Fractons and Higher Rank Gauge Theories

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KITP spinquant19, November 6, 2019

PRB **97**, 235112 (2018)

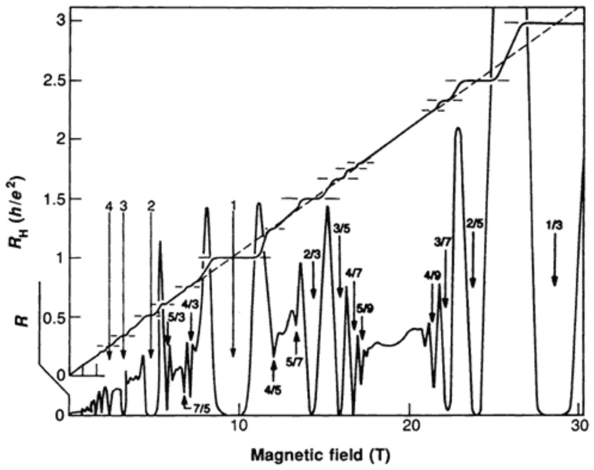
Collaboration: Maissam Barkeshli

My goals for this talk

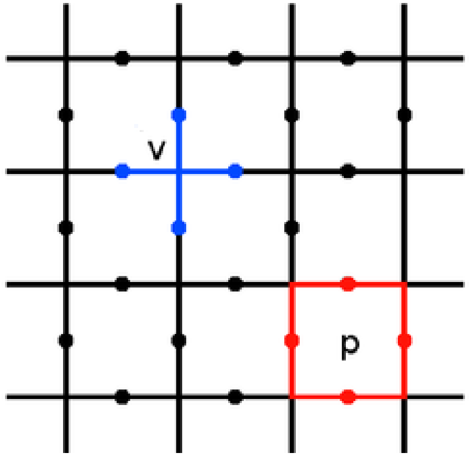
- Broad overview of the field of fractons, clarify terminology
- Why should we care about fractons?
- See how fractons arise in a few explicit settings
- Connections between two settings with fractonic physics

Topological order

- Described by topological quantum field theory
- Protected O(1) ground state degeneracy depending on topology of space – could be a good qubit
- Excitations have anyonic statistics
- Examples: fractional quantum Hall, toric code, “quantum double” models

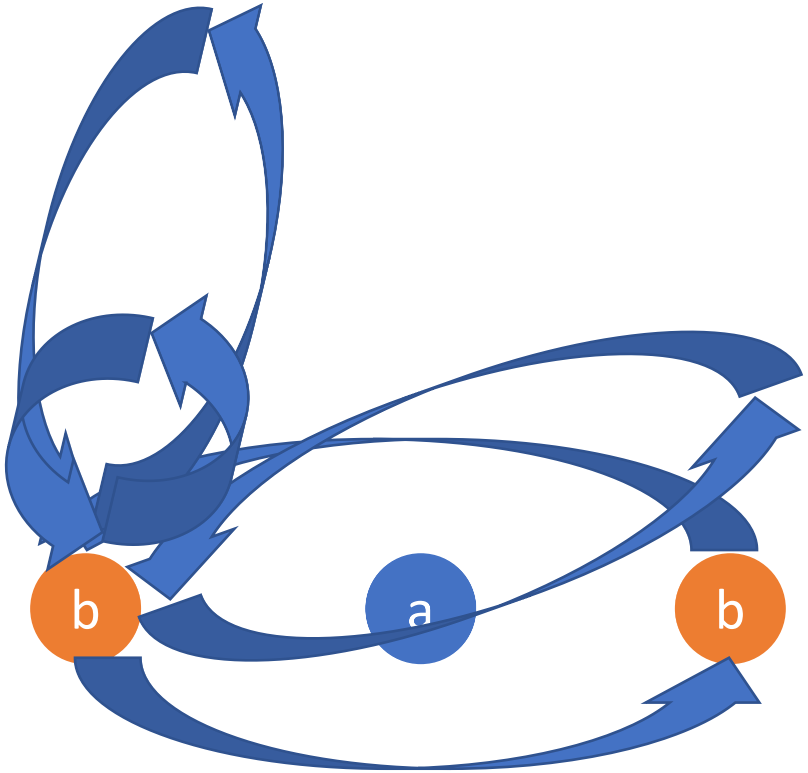


Willett et. al. PRL **59** 15 (1987)



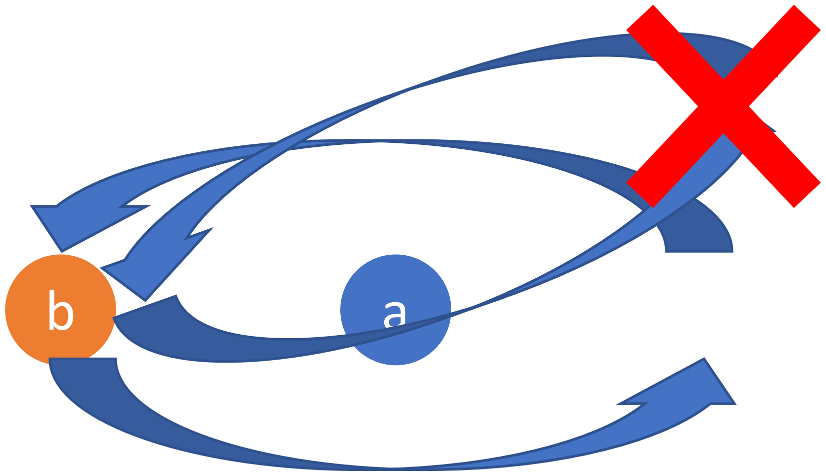
Wikipedia

No point-like anyons in 3D



In 3D, two exchanges = identity, so no (point-like) anyons…

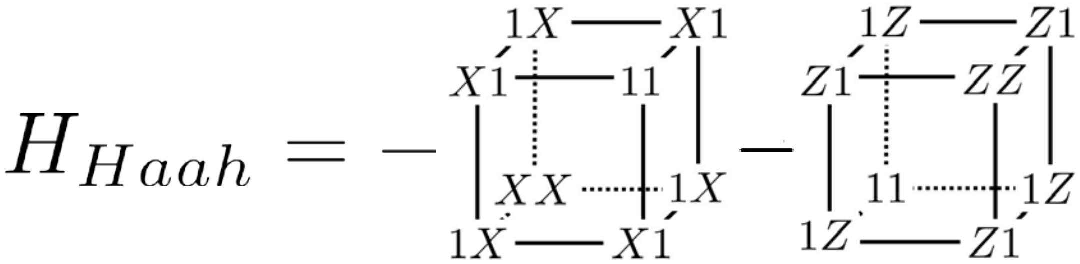
No point-like anyons in 3D



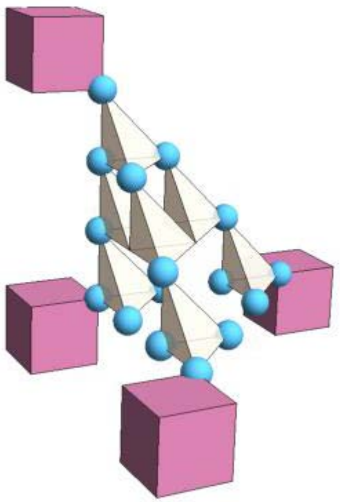
In 3D, two exchanges = identity, so no (point-like) anyons…

Fractons are a loophole and allow particle “braiding” in 3D!

Haah’s Code – a “gapped fracton phase”



- Haah’s aim: a self-correcting quantum memory in 3D
- What he found:
  - (Complicated) protected ground state degeneracy on the 3-torus
  - Excitations (“fractons”) created by fractal-shaped operators
  - No local operator can move an isolated fracton
  - “Marginally” self-correcting memory



Haah PRA **83** 042330 (2011)
Bravyi, Haah PRL 111 200501 (2013)

Gapped fracton phases are totally new phases

	Topological order	Gapped fracton phases
Dimension	≥ 2	≥ 3
Gap	Yes	Yes
Protected ground state degeneracy	$\mathcal{O}(1)$	$\sim e^{cL}$
Particle mobility	Fully mobile	Subdimensional
“Anyonic” “braiding” processes (3D)	Particle-string, string-string	Particle-particle
General theory	Topological quantum field theory	???

