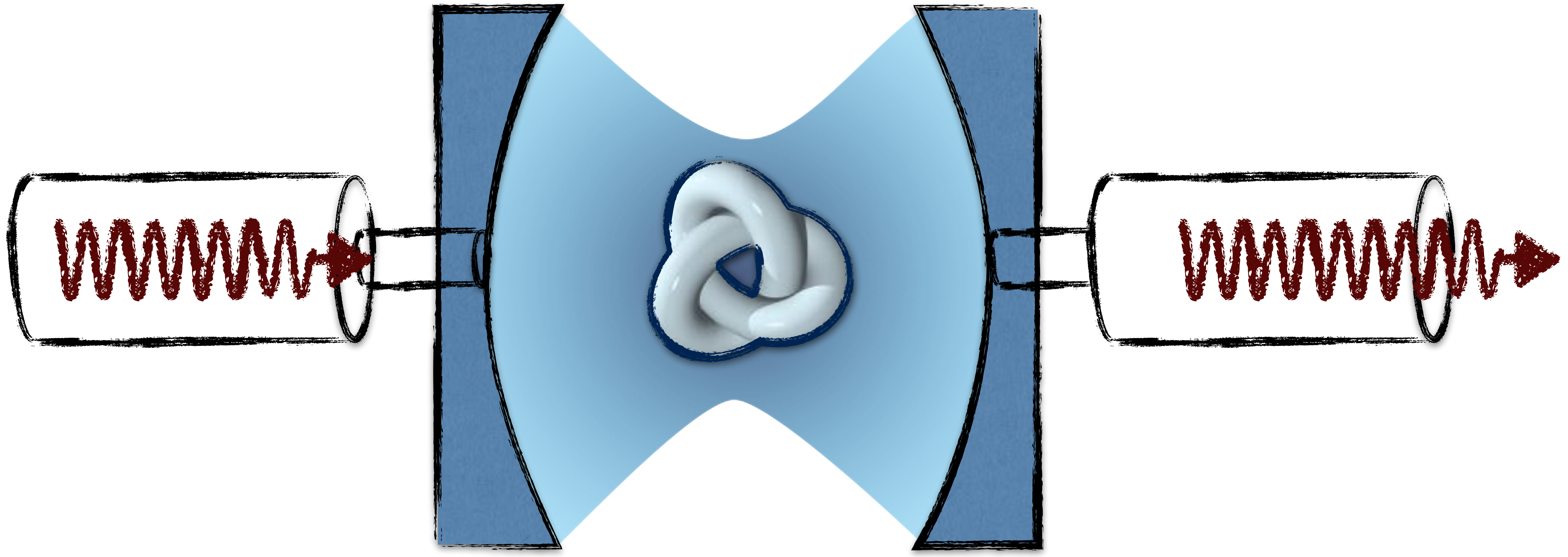
Measuring a Topological Flow:
from the Quantized Hall Effect to circuit QED



C. Dutreix (Bordeaux),
Q. Ficheux (JQI)
P. Delplace, B. Huard, J. Luneau, D. Carpentier (ENS de Lyon)

KITP, SpinQuant19
Spin and Heat Transport in Quantum and Topological Materials

Topological States of Matter

Topological Electronic States

- Quantum Hall Effect
  (electrons in D=2, strong Magnetic field) — D. Thouless, M. Kohmoto, MP Nightingale, and M Den Nijs (1982)
- Quantum Spin Hall Effect
  (electrons in D=2, strong spin-orbit) — C. Kane and G. Mele (2005); A. Bernevig, T. Hugues and S.C. Zhang (2006); L. Molenkamp *et al.* (2007)
- Topological Insulators in D=3
  (Bi_2Se_3, Bi_2Te_3, Sb_2Te_3, …) — L. Fu, C. Kane et G. Mele (2007); J. Moore and L. Balents (2007); R. Roy (2009)
- Topological Crystalline Insulators — L. Fu (2011)
- Topological Superconductors — Kitaev, 2008; Schnyder, Ryu, Furusaki, Ludwig 2008
- Higher Order Topological Insulators — F. Schindler *et al.* (2018)
- Topological Quantum Chemistry — T. Zhang *et al* (2019); F. Tang *et al* (2019); M.G Vergniory *et al* (2019)
- Interacting Electronic and Magnetic Phases

topology ↔ robustness

Topological waves beyond electrons

- Topological Optical Metamaterials — Haldane, Raghu (2008)
- Topological Mechanical Lattices — Prodhan, Prodhan, (2009); Kane, Lubensky (2014)
- Topological equatorial waves — Delplace, Marston, Venaille (2017)

Topological States of Matter and Surface (edge) States

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(electrons in D=2, strong Magnetic field)

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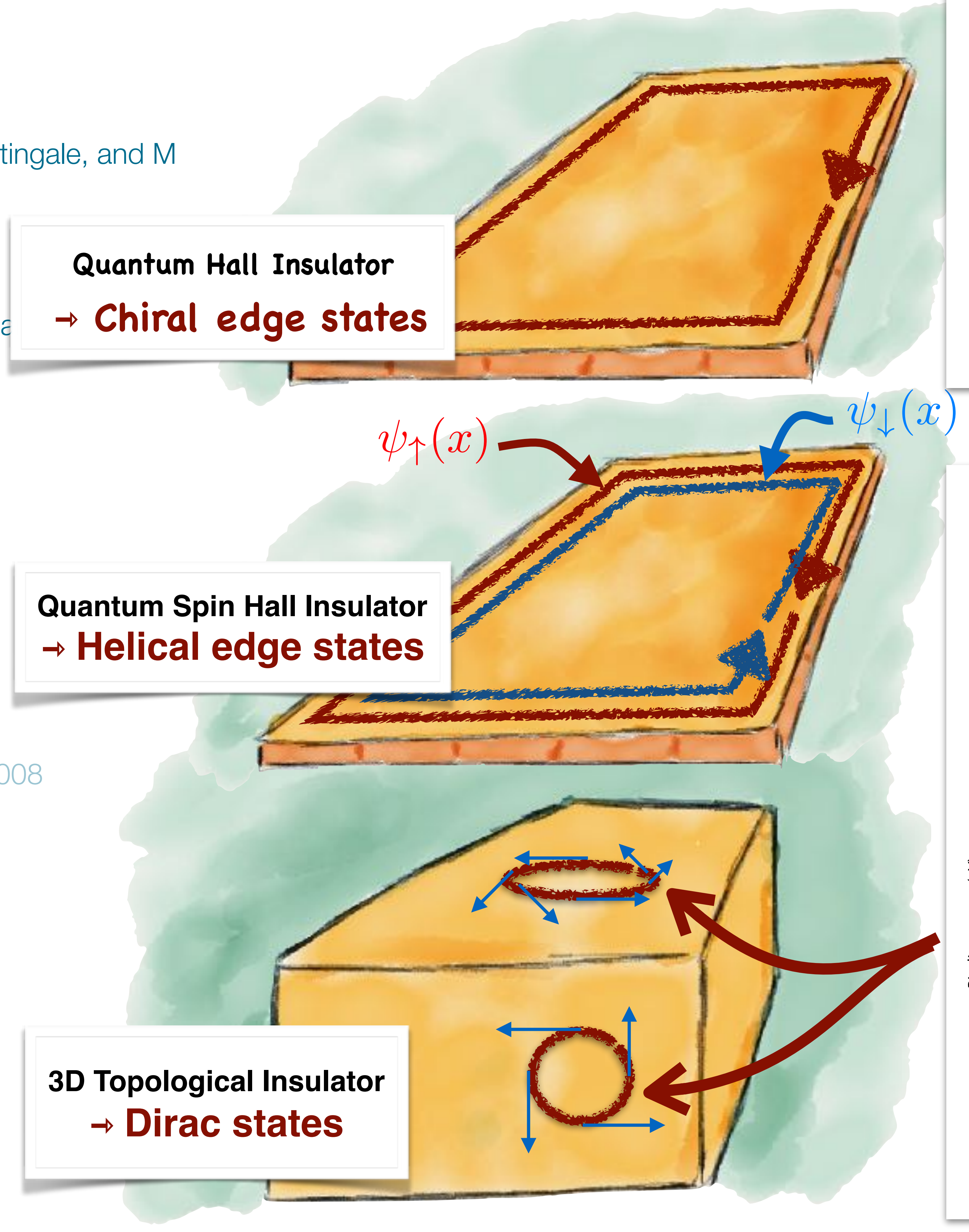
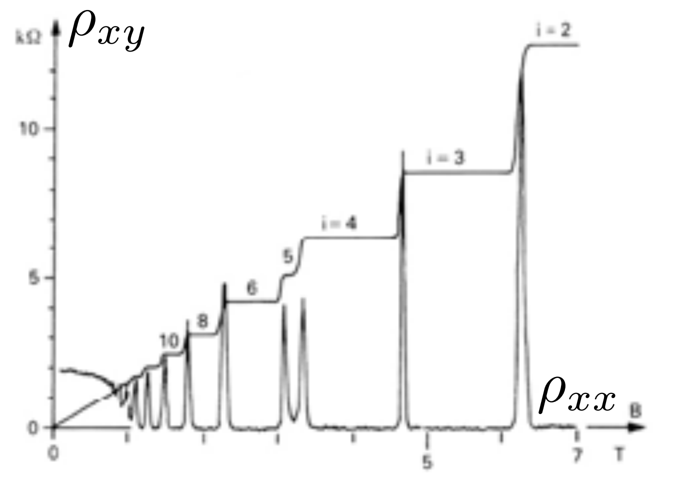
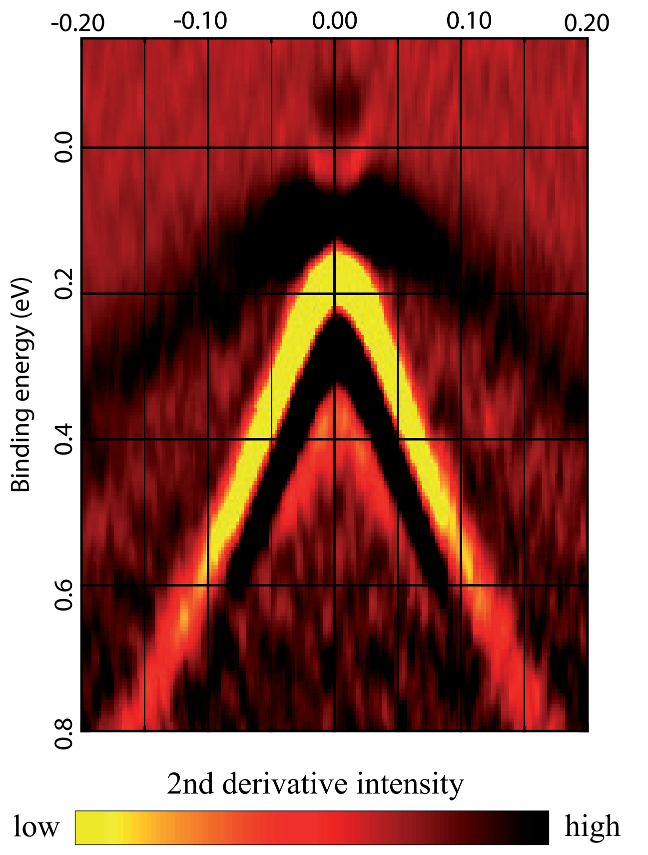


Transport

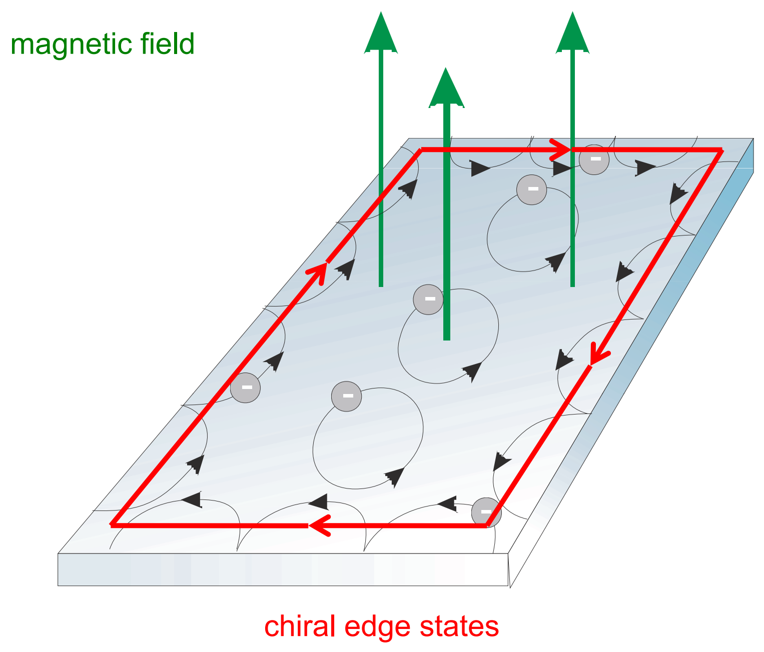


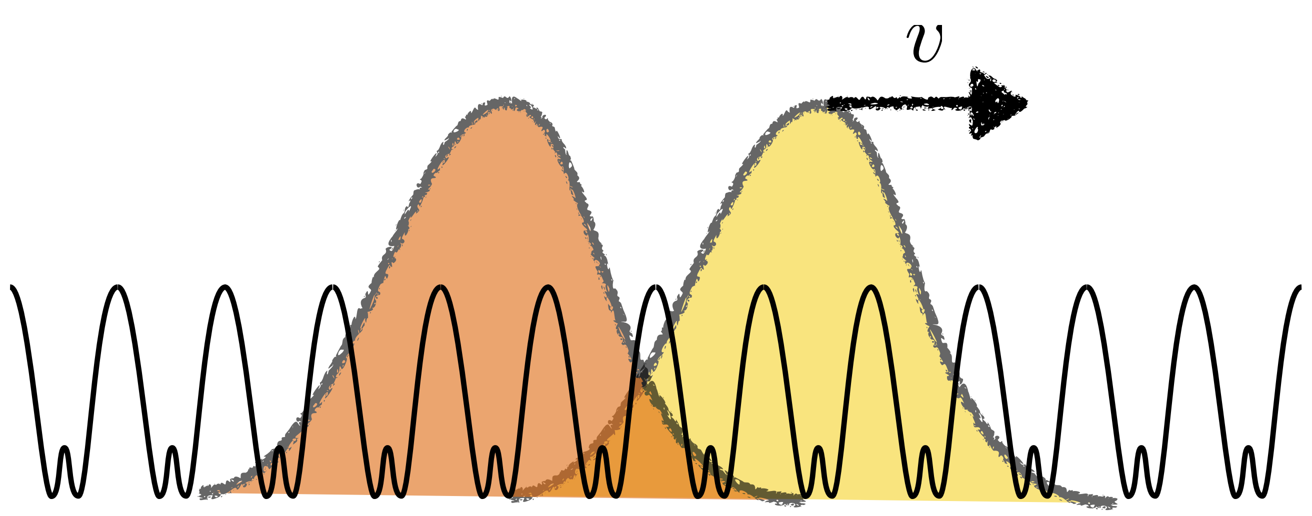
Angle **R**esolved **P**hoto**E**mission **S**pectroscopy

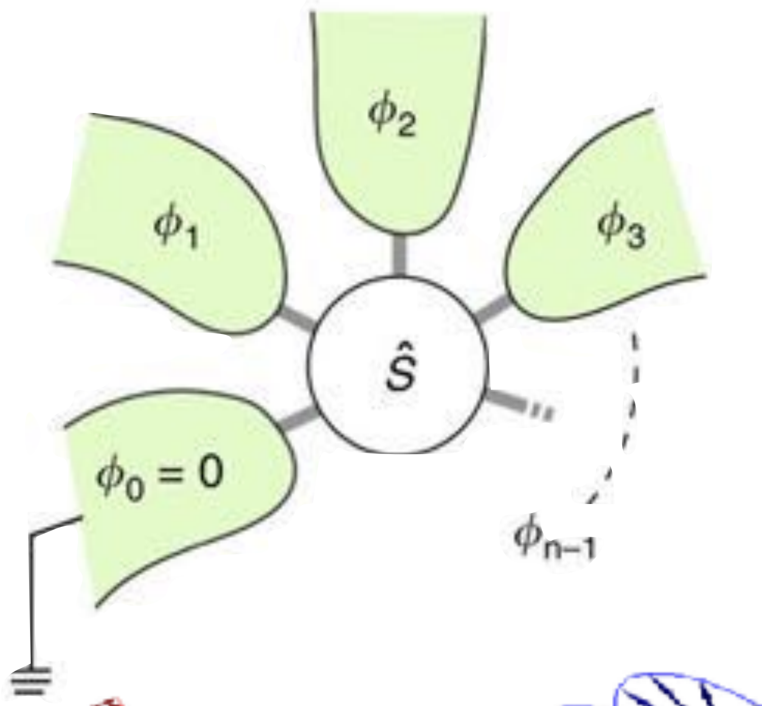


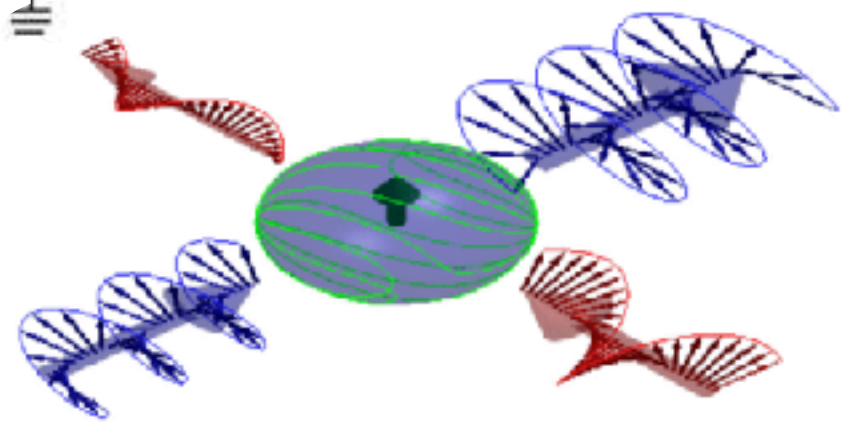
→ Physical properties via surface states

Topological Pumping beyond Quantum Hall Effect









Quantum Hall Effect

- dimension **d=2**
- quantized Hall conductivity

$$\sigma_{xy} = n\frac{e^2}{h}$$

D. Thouless, M. Kohmoto, MP Nightingale, and M Den Nijs (1982)

Q. Niu, D. J. Thouless, and Y.-S. Wu (1985)

Thouless pump

- dimension **d=1**
- time-dependent periodic potential

$$V(x,t) = V(x+a,t) = V(x,t+T)$$

- velocity : $v = n\ a/T$

D. Thouless (1983)

Q. Niu, D. Thouless (1984)

M. Lohse *et al.* (2016)

Multi-terminal Josephson junctions

- dimension **d=0**
- quantized transconductance

$$I_\alpha = -n_{\alpha\beta}\frac{4e^2}{h}V_\beta$$

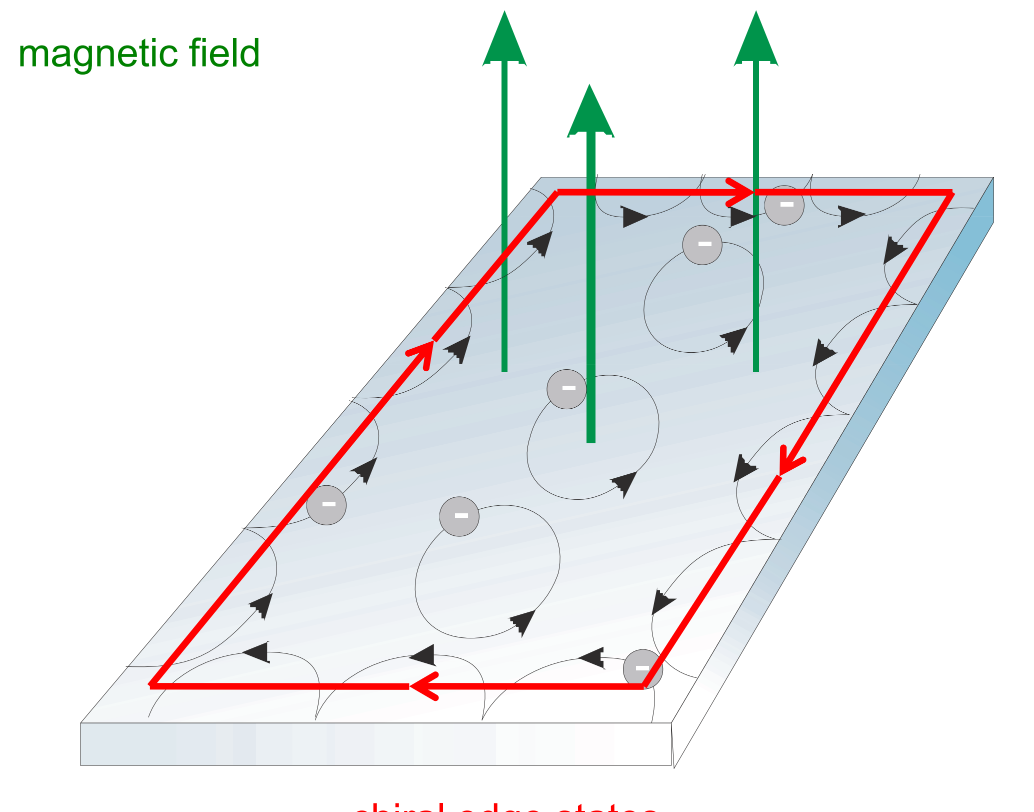
R.-P. Riwar, M. Houzet, J.S. Meyer, and Y.V. Nazarov (2016)

Topological frequency converter

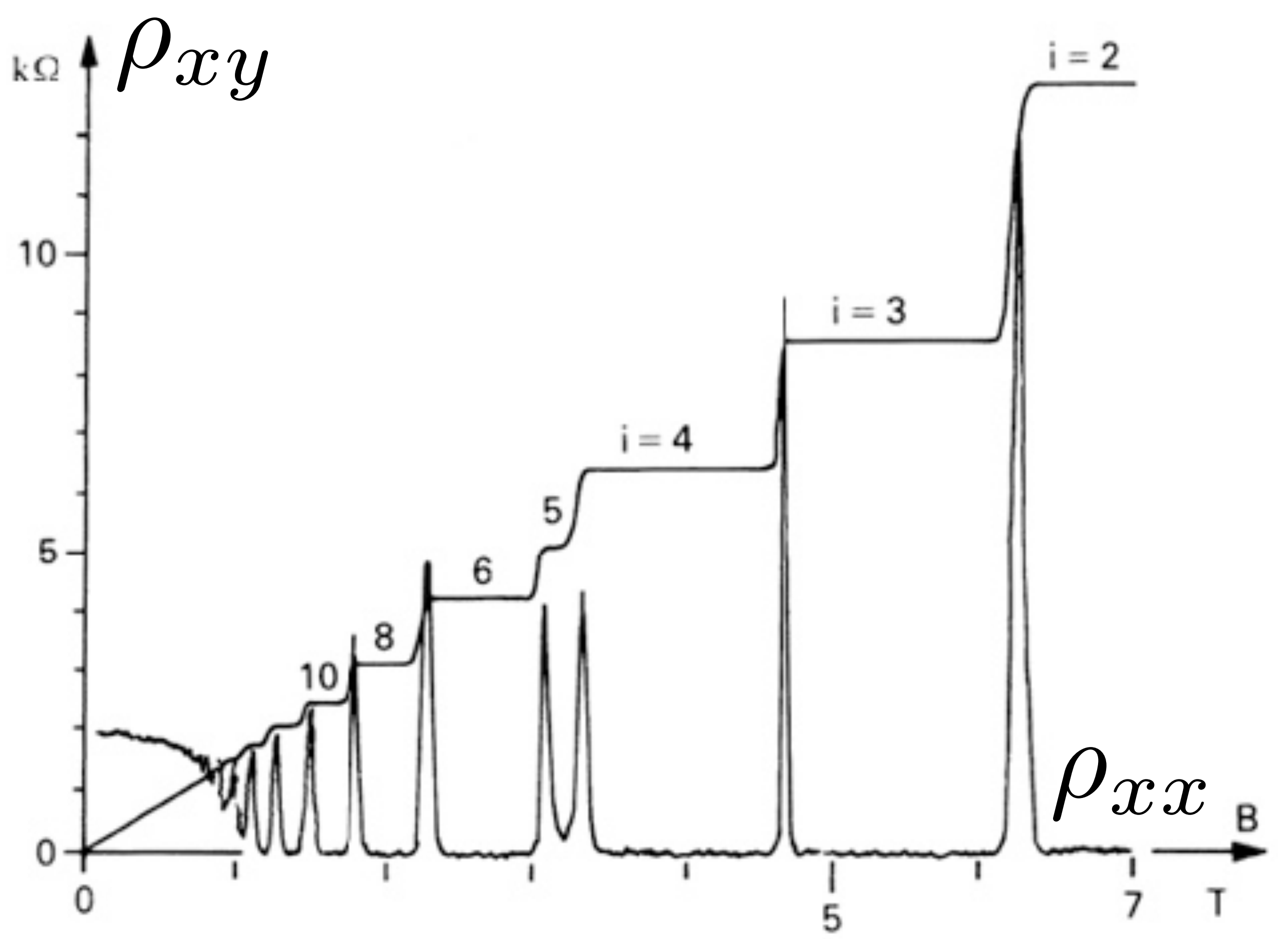
- dimension **d=0**
- time-dependent **(**Floquet theory**)**

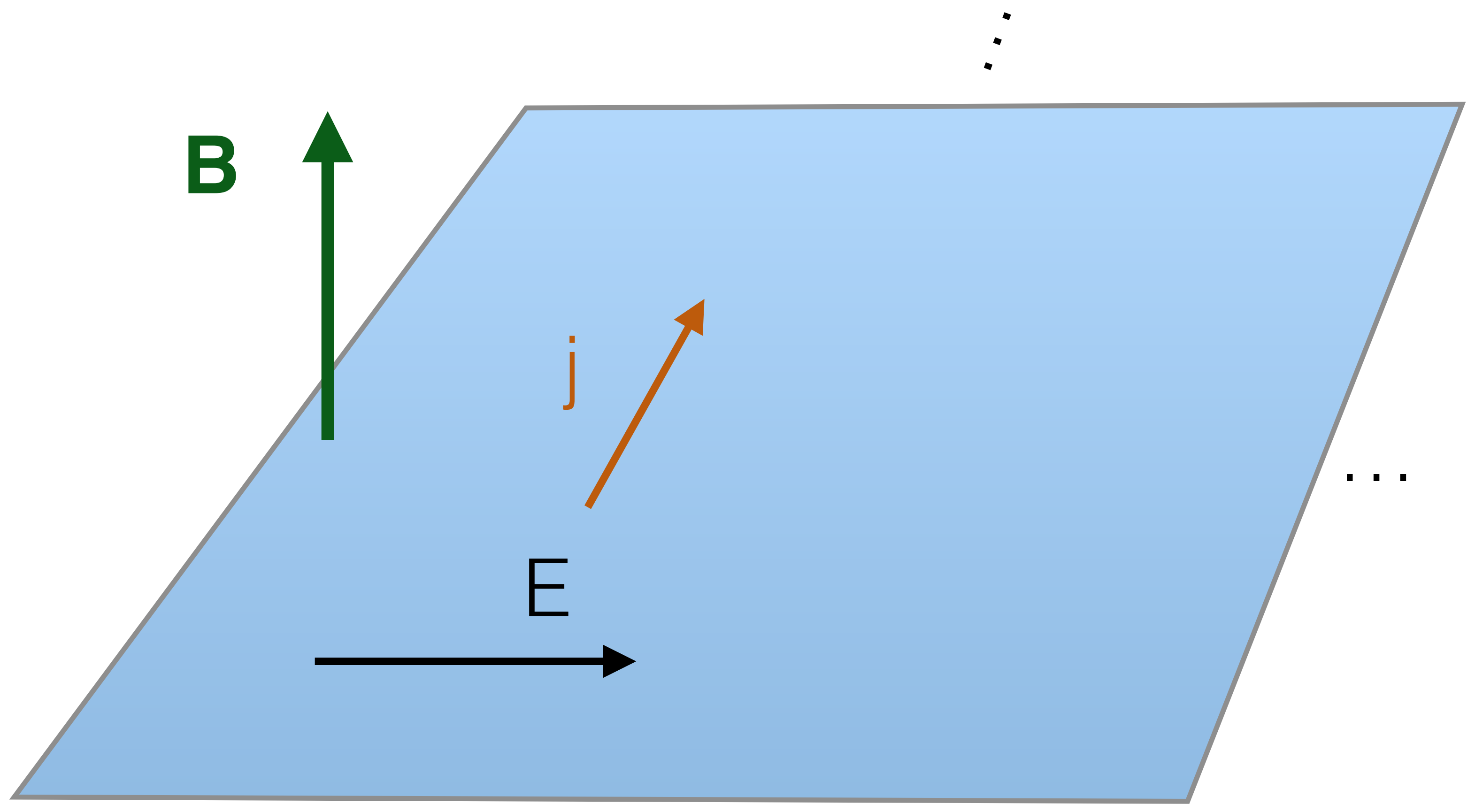
I. Martin, G. Refael, B. Halperin (2017)

Topological Response of the Quantum Hall Effect



2DEG (Heterojunction GaAs/AlGaAs)





linear response theory (Kubo) :

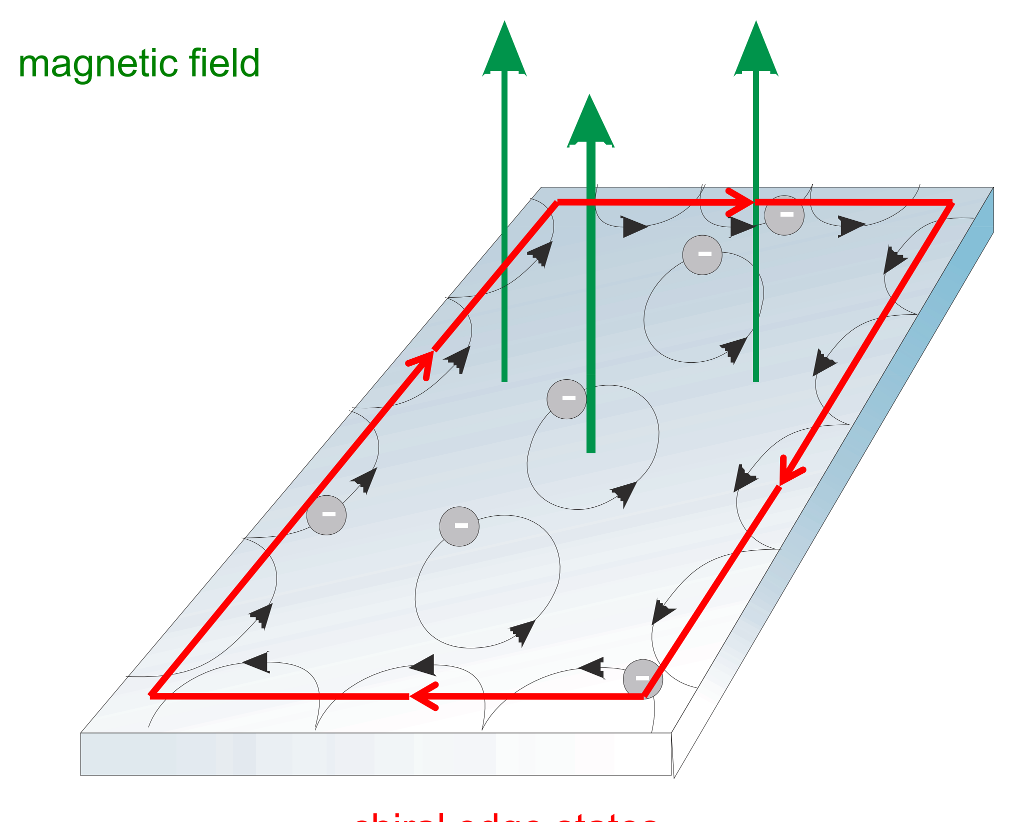
Thouless *et al.*. PRL **49** (1982)

$$\mathbf{j} = \sigma \mathbf{E} \qquad \sigma_{xx} = 0, \;\; \sigma_{xy} = n\frac{e^2}{h}$$

n is a topological invariant : Chern number

(quantized Hall effect with high precision)

