
Spherical dots ⇒ 50% selection rules for nonresonant interaction

• Phase shift of photon for virtual process involving heavy holes is 3 times larger than for virtual process involving light holes
⇒ net Faraday rotation of polarization of photon

• Phase shift depends on spin state ⇒ entanglement of photon with spin

Optical selection rules
⇒ single-photon Faraday rotation

Leuenberger, Flatte, Awschalom, Phys. Rev. Lett. **94**, 107401 (2005)



$\sigma^+_{(z)}$ **right circularly polarized photon**

$\sigma^-_{(z)}$ **left circularly polarized photon**

Non-spherical dots (strong anisotropy splits heavy and light holes)
⇒ 100% selection rules for nonresonant interaction ⇒ accelerated Faraday rotation

- Phase shift of photon for virtual process depends on spin state
⇒ Faraday rotation of linear polarization of photon
- Direction of Faraday rotation depends on spin state ⇒ entanglement of photon with spin

Single-qubit teleportation

Spin state of excess electron in quantum dot of destination: $\left|\psi_{\mathrm{e}'}^{(1)}\right\rangle = \left|\leftarrow'\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\uparrow'\right\rangle + \left|\downarrow'\right\rangle\right)$

Incoming photon is linearly polarized in, e.g. x direction: $\left|\psi_{\mathrm{pe}'}^{(1)}(t_A)\right\rangle = \left|\leftrightarrow\right\rangle\left|\leftarrow'\right\rangle$

After interaction of photon with quantum dot: $\left|\psi_{\mathrm{pe}'}^{(1)}(t_A+T)\right\rangle = e^{iS_0^{\mathrm{hh}}}\left|\psi_{\mathrm{hh}}^{(1)}\right\rangle + e^{iS_0^{\mathrm{lh}}}\left|\psi_{\mathrm{lh}}^{(1)}\right\rangle$

Process involving heavy/light hole: $\left|\psi_{\mathrm{hh}}^{(1)}\right\rangle = \left(\left|\sigma_{(z)}^{+}\right\rangle\left|\uparrow'\right\rangle + \left|\sigma_{(z)}^{-}\right\rangle\left|\downarrow'\right\rangle\right)/2 \quad \left|\psi_{\mathrm{lh}}^{(1)}\right\rangle = \left(\left|\sigma_{(z)}^{-}\right\rangle\left|\uparrow'\right\rangle + \left|\sigma_{(z)}^{+}\right\rangle\left|\downarrow'\right\rangle\right)/2$

Conditional phase shift for maximum entanglement: $S_0 = S_0^{\mathrm{hh}} - S_0^{\mathrm{lh}} = \pi/2$

The photon is now sent to origin D.
Photon at D and spin at D‘ are in a Bell state (EPR pair):

$$\left|\psi_{\mathrm{pe}'}^{(1)}(t_A+T)\right\rangle = \left(\left|\leftrightarrow\right\rangle\left|\uparrow'\right\rangle + \left|\leftrightarrow\right\rangle\left|\downarrow'\right\rangle\right)/\sqrt{2}$$

(similar to distribution of EPR pair produced by e.g. parametric down-conversion in traditional EPR teleportation)

Preparation of spin at D: $\left|\psi_{\mathrm{e}}^{(1)}\right\rangle = \alpha\left|\uparrow\right\rangle + \beta\left|\downarrow\right\rangle$

When teleporting a qubit from D to D‘ is desired, photon at D interacts with spin at D:

$$\left|\psi_{\mathrm{epe}'}^{(1)}(t_C)\right\rangle = \left(\alpha\left|\uparrow\right\rangle\left|\nwarrow\!\!\!\searrow\right\rangle\left|\uparrow'\right\rangle + \alpha\left|\uparrow\right\rangle\left|\swarrow\!\!\!\nearrow\right\rangle\left|\downarrow'\right\rangle + \beta\left|\downarrow\right\rangle\left|\nwarrow\!\!\!\searrow\right\rangle\left|\uparrow'\right\rangle + \beta\left|\downarrow\right\rangle\left|\swarrow\!\!\!\nearrow\right\rangle\left|\downarrow'\right\rangle\right)/\sqrt{2}$$

$\Rightarrow$ Formation of GHZ state of spin at D, photon, and spin at D‘

Collapse of the GHZ state

Traditional teleportation: - No interaction of spin at D and photon

Theory: Bennett et al., PRL **70**, 1895 (1993) - Bell measurement of spin at D and photon

Experiment: Bouwmeester et al., Nature **390**, 575 (1997)

Teleportation scheme here: Single-particle measurement of spin at D and photon
$\Leftrightarrow$ collapse of GHZ state

$$\left|\psi^{(1)}_{\mathrm{epe'}}(t_C+T)\right\rangle = \left[\left|\updownarrow\right\rangle\left(-\alpha\left|\uparrow\right\rangle\left|\uparrow'\right\rangle+\beta\left|\downarrow\right\rangle\left|\downarrow'\right\rangle\right)+\left|\leftrightarrow\right\rangle\left(\alpha\left|\uparrow\right\rangle\left|\downarrow'\right\rangle+\beta\left|\downarrow\right\rangle\left|\uparrow'\right\rangle\right)\right]/\sqrt{2}$$

$$=\left\{\left|\updownarrow\right\rangle\left[\left|\leftarrow\right\rangle\left(-\alpha\left|\uparrow'\right\rangle+\beta\left|\downarrow'\right\rangle\right)+\left|\rightarrow\right\rangle\left(-\alpha\left|\uparrow'\right\rangle-\beta\left|\downarrow'\right\rangle\right)\right]\right.$$

$$\left.+\left|\leftrightarrow\right\rangle\left[\left|\leftarrow\right\rangle\left(\beta\left|\uparrow'\right\rangle+\alpha\left|\downarrow'\right\rangle\right)+\left|\rightarrow\right\rangle\left(\beta\left|\uparrow'\right\rangle-\alpha\left|\downarrow'\right\rangle\right)\right]\right\}/\sqrt{2}$$

Measurement of linear polarization of photon:

$$\left|\psi^{(1)}_{\mathrm{ee'1}}(t_D)\right\rangle=-\alpha\left|\uparrow\right\rangle\left|\uparrow'\right\rangle+\beta\left|\downarrow\right\rangle\left|\downarrow'\right\rangle \quad \text{or} \quad \left|\psi^{(1)}_{\mathrm{ee'2}}(t_D)\right\rangle=\alpha\left|\uparrow\right\rangle\left|\downarrow'\right\rangle+\beta\left|\downarrow\right\rangle\left|\uparrow'\right\rangle$$

Single-spin detection of D in x direction (possible with a single photon) $\Rightarrow$

$$\left|\psi^{(1)}_{\mathrm{e'1}}(t_D)\right\rangle=-\alpha\left|\uparrow'\right\rangle+\beta\left|\downarrow'\right\rangle \quad \text{or} \quad \left|\psi^{(1)}_{\mathrm{e'3}}(t_D)\right\rangle=\beta\left|\uparrow'\right\rangle+\alpha\left|\downarrow'\right\rangle$$

$$\text{or} \quad \left|\psi^{(1)}_{\mathrm{e'2}}(t_D)\right\rangle=-\alpha\left|\uparrow'\right\rangle-\beta\left|\downarrow'\right\rangle \quad \text{or} \quad \left|\psi^{(1)}_{\mathrm{e'4}}(t_D)\right\rangle=\beta\left|\uparrow'\right\rangle-\alpha\left|\downarrow'\right\rangle$$

Knowledge of detection outcome $\Rightarrow$ teleportation from D to D‘ complete

Nonperturbative calculation of the single-photon Faraday rotation

Full Hamiltonian: $H = H_L + V$

Luttinger-Kohn Hamiltonian for spherical dots: $H_L = (\gamma_1 + \frac{5}{2}\gamma)\frac{p^2}{2m_0} - \frac{\gamma}{m_0}(\mathbf{pJ})^2$

