Nonequilibrium quantum criticality in a driven magnon system

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S. Takei, Phys. Rev. B **100**, 134440 (2019)

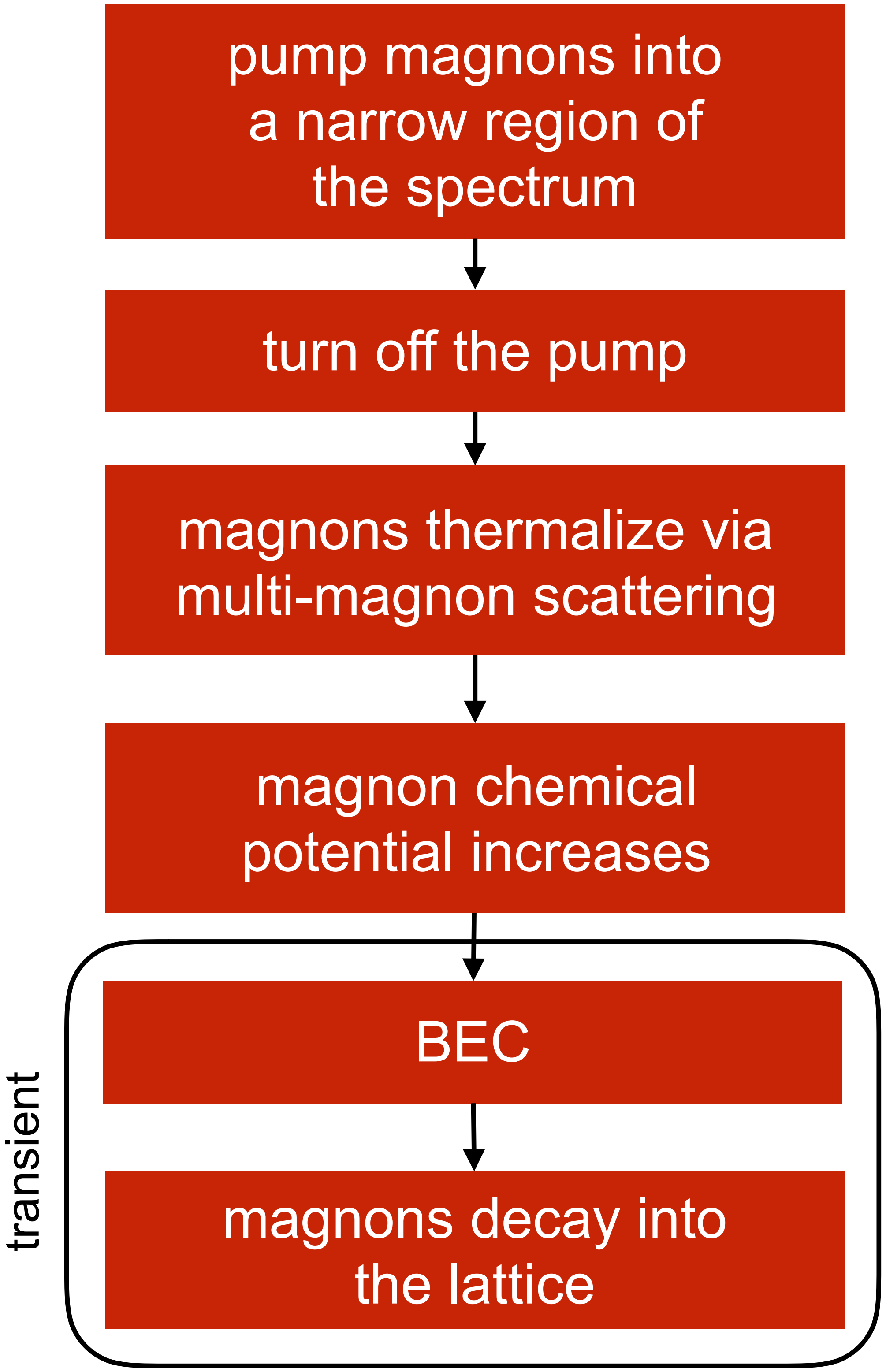
Spin and Heat Transport in Quantum and Topological Materials, KITP, Dec. 11, 2019

outline

- spintronics device: electrically-induced magnon BEC in a driven-dissipative environment
- Keldysh theory of nonequilibrium magnons near BEC instability
- magnon spin conductivity
- connection to experiment

parametrically-pumped magnon BEC

- room temperature magnon BEC using microwave pumping



pump magnons into a narrow region of the spectrum

turn off the pump

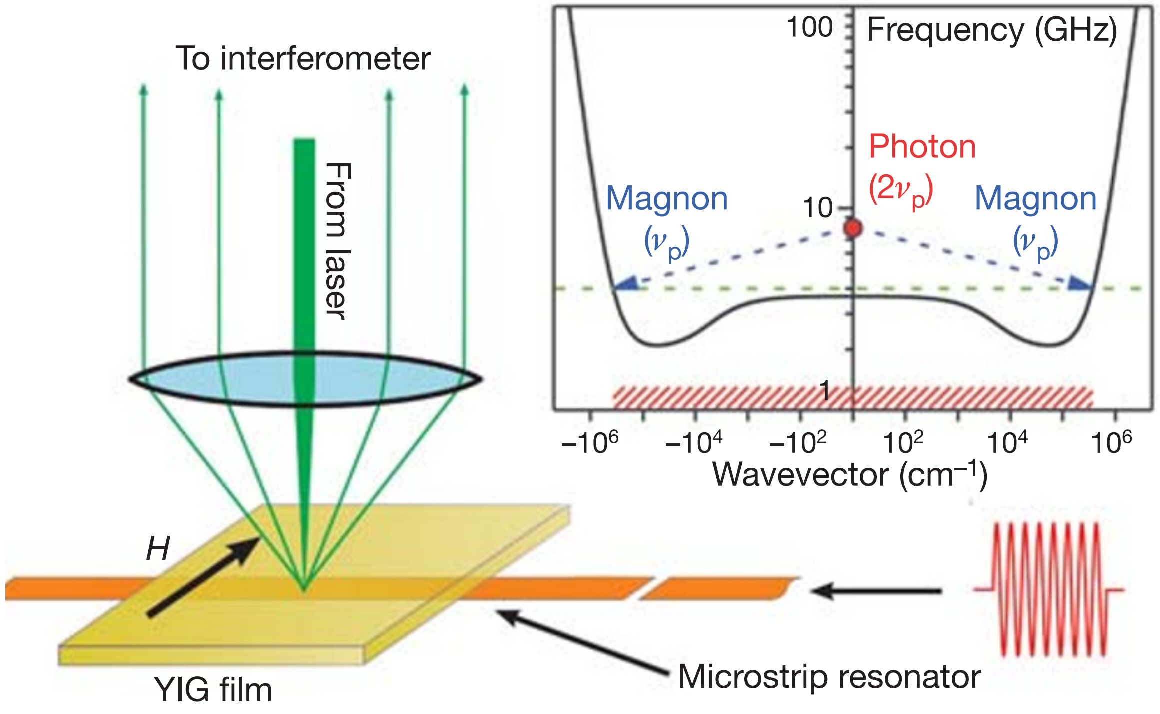
magnons thermalize via multi-magnon scattering

magnon chemical potential increases

BEC

magnons decay into the lattice

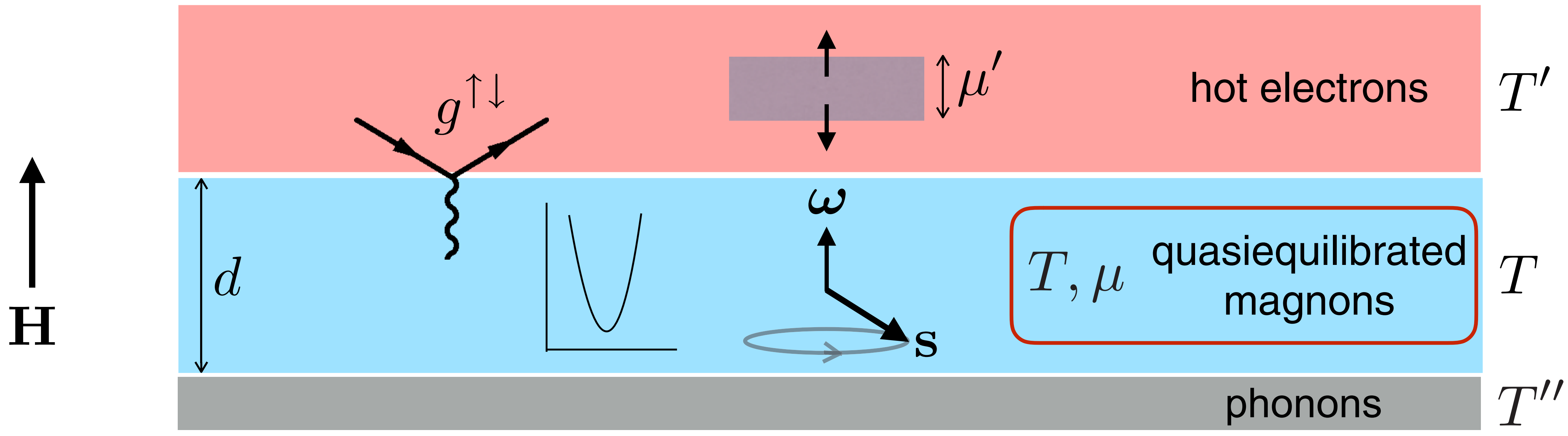
transient



S. O. Demokritov *et al*., Nature **443**, 430 (2006)
V. E. Demidov *et al*., Phys. Rev. Lett. **99**, 037205 (2007)
V. E. Demidov *et al*., Phys. Rev. Lett. **100**, 047205 (2008)
A. A. Serga *et al*., Nature Commun. **5**, 3452 EP (2014)
V. E. Demidov *et al*., Nature Commun. **8**, 1579 (2017)

electrically-pumped magnon BEC

- magnon BEC via electrical pumping and spin Hall effect.
- **nonequilibrium steady-state**: spin/energy loss due to Gilbert damping is precisely compensated by spin/energy injection via spin Hall effect and/or spin Seebeck effect.



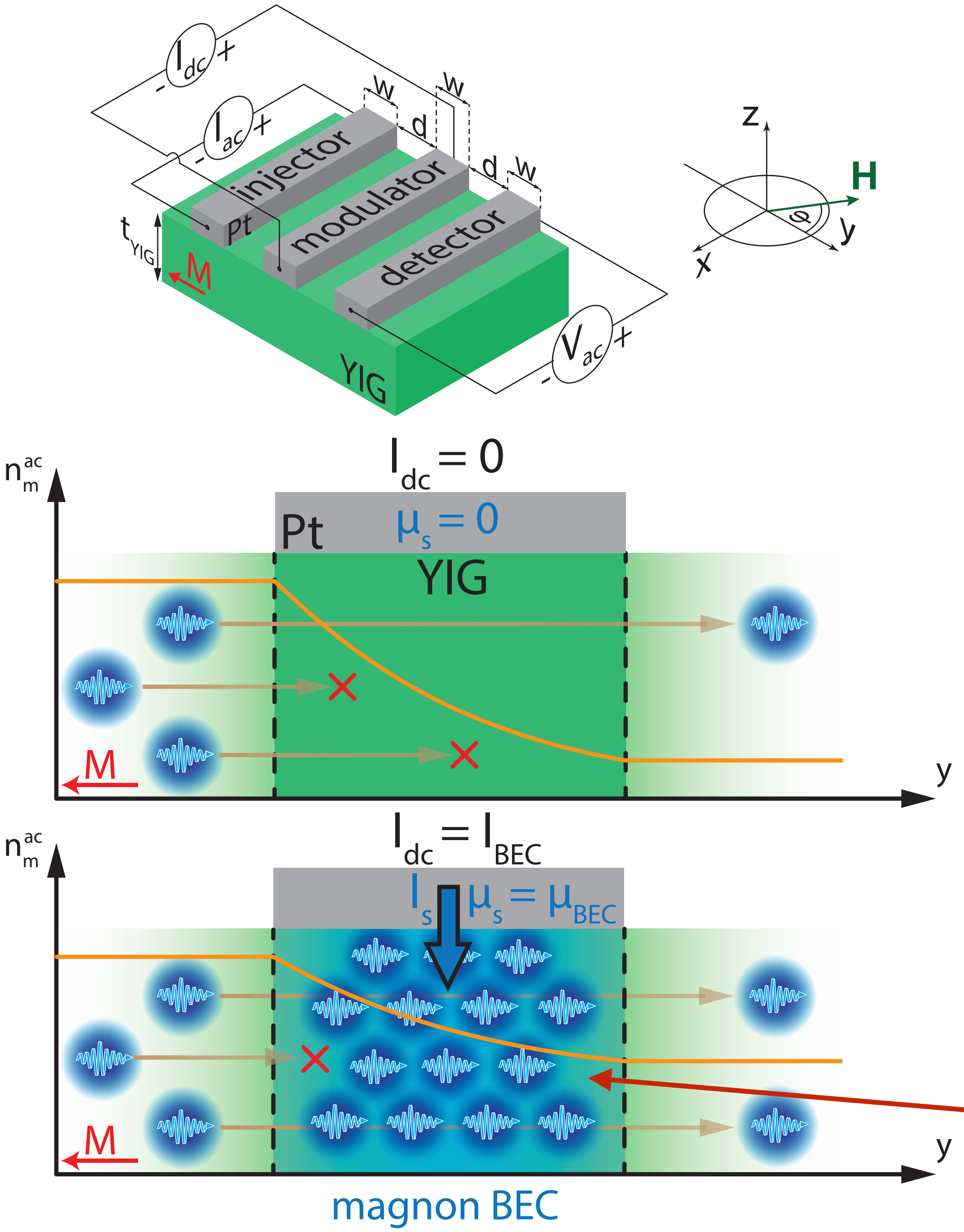
- quasi-equilibrated magnons: in the limit of strong magnon-magnon interactions magnons can be well-described by Bose-Einstein distribution with an effective chemical potential and temperature.

$$N(\Omega) = \frac{1}{e^{(\hbar\Omega-\mu)/k_B T} - 1}$$

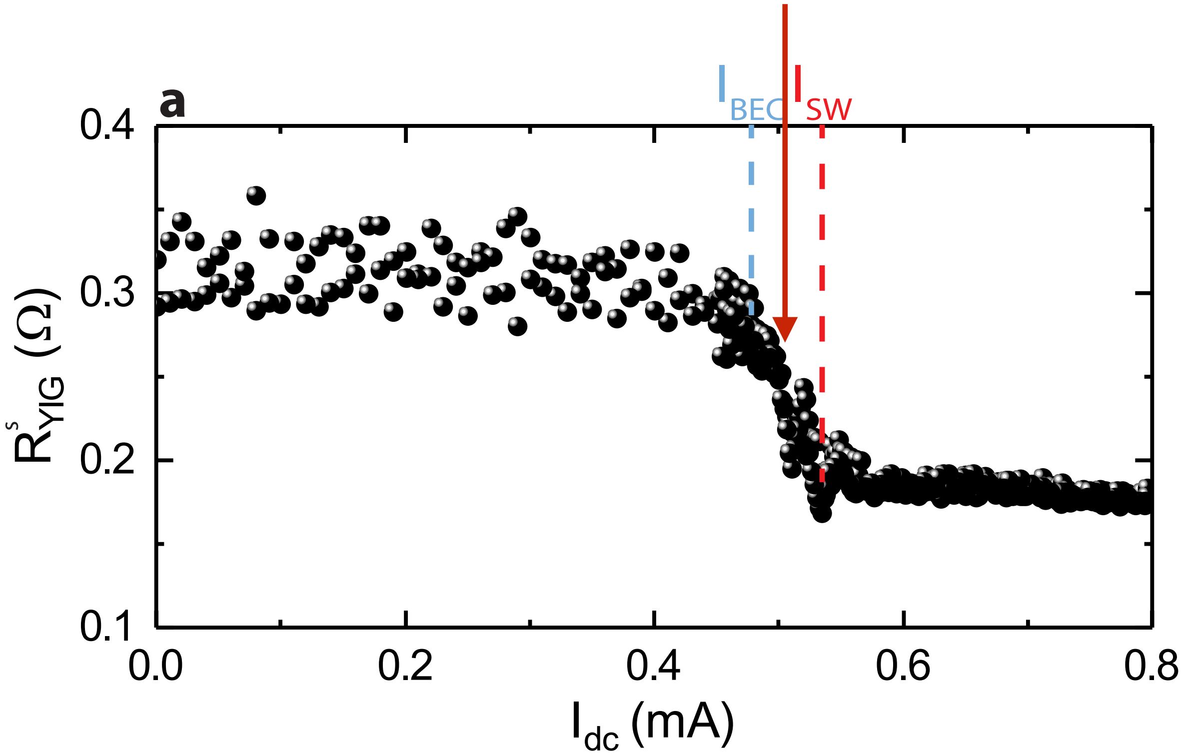
S. A. Bender *et al.*, Phys. Rev. Lett. **108**, 246601 (2012)
S. A. Bender *et al.*, Phys. Rev. B **90**, 094409 (2014)
E. L. Fjærbu *et al.*, Phys. Rev. B **95**, 144408 (2017)

experiment

- magnon BEC via spin Hall effect in a three-terminal device.
- reach critical magnon density by increasing the current in the modulator.
- measure two-terminal magnon conductivity by adjusting the current in the modulator.