Topological Response of the Quantum Hall Effect



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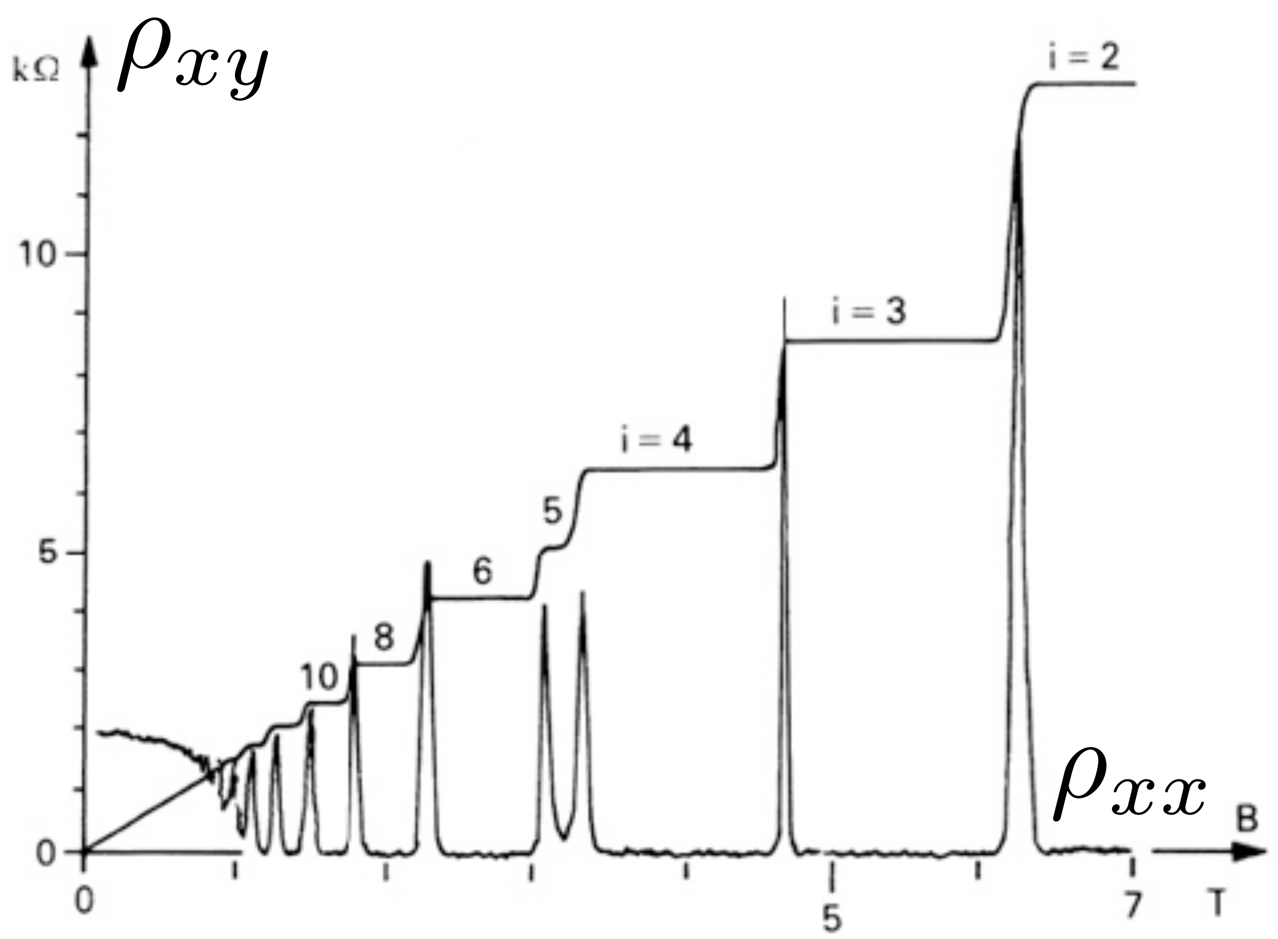
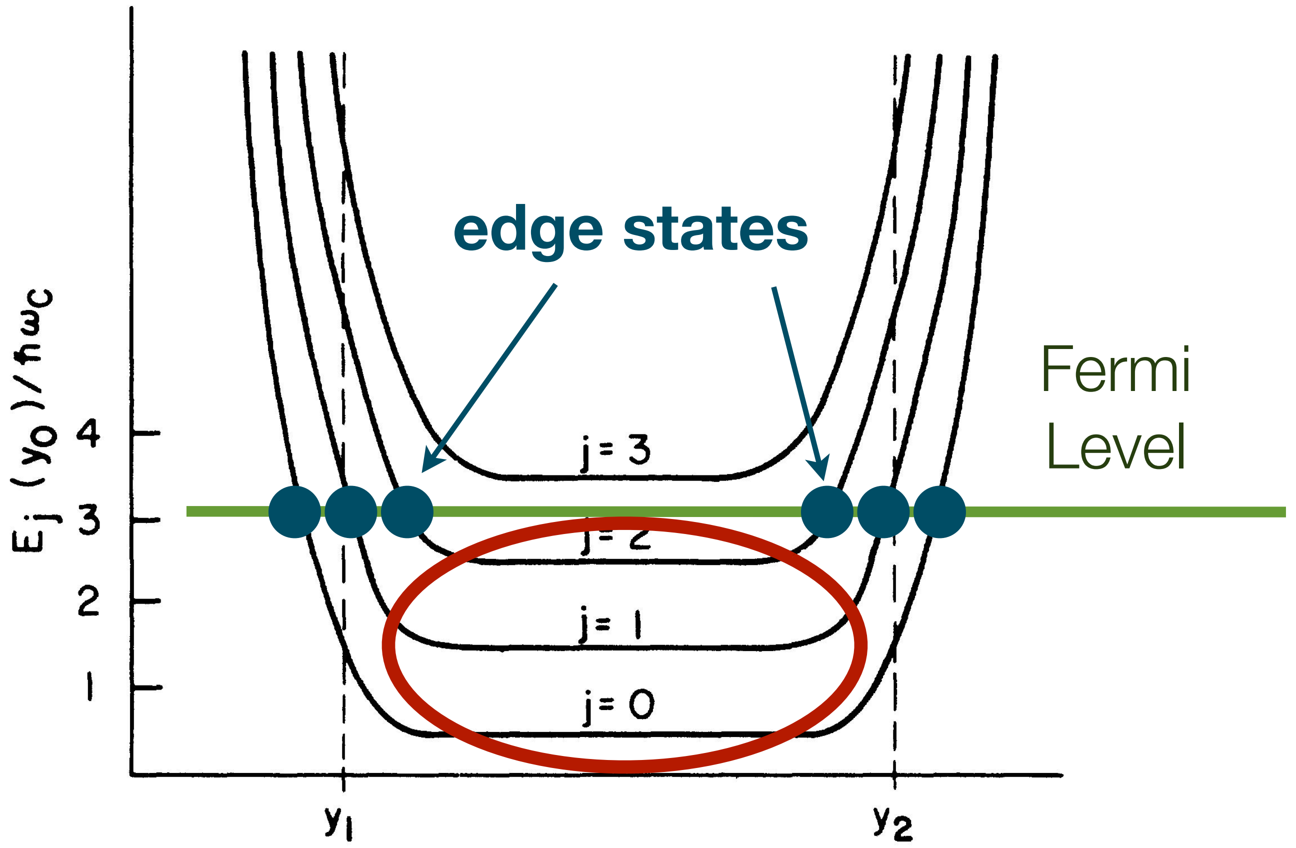
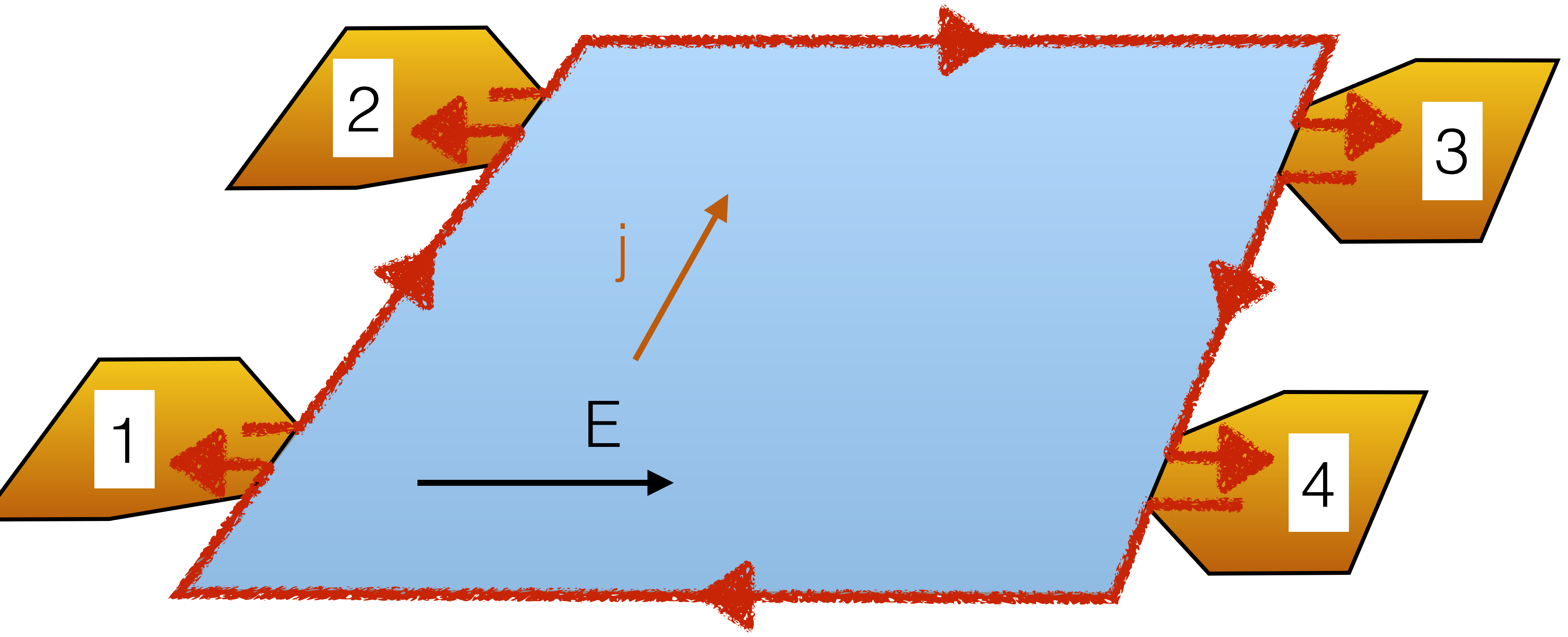






Landauer formalism

M. Büttiker, PRB **38** (1988)

➡ scattering matrix between the leads

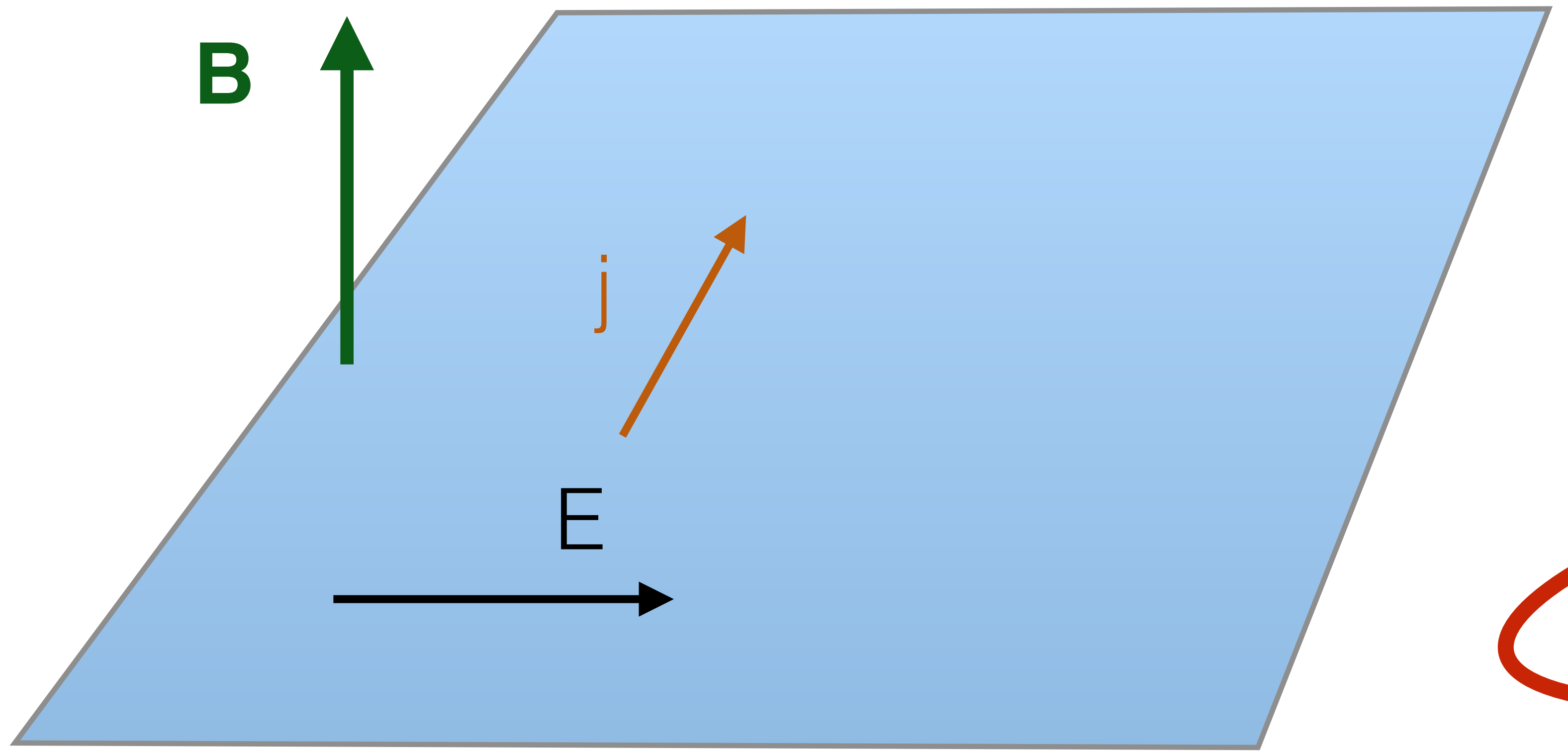
$$S = \begin{pmatrix} 0 & n & 0 & 0 \\ 0 & 0 & n & 0 \\ 0 & 0 & 0 & n \\ n & 0 & 0 & 0 \end{pmatrix}$$

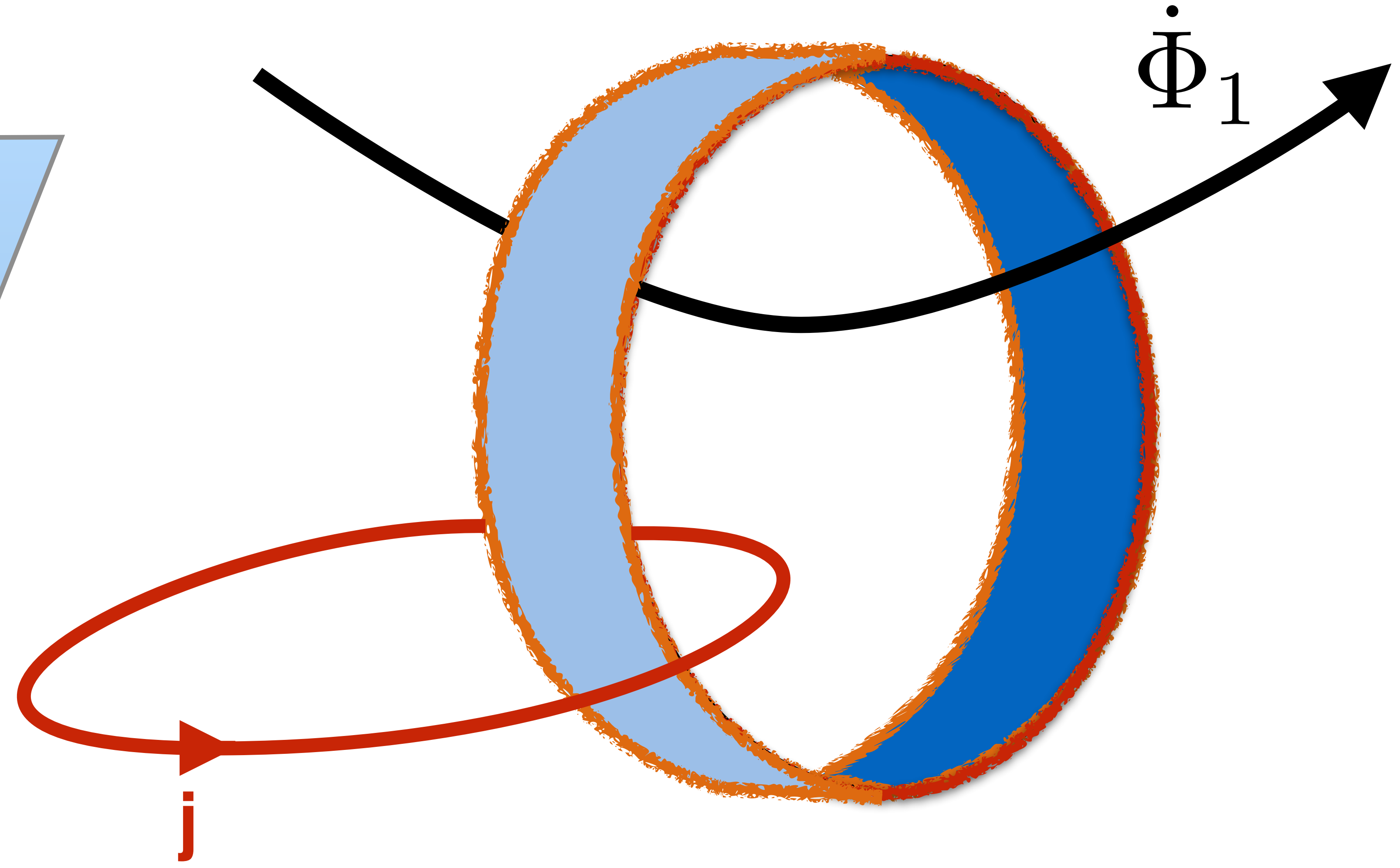
➡ conductance $G_{13,24} = n\dfrac{e^2}{h}$

n : number of (robust) edge modes

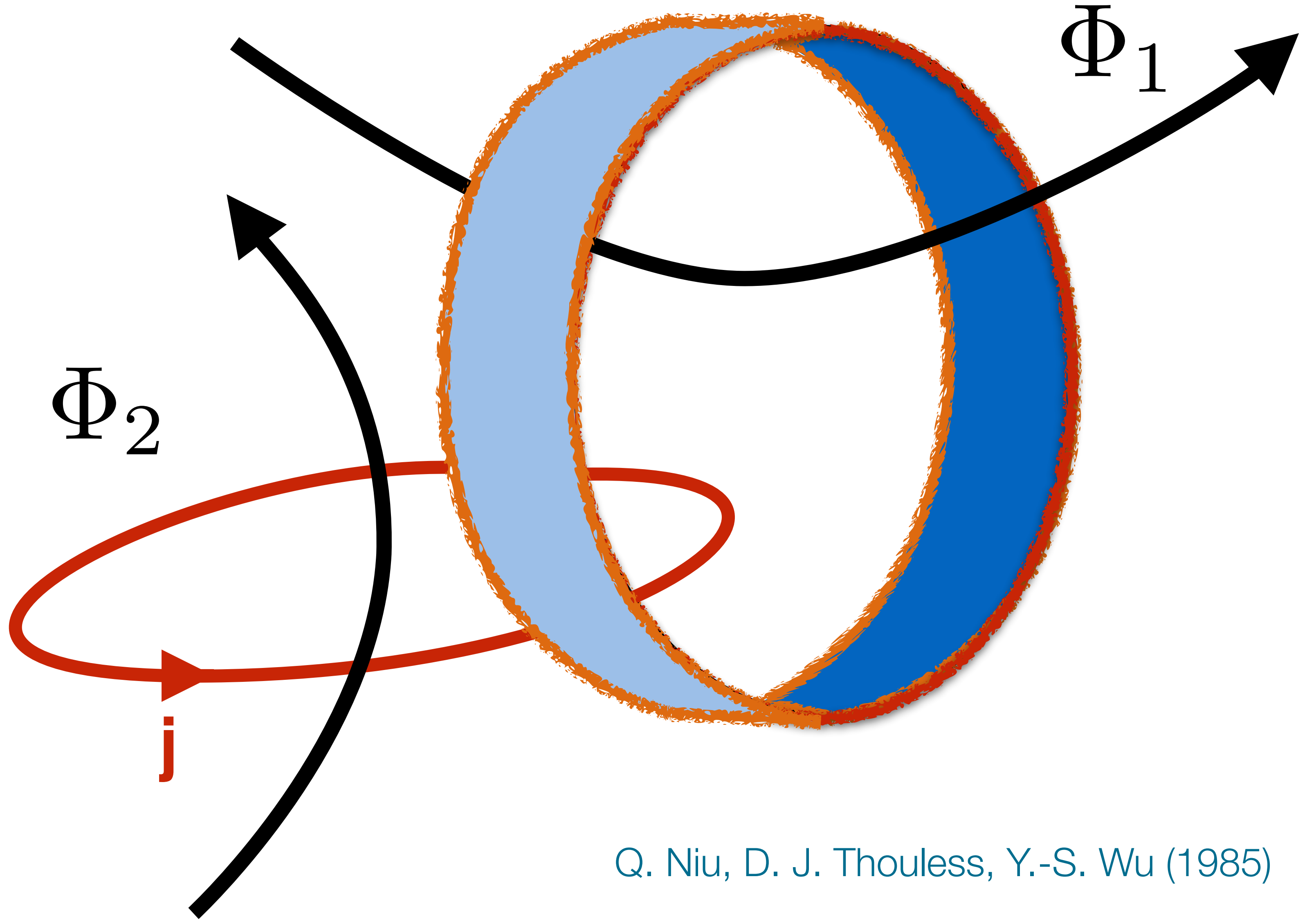
- robustness is related to nature of edge modes (chiral)
- determines transport properties

Response of the Quantum Hall Effect as Topological Pumping





B. Laughlin (1981)



Q. Niu, D. J. Thouless, Y.-S. Wu (1985)

J. E. Avron, R. Seiler, and L. G. Yaffe (1987)

Laughlin Gedanken experiment

➡ quantized charge transferred after unit flux insertion

Hall conductance as a topological object

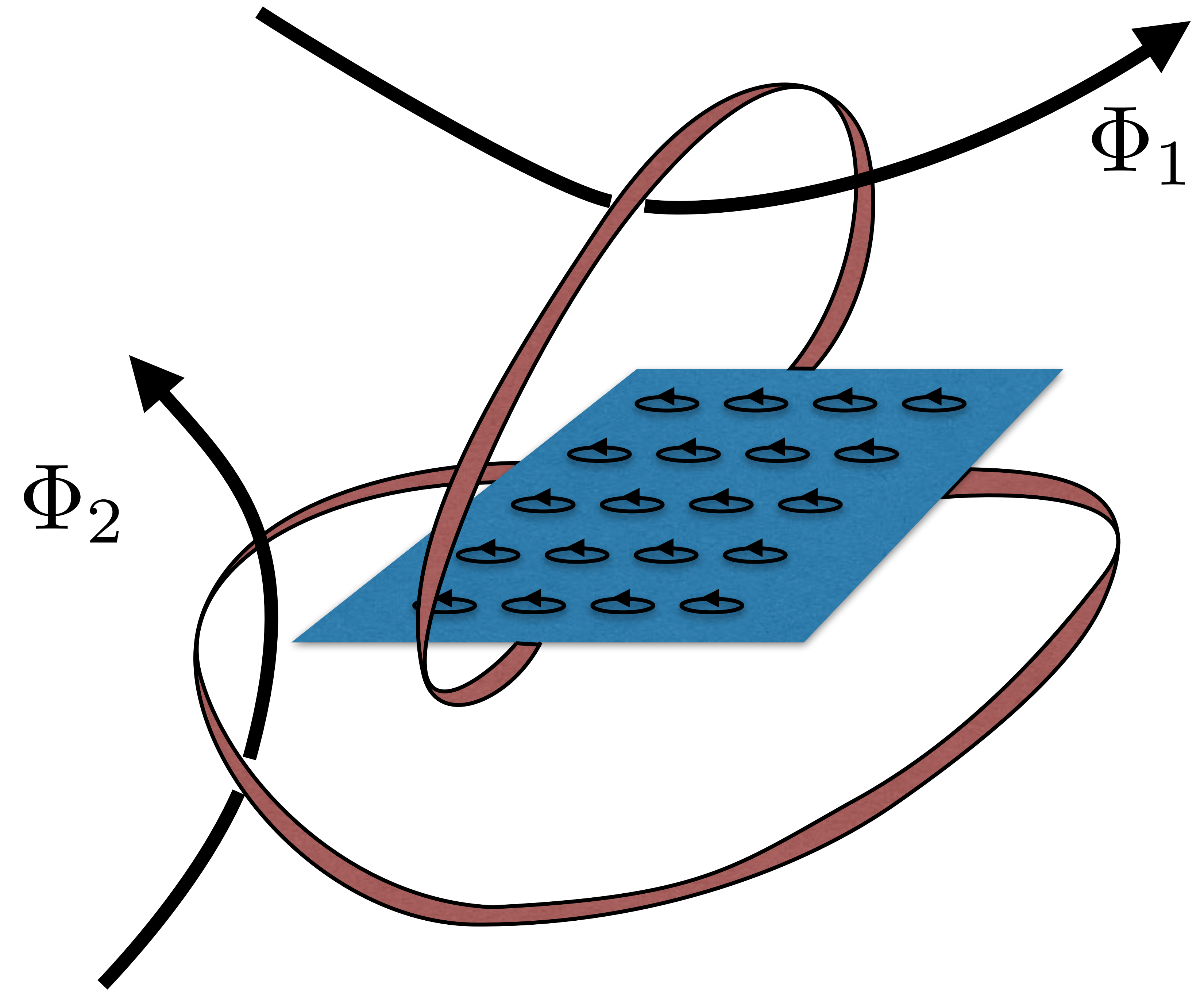
➡ current $\langle \Psi | j | \Psi \rangle = -\hbar F \dot{\Phi}_1$

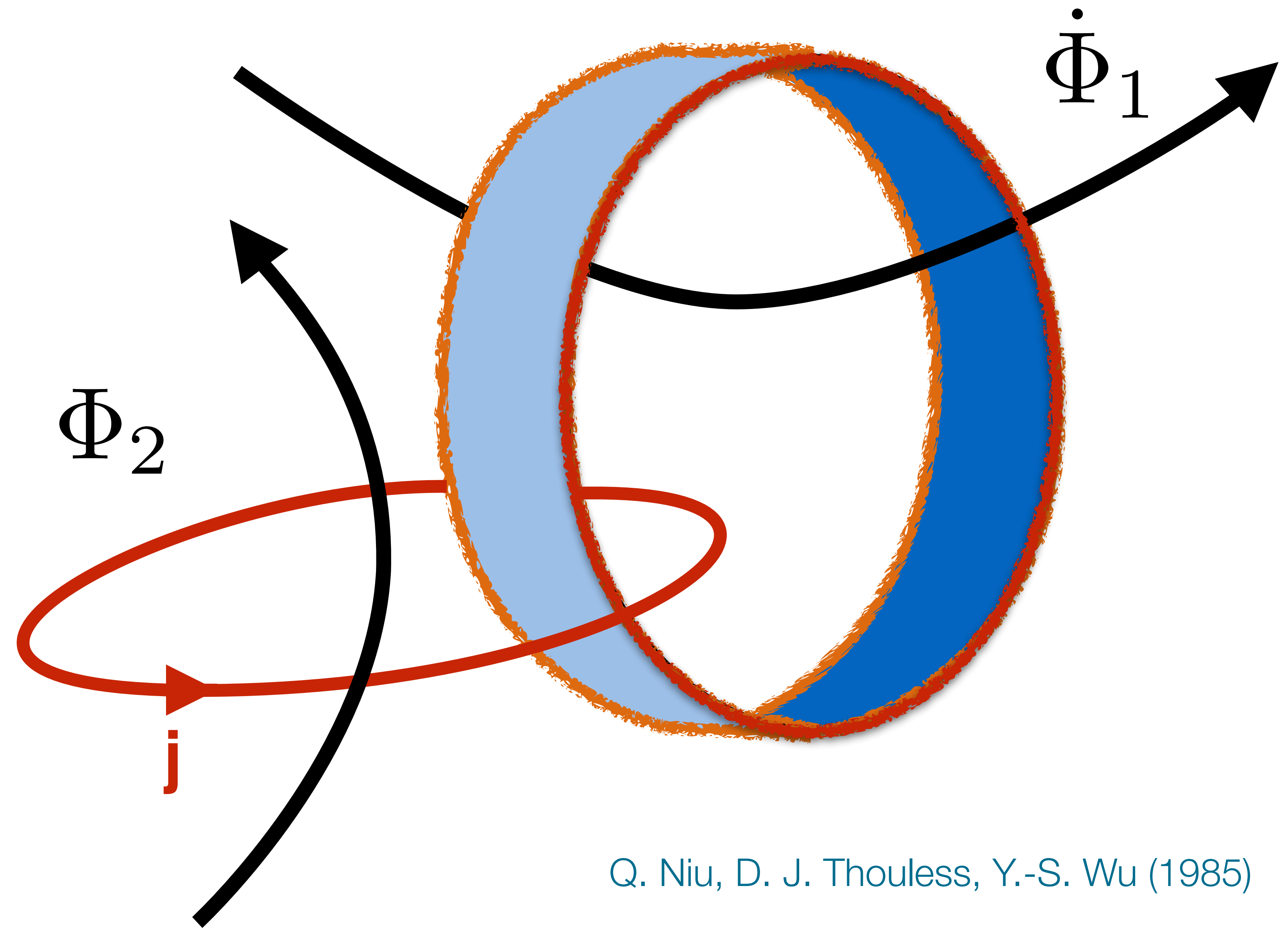
➡ Berry curvature

$$F = 2\ \mathrm{Im}\, \langle \partial_{\Phi_2} \Psi | \partial_{\Phi_1} \Psi \rangle$$

Many-body QH ground state $|\Psi\rangle$

Response of the Quantum Hall Effect as Topological Pumping





Q. Niu, D. J. Thouless, Y.-S. Wu (1985)

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Each electric branch : $\mathcal{H}_\alpha = \dfrac{Q_\alpha^2}{2C_\alpha} + \dfrac{\Phi_\alpha^2}{2L_\alpha}$

$$I_\alpha = \dot{Q}_\alpha = \frac{\partial \mathcal{H}_\alpha}{\partial \Phi_\alpha} = \frac{\Phi_\alpha}{L_\alpha}$$

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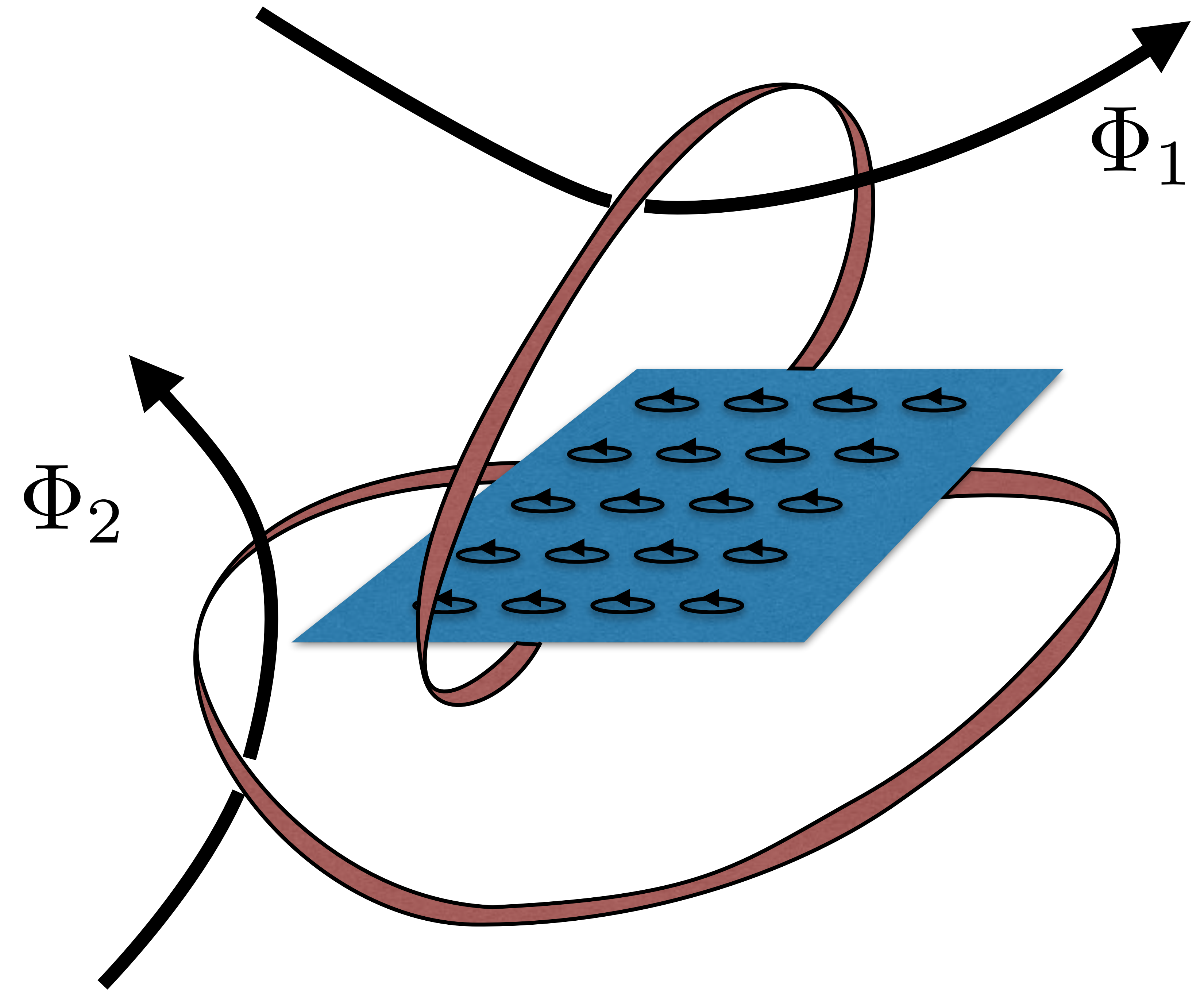

Many-body QH ground state $|\Psi\rangle$

Coupling (generalized boundary condition) :

$$\Psi(x_1, y_1, \ldots, x_i + L_x, y_i, \ldots,) = e^{i2\pi\frac{\Phi_1}{\phi_0}}\ \Psi(x_1, y_1, \ldots, x_i, y_i, \ldots,)$$

$$\phi_0 = h/e$$

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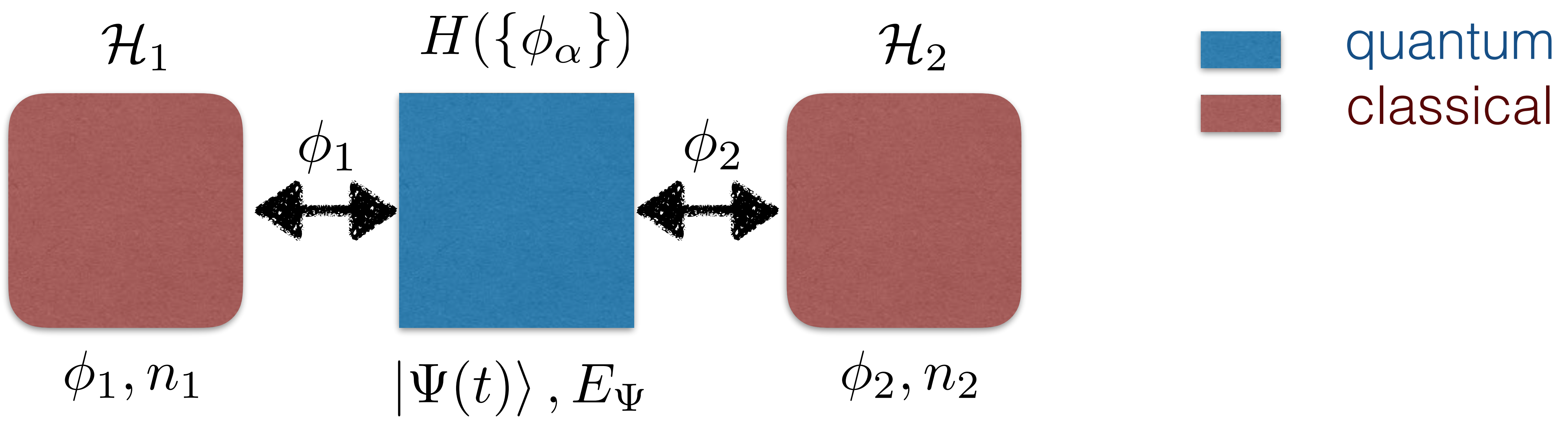
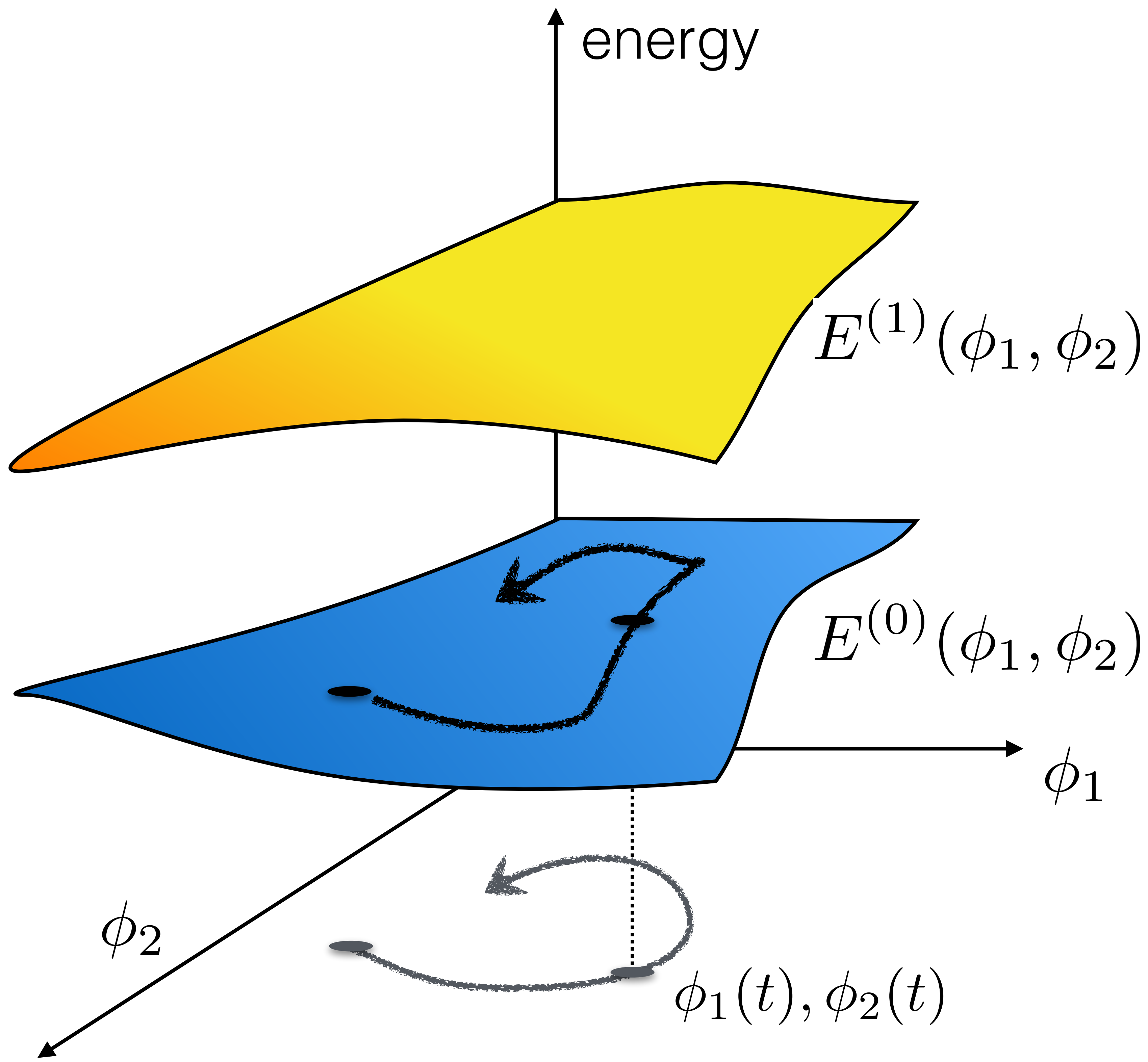
Natural variables (2π periodic coupling) :