Fracton excitations

$$\int \rho(x)\, d^3x = \text{constant}$$

$$\int x\,\rho(x)\, d^3x = \text{constant}$$

“Fractons” are (point) excitations that cannot move alone.

“Lineons” can only move in 1D, “planons” in 2D.

“Subdimensional particles” is the umbrella term.

Higher-rank gauge theory

$$\rho = \partial_i E_i$$

$$A_i \to A_i - \partial_i \lambda$$

$$\int \rho(x) d^3x = \int \partial_i E_i d^3x = \text{bdry term}$$

$$E_{ij} = E_{ji}$$

$$\rho = \partial_i \partial_j E_{ij}$$

$$\int \rho(x) d^3x = \int \partial_i \partial_j E_{ij} d^3x = \text{bdry term}$$

$$A_{ij} \to A_{ij} + \partial_i \partial_j \lambda$$

$$\int x_i \rho(x) d^3x = -\int \partial_j E_{ij} d^3x = \text{bdry term}$$

New classes of (Lorentz-violating) gauge theories! Can have fractons!

A (heavily) abbreviated history of fractons

- 2006: higher rank gauge theories first discussed
  Xu PRB **74** 224433 (2006), Xu cond-mat/0602443
- 2011-2012: early fracton models in spin glasses
  Chamon PRL **94** 040402 (2011), Bravyi, Leemhuis, Terhal Ann. Phys. **326** 839 (2011); Castelnovo, Chamon Phil. Mag. **92**, 304 (2012)
- 2011: Haah's code
  Haah PRA **83** 042330 (2011)
- 2015-16: (Gapped) fracton phenomenology, simple models found
  Vijay, Haah, Fu PRB **92** 235136 (2015) and PRB **94** 235157 (2016)
- 2016-17: Subdimensional particles found in higher-rank gauge theories
  Pretko PRB **95** 115139 (2017) and PRB **96** 035119 (2017)
- 2017-present: Field grows rapidly
  Review: Ann. Rev. Cond. Mat. **10**, 295 (2019)

arXiv

Showing 1–50 of 130 results for all: fracton

Other uses of the word “fracton”

- Mobility restrictions protected by subsystem symmetries (Princeton group and others)
- Extra conservation laws put in by hand (Boulder group)
- Energetic suppression of mobility (Boulder group)

Interesting properties, especially dynamics! I won’t talk about them.

So why care about fractons?

- New phases of matter
  - With quantum information applications?
- New types of gauge theories
- Very unusual and interesting phenomenology and dynamics

Map for the rest of this talk

	Topological order	Gapped fracton phases	Higher-rank gauge theory
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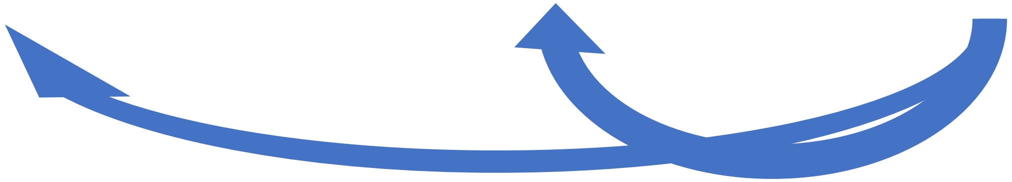
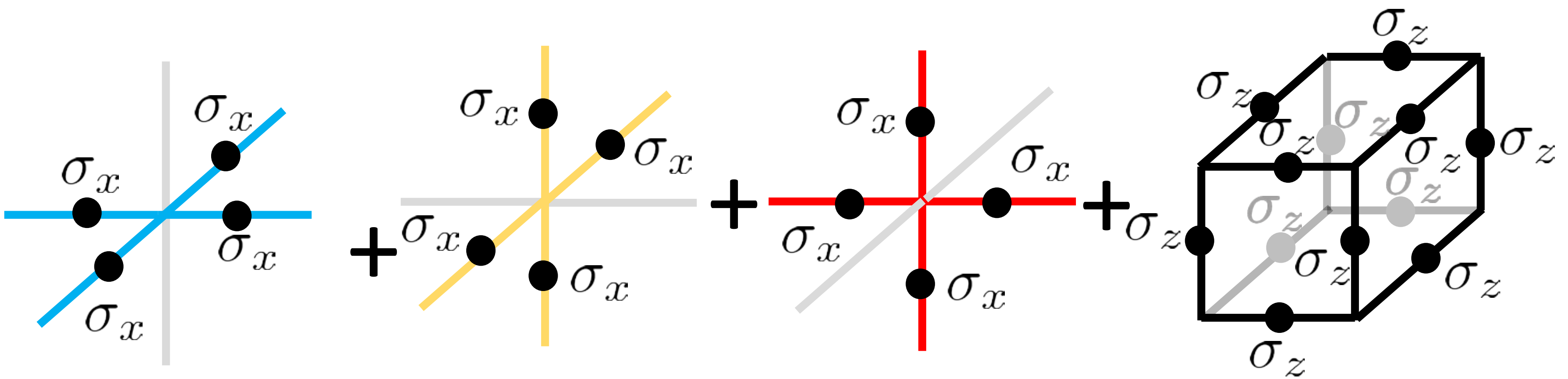


Higgs mechanism

X-Cube – a simple fracton model

Spin-1/2 on links of cubic lattice

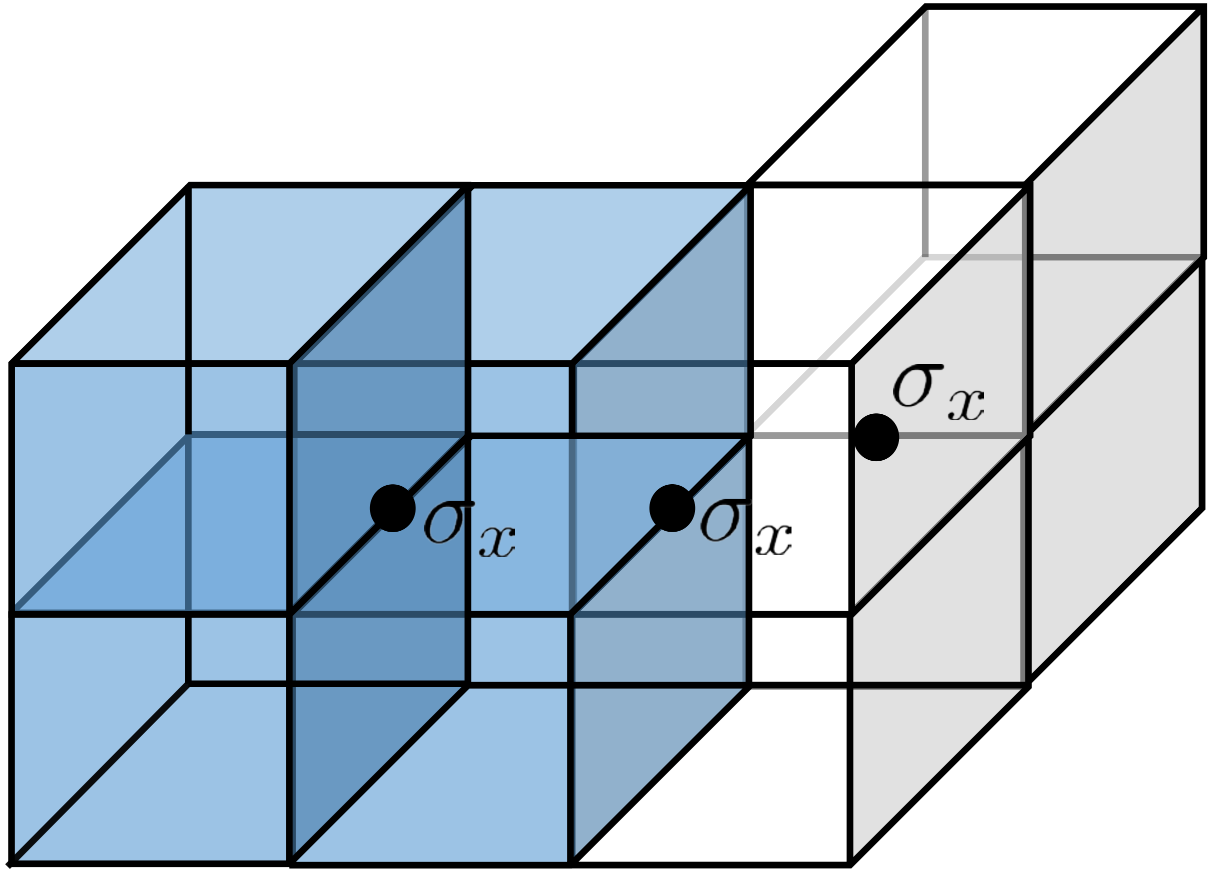
$$H = -\sum$$



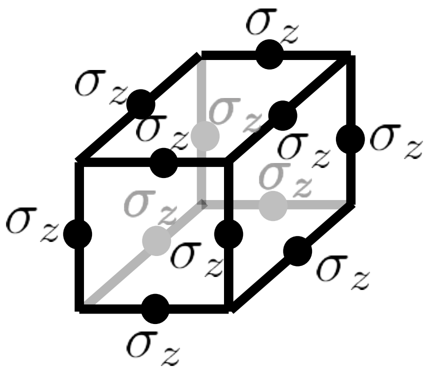
- Gapped
- Ground states have +1 eigenvalue for every term in Hamiltonian
- “Topological” degeneracy 2^{6L-3}
- Contains 0D, 1D, and 2D excitations