drop in spin resistance at BEC instability

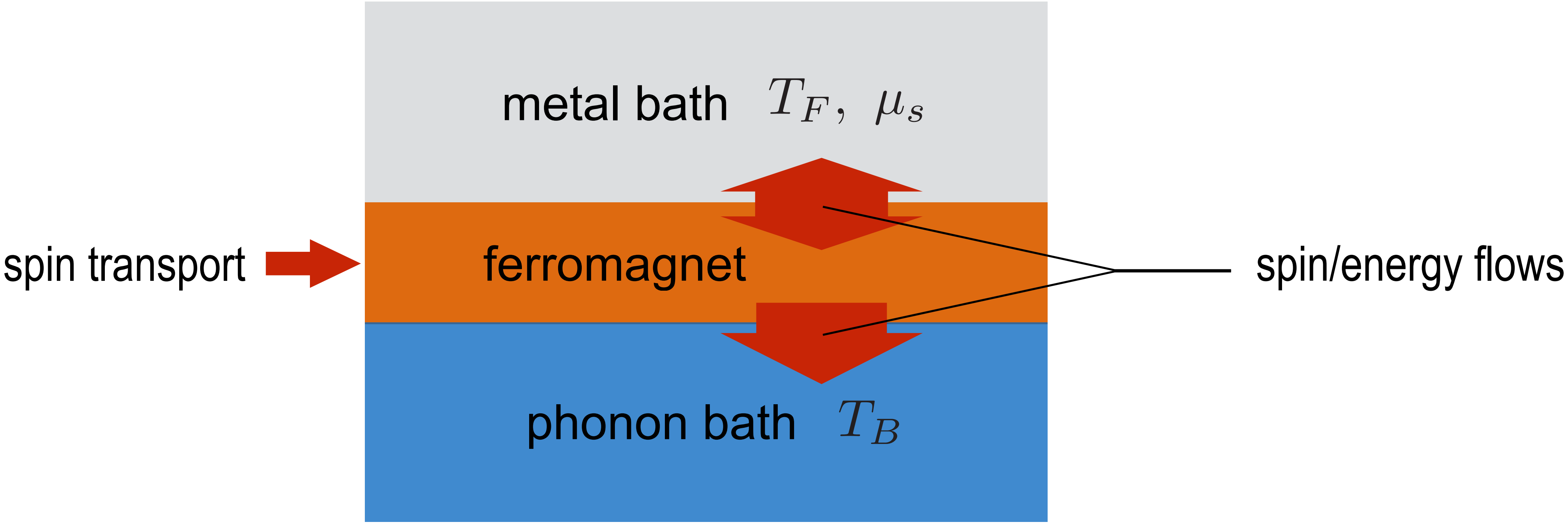


spin resistance in the active region (beneath the modulator contact) drops by two orders of magnitude

T. Wimmer *et al.*, arXiv:1812.01334

nonequilibrium quantum criticality

- BEC of 3d ferromagnetic exchange magnons ↔ vacuum-superfluid transition in 3d dilute Bose gas (**3d BEC universality class**).
- **question:** how is BEC quantum critical point modified in the case of nonequilibrium open quantum systems?



- **goal:** microscopic theory for BEC quantum phase transition of magnons coupled to two baths with mismatched temperatures and/or subjected to spin bias.
  - ➡ explicitly model the baths and the coupling to the baths.
  - ➡ baths held at temperatures T_B and T_F.
  - ➡ model spin bias by spin-split chemical potentials in the metal: $\mu_s \equiv \mu_\uparrow - \mu_\downarrow$.
  - ➡ focus exclusively on the normal (uncondensed) phase.
- **connection to experiment:** spin conductivity of magnons in a driven-dissipative steady-state.

3d BEC universality class

- BEC quantum critical point: $z = 2$ theory

$$\mathscr{L} = \psi^* i\hbar\partial_t\psi - \mu_0|\psi|^2 - \frac{\hbar^2}{2m}|\nabla\psi|^2 + \frac{u}{2}|\psi|^4$$

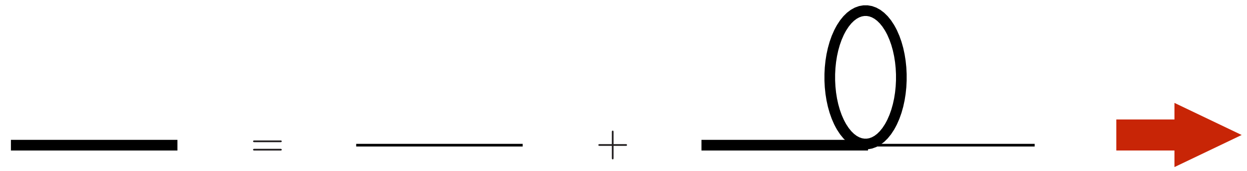
$$\dim[u] = 2 - d$$

- $d = 3$ is above the upper critical dimension → treat magnon-magnon interactions perturbatively.
- thermal crossover behavior in the normal phase of a weakly interacting dilute Bose gas

K. K. Singh, Phys. Rev. B **12**, 2819 (1975)
K. K. Singh, Phys. Rev. B 17, 324 (1978)
R. J. Creswick and F. W. Wiegel, Phys. Rev. A **28**, 1579 (1983)
M. Rasolt *et al.*, Phys. Rev. Lett. **53**, 798 (1984)
P. B. Weichman *et al.*, Phys. Rev. B **33**, 4632 (1986)

- one-loop self-consistent Hartree-Fock approximation leads to self-consistent condition for the boson chemical potential:

S. Sachdev, *Quantum Phase Transitions*, Ch. 16 (Cambridge University Press, Cambridge, 2011)

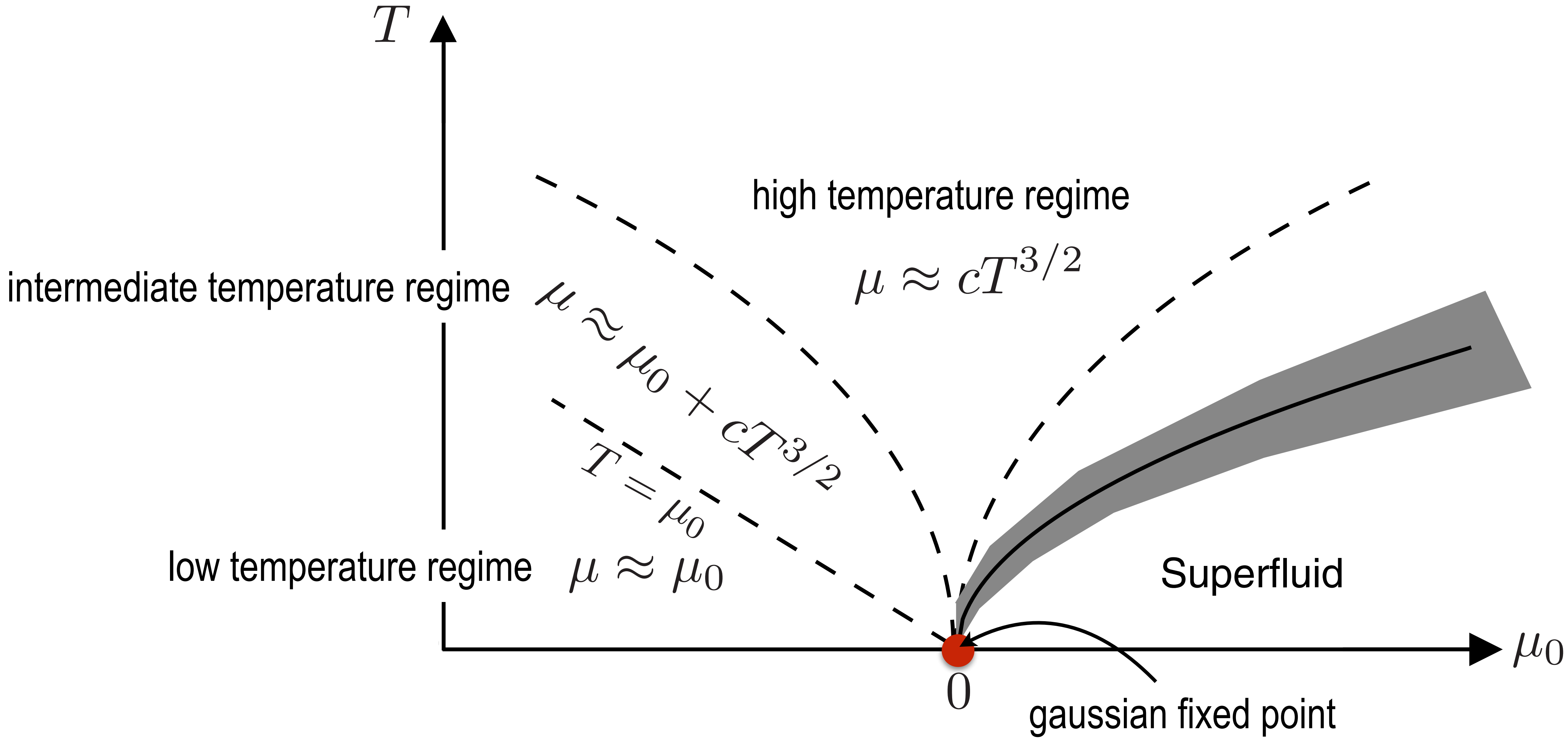


$$\mu = \mu_0 + c\frac{\zeta_{3/2}(e^{-\mu/k_B T})}{\zeta_{3/2}(1)}T^{3/2}$$

3d BEC universality class

- thermal crossovers for the equilibrium 3d BEC universality class:

$$\mu = \mu_0 + c\frac{\zeta_{3/2}(e^{-\mu/k_BT})}{\zeta_{3/2}(1)}T^{3/2}$$

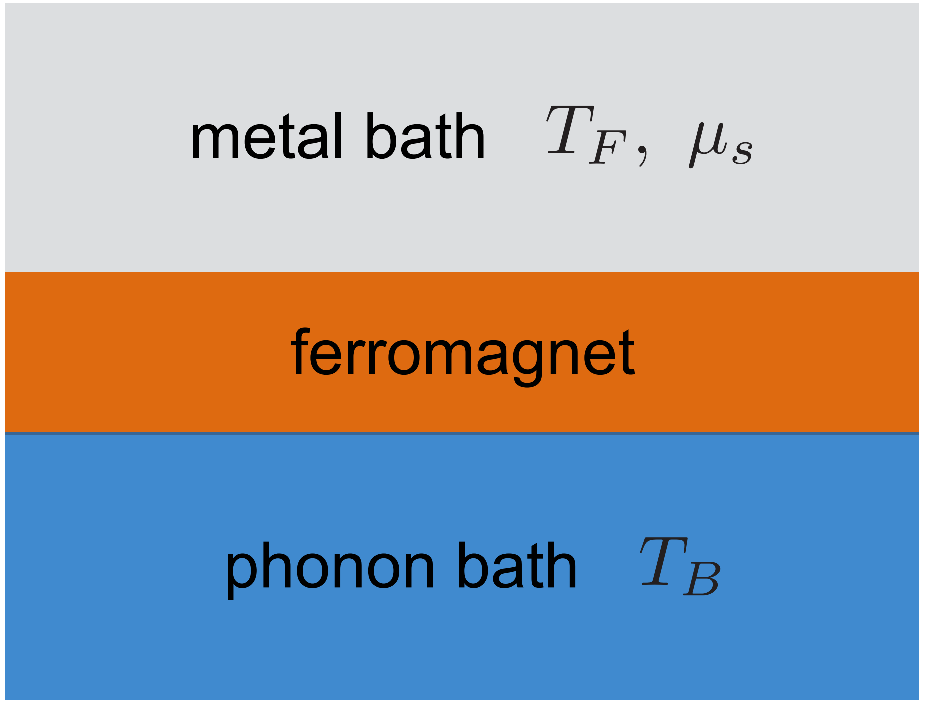


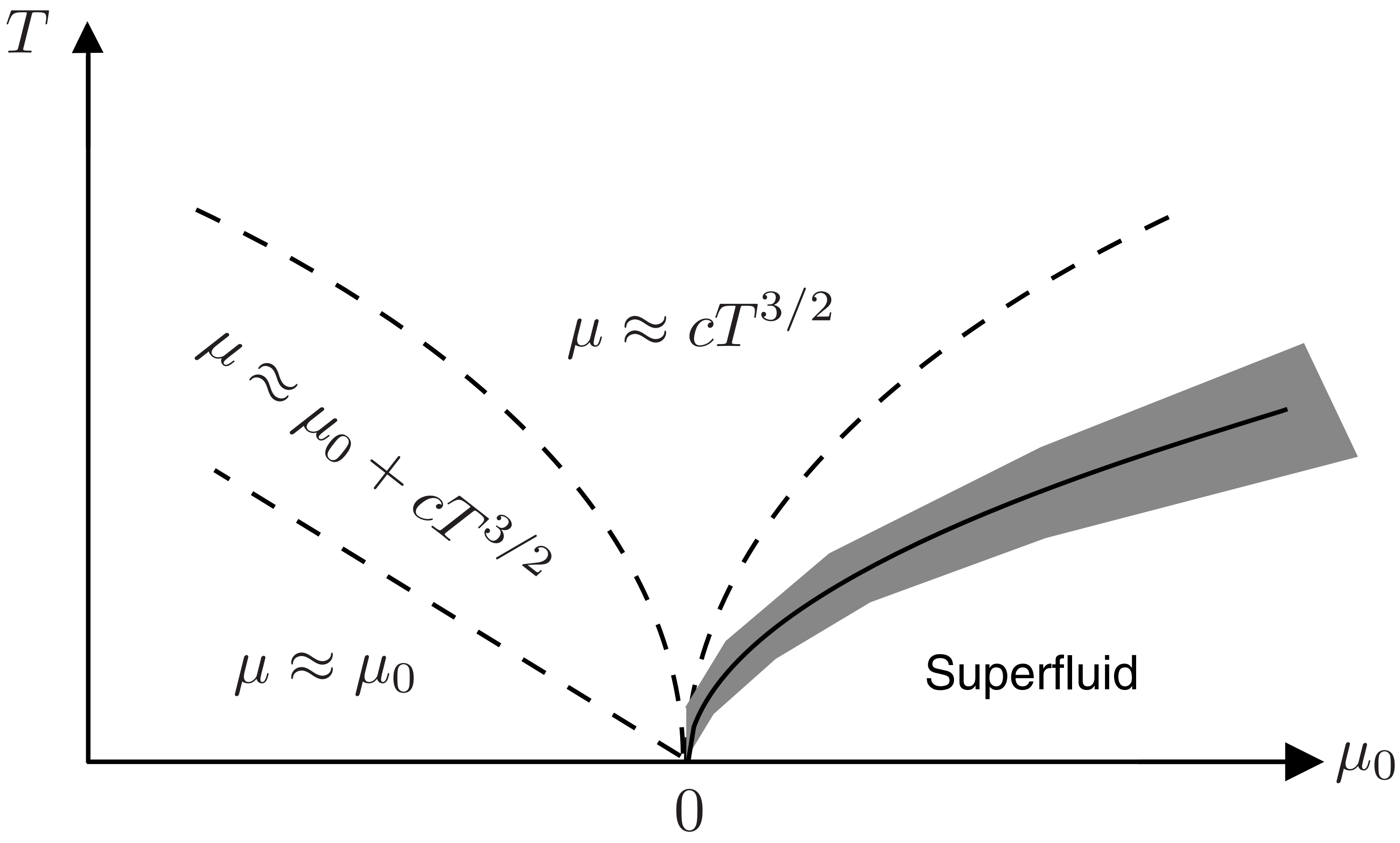
large area of the phase diagram can be understood via perturbation theory with respect to magnon-magnon interactions

S. Sachdev, *Quantum Phase Transitions*, Ch. 16
(Cambridge University Press, Cambridge, 2011)

questions

- how are thermal crossovers modified in the presence of two bath temperatures?
- how do these thermal crossovers affect magnon spin conductivity?
- how does magnon spin conductivity behave at finite spin bias?