- phase $\phi_\alpha = 2\pi\Phi_\alpha/\phi_0$
- charge $n_\alpha = Q_\alpha \phi_0/(2\pi)$

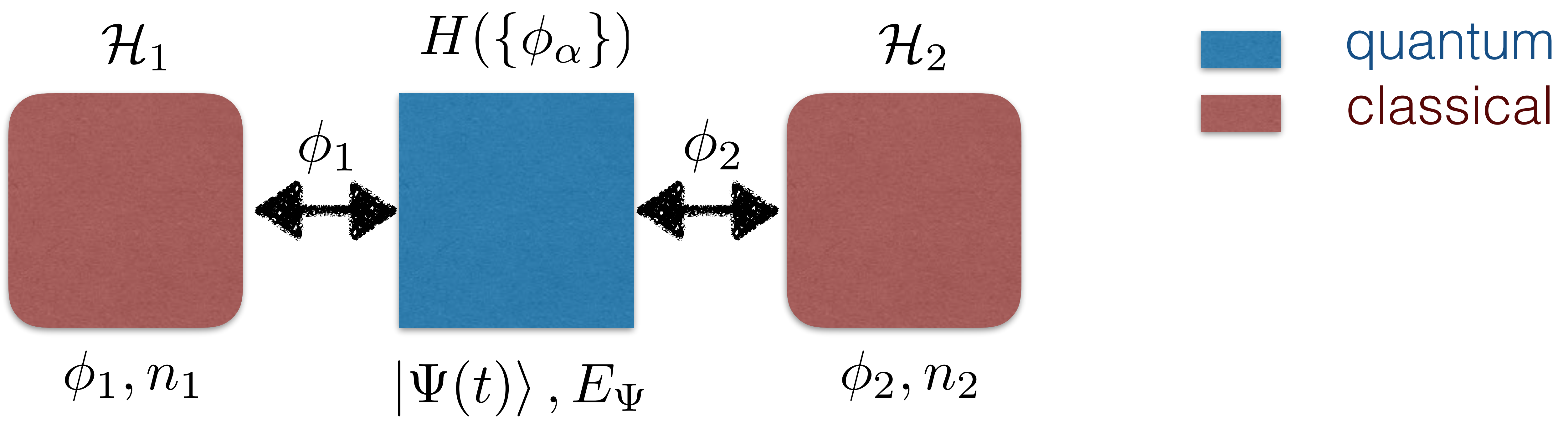
General Formalism for geometrical pumping

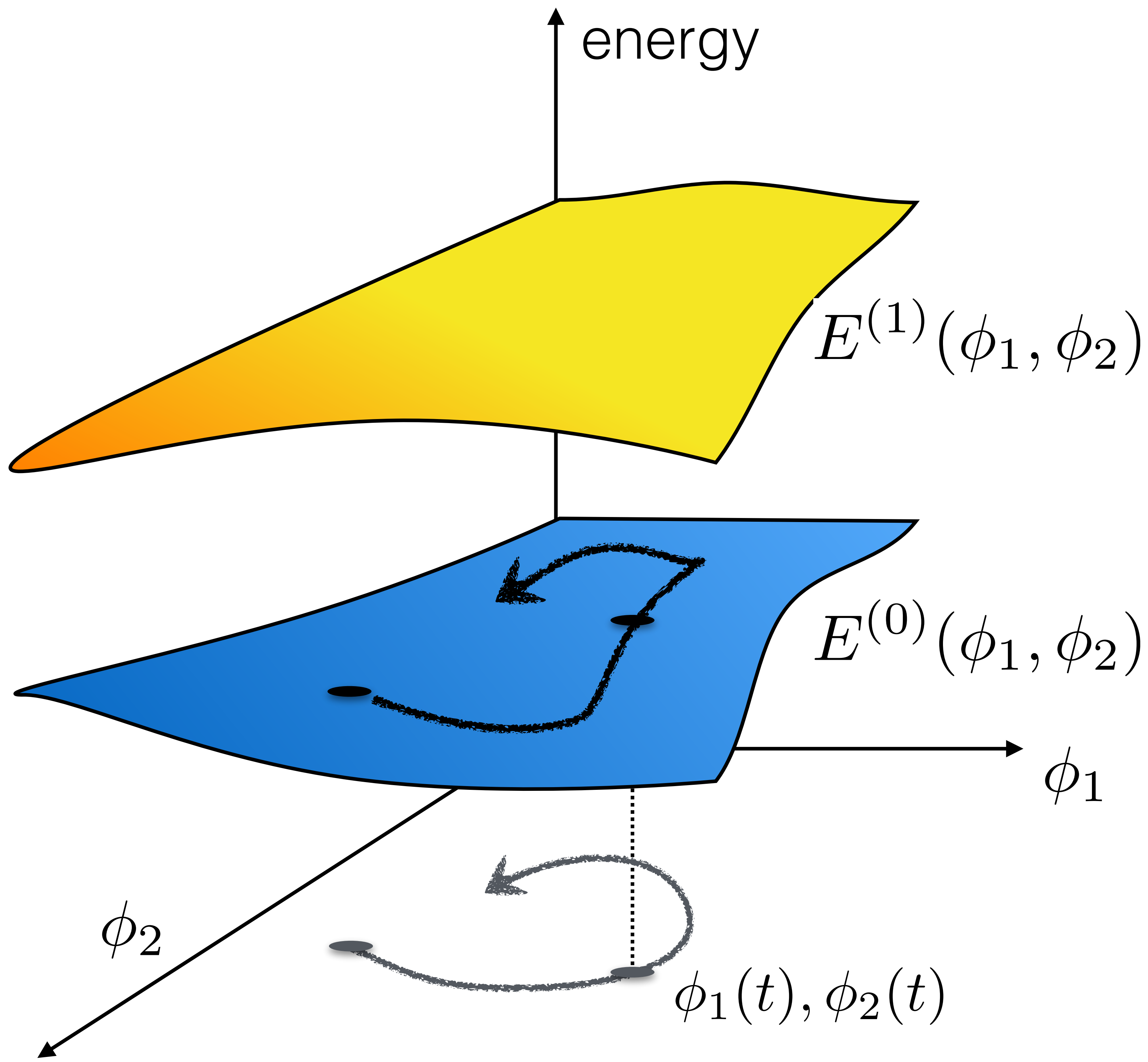




Adiabatic evolution for quantum system in $|\Psi(\phi_1(t), \phi_2(t))\rangle$

General Formalism for geometrical pumping





Adiabatic evolution for quantum system in $|\Psi(\phi_1(t), \phi_2(t))\rangle$

Equations of motion for baths:

$$\dot{n}_\alpha = -\frac{\partial \mathcal{H}_\alpha}{\partial \phi_\alpha}$$

$$\dot{\phi}_\alpha = \frac{\partial \mathcal{H}_\alpha}{\partial n_\alpha}$$

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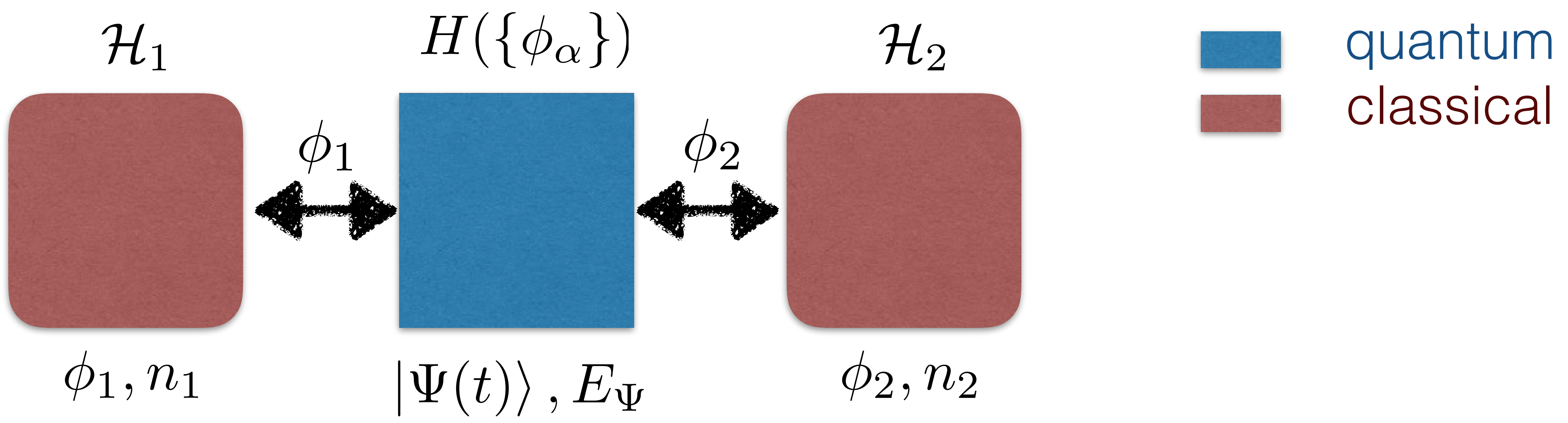

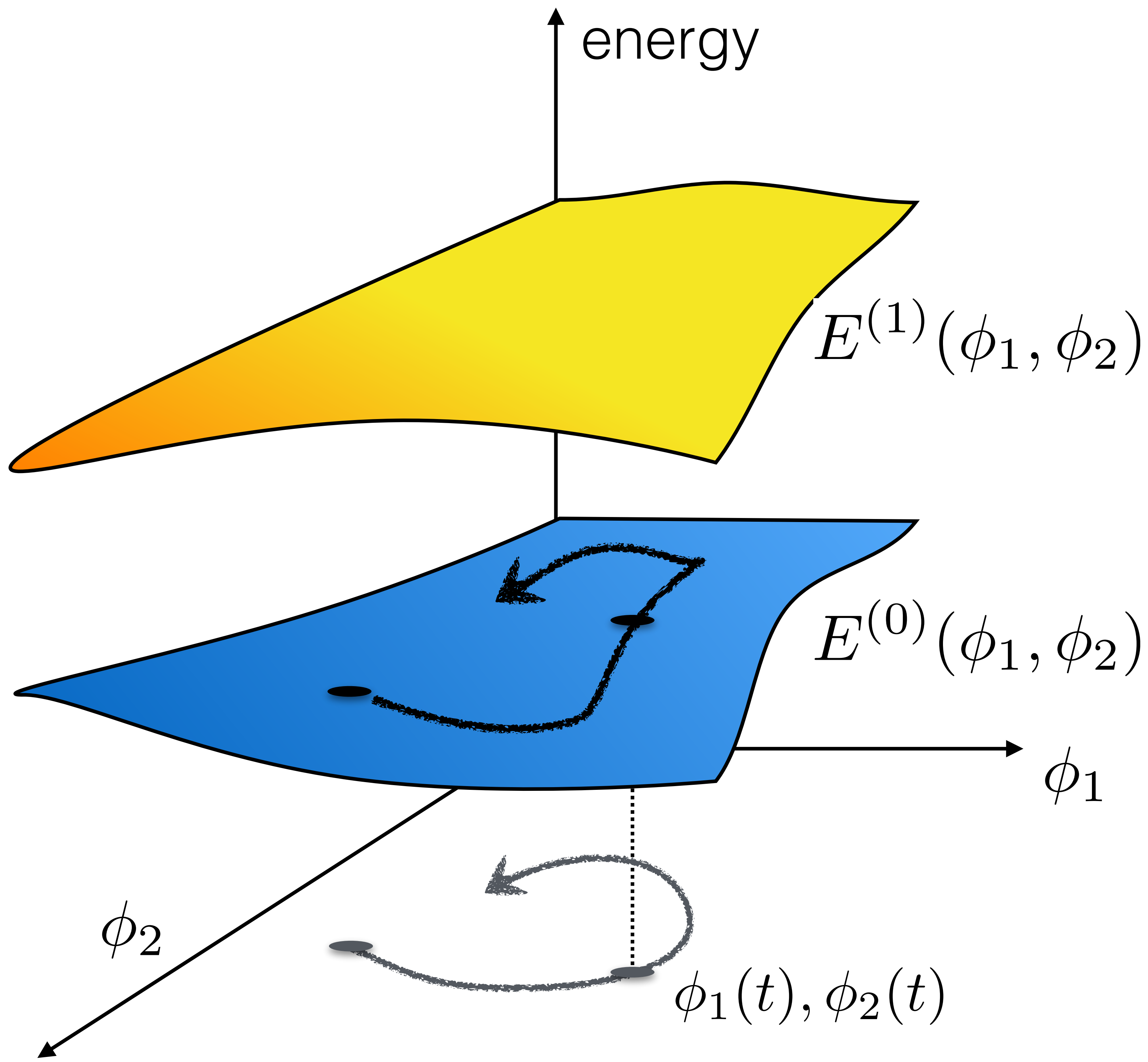

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$$\dot{\phi}_\alpha = \frac{\partial \mathcal{H}_\alpha}{\partial n_\alpha}$$

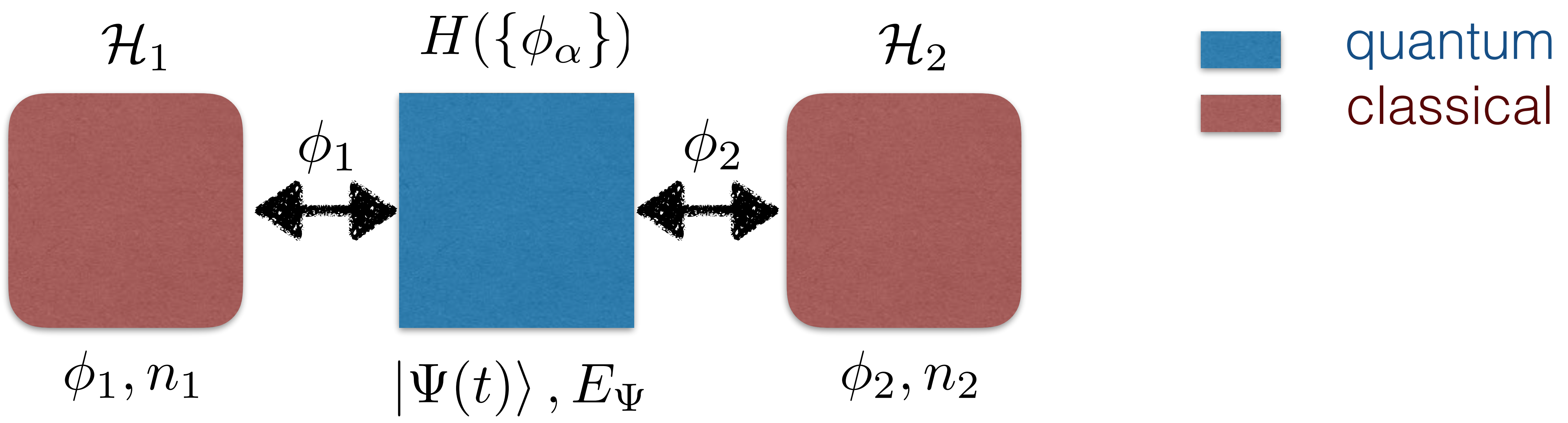
Hellmann–Feynman theorem

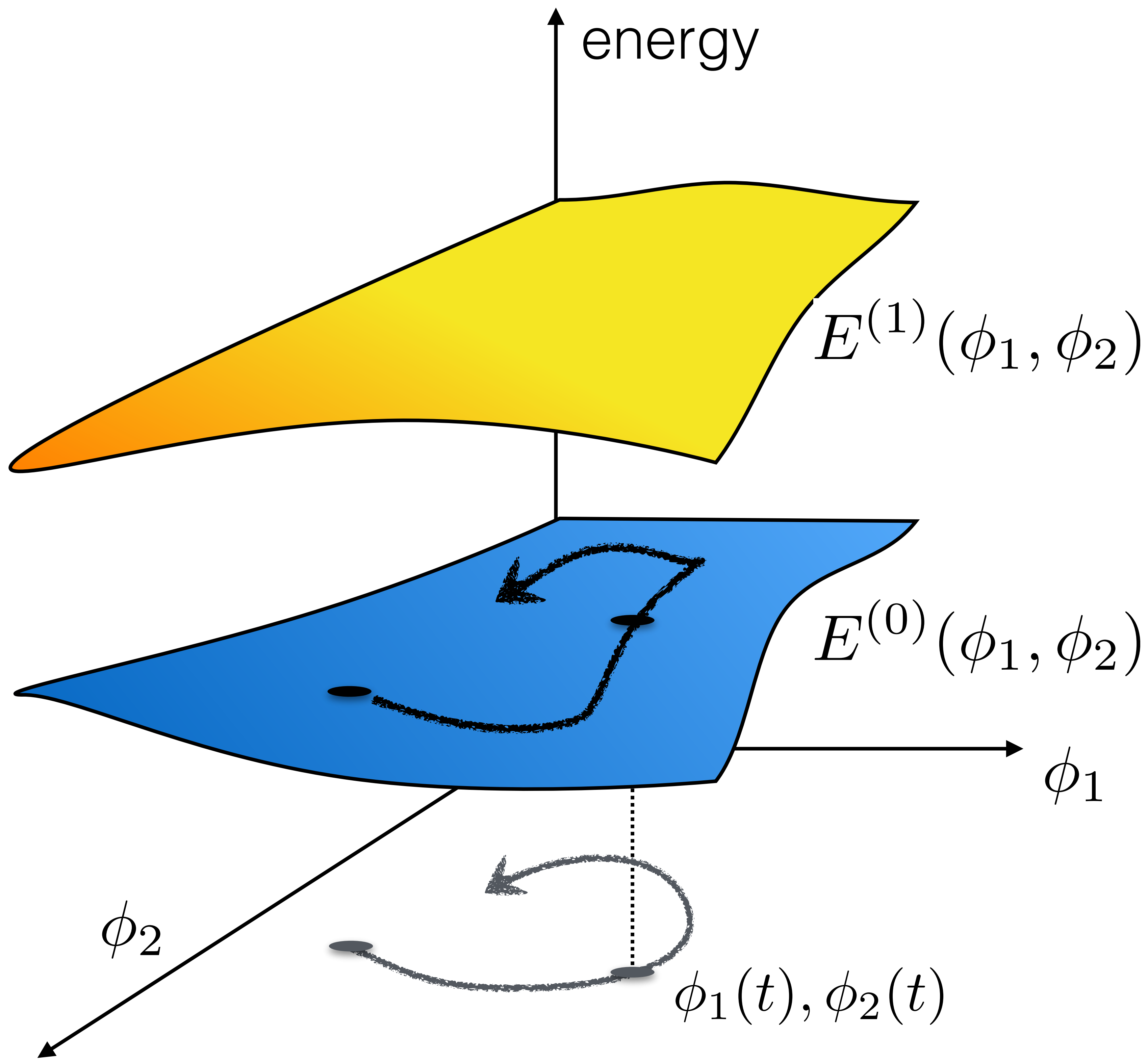
additional « geometrical » contribution
(cf Born-Oppenheimer)

- Berry curvature : « geometry » of the eigenstate $|\Psi(\phi_1, \phi_2)\rangle$

$$F^{(\Psi)}_{\alpha,\beta} = 2 \, \mathrm{Im}\langle \partial_{\phi_\alpha} \Psi | \partial_{\phi_\beta} \Psi \rangle$$

General Formalism for geometrical pumping





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$$\dot{\phi}_\alpha = \frac{\partial \mathcal{H}_\alpha}{\partial n_\alpha}$$

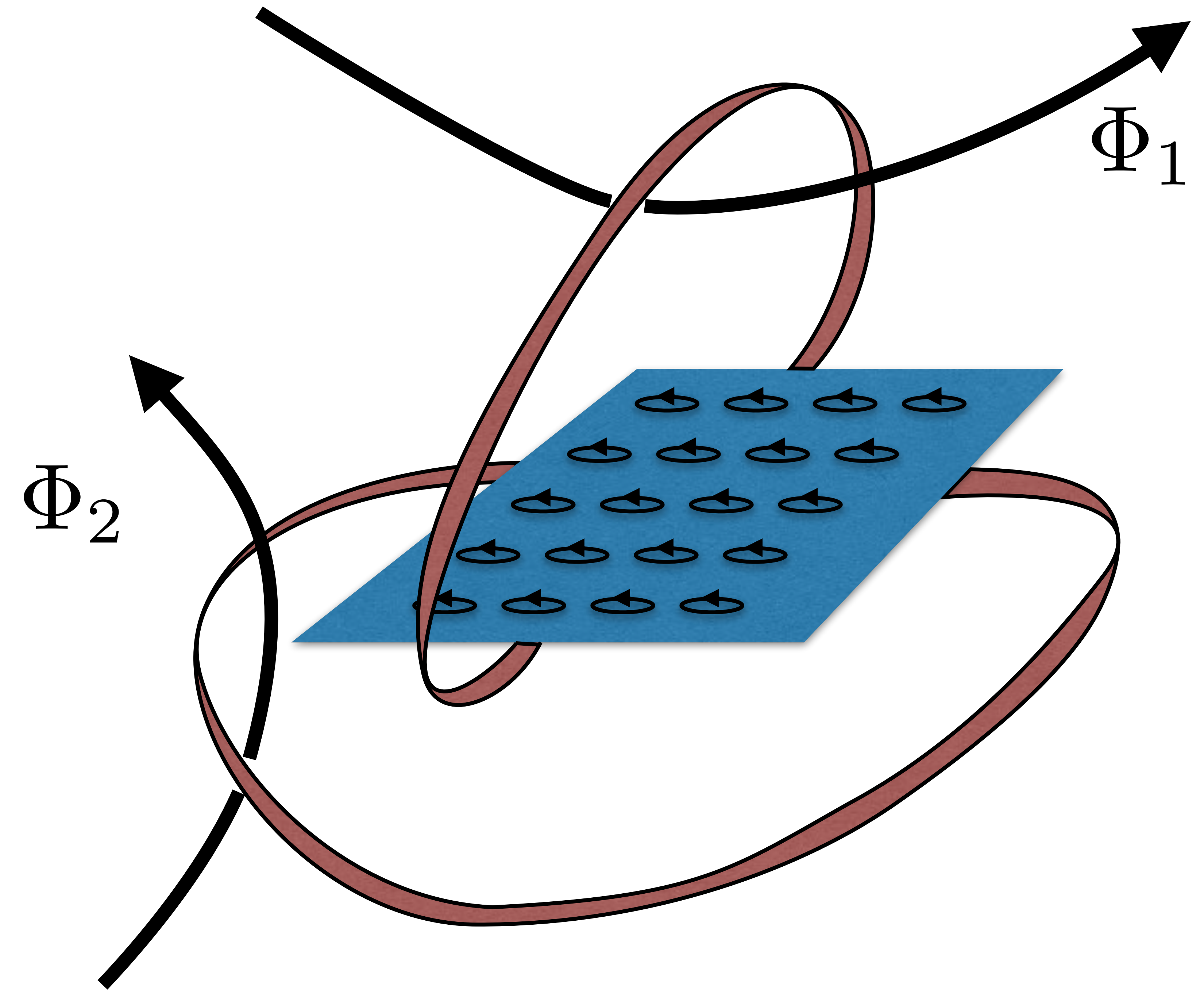
If $H(\phi_1,\phi_2)$ is 2π periodic $\quad c^{(\Psi)}_{12} = \frac{1}{2\pi}\int d\phi_1 d\phi_2 \; F^{(\Psi)}_{1,2}$ is an integer (Chern number)

➡ Average Berry curvature $\overline{F^{(\Psi)}_{1,2}} = \frac{1}{2\pi} c^{(\Psi)}_{12}$

➡ Average (topological) flux $\overline{\dot{n}_1} = \frac{\hbar}{2\pi} c^{(\Psi)}_{12} \dot{\phi}_2$

Fluctuations of flux : $\quad \delta\dot{n}_1 = -\frac{\partial E_\Psi}{\partial \phi_1} - \hbar \; \delta F^{(\Psi)}_{1,2} \; \dot{\phi}_2$

Response of the Quantum Hall Effect as Topological Pumping



Many-body QH ground state $|\Psi\rangle$

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Natural variables (2π periodic coupling) :

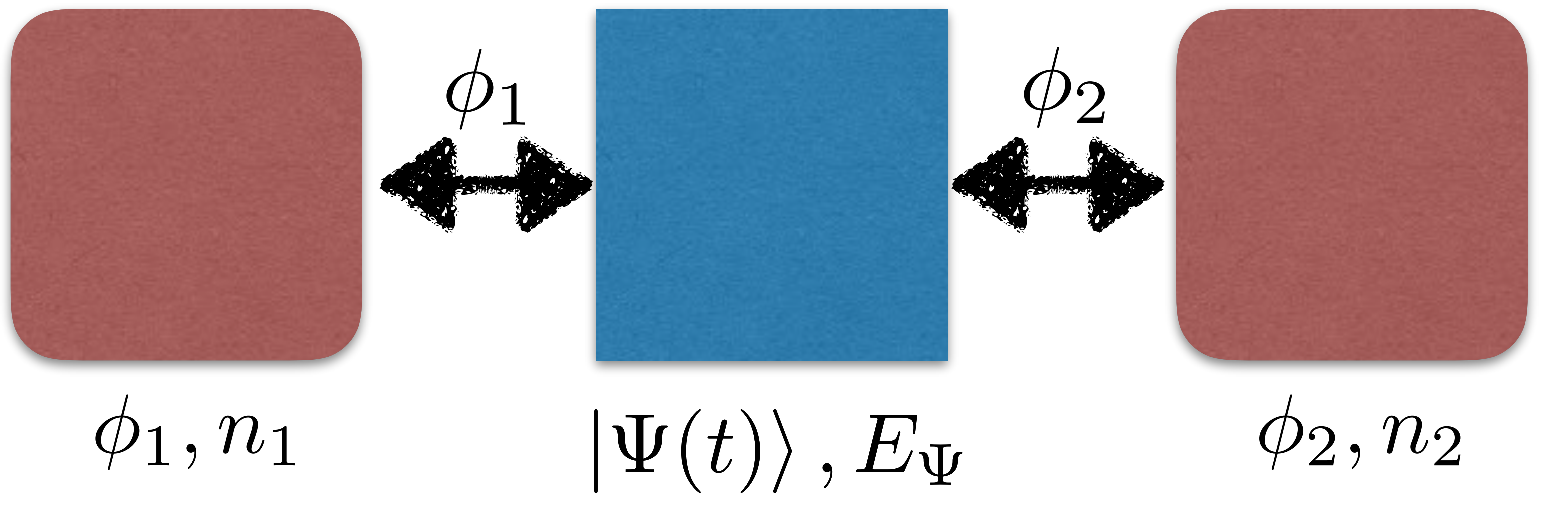
- phase $\phi_\alpha = 2\pi\Phi_\alpha/\phi_0$
- charge $n_\alpha = Q_\alpha\phi_0/(2\pi)$

$$\dot{n}_1 = -\hbar F_{12}\dot{\phi}_2 \quad \rightarrow \quad I_1 = -c_{12}\frac{e^2}{h}V_2 \equiv \sigma_{12}V_2$$

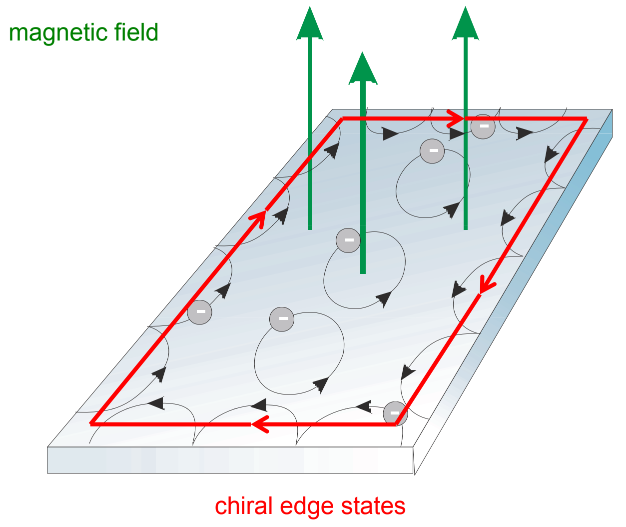
$$\overline{F_{1,2}^{(\Psi)}} = \frac{1}{2\pi}c_{12}^{(\Psi)}$$

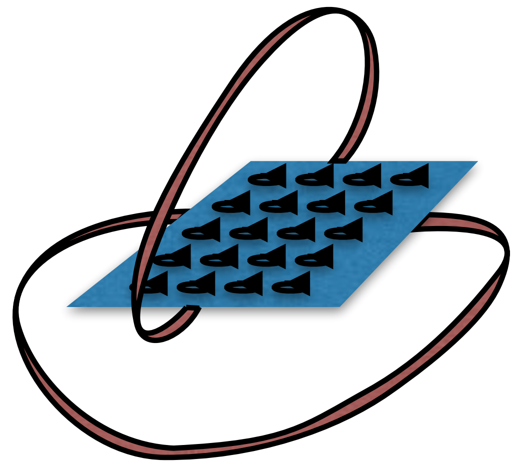
$$\dot{\phi}_2 = 2\pi e/hV_2$$

Topological Pumping via circuit QED



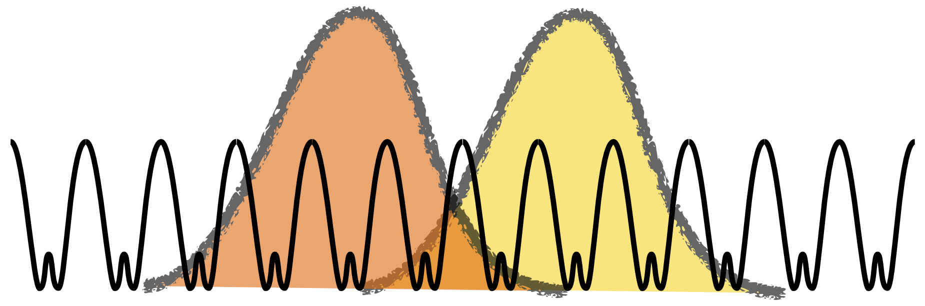
$$\dot{n}_1 = \hbar F_{12}\dot{\phi}_2 = \frac{\hbar}{2\pi}c_{12}\dot{\phi}_2$$

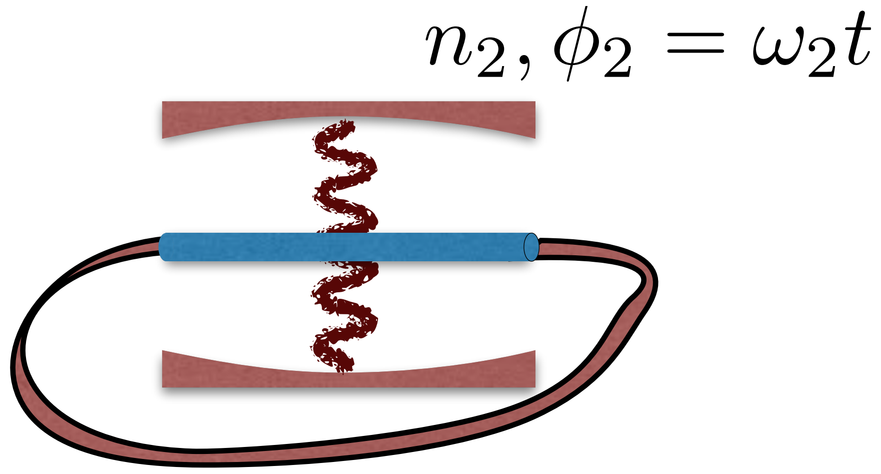




Quantum Hall Effect (d=2)

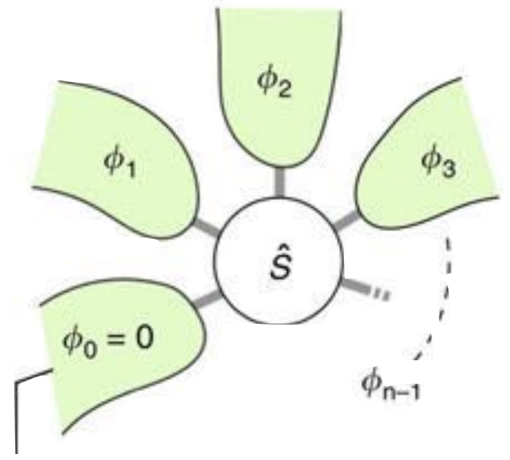
$$I_1 = -c_{12}\frac{e^2}{h}V_2$$

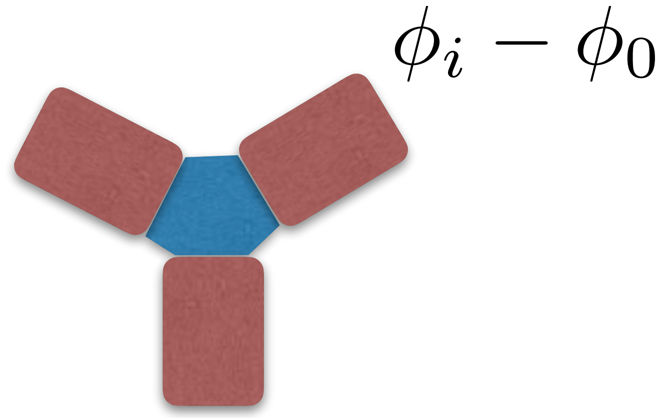




Thouless pump (d=1)

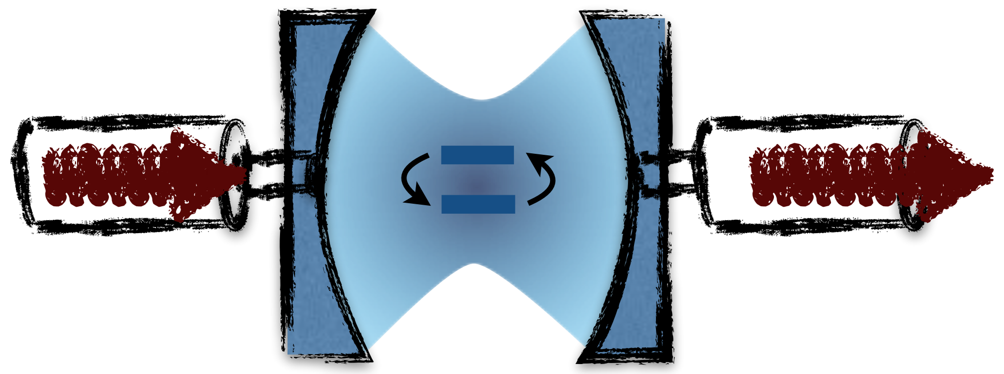
$$j_1 = c_{12}\frac{2\pi a}{T_2}$$





Multi-terminal Josephson junctions

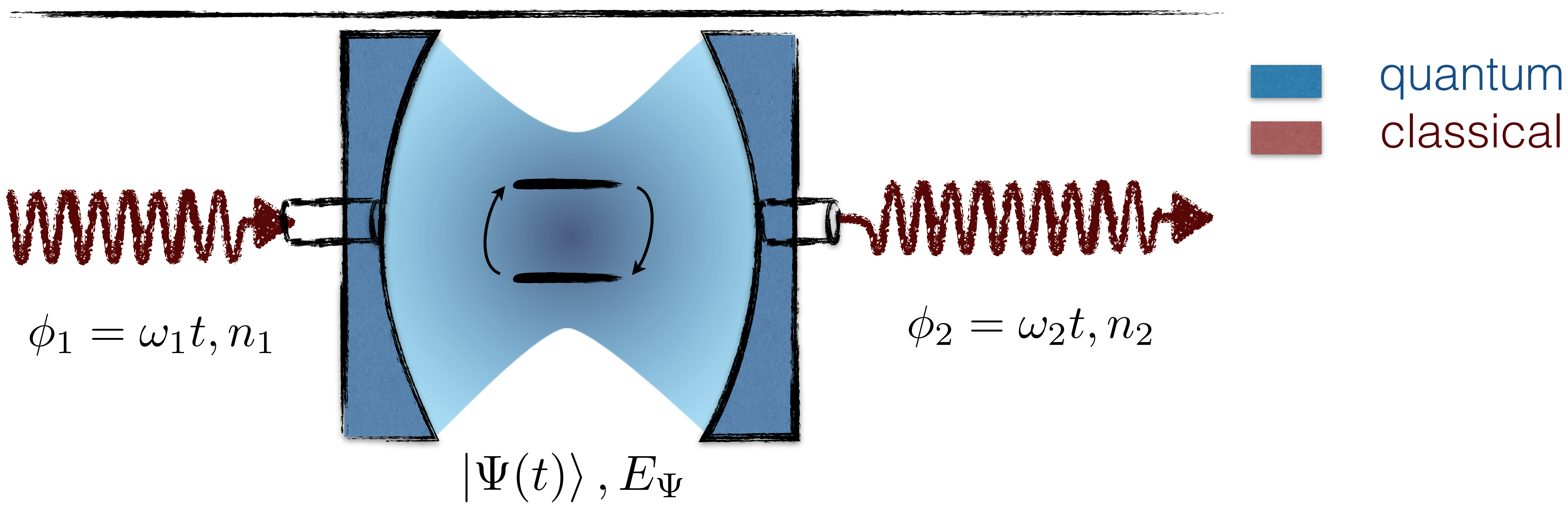
$$I_\alpha = -n_{\alpha\beta}\frac{4e^2}{h}V_\beta$$

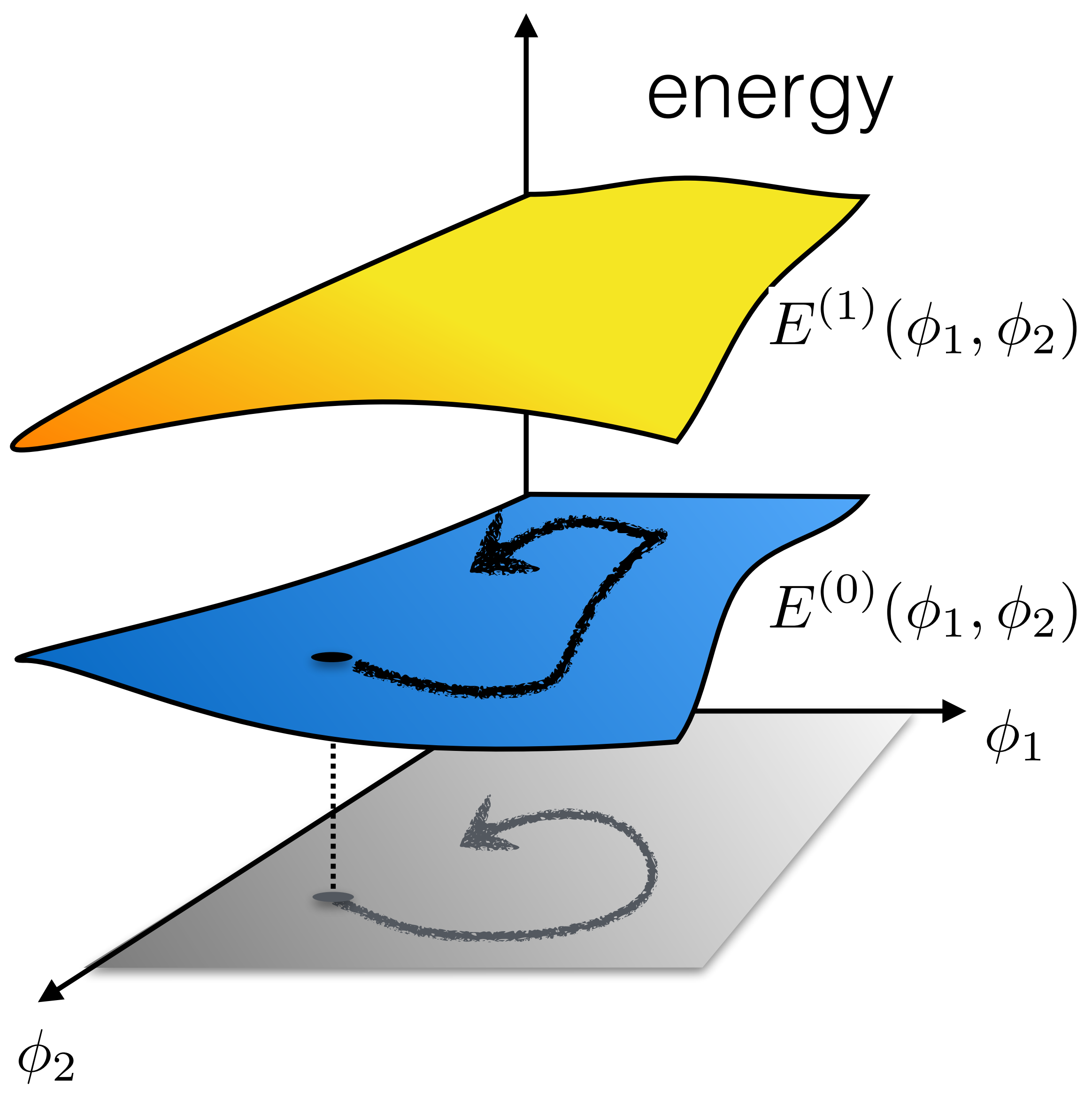


circuit QED

?