Castelnovo, Chamon Phil. Mag. **92**, 304 (2012)
Vijay, Haah, Fu PRB **94** 235157 (2016)

Planon excitations of X-Cube



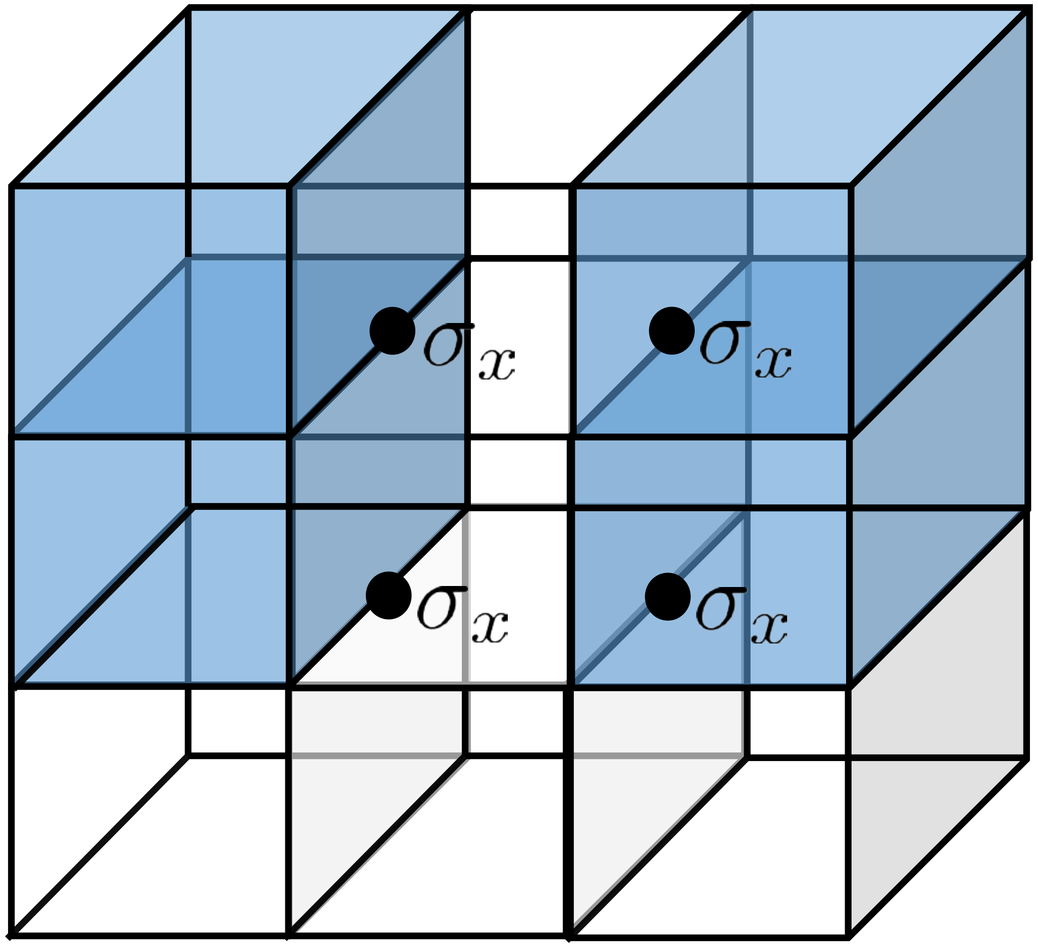
$$H = -\sum$$



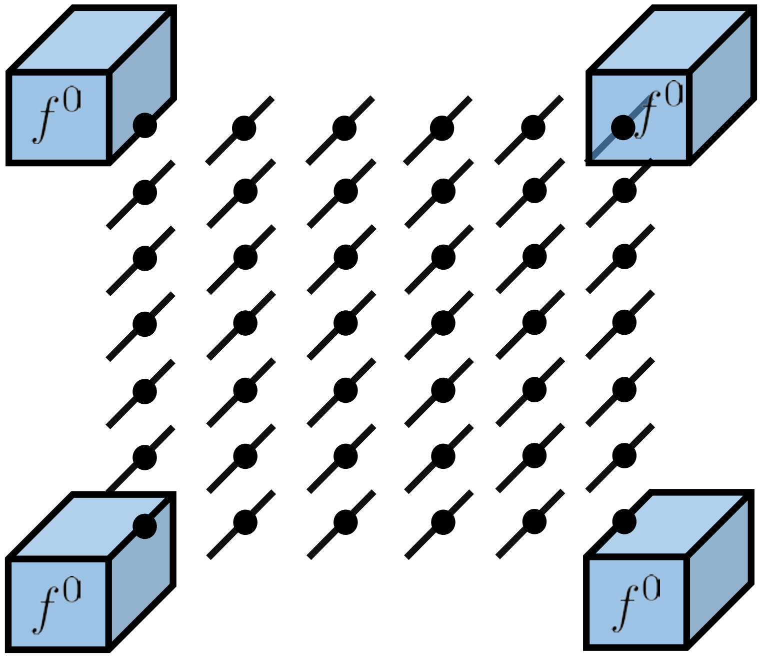
$$+\cdots$$

2-cube bound states mobile in xy-plane

Fracton excitations of X-Cube



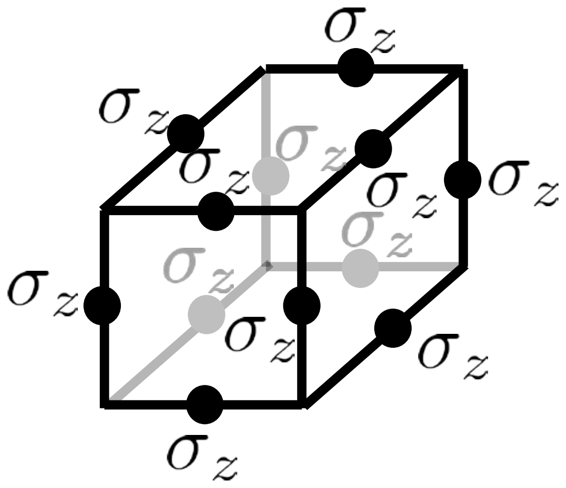
Fracton excitations of X-Cube



Isolated cube excitations ("fractons") live at the corners of membrane operators and are immobile.