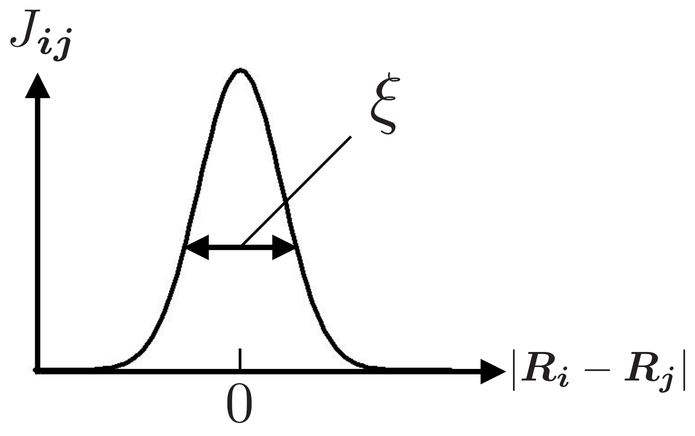




model: ferromagnet

- easy-plane ferromagnet in a perpendicular magnetic field and with short-ranged exchange coupling:

$$H_F = -\frac{1}{2}\sum_{\boldsymbol{ij}} J_{\boldsymbol{ij}}\boldsymbol{S_i}\cdot\boldsymbol{S_j} + \sum_{\boldsymbol{i}}(\hbar\gamma B + KS_{\boldsymbol{i}}^z)S_{\boldsymbol{i}}^z$$



$$J_{\boldsymbol{ij}} = Je^{-\frac{|\boldsymbol{R_i}-\boldsymbol{R_j}|^2}{2\xi^2}} \quad \Rightarrow \quad J_{\boldsymbol{q}} = J_{\boldsymbol{q}=0}e^{-\frac{q^2\xi^2}{2}}$$

- boson representation:

E. G. Batyev, Sov. Phys. JETP **62**, 173 (1985)

$$\varepsilon_{\boldsymbol{q}} = S(J_{\boldsymbol{0}} - J_{\boldsymbol{q}}) \approx \frac{J_{\boldsymbol{0}}S\xi^2}{2}q^2 \qquad \mu_0 = \hbar\gamma B - SK$$

$$H_F = \sum_{\boldsymbol{q}}(\varepsilon_{\boldsymbol{q}} + \mu_0)a_{\boldsymbol{q}}^\dagger a_{\boldsymbol{q}} + \frac{1}{\mathcal{N}}\sum_{\{\boldsymbol{q}_n\}} V_{\boldsymbol{q}_1\boldsymbol{q}_3}a_{\boldsymbol{q}_1}^\dagger a_{\boldsymbol{q}_2}^\dagger a_{\boldsymbol{q}_3}a_{\boldsymbol{q}_4}\delta_{\boldsymbol{q}_1+\boldsymbol{q}_2,\boldsymbol{q}_3+\boldsymbol{q}_4}$$

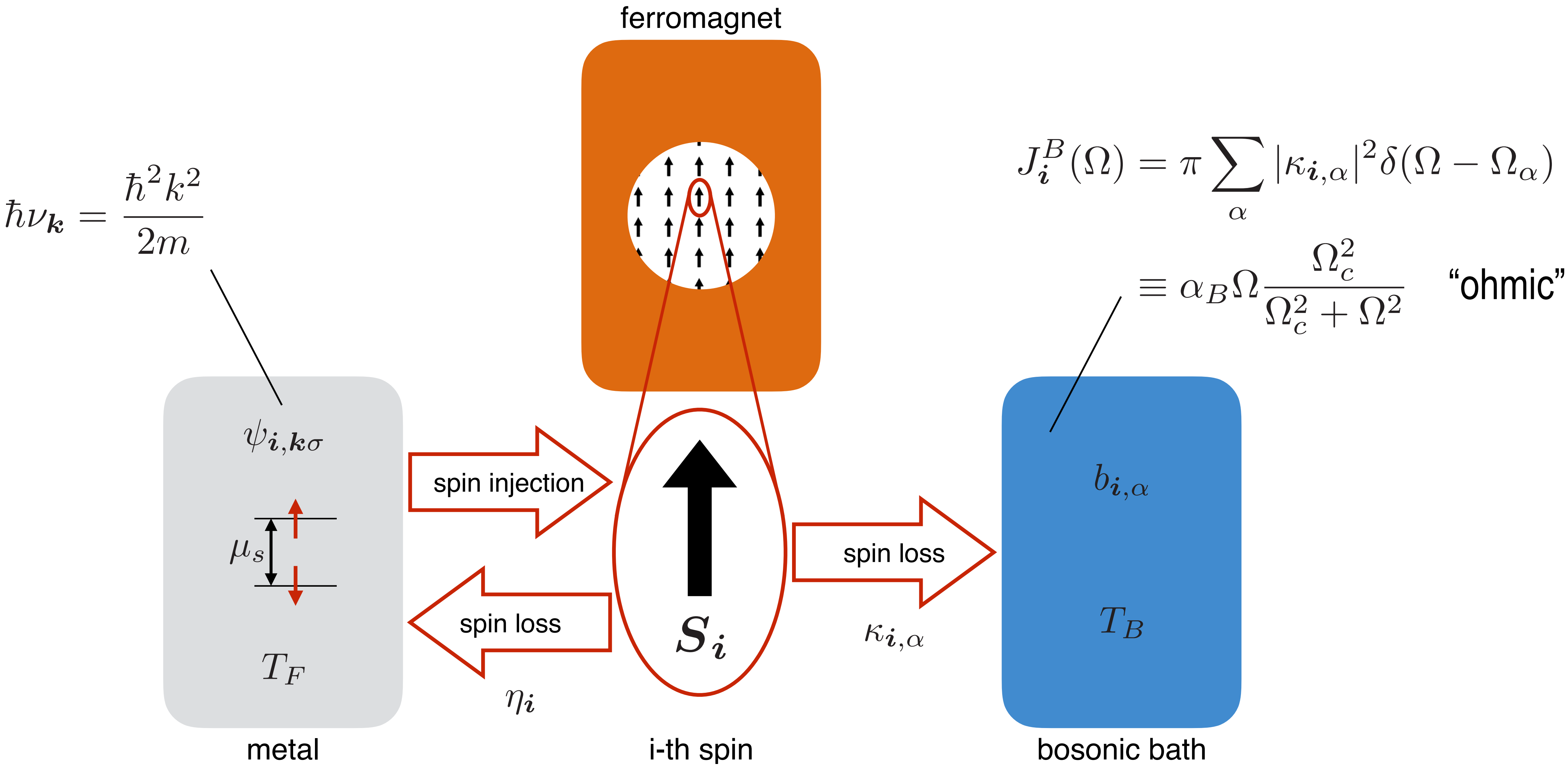
$$V_{\boldsymbol{q}_1\boldsymbol{q}_3} = \frac{K - J_{\boldsymbol{q}_1-\boldsymbol{q}_3}}{2} + \lambda(J_{\boldsymbol{q}_1} + J_{\boldsymbol{q}_3})\ , \qquad \lambda = S\left(1 - \sqrt{1 - \frac{1}{2S}}\right)$$

model: baths

- each atomic spin couples to its own boson and metal baths.

$$H_{\boldsymbol{i}}^B = \sum_\alpha \hbar\Omega_\alpha b_{\boldsymbol{i},\alpha}^\dagger b_{\boldsymbol{i},\alpha} + \hbar\sum_\alpha \left[\kappa_{\boldsymbol{i},\alpha} a_{\boldsymbol{i}} b_{\boldsymbol{i},\alpha}^\dagger + h.c.\right]$$

$$H_{\boldsymbol{i}}^m = \sum_{\boldsymbol{k},\sigma} \hbar\nu_{\boldsymbol{k}} \psi_{\boldsymbol{i},\boldsymbol{k}\sigma}^\dagger \psi_{\boldsymbol{i},\boldsymbol{k}\sigma} + \sum_{\boldsymbol{k},\boldsymbol{k}'} \left[\eta_{\boldsymbol{i}} \psi_{\boldsymbol{i},\boldsymbol{k}\uparrow}^\dagger \psi_{\boldsymbol{i},\boldsymbol{k}'\downarrow} a_{\boldsymbol{i}} + h.c.\right]$$

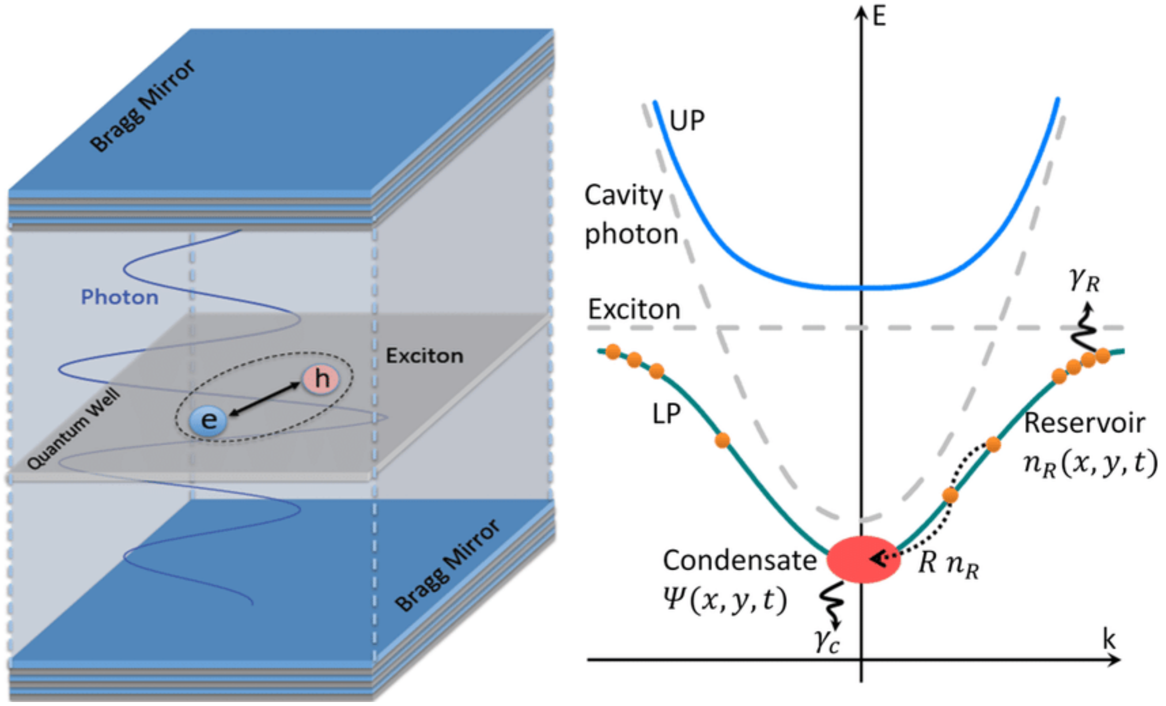


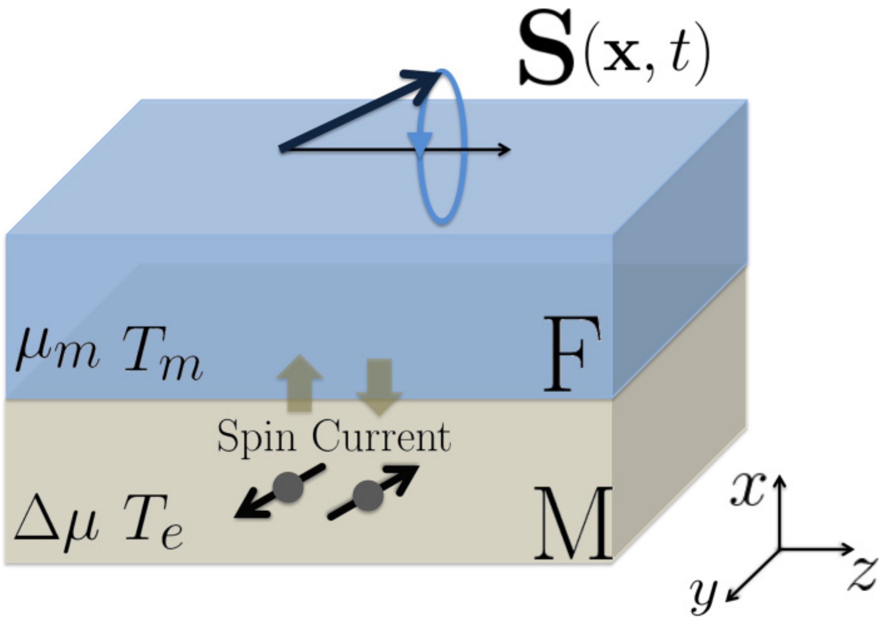
related works

- **formalism of choice**: Keldysh path integral formalism.