Topological Pump : recipe





$$\dot{n}_1 = \hbar F_{12}\dot{\phi}_2 = \frac{\hbar}{2\pi} c_{12}\dot{\phi}_2$$

$$\overline{F^{(\Psi)}_{1,2}} = \frac{1}{2\pi} c^{(\Psi)}_{12}$$

$$c^{(\Psi)}_{12} = \frac{1}{2\pi}\int d\phi_1 d\phi_2 \; F^{(\Psi)}_{1,2}$$

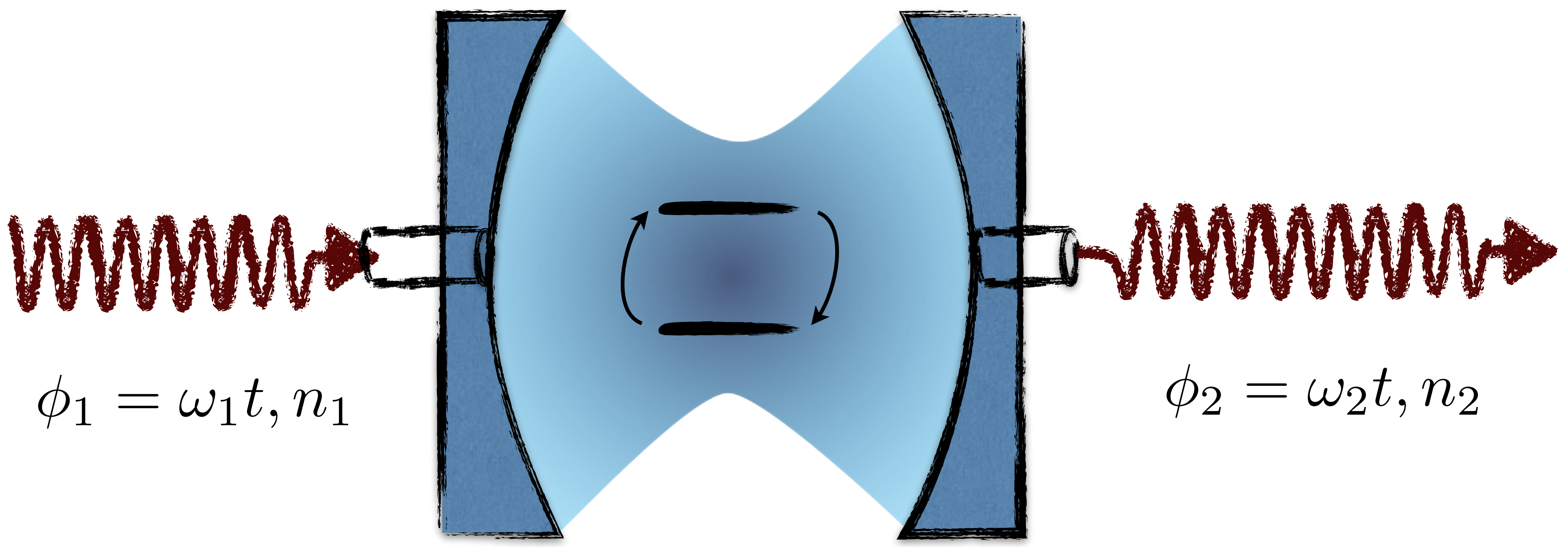
Adiabatic dynamics

- gapped Hamiltonian $H(\{\phi_\alpha\})$ for every $\{\phi_\alpha\}$

Topological (Berry) pump

- choice of (relative) couplings to $\phi_1, \phi_2 \leftrightarrow$ Berry curvature $F^{(\Psi)}_{1,2}$ (depends on dimension)
- induce a Chern number (topological gap opening near level crossing)
- ergodic » : samples the phase space

Topological Pump : 2-level system (qubit)



quantum
classical

$\phi_1 = \omega_1 t, n_1$ $\phi_2 = \omega_2 t, n_2$

$|\Psi(t)\rangle \, , E_\Psi$

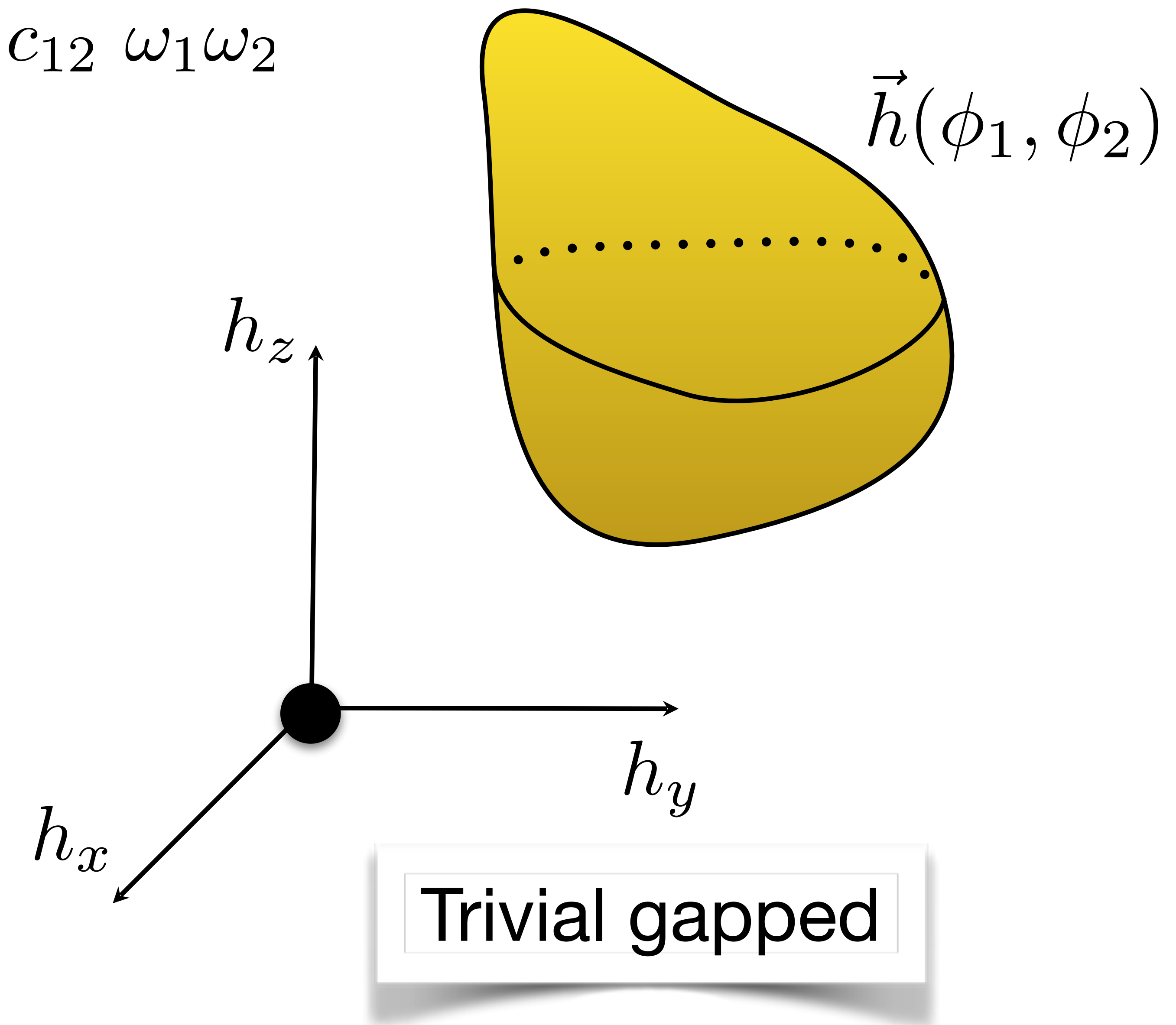
$$H(\phi_1,\phi_2) = \hbar \begin{pmatrix} h_z(\phi_1,\phi_2) & h_x(\phi_1,\phi_2) - i h_y(\phi_1,\phi_2) \\ h_x(\phi_1,\phi_2) + i h_y(\phi_1,\phi_2) & -h_z(\phi_1,\phi_2) \end{pmatrix}$$

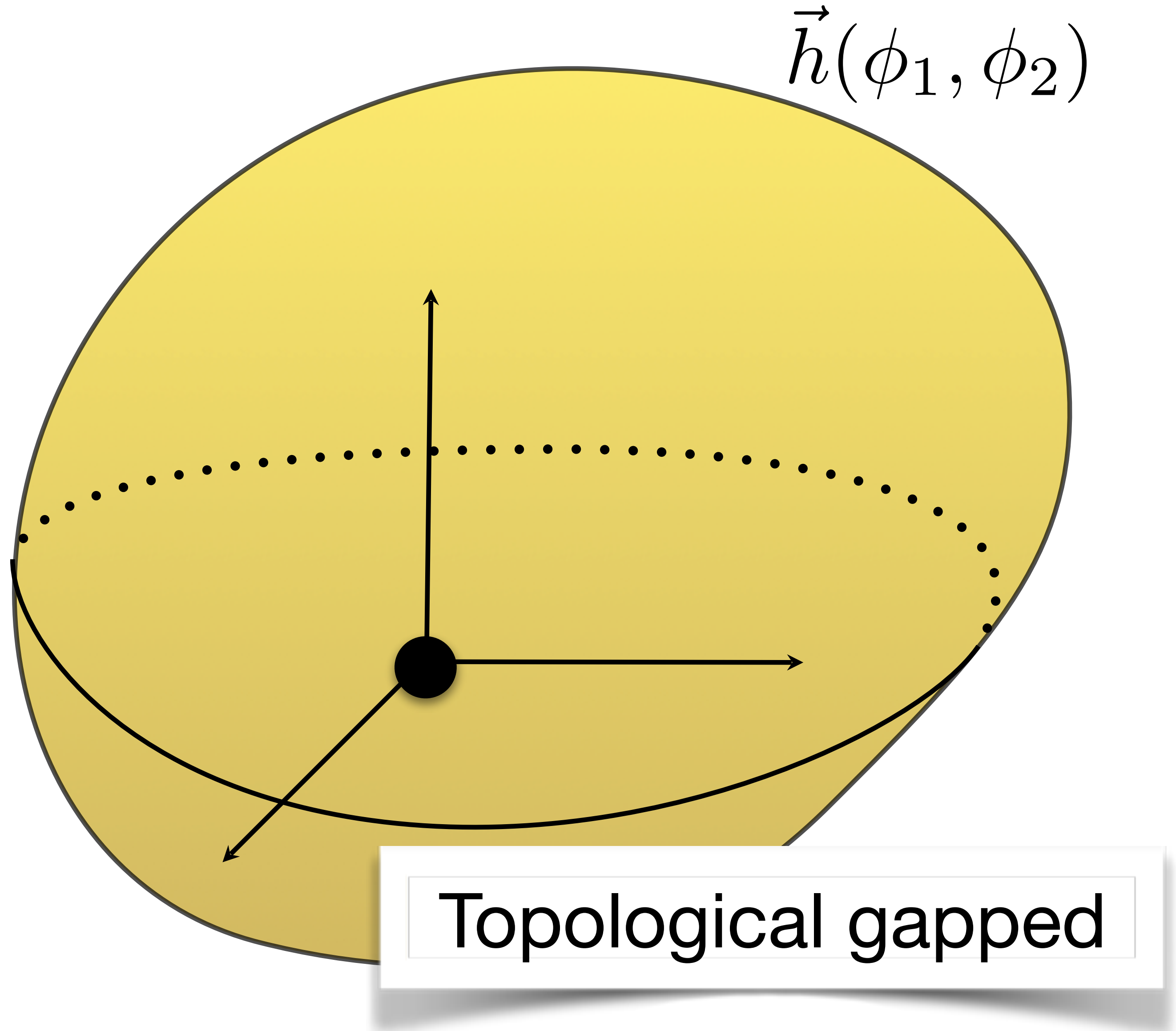
Power transfer :

$$\dot{n}_1 = \hbar F_{12}\dot{\phi}_2 = \frac{\hbar}{2\pi} c_{12}\dot{\phi}_2 \quad \Rightarrow \quad \Delta\mathcal{E}_1 = \dot{n}_1\omega_1 = \frac{\hbar}{2\pi} c_{12}\ \omega_1\omega_2$$

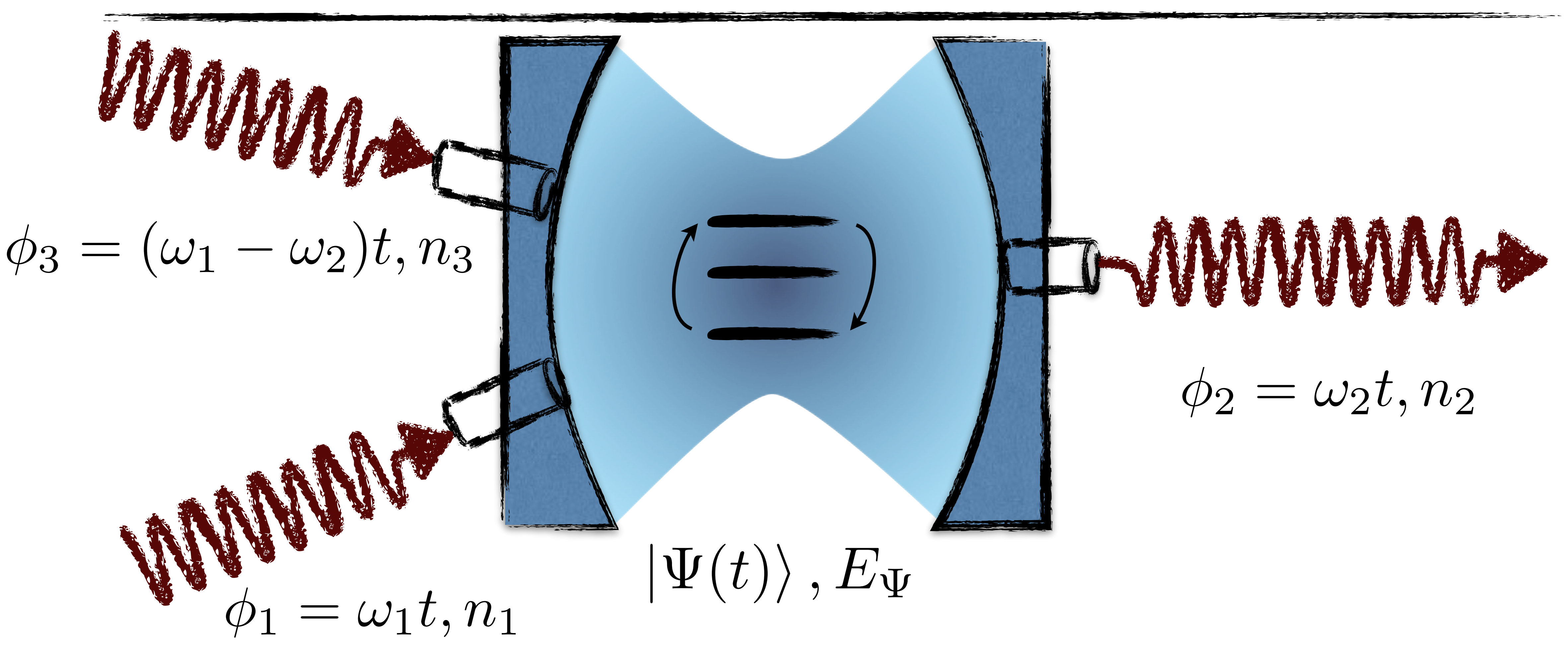
Condition of topology :

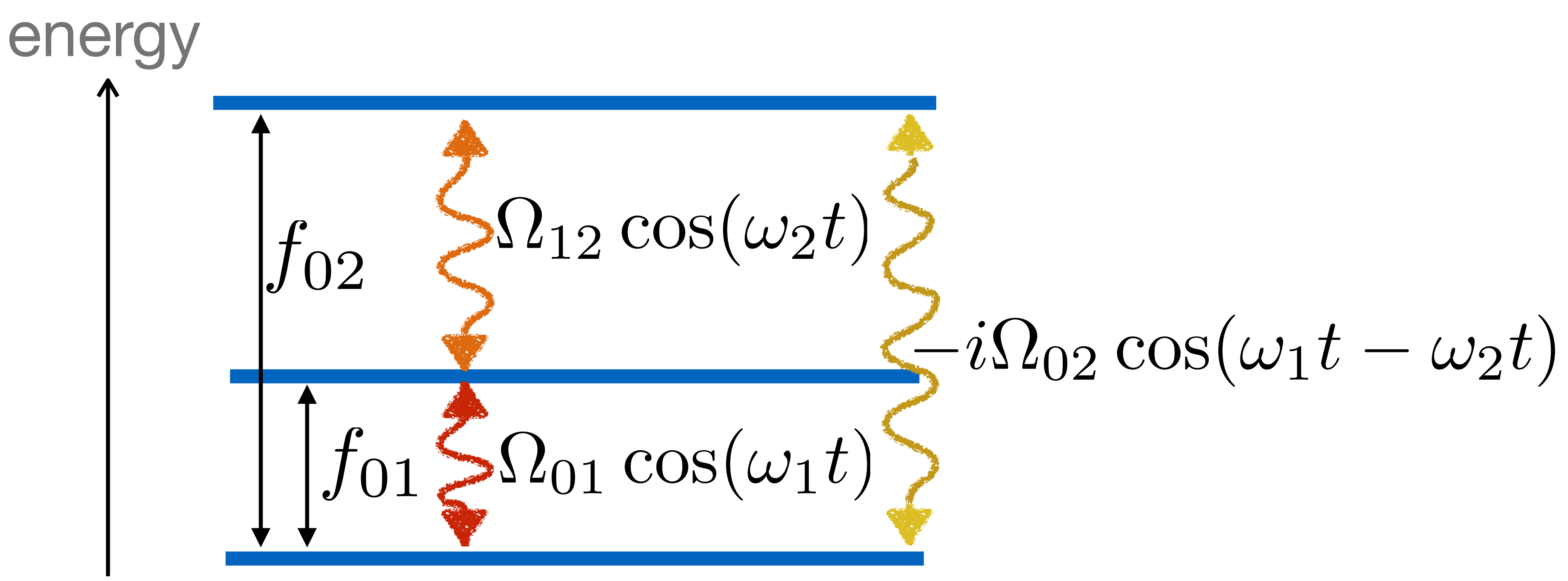
necessary condition :
all couplings must change sign !
... hard





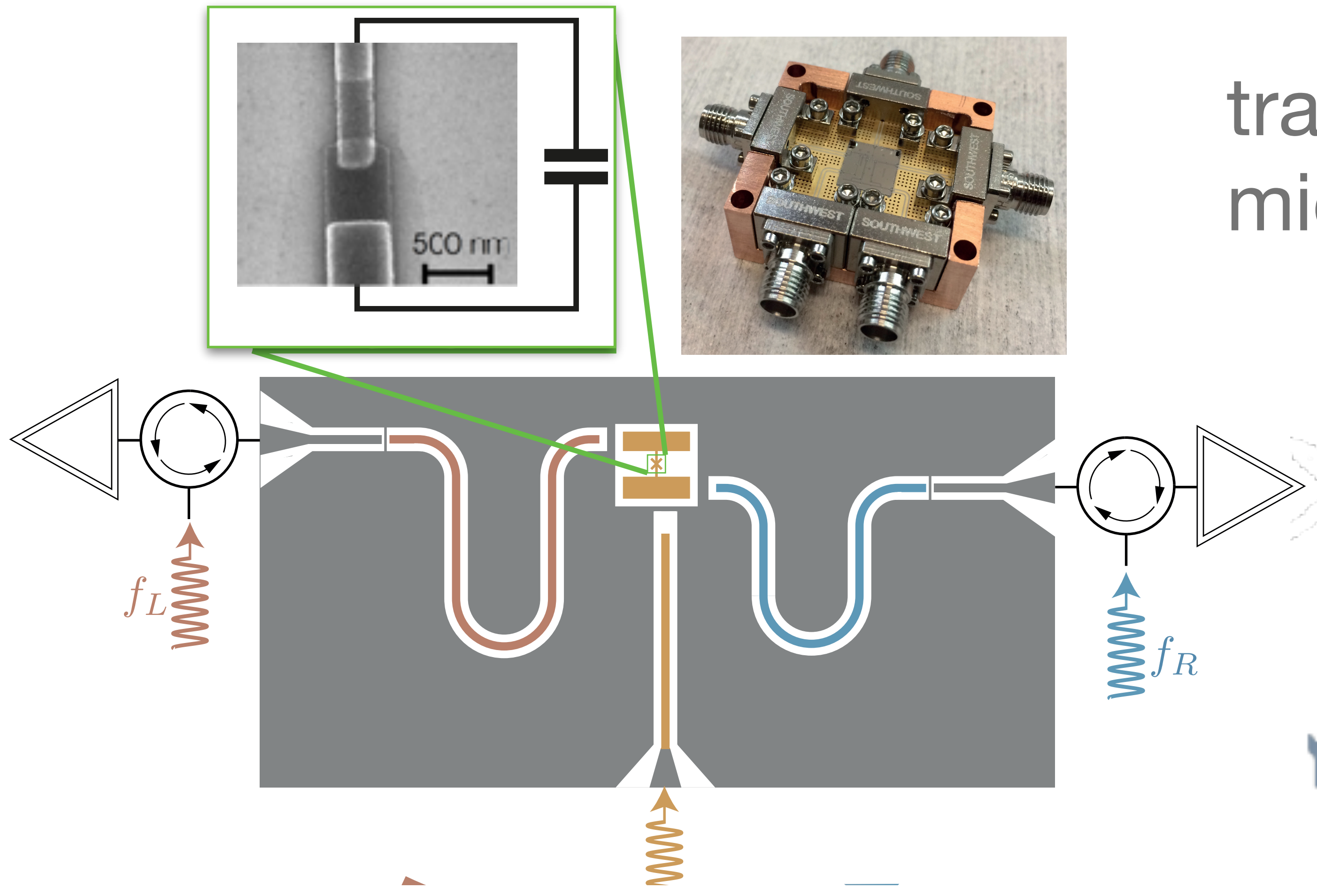
Topological Pump : 3-level system (qutrit)



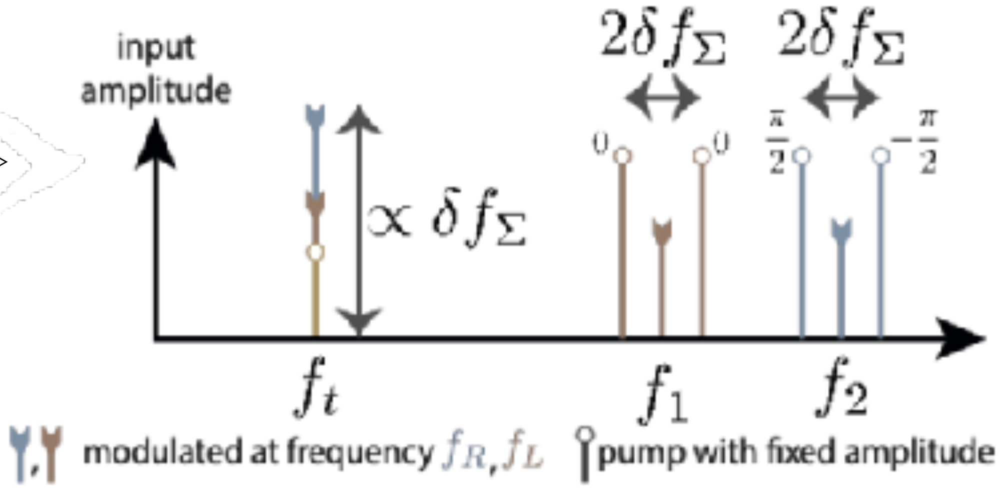


$$\Omega_{01}, \Omega_{02} < f_{01} < f_{12}, \Omega_{12} < f_{12}$$

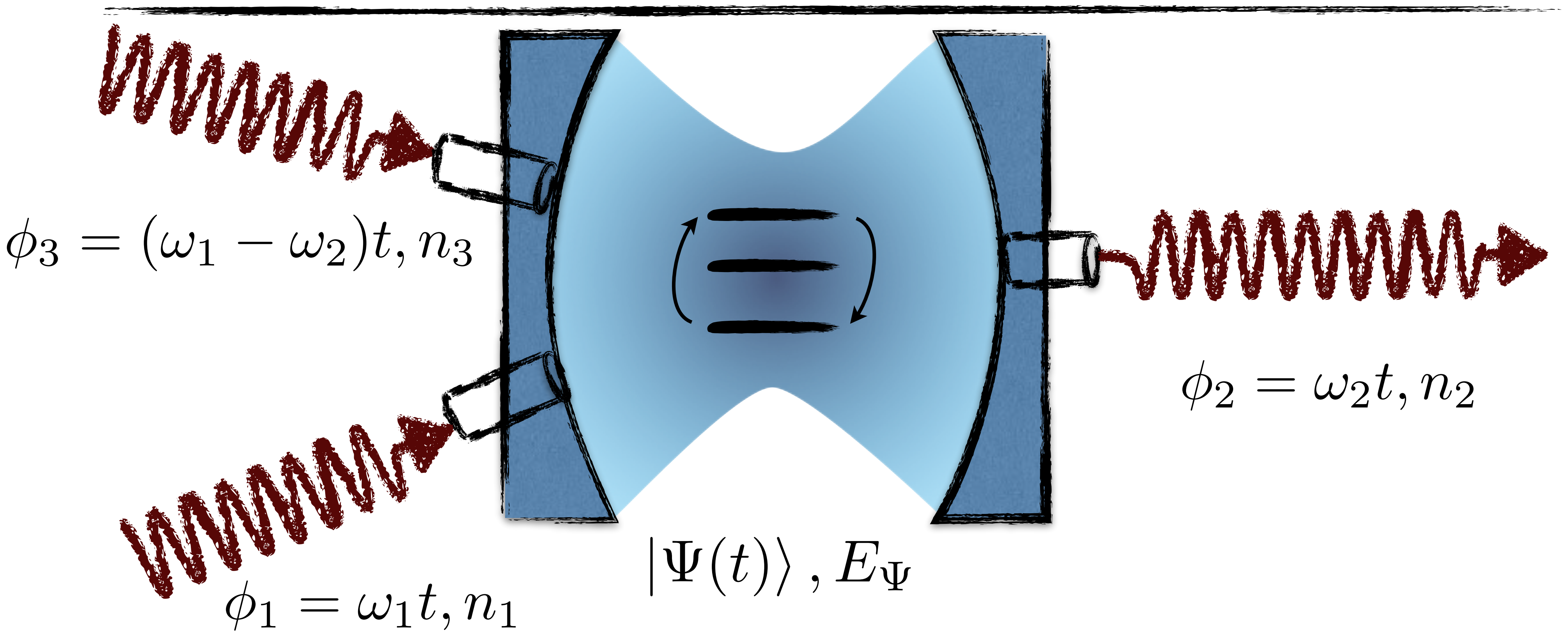
$$H(\phi_1,\phi_2) = \hbar \begin{pmatrix} 0 & \Omega_{01}\cos(\phi_1) & -i\Omega_{02}\cos(\phi_1-\phi_2) \\ \Omega_{01}\cos(\phi_1) & f_{01} & \Omega_{12}\cos(\phi_2) \\ i\Omega_{02}\cos(\phi_1-\phi_2) & \Omega_{12}\cos(\phi_2) & f_{02} \end{pmatrix}$$

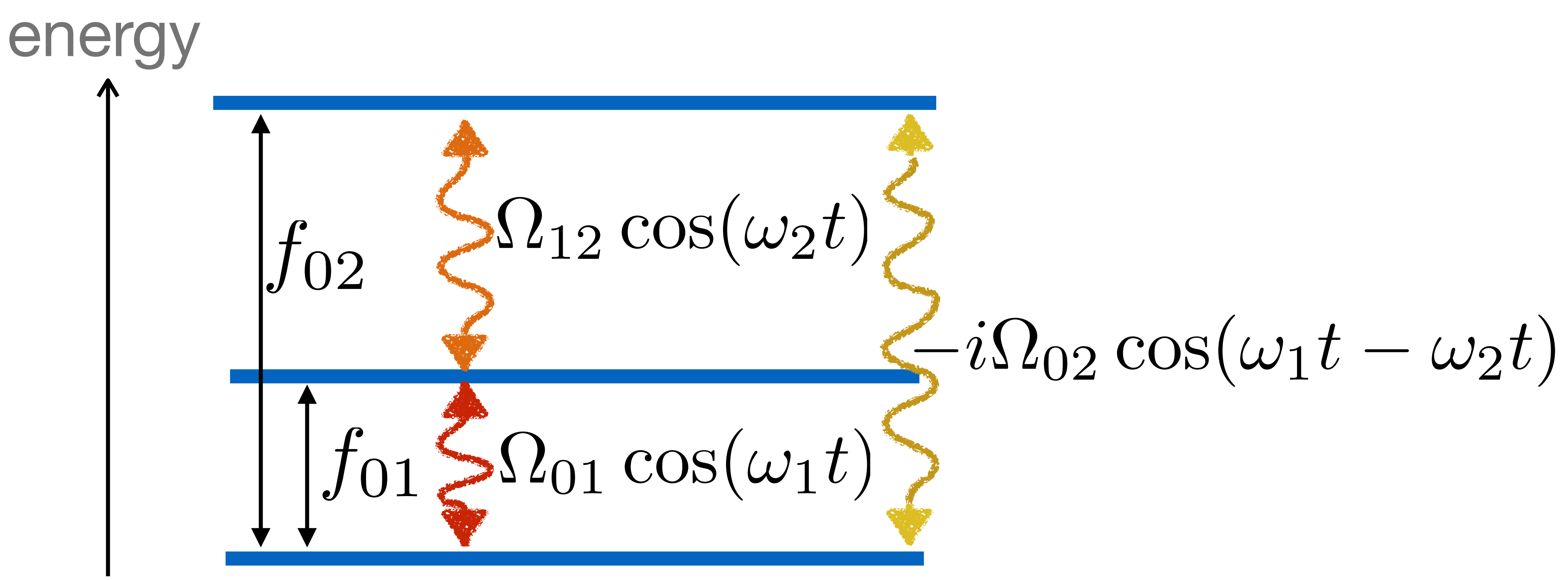


transmon qubit coupled to microwave cavities



Topological Pump : 3-level system (qutrit)

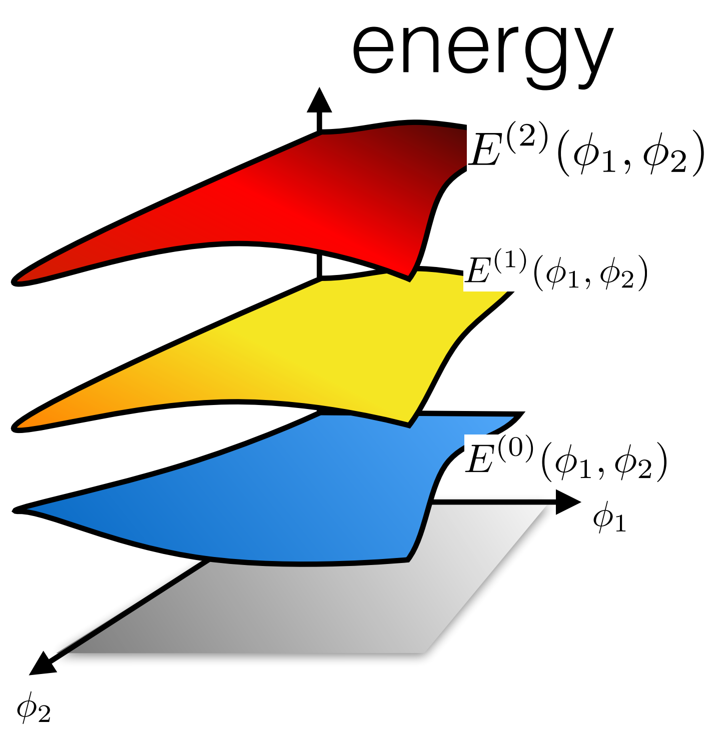




$$\Omega_{01}, \Omega_{02} < f_{01} < f_{12}, \Omega_{12} < f_{12}$$

$$H(k_x, k_y) = \hbar \begin{pmatrix} 0 & \Omega_{01}\cos(k_x) & -i\Omega_{02}\cos(k_x - k_y) \\ \Omega_{01}\cos(k_x) & f_{01} & \Omega_{12}\cos(k_y) \\ i\Omega_{02}\cos(k_x - k_y) & \Omega_{12}\cos(k_y) & f_{02} \end{pmatrix}$$