Subsystem conservation laws

$\prod_{\text{plane}}$



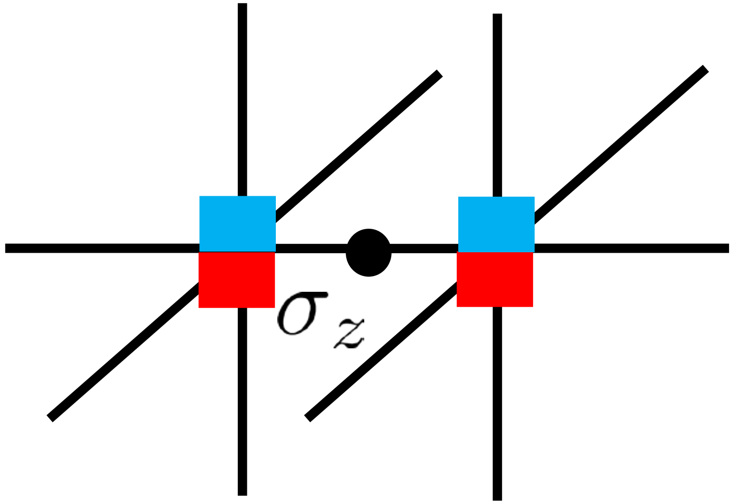
$= 1$

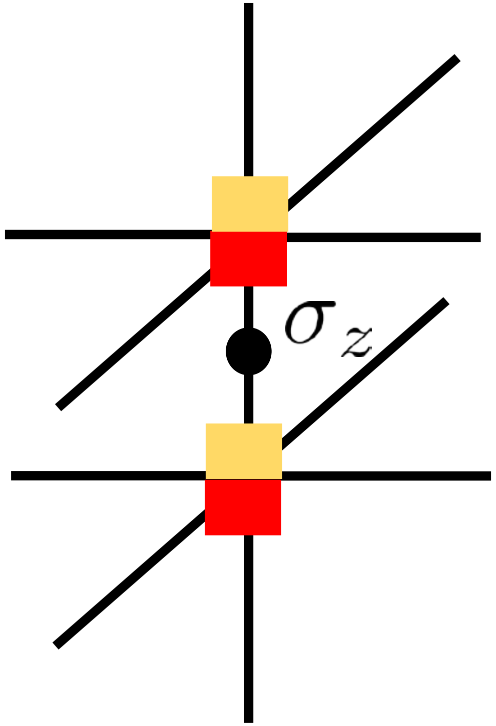
Fracton (cubes with -1) parity is conserved in each plane. Moving a fracton violates this conservation law – hence immobile.

Conservation law comes from *operator structure* arising from gauging subsystem symmetries

Vijay, Haah, Fu PRB **94** 235157 (2016)

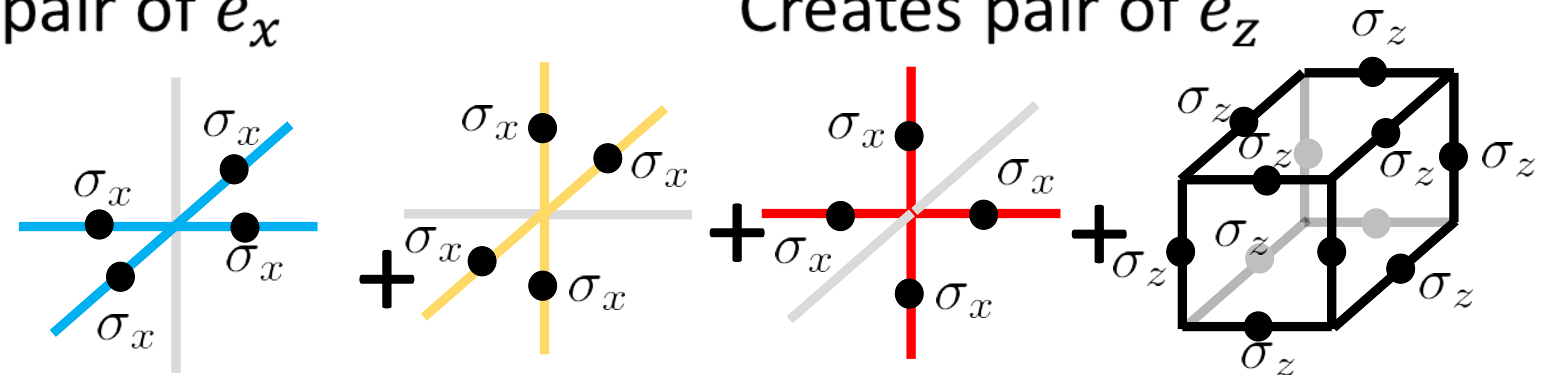
Lineon excitations of X-Cube



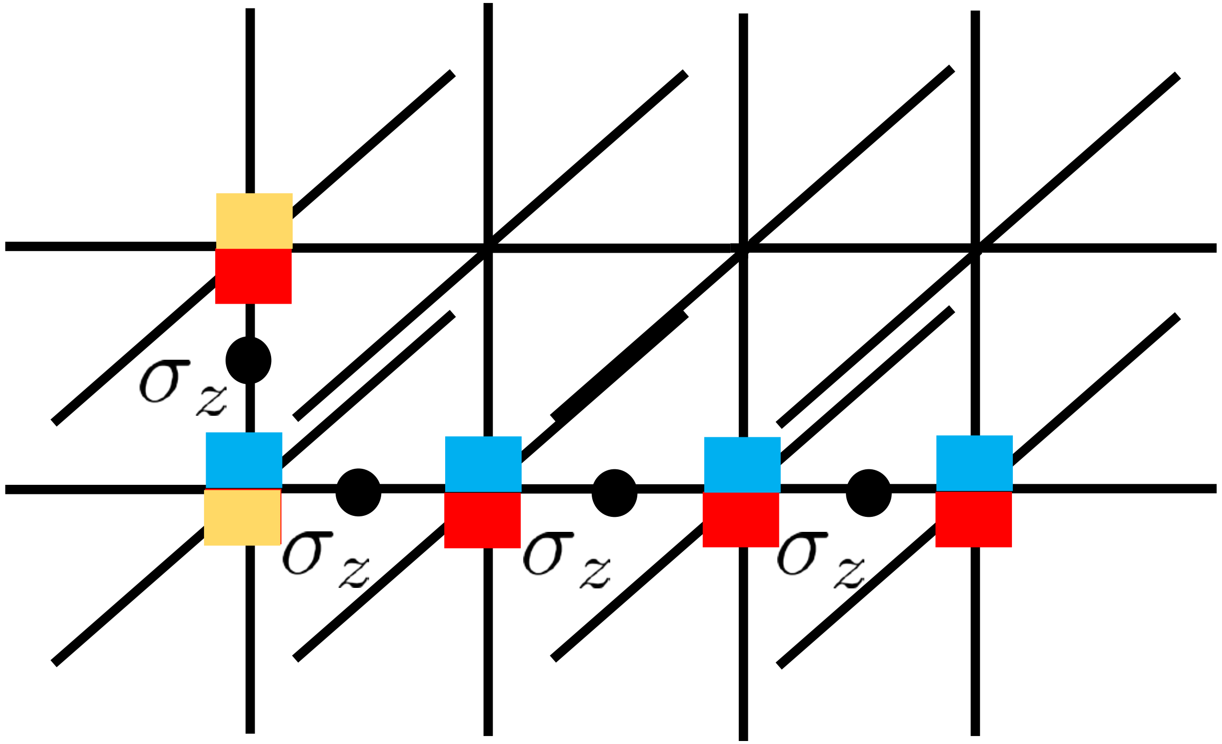


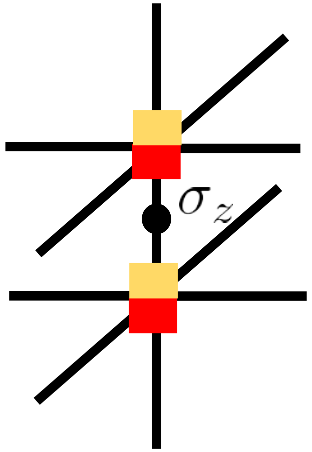
Creates pair of e_x

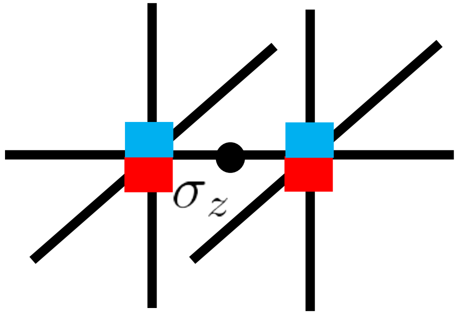
Creates pair of e_z



Lineon excitations of X-Cube







e_x only mobile in x direction!

An aside: “type-I” and “type-II” fractons

- “Type-I”: immobile excitations have partially mobile bound states (e.g. X-cube)
- “Type-II”: all point-like excitations immobile (e.g. Haah’s code)

Vijay, Haah, Fu PRB **94** 235157 (2016)

Not an exhaustive classification!