Interaction Hamiltonian: $V = \frac{e}{2mc}\mathbf{A}\cdot\mathbf{p}$

Vector potential of electromagnetic field of photon: $\mathbf{A}(t) = \tilde{A}(t)\sum_\sigma \boldsymbol{\varepsilon}_{q\sigma}\left(a_{q\sigma}e^{-i\omega t} - a^\dagger_{q\sigma}e^{+i\omega t}\right)$

$V_{\mathrm{hh}} = \langle I;0|V|g;i\rangle = \frac{e}{mc}A_0 p_{cv} = \sqrt{3}V_{\mathrm{lh}} = 50\ \mu\mathrm{eV}$

Full Hamiltonian in rotating frame: $H = \begin{pmatrix} E_0 & V_{\mathrm{hh}} & 0 & 0 \\ V_{\mathrm{hh}} & E_0 + \hbar\omega_{\mathrm{d}} & 0 & 0 \\ 0 & 0 & E_0 & V_{\mathrm{lh}} \\ 0 & 0 & V_{\mathrm{lh}} & E_0 + \hbar\omega_{\mathrm{d}} \end{pmatrix}$

$E_{\mathrm{gap}} = \hbar(\omega + \omega_{\mathrm{d}}) = 1.4\ \mathrm{eV}$

$1.5\ \mathrm{meV} = \hbar\omega_{\mathrm{d}} \gg \hbar\Gamma = 10\ \mu\mathrm{eV}$

Basis states: $|\uparrow\rangle|\sigma^+_{(z)}\rangle; |\uparrow, \text{hh-exciton}\rangle; |\uparrow\rangle|\sigma^-_{(z)}\rangle; |\uparrow, \text{lh-exciton}\rangle$

Initial state: $|\psi(t_A)\rangle = |\uparrow\rangle|\leftrightarrow\rangle = \left(|\uparrow\rangle|\sigma^+_{(z)}\rangle + |\uparrow\rangle|\sigma^-_{(z)}\rangle\right)/\sqrt{2}$

$V_{\mathrm{lh}}/\hbar\omega_{\mathrm{d}} \ll 1 \Rightarrow |\uparrow\rangle|\sigma^+_{(z)}\rangle$ and $|\uparrow\rangle|\sigma^-_{(z)}\rangle$ remain fully populated.

After interaction time *T*: $|\psi(t_A + T)\rangle = e^{-\frac{i}{\hbar}E_0 T}\left[e^{-i\Omega_{\mathrm{hh}}T}|\uparrow\rangle|\sigma^+_{(z)}\rangle + e^{-i\Omega_{\mathrm{lh}}T}|\uparrow\rangle|\sigma^-_{(z)}\rangle\right]/\sqrt{2}$

Interaction frequencies: $\Omega_{\mathrm{hh}} = \frac{V_{\mathrm{hh}}^2}{\hbar^2\omega_{\mathrm{d}}} = \frac{3V_{\mathrm{lh}}^2}{\hbar^2\omega_{\mathrm{d}}}$ $\quad \Omega_{\mathrm{lh}} = \frac{V_{\mathrm{lh}}^2}{\hbar^2\omega_{\mathrm{d}}}$ $\quad T = 1\ \mathrm{ns}$

$\Rightarrow$ phase shifts are different: $S_0^{\mathrm{hh}} = \Omega_{\mathrm{hh}}T$ $\quad S_0^{\mathrm{lh}} = \Omega_{\mathrm{lh}}T$ $\quad \Rightarrow$ Faraday rotation!

Single-photon Faraday rotation

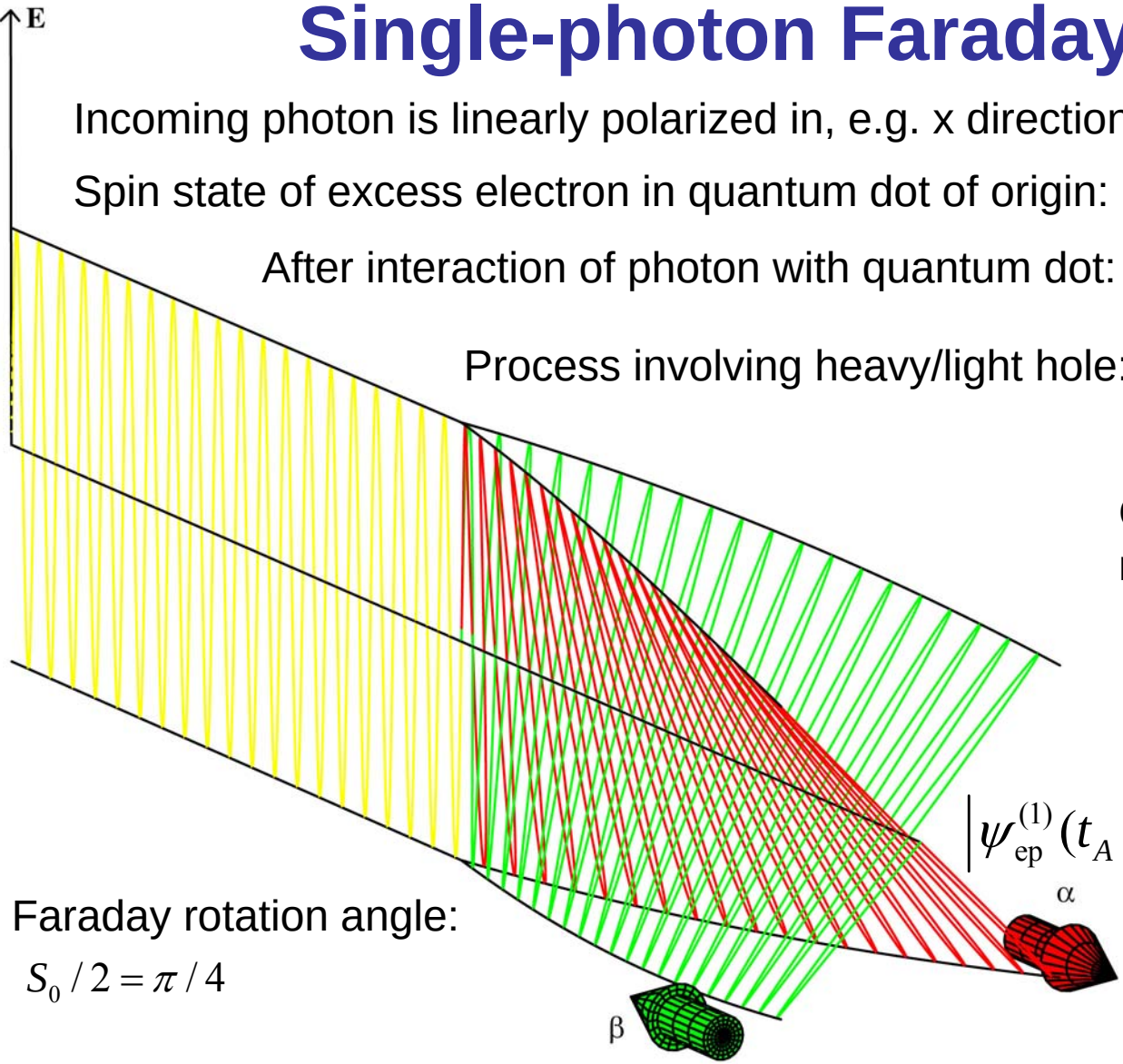
Incoming photon is linearly polarized in, e.g. x direction: $\left|\psi_{\mathrm{ep}}^{(1)}(t_A)\right\rangle = \left(\alpha\left|\uparrow\right\rangle + \beta\left|\downarrow\right\rangle\right)\left|\leftrightarrow\right\rangle$

