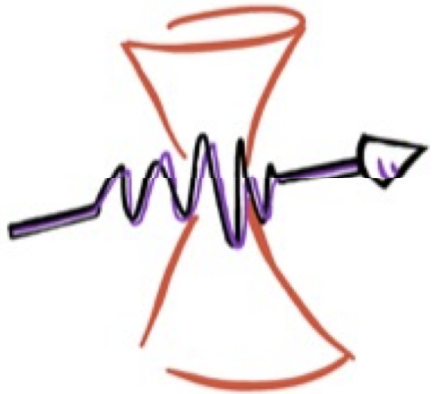




Quantum Spin Liquids: Signatures of Fractionalization



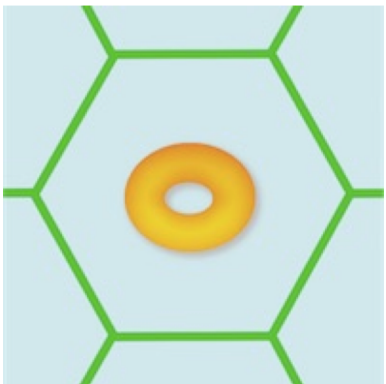
Nandini Trivedi
Physics Department
The Ohio State University

trivedi.15@osu.edu

<http://trivediresearch.org.ohio-state.edu/>



Center of Emergent Materials
NSF MRSEC – DMR



Spin and Heat Transport in Quantum and Topological Materials
Topological Quantum Matter: Concepts and Realizations
November 11-15, 2019



Gordon Research Conference

Correlated Electron Systems

Topology and Correlations: Long-Range
Entanglement in Many-Body Systems

June 28 - July 3, 2020

Mount Holyoke College, South Hadley, MA

<https://www.grc.org/correlated-electron-systems-conference/2020/>

Chairs: Nandini Trivedi and James G. Analytis

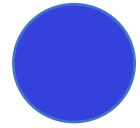
Roadmap

- Big picture
- 2D: Kitaev Model
 - ❖ Discovery of a new gapless QSL with a spinon Fermi surface
 - ❖ Spectrum of 1 spin flip and 2 spin flip excitations
- QSL Materials
- How do you detect a QSL?
- Going forward....



Big Picture

Quantum Matter: Fractionalization



electron



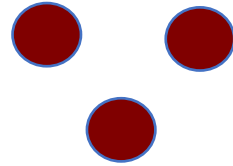
Electron=charge+spin



Spin wave (magnon) $S=1$

$$\nabla \cdot B = 0$$

No monopoles



fractionalization

Fractional
Quantum Hall effect



spin-charge separation

Luttinger Liquids

Fractionalization: $S=1/2$ spinons:
(neutral) spinon Fermi surface

Monopoles in spin ice

Quantum Spin Liquid

- ❑ Landau paradigm: spontaneously broken symmetry →
local order parameter m
bosonic excitations: magnons (for continuous spins)
- ❑ Topological Paradigm

"IQHE"

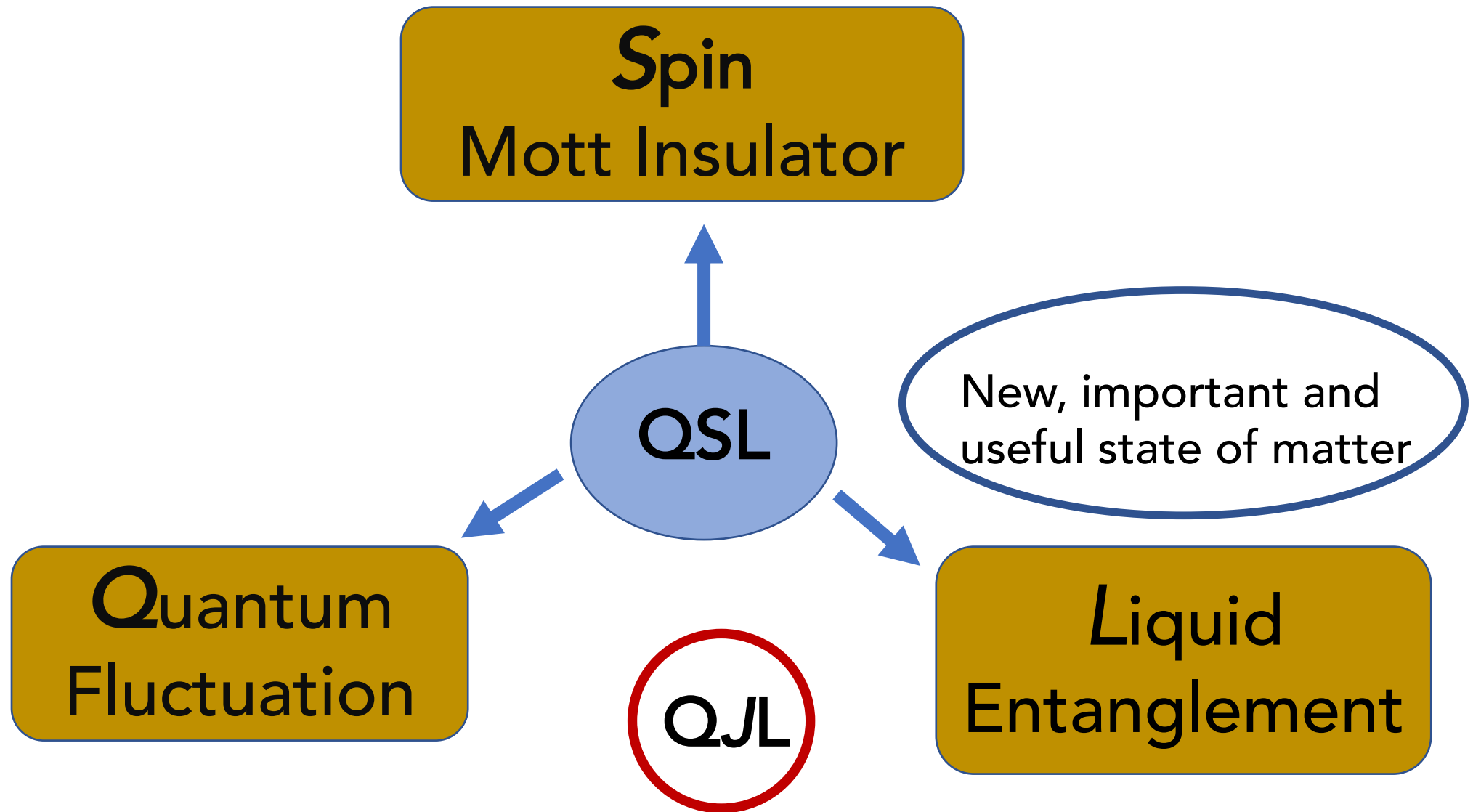
Topological Insulators
Topological Superconductors
Topological Weyl and Dirac Semimetals
Topological magnons

"FQHE"

- Ground state degeneracy
- Long range entanglement
- Fractionalized Excitations
- Topological Order

Important for storing information non-locally;
robust against decoherence

Review: Savary and Balents, Repts. on
Progress in Physics 80, 016502 (2017)



Burning question....

Have we seen a quantum spin liquid?



A. Kitaev, *Annals of Physics*
321, 2-111 (2006)

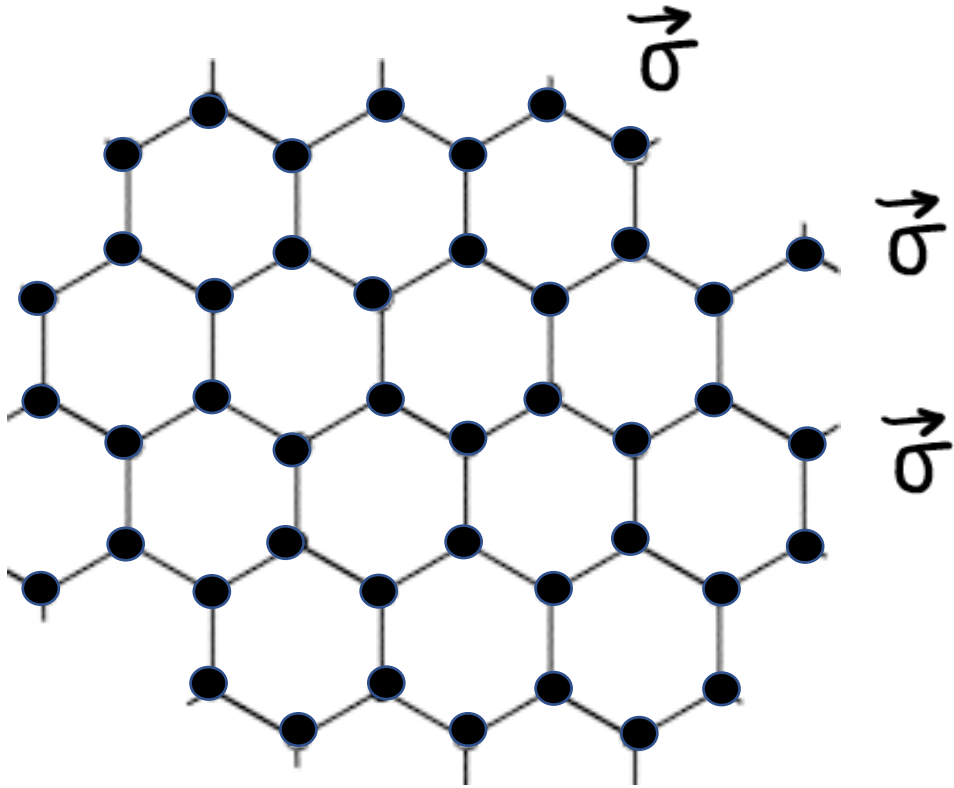


G. Jackeli and G. Khaliullin, *PRL*
102, 017205 (2009)



Kasahara, Ohnishi, Kurita, Tanaka, Nasu,
Motome, Shibauchi, Matsuda, *Nature* 559,
227-231 (2018)

Kitaev Model

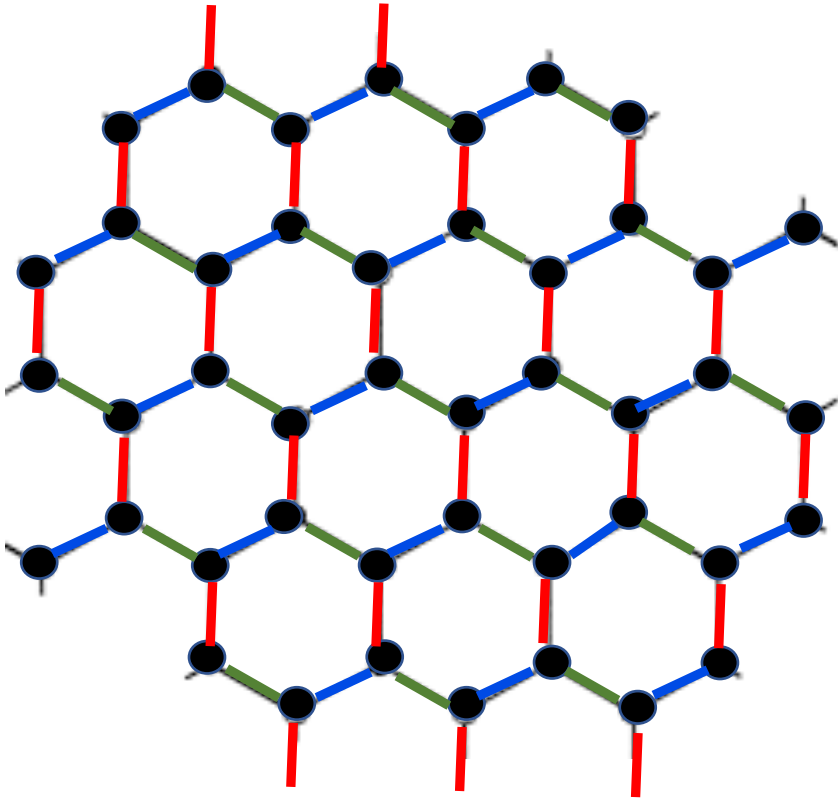


put qubit on each site

Honey comb lattice

Bipartite lattice; no geometric frustration

Kitaev Model: bond-dependent interactions



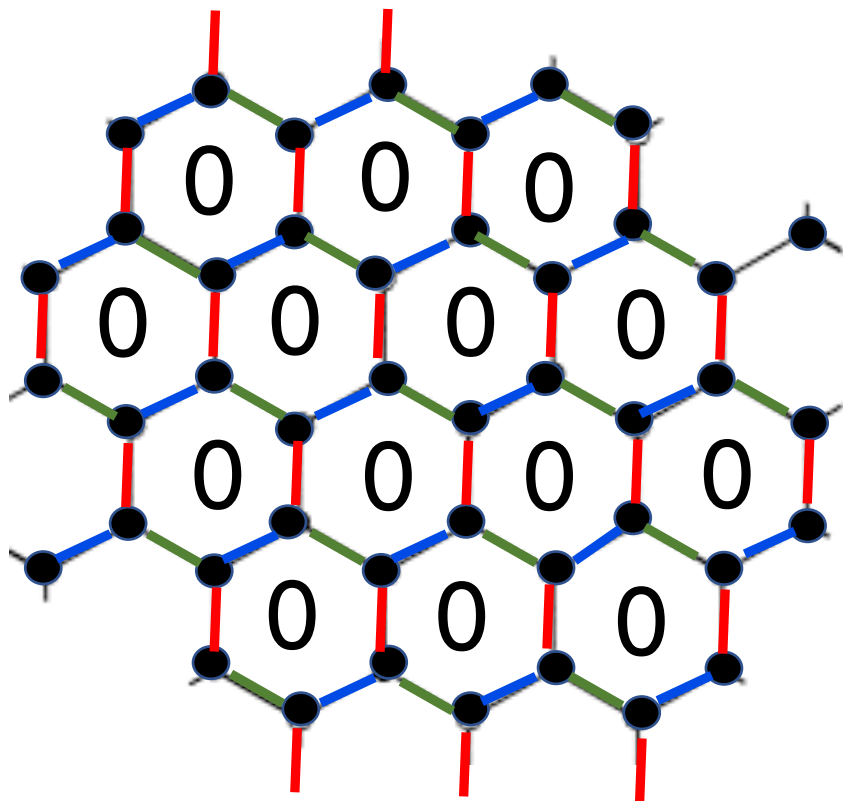
$$\mathcal{H} = K \left[\sum_{\langle ij \rangle \in x} \sigma_i^x \sigma_j^x + \sum_{\langle ij \rangle \in y} \sigma_i^y \sigma_j^y + \sum_{\langle ij \rangle \in z} \sigma_i^z \sigma_j^z \right]$$

Parton construction:

$$\sigma^{\alpha} = i b^{\alpha} c$$

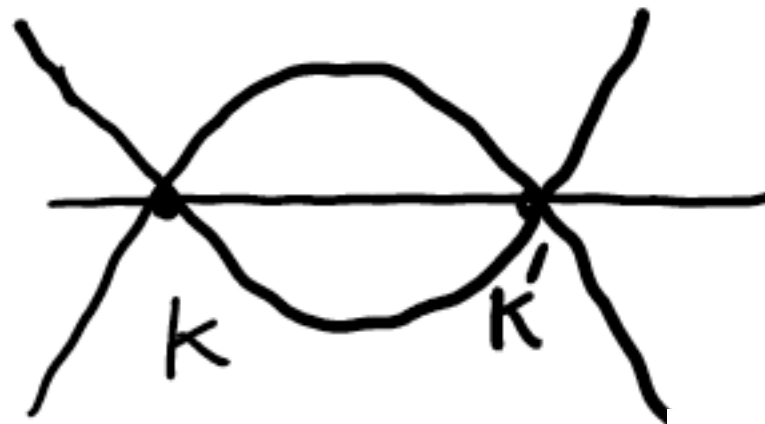
$$\mathcal{H} = K \frac{i}{2} \sum_{\langle ij \rangle} \hat{u}_{ij} c_i c_j$$

Ground State:

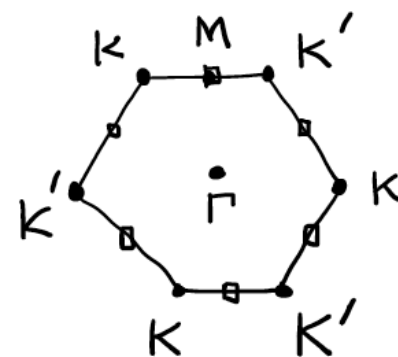


All plaquettes have zero flux

c-Majoranas

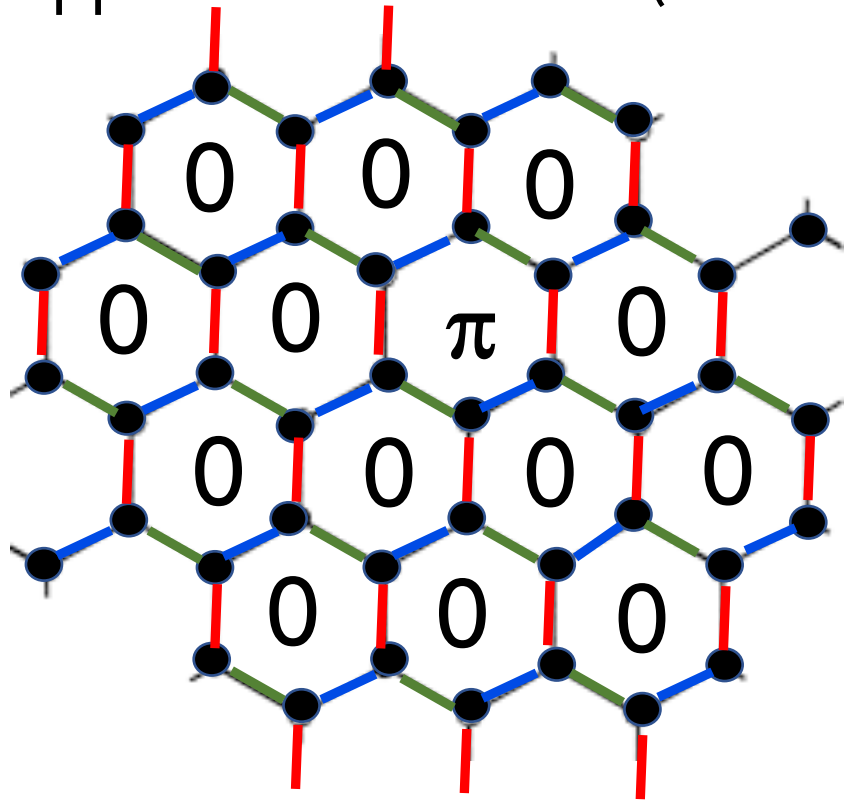


c-majorana
fermions have a
Dirac dispersion

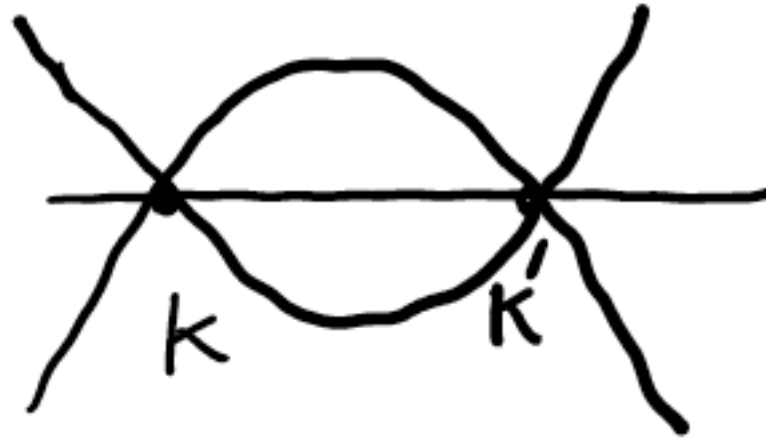


Excitations:

(1) Gapped flux excitation (visons)



(2) Gapless majorana fermions



Gapless Z_2 Quantum Spin Liquid

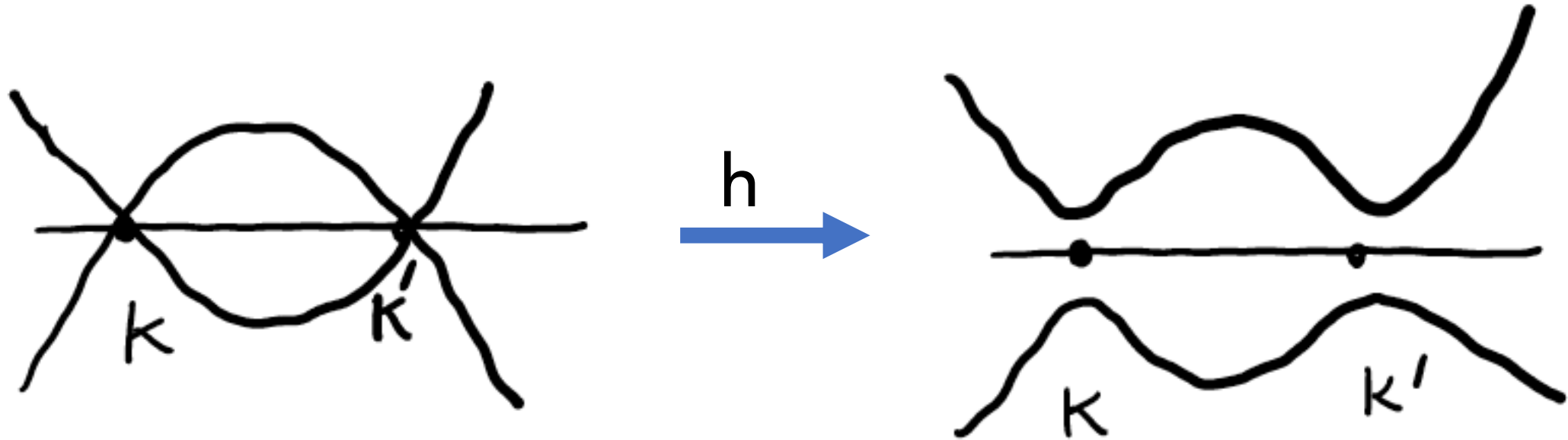
Now add a magnetic field...

$$H = H_K + h \sum_{i\alpha} S_i^\alpha$$

Focus on here:

AF Kitaev interactions and field along $h \parallel [111]$

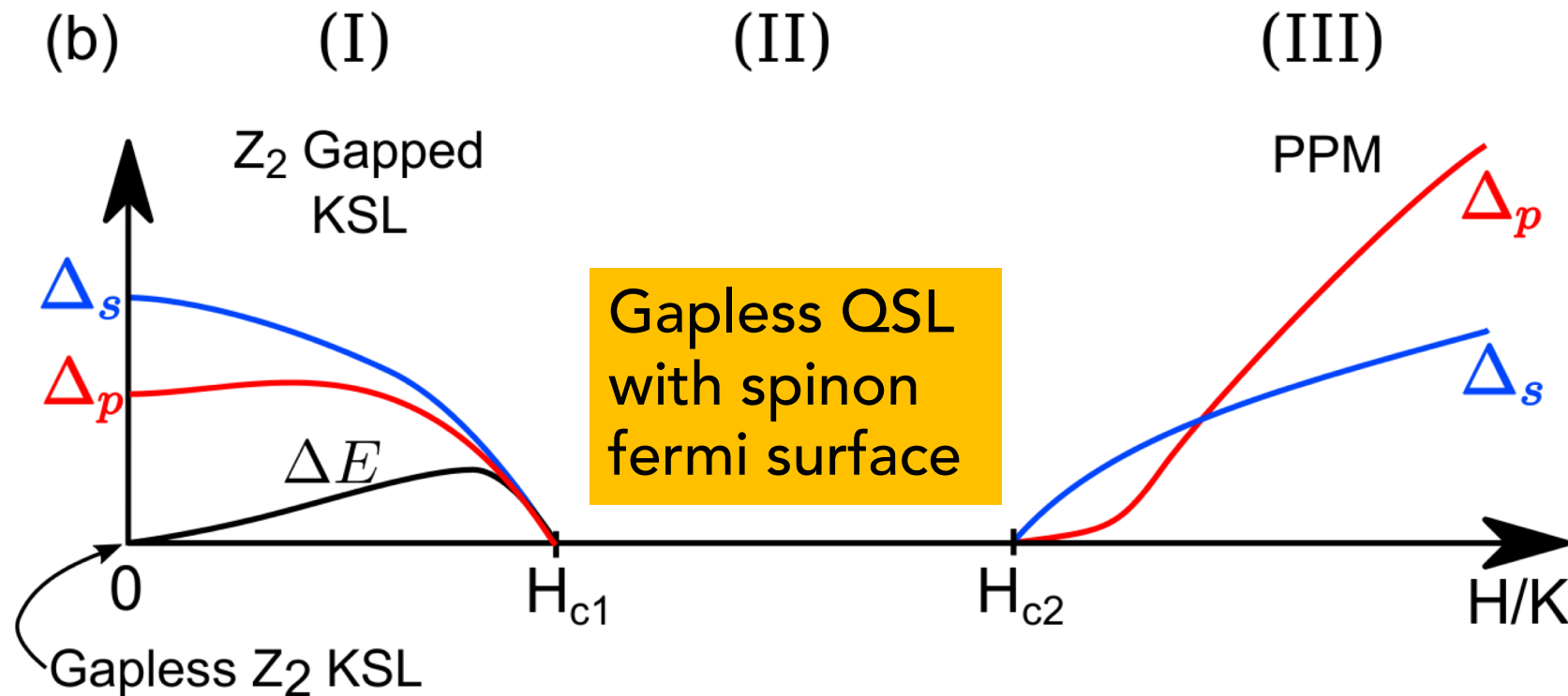
Non-abelian gapped Kitaev spin liquid:



- Majorana fermions get gapped
- Chern insulator
- Chiral edge mode with thermal Hall conductance

$$\kappa_{xy}/T = (1/2) (\pi/6) (k_B^2/\hbar)$$

Our Main Results: Kitaev Model in a Magnetic Field



Δ_s : single spin flip energy
 Δ_p : 2-spin flip energy

Results based on exact diagonalization ED and Density matrix renormalization group (DMRG)



David Ronquillo

Field-orientation-dependent spin dynamics of the Kitaev honeycomb model

Phys. Rev. B 99, 140413(R) (2019)



Adu Vengal



Nirav Patel

Magnetic field induced intermediate gapless spin-liquid phase with a spinon Fermi surface

PNAS 201821406 (2019)



Subhasree Pradhan

Two-Magnon Bound States in the Kitaev Model in a [111]-Field

[arXiv:1908.10877](https://arxiv.org/abs/1908.10877)

Related work:

Z. Zhu, et al., Phys. Rev. B 97, 241110 (2018)

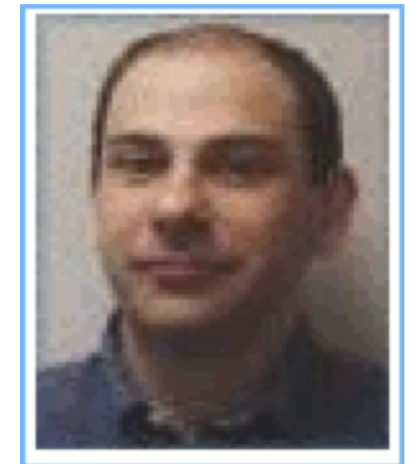
M. Gohlke, et al., Phys. Rev. B 98, 014418 (2018)