DB, Barkeshli PRB **100** 155146 (2019); Prem, Williamson 1905.06309

A few recent/ongoing directions

- Non-Abelian gapped fracton phases

Vijay, Fu 1706.07070; Song et. al. PRB 99 155118 (2019); Prem et. al. PRX **9** 021010 (2019); DB, Barkeshli PRB **100** 155146 (2019); Prem, Williamson 1905.06309

- Highly general pictures of gapped fracton phases

Pai, Hermele 1903.11625; Aasen et. al. (in preparation)

- Understanding the role of geometry

Shirley et. al. PRX **8** 031051 (2018) and others; DB, Iadecola PRB **99** 125132 (2019); Slagle, Aasen, Williamson SciPost Physics **6** 043 (2019)

Map for this talk

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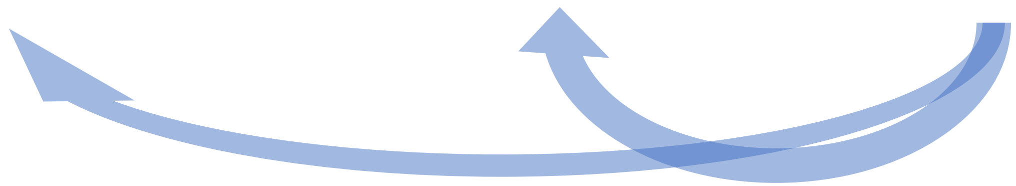
General theory	Topological quantum field theory	???	N/A

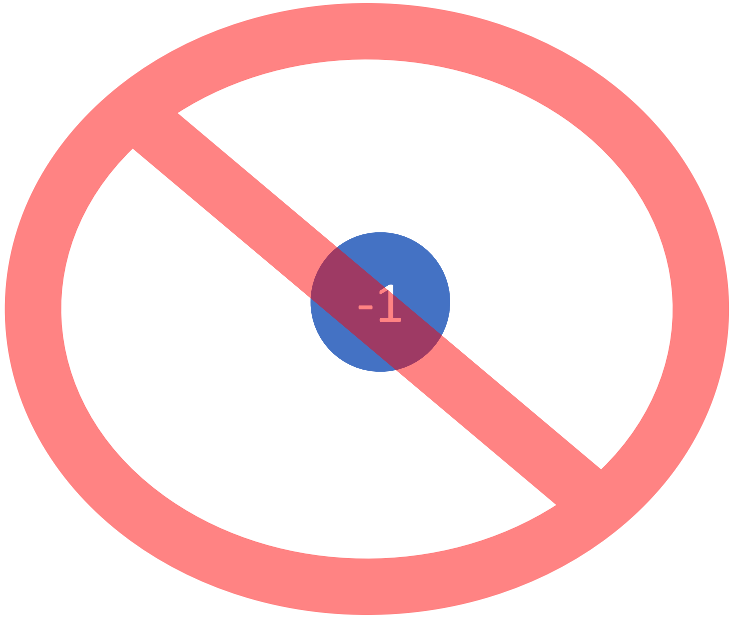


Higgs mechanism

Local operators in conventional gauge theory

$$\rho = \partial_i E_i$$

$$\int \rho(x) d^3x = \int \partial_i E_i d^3x = \text{bdry term}$$



No local operator creates an isolated charge.

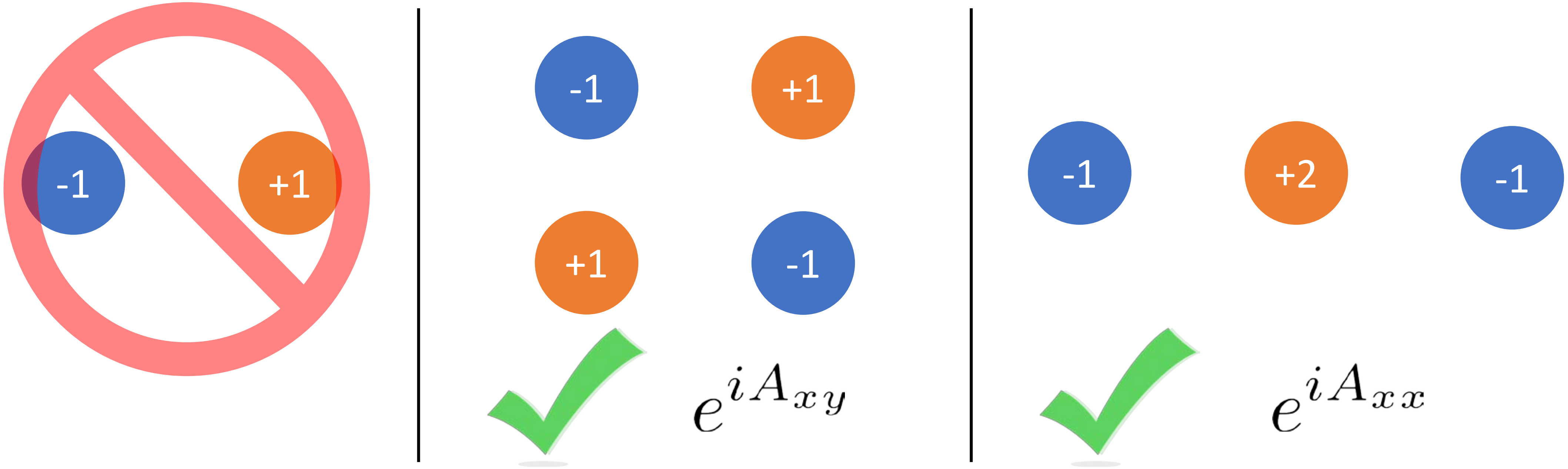
(Short) Wilson line operator e^{iA_x} creates a dipole (on the lattice).

Local operators in the scalar charge theory

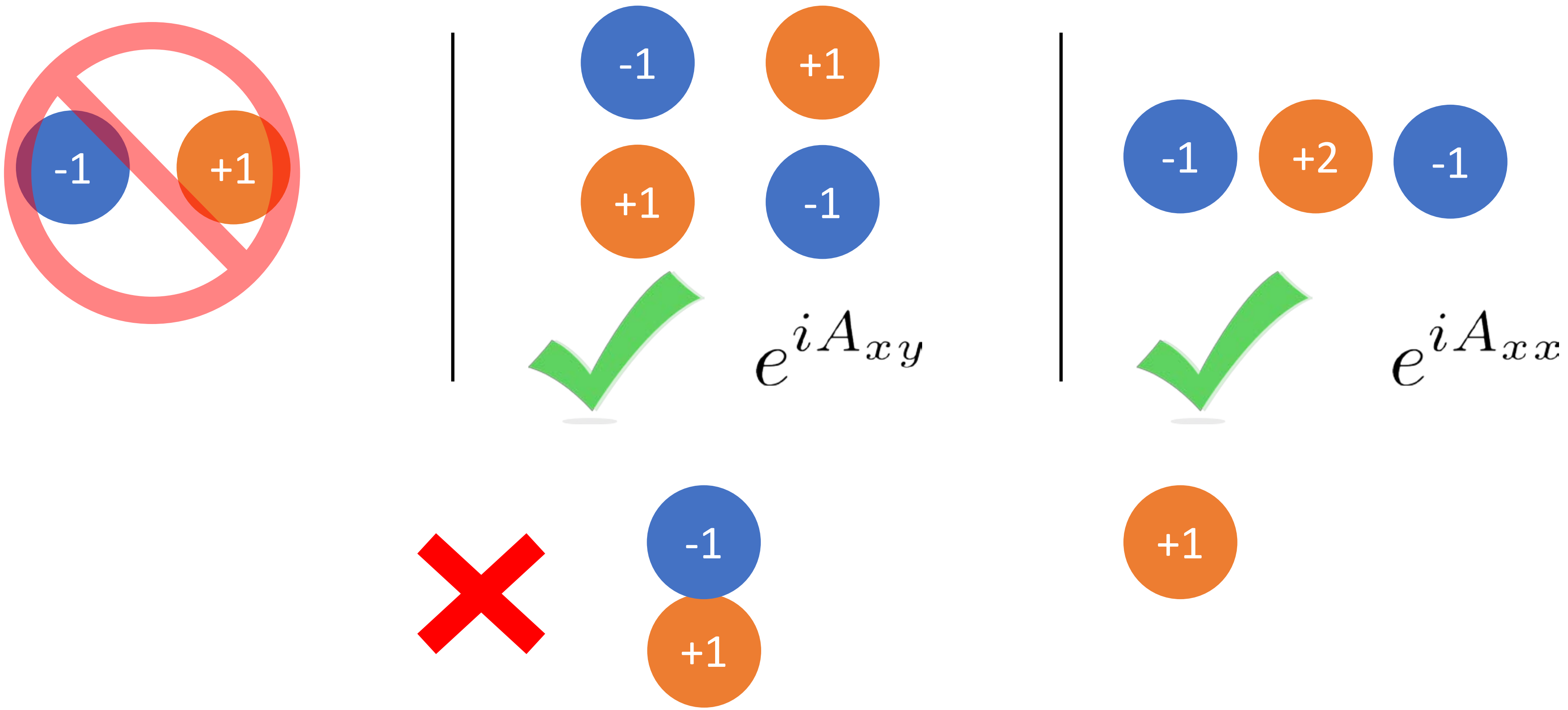
$$\rho = \partial_i \partial_j E_{ij}$$

$$\int \rho(x) d^3x = \int \partial_i \partial_j E_{ij} d^3x = \text{bdry term}$$