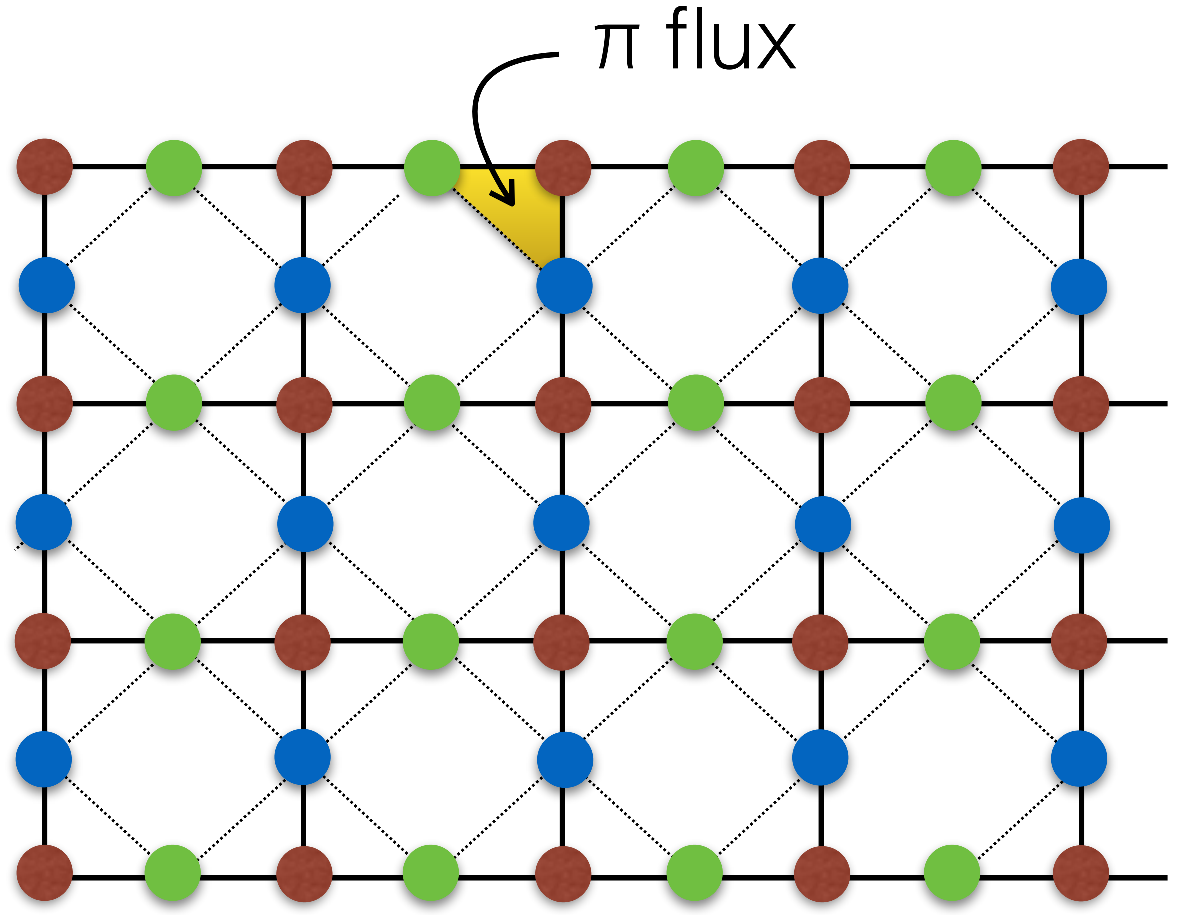
Condition of topology :



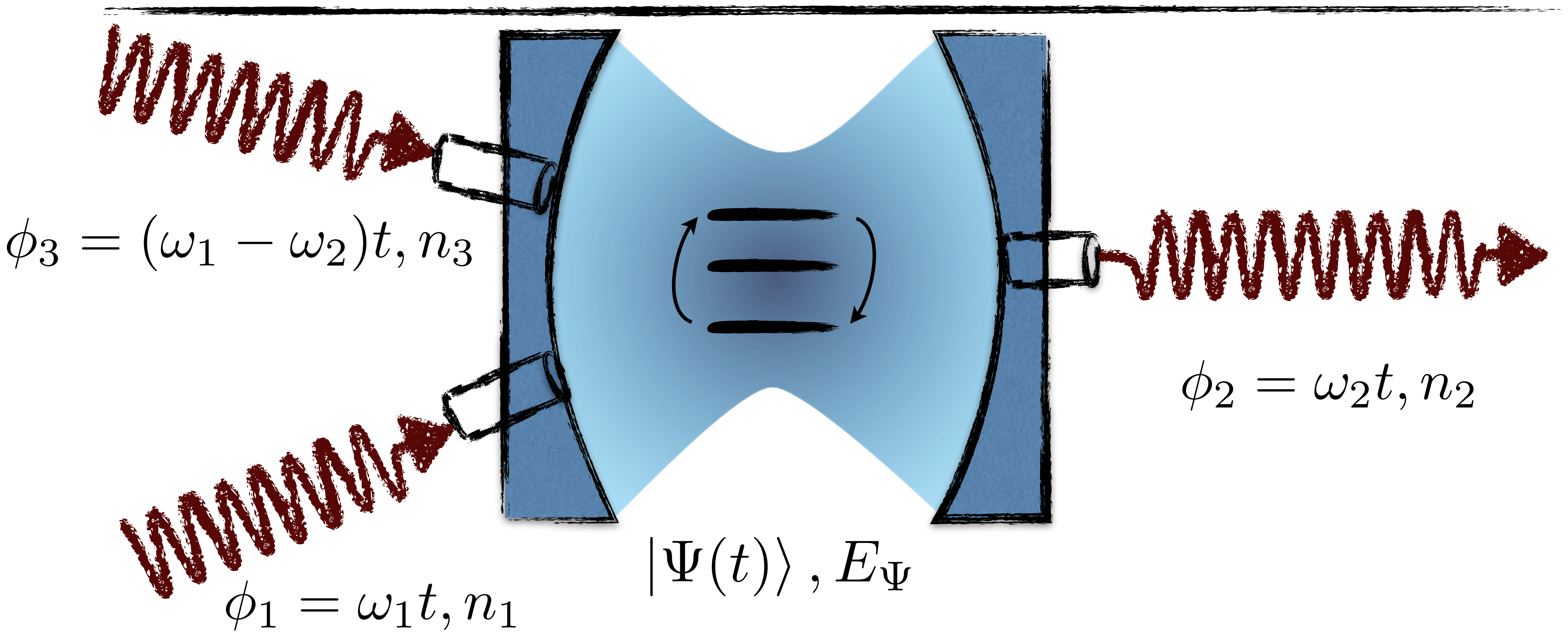
- analysis of topology via Haldane model on Lieb lattice

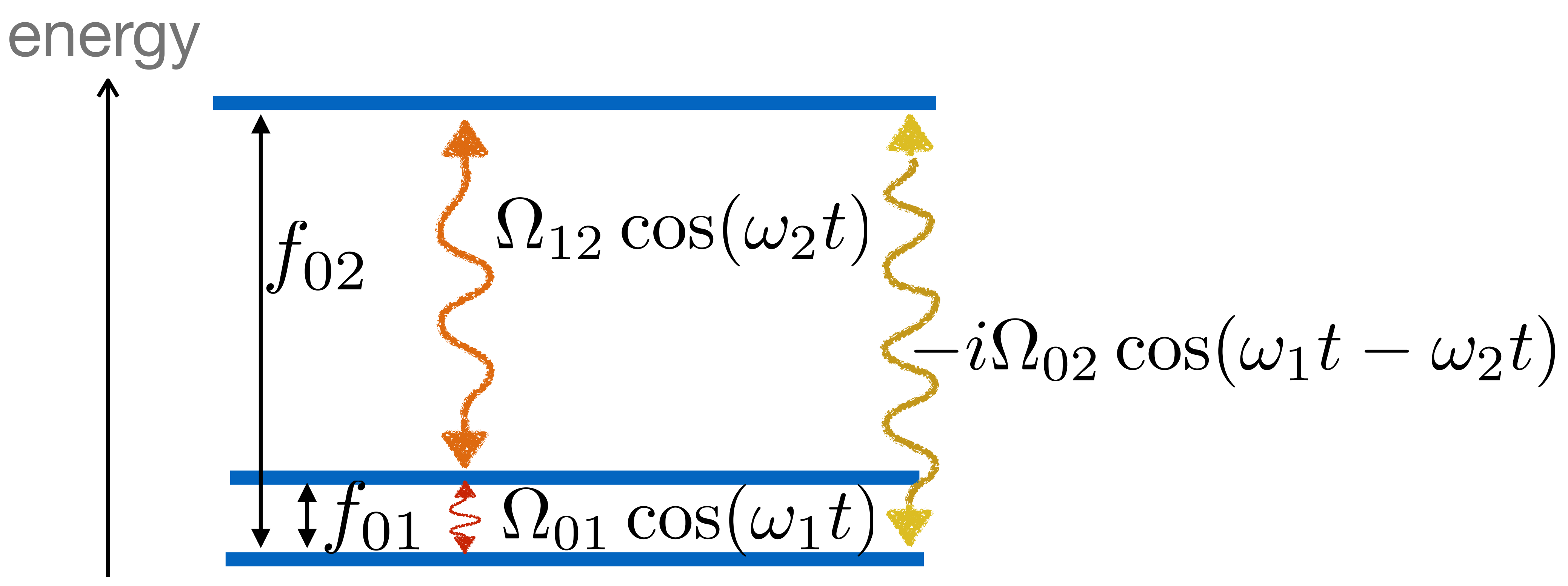
  $H(\phi_1, \phi_2) \leftrightarrow H(k_x, k_y)$

- 2 lowers bands « topological » (Chern number $c = \pm 4$)
  if $\Omega_{12} \geq \sqrt{f_{01} f_{02}}$



Topological Pump : effective 2-level system (qutrit)





$$\Omega_{01}, \Omega_{02} < f_{01} < f_{12}, \Omega_{12} < f_{12}$$

$$H_{eff}(\phi_1,\phi_2) = \hbar \begin{pmatrix} h_z(\phi_1,\phi_2) & h_x(\phi_1,\phi_2) - ih_y(\phi_1,\phi_2) \\ h_x(\phi_1,\phi_2) + ih_y(\phi_1,\phi_2) & -h_z(\phi_1,\phi_2) \end{pmatrix}$$

Perturbation theory

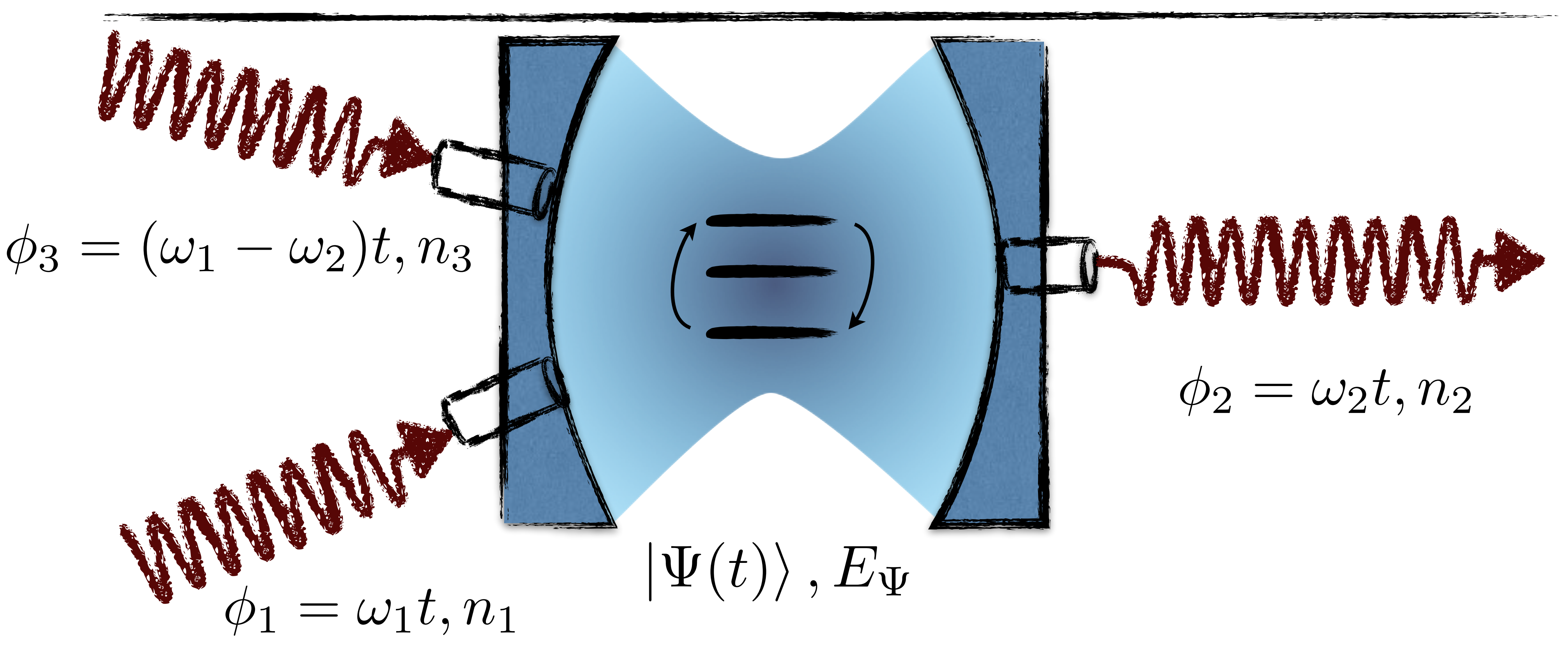
if $f_{01} \ll f_{02}$

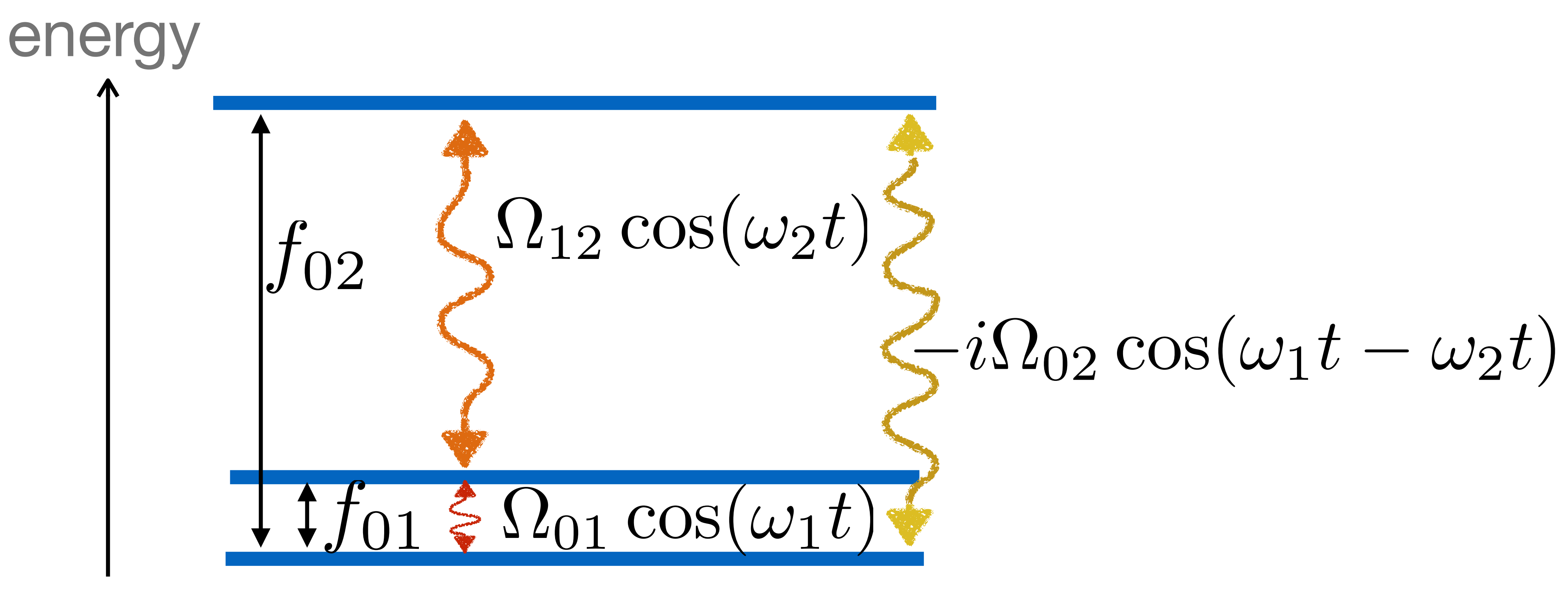
$$h_x = \Omega_{01}\cos(\phi_a) \; ;$$

$$h_y = -\frac{\Omega_{02}\Omega_{12}}{f_{02}}\cos(\phi_a - \phi_b)\cos(\phi_b) \; ;$$

$$h_z = -\frac{1}{2}f_{01} - \frac{1}{2}\frac{\Omega_{02}\Omega_{02}}{f_{02}}\cos^2(\phi_a - \phi_b) + \frac{1}{2}\frac{\Omega_{12}\Omega_{12}}{f_{12}}\cos^2(\phi_b)$$

Topological Pump : effective 2-level system (qutrit)



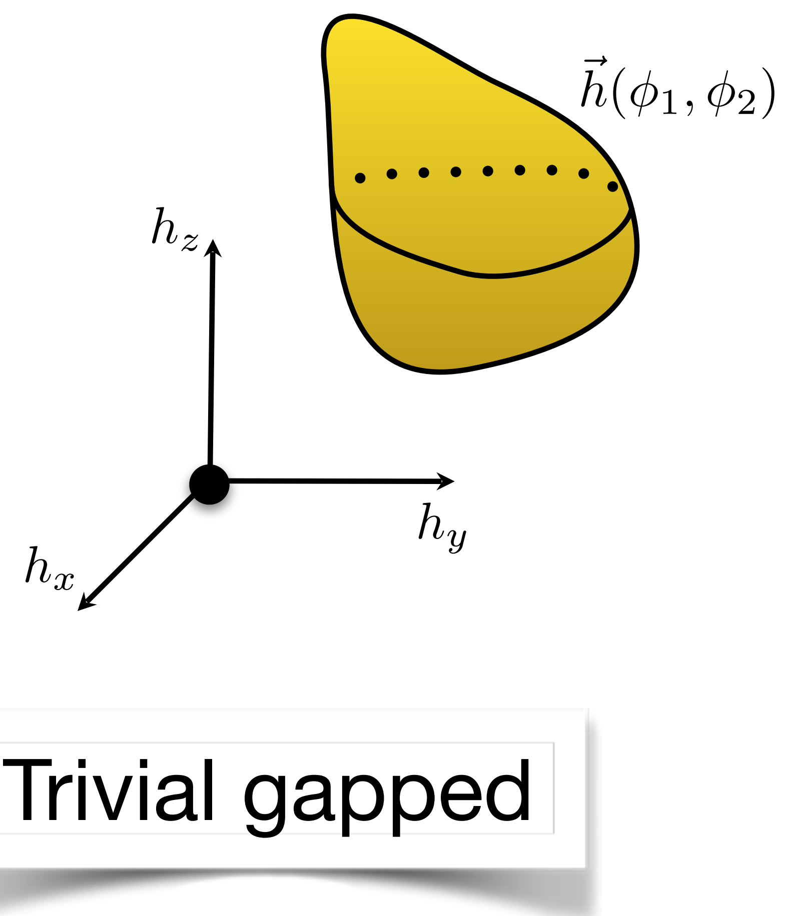


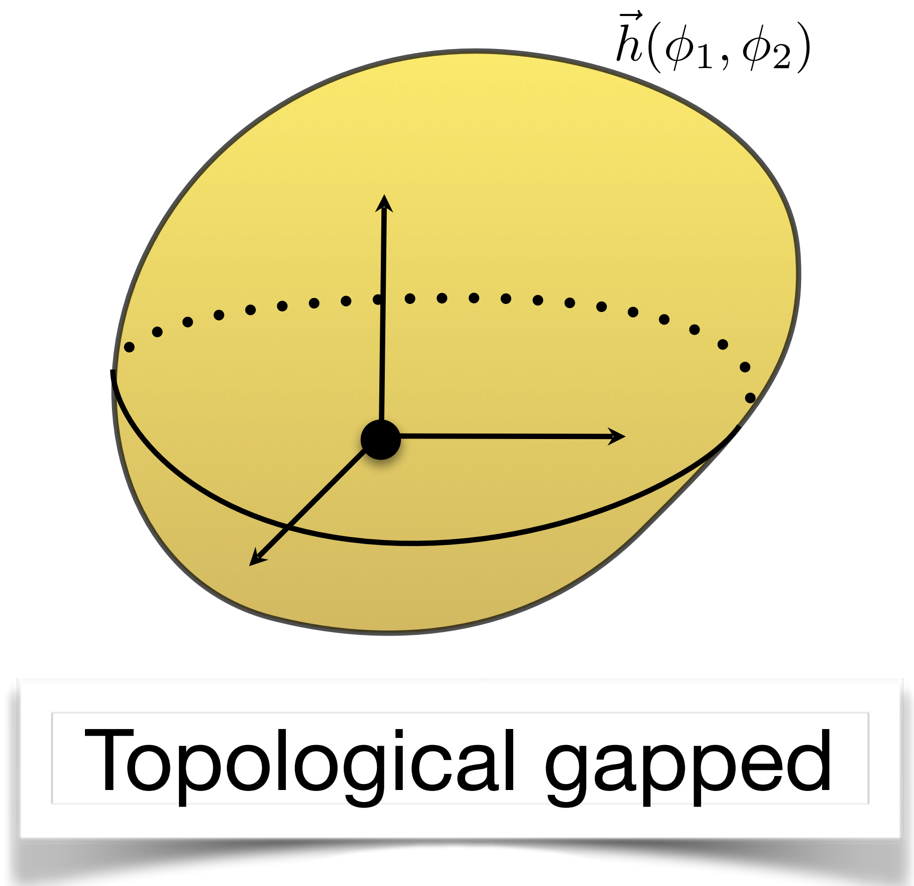
$$\Omega_{01}, \Omega_{02} < f_{01} < f_{12}, \Omega_{12} < f_{12}$$

$$H_{eff}(\phi_1,\phi_2) = \hbar \begin{pmatrix} h_z(\phi_1,\phi_2) & h_x(\phi_1,\phi_2) - ih_y(\phi_1,\phi_2) \\ h_x(\phi_1,\phi_2) + ih_y(\phi_1,\phi_2) & -h_z(\phi_1,\phi_2) \end{pmatrix}$$

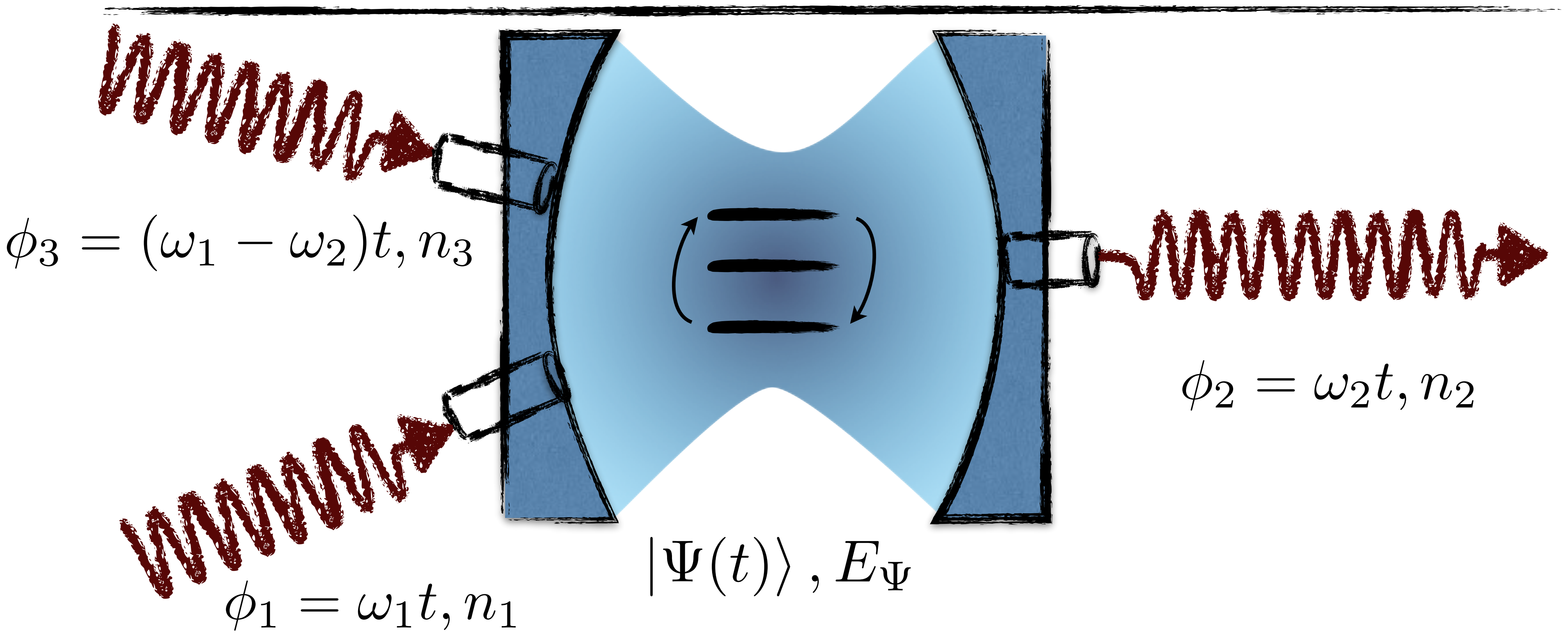
Condition of topology :

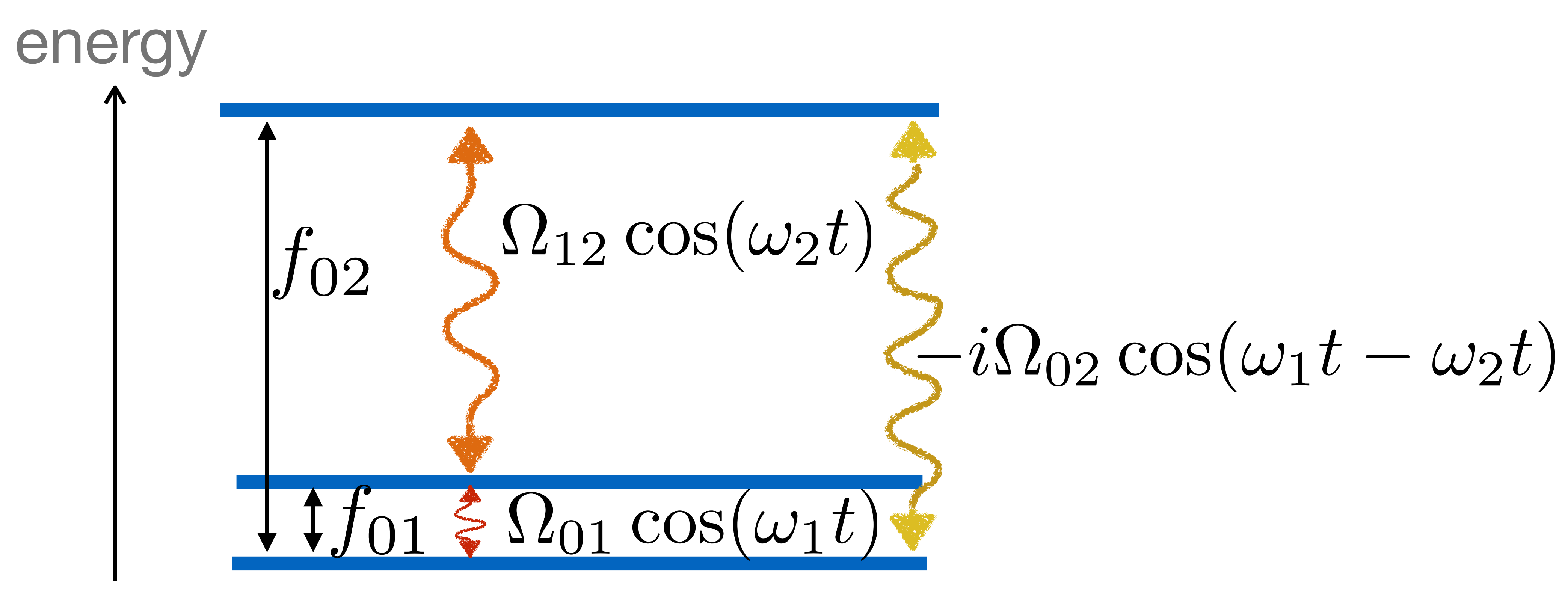
2 lowers bands « topological » (Chern number $c = \pm 4$)
if $\Omega_{12} \geq \sqrt{f_{01} f_{02}}$





Topological Pump : effective 2-level system (qutrit)





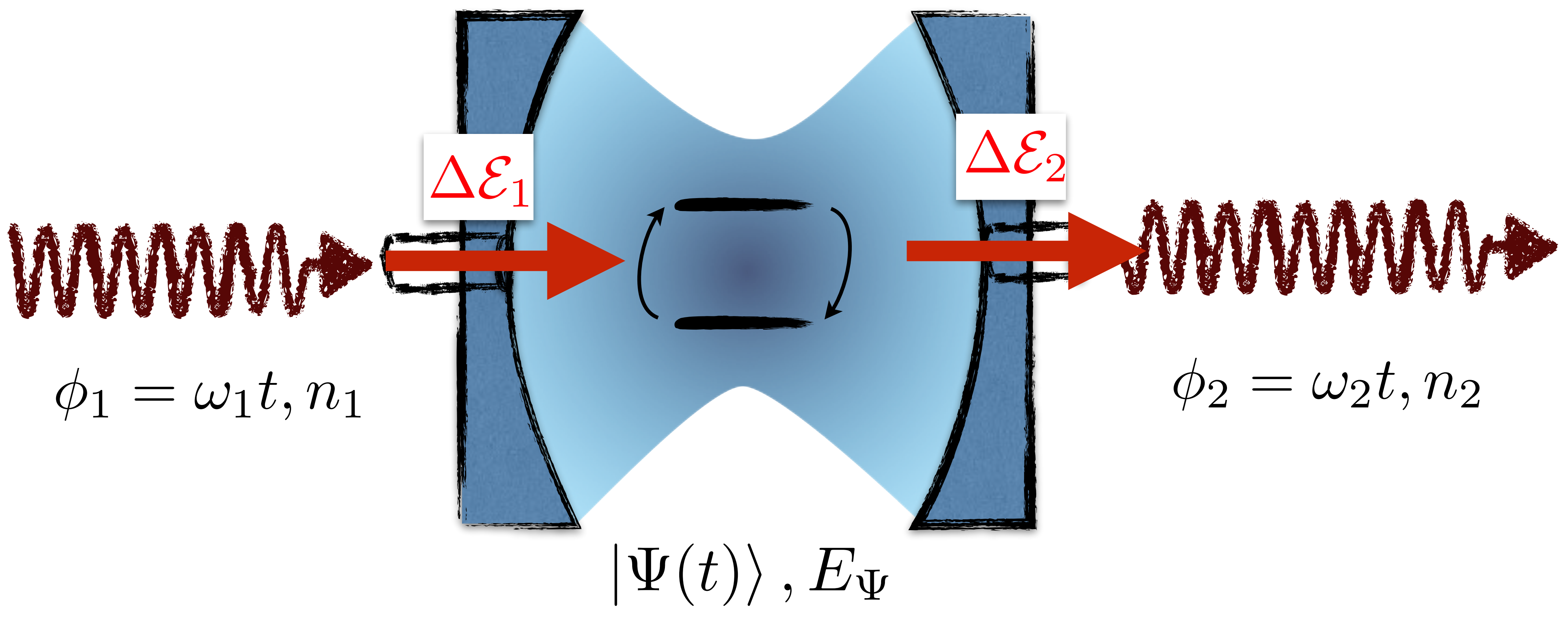
$$\Omega_{01}, \Omega_{02} < f_{01} < f_{12}, \Omega_{12} < f_{12}$$

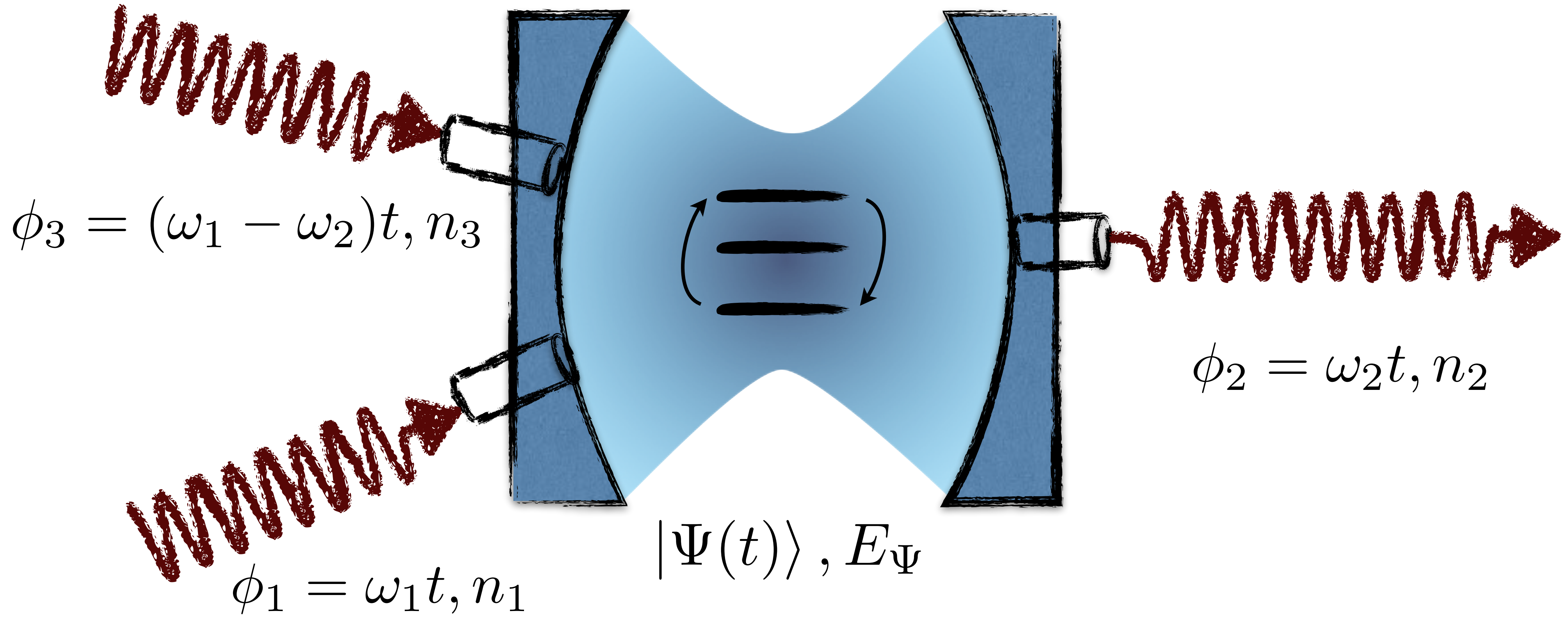
$$H(\phi_1,\phi_2) = \hbar \begin{pmatrix} 0 & \Omega_{01}\cos(\phi_1) & -i\Omega_{02}\cos(\phi_1-\phi_2) \\ \Omega_{01}\cos(\phi_1) & f_{01} & \Omega_{12}\cos(\phi_2) \\ i\Omega_{02}\cos(\phi_1-\phi_2) & \Omega_{12}\cos(\phi_2) & f_{02} \end{pmatrix}$$

f_{01}	$2\pi \times 200$ MHz
f_{02}	$2\pi \times 5$ GHz
Ω_{01}	$2\pi \times 100$ MHz
Ω_{02}	$2\pi \times 1$ GHz
Ω_{12}	$2\pi \times 1.2$ GHz
$\omega_1 = 2\omega_2$	$2\pi \times 20$MHz