Spin state of excess electron in quantum dot of origin: $\left|\psi_{\mathrm{e}}^{(1)}\right\rangle = \alpha\left|\uparrow\right\rangle + \beta\left|\downarrow\right\rangle$

After interaction of photon with quantum dot: $\left|\psi_{\mathrm{ep}}^{(1)}(t_A+T)\right\rangle = e^{iS_0^{\mathrm{hh}}}\left|\psi_{\mathrm{hh}}^{(1)}\right\rangle + e^{iS_0^{\mathrm{lh}}}\left|\psi_{\mathrm{lh}}^{(1)}\right\rangle$

Process involving heavy/light hole: $\left|\psi_{\mathrm{hh}}^{(1)}\right\rangle = \left(\alpha\left|\uparrow\right\rangle\left|\sigma_{(z)}^{+}\right\rangle + \beta\left|\downarrow\right\rangle\left|\sigma_{(z)}^{-}\right\rangle\right)/\sqrt{2}$

$\left|\psi_{\mathrm{lh}}^{(1)}\right\rangle = \left(\alpha\left|\uparrow\right\rangle\left|\sigma_{(z)}^{-}\right\rangle + \beta\left|\downarrow\right\rangle\left|\sigma_{(z)}^{+}\right\rangle\right)/\sqrt{2}$



Conditional phase shift for maximum entanglement:

$$S_0 = S_0^{\mathrm{hh}} - S_0^{\mathrm{lh}} = \pi/2$$

$\Rightarrow$ Faraday rotation of photon polarization:

$$\left|\psi_{\mathrm{ep}}^{(1)}(t_A+T)\right\rangle = \alpha\left|\uparrow\right\rangle\left|\leftrightarrow\right\rangle + \beta\left|\downarrow\right\rangle\left|\leftrightarrow\right\rangle$$

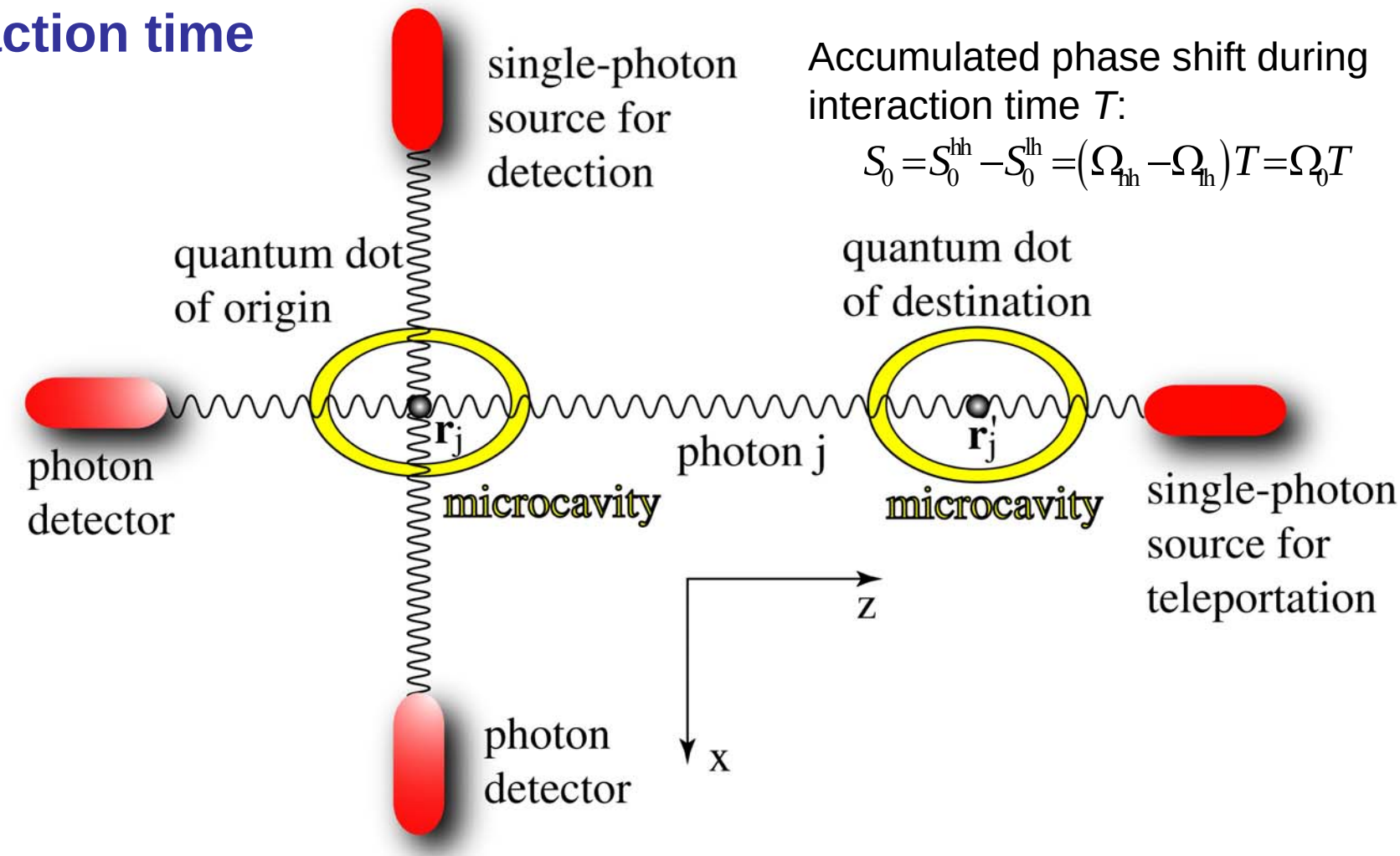
Faraday rotation angle:

$S_0/2 = \pi/4$

Measurement of spin in x direction $\Rightarrow$ photon projected onto $\left|\psi_{\mathrm{p}}^{(1)}(t_B)\right\rangle = \alpha\left|\nwarrow\!\!\searrow\right\rangle \pm \beta\left|\swarrow\!\!\nearrow\right\rangle$

$\Rightarrow$ optospintronic link between photons and electron spins

Enclosing dot in a microcavity permits precise control of the interaction time



Accumulated phase shift during interaction time *T*:

$$S_0 = S_0^{hh} - S_0^{lh} = (\Omega_{hh} - \Omega_{lh})T = \Omega_0 T$$

$3.5\ \mu\mathrm{m}^3$ volume cavity $\Rightarrow$ $\Omega_0 = \frac{\pi}{2} \times 10^9\ \mathrm{s}^{-1}$ and $T = 1\ \mathrm{ns}$ $\Rightarrow$ $S_0 = \pi/2$ maximum entanglement