$$\int x_i \rho(x) d^3x = -\int \partial_j E_{ij} d^3x = \text{bdry term}$$

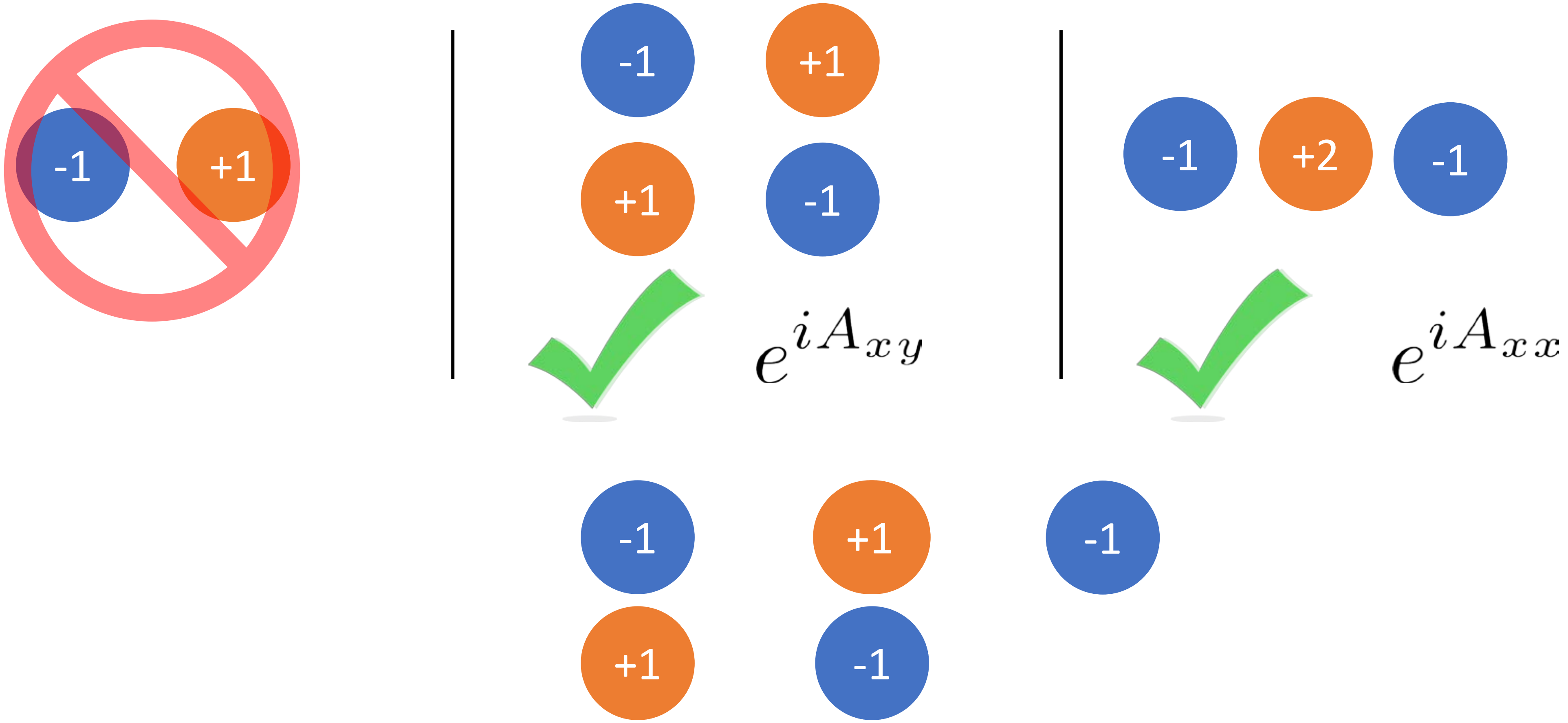


Local picture of mobility restrictions



Charge hopping operator is not allowed.

Local picture of mobility restrictions



Local operators are dipole hopping operators.

Lattice formulation of scalar charge theory

Hilbert space: $U(1)$ rotor $e^{iA_{ij}}$ on each plaquette of square lattice, three rotors $e^{iA_{ii}}$ per site. Integer-valued conjugate E_{ij}

$$H = U\left(\sum \Delta_i \Delta_j E_{ij}\right)^2 - \frac{1}{g^2}\sum \cos B_{ij}$$

$$[A_{ij}, E_{kl}] = \frac{i}{2}\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right) \qquad B_{ij} = \epsilon^{iab}\Delta_a A_{bj}$$

Other higher-rank gauge theories

- 2D scalar charge theory
  - Dual to 2D elasticity theory

  Pretko, Radzihovsky PRL **120** 195301 (2017), see also various works of Gromov, Radzihovsky groups
- Vector charge theory: $\rho_i = \partial_i E_{ij}$
  - Connections to linearized gravity

  Xu PRB **74** 224433 (2006), Xu cond-mat/0602443
  - Spin ice realization

  Yan et. al. 1902.10934
- Traceless theories: $\sum_i E_{ii} = 0$
- “Generalized” gauge theories – remove symmetric tensor structure

DB, Barkeshli 1806.01855
  - “Multipole algebra” structure

  Gromov PRX **9** 031035 (2019)

Xu PRB **74** 224433 (2006),
Rasmussen, You, Xu 1601.08235
Pretko PRB **95** 115139 (2017)

A few recent/ongoing directions

- Higher-rank gauge theory in curved space

Slagle, Prem, Pretko Ann. Phys. 410 167910 (2019); Gromov PRL **122** 076403 (2019)

- Chern-Simons and BF-like theories from higher-rank gauge theories

Pretko PRB **96** 125151 (2017); Slagle, Kim PRB **96** 195139 (2017); You et. al. 1904.11530

- High-energy/mathematical physics interest in higher-rank gauge theory

Seiberg 1909.10544; Radicevic 1910.06336; Wang, Xu 1909.13879

- Holographic toy models

Yan 1911.01007

Map for this talk

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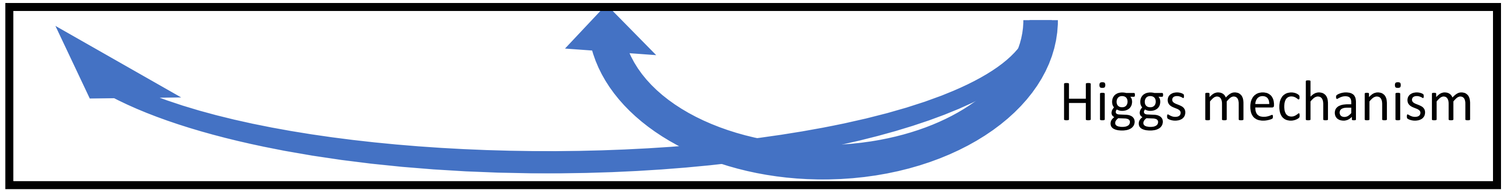
“Anyonic” “braiding” processes (3D)	Particle-string, string-string	Particle-particle	???
General theory	Topological quantum field theory	???	N/A