C. Hickey and S. Trebst, Nat. Comm. 10, 530 (2019)

H.C. Jiang et al. arXiv 1809.08247

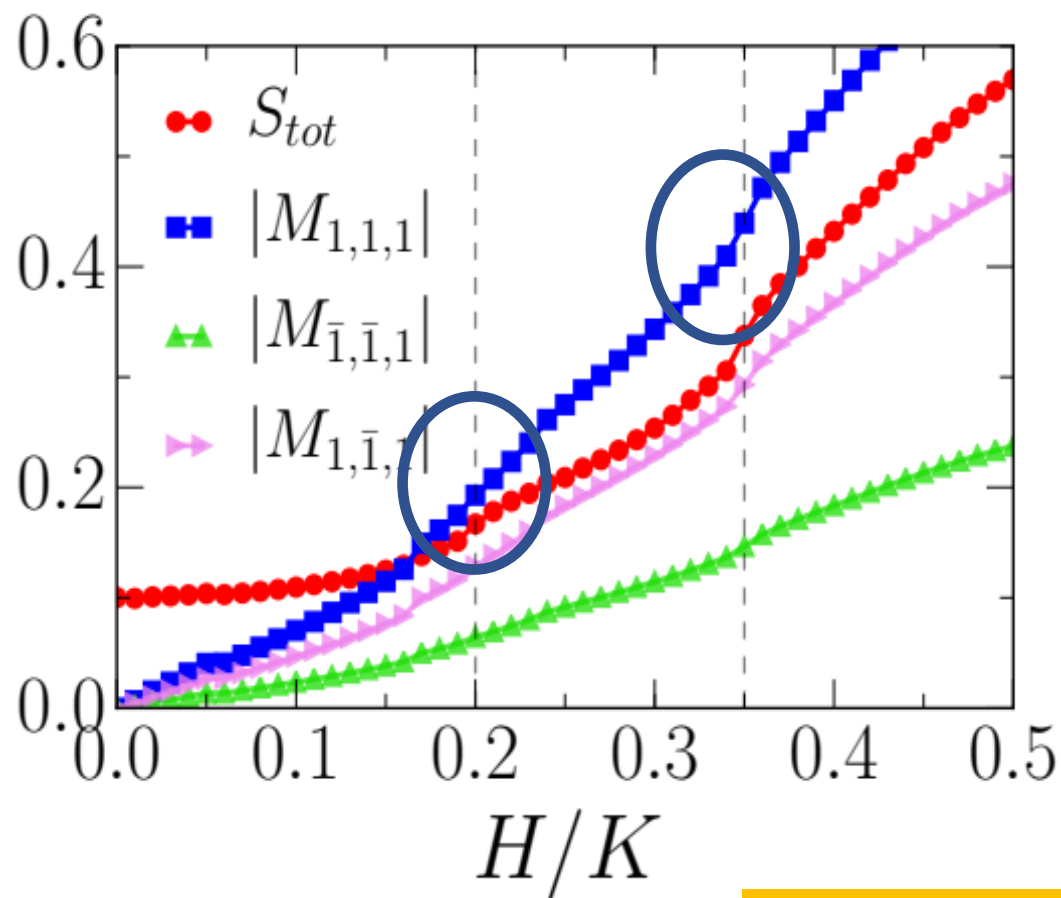
Y. Motome and J. Nasu, arXiv:1909.02234

Gonzalo Alvarez
ORNL

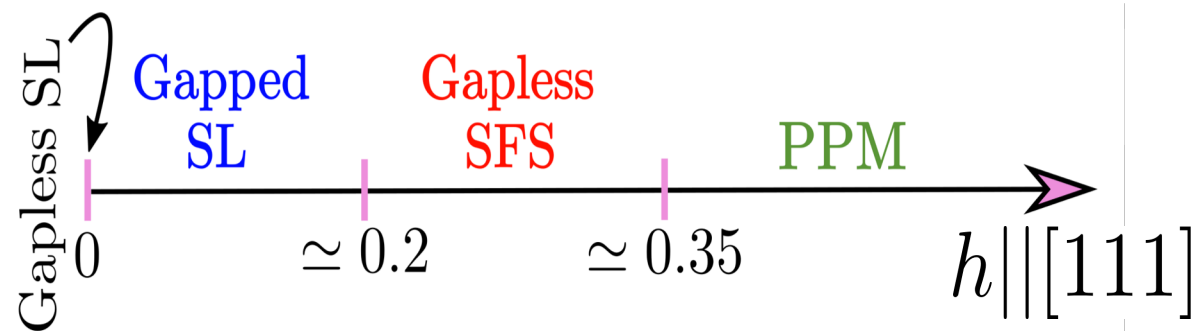


Evidence for TWO phase transitions

Kitaev Model + Magnetic field: $h||[111]$ magnetization

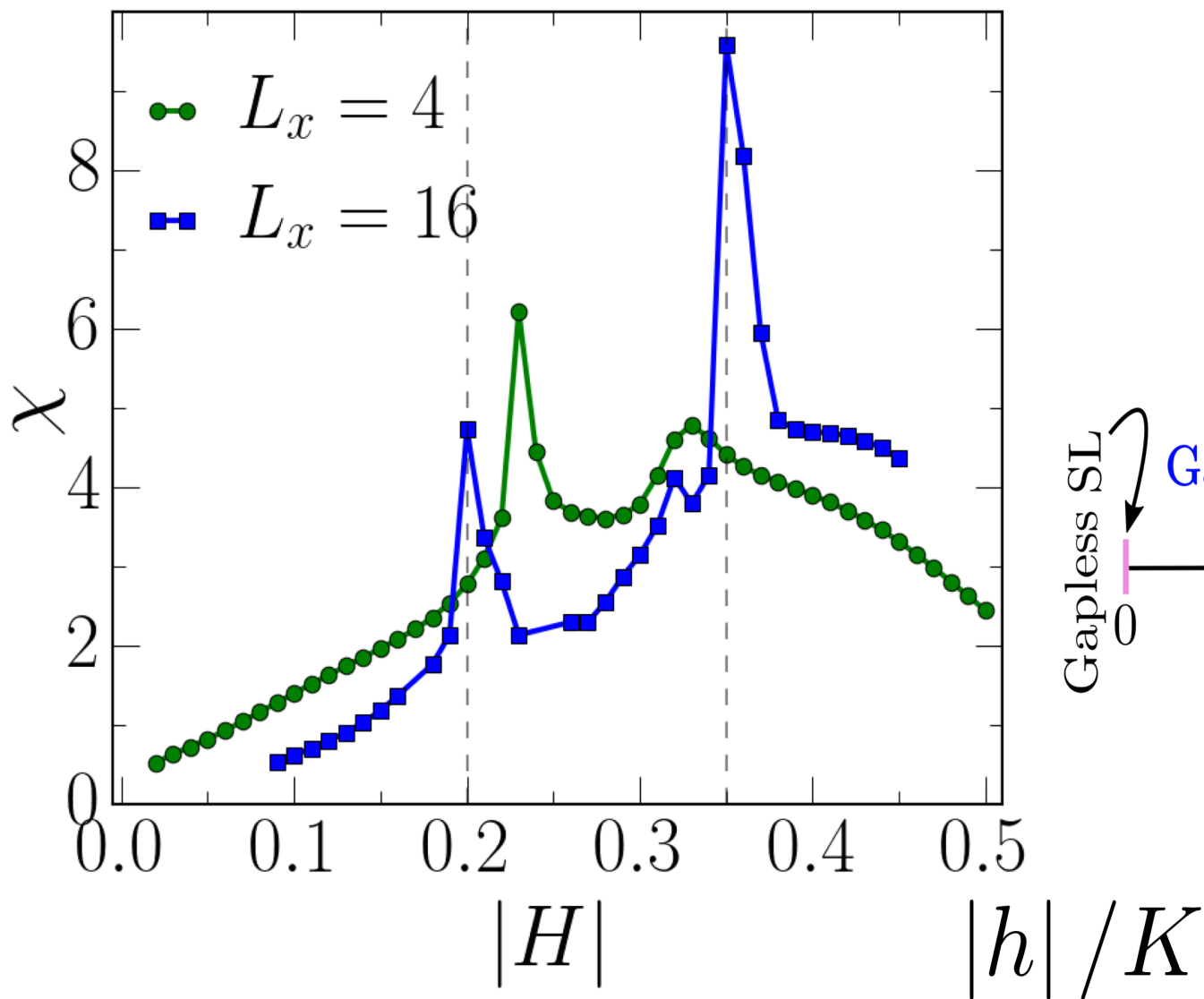


$$H = H_K + h \sum_{i\alpha} S_i^\alpha$$

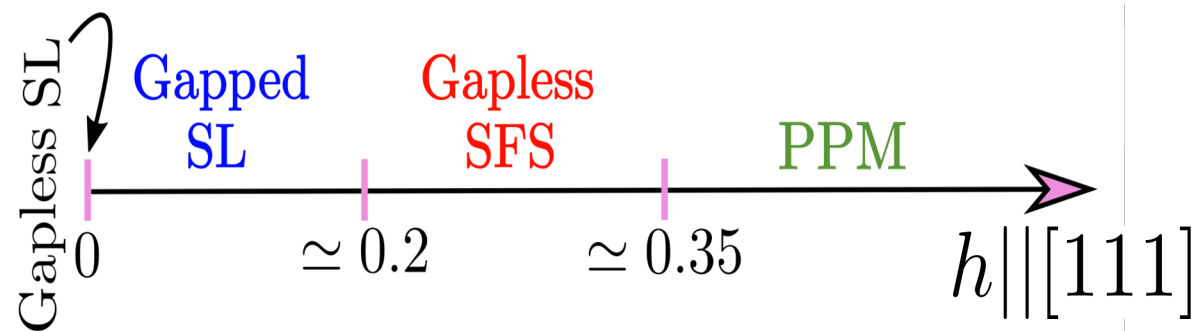


Density Matrix Renormalization Group calculations with 160 spins

Kitaev Model + Magnetic field: $h||[111]$ susceptibility

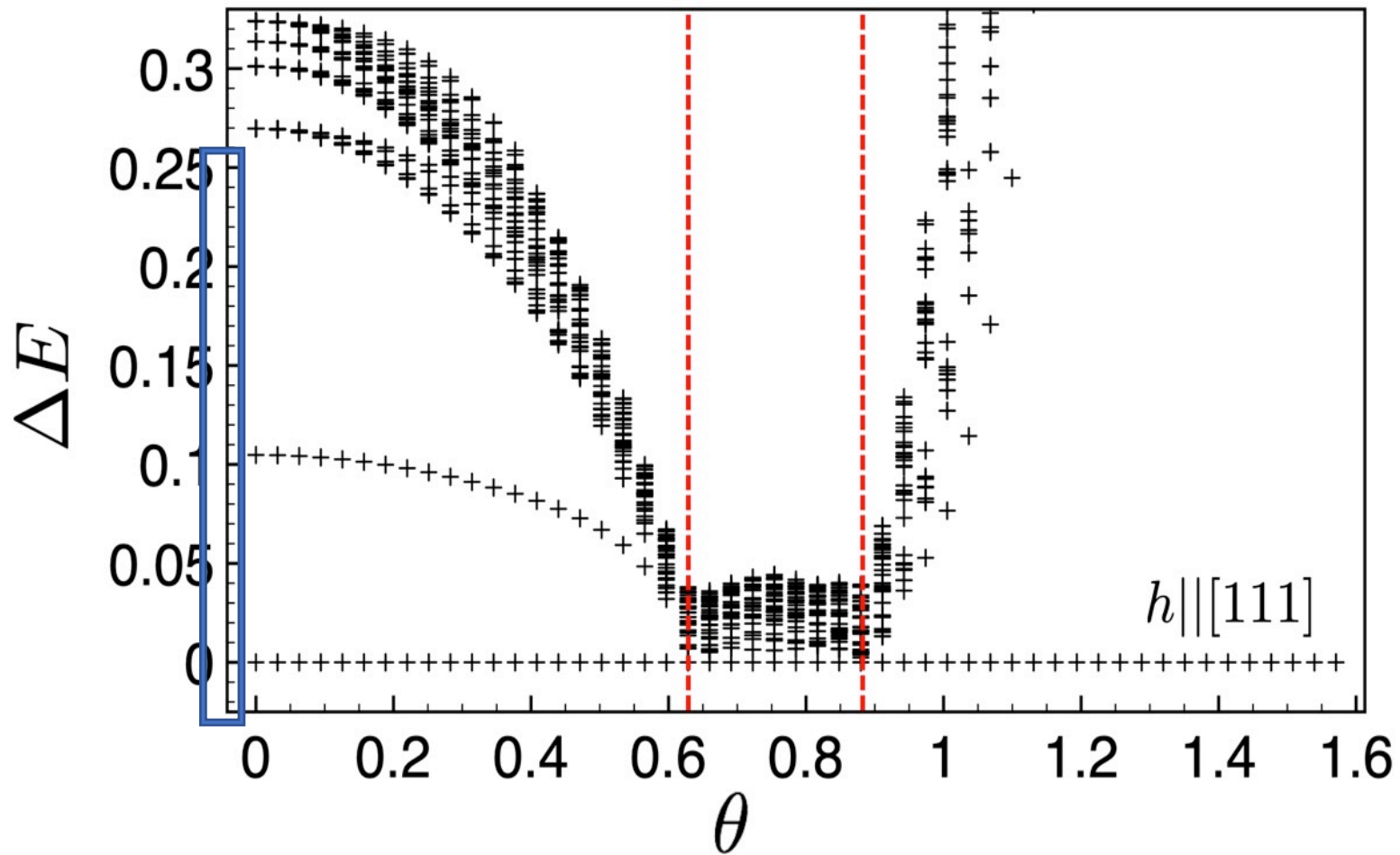
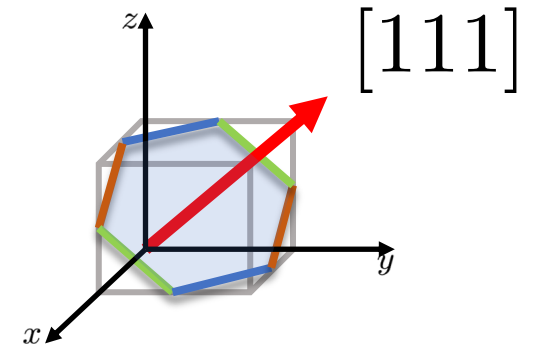


$$H = H_K + h \sum_{i\alpha} S_i^\alpha$$



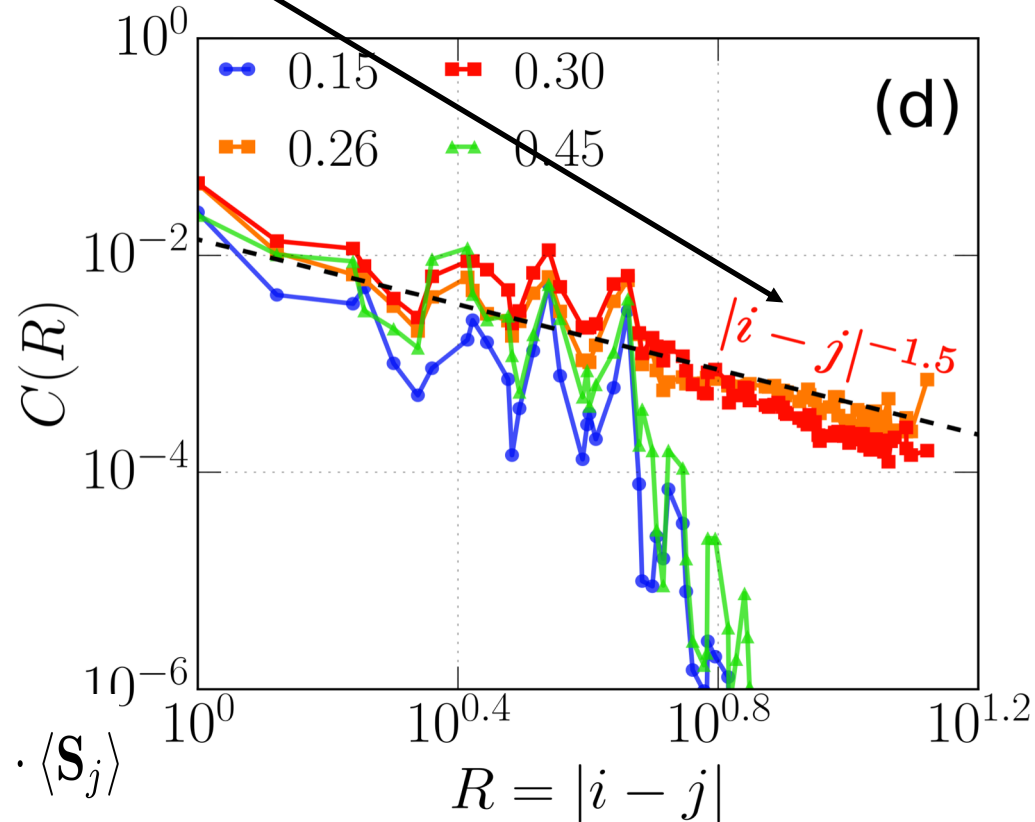
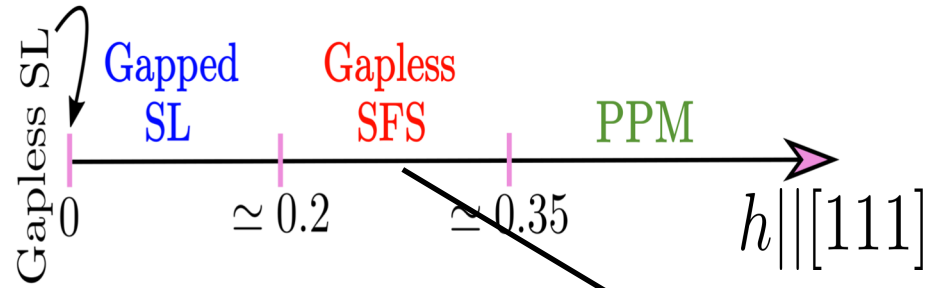
Evidence for gapless intermediate phase

Energy Spectra in a field



$$\theta \sim |\vec{h}|/K$$

Spin-Spin Correlations in Intermediate phase



Distinct power law decay of real-space spin-spin correlations!

$$C(R) = \frac{1}{N_R} \sum_{R=|i-j|} \langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle - \langle \mathbf{S}_i \rangle \cdot \langle \mathbf{S}_j \rangle$$

Evidence for QSL

Topological Entanglement Entropy

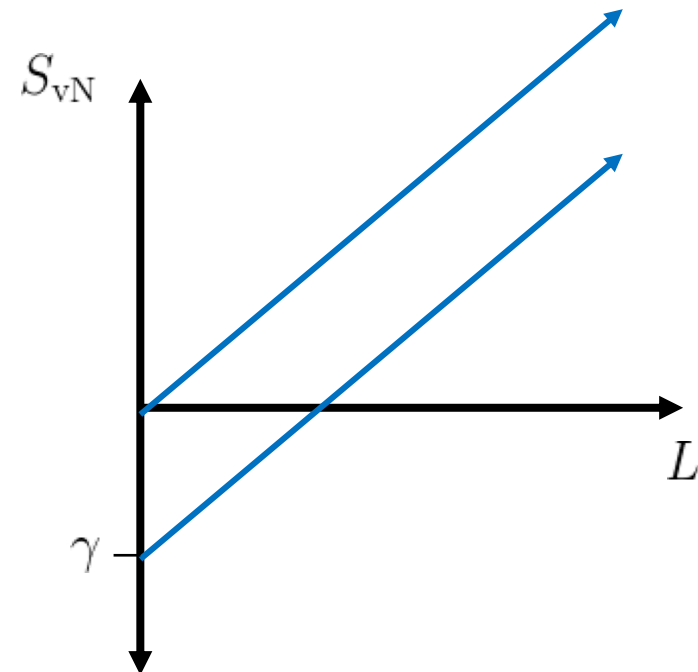
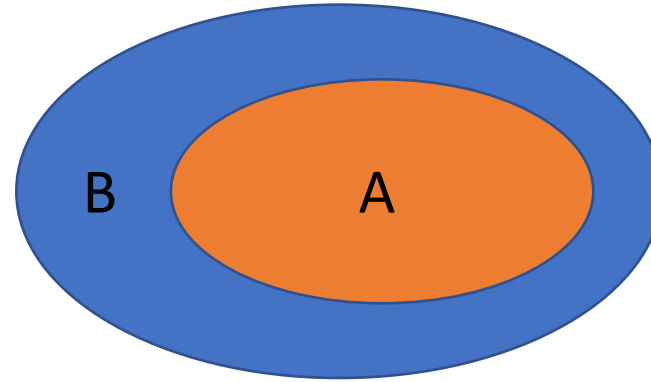
$$S_{\text{vN}} = \text{Tr}(\rho_A \log_2 \rho_A)$$

$$\rho_A \equiv \text{Tr}_B(\rho)$$

$$S_{\text{vN}} = \alpha L \underbrace{-\gamma}_{\text{topo}} + \mathcal{O}(1/L)$$

$$S_{\text{topo}} = -\gamma = -\ln \mathcal{D}$$

$$\mathcal{D} \equiv \sqrt{\sum_i d_i^2}$$

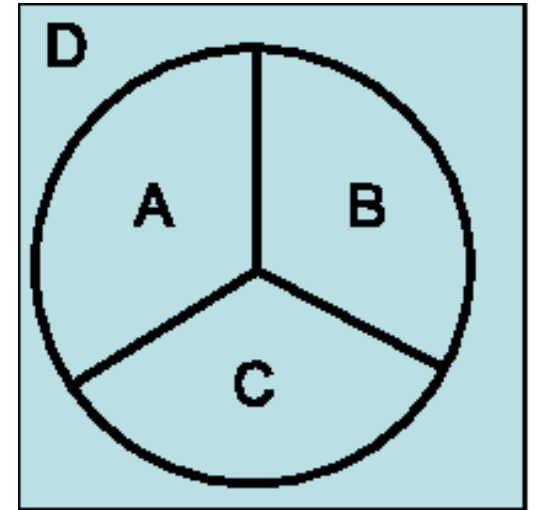


Gapped Non abelian KSL

| | | |
|---------|--------------------------------|---------------|
| Vacuum | $1 \sim d_1 = 1$ | } abelian |
| Fermion | $\epsilon \sim d_\epsilon = 1$ | |
| Vortex | $v \sim d_v = \sqrt{2} > 1$ | } Non abelian |

$$\begin{aligned}\rightarrow D &= \sqrt{d_1^2 + d_\epsilon^2 + d_v^2} \\ &= \sqrt{1 + 1 + 2} \\ &= 2\end{aligned}$$

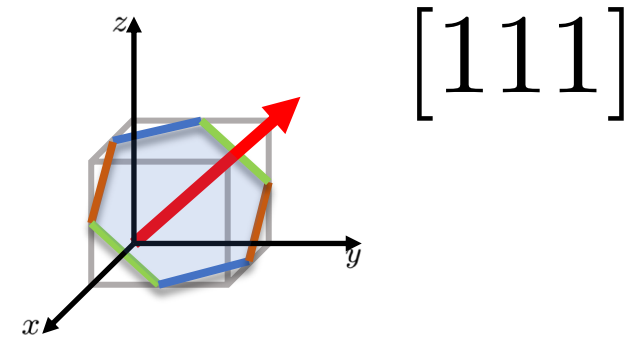
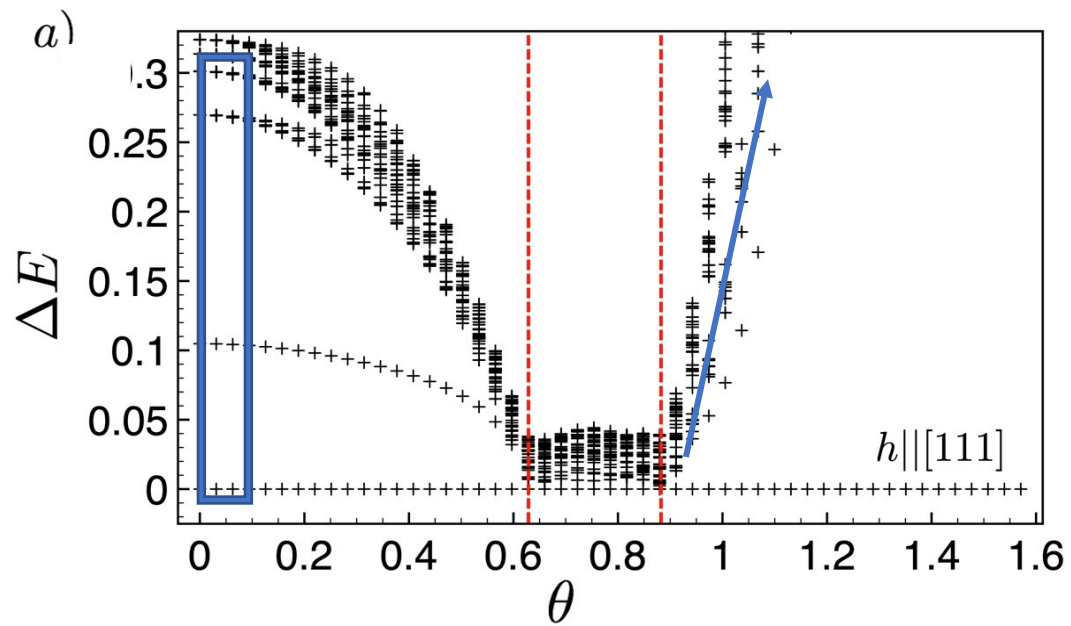
$$\rightarrow \gamma = \log D = \log 2$$