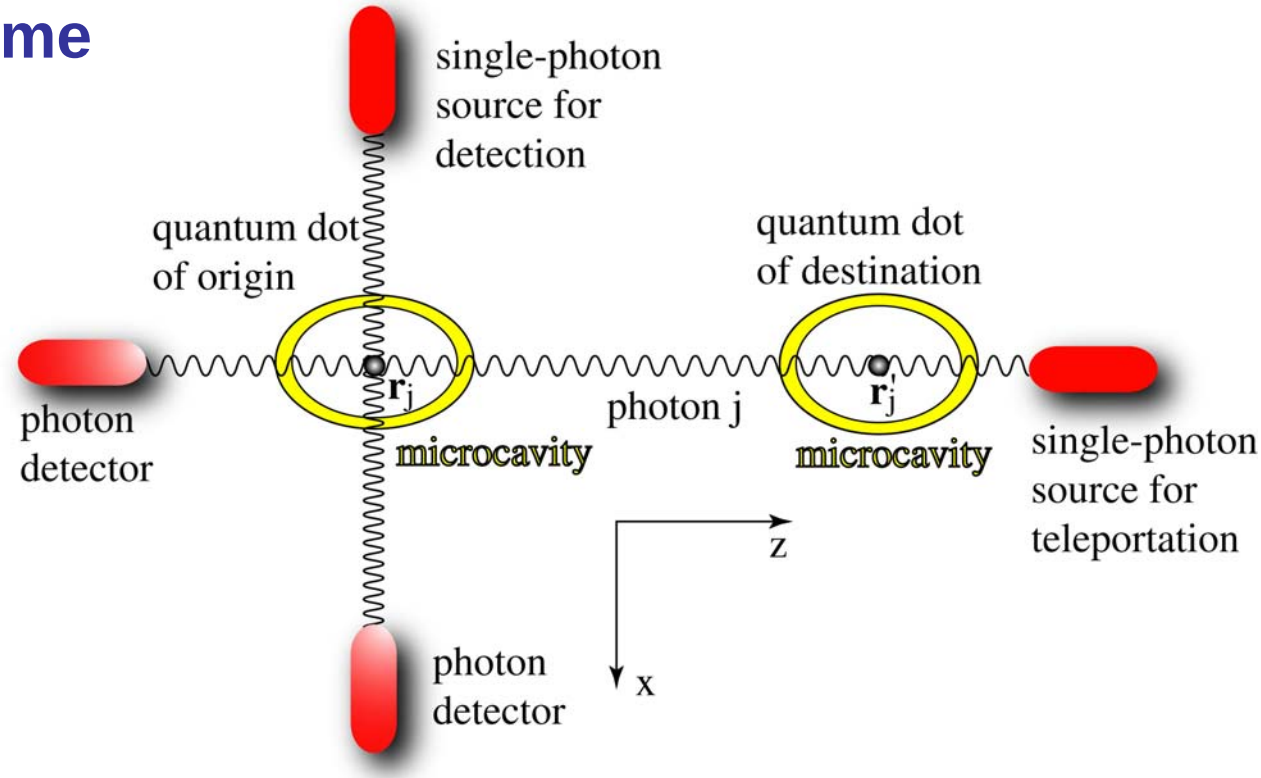
T controlled by active *Q*-switching with 1 ps resolution $\Rightarrow$ phase error 1 ps/ 1 ns = 0.1%

$Q = 1.25 \times 10^8$ $\Rightarrow$ $\tau = 43\ \mathrm{ns}$ $\Rightarrow$ escape probability $1 - e^{-T/\tau} = 2\%$

$Q = 10^7$ $\Rightarrow$ $1 - e^{-T/\tau} = 4\%$

$Q = 10^6$ $\Rightarrow$ $1 - e^{-T/\tau} = 37\%$

Enclosing dot in a microcavity permits precise control of the interaction time



$0.04\ \mu\mathrm{m}^3$ volume cavity $\Rightarrow$ $T = 50\ \mathrm{ps} \Rightarrow S_0 = \pi/2$ maximum entanglement

Naruse et al., Appl. Phys. Lett. **83**, 4869 (2003):

T controlled by active *Q*-switching with 100 fs resolution $\Rightarrow$ phase error 100 fs/ 50 ps = 10^{-3}

Phase error due to bandwidth of photon: $\dfrac{\Gamma_{\mathrm{photon}} T/4\hbar}{\hbar\omega_{\mathrm{d}} T/4\hbar} = \dfrac{\Gamma_{\mathrm{photon}}}{\hbar\omega_{\mathrm{d}}} = 1.6\times10^{-4}$

In 2D photonic crystal: Lodahl et al., Nature **430**, 654 (2004): $\tau = 20\ \mathrm{ns}$

$\Rightarrow$ escape probability $1 - e^{-T/\tau} = 2.6\times10^{-3}$ $\tau = 700\ \mathrm{ns} \Rightarrow 1 - e^{-T/\tau} = 0.7\times10^{-4}$

Two-qubit teleportation

Two-qubit state in quantum dots of D‘: $\left|\psi_{\mathrm{e}'}^{(2)}(t_A)\right\rangle = \left|\leftarrow'\right\rangle\left|\leftarrow'\right\rangle$

Incoming photons linearly polarized: $\left|\psi_{\mathrm{pe}'}^{(1)}(t_A)\right\rangle = \left|\leftrightarrow\right\rangle\left|\leftrightarrow\right\rangle\left|\leftarrow'\right\rangle\left|\leftarrow'\right\rangle$

After interaction of photons with qubits: $\left|\psi_{\mathrm{pe}'}^{(2)}(t_A+T)\right\rangle = e^{2iS_0^{\mathrm{hh}}}\left|\psi_{\mathrm{hh}}^{(2)}\right\rangle + e^{i\left(S_0^{\mathrm{hh}}+S_0^{\mathrm{lh}}\right)}\left|\psi_{\mathrm{hh,lh}}^{(2)}\right\rangle + e^{2iS_0^{\mathrm{lh}}}\left|\psi_{\mathrm{lh}}^{(2)}\right\rangle$

Both photon interactions involve heavy holes:

$$\left|\psi_{\mathrm{hh}}^{(2)}\right\rangle = \left(\left|\sigma_{(z)}^{+}\sigma_{(z)}^{+}\right\rangle\left|\uparrow'\uparrow'\right\rangle + \left|\sigma_{(z)}^{+}\sigma_{(z)}^{-}\right\rangle\left|\uparrow'\downarrow'\right\rangle + \left|\sigma_{(z)}^{-}\sigma_{(z)}^{+}\right\rangle\left|\downarrow'\uparrow'\right\rangle + \left|\sigma_{(z)}^{-}\sigma_{(z)}^{-}\right\rangle\left|\downarrow'\downarrow'\right\rangle\right)/4$$

One photon interaction involves a heavy hole, the other photon interaction a light hole:

$$\left|\psi_{\mathrm{hh,lh}}^{(2)}\right\rangle = \left[\left(\left|\sigma_{(z)}^{+}\sigma_{(z)}^{-}\right\rangle + \left|\sigma_{(z)}^{-}\sigma_{(z)}^{+}\right\rangle\right)\left(\left|\uparrow'\uparrow'\right\rangle + \left|\downarrow'\downarrow'\right\rangle\right) + \left(\left|\sigma_{(z)}^{+}\sigma_{(z)}^{+}\right\rangle + \left|\sigma_{(z)}^{-}\sigma_{(z)}^{-}\right\rangle\right)\left(\left|\uparrow'\downarrow'\right\rangle + \left|\downarrow'\uparrow'\right\rangle\right)\right]/4$$

Both photon interactions involve light holes:

$$\left|\psi_{\mathrm{lh}}^{(2)}\right\rangle = \left(\left|\sigma_{(z)}^{-}\sigma_{(z)}^{-}\right\rangle\left|\uparrow'\uparrow'\right\rangle + \left|\sigma_{(z)}^{-}\sigma_{(z)}^{+}\right\rangle\left|\uparrow'\downarrow'\right\rangle + a_{-+}\left|\sigma_{(z)}^{+}\sigma_{(z)}^{-}\right\rangle\left|\downarrow'\uparrow'\right\rangle + a_{--}\left|\sigma_{(z)}^{+}\sigma_{(z)}^{+}\right\rangle\left|\downarrow'\downarrow'\right\rangle\right)/4$$

Conditional phase shift for maximum entanglement: $S_0 = S_0^{\mathrm{hh}} - S_0^{\mathrm{lh}} = \pi/2$

$\Rightarrow$ Faraday rotation of photon polarization:

$$\left|\psi_{\mathrm{pe}'}^{(2)}(t_A+T)\right\rangle = \left(\left|\leftrightarrow\leftrightarrow\right\rangle\left|\uparrow'\uparrow'\right\rangle + \left|\leftrightarrow\leftrightarrow\right\rangle\left|\uparrow'\downarrow'\right\rangle + \left|\leftrightarrow\leftrightarrow\right\rangle\left|\downarrow'\uparrow'\right\rangle + \left|\leftrightarrow\leftrightarrow\right\rangle\left|\downarrow'\downarrow'\right\rangle\right)/2$$