A. Kamenev, *Field Theory of Non-Equilibrium Systems* (Cambridge University Press, Cambridge, 2011)

  - ➡ effects of the baths on the active magnon system can be conveniently included by integrating out the baths.
- BEC of exciton-polaritons in the presence of continuous-wave pumping and photon losses through cavity mirrors.
  - ➡ I. Carusotto and C. Ciuti, Rev. Mod. Phys. 85, 299 (2013)
  - ➡ M. H. Szymańska, J. Keeling and P. B. Littlewood, arXiv:1206.1784





- magnon BEC in ferromagnetic insulator
  - ➡ R. E. Troncoso, A. Brataas, and R. A. Duine, Phys. Rev. B **99**, 104426 (2019)

integrating out baths

- total Keldysh partition function (magnon + baths):

$$Z = \int \mathcal{D}\{a_{\boldsymbol{q}}(\Omega), b_{\boldsymbol{i},\alpha}(\Omega), \psi_{\boldsymbol{i},\boldsymbol{k}\sigma}(\omega)\} \exp\left\{\iota\mathcal{S}_F + \iota\sum_{\boldsymbol{i}}(\mathcal{S}_{\boldsymbol{i}}^B + \mathcal{S}_{\boldsymbol{i}}^m)\right\}$$

$$\to \int \mathcal{D}\{a_{\boldsymbol{q}}(\Omega)\} \exp\{\iota\bar{\mathcal{S}}_F\}$$

- effective magnon action:

$$\bar{\mathcal{S}}_F = \sum_{\boldsymbol{q}} \int \frac{d\Omega}{2\pi} \begin{pmatrix} a_{\boldsymbol{q}}^{c*}(\Omega) & a_{\boldsymbol{q}}^{q*}(\Omega) \end{pmatrix} \begin{pmatrix} D_{\boldsymbol{q}}^K(\Omega) & D_{\boldsymbol{q}}^R(\Omega) \\ D_{\boldsymbol{q}}^A(\Omega) & 0 \end{pmatrix}^{-1} \begin{pmatrix} a_{\boldsymbol{q}}^c(\Omega) \\ a_{\boldsymbol{q}}^q(\Omega) \end{pmatrix}$$

$$- \frac{1}{\mathcal{N}} \sum_{\{\boldsymbol{q}_n\}} \int_{-\infty}^{\infty} dt\ V_{\boldsymbol{q}_1\boldsymbol{q}_3} \Big[a_{\boldsymbol{q}_1}^{c*}(t) a_{\boldsymbol{q}_2}^{c*}(t) a_{\boldsymbol{q}_3}^{c}(t) a_{\boldsymbol{q}_4}^{q}(t)$$

$$+ a_{\boldsymbol{q}_1}^{q*}(t) a_{\boldsymbol{q}_2}^{q*}(t) a_{\boldsymbol{q}_3}^{q}(t) a_{\boldsymbol{q}_4}^{c}(t) + c.c. \Big] \delta_{\boldsymbol{q}_1+\boldsymbol{q}_2,\boldsymbol{q}_3+\boldsymbol{q}_4}$$

nonequilibrium magnons

- retarded magnon propagator:

$$D^R_{\boldsymbol{q}}(\Omega) = \frac{1}{\Omega - (\varepsilon_{\boldsymbol{q}} + \mu_0)/\hbar - \Sigma^R(\Omega) - \Pi^R(\Omega)}$$

$$= \frac{1}{\Omega - (\varepsilon_{\boldsymbol{q}} + \mu_0)/\hbar + \imath\alpha_B\Omega + \imath\alpha_F(\Omega - \mu_s/\hbar)}$$

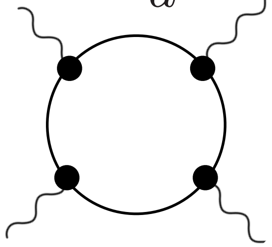
$$\alpha_F = \pi|\eta_{\boldsymbol{i}}|^2\rho_0^2\hbar^2$$

- Keldysh magnon propagator:

$$D^K_{\boldsymbol{q}}(\Omega) = \frac{\imath}{2}\frac{\Pi^K(\Omega) + \Sigma^K(\Omega)}{\alpha_B\Omega + \alpha_F(\Omega - \mu_s/\hbar)}[D^R_{\boldsymbol{q}}(\Omega) - D^A_{\boldsymbol{q}}(\Omega)]$$

$$\equiv [2N(\Omega) + 1][D^R_{\boldsymbol{q}}(\Omega) - D^A_{\boldsymbol{q}}(\Omega)]$$

- quartic correction induced by metallic bath:



$$\mathcal{S}_d^{(4)} = -\int dt \sum_{\boldsymbol{i}} \Big[u_1 a^{*c}_{\boldsymbol{i}} a^{*c}_{\boldsymbol{i}} a^c_{\boldsymbol{i}} a^q_{\boldsymbol{i}} - u_1 a^{*c}_{\boldsymbol{i}} a^{*q}_{\boldsymbol{i}} a^c_{\boldsymbol{i}} a^c_{\boldsymbol{i}} + u_2 a^{*q}_{\boldsymbol{i}} a^{*q}_{\boldsymbol{i}} a^c_{\boldsymbol{i}} a^q_{\boldsymbol{i}} - u_2 a^{*c}_{\boldsymbol{i}} a^{*q}_{\boldsymbol{i}} a^q_{\boldsymbol{i}} a^q_{\boldsymbol{i}}$$

$$+ u_3 a^{*c}_{\boldsymbol{i}} a^{*c}_{\boldsymbol{i}} a^q_{\boldsymbol{i}} a^q_{\boldsymbol{i}} + u_3 a^{*q}_{\boldsymbol{i}} a^{*q}_{\boldsymbol{i}} a^c_{\boldsymbol{i}} a^c_{\boldsymbol{i}} + u_4 a^{*c}_{\boldsymbol{i}} a^{*q}_{\boldsymbol{i}} a^c_{\boldsymbol{i}} a^q_{\boldsymbol{i}} + u_5 a^{*q}_{\boldsymbol{i}} a^{*q}_{\boldsymbol{i}} a^q_{\boldsymbol{i}} a^q_{\boldsymbol{i}}\Big]$$

- ➡ complex quartic vertices: $u_i \propto i\alpha_F^2$
- ➡ L. M. Sieberer, M. Buchhold and S. Diehl, Rep. Prog. Phys. **79** 096001 (2016): phenomenological discussion of complex quartic vertices

ignore quartic corrections induced by the metal

S. Takei, Phys. Rev. B **100**, 134440 (2019)

nonequilibrium magnons

- magnon distribution function:

$$N(\Omega) = \frac{1}{\alpha\Omega - \alpha_F \mu_s/\hbar} \left[\frac{\alpha_B \Omega}{e^{\hbar\Omega/k_B T_B} - 1} + \frac{\alpha_F(\Omega - \mu_s/\hbar)}{e^{(\hbar\Omega - \mu_s)/k_B T_F} - 1} \right]$$

Magnon distribution function is generally non-thermal

- equal bath temperatures and zero spin bias, i.e., $T_B = T_F \equiv T$, $\mu_s = 0$:

$$N(\Omega) = \frac{1}{e^{\hbar\Omega/k_B T} - 1}$$

- mismatched bath temperatures and zero spin bias, i.e., $T_B \neq T_F$, $\mu_s = 0$:

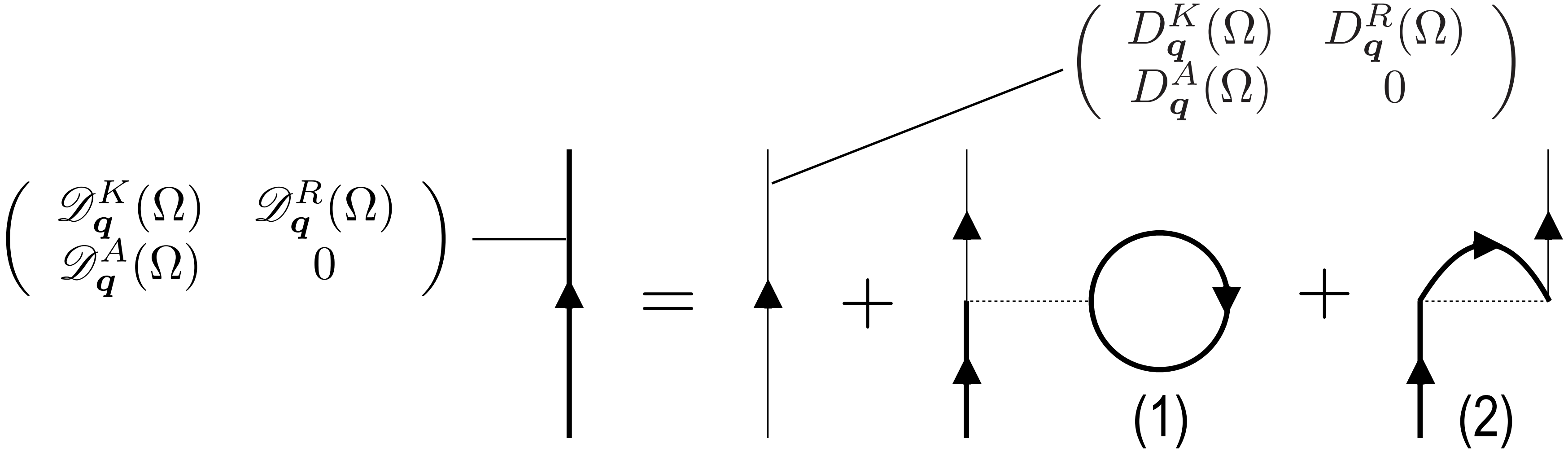
$$N(\Omega) = \frac{\alpha_B}{\alpha} \frac{1}{e^{\hbar\Omega/k_B T_B} - 1} + \frac{\alpha_F}{\alpha} \frac{1}{e^{\hbar\Omega/k_B T_F} - 1} \qquad \alpha = \alpha_B + \alpha_F$$

Even when electrical pumping is zero, magnon distribution function is in general given by linear combination of two bath distribution functions weighted by the respective Gilbert damping parameters associated with each of the baths.

S. Takei, Phys. Rev. B **100**, 134440 (2019)

mean-field theory

- self-consistent Hartree-Fock approximation to one-loop order:



$$\begin{pmatrix} \mathscr{D}^K_{\boldsymbol{q}}(\Omega) & \mathscr{D}^R_{\boldsymbol{q}}(\Omega) \\ \mathscr{D}^A_{\boldsymbol{q}}(\Omega) & 0 \end{pmatrix} \qquad \begin{pmatrix} D^K_{\boldsymbol{q}}(\Omega) & D^R_{\boldsymbol{q}}(\Omega) \\ D^A_{\boldsymbol{q}}(\Omega) & 0 \end{pmatrix}$$

- Hartree-Fock self-energy:

$$\tilde{\Sigma}^R_{\boldsymbol{q}} = \tilde{\Sigma}^A_{\boldsymbol{q}} = -\frac{4}{\mathcal{N}}\frac{1}{\hbar}\sum_{\boldsymbol{q}'}\int\frac{d\Omega}{2\pi}N(\Omega)\,(\overset{(2)}{V_{\boldsymbol{q}\boldsymbol{q}'}} + \overset{(1)}{V_{\boldsymbol{q}'\boldsymbol{q}'}})\,\mathrm{Im}\mathscr{D}^R_{\boldsymbol{q}'}(\Omega) \qquad \tilde{\Sigma}^K_{\boldsymbol{q}} = 0$$

- self-consistent condition for the magnon spectrum:

$$\tilde{\varepsilon}_{\boldsymbol{q}} + \mu = \varepsilon_{\boldsymbol{q}} + \mu_0 - \frac{4}{\mathcal{N}}\sum_{\boldsymbol{q}'}\int\frac{d\Omega}{2\pi}N(\Omega)\,(V_{\boldsymbol{q}\boldsymbol{q}'} + V_{\boldsymbol{q}'\boldsymbol{q}'})\,\mathrm{Im}\{\mathscr{D}^R_{\boldsymbol{q}'}(\Omega)\}$$

- For dilute magnons $n \ll S$, renormalization of the magnon effective mass is negligible so we consider only the self-consistency condition for the magnon gap:

$$\mu = \mu_0 + 2n[K + J_{\boldsymbol{0}}(4\lambda - 1)] \qquad n = -\frac{2}{\mathcal{N}}\sum_{\boldsymbol{q}'}\int\frac{d\Omega}{2\pi}N(\Omega)\mathrm{Im}\{\mathscr{D}^R_{\boldsymbol{q}'}(\Omega)\}$$

S. Takei, Phys. Rev. B **100**, 134440 (2019)

critical line (zero pumping)

- locus of critical magnetic field giving $\mu = 0$:

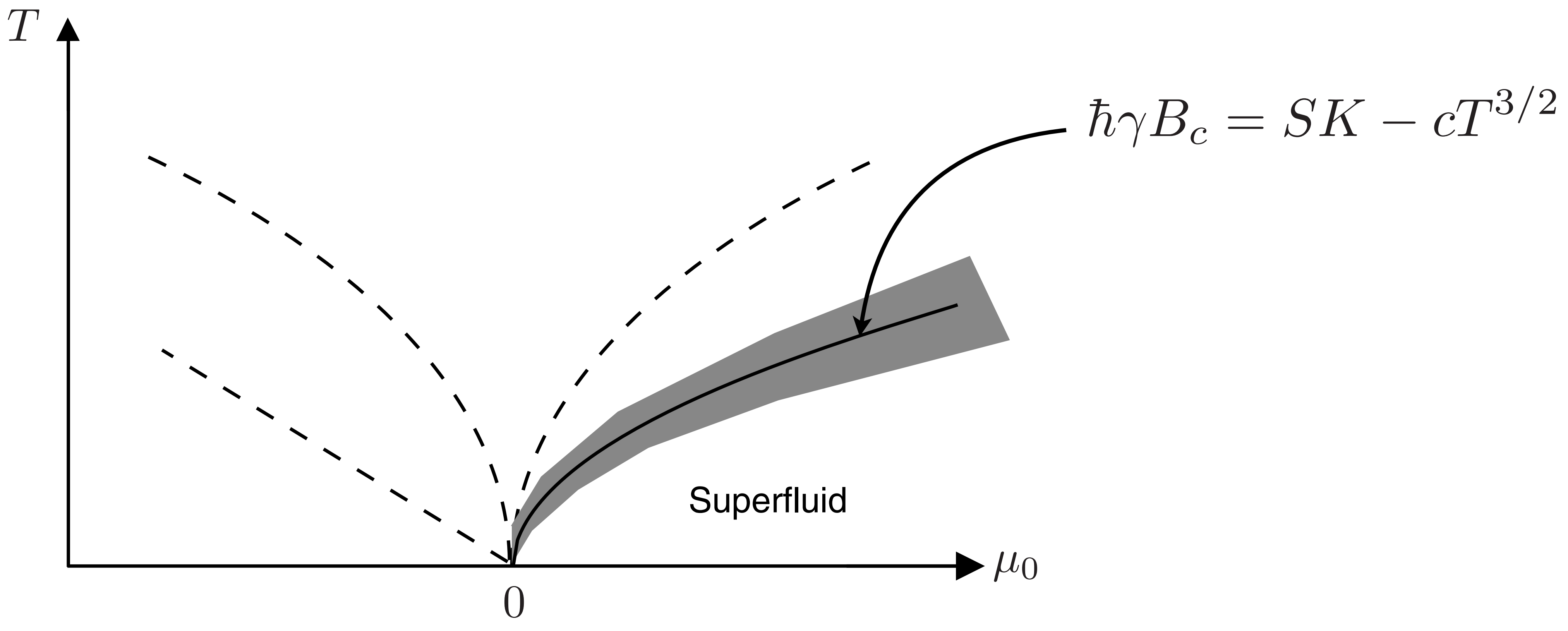
$$\Gamma_\alpha = (1+\alpha^2)^{1/4}\cos[(\tan^{-1}\alpha)/2]$$

$$c = [K + J_{\mathbf{0}}(4\lambda - 1)]\frac{\zeta_{3/2}(1)}{\sqrt{2}}\left(\frac{k_B}{\pi J_{\mathbf{0}} S}\right)^{3/2}$$

$$\hbar\gamma B_c = SK - \Gamma_\alpha c\left[\frac{\alpha_B}{\alpha}T_B^{3/2} + \frac{\alpha_F}{\alpha}T_F^{3/2}\right]$$

$$\alpha = \alpha_B + \alpha_F$$

- $T_B = T_F \equiv T$ and $\alpha = 0$ gives the equilibrium dilute Bose gas result (i.e., “Sachdev’s book”)