...challenging experimentally

Topological Pump : effective 2-level system (qutrit)





Topological transfer :

« mode number » transfer $\dot{n}_1 = \hbar F^{12} \omega_2$

$\dot{n}_2 = -\hbar F^{12} \omega_1$

Energy exchange (power): $\Delta\mathcal{E}_i = \dot{n}_i \omega_i$

➠ $\Delta\mathcal{E}_1 = -\Delta\mathcal{E}_2$

« mode number » transfer $\dot{n}_1 + \dot{n}_3 = \hbar F^{12} \omega_2$

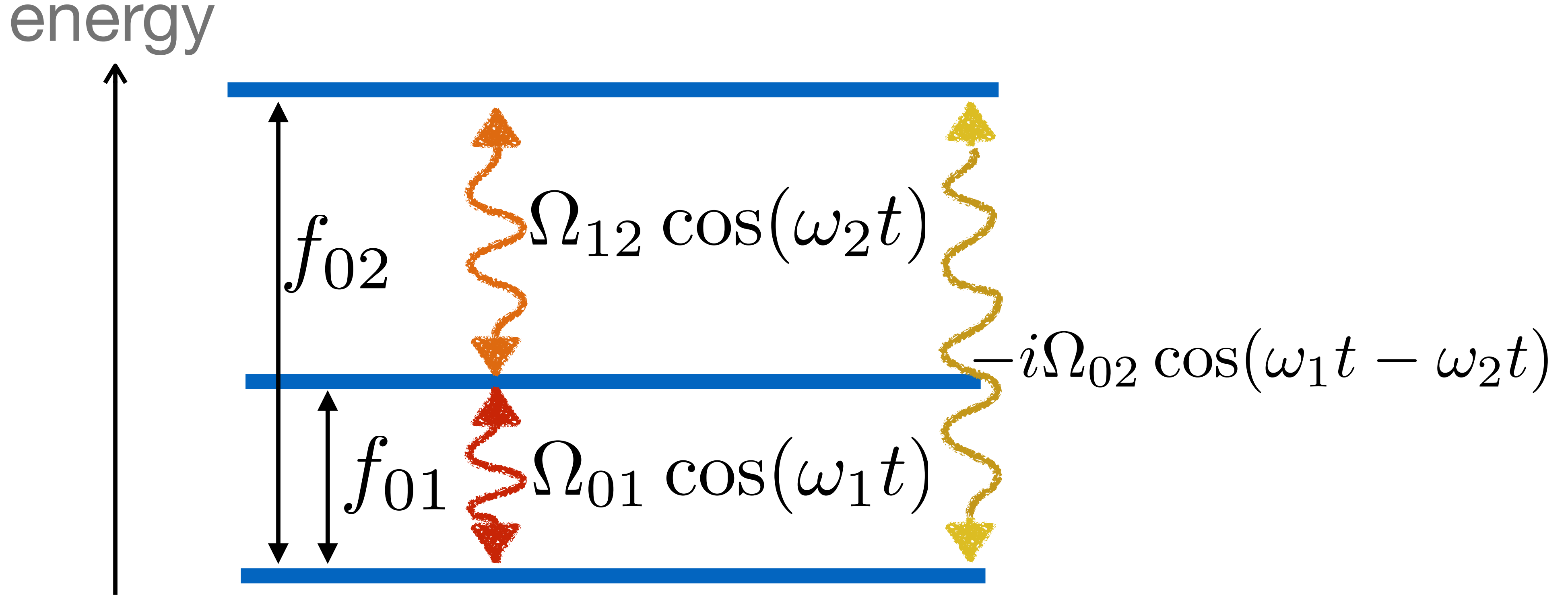
$\dot{n}_2 - \dot{n}_3 = -\hbar F^{12} \omega_1$

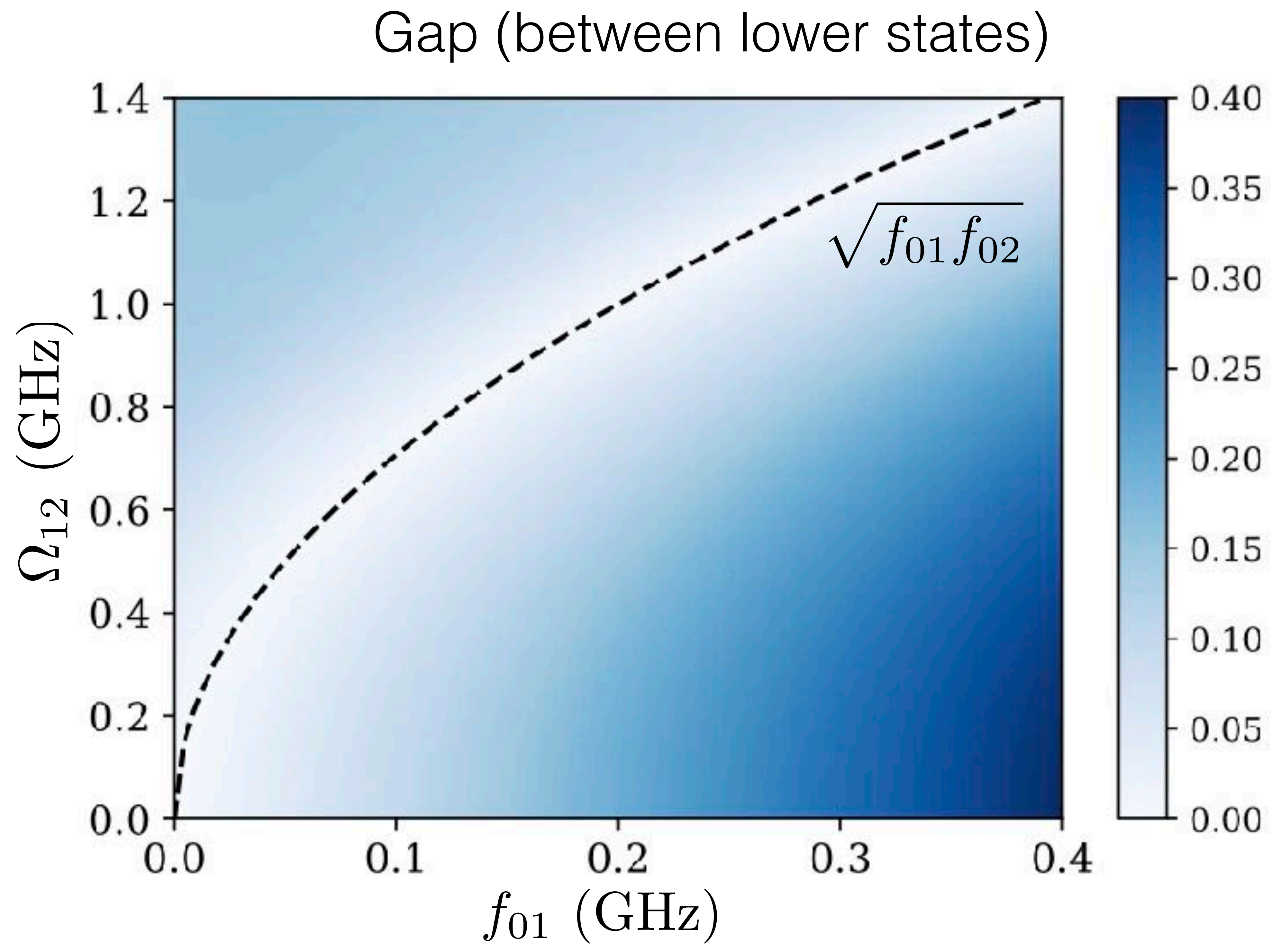
Energy exchange : $\Delta\mathcal{E}_i = \dot{n}_i \omega_i$

Topological rate : $\dfrac{\Delta\mathcal{E}_1}{\omega_1} + \dfrac{\Delta\mathcal{E}_3}{\omega_1 - \omega_2} = \dfrac{\hbar}{2\pi} c_{12}\, \omega_2$

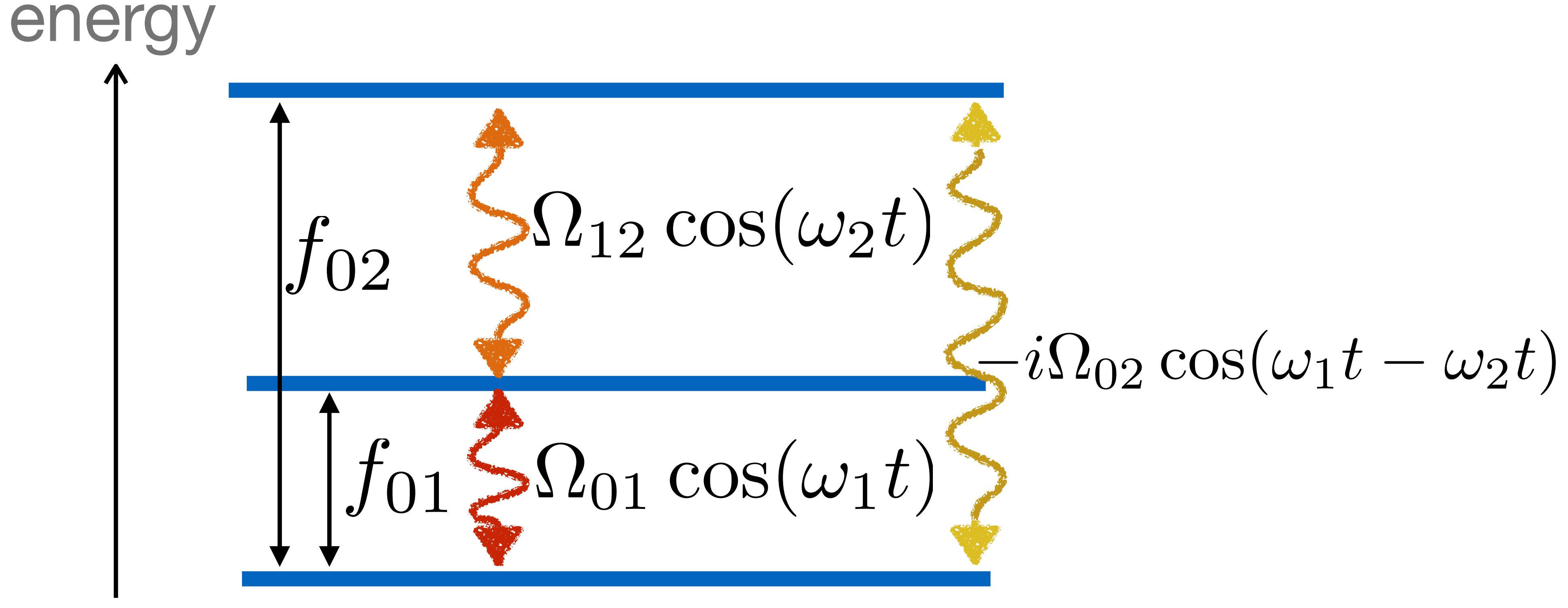
Topological Pump : effective 2 levels (qubit)

f_{01}	$2\pi \times 200$ MHz
f_{02}	$2\pi \times 5$ GHz
Ω_{01}	$2\pi \times 100$ MHz
Ω_{02}	$2\pi \times 1$ GHz
Ω_{12}	$2\pi \times 1.2$ GHz
$\omega_1 = 2\omega_2$	$2\pi \times 20$MHz

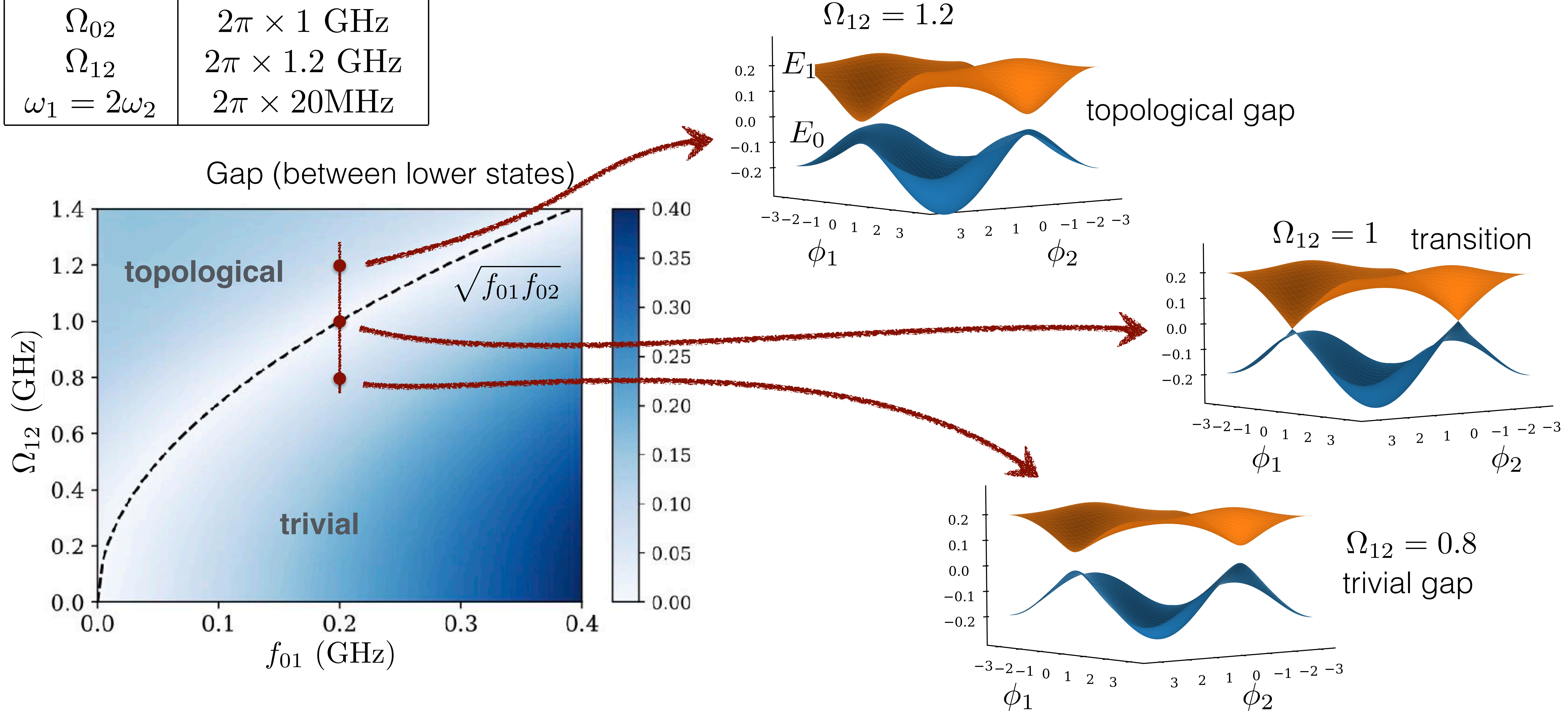




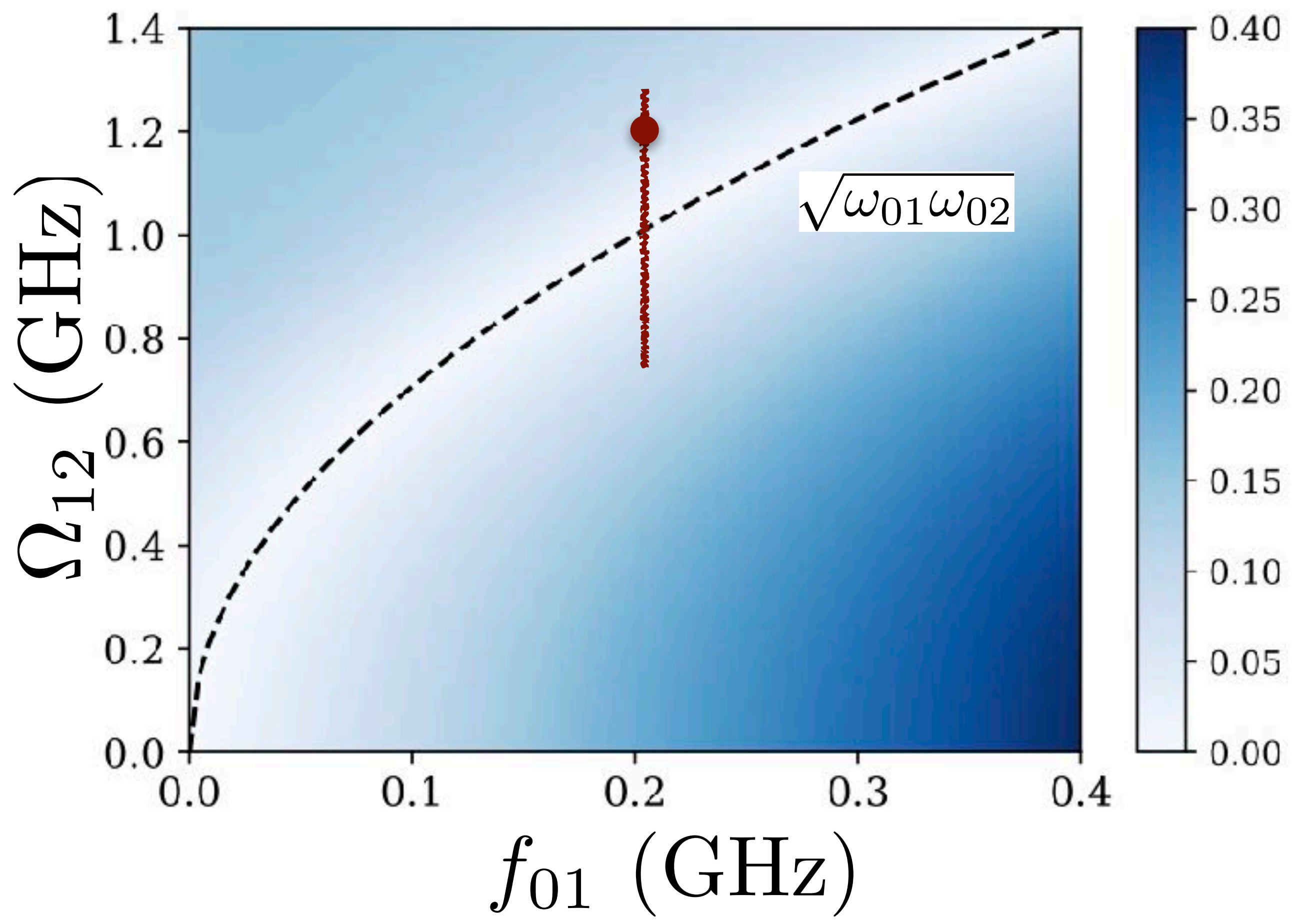
Topological Pump : effective 2 levels (qubit)



f_{01}	$2\pi \times 200$ MHz
f_{02}	$2\pi \times 5$ GHz
Ω_{01}	$2\pi \times 100$ MHz
Ω_{02}	$2\pi \times 1$ GHz
Ω_{12}	$2\pi \times 1.2$ GHz
$\omega_1 = 2\omega_2$	$2\pi \times 20$MHz

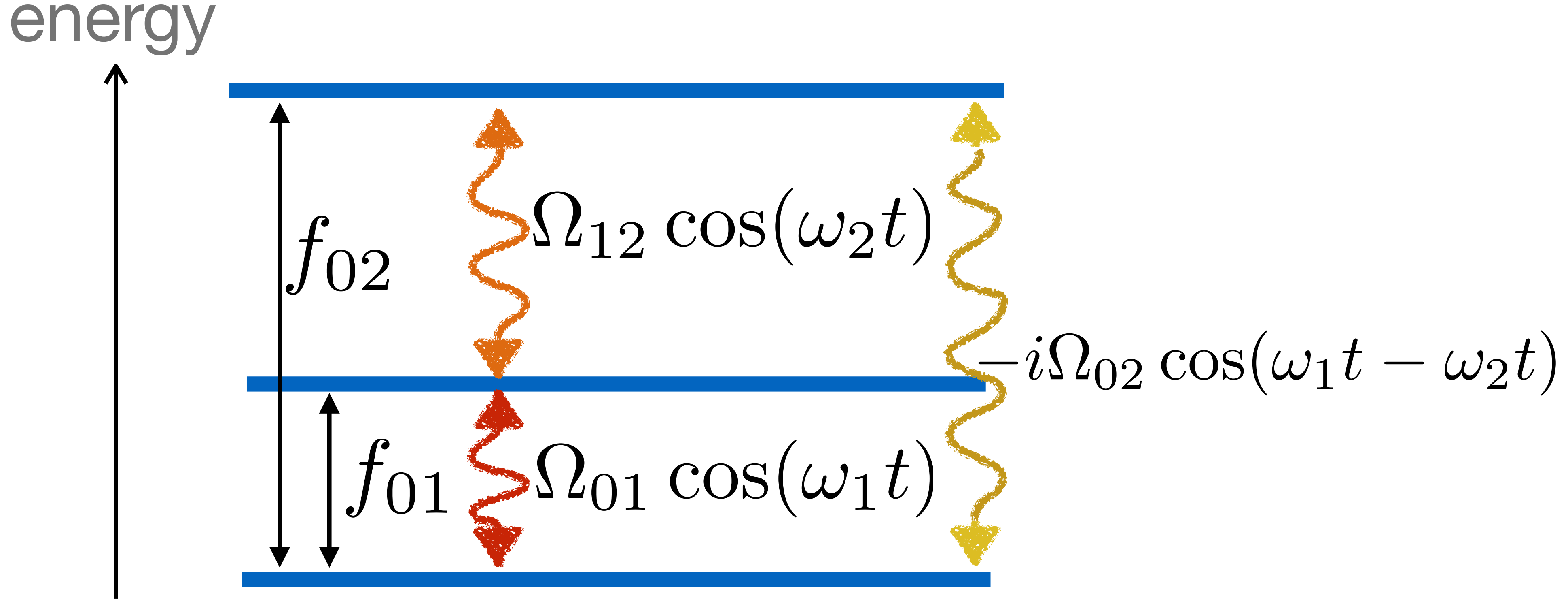


Topological Pump : effective 2 levels (qubit)

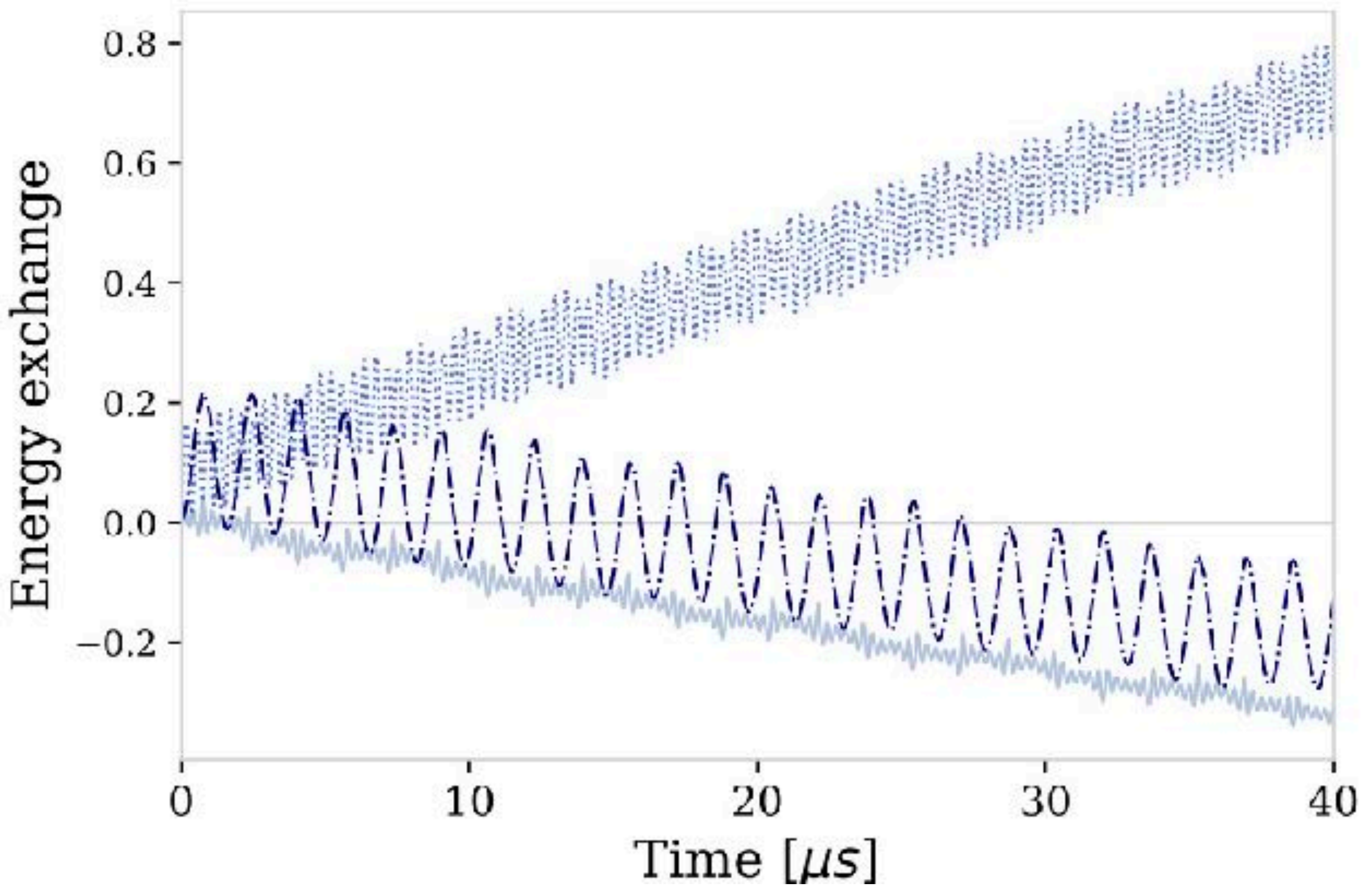


$\omega_1 = 10$ MHz
$\omega_2 \simeq 8$ MHz
$\Omega_{12} = 1.2$

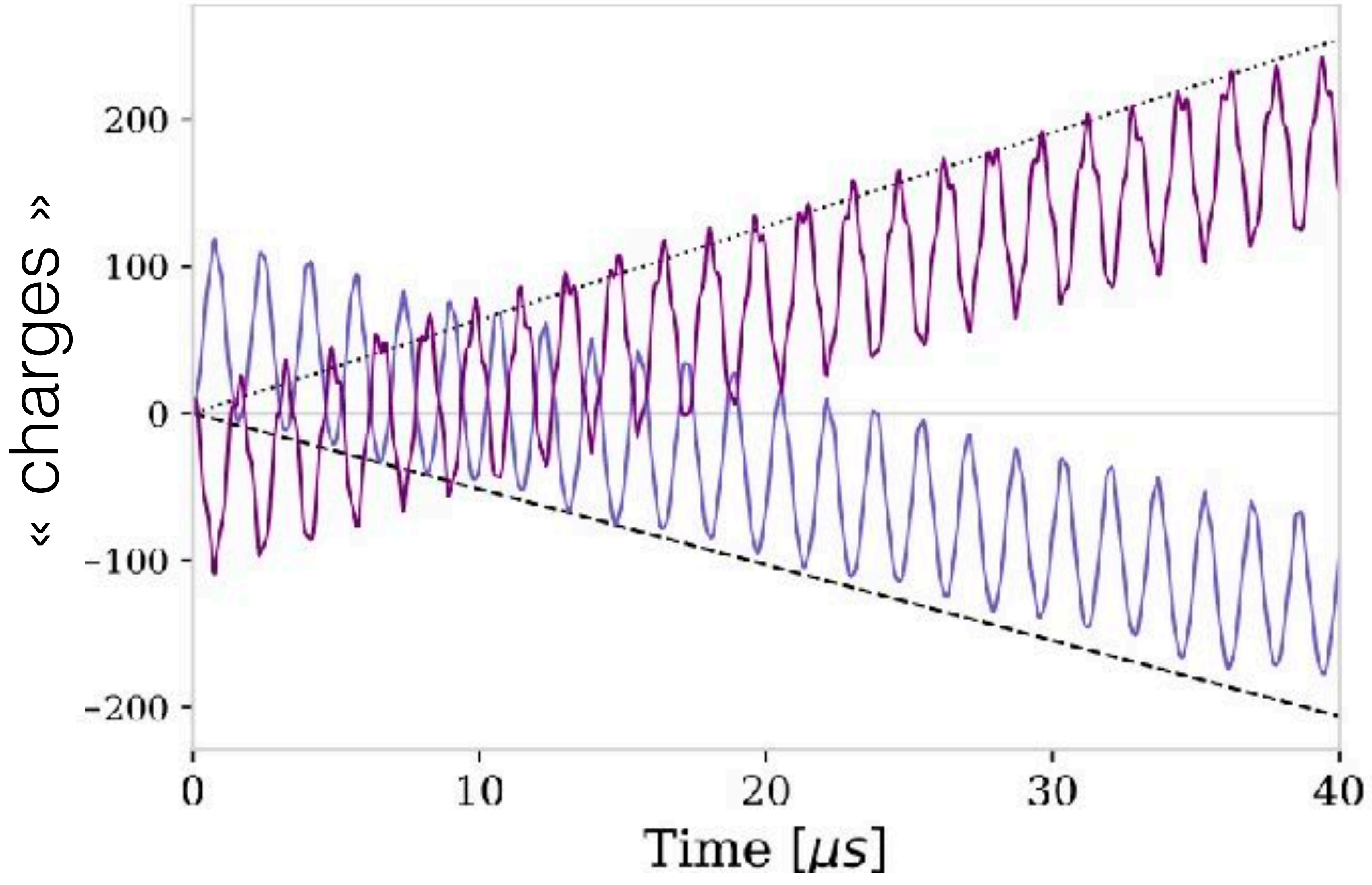
Prepare qutrit in ground state $|\psi_0\rangle$



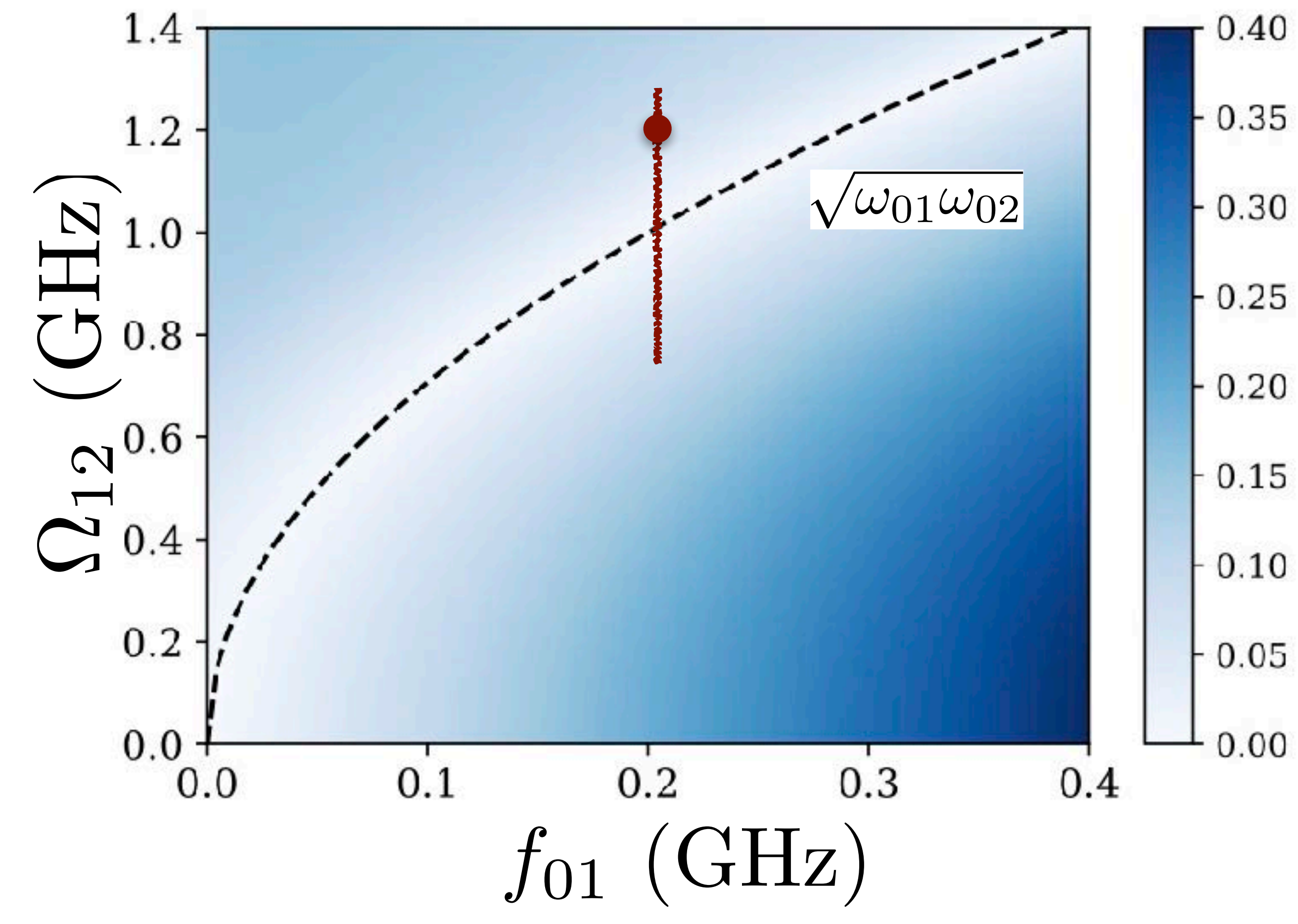
Energy exchange $\Delta\mathcal{E}_i$



« charges » $n_1 + n_3, n_2 - n_3$



Topological Pump : effective 2 levels (qubit)