Formation of two entangled GHZ states

Two-qubit state in quantum dots of origin: $\left|\psi_{\mathrm{e}}^{(2)}(t_C)\right\rangle = a_{\uparrow\uparrow}\left|\uparrow\uparrow\right\rangle + a_{\uparrow\downarrow}\left|\uparrow\downarrow\right\rangle + a_{\downarrow\uparrow}\left|\downarrow\uparrow\right\rangle + a_{\downarrow\downarrow}\left|\downarrow\downarrow\right\rangle$

Each photon interacts with each spin at D separately:

$$\begin{aligned}
\left|\psi_{\mathrm{epe'}}^{(2)}(t_C)\right\rangle = &\Big(a_{\uparrow\uparrow}\left|\uparrow\uparrow\right\rangle\left|\nwarrow\!\!\!\!\searrow\ \nwarrow\!\!\!\!\searrow\right\rangle\left|\uparrow'\uparrow'\right\rangle + a_{\uparrow\uparrow}\left|\uparrow\uparrow\right\rangle\left|\nwarrow\!\!\!\!\searrow\ \swarrow\!\!\!\!\nearrow\right\rangle\left|\uparrow'\downarrow'\right\rangle + a_{\uparrow\uparrow}\left|\uparrow\uparrow\right\rangle\left|\swarrow\!\!\!\!\nearrow\ \nwarrow\!\!\!\!\searrow\right\rangle\left|\downarrow'\uparrow'\right\rangle + a_{\uparrow\uparrow}\left|\uparrow\uparrow\right\rangle\left|\swarrow\!\!\!\!\nearrow\ \swarrow\!\!\!\!\nearrow\right\rangle\left|\downarrow'\downarrow'\right\rangle\Big)/2 \\
&+\Big(a_{\uparrow\downarrow}\left|\uparrow\downarrow\right\rangle\left|\nwarrow\!\!\!\!\searrow\ \nwarrow\!\!\!\!\searrow\right\rangle\left|\uparrow'\uparrow'\right\rangle + a_{\uparrow\downarrow}\left|\uparrow\downarrow\right\rangle\left|\nwarrow\!\!\!\!\searrow\ \swarrow\!\!\!\!\nearrow\right\rangle\left|\uparrow'\downarrow'\right\rangle + a_{\uparrow\downarrow}\left|\uparrow\downarrow\right\rangle\left|\swarrow\!\!\!\!\nearrow\ \nwarrow\!\!\!\!\searrow\right\rangle\left|\downarrow'\uparrow'\right\rangle + a_{\uparrow\downarrow}\left|\uparrow\downarrow\right\rangle\left|\swarrow\!\!\!\!\nearrow\ \swarrow\!\!\!\!\nearrow\right\rangle\left|\downarrow'\downarrow'\right\rangle\Big)/2 \\
&+\Big(a_{\downarrow\uparrow}\left|\downarrow\uparrow\right\rangle\left|\nwarrow\!\!\!\!\searrow\ \nwarrow\!\!\!\!\searrow\right\rangle\left|\uparrow'\uparrow'\right\rangle + a_{\downarrow\uparrow}\left|\downarrow\uparrow\right\rangle\left|\nwarrow\!\!\!\!\searrow\ \swarrow\!\!\!\!\nearrow\right\rangle\left|\uparrow'\downarrow'\right\rangle + a_{\downarrow\uparrow}\left|\downarrow\uparrow\right\rangle\left|\swarrow\!\!\!\!\nearrow\ \nwarrow\!\!\!\!\searrow\right\rangle\left|\downarrow'\uparrow'\right\rangle + a_{\downarrow\uparrow}\left|\downarrow\uparrow\right\rangle\left|\swarrow\!\!\!\!\nearrow\ \swarrow\!\!\!\!\nearrow\right\rangle\left|\downarrow'\downarrow'\right\rangle\Big)/2 \\
&+\Big(a_{\downarrow\downarrow}\left|\downarrow\downarrow\right\rangle\left|\nwarrow\!\!\!\!\searrow\ \nwarrow\!\!\!\!\searrow\right\rangle\left|\uparrow'\uparrow'\right\rangle + a_{\downarrow\downarrow}\left|\downarrow\downarrow\right\rangle\left|\nwarrow\!\!\!\!\searrow\ \swarrow\!\!\!\!\nearrow\right\rangle\left|\uparrow'\downarrow'\right\rangle + a_{\downarrow\downarrow}\left|\downarrow\downarrow\right\rangle\left|\swarrow\!\!\!\!\nearrow\ \nwarrow\!\!\!\!\searrow\right\rangle\left|\downarrow'\uparrow'\right\rangle + a_{\downarrow\downarrow}\left|\downarrow\downarrow\right\rangle\left|\swarrow\!\!\!\!\nearrow\ \swarrow\!\!\!\!\nearrow\right\rangle\left|\downarrow'\downarrow'\right\rangle\Big)/2
\end{aligned}$$

After interaction: $\left|\psi_{\mathrm{epe'}}^{(2)}(t_C+T)\right\rangle = \frac{1}{2}\left(\left|\updownarrow\updownarrow\right\rangle\left|\psi_{\mathrm{ee'1}}^{(2)}\right\rangle + \left|\updownarrow\leftrightarrow\right\rangle\left|\psi_{\mathrm{ee'2}}^{(2)}\right\rangle + \left|\leftrightarrow\updownarrow\right\rangle\left|\psi_{\mathrm{ee'3}}^{(2)}\right\rangle + \left|\leftrightarrow\leftrightarrow\right\rangle\left|\psi_{\mathrm{ee'4}}^{(2)}\right\rangle\right)$

Measurement of linear polarization of the two photons:

$$\left|\psi_{\mathrm{ee'1}}^{(2)}(t_D)\right\rangle = a_{\uparrow\uparrow}\left|\uparrow\uparrow\right\rangle\left|\uparrow'\uparrow'\right\rangle - a_{\uparrow\downarrow}\left|\uparrow\downarrow\right\rangle\left|\uparrow'\downarrow'\right\rangle - a_{\downarrow\uparrow}\left|\downarrow\uparrow\right\rangle\left|\downarrow'\uparrow'\right\rangle + a_{\downarrow\downarrow}\left|\downarrow\downarrow\right\rangle\left|\downarrow'\downarrow'\right\rangle$$

$$\left|\psi_{\mathrm{ee'2}}^{(2)}(t_D)\right\rangle = a_{\uparrow\downarrow}\left|\uparrow\downarrow\right\rangle\left|\uparrow'\uparrow'\right\rangle + a_{\uparrow\uparrow}\left|\uparrow\uparrow\right\rangle\left|\uparrow'\downarrow'\right\rangle - a_{\downarrow\downarrow}\left|\downarrow\downarrow\right\rangle\left|\downarrow'\uparrow'\right\rangle - a_{\downarrow\uparrow}\left|\downarrow\uparrow\right\rangle\left|\downarrow'\downarrow'\right\rangle$$

$$\left|\psi_{\mathrm{ee'3}}^{(2)}(t_D)\right\rangle = a_{\downarrow\uparrow}\left|\downarrow\uparrow\right\rangle\left|\uparrow'\uparrow'\right\rangle - a_{\downarrow\downarrow}\left|\downarrow\downarrow\right\rangle\left|\uparrow'\downarrow'\right\rangle + a_{\uparrow\uparrow}\left|\uparrow\uparrow\right\rangle\left|\downarrow'\uparrow'\right\rangle - a_{\uparrow\downarrow}\left|\uparrow\downarrow\right\rangle\left|\downarrow'\downarrow'\right\rangle$$

$$\left|\psi_{\mathrm{ee'4}}^{(2)}(t_D)\right\rangle = a_{\downarrow\downarrow}\left|\downarrow\downarrow\right\rangle\left|\uparrow'\uparrow'\right\rangle + a_{\downarrow\uparrow}\left|\downarrow\uparrow\right\rangle\left|\uparrow'\downarrow'\right\rangle + a_{\uparrow\downarrow}\left|\uparrow\downarrow\right\rangle\left|\downarrow'\uparrow'\right\rangle + a_{\uparrow\uparrow}\left|\uparrow\uparrow\right\rangle\left|\downarrow'\downarrow'\right\rangle$$