Kitaev-Preskill Construction

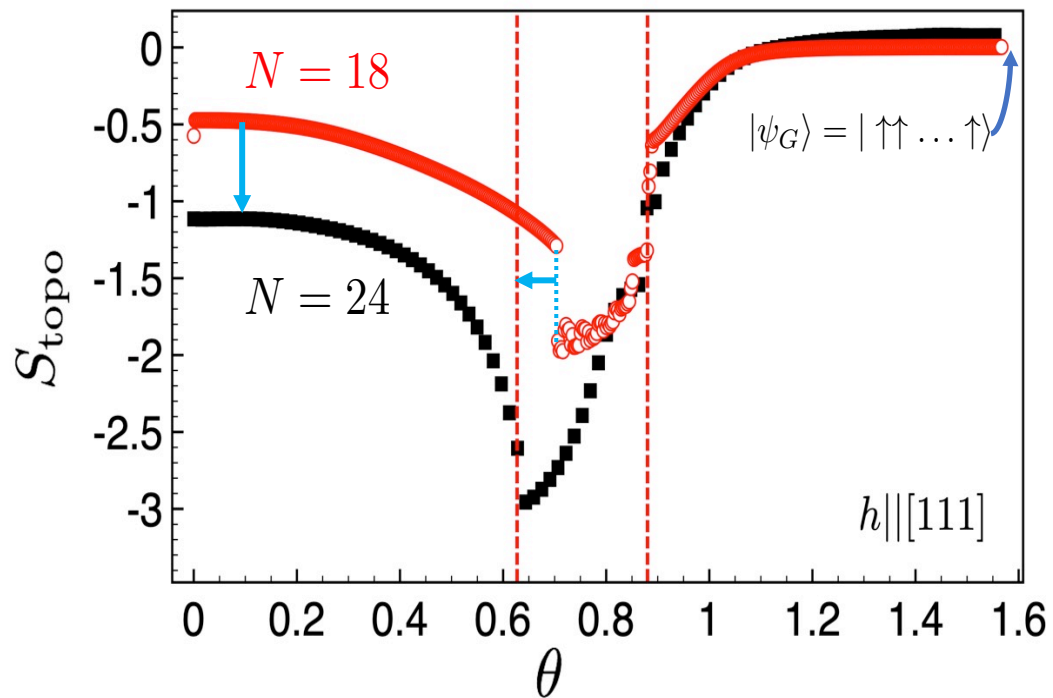
$$S_{\text{topo}} = S_A + S_B + S_C - S_{AB} - S_{BC} - S_{CA} + S_{ABC}$$

Energy Spectra

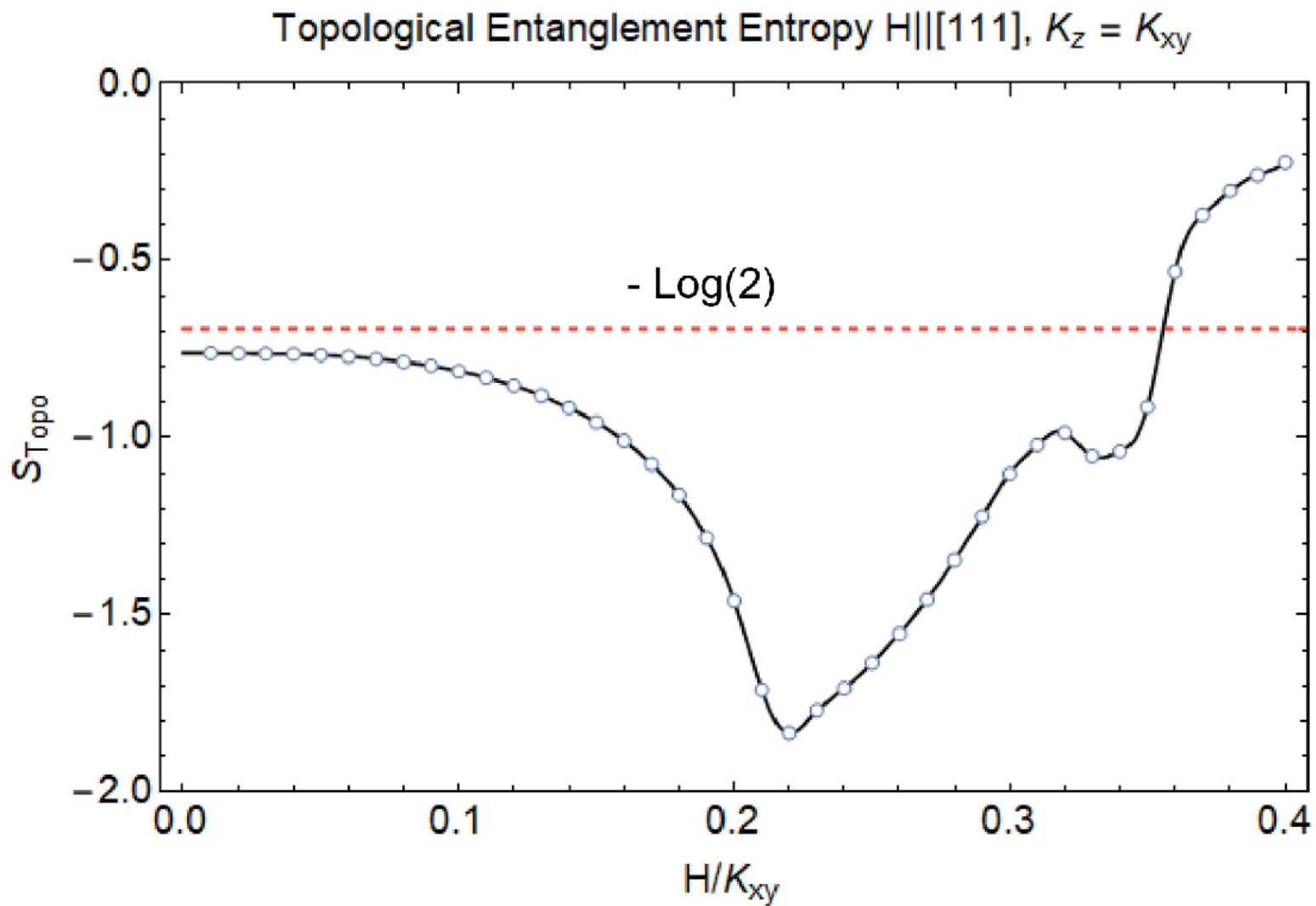


$$\theta \sim |\vec{h}|/K$$

TEE



Ronquillo, Vengal, Trivedi,
PRB 99, 140413(R) (2019)

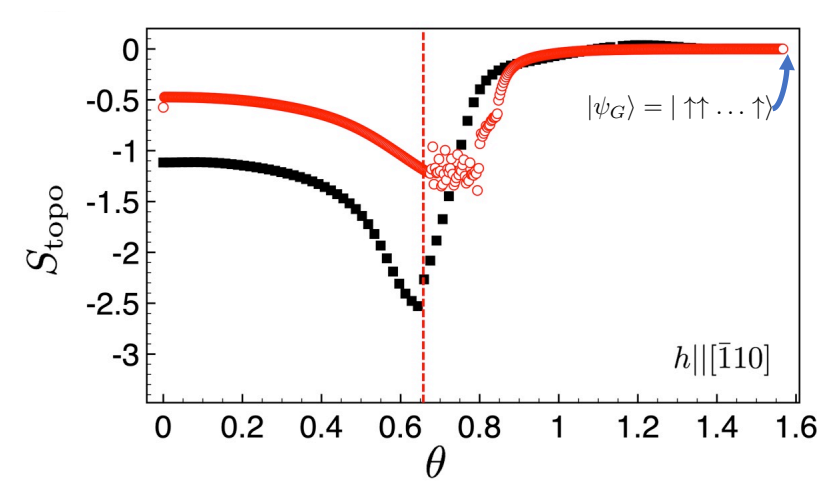
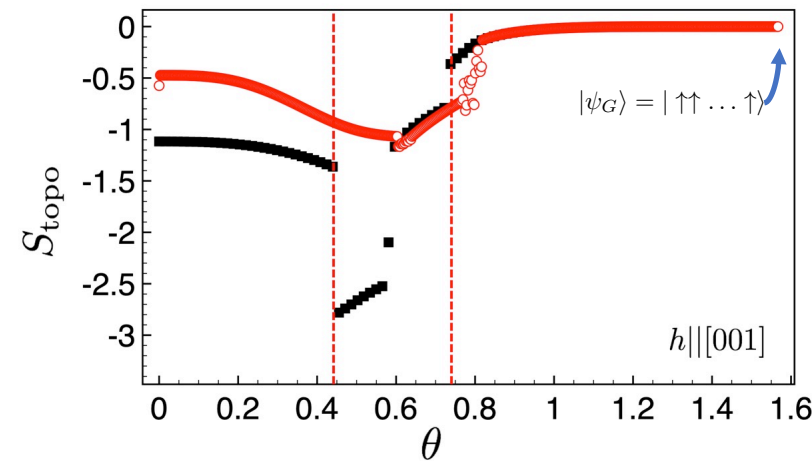
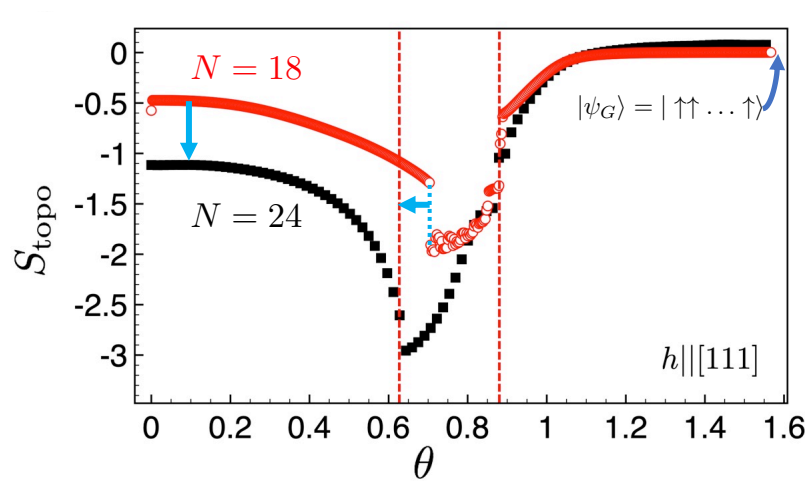
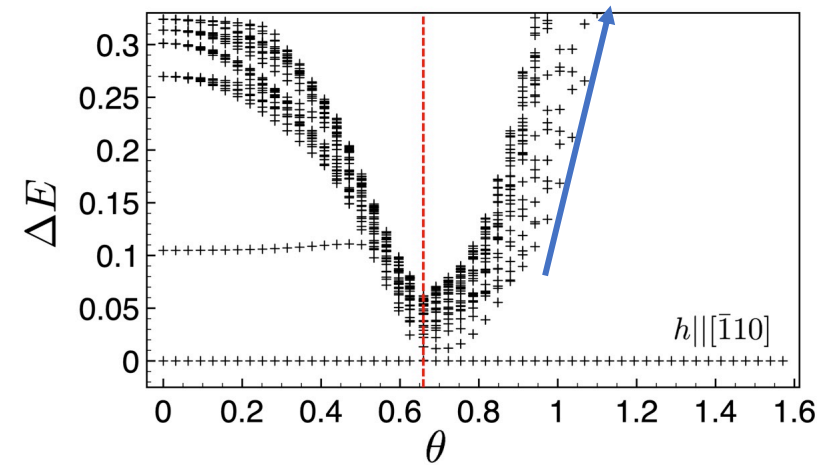
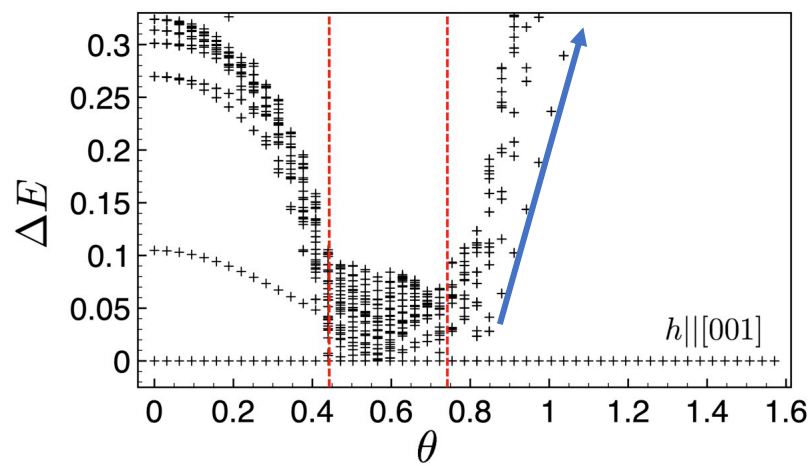
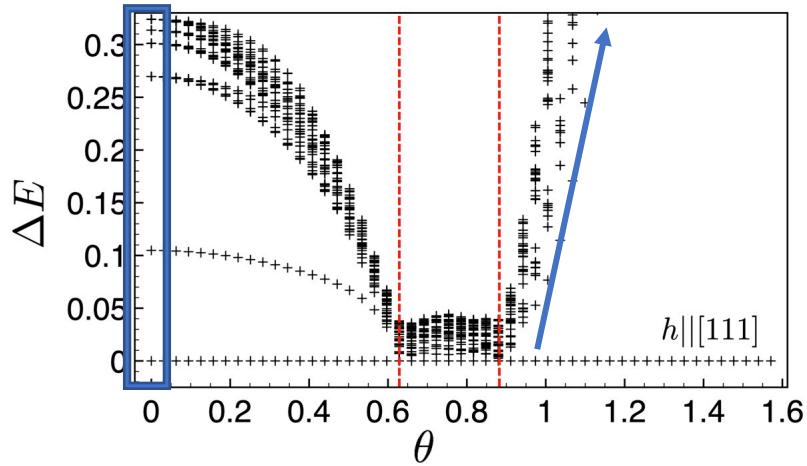
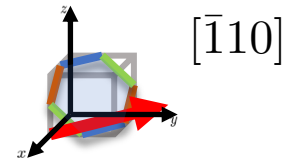
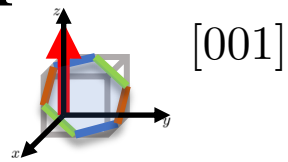
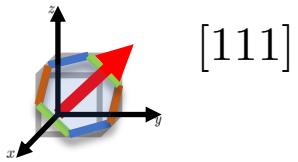