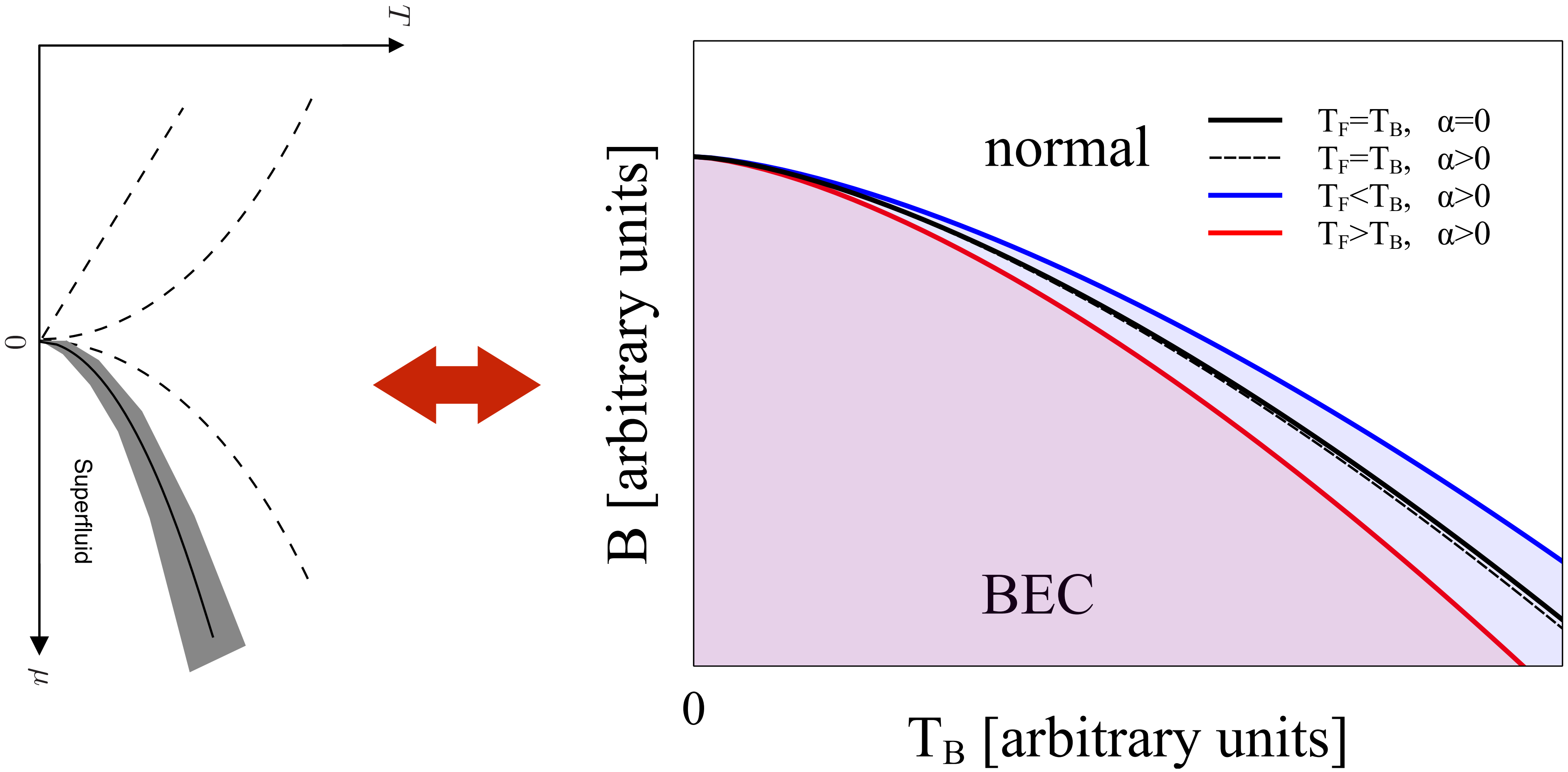
$$\hbar\gamma B_c = SK - cT^{3/2}$$

S. Takei, Phys. Rev. B **100**, 134440 (2019)

critical line (zero pumping)

- critical line for open magnon system and general bath temperatures:

$$\hbar\gamma B_c = SK - \Gamma_\alpha c\left[\frac{\alpha_B}{\alpha}T_B^{3/2} + \frac{\alpha_F}{\alpha}T_F^{3/2}\right]$$



➡ $T_B = T_F$ and $\alpha = 0 \rightarrow$ equilibrium dilute Bose gas result (Sachdev).

➡ $T_B = T_F$ and $\alpha > 0 \rightarrow$ BEC region shrinks.

➡ $T_F = 0.8T_B$ and $\alpha > 0 \rightarrow$ BEC region grows.

➡ $T_F = 1.2T_B$ and $\alpha > 0 \rightarrow$ BEC region shrinks.

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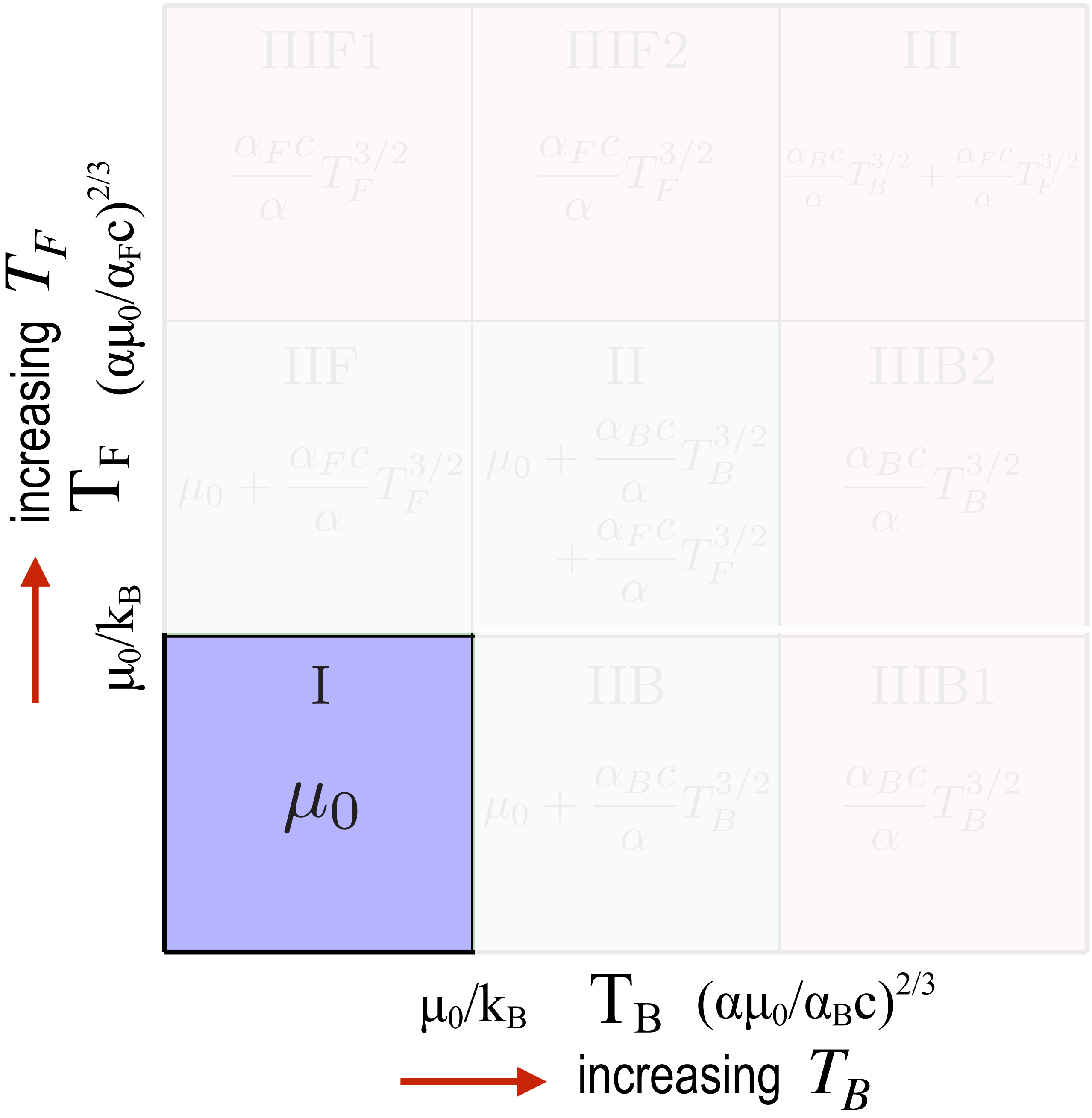
thermal crossovers (zero pumping)

- self-consistent equation for the magnon gap:

$$\mu = \mu_0 + \cancel{\frac{\alpha_B}{\alpha}\left[\frac{\zeta_{3/2}(e^{-\mu/k_B T_B})}{\zeta_{3/2}(1)}\right]} cT_B^{3/2} + \cancel{\frac{\alpha_F}{\alpha}\left[\frac{\zeta_{3/2}(e^{-\mu/k_B T_F})}{\zeta_{3/2}(1)}\right]} cT_F^{3/2}$$

- Regime I: low bath temperatures $T_B \ll \mu_0/k_B, \quad T_F \ll \mu_0/k_B$:



S. Takei, arXiv:1907.01043

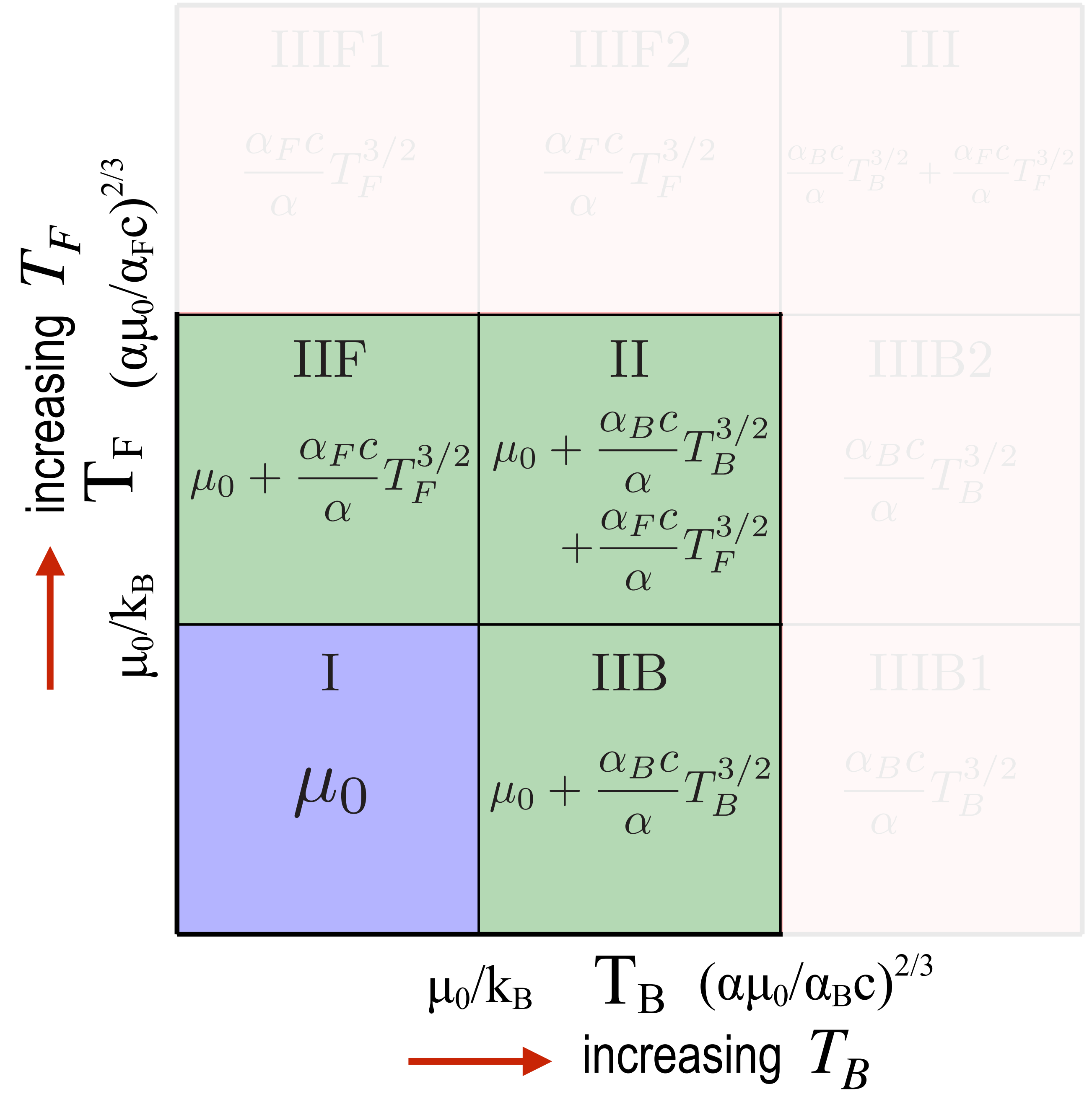
thermal crossovers (zero pumping)

- self-consistent equation for the magnon gap:

$$\mu = \mu_0 + \underbrace{\frac{\alpha_B}{\alpha}\left[\frac{\zeta_{3/2}(e^{-\mu/k_B T_B})}{\zeta_{3/2}(1)}\right] c T_B^{3/2} + \frac{\alpha_F}{\alpha}\left[\frac{\zeta_{3/2}(e^{-\mu/k_B T_F})}{\zeta_{3/2}(1)}\right] c T_F^{3/2}}_{\text{small correction}}$$

- Regime I: low bath temperatures
  $T_B \ll \mu_0/k_B, \quad T_F \ll \mu_0/k_B.$

- Regime II: intermediate bath temperatures
  $\mu_0/k_B \ll T_B \ll (\alpha\mu_0/\alpha_B c)^{2/3}$ OR
  $\mu_0/k_B \ll T_F \ll (\alpha\mu_0/\alpha_F c)^{2/3}.$



IIIF1 $\frac{\alpha_F c}{\alpha} T_F^{3/2}$	IIIF2 $\frac{\alpha_F c}{\alpha} T_F^{3/2}$	III $\frac{\alpha_B c}{\alpha} T_B^{3/2} + \frac{\alpha_F c}{\alpha} T_F^{3/2}$
IIF $\mu_0 + \frac{\alpha_F c}{\alpha} T_F^{3/2}$	II $\mu_0 + \frac{\alpha_B c}{\alpha} T_B^{3/2} + \frac{\alpha_F c}{\alpha} T_F^{3/2}$	IIIB2 $\frac{\alpha_B c}{\alpha} T_B^{3/2}$
I μ_0	IIB $\mu_0 + \frac{\alpha_B c}{\alpha} T_B^{3/2}$	IIIB1 $\frac{\alpha_B c}{\alpha} T_B^{3/2}$

S. Takei, arXiv:1907.01043

thermal crossovers (zero pumping)

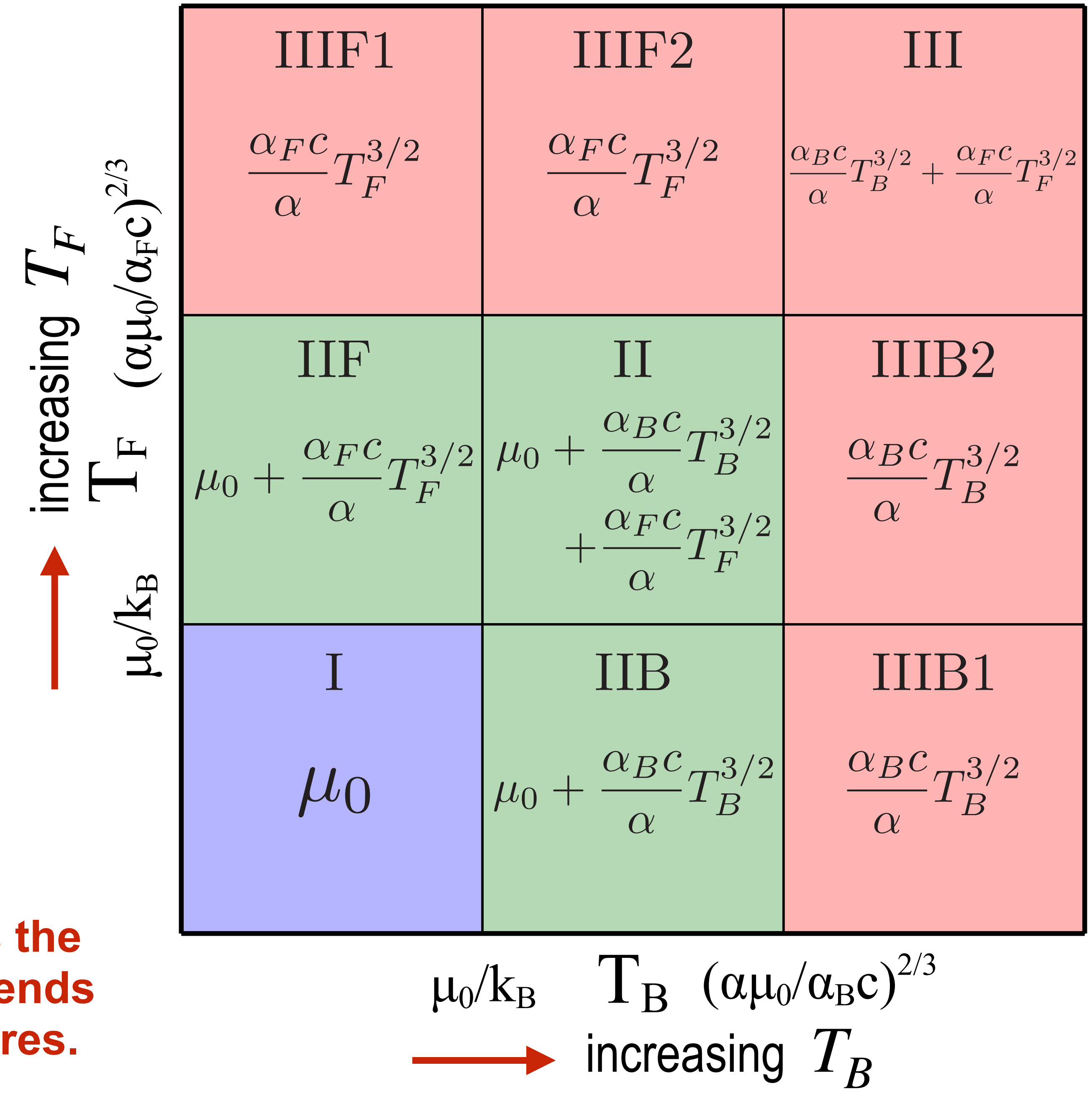
- self-consistent equation for the magnon gap:

$$\mu = \cancel{\mu_0} + \underbrace{\frac{\alpha_B}{\alpha}\left[\frac{\zeta_{3/2}(e^{-\mu/k_BT_B})}{\zeta_{3/2}(1)}\right]cT_B^{3/2} + \frac{\alpha_F}{\alpha}\left[\frac{\zeta_{3/2}(e^{-\mu/k_BT_F})}{\zeta_{3/2}(1)}\right]cT_F^{3/2}}_{\text{dominant}}$$