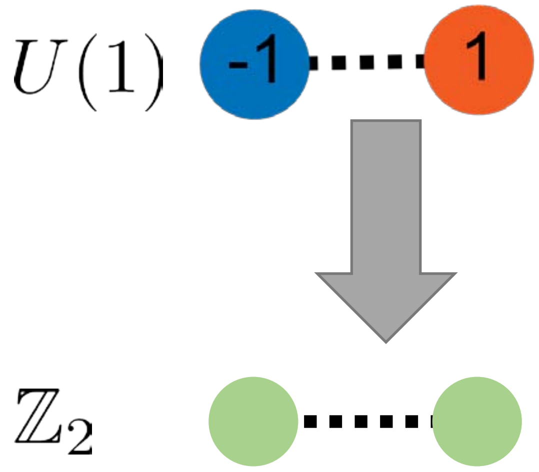


The Higgs mechanism in conventional gauge theory

Condense charge 2 matter in conventional $U(1)$ gauge theory

$$\rho = \partial_i E_i$$

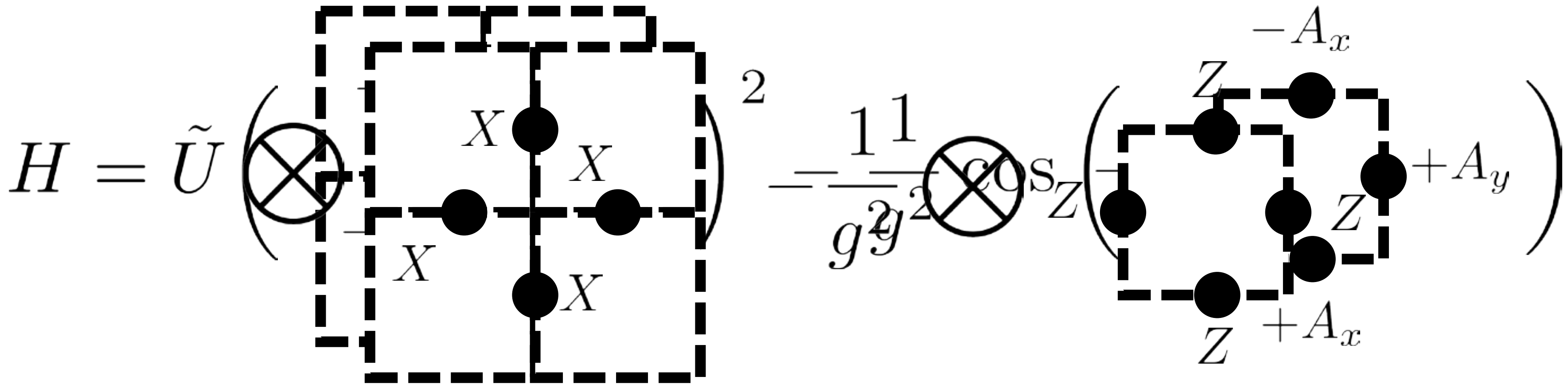
$$(-1)^{\rho} = (-1)^{\partial_i E_i}$$



$$(-1)^{E_i} \Rightarrow X$$

$$e^{iA_i} \Rightarrow Z$$

The Higgs mechanism in conventional gauge theory



$$(-1)^{E_i} \Rightarrow X$$

$$e^{iA_i} \Rightarrow Z$$

$$[A_j, E_k] = i\delta_{jk}$$

$$\{X, Z\} = 0$$

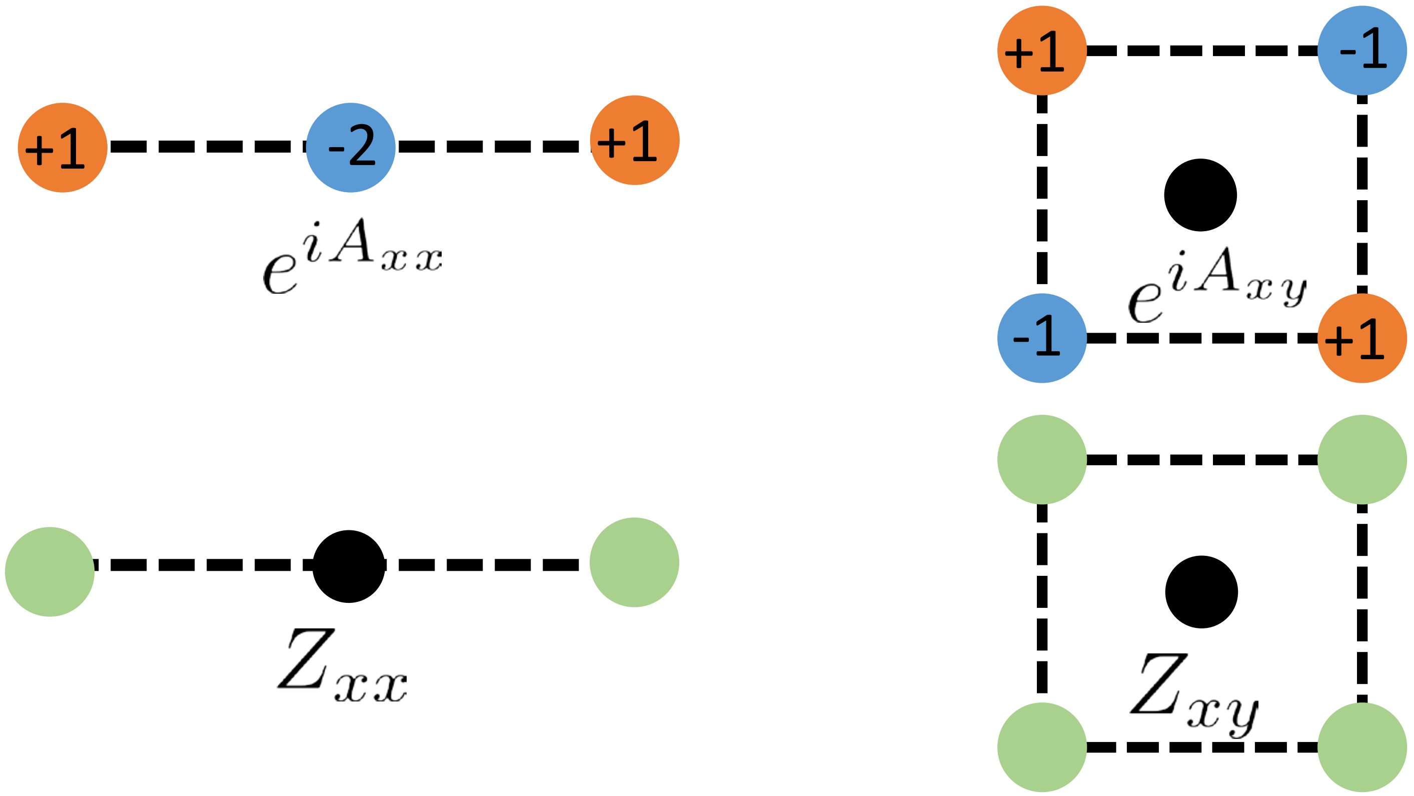
$$X^2 = Z^2 = 1$$

$$A_j \simeq A_j + 2\pi$$

Higgsing in the scalar charge theory

Scalar charge theory has fractonic charges that are *mobile* after Higgsing

$$\rho = \sum_{ij} \partial_i \partial_j E_{ij}$$

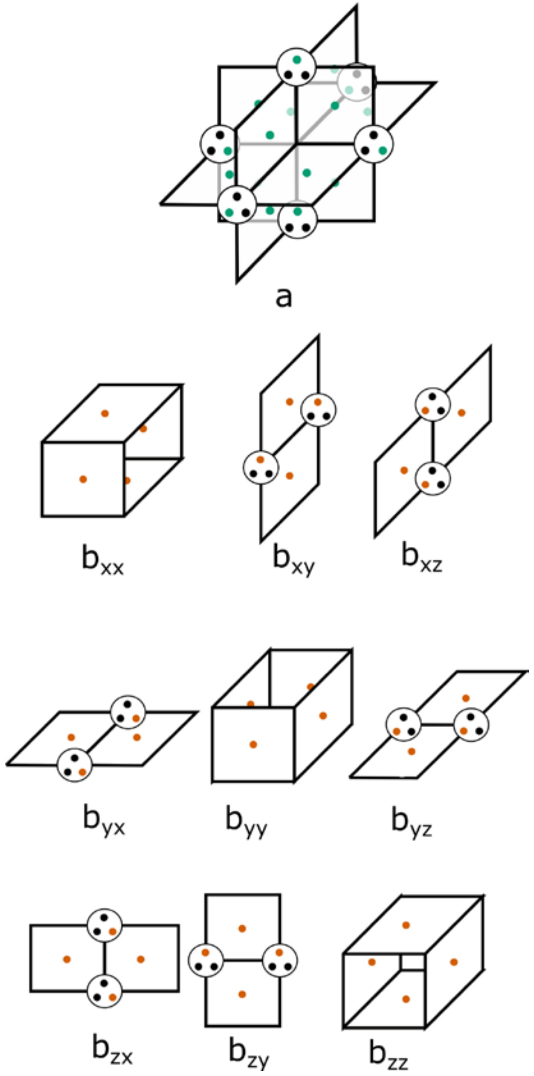


DB, Barkeshli PRB **97**, 235112 (2018)
Ma, Hermele, Chen PRB **98** 035111 (2018)

Higgsing in the scalar charge theory

Discrete scalar charge theory: 3 spins per site, one per plaquette.