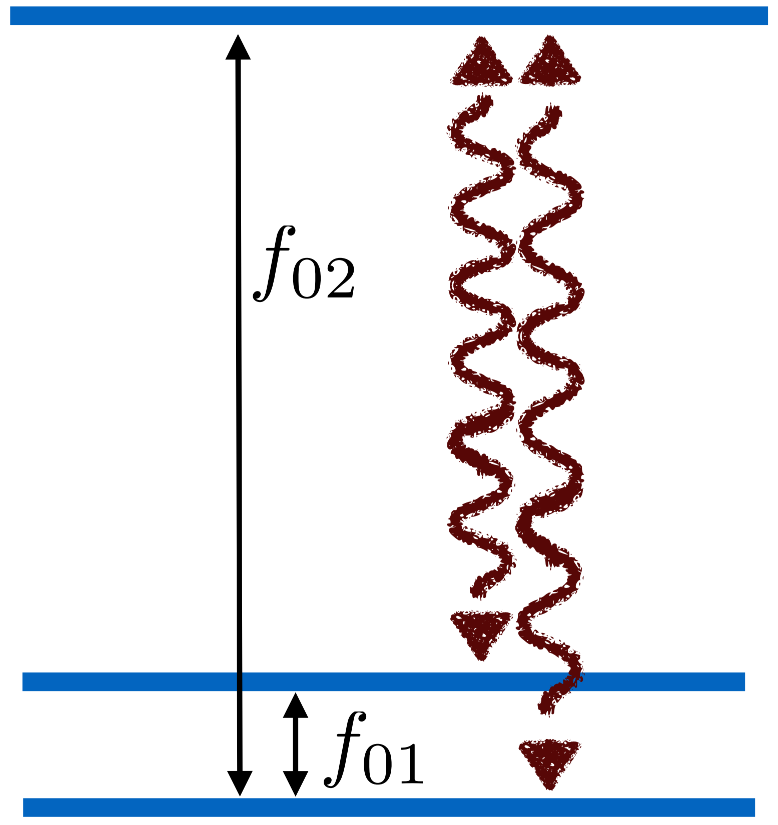


$\omega_1 = 10$ MHz

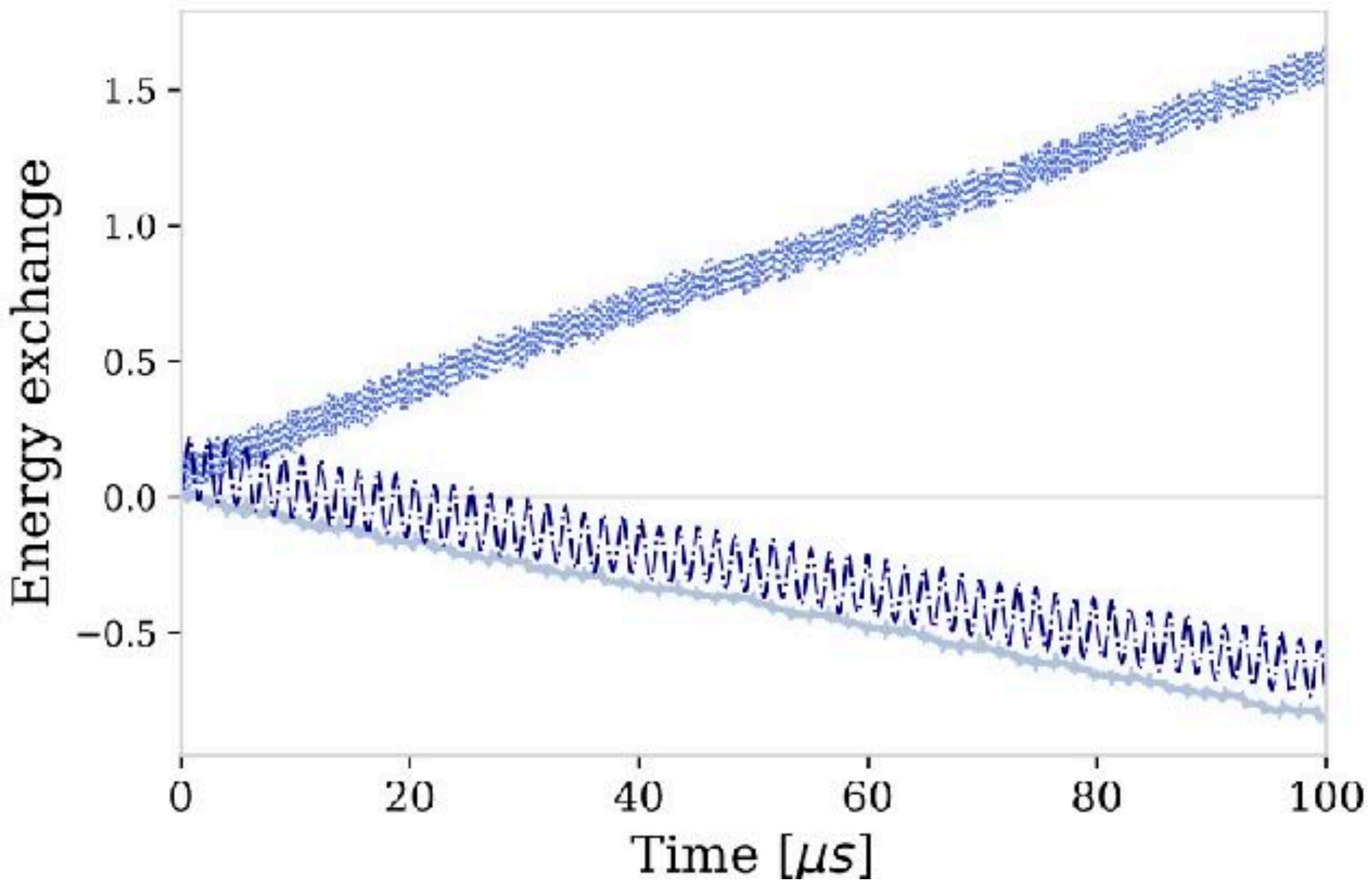
$\omega_2 \simeq 8$ MHz

$\Omega_{12} = 1.2$

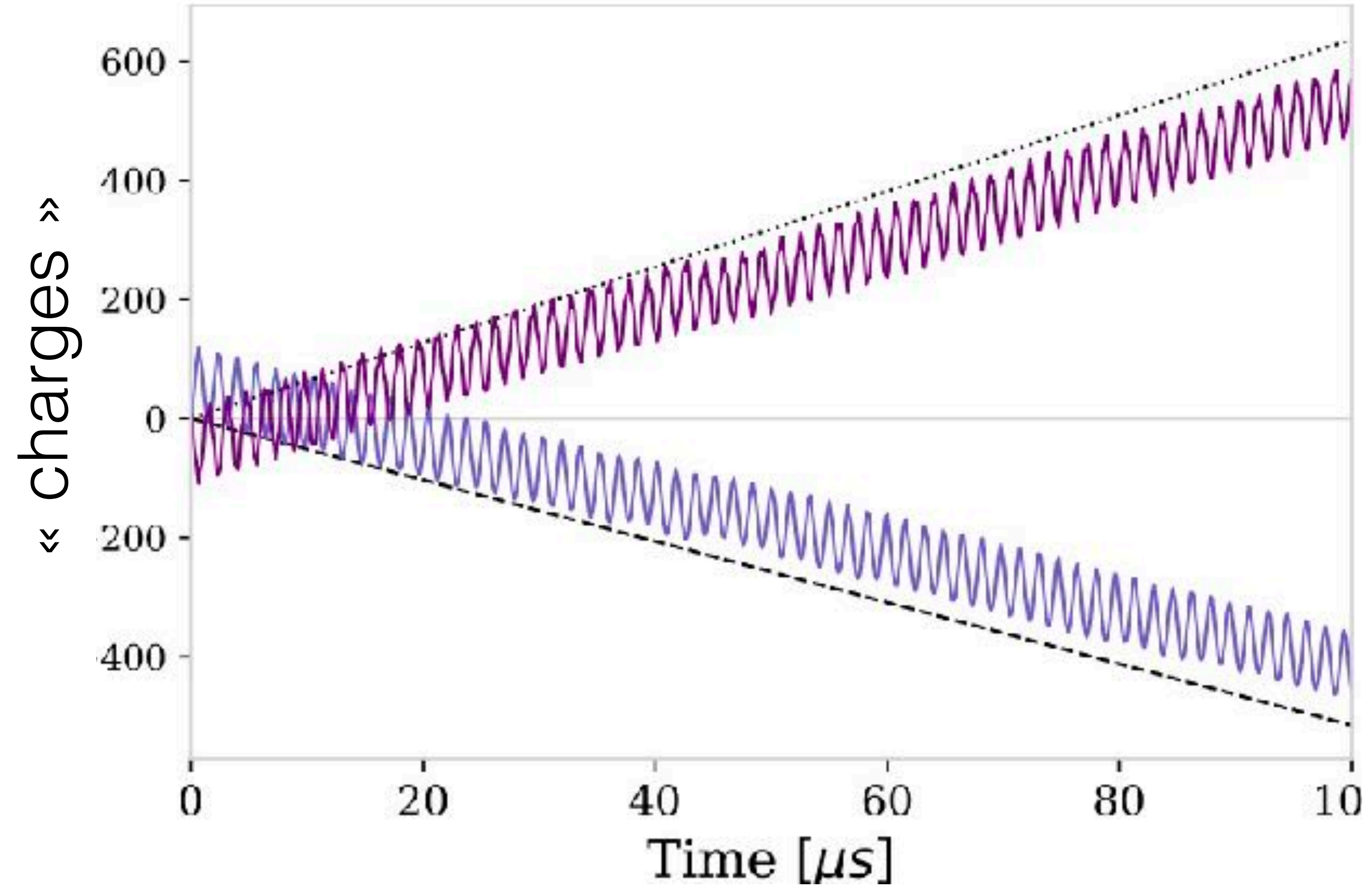
Prepare qutrit in ground state $|\psi_0\rangle$

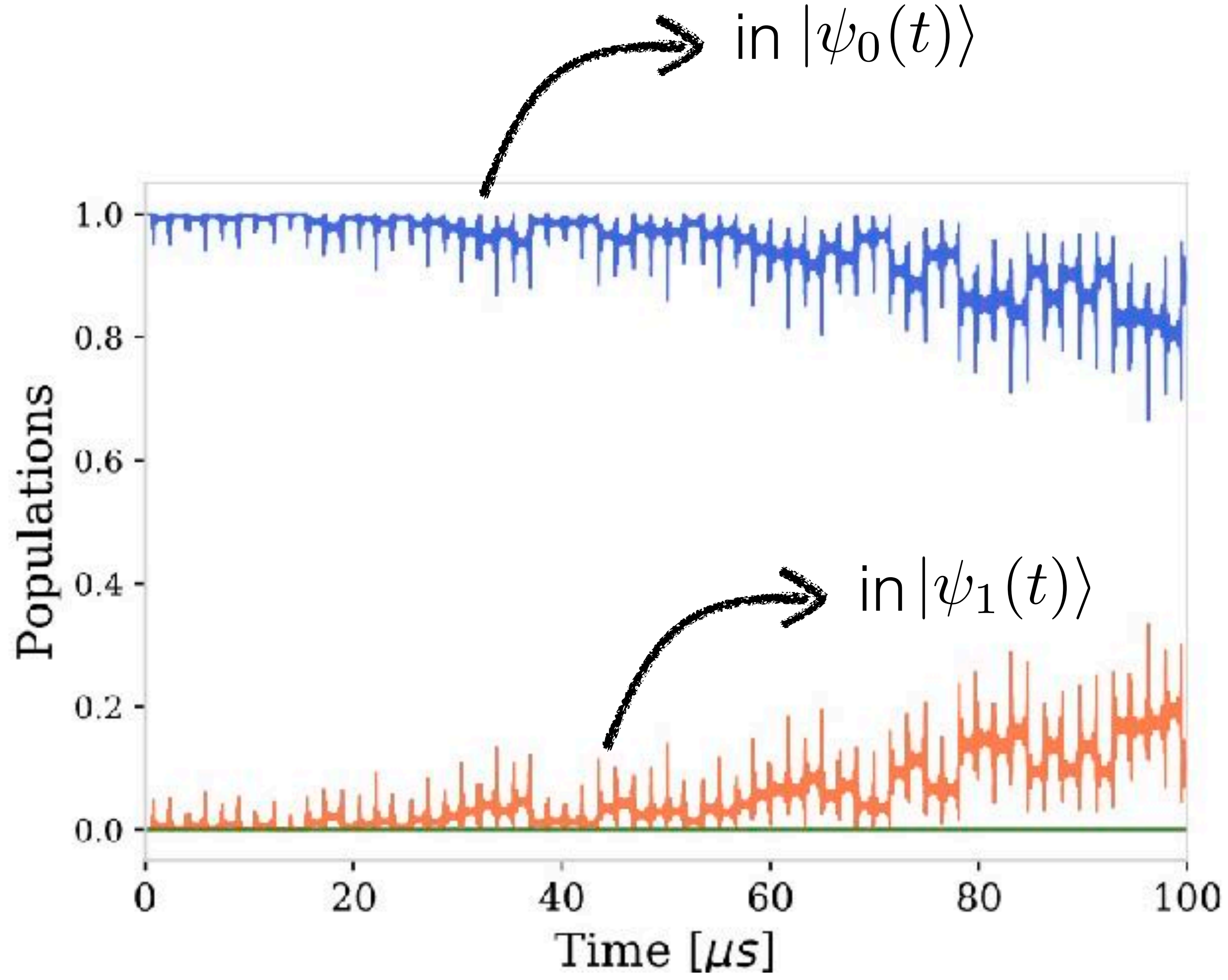


Energy exchange $\Delta\mathcal{E}_i$

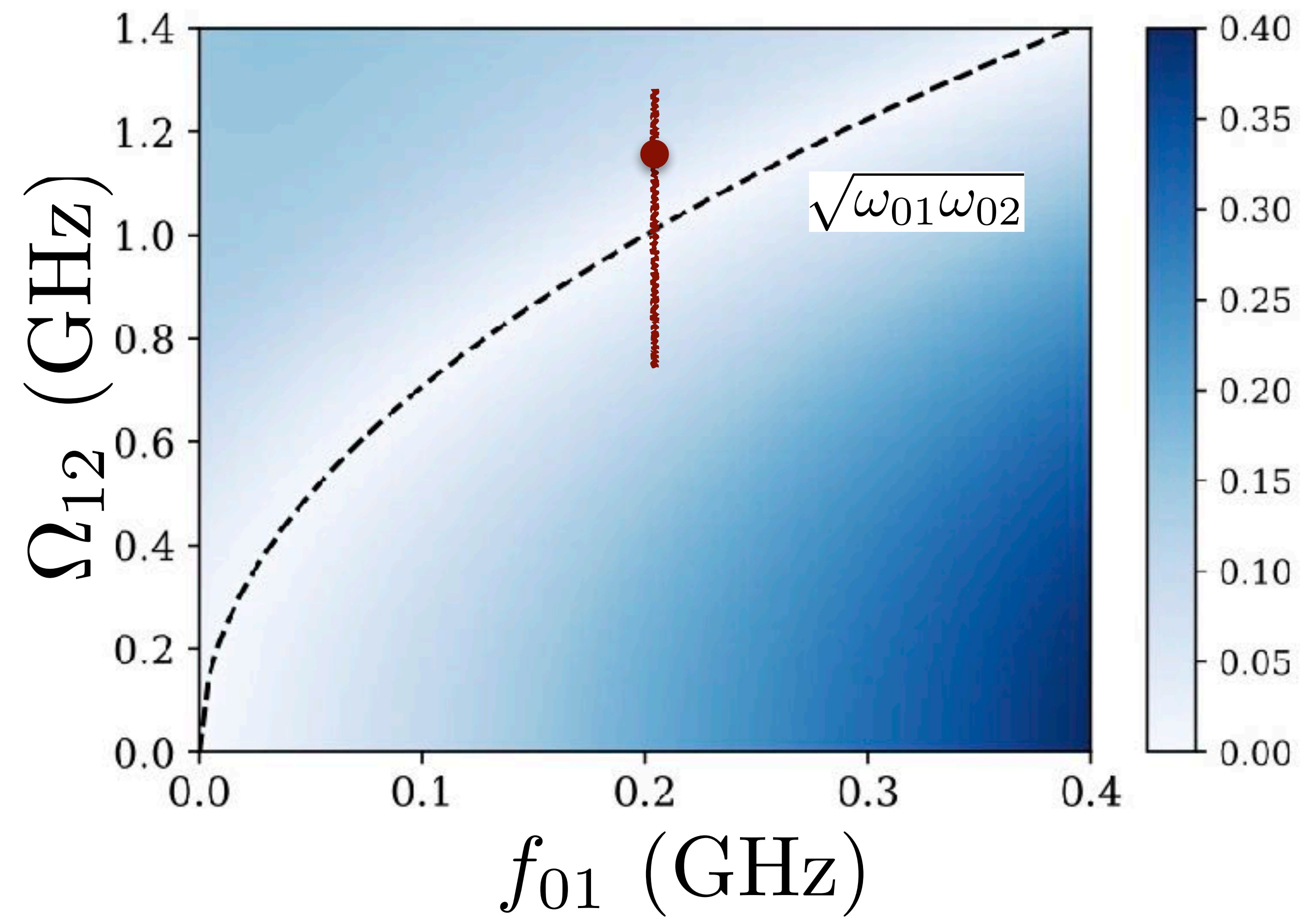


« charges » $n_1 + n_3, n_2 - n_3$





Topological Pump : effective 2 levels (qubit)

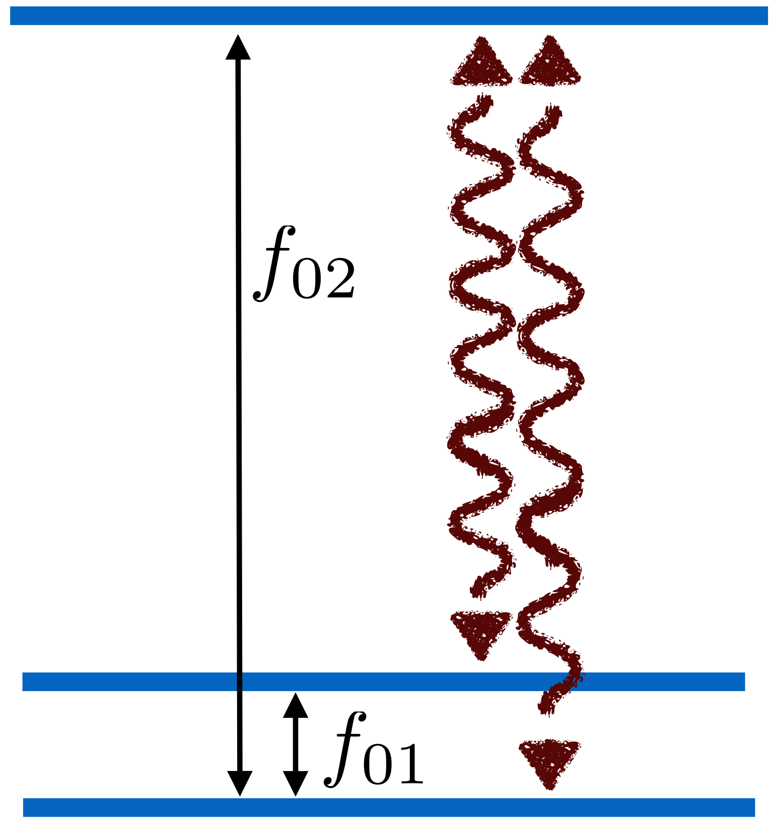


$\omega_1 = 10$ MHz

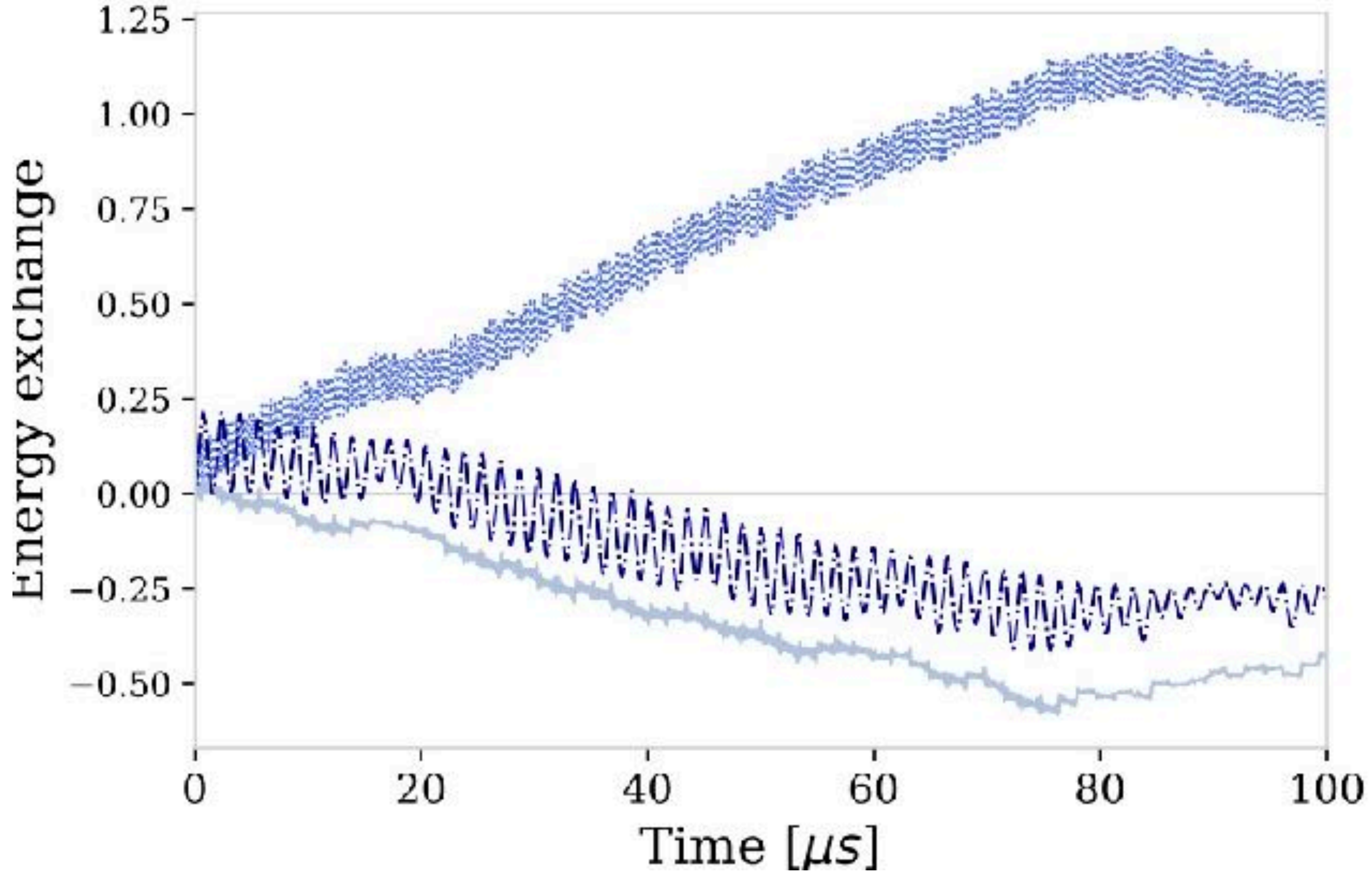
$\omega_2 \simeq 8$ MHz

$\Omega_{12} = 1.15$

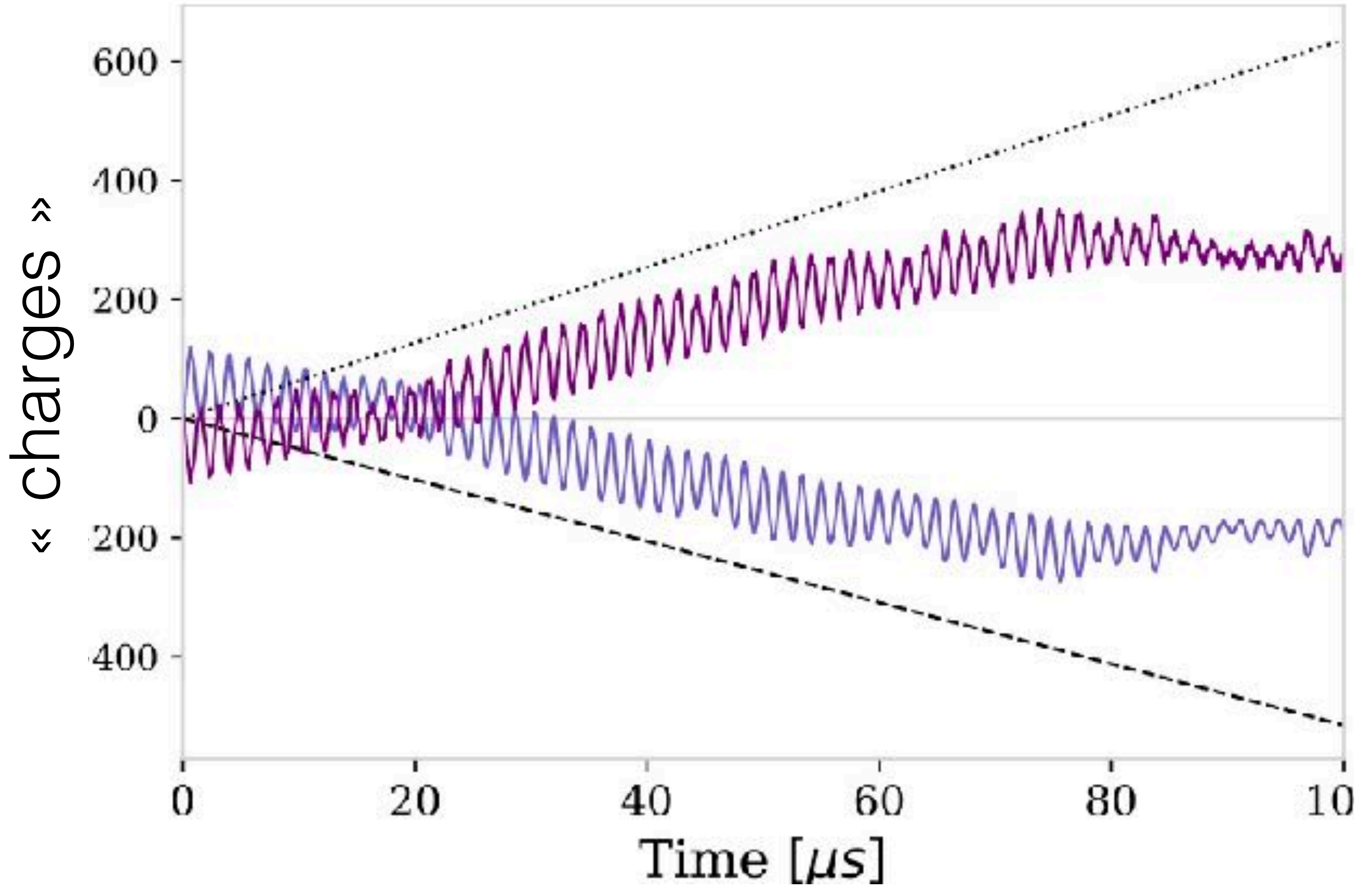
Prepare qutrit in ground state $|\psi_0\rangle$

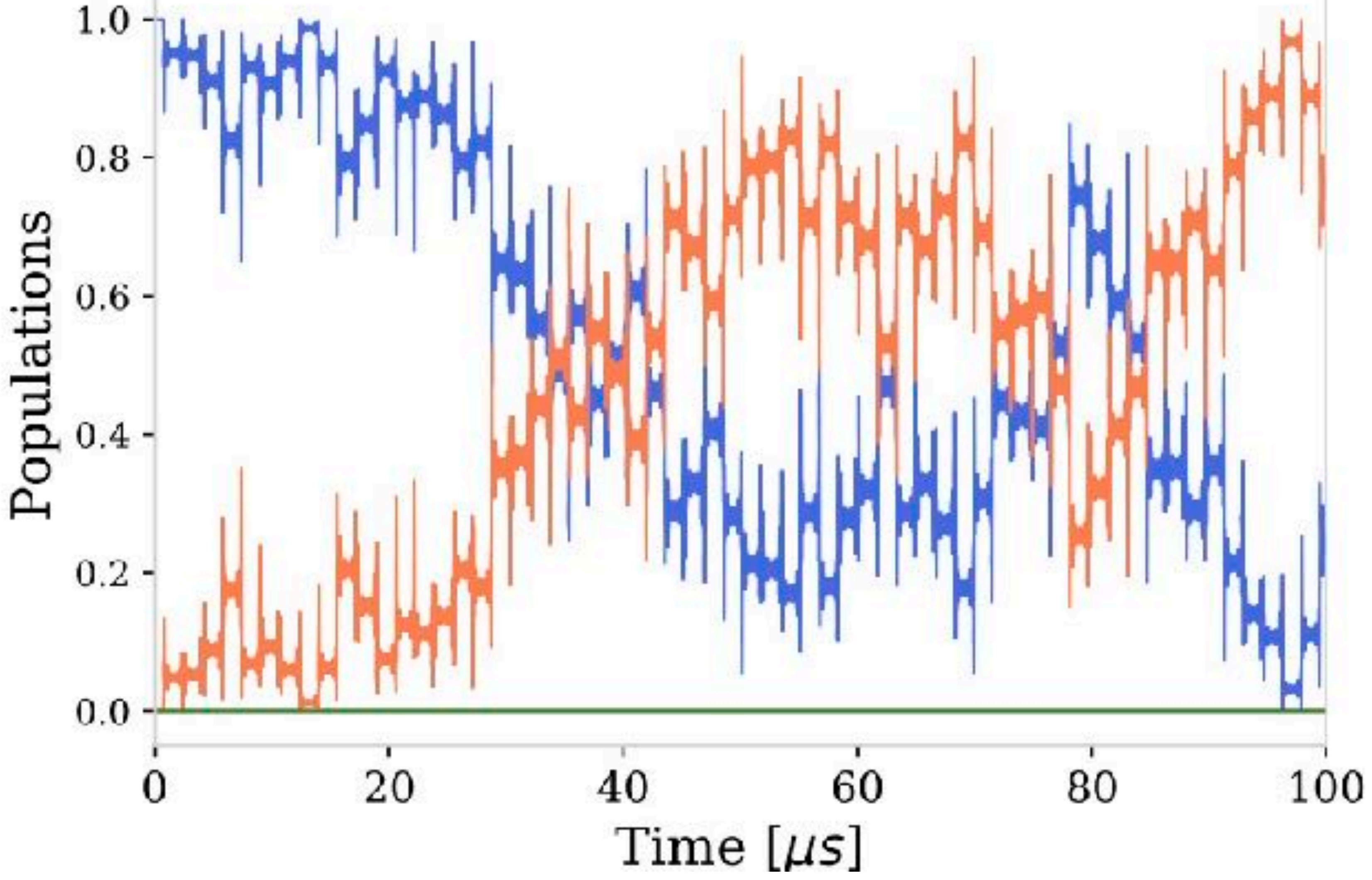


Energy exchange $\Delta\mathcal{E}_i$

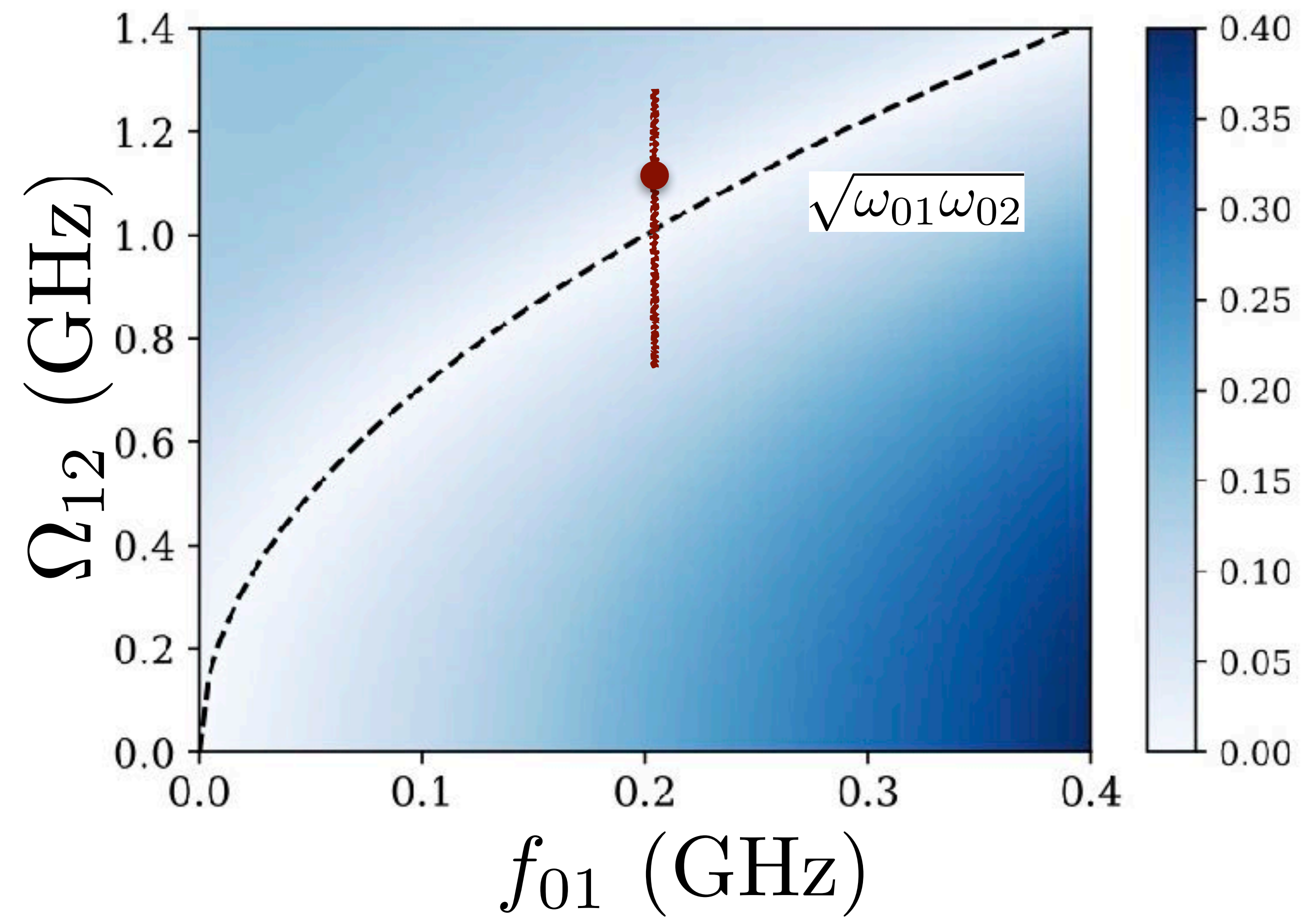


« charges » $n_1 + n_3, n_2 - n_3$



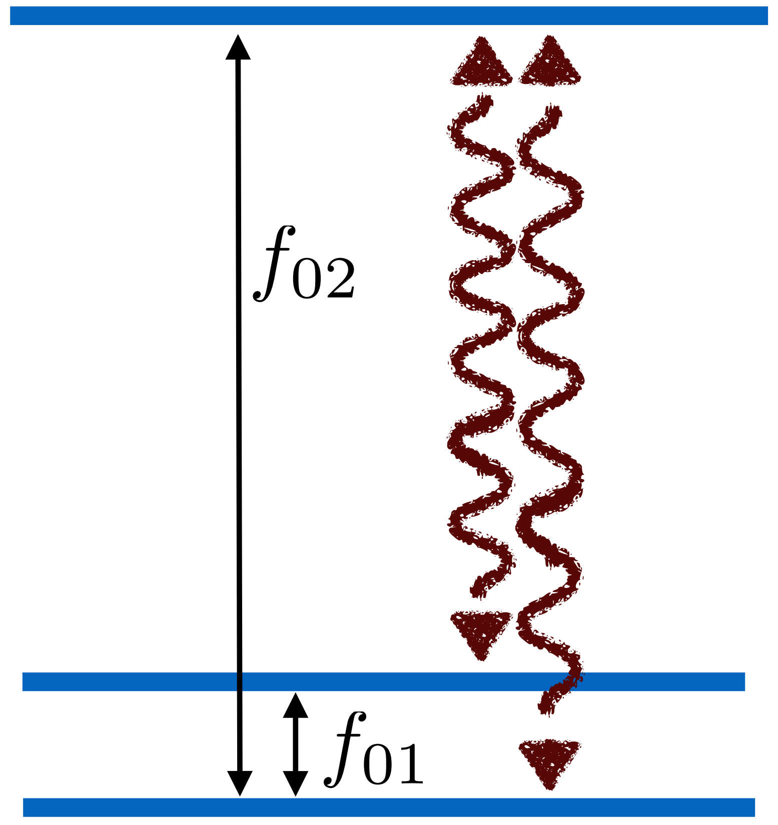


Topological Pump : effective 2 levels (qubit)