- Regime I: low bath temperatures
  $T_B \ll \mu_0/k_B, \quad T_F \ll \mu_0/k_B.$

- Regime II: intermediate bath temperatures
  $\mu_0/k_B \ll T_B \ll (\alpha\mu_0/\alpha_B c)^{2/3}$ OR
  $\mu_0/k_B \ll T_F \ll (\alpha\mu_0/\alpha_F c)^{2/3}.$

- Regime III: high bath temperatures
  $(\alpha\mu_0/\alpha_B c)^{2/3} \ll T_B$ OR
  $(\alpha\mu_0/\alpha_F c)^{2/3} \ll T_F.$



IIIF1 $\frac{\alpha_F c}{\alpha}T_F^{3/2}$	IIIF2 $\frac{\alpha_F c}{\alpha}T_F^{3/2}$	III $\frac{\alpha_B c}{\alpha}T_B^{3/2}+\frac{\alpha_F c}{\alpha}T_F^{3/2}$
IIF $\mu_0+\frac{\alpha_F c}{\alpha}T_F^{3/2}$	II $\mu_0+\frac{\alpha_B c}{\alpha}T_B^{3/2}$ $+\frac{\alpha_F c}{\alpha}T_F^{3/2}$	IIIB2 $\frac{\alpha_B c}{\alpha}T_B^{3/2}$
I μ_0	IIB $\mu_0+\frac{\alpha_B c}{\alpha}T_B^{3/2}$	IIIB1 $\frac{\alpha_B c}{\alpha}T_B^{3/2}$

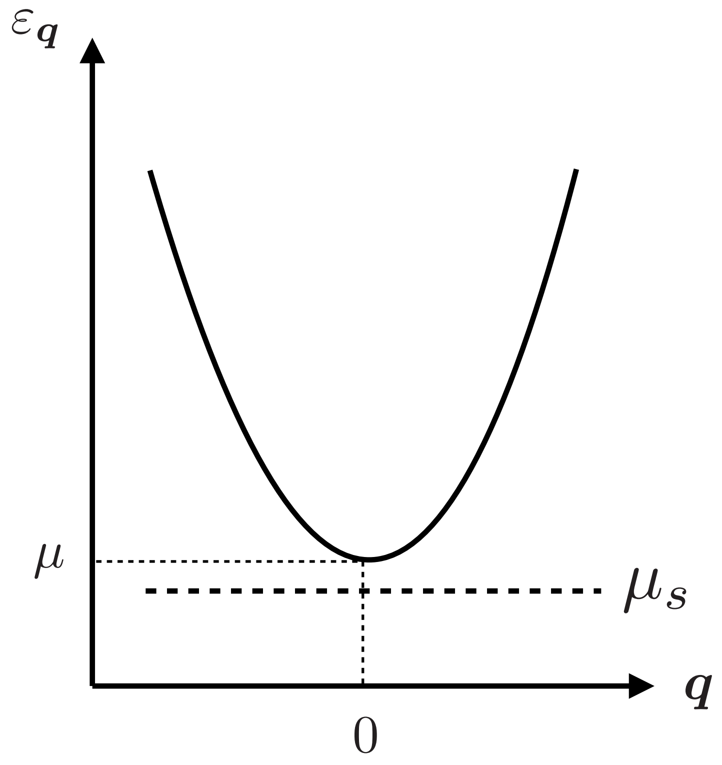
The bath temperature that ultimately determines the magnon gap in the high temperature regime depends on the relative magnitudes of the two temperatures.

S. Takei, arXiv:1907.01043

BEC instability by pumping

- BEC instability: imaginary part of the lowest energy pole changes sign from negative to positive [c.f. M. H. Szymańska, J. Keeling and P. B. Littlewood, arXiv:1206.1784].

$$\mathscr{D}^R_{\boldsymbol{q}}(\Omega) = \frac{1}{\Omega - (\varepsilon_{\boldsymbol{q}} + \mu)/\hbar + \boxed{\imath\alpha_B\Omega + \imath\alpha_F(\Omega - \mu_s/\hbar)}}$$



- condition for critical spin bias:

$$\alpha_B\mu + \alpha_F(\mu - \mu_s^c) = 0$$

➡ $$\mu_s^c = \underline{\left(1 + \frac{\alpha_B}{\alpha_F}\right)}\mu$$

elevated instability
[S. A. Bender et al., Phys. Rev. B **90**, 094409 (2014)]

- this instability is also signaled by divergence in the magnon distribution function.

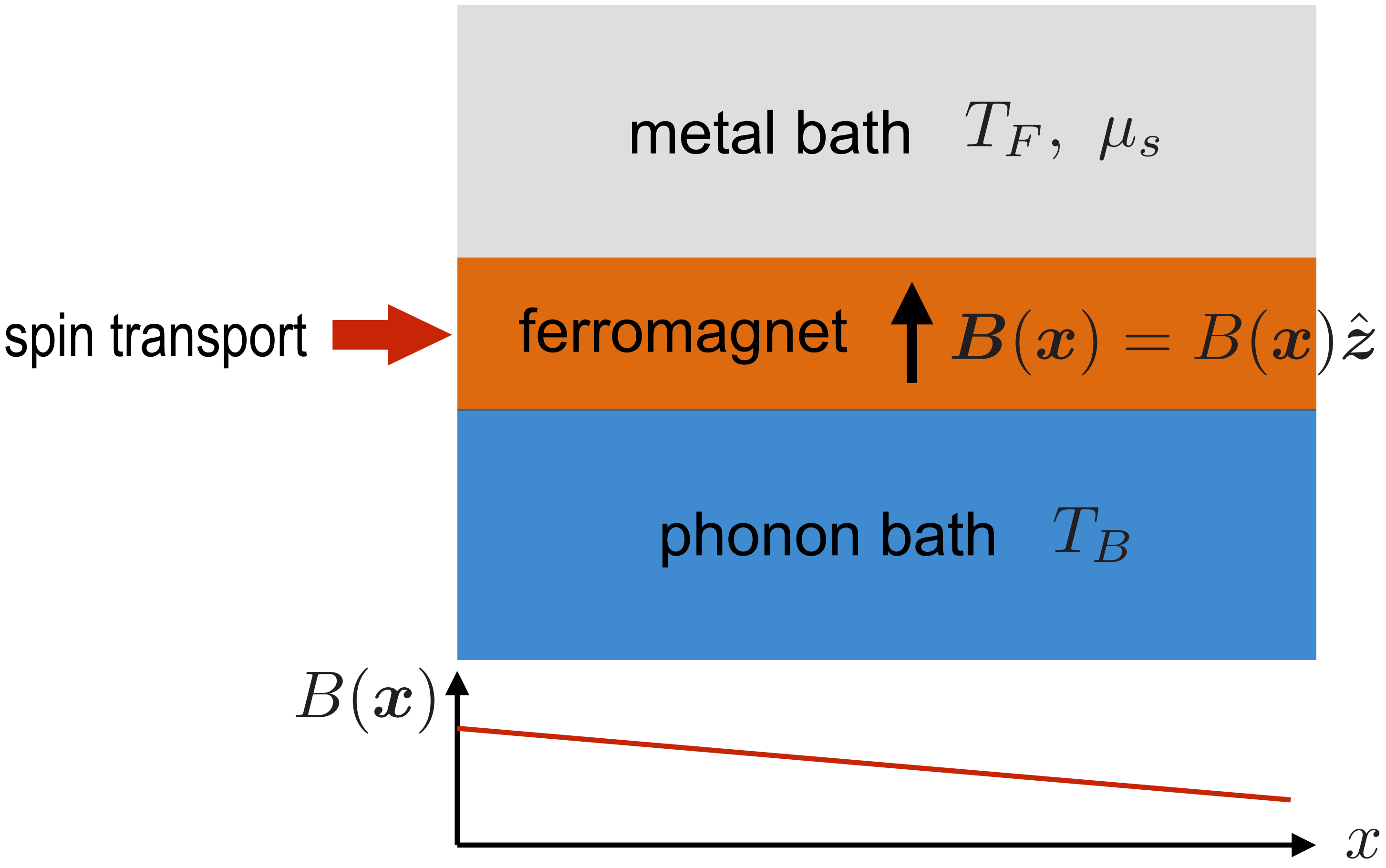
$$N(\Omega) = \frac{1}{\underline{\alpha\Omega - \alpha_F\mu_s/\hbar}}\left[\frac{\alpha_B\Omega}{e^{\hbar\Omega/k_BT_B} - 1} + \frac{\alpha_F(\Omega - \mu_s/\hbar)}{e^{(\hbar\Omega - \mu_s)/k_BT_F} - 1}\right]$$

divergence

S. Takei, Phys. Rev. B **100**, 134440 (2019)

spin conductivity

- dc spin conductivity for spin current polarized along z axis, computed within the self-consistent Hartree-Fock approximation.



- magnon current flowing in response to magnetic field gradient along $\hat{\boldsymbol{x}}$

$$H'(t) = \hbar\gamma \int d^3\boldsymbol{x}\, B(\boldsymbol{x}, t) a^\dagger(\boldsymbol{x}) a(\boldsymbol{x})$$

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spin conductivity

- Kubo formula for magnon current (one-loop calculation):

$$J(\boldsymbol{x},t)=\hbar\gamma\int d^3\boldsymbol{x}'\int dt'\chi(\boldsymbol{x}-\boldsymbol{x}',t-t')B(\boldsymbol{x}',t')$$

$$\chi(\boldsymbol{x}-\boldsymbol{x}',t-t')=-\frac{\iota}{\hbar}\theta(t-t')\langle[j(\boldsymbol{x},t),\varrho(\boldsymbol{x}',t')]\rangle_{\text{Noneq HF}}$$

➡ magnon density: $\varrho(\boldsymbol{x},t)=a^\dagger(\boldsymbol{x},t)a(\boldsymbol{x},t)$

➡ magnon current: $j(\boldsymbol{x},t)=i\frac{J_{\mathbf{0}}S\xi^2}{2}[\partial_x a^\dagger(\boldsymbol{x})]a(\boldsymbol{x})+h.c.$

- spin conductivity:

$$J_{\boldsymbol{q}}(\omega)=\sigma(\boldsymbol{q},\omega)(-iq_x)\hbar\gamma B_{\boldsymbol{q}}(\omega)\ ,\quad \sigma(\boldsymbol{q},\omega)=\frac{\chi(\boldsymbol{q},\omega)}{-iq_x}$$

- dc limit:

$$\sigma_0=-\frac{2}{3}\left(\frac{J_{\mathbf{0}}S\xi^2}{2\hbar}\right)^2\int\frac{d\Omega}{2\pi}\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\partial_\Omega N(\Omega)k^2[2\mathrm{Im}\mathcal{D}^R_{\boldsymbol{k}}(\Omega)]^2$$

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spin conductivity (zero pumping)

- dc spin conductivity for zero spin bias, i.e., $\mu_s = 0$

$$\sigma_0 = \frac{1}{24\pi^2\alpha}\left[\underbrace{\frac{\alpha_B}{\alpha}\sqrt{\frac{2k_BT_B}{J_{\mathbf{0}}S\xi^2}}\,\mathfrak{S}\left(\frac{\mu}{k_BT_B}\right)}_{\text{thermal contribution from bosonic bath}} + \underbrace{\frac{\alpha_F}{\alpha}\sqrt{\frac{2k_BT_F}{J_{\mathbf{0}}S\xi^2}}\,\mathfrak{S}\left(\frac{\mu}{k_BT_F}\right)}_{\text{thermal contribution from metallic bath}}\right]$$

$$\mathfrak{S}(s) \approx \begin{cases} \frac{3\sqrt{\pi}}{4}\frac{e^{-s}}{s} & , \quad s \gg 1 \\ \frac{3\pi}{2}\frac{1}{\sqrt{s}} & , \quad s \ll 1 \end{cases}$$

- equal bath temperatures ($T_B = T_F \equiv T$):

$$\sigma_0 = \frac{1}{24\pi^2\alpha}\sqrt{\frac{2k_BT}{J_{\mathbf{0}}S\xi^2}} \times \begin{cases} \frac{3\sqrt{\pi}}{4}\frac{k_BT}{\mu}e^{-\mu/k_BT} \, , & \frac{\mu}{k_BT} \gg 1 \\ \frac{3\pi}{2}\sqrt{\frac{k_BT}{\mu}} \, , & \frac{\mu}{k_BT} \ll 1 \end{cases}$$