$a_{\uparrow\uparrow} = a_{\uparrow\downarrow} = a_{\downarrow\uparrow} = a_{\downarrow\downarrow} = \frac{1}{2}$ $\Rightarrow$ production of four-qubit Bell states between D and D‘

Collapse of the GHZ states

$$\left|\psi^{(2)}_{ee'1}(t_D)\right\rangle = a_{\uparrow\uparrow}\left|\uparrow\uparrow\right\rangle\left|\uparrow'\uparrow'\right\rangle - a_{\uparrow\downarrow}\left|\uparrow\downarrow\right\rangle\left|\uparrow'\downarrow'\right\rangle - a_{\downarrow\uparrow}\left|\downarrow\uparrow\right\rangle\left|\downarrow'\uparrow'\right\rangle + a_{\downarrow\downarrow}\left|\downarrow\downarrow\right\rangle\left|\downarrow'\downarrow'\right\rangle$$

$$= \left|\leftarrow\leftarrow\right\rangle\left(a_{\uparrow\uparrow}\left|\uparrow'\uparrow'\right\rangle - a_{\uparrow\downarrow}\left|\uparrow'\downarrow'\right\rangle - a_{\downarrow\uparrow}\left|\downarrow'\uparrow'\right\rangle + a_{\downarrow\downarrow}\left|\downarrow'\downarrow'\right\rangle\right) + \left|\leftarrow\rightarrow\right\rangle\left(a_{\uparrow\uparrow}\left|\uparrow'\uparrow'\right\rangle + a_{\uparrow\downarrow}\left|\uparrow'\downarrow'\right\rangle - a_{\downarrow\uparrow}\left|\downarrow'\uparrow'\right\rangle - a_{\downarrow\downarrow}\left|\downarrow'\downarrow'\right\rangle\right)$$

$$+ \left|\rightarrow\leftarrow\right\rangle\left(a_{\uparrow\uparrow}\left|\uparrow'\uparrow'\right\rangle - a_{\uparrow\downarrow}\left|\uparrow'\downarrow'\right\rangle + a_{\downarrow\uparrow}\left|\downarrow'\uparrow'\right\rangle - a_{\downarrow\downarrow}\left|\downarrow'\downarrow'\right\rangle\right) + \left|\rightarrow\rightarrow\right\rangle\left(a_{\uparrow\uparrow}\left|\uparrow'\uparrow'\right\rangle + a_{\uparrow\downarrow}\left|\uparrow'\downarrow'\right\rangle + a_{\downarrow\uparrow}\left|\downarrow'\uparrow'\right\rangle + a_{\downarrow\downarrow}\left|\downarrow'\downarrow'\right\rangle\right)$$

$$\left|\psi^{(2)}_{ee'2}(t_D)\right\rangle = a_{\uparrow\downarrow}\left|\uparrow\downarrow\right\rangle\left|\uparrow'\uparrow'\right\rangle + a_{\uparrow\uparrow}\left|\uparrow\uparrow\right\rangle\left|\uparrow'\downarrow'\right\rangle - a_{\downarrow\downarrow}\left|\downarrow\downarrow\right\rangle\left|\downarrow'\uparrow'\right\rangle - a_{\downarrow\uparrow}\left|\downarrow\uparrow\right\rangle\left|\downarrow'\downarrow'\right\rangle$$

$$= \left|\leftarrow\leftarrow\right\rangle\left(a_{\uparrow\downarrow}\left|\uparrow'\uparrow'\right\rangle + a_{\uparrow\uparrow}\left|\uparrow'\downarrow'\right\rangle - a_{\downarrow\downarrow}\left|\downarrow'\uparrow'\right\rangle - a_{\downarrow\uparrow}\left|\downarrow'\downarrow'\right\rangle\right) + \left|\leftarrow\rightarrow\right\rangle\left(-a_{\uparrow\downarrow}\left|\uparrow'\uparrow'\right\rangle + a_{\uparrow\uparrow}\left|\uparrow'\downarrow'\right\rangle + a_{\downarrow\downarrow}\left|\downarrow'\uparrow'\right\rangle - a_{\downarrow\uparrow}\left|\downarrow'\downarrow'\right\rangle\right)$$

$$+ \left|\rightarrow\leftarrow\right\rangle\left(a_{\uparrow\downarrow}\left|\uparrow'\uparrow'\right\rangle + a_{\uparrow\uparrow}\left|\uparrow'\downarrow'\right\rangle + a_{\downarrow\downarrow}\left|\downarrow'\uparrow'\right\rangle + a_{\downarrow\uparrow}\left|\downarrow'\downarrow'\right\rangle\right) + \left|\rightarrow\rightarrow\right\rangle\left(-a_{\uparrow\downarrow}\left|\uparrow'\uparrow'\right\rangle + a_{\uparrow\uparrow}\left|\uparrow'\downarrow'\right\rangle - a_{\downarrow\downarrow}\left|\downarrow'\uparrow'\right\rangle + a_{\downarrow\uparrow}\left|\downarrow'\downarrow'\right\rangle\right)$$

$$\left|\psi^{(2)}_{ee'3}(t_D)\right\rangle = a_{\downarrow\uparrow}\left|\downarrow\uparrow\right\rangle\left|\uparrow'\uparrow'\right\rangle - a_{\downarrow\downarrow}\left|\downarrow\downarrow\right\rangle\left|\uparrow'\downarrow'\right\rangle + a_{\uparrow\uparrow}\left|\uparrow\uparrow\right\rangle\left|\downarrow'\uparrow'\right\rangle - a_{\uparrow\downarrow}\left|\uparrow\downarrow\right\rangle\left|\downarrow'\downarrow'\right\rangle$$

$$= \left|\leftarrow\leftarrow\right\rangle\left(a_{\downarrow\uparrow}\left|\uparrow'\uparrow'\right\rangle - a_{\downarrow\downarrow}\left|\uparrow'\downarrow'\right\rangle + a_{\uparrow\uparrow}\left|\downarrow'\uparrow'\right\rangle - a_{\uparrow\downarrow}\left|\downarrow'\downarrow'\right\rangle\right) + \left|\leftarrow\rightarrow\right\rangle\left(a_{\downarrow\uparrow}\left|\uparrow'\uparrow'\right\rangle + a_{\downarrow\downarrow}\left|\uparrow'\downarrow'\right\rangle + a_{\uparrow\uparrow}\left|\downarrow'\uparrow'\right\rangle + a_{\uparrow\downarrow}\left|\downarrow'\downarrow'\right\rangle\right)$$

$$+ \left|\rightarrow\leftarrow\right\rangle\left(-a_{\downarrow\uparrow}\left|\uparrow'\uparrow'\right\rangle + a_{\downarrow\downarrow}\left|\uparrow'\downarrow'\right\rangle + a_{\uparrow\uparrow}\left|\downarrow'\uparrow'\right\rangle - a_{\uparrow\downarrow}\left|\downarrow'\downarrow'\right\rangle\right) + \left|\rightarrow\rightarrow\right\rangle\left(-a_{\downarrow\uparrow}\left|\uparrow'\uparrow'\right\rangle - a_{\downarrow\downarrow}\left|\uparrow'\downarrow'\right\rangle + a_{\uparrow\uparrow}\left|\downarrow'\uparrow'\right\rangle + a_{\uparrow\downarrow}\left|\downarrow'\downarrow'\right\rangle\right)$$