Topological entanglement entropy: Information resource



Ian Osbourne

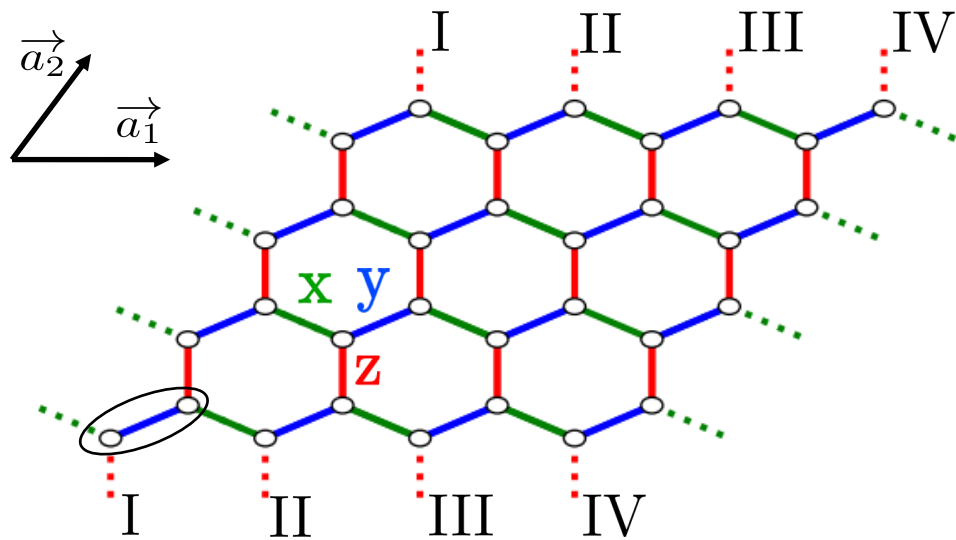
Energy Spectra & TEE



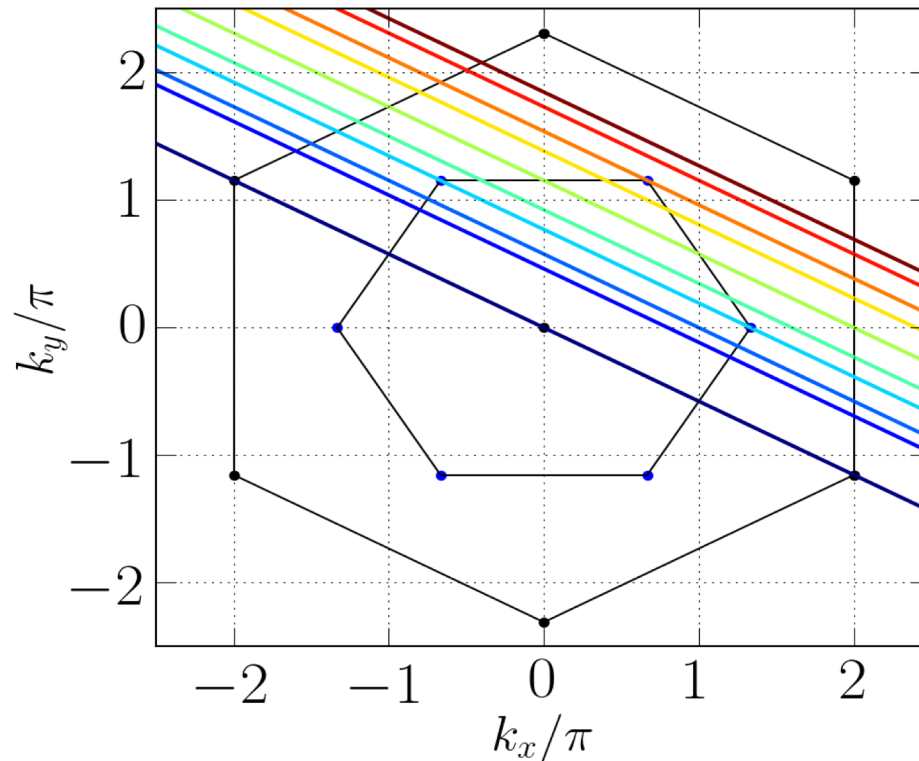
$$\theta \sim |\vec{h}|/K$$

Evidence for spinon Fermi surface

Kitaev Model: spin structure factor



Brillouin Zone: Momenta cuts

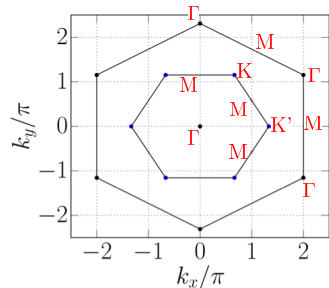
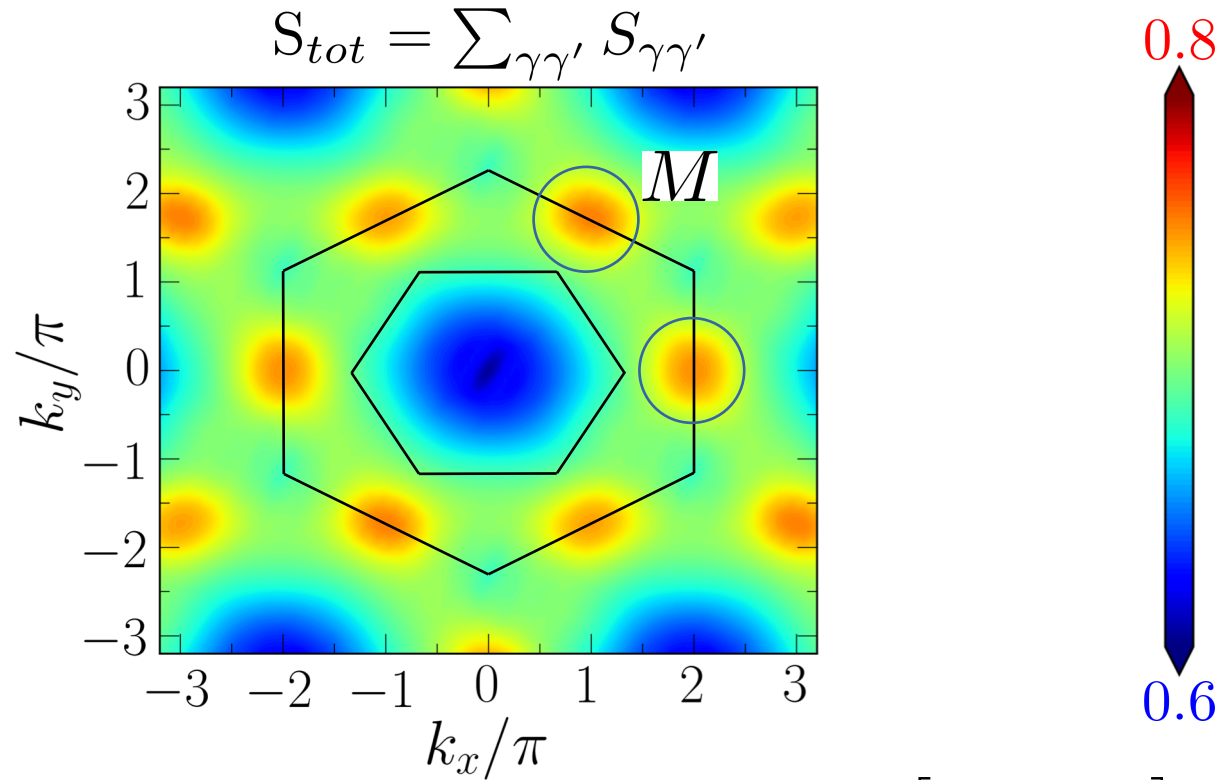


$$H_K = K \sum_{\langle ij \rangle} S_i^x S_j^x + S_i^y S_j^y + S_i^z S_j^z$$

DMRG++ Open Source:

<https://web.ornl.gov/~gz1/dmrgPlusPlus/>

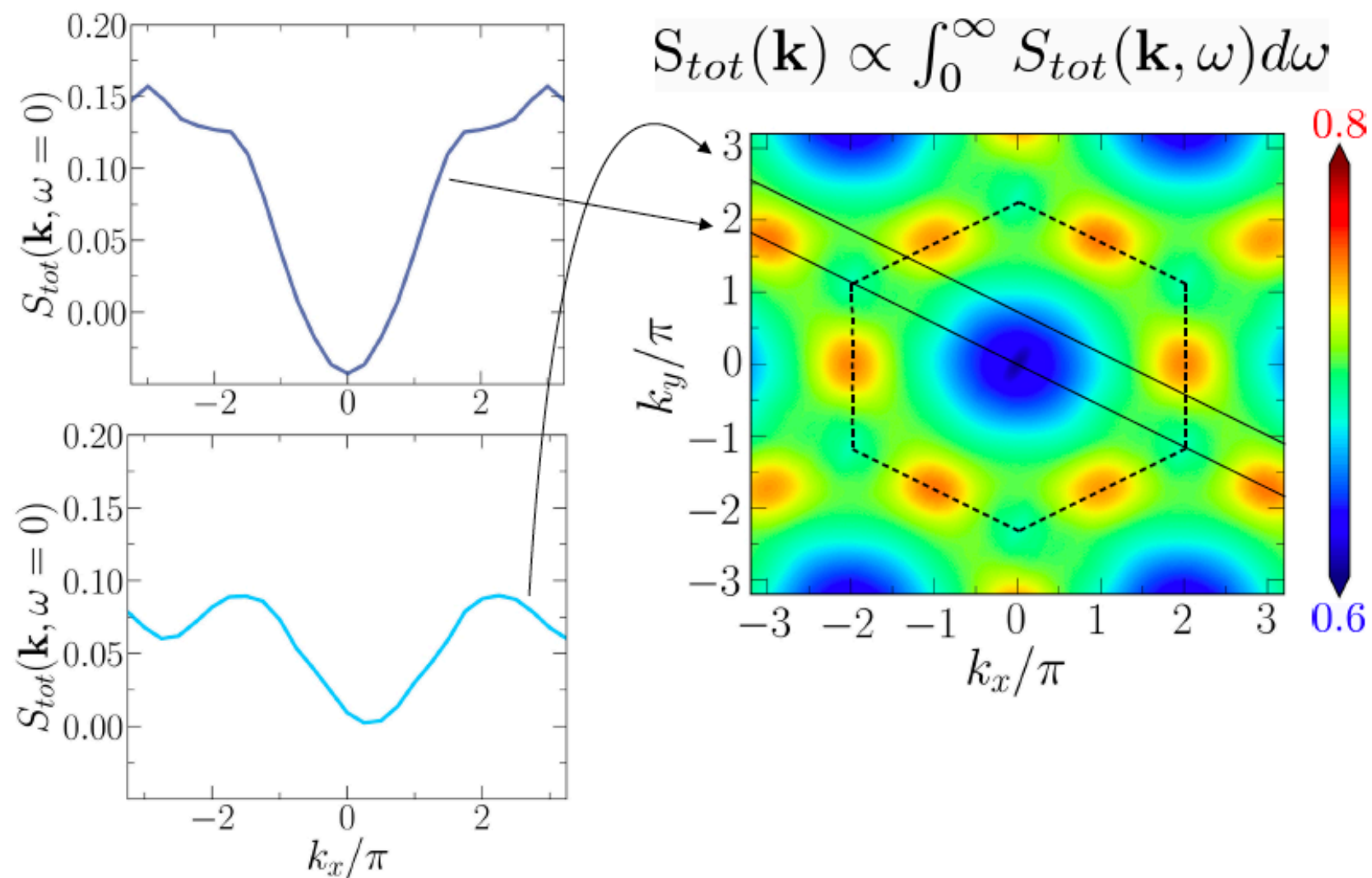
Structure Factor $S(\mathbf{k})$ – Intermediate phase



$$S(\mathbf{k}) = \begin{bmatrix} S_{AA} & S_{AB} \\ S_{BA} & S_{BB} \end{bmatrix}$$

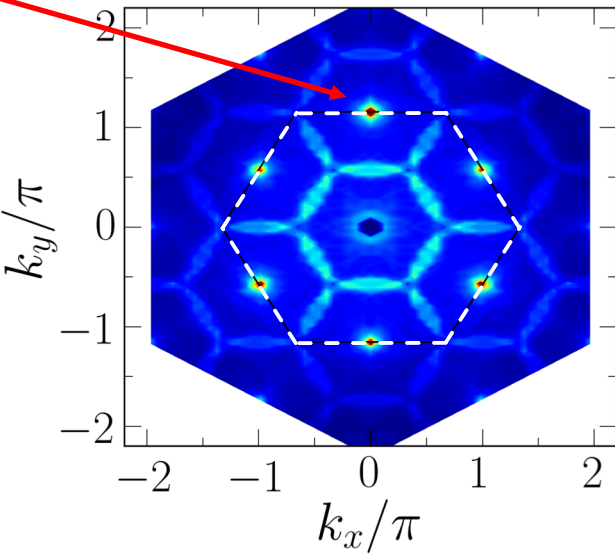
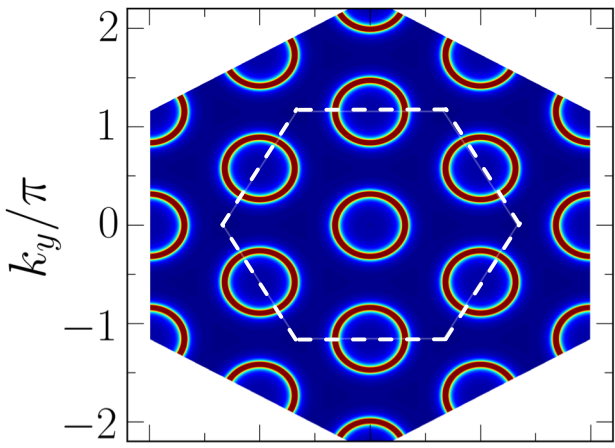
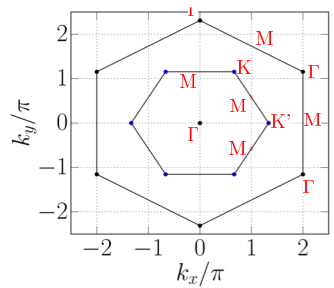
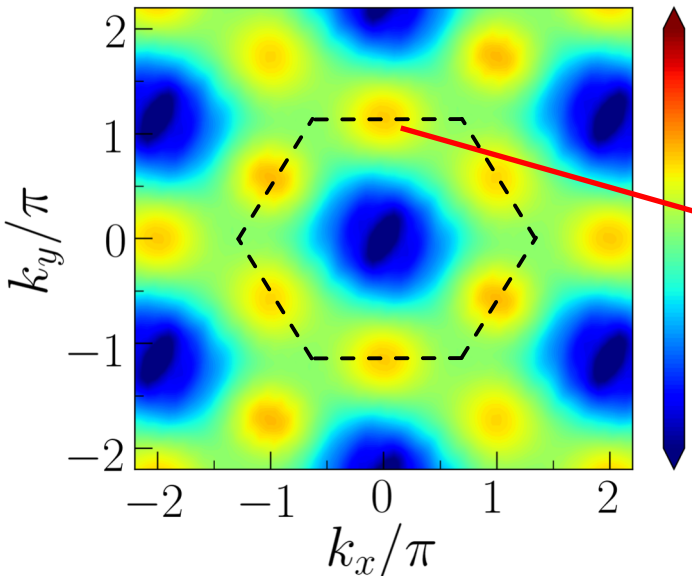
$$S_{\gamma\gamma'}(\mathbf{k}) = \frac{1}{L^2} \sum_{i \in \gamma, j \in \gamma'} e^{-i\mathbf{k} \cdot \mathbf{r}_{ij}} [\langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle - \langle \mathbf{S}_i \rangle \cdot \langle \mathbf{S}_j \rangle]$$

Assumption: $S_{tot}(\mathbf{k}) \simeq S_{tot}(\mathbf{k}, \omega = 0)$ ✓



$S(\mathbf{k}) \rightarrow$ spinon Fermi surface

DMRG Results
 $S_{tr} = \sum_{\gamma} S_{\gamma\gamma}$

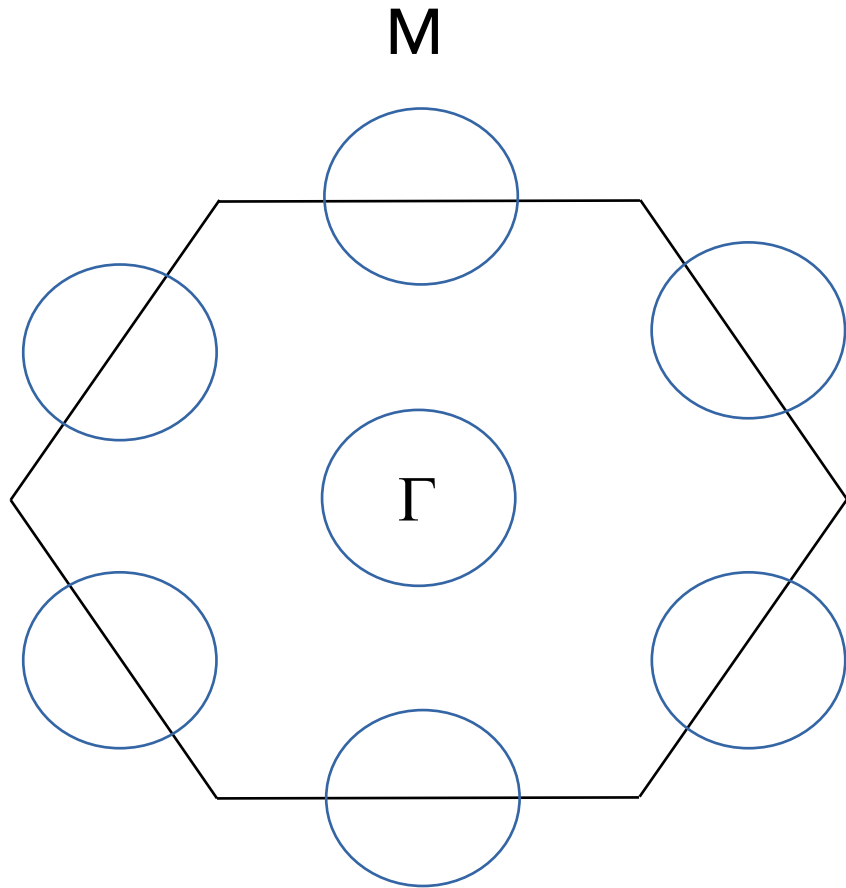


$A(\mathbf{k}, \omega = 0)$
 Conjectured
 Fermi
 surface

$\omega = 0$

$$\mathfrak{S}(\mathbf{k}) = \sum_{\mathbf{q}} A(\mathbf{k} + \mathbf{q})A(\mathbf{q})$$