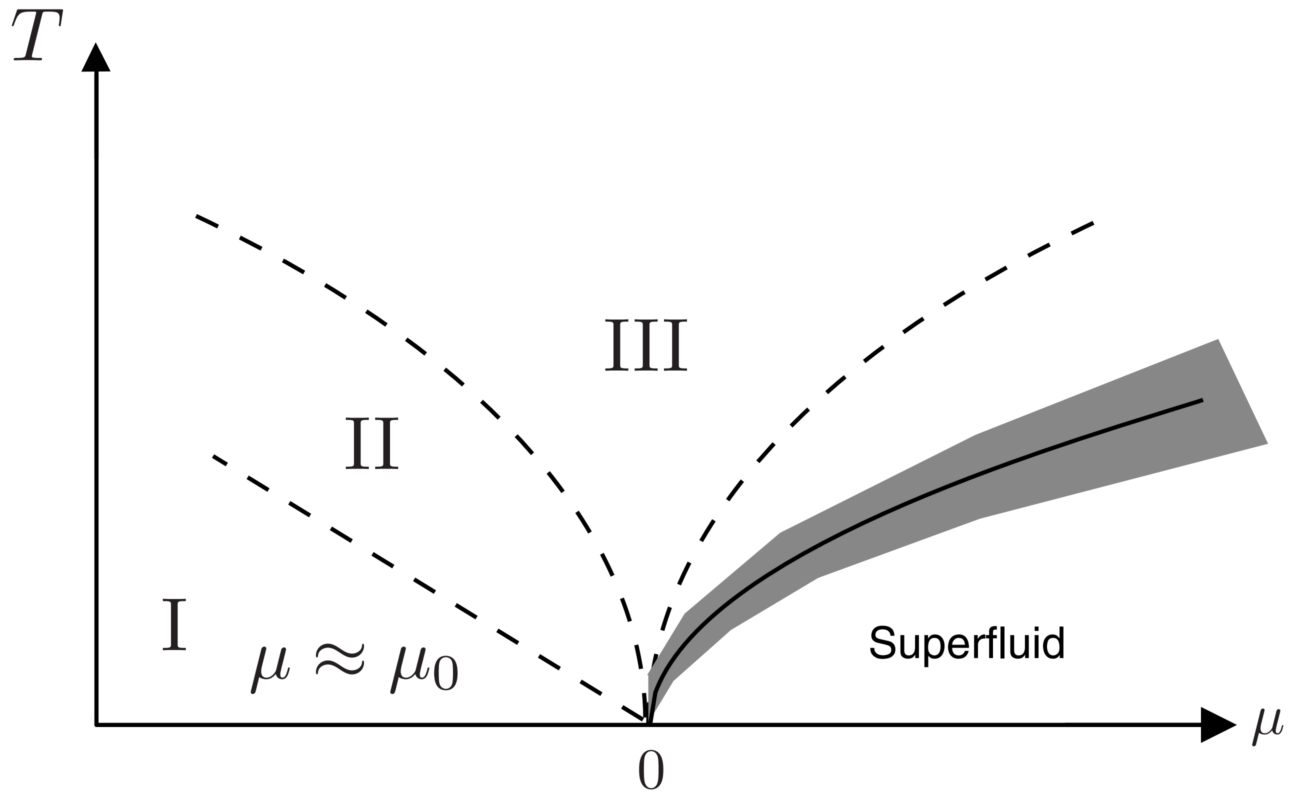
low temperatures

- low temperature regime I, i.e., $T \ll \mu_0/k_B$

$$\sigma_0 = \frac{1}{24\pi^2\alpha}\sqrt{\frac{2k_BT}{J_{\mathbf{0}}S\xi^2}} \times \begin{cases} \boxed{\frac{3\sqrt{\pi}}{4}\frac{k_BT}{\mu}e^{-\mu/k_BT} \,, \qquad \frac{\mu}{k_BT} \gg 1} \\ \frac{3\pi}{2}\sqrt{\frac{k_BT}{\mu}} \,, \qquad \frac{\mu}{k_BT} \ll 1 \end{cases}$$

$$\Downarrow$$

$$\sigma_0 \approx \frac{\sqrt{2}}{32\alpha\xi}\left(\frac{J_{\mathbf{0}}S}{\mu_0}\right)\left(\frac{k_BT}{\pi J_{\mathbf{0}}S}\right)^{3/2} e^{-\mu_0/k_BT}$$



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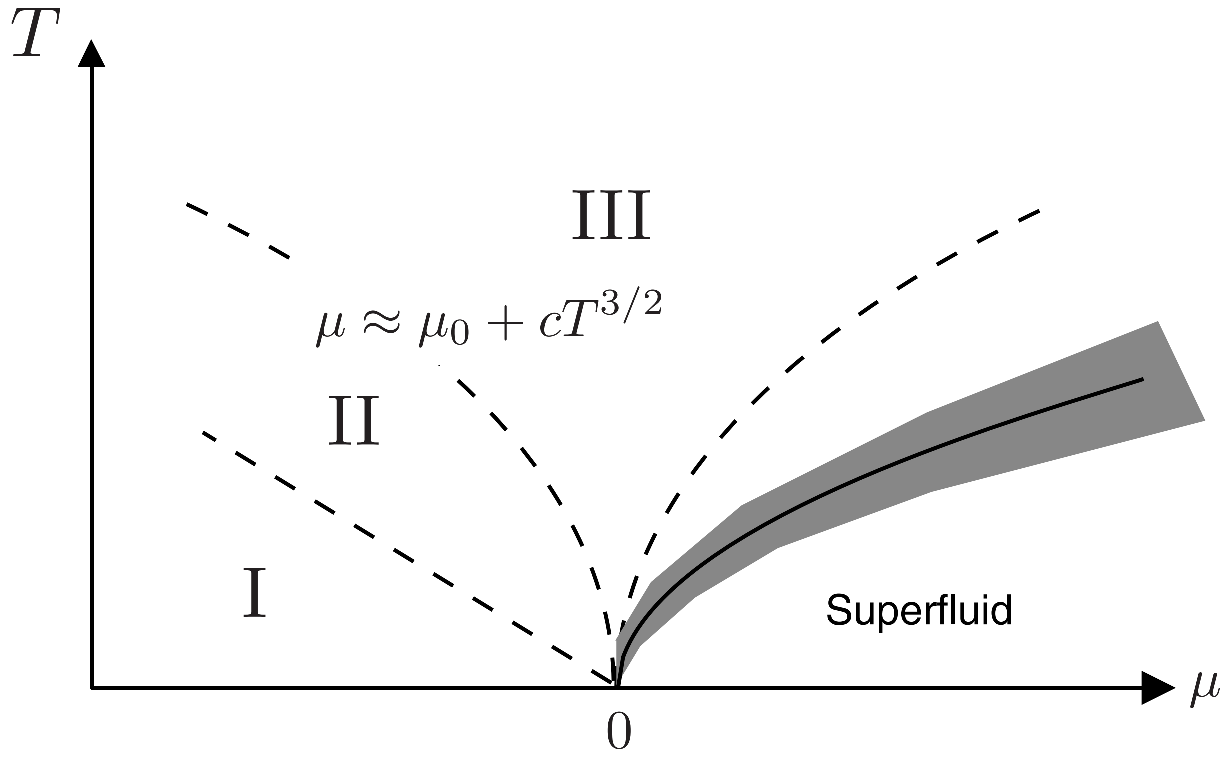
higher temperatures

- intermediate to high temperature regimes II and III (i.e., $T \gg \mu_0/k_B$)

$$\sigma_0 = \frac{1}{24\pi^2\alpha}\sqrt{\frac{2k_BT}{J_{\mathbf{0}}S\xi^2}} \times \begin{cases} \frac{3\sqrt{\pi}}{4}\frac{k_BT}{\mu}e^{-\mu/k_BT}\,, & \frac{\mu}{k_BT} \gg 1 \\ \boxed{\frac{3\pi}{2}\sqrt{\frac{k_BT}{\mu}}\,, \qquad \frac{\mu}{k_BT} \ll 1} \end{cases}$$

$$\Downarrow$$

$$\sigma_0 \approx \frac{\sqrt{2}}{16(\alpha_B+\alpha_F)\xi_J}\sqrt{\frac{J_{\mathbf{0}}S}{\mu_0+cT^{3/2}}}\left(\frac{k_BT}{\pi J_{\mathbf{0}}S}\right)$$



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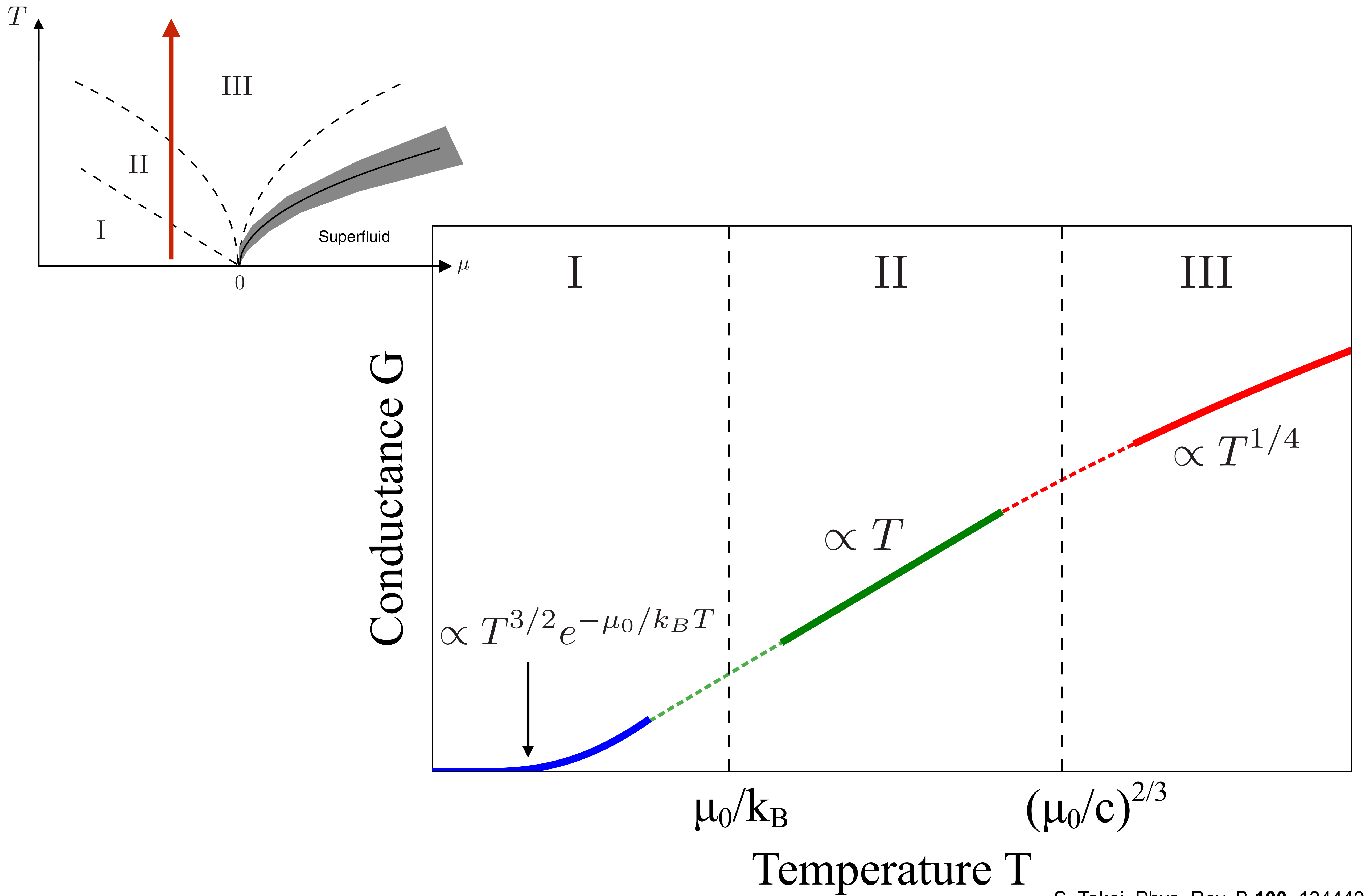
spin conductivity (zero pumping)

- thermal crossovers in spin conductivity



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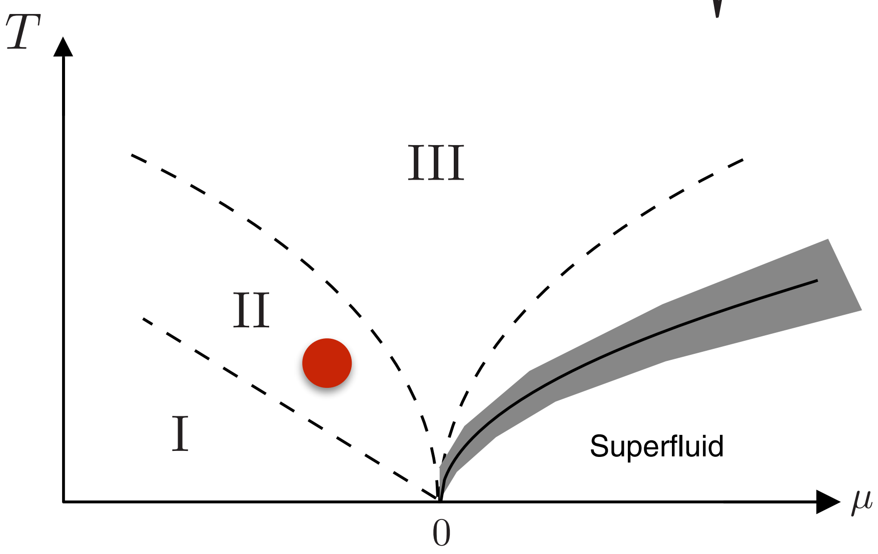
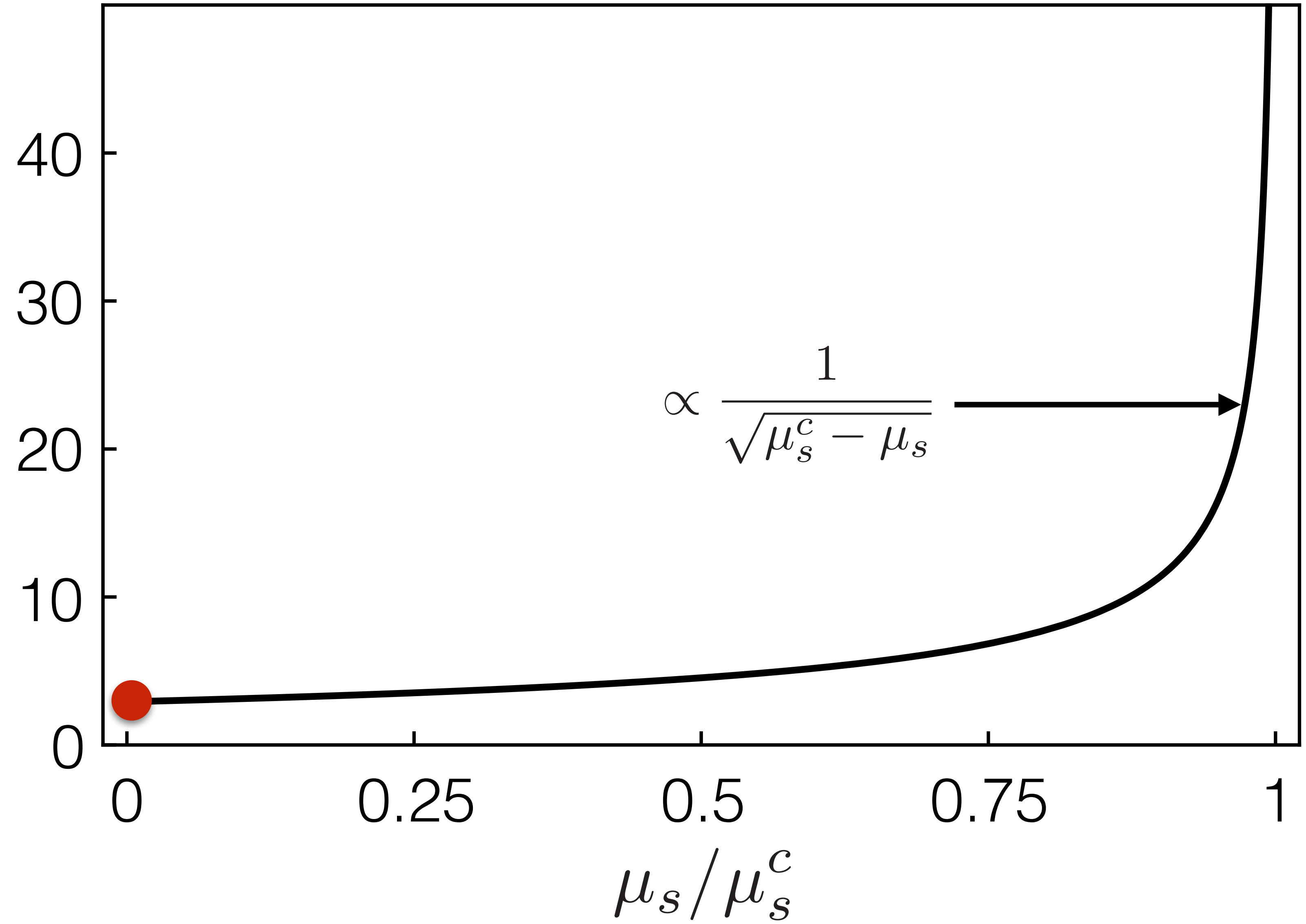
spin conductivity at finite spin bias

- spin conductivity for fixed magnetic field μ_0 and fixed bath temperatures $T_B = T_F = T$.
- square-root divergence in spin conductivity as $\mu_s \rightarrow \mu_s^c$.