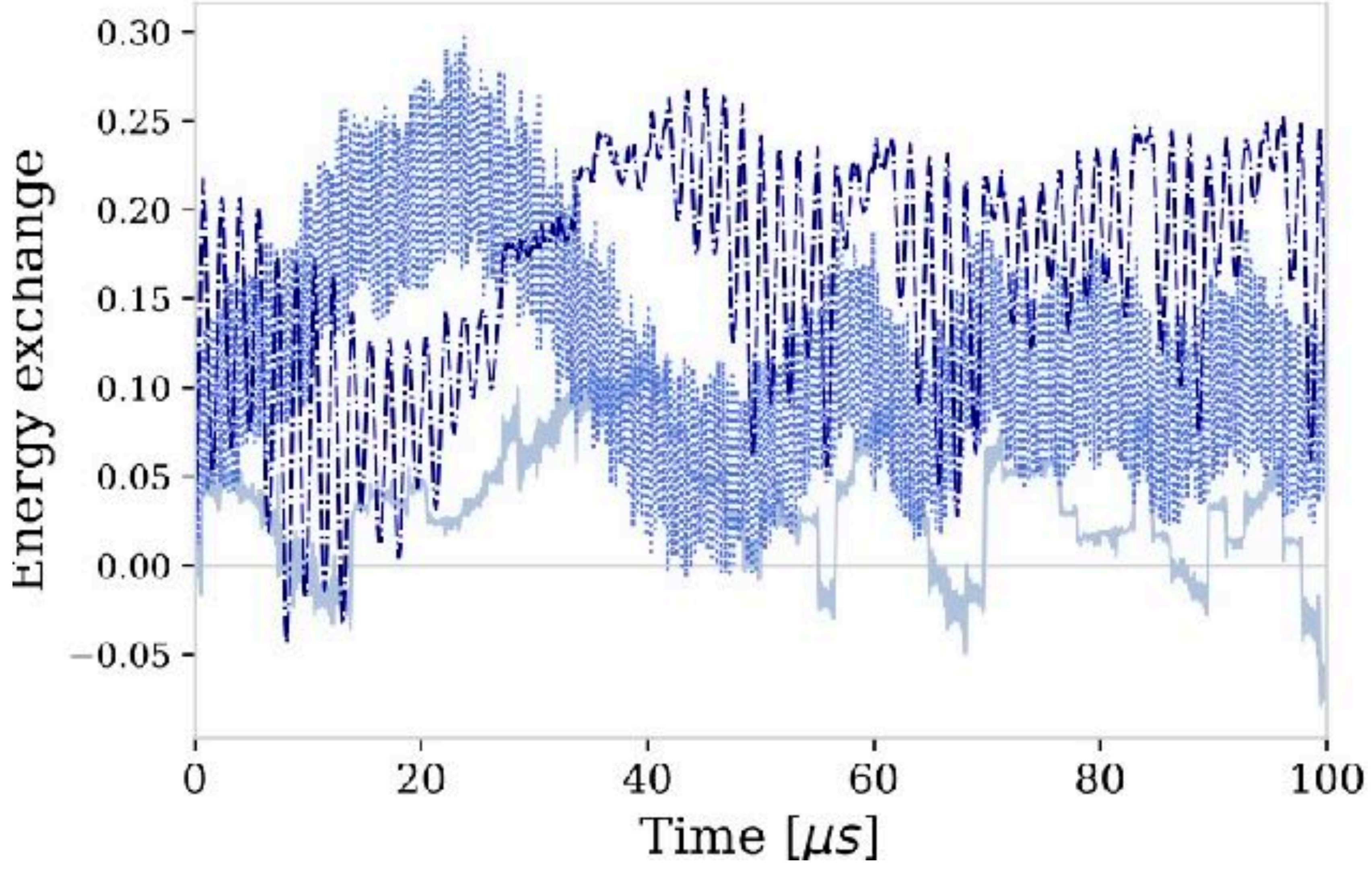


$\omega_1 = 10$ MHz
$\omega_2 \simeq 8$ MHz
$\Omega_{12} = 1.10$

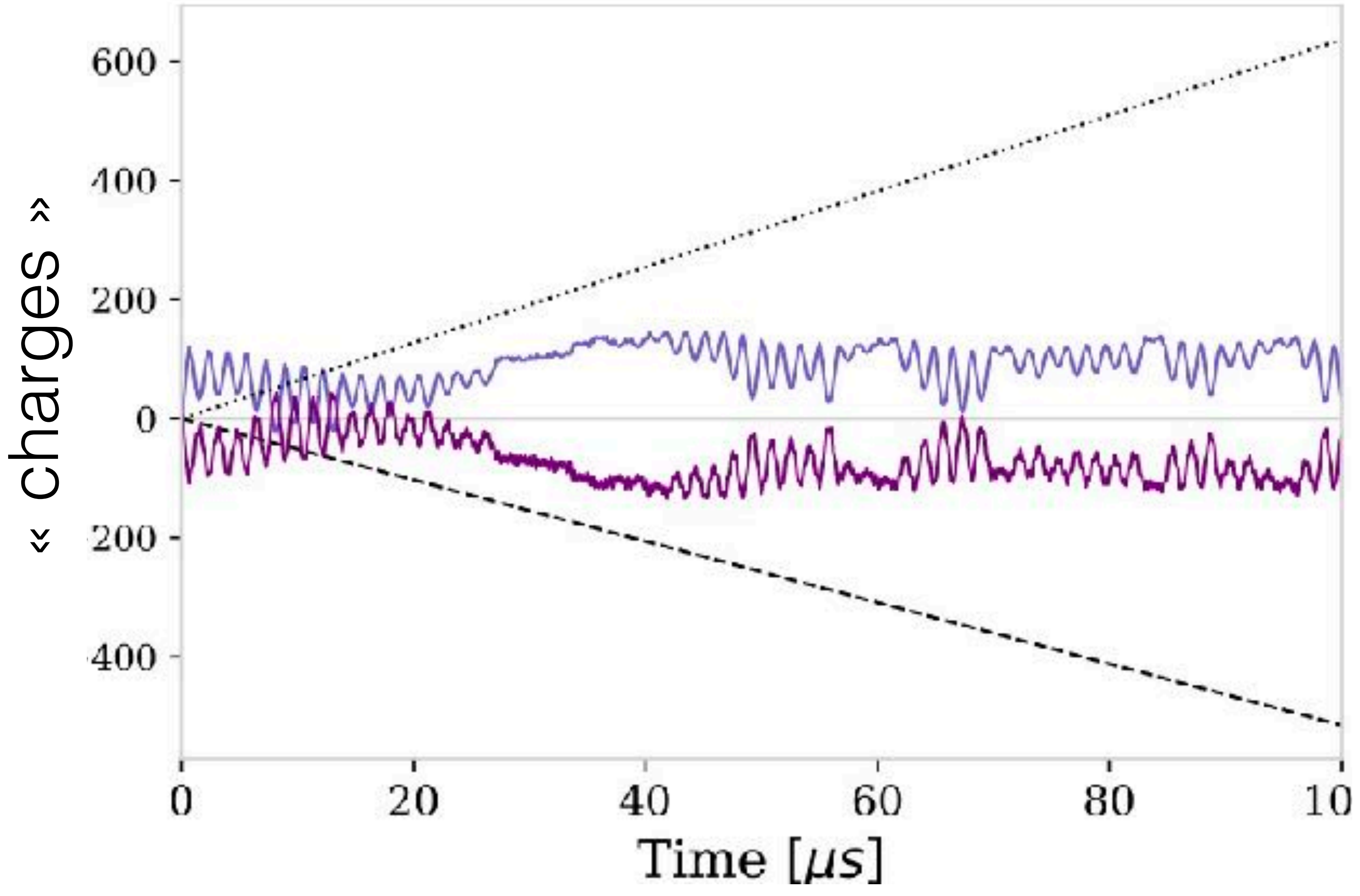
Prepare qutrit in ground state $|\psi_0\rangle$

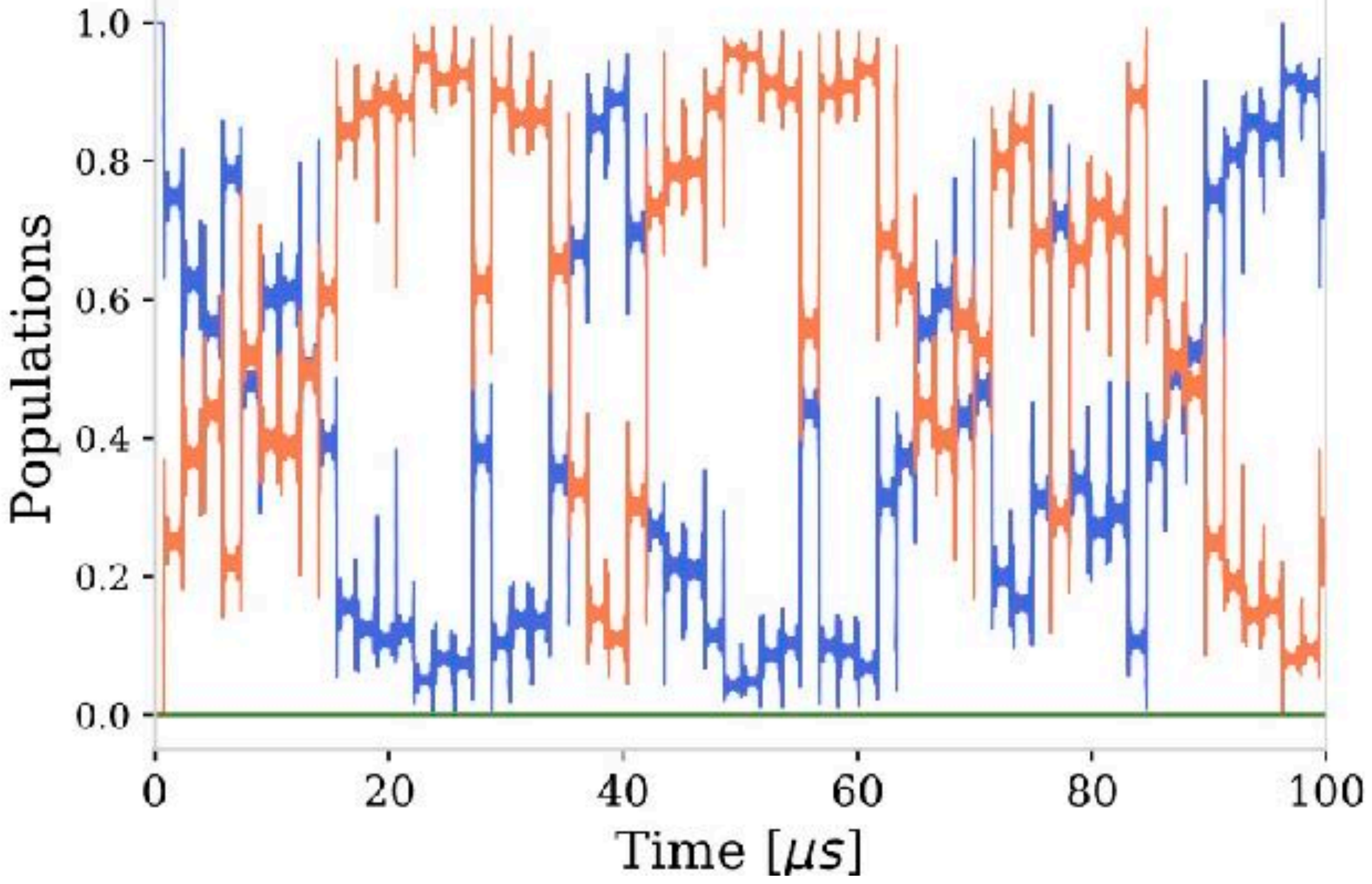


Energy exchange $\Delta\mathcal{E}_i$

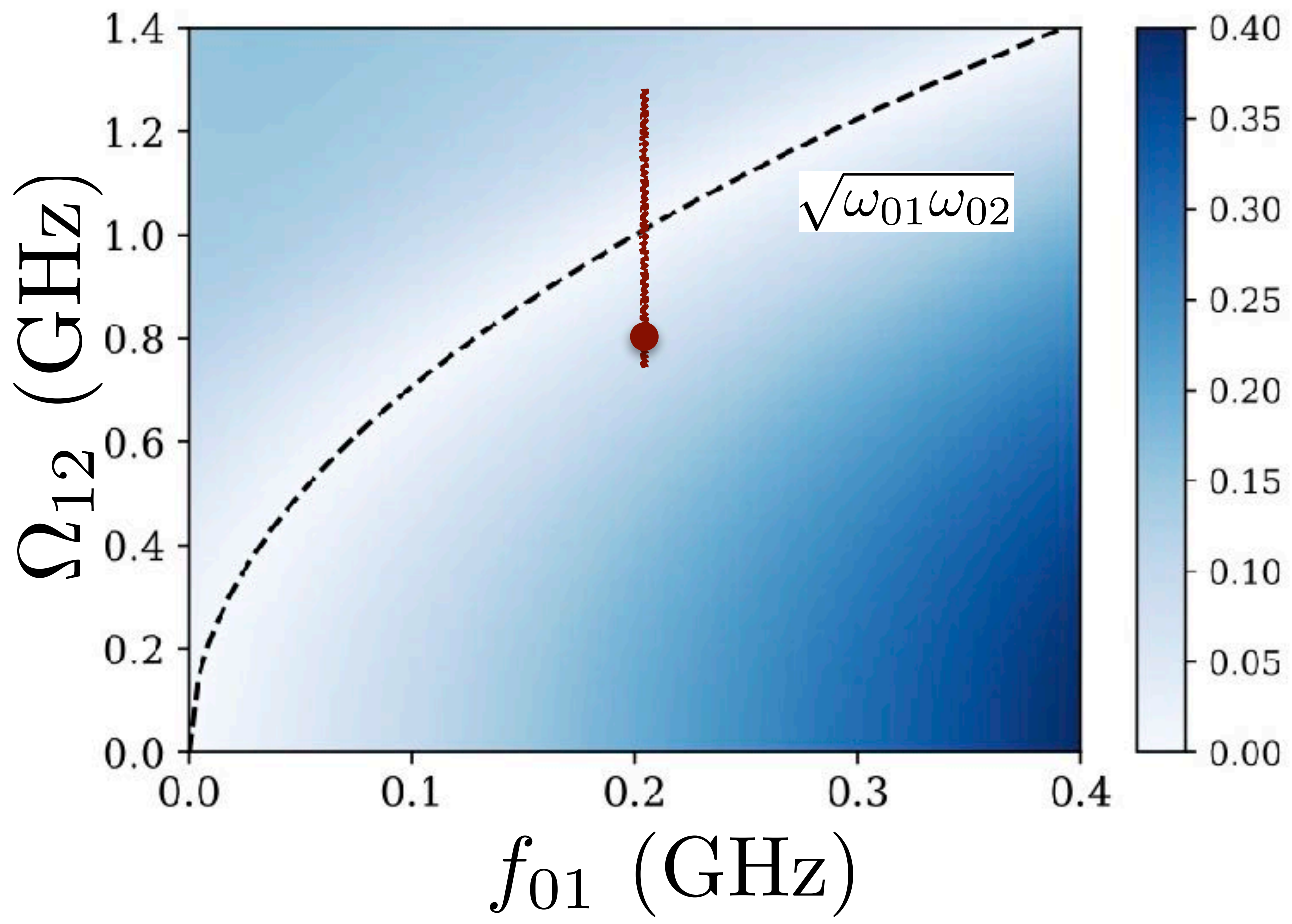


« charges » $n_1 + n_3, n_2 - n_3$





Topological Pump : effective 2 levels (qubit)

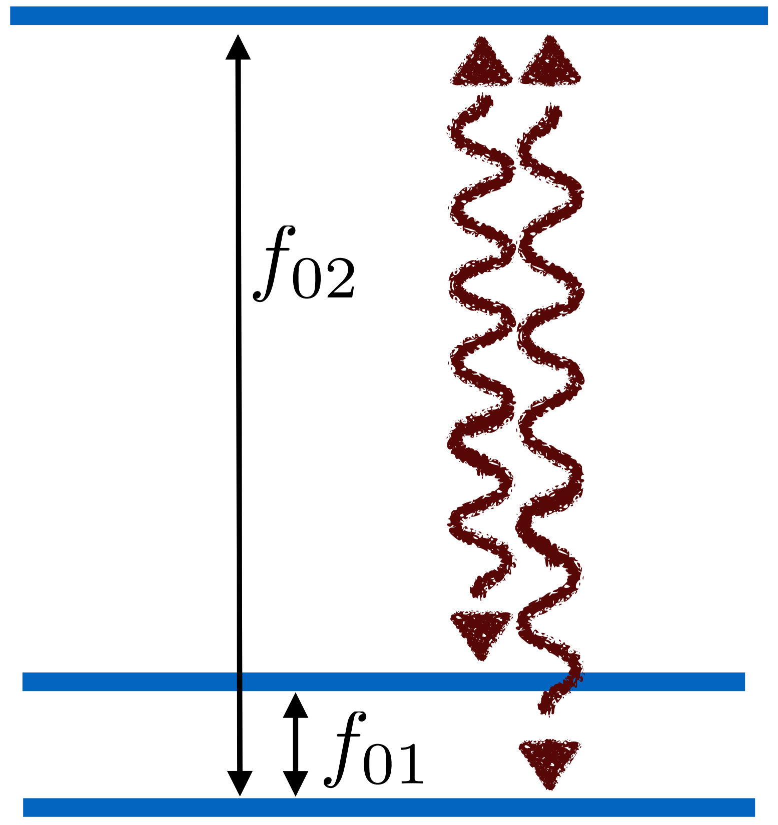


$\omega_1 = 10$ MHz

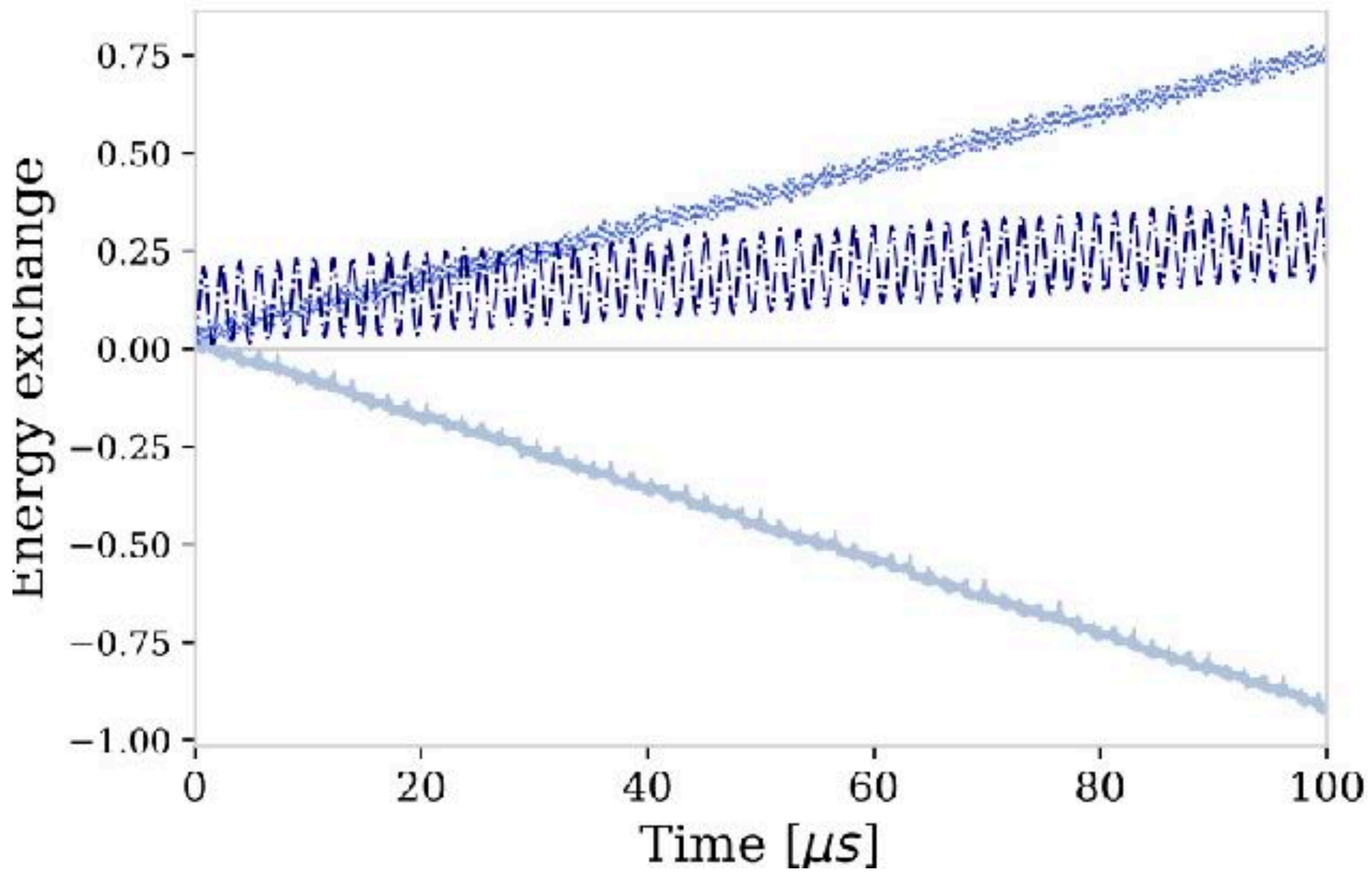
$\omega_2 \simeq 8$ MHz

$\Omega_{12} = 0.80$

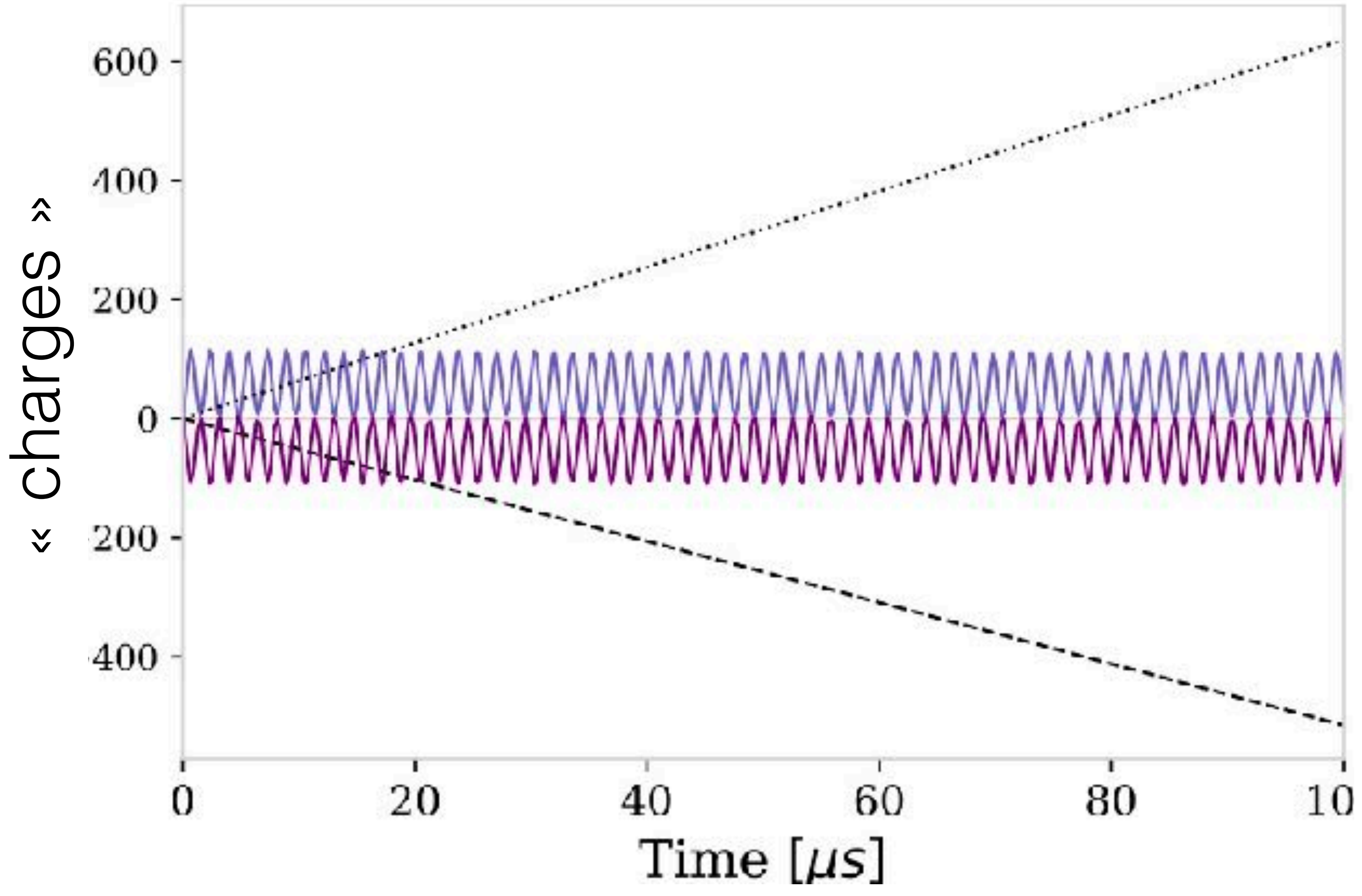
Prepare qutrit in ground state $|\psi_0\rangle$



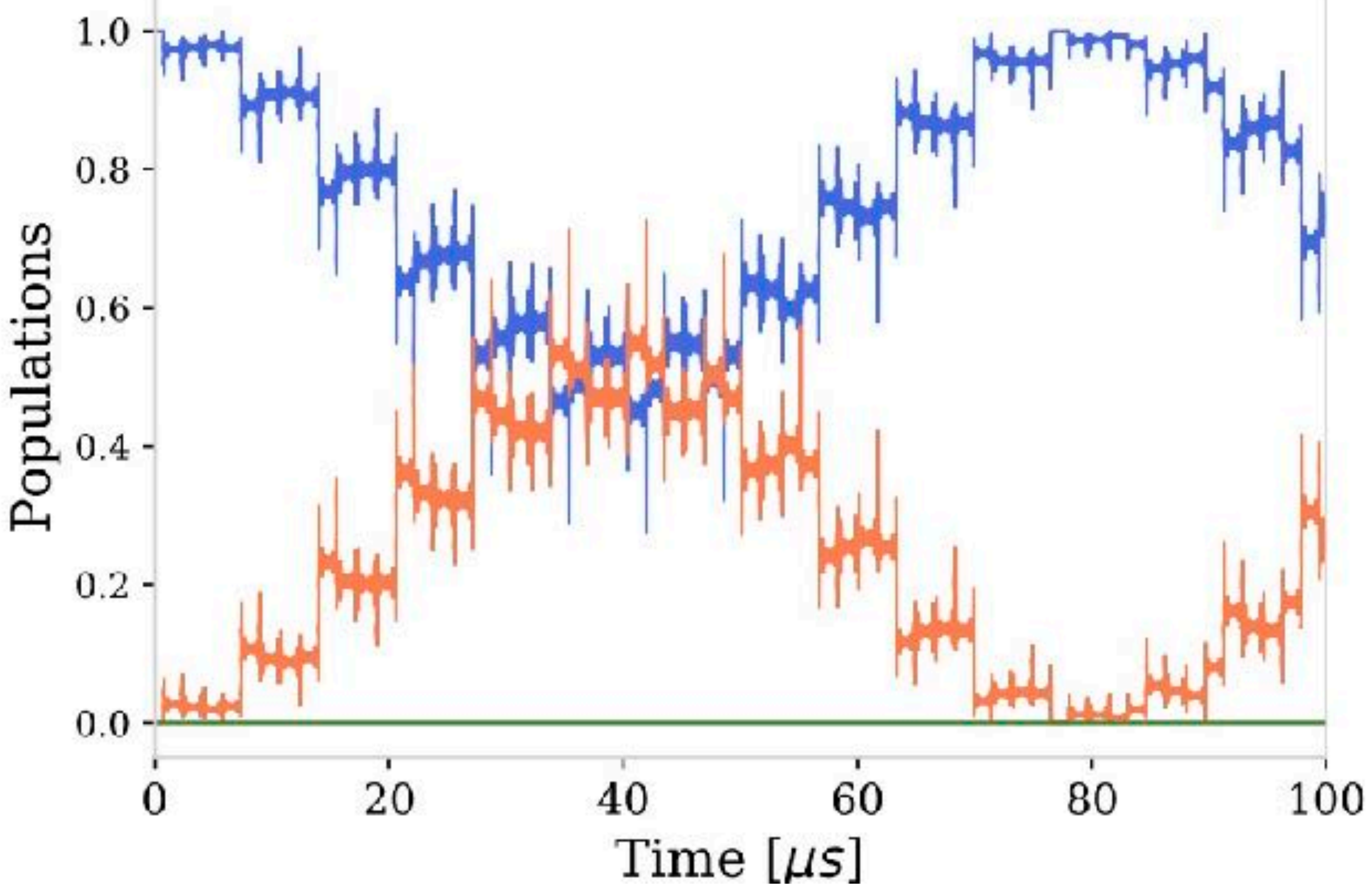
Energy exchange $\Delta\mathcal{E}_i$



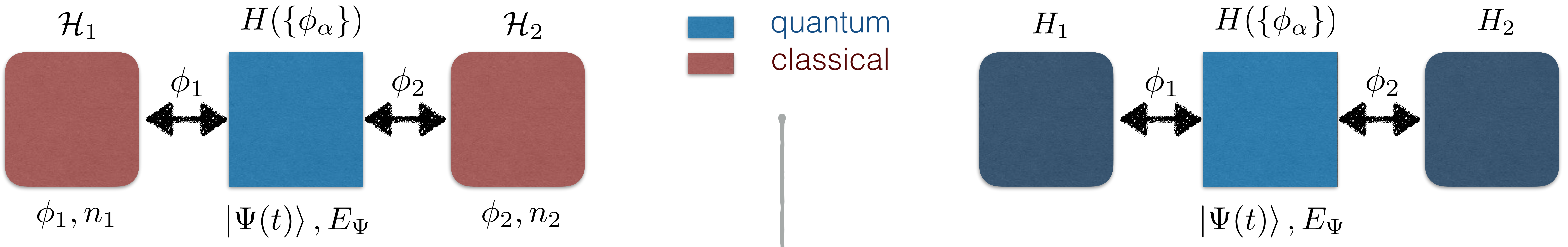
« charges » $n_1 + n_3, n_2 - n_3$



Populations



Quantum cavities ?



Adiabatic evolution for quantum system in $|\Psi(\phi_1(t), \phi_2(t))\rangle$

Equations of motion for baths:

$$\dot{n}_\alpha = -\frac{\partial \mathcal{H}_\alpha}{\partial \phi_\alpha} - \langle \Psi(t)| \frac{\partial H}{\partial \phi_\alpha} |\Psi(t)\rangle$$

$$= -\frac{\partial \mathcal{H}_\alpha}{\partial \phi_\alpha} - \frac{\partial E_\Psi}{\partial \phi_\alpha} - \hbar \sum_\beta F^{(\Psi)}_{\alpha,\beta}\ \dot{\phi}_\beta$$

$$\dot{\phi}_\alpha = \frac{\partial \mathcal{H}_\alpha}{\partial n_\alpha}$$

Quantum rotators: $[\hat{n}_\alpha, \hat{\phi}_\alpha] = i\hbar$

$$H_{tot} = H + \sum_i \omega_i \hat{n}_i$$

$$\frac{d}{dt} \langle \Psi_{tot}|\hat{n}_i|\Psi_{tot}\rangle = \frac{1}{i\hbar} \langle \Psi_{tot}|[\hat{n}_i, H_{tot}]|\Psi_{tot}\rangle$$

Perfect correlation bath / qubit :

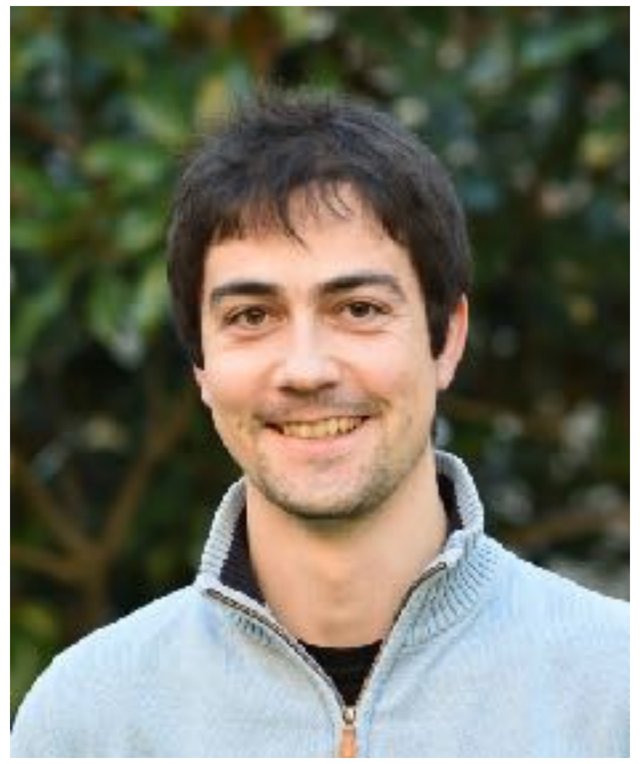
$$|\Psi_{tot}\rangle = \int d\phi_1 d\phi_2\ u_n(\phi_1, \phi_2)\ \ |\phi_2\rangle \otimes |\phi_2\rangle \otimes |\Psi(\phi_1, \phi_2)\rangle$$

➡ smoothes the Berry curvature (increase quantization)

$$\frac{d}{dt} \langle n_\alpha \rangle = -\sum_\beta \int d\phi_1 d\phi_2 |u_n(\phi)|^2 F^{(\Psi)}_{\alpha,\beta}\ \hbar\omega_\beta$$

Decorrelation bath / qubit : decreases pumping

Summary



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Tommaso Roscilde
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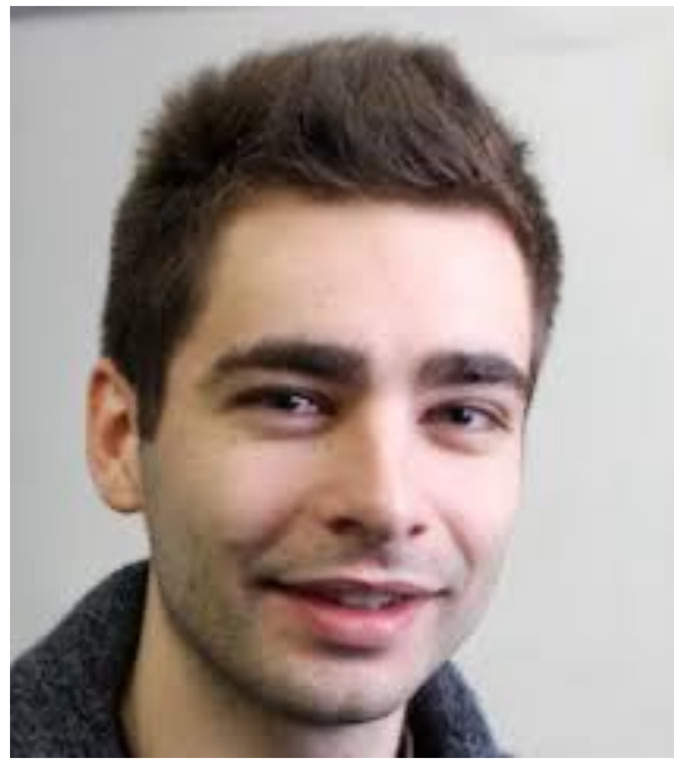
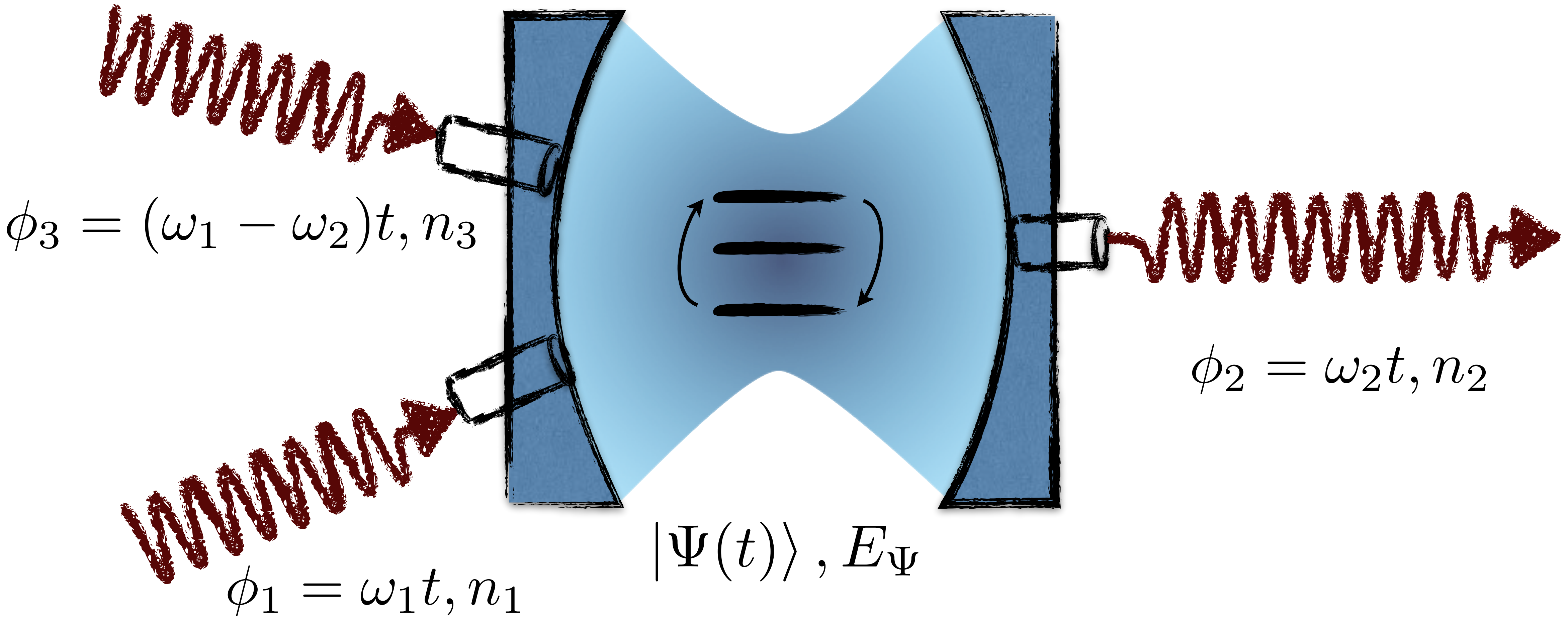


Quentin Ficheux
(JQI)

Benjamin Huard
(ENS Lyon)

C. Dutreix et al., in preparation



Thank you for your attention