$$\left|\psi^{(2)}_{ee'4}(t_D)\right\rangle = a_{\downarrow\downarrow}\left|\downarrow\downarrow\right\rangle\left|\uparrow'\uparrow'\right\rangle + a_{\downarrow\uparrow}\left|\downarrow\uparrow\right\rangle\left|\uparrow'\downarrow'\right\rangle + a_{\uparrow\downarrow}\left|\uparrow\downarrow\right\rangle\left|\downarrow'\uparrow'\right\rangle + a_{\uparrow\uparrow}\left|\uparrow\uparrow\right\rangle\left|\downarrow'\downarrow'\right\rangle$$

$$= \left|\leftarrow\leftarrow\right\rangle\left(a_{\downarrow\downarrow}\left|\uparrow'\uparrow'\right\rangle + a_{\downarrow\uparrow}\left|\uparrow'\downarrow'\right\rangle + a_{\uparrow\downarrow}\left|\downarrow'\uparrow'\right\rangle + a_{\uparrow\uparrow}\left|\downarrow'\downarrow'\right\rangle\right) + \left|\leftarrow\rightarrow\right\rangle\left(-a_{\downarrow\downarrow}\left|\uparrow'\uparrow'\right\rangle + a_{\downarrow\uparrow}\left|\uparrow'\downarrow'\right\rangle - a_{\uparrow\downarrow}\left|\downarrow'\uparrow'\right\rangle + a_{\uparrow\uparrow}\left|\downarrow'\downarrow'\right\rangle\right)$$

$$+ \left|\rightarrow\leftarrow\right\rangle\left(-a_{\downarrow\downarrow}\left|\uparrow'\uparrow'\right\rangle - a_{\downarrow\uparrow}\left|\uparrow'\downarrow'\right\rangle + a_{\uparrow\downarrow}\left|\downarrow'\uparrow'\right\rangle + a_{\uparrow\uparrow}\left|\downarrow'\downarrow'\right\rangle\right) + \left|\rightarrow\rightarrow\right\rangle\left(a_{\downarrow\downarrow}\left|\uparrow'\uparrow'\right\rangle - a_{\downarrow\uparrow}\left|\uparrow'\downarrow'\right\rangle - a_{\uparrow\downarrow}\left|\downarrow'\uparrow'\right\rangle + a_{\uparrow\uparrow}\left|\downarrow'\downarrow'\right\rangle\right)$$

Knowledge of detection outcome $\Rightarrow$ teleportation from D to D‘ complete

Generalization to *n* qubits is straightforward.

Complexity of *n*-qubit GHZ teleportation scales linearly with *n*.

Production of entangled photons

Independent nonresonant interaction of two photons with two quantum dots:

$$\left|\psi_{\mathrm{ep}}^{(2)}(t_A+T)\right\rangle = a_{\uparrow\uparrow}\left|\uparrow\uparrow\right\rangle\left|\leftrightarrow\leftrightarrow\right\rangle + a_{\uparrow\downarrow}\left|\uparrow\downarrow\right\rangle\left|\leftrightarrow\leftrightarrow\right\rangle + a_{\downarrow\uparrow}\left|\downarrow\uparrow\right\rangle\left|\leftrightarrow\leftrightarrow\right\rangle + a_{\downarrow\downarrow}\left|\downarrow\downarrow\right\rangle\left|\leftrightarrow\leftrightarrow\right\rangle$$

Change to S_x representation:

$$\left|\psi_{\mathrm{ep}}^{(2)}(t_A+T)\right\rangle = \frac{1}{2}\left(\left|\leftarrow\leftarrow\right\rangle\left|\psi_{\mathrm{p1}}^{(2)}\right\rangle + \left|\leftarrow\rightarrow\right\rangle\left|\psi_{\mathrm{p2}}^{(2)}\right\rangle + \left|\rightarrow\leftarrow\right\rangle\left|\psi_{\mathrm{p3}}^{(2)}\right\rangle + \left|\rightarrow\rightarrow\right\rangle\left|\psi_{\mathrm{p4}}^{(2)}\right\rangle\right)$$

Measuring spins in x direction:

$$\left|\psi_{\mathrm{p1}}^{(2)}\right\rangle = a_{\uparrow\uparrow}\left|\nwarrow\nwarrow\right\rangle + a_{\uparrow\downarrow}\left|\nwarrow\swarrow\right\rangle + a_{\downarrow\uparrow}\left|\swarrow\nwarrow\right\rangle + a_{\downarrow\downarrow}\left|\swarrow\swarrow\right\rangle$$

or $$\left|\psi_{\mathrm{p2}}^{(2)}\right\rangle = a_{\uparrow\uparrow}\left|\nwarrow\nwarrow\right\rangle - a_{\uparrow\downarrow}\left|\nwarrow\swarrow\right\rangle + a_{\downarrow\uparrow}\left|\swarrow\nwarrow\right\rangle - a_{\downarrow\downarrow}\left|\swarrow\swarrow\right\rangle$$

or $$\left|\psi_{\mathrm{p3}}^{(2)}\right\rangle = a_{\uparrow\uparrow}\left|\nwarrow\nwarrow\right\rangle + a_{\uparrow\downarrow}\left|\nwarrow\swarrow\right\rangle - a_{\downarrow\uparrow}\left|\swarrow\nwarrow\right\rangle - a_{\downarrow\downarrow}\left|\swarrow\swarrow\right\rangle$$

or $$\left|\psi_{\mathrm{p4}}^{(2)}\right\rangle = a_{\uparrow\uparrow}\left|\nwarrow\nwarrow\right\rangle - a_{\uparrow\downarrow}\left|\nwarrow\swarrow\right\rangle - a_{\downarrow\uparrow}\left|\swarrow\nwarrow\right\rangle + a_{\downarrow\downarrow}\left|\swarrow\swarrow\right\rangle$$

see also Leuenberger, Flatte, Awschalom, Europhys. Lett. **71**, 387 (2005).

Knowledge of detection outcome $\Rightarrow$ „direct teleportation“ from spins to photons

$a_{\uparrow\uparrow} = a_{\downarrow\downarrow} = 0,\ a_{\uparrow\downarrow} = a_{\downarrow\uparrow} = \frac{1}{\sqrt{2}}$ $\Rightarrow$ production of Bell states $\left|\Psi_{\mathrm{p}}^{(\pm)}\right\rangle = \left(\left|\nwarrow\swarrow\right\rangle \pm \left|\swarrow\nwarrow\right\rangle\right)/\sqrt{2}$

$a_{\uparrow\uparrow} = a_{\downarrow\downarrow} = \frac{1}{\sqrt{2}},\ a_{\uparrow\downarrow} = a_{\downarrow\uparrow} = 0$ $\Rightarrow$ production of Bell states $\left|\Phi_{\mathrm{p}}^{(\pm)}\right\rangle = \left(\left|\nwarrow\nwarrow\right\rangle \pm \left|\swarrow\swarrow\right\rangle\right)/\sqrt{2}$