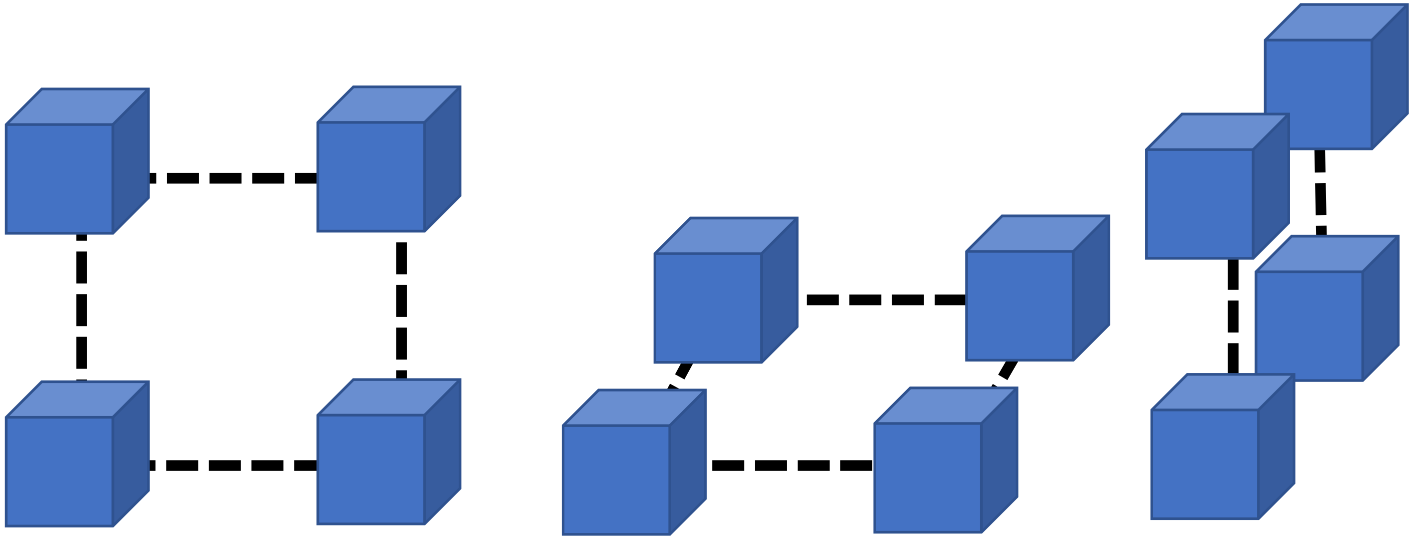
Can show: topological order is 4 copies of 3D toric code



DB, Barkeshli PRB **97**, 235112 (2018)
Ma, Hermele, Chen PRB **98** 035111 (2018)

Generating a $U(1)$ theory that Higgses to X-Cube?

Valid X-Cube fracton configurations are **only**
$U(1)$ charge



$$\rho = \partial_x \partial_y E_{xy} + \partial_x \partial_z E_{xz} + \partial_y \partial_z E_{yz}$$

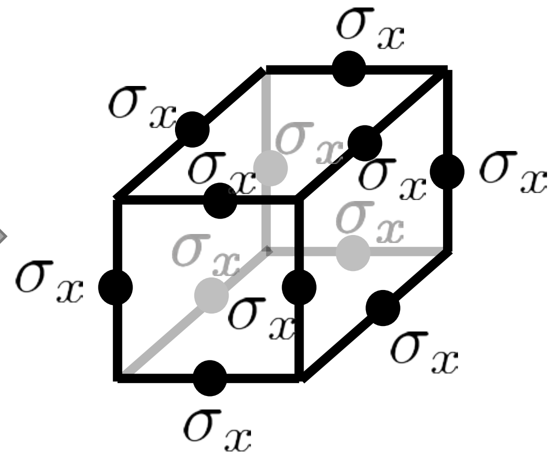
(0,1) or off-diagonal or “hollow” scalar charge theory

DB, Barkeshli PRB **97**, 235112 (2018)
Ma, Hermele, Chen PRB **98** 035111 (2018)

(0,1) scalar charge theory Higgses to X-Cube

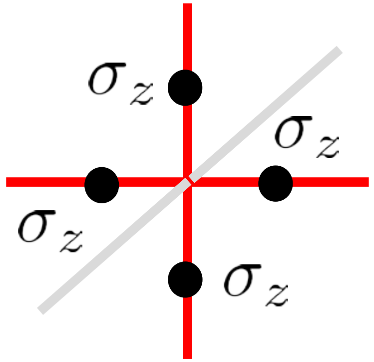
$$\rho = \partial_x \partial_y E_{xy} + \partial_x \partial_z E_{xz} + \partial_y \partial_z E_{yz}$$

Higgs



$$\cos(B_i) = \cos\left(\sum_{jk} \epsilon^{ijk} A_{ki}\right)$$

Higgs



DB, Barkeshli PRB **97**, 235112 (2018)
Ma, Hermele, Chen PRB **98** 035111 (2018)

Higgsing symmetric tensor gauge theories

$U(1)$ **Charge Type**	(m, n)	**Higgs Phase**
$d = 2$ scalar		
	$(2r+1,\ 2s+1)$	$\mathbb{Z}_2^3$ topological order
	$(2r, 2s+1)$	Trivial
	$(2r+1, 2s+2)$	Trivial
	$(1, 0)$	$\mathbb{Z}_2^4$ topological order
$d = 2$ vector		
	$(2r+1,\ 2s+1)$	$\mathbb{Z}_2^3$ topological order
	$(2r+2,\ 2s+1)$	$\mathbb{Z}_2^4$ topological order
	$(2r+1,\ 2s)$	Trivial
	$(0, 1)$	Trivial
$d = 3$ scalar		
	$(2r+1,\ 2s+1)$	$\mathbb{Z}_2^4$ topological order
	$(2r, 2s+1)$	X-Cube fracton order
	$(2r+1, 2s+2)$	Trivial
	$(1, 0)$	$\mathbb{Z}_2^8$ topological order
$d = 3$ vector		
	$(2r+1,\ 2s+1)$	$\mathbb{Z}_2^7$ topological order
	$(4r+2,\ 2s+1)$	$\mathbb{Z}_2$ topological order
	$(4r,\ 2s+1)$	Trivial
	$(2r+1,\ 2s)$	Trivial

DB, Barkeshli PRB **97**, 235112 (2018)

Broad challenges for the future

- What is a general, abstract theory of gapped fracton phases?
- How does “braiding” work with fractons?
- Can we go from local operator structure to mobility constraints?
- How can we realize gapped fracton phases or (quantum) higher-rank gauge theories experimentally?
- How does (thermal) transport behave in the presence of fractons?

Concluding map

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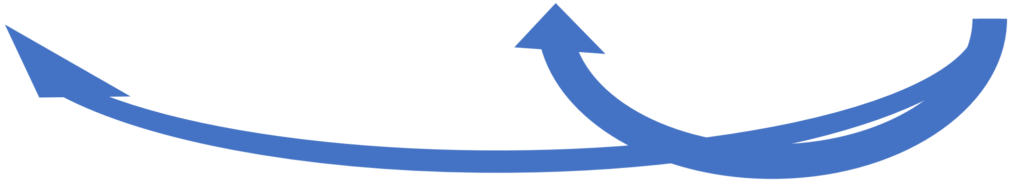

Higgs mechanism