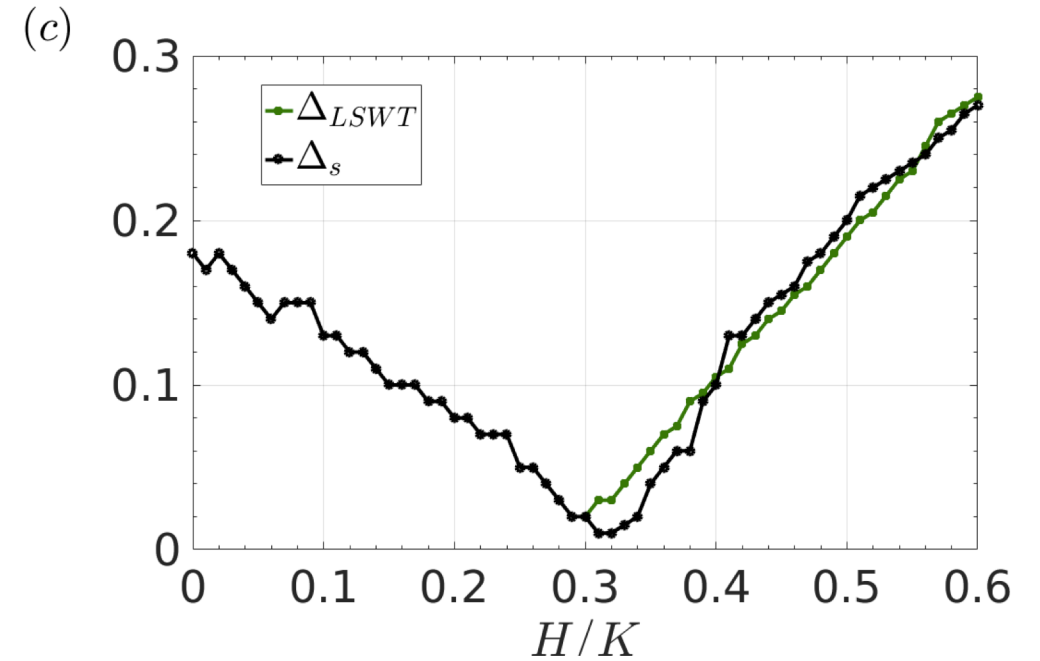
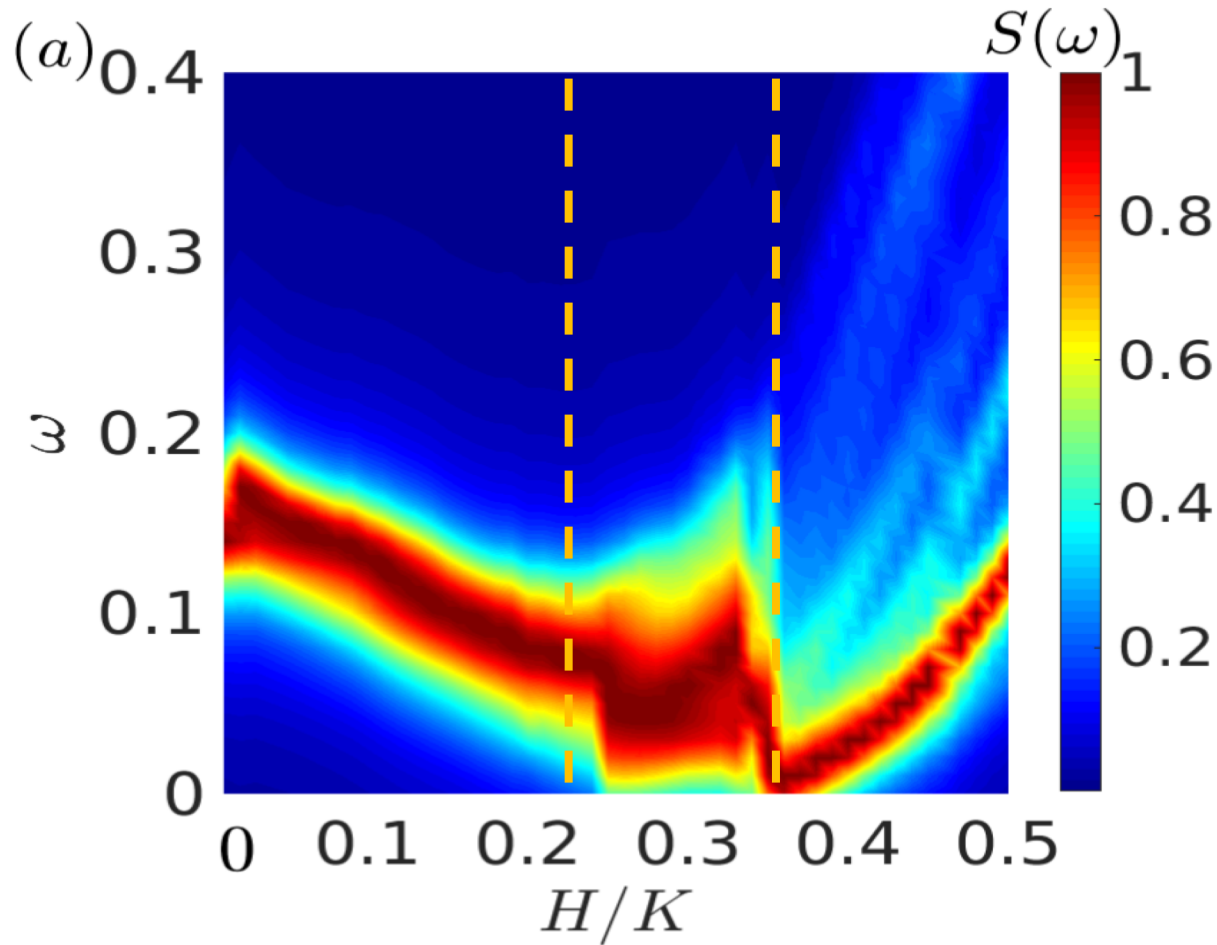
“Fermi Surface” of spinons in a Mott insulator!



Test using VMC
on projected
wave function

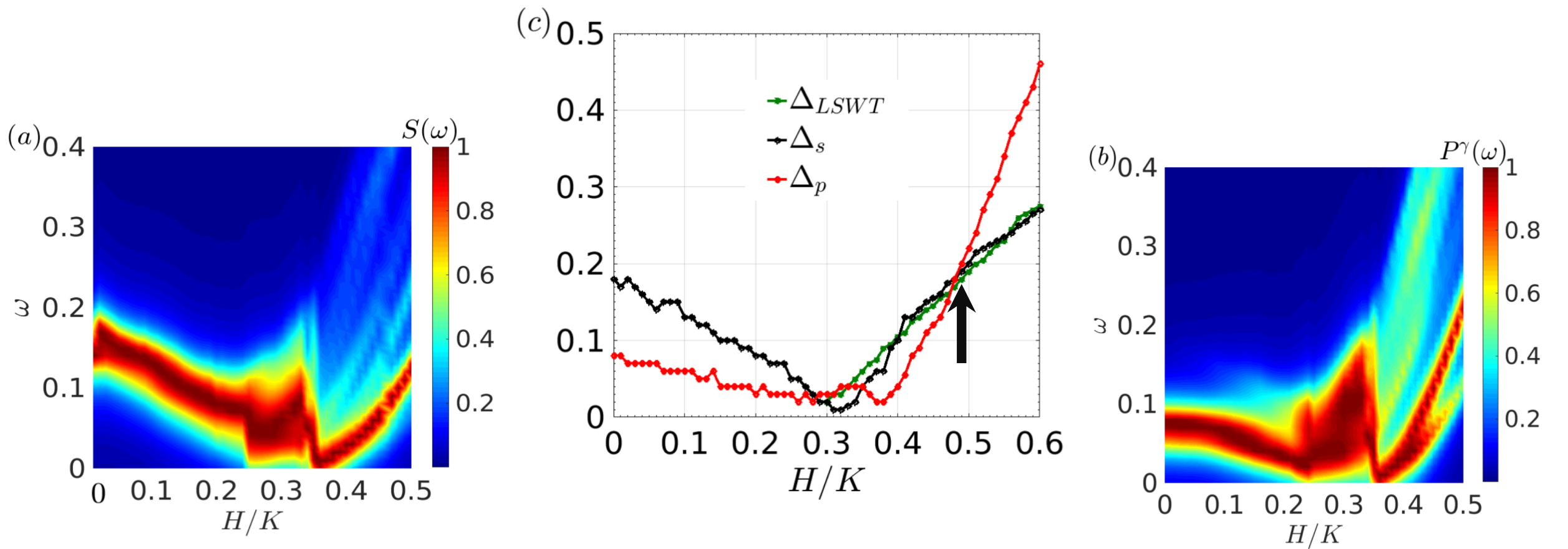
Singularities at all the M points related
by C3 Rotations and Translations

1-spin flip spectral functions



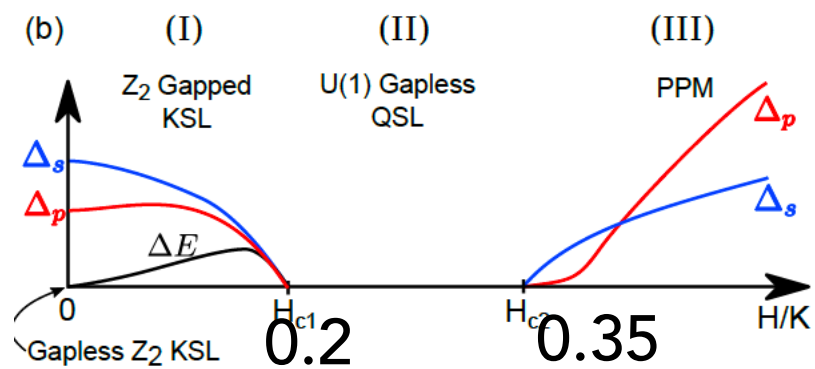
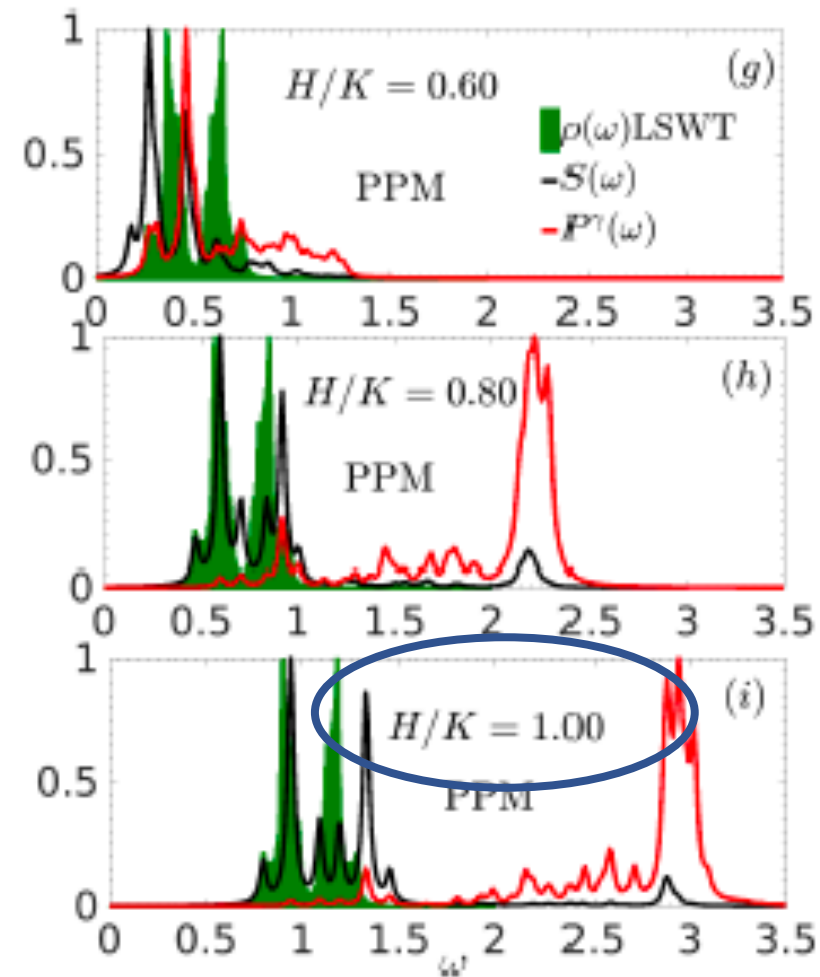
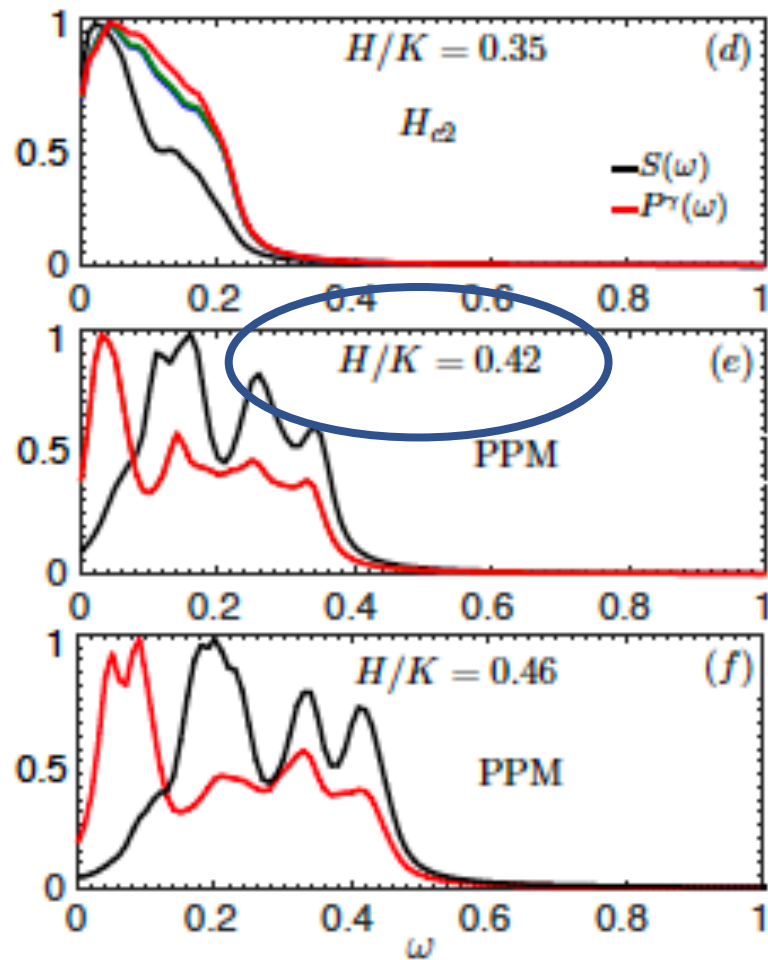
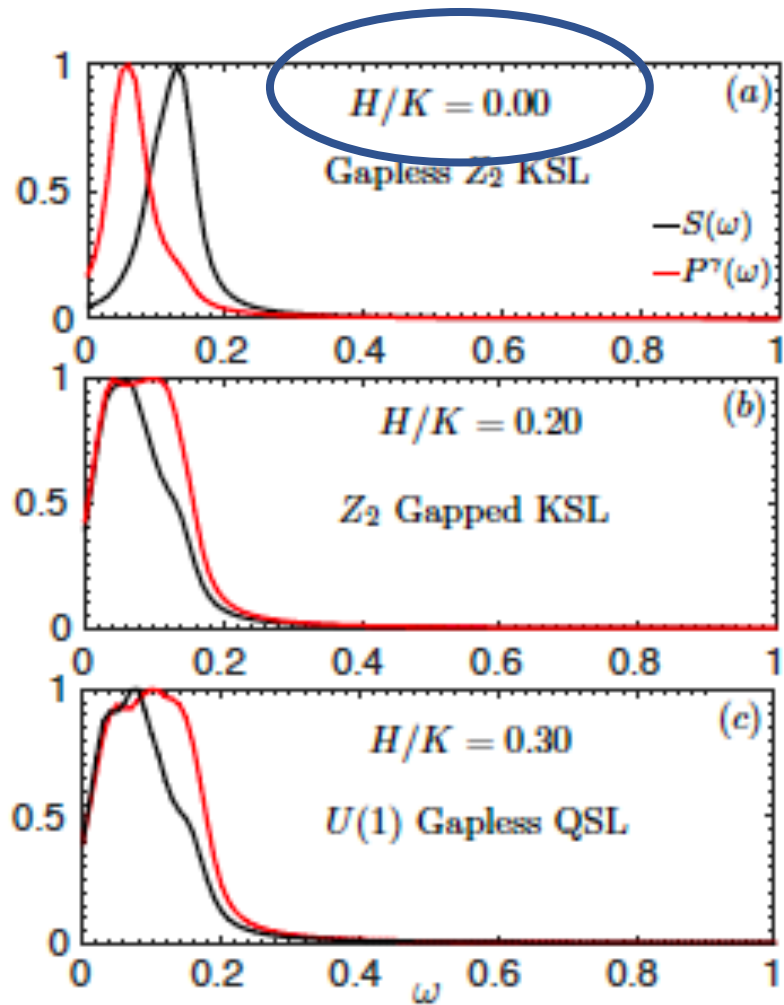
$$S(\omega) = \frac{-1}{N\pi} \text{Im} \left[\sum_{\substack{m \neq 0, i \\ \alpha = +, -, z}} \frac{|\langle 0 | S_i^\alpha | m \rangle|^2}{\omega + E_0 - E_m + i\eta} \right]$$

1-spin flip and 2 spin-flip spectral functions



$$S(\omega) = \frac{-1}{N\pi} \text{Im} \left[\sum_{\substack{m \neq 0, i \\ \alpha = +, -, z}} \frac{|\langle 0 | S_i^\alpha | m \rangle|^2}{\omega + E_0 - E_m + i\eta} \right]$$