$$\sigma_0 = \frac{1}{6\pi^2}\sqrt{\frac{2}{J_{\mathbf{0}} S \xi_J^2}} \int_{\mu/\hbar}^{\infty} d\Omega \; N(\Omega) \frac{\partial}{\partial\Omega} \frac{(\hbar\Omega-\mu)^{3/2}}{\alpha\hbar\Omega-\alpha_F\mu_s} \propto \frac{1}{\sqrt{\mu_s^c-\mu_s}}$$





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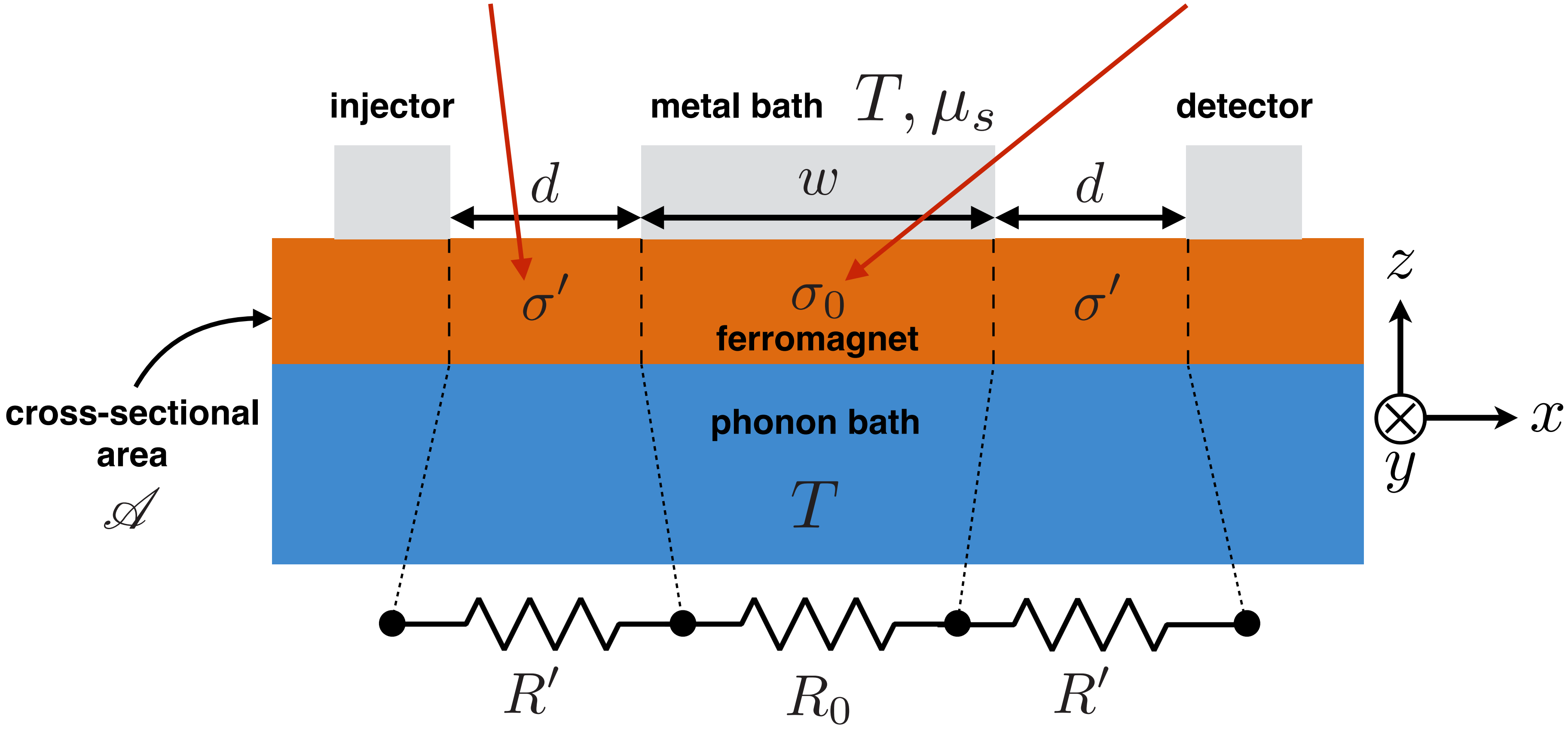
experiment? (zero spin bias)

- inspired by three-terminal spin transport experiment (arXiv:1812.01334)
- focus on equal bath temperatures ($T_B = T_F \equiv T$).

$$\sigma' = \sigma_0\big|_{\alpha_F=0} = \frac{1}{24\pi^2\alpha_B}\sqrt{\frac{2k_BT}{J_0S\xi^2}}\,\mathfrak{S}\left(\frac{\mu}{k_BT}\right) \qquad \sigma_0 = \frac{1}{24\pi^2\alpha}\sqrt{\frac{2k_BT}{J_0S\xi^2}}\,\mathfrak{S}\left(\frac{\mu}{k_BT}\right)$$



- a simple model for the total spin resistance between injector and detector contacts:

$$R = 2R' + R_0 = \frac{2d}{\sigma'\mathscr{A}} + \frac{w}{\sigma_0\mathscr{A}}$$

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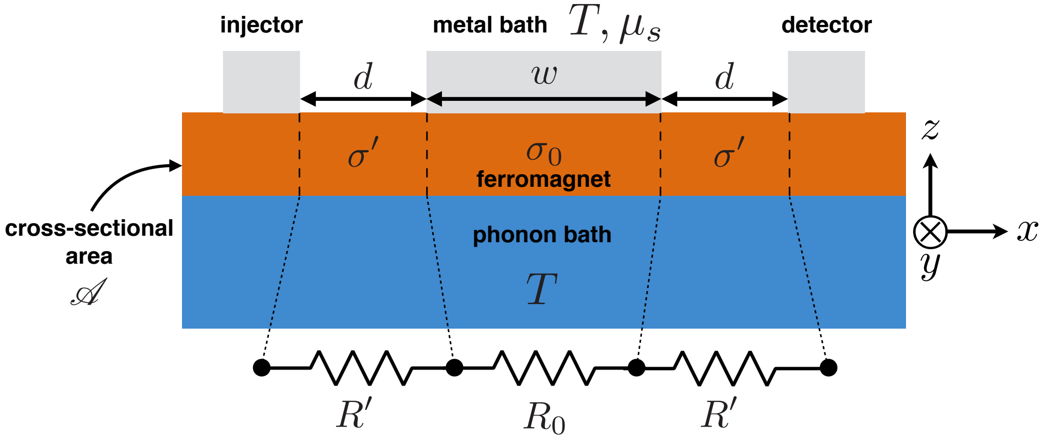
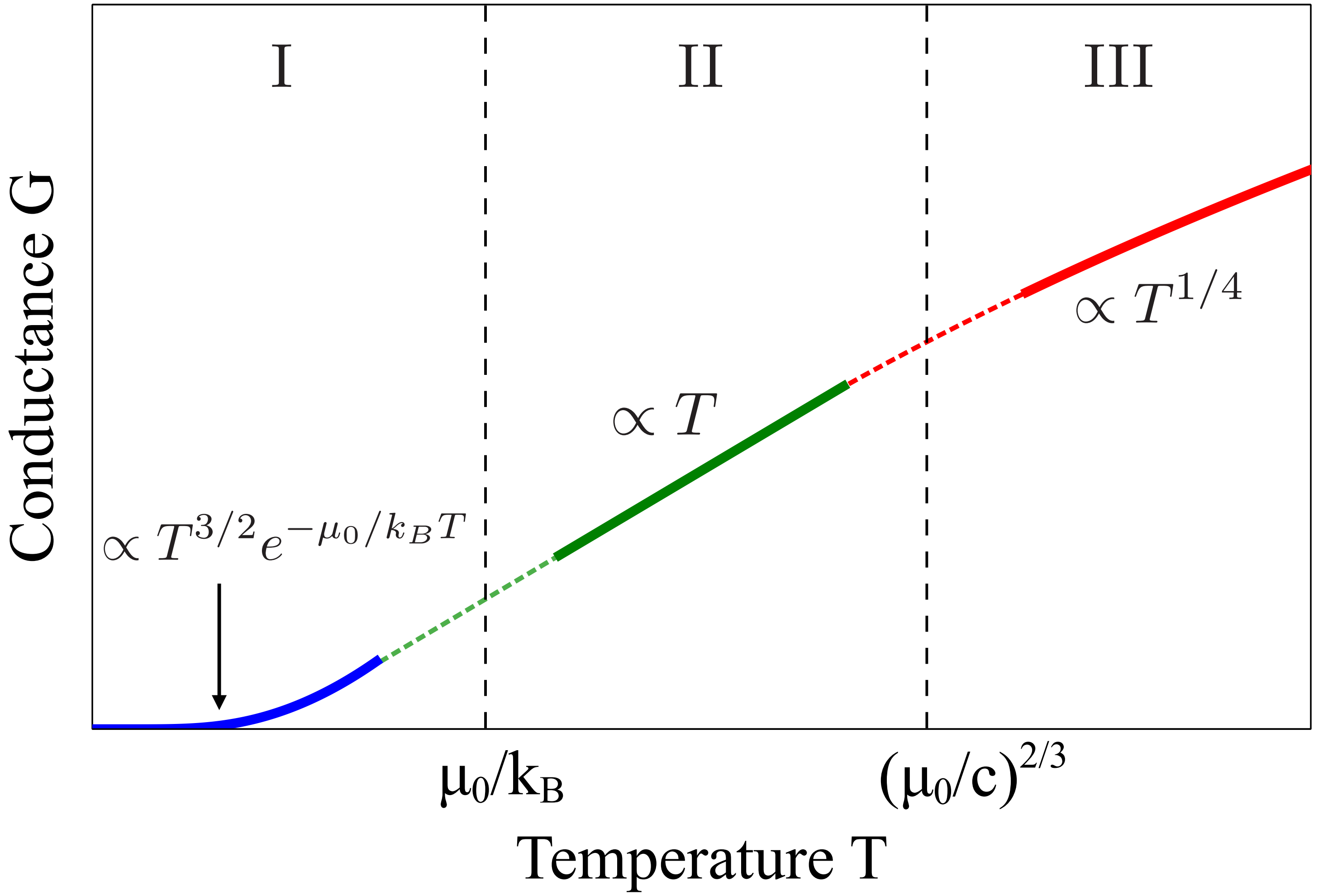
experiment? (zero spin bias)

- total spin conductance

$$G = \frac{1}{R} = \frac{\mathscr{A}}{24\pi^2(2d\alpha_B + w\alpha)}\sqrt{\frac{2k_BT}{J_{\mathbf{0}}S\xi^2}}\,\mathfrak{S}\left(\frac{\mu}{k_BT}\right)$$

$$\propto \sqrt{T} \times \begin{cases} \frac{3\sqrt{\pi}}{4}\frac{k_BT}{\mu_0}e^{-\mu_0/k_BT}\,, & \frac{\mu_0}{k_BT} \gg 1 \\ \frac{3\pi}{2}\sqrt{\frac{k_BT}{\mu_0+cT^{3/2}}}\,, & \frac{\mu_0}{k_BT} \ll 1 \end{cases}$$





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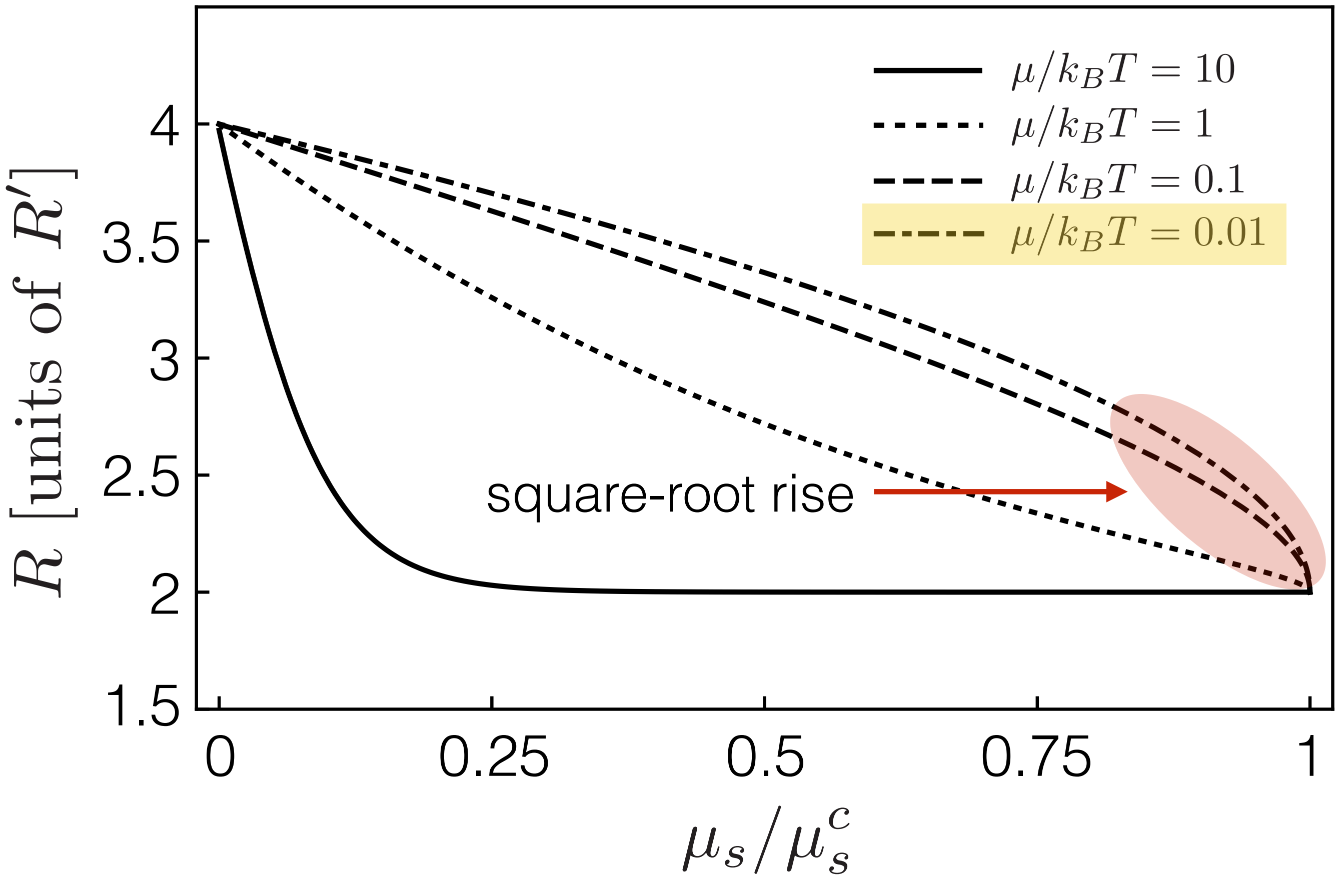
experiment? (finite spin bias)

- fix magnetic field μ_0 and fix bath temperatures $T_B = T_F = T$.
- increase spin bias $\mu_s \to \mu_s^c$.
- total spin resistance:

$$R = 2R' + R_0 = \frac{2d}{\sigma' \mathscr{A}} + \underline{\frac{w}{\sigma_0 \mathscr{A}}}$$

$$\propto \sqrt{\mu_s - \mu_s^c}$$



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summary

- nonequilibrium steady-state of magnons coupled to phonon and metal baths with two different temperatures and subjected to electrical spin injection from the metal.
- perturb around the 3d BEC quantum critical point
- magnon distribution function is generally non-thermal → spin Seebeck effect alone cannot trigger BEC.
- two bath temperatures enriches thermal crossover behavior and the corresponding thermal crossover in spin conductivity
- square-root divergence in bulk dc spin conductivity as electrically-induced BEC instability is approached

- the role of magnon-magnon interactions as one approaches the Ginzburg region — strong-coupling fixed point and its relevance to experiment?
- nonequilibrium RG
- dipolar interactions → how they affect universality class.