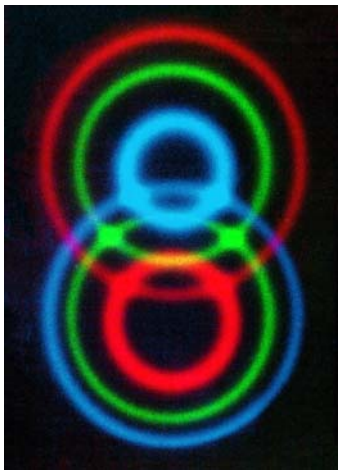
Conclusions

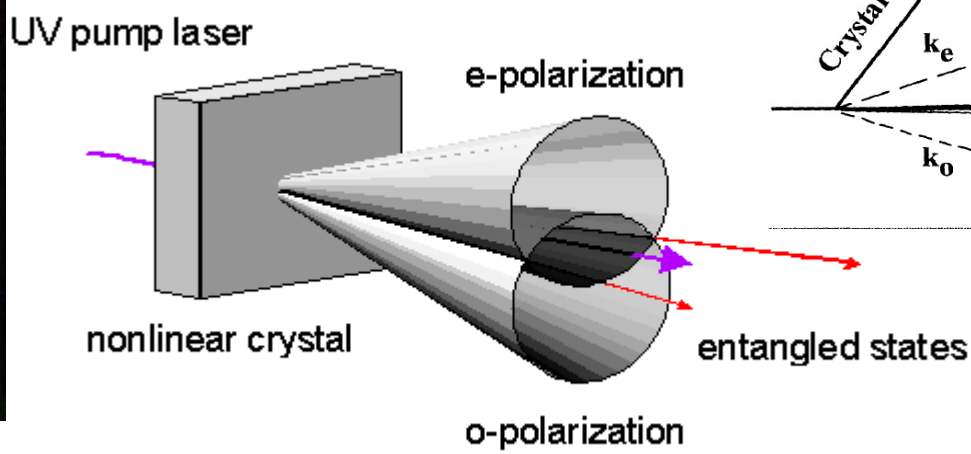
- The many-qubit state of one quantum dot system (origin) can be teleported to a second quantum dot system (destination) via nonresonant interaction of single photons with the quantum dots (producing qubit-photon-qubit entanglement, i.e. GHZ states).
- Single-photon measurements are sufficient to teleport not only a single-qubit state but also a many-qubit state (no Bell measurements are required). Reason: GHZ entanglement
- The state of the excess electron selects the circular polarization of the photons that gets entangled with the many-qubit state via conditional Faraday rotation.
- GHZ teleportation scheme opens up the possibility to build a Quantum Dynamic RAM (QDRAM) where the quantum error corrections are applied to the many-qubit state (short spin decoherence time of electrons).

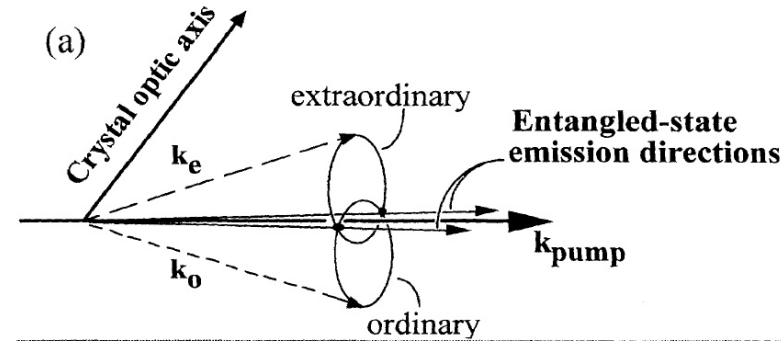
Parametric down-conversion

Kwiat, Mattle, Weinfurter, Zeilinger, Sergienko, Shih, PRL **75**, 4337 (1995)

Noncollinear type-II phase matching:







Nonlinear crystal: β-Bariumborate [β-$Ba(B_2O_4)$ or β-$Ba_3(B_3O_6)_2$]

Output state on overlapping cones: $|\psi\rangle_{12} = \frac{1}{\sqrt{2}}\left(|\leftrightarrow\rangle_1|\updownarrow\rangle_2 + e^{i\varphi}|\updownarrow\rangle_1|\leftrightarrow\rangle_2\right) \Rightarrow |\Psi^{\pm}\rangle_{12}$

Ordinary and extraordinary polarization: vertical and horizontal

φ is controlled by birefringent phase shifter or by rotating the crystal.

λ/2 plate for one photon produces: $|\psi\rangle_{12} = \frac{1}{\sqrt{2}}\left(|\leftrightarrow\rangle_1|\leftrightarrow\rangle_2 + e^{i\varphi}|\updownarrow\rangle_1|\updownarrow\rangle_2\right) \Rightarrow |\Phi^{\pm}\rangle_{12}$

Efficiency: 2 photons out of 10^6 are entangled.

Energy conservation: $\omega_0 = \omega_1 + \omega_2$ Momentum conservation: $\mathbf{k}_0 = \mathbf{k}_1 + \mathbf{k}_2$

Degenerate case: $\omega_1 = \omega_2 = \omega_0 / 2$

Quantum mechanics of PDC

Incident classical pump light field (UV): $\mathbf{V}(\mathbf{r},t) = \mathbf{V}e^{i(\mathbf{k}_0\bullet\mathbf{r}-\omega_0 t)}$

Interaction Hamiltonian in the interaction picture:

$$\hat{H}_I(t) = \int_v \frac{d^3x}{L^3} \sum_{\mathbf{k}_1,s_1} \sum_{\mathbf{k}_2,s_2} \chi_{ijl}(\omega_0,\omega_1,\omega_2) \left(\boldsymbol{\varepsilon}_{\mathbf{k}_1 s_1}\right)_i \left(\boldsymbol{\varepsilon}_{\mathbf{k}_2 s_2}\right)_j \mathbf{V}_l a^\dagger_{\mathbf{k}_1 s_1} a^\dagger_{\mathbf{k}_2 s_2} e^{i[(\mathbf{k}_0-\mathbf{k}_1-\mathbf{k}_2)\bullet\mathbf{r}-(\omega_0-\omega_1-\omega_2)t]} + \text{H.C.}$$

Initial state of the quantum field: $\left|\psi(t=0)\right\rangle_{12} = \left|\psi_{\text{vac}}\right\rangle$

$$\left|\psi(t)\right\rangle_{12} = \exp\left[-\frac{i}{\hbar}\int_0^t dt'\,\hat{H}_I(t')\right]\left|\psi_{\text{vac}}\right\rangle$$

$$= \left|\psi_{\text{vac}}\right\rangle - \frac{i}{\hbar}\frac{1}{L^3}\sum_{\mathbf{k}_1,s_1}\sum_{\mathbf{k}_2,s_2}\chi_{ijl}(\omega_0,\omega_1,\omega_2)\left(\boldsymbol{\varepsilon}_{\mathbf{k}_1 s_1}\right)_i\left(\boldsymbol{\varepsilon}_{\mathbf{k}_2 s_2}\right)_j \mathbf{V}_l a^\dagger_{\mathbf{k}_1 s_1} a^\dagger_{\mathbf{k}_2 s_2} e^{i(\mathbf{k}_0-\mathbf{k}_1-\mathbf{k}_2)\bullet\mathbf{R}}$$

$$\times\prod_{m=1}^{3}\left[\frac{\sin[\frac{1}{2}(\mathbf{k}_0-\mathbf{k}_1-\mathbf{k}_2)_m l_m]}{\frac{1}{2}(\mathbf{k}_0-\mathbf{k}_1-\mathbf{k}_2)_m}\right] e^{-\frac{i}{2}(\omega_0-\omega_1-\omega_2)t}\,\frac{\sin[\frac{1}{2}(\omega_0-\omega_1-\omega_2)t]}{\frac{1}{2}(\omega_0-\omega_1-\omega_2)t}\left|\mathbf{k}_1 s_1,\mathbf{k}_2 s_2\right\rangle + O^2(\mathbf{V})$$

R is the midpoint of the nonlinear medium which has the shape of a rectangular parallelopiped of sides l_1, l_2, l_3.

Type-II phase matching: center of $\leftrightarrow$ and $\updownarrow$ cones or not collinear.