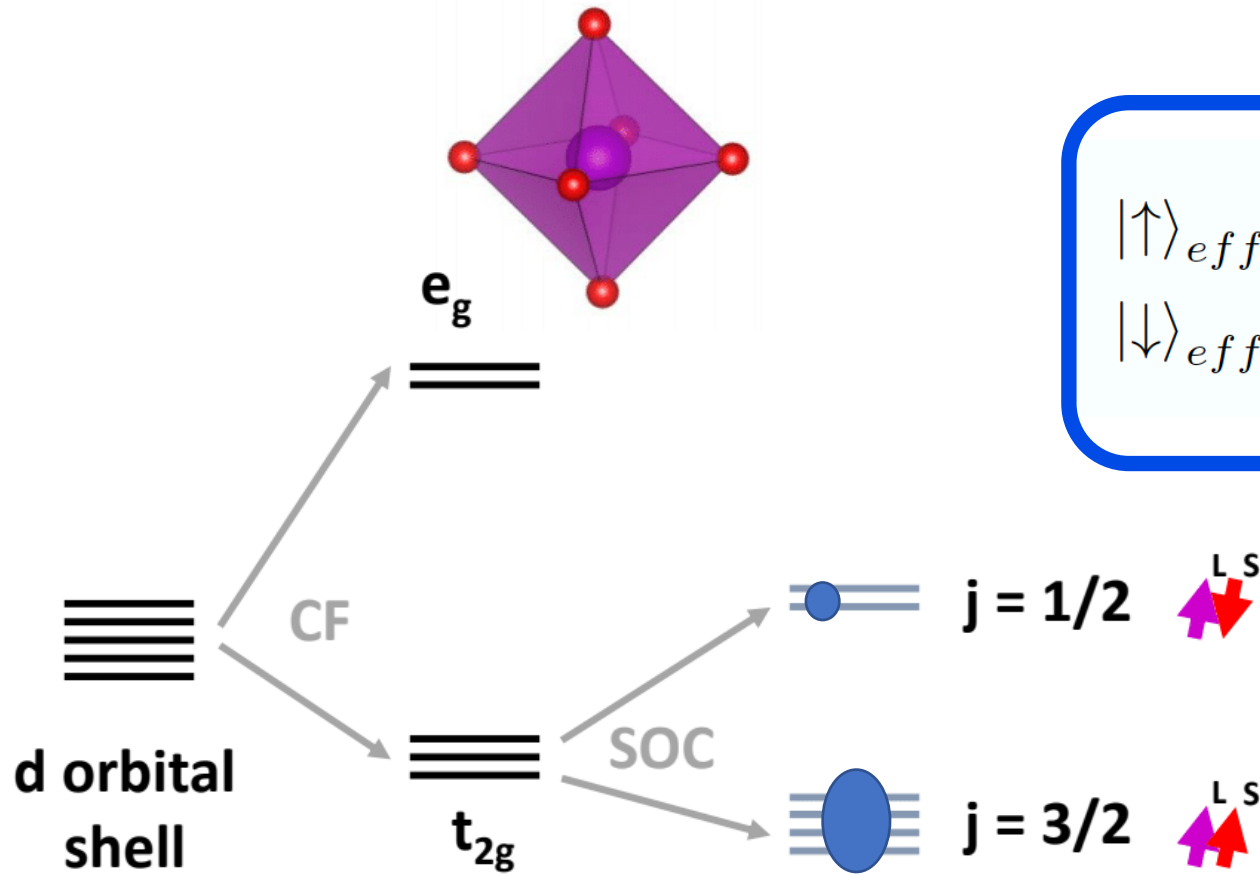
$$P^\gamma(\omega) = \frac{-1}{N\pi} \text{Im} \left[\sum_{\substack{m \neq 0, i \\ \alpha = +, -, z}} \frac{|\langle 0 | S_i^\alpha S_{i+\gamma}^\alpha | m \rangle|^2}{\omega + E_0 - E_m + i\eta} \right]$$



Main ingredients for Kitaev materials

Spin-Orbit coupled Mott Insulators

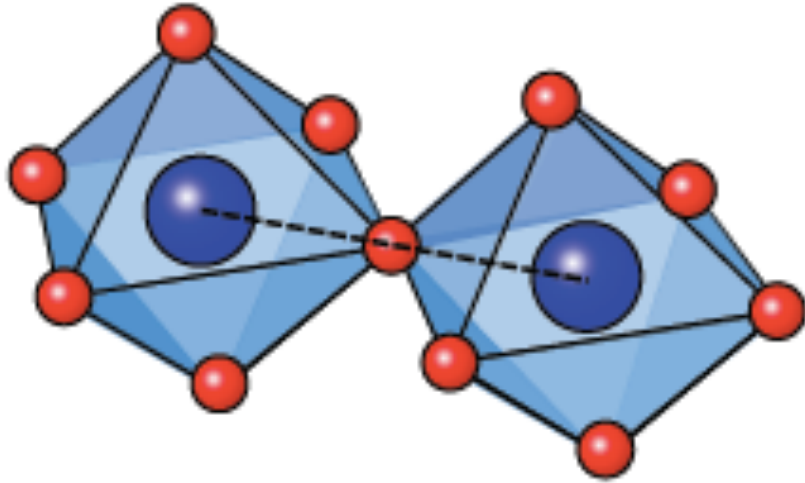
| | | | |
|----|----|----|----|
| Mo | Tc | Ru | Rh |
| W | Re | Os | Ir |



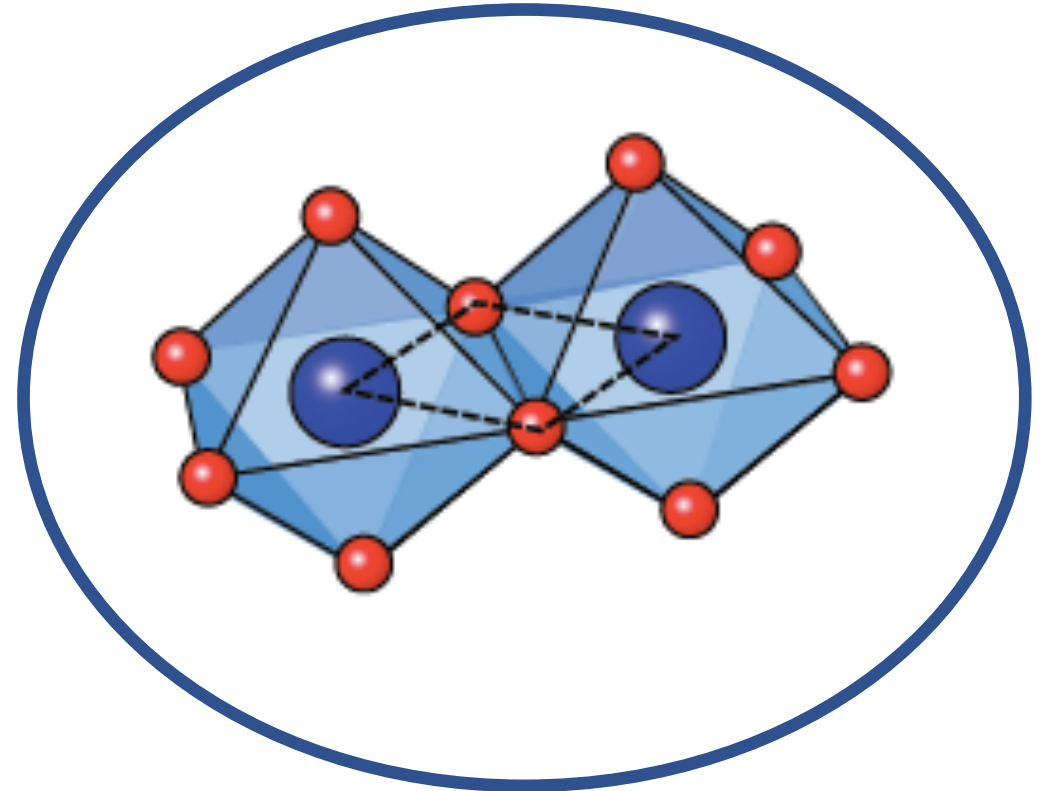
$$|\uparrow\rangle_{eff} \sim i |zx, \downarrow\rangle + |yz, \downarrow\rangle + |xy, \uparrow\rangle$$

$$|\downarrow\rangle_{eff} \sim -i |zx, \uparrow\rangle + |yz, \uparrow\rangle - |xy, \downarrow\rangle$$

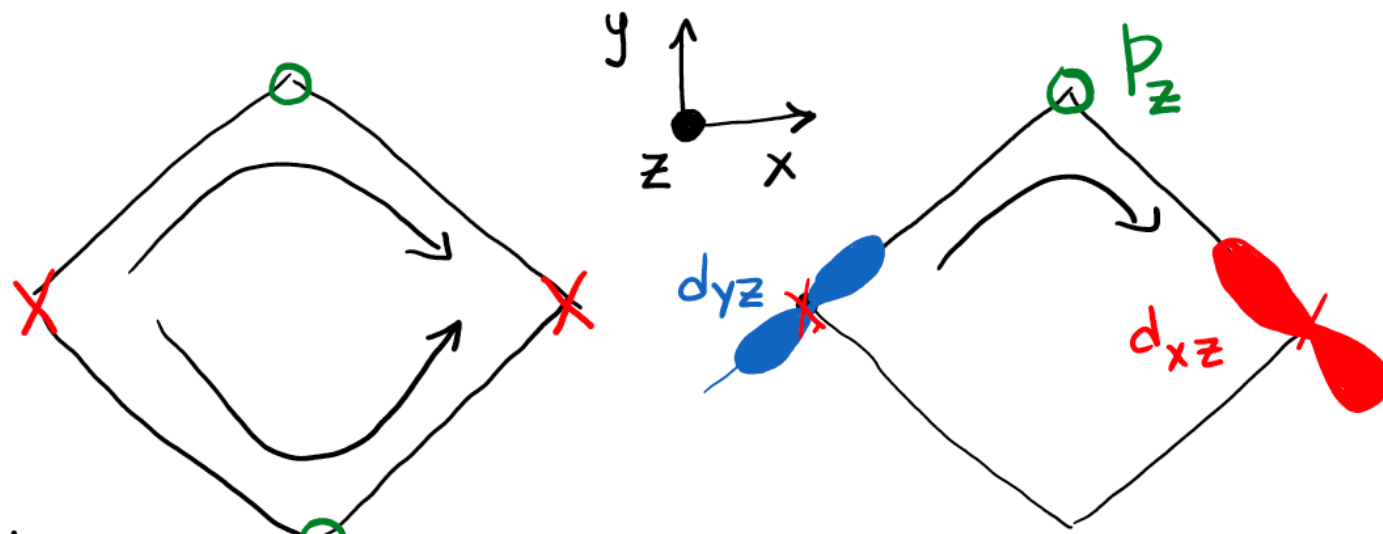
I: corner-sharing



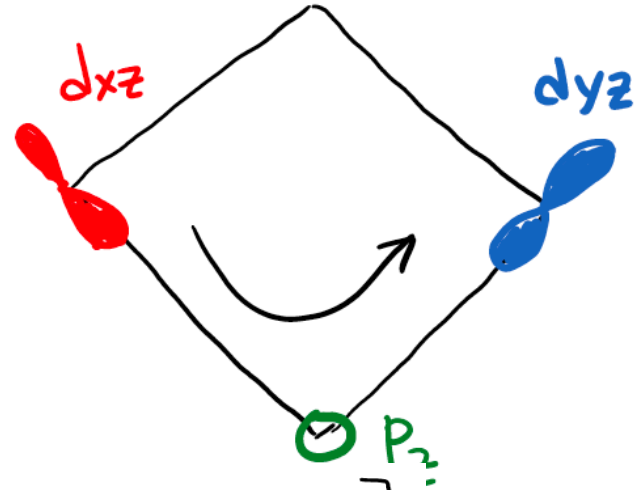
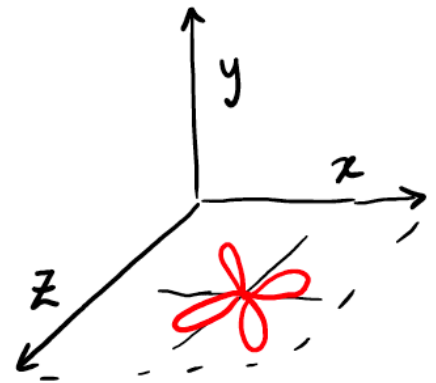
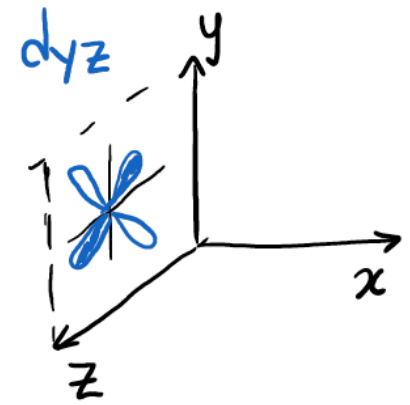
II: edge-sharing



Destructive interference between the two pathways generates bond-dependent interactions



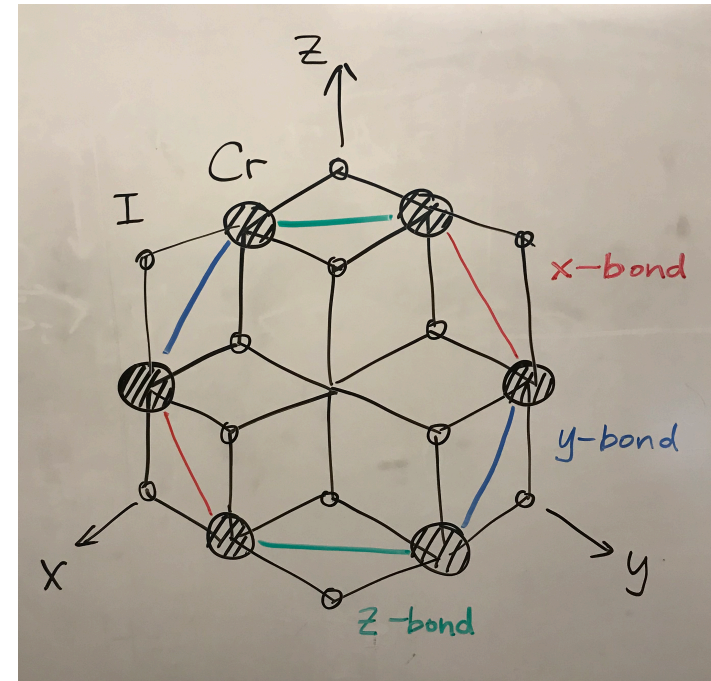
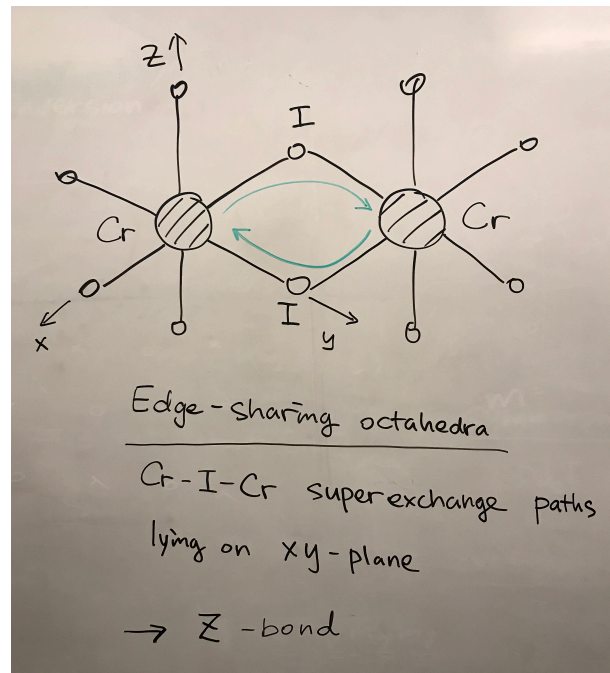
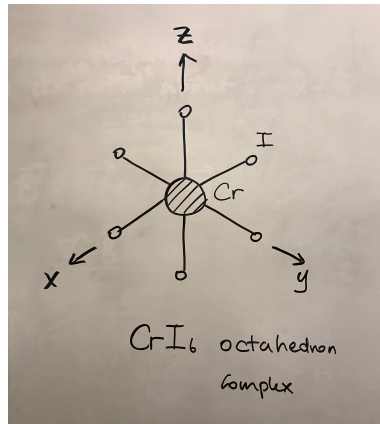
hopping via p_z
orbital on ligand
changes $d_{yz} \rightarrow d_{xz}$



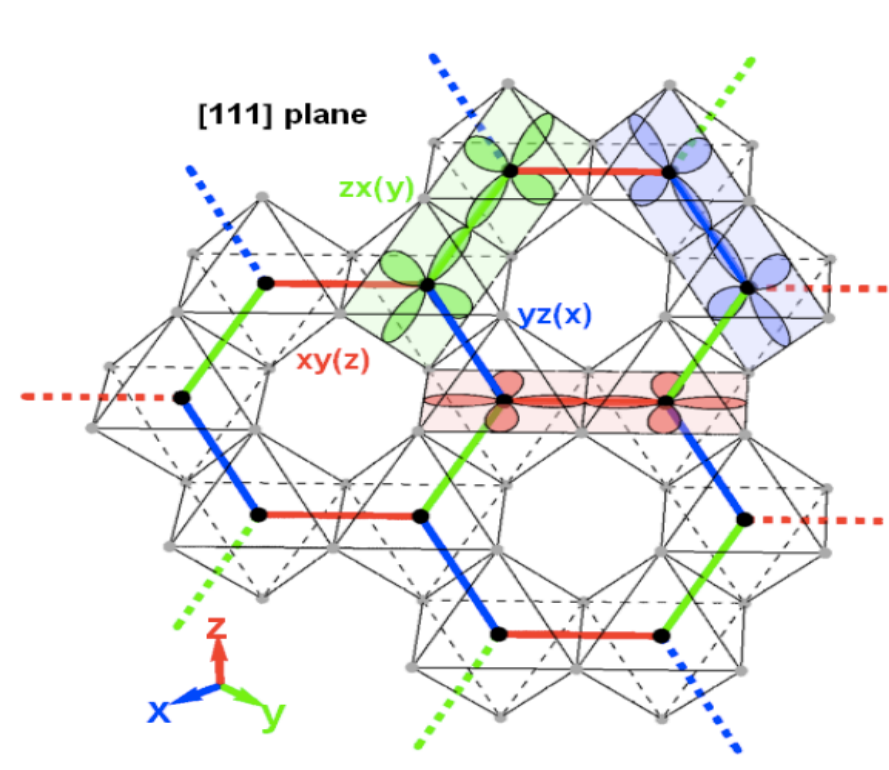
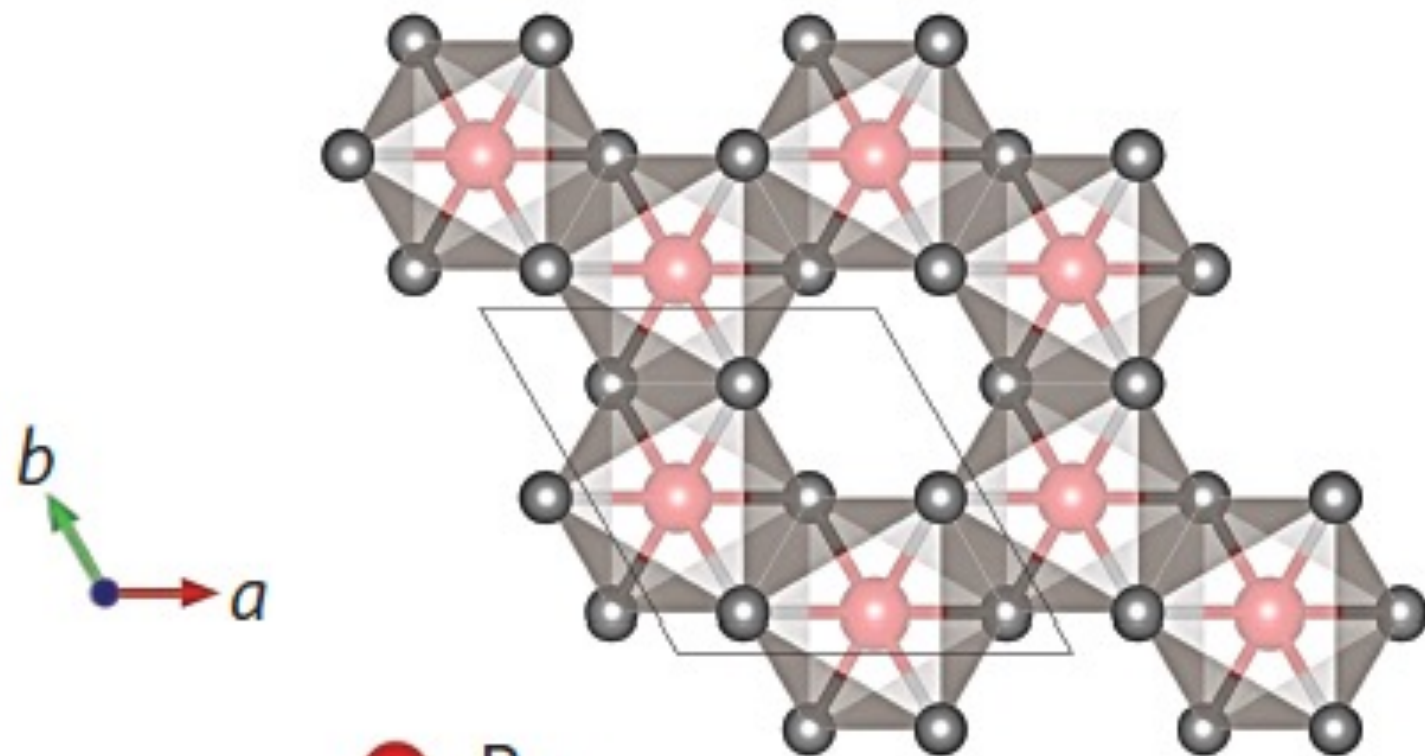
$d_{xz} \rightarrow d_{yz}$

$$|J_z = +\frac{1}{2}\rangle = \frac{1}{\sqrt{3}} \left[|d_{xy}\uparrow\rangle + |d_{yz}\downarrow\rangle + i |d_{xz}\downarrow\rangle \right]$$

Crystal structure of α - RuCl_3 → candidate Kitaev material

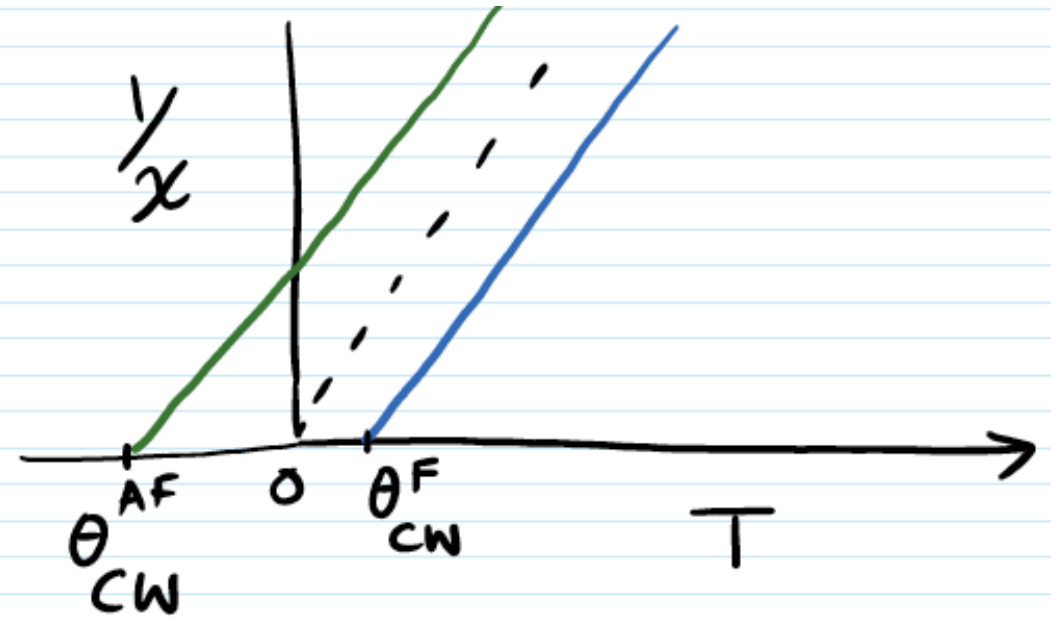
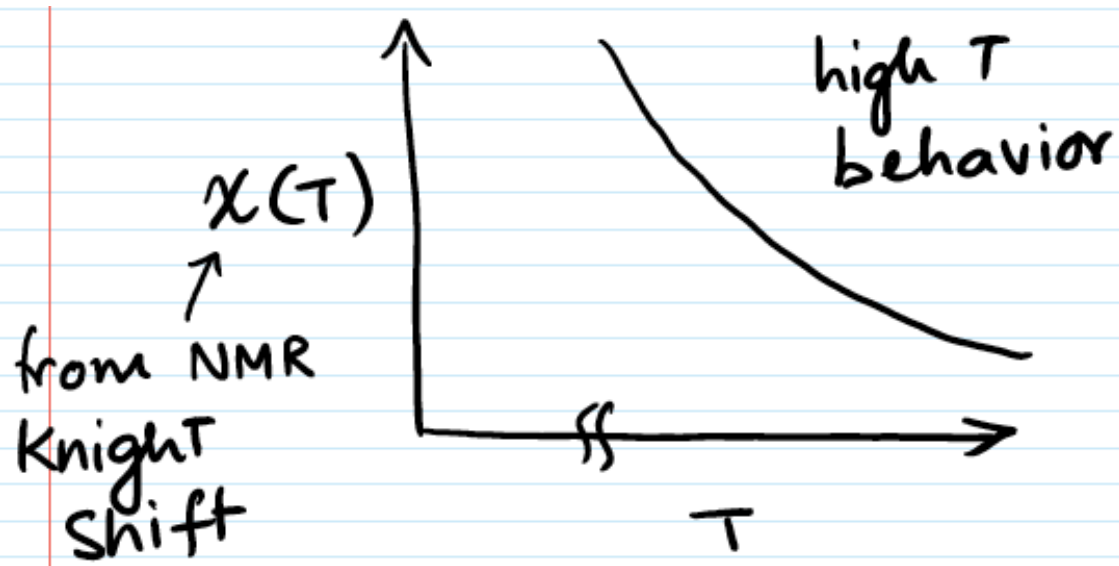


α -RuCl₃



How do you detect a QSL?

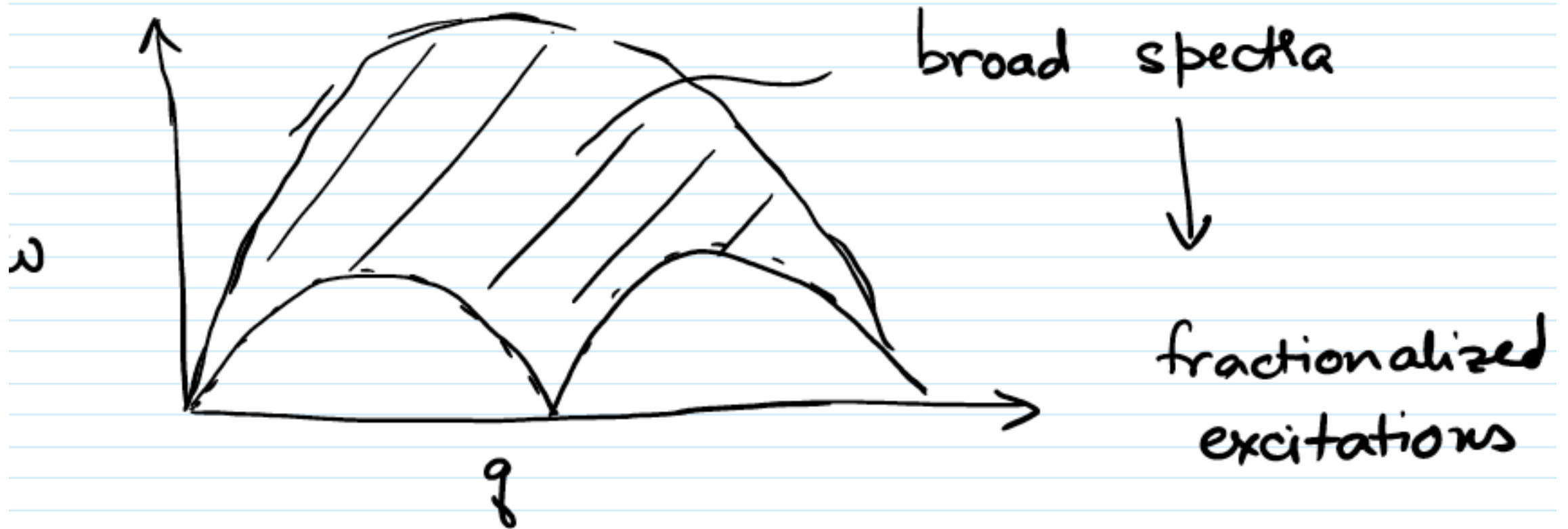
Signatures of a QSL: large frustration parameter



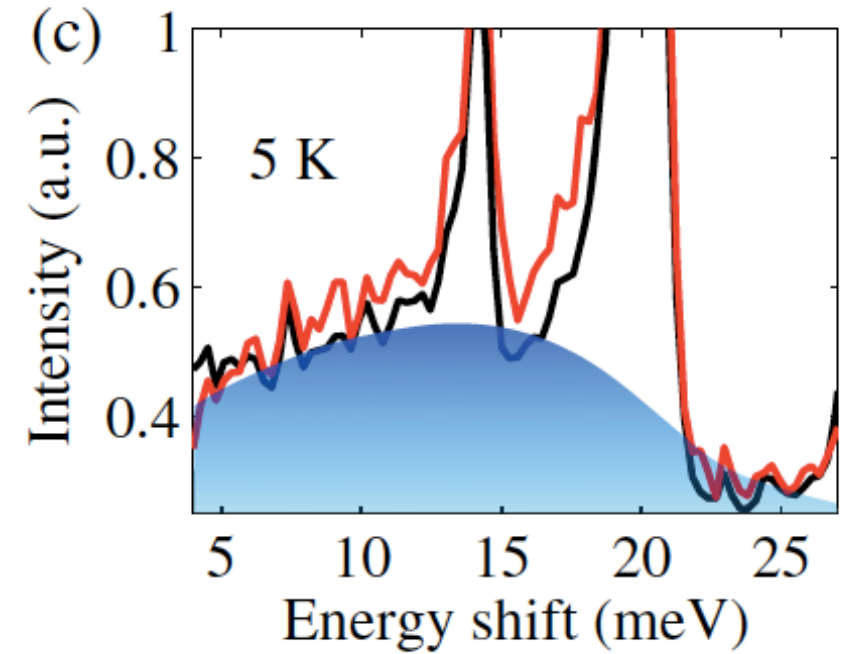
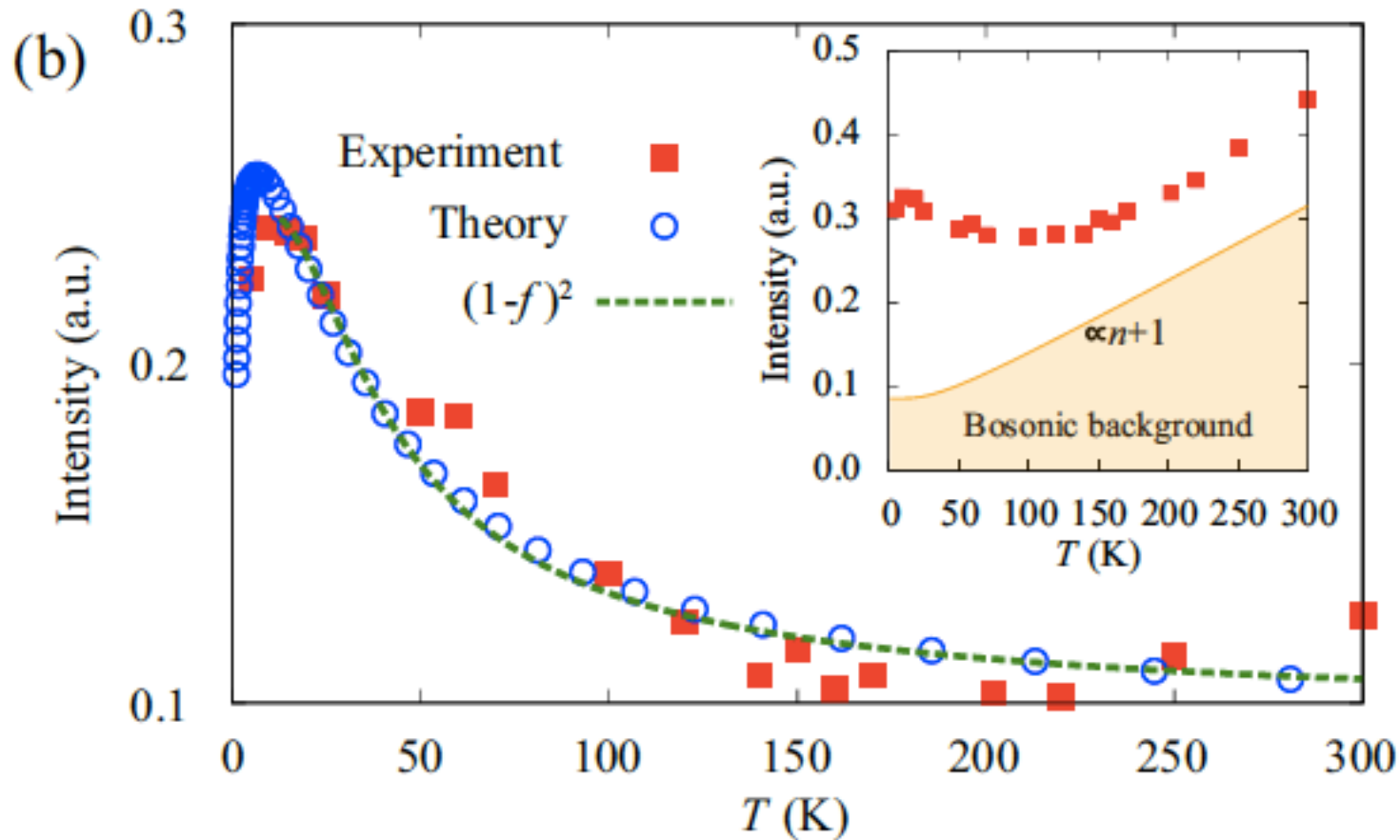
$$\chi(T) = \frac{C}{T - \theta_{CW}}$$

$$f = \frac{|\theta_{CW}|}{T_c}$$

Signatures of a QSL: broad spectrum of spin excitations



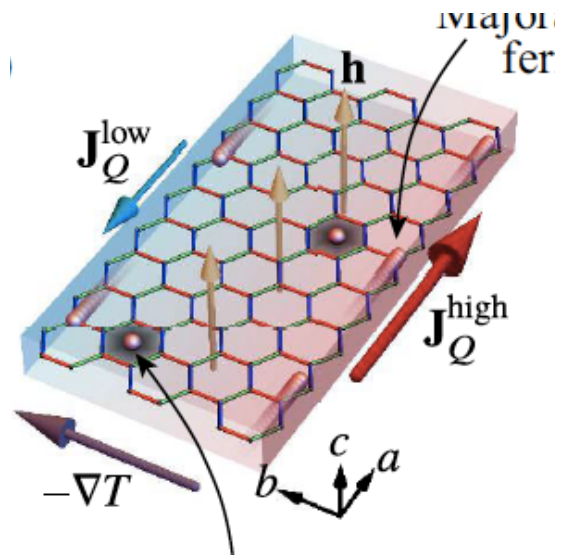
Signatures of a QSL: fermionic nature of spin excitations in Raman



$$\mathcal{R} = \sum_{\langle ij \rangle_{\mu}} (\boldsymbol{\epsilon}_{\text{in}} \cdot \mathbf{d}^{\mu})(\boldsymbol{\epsilon}_{\text{out}} \cdot \mathbf{d}^{\mu}) J_{\mu} S_i^{\mu} S_j^{\mu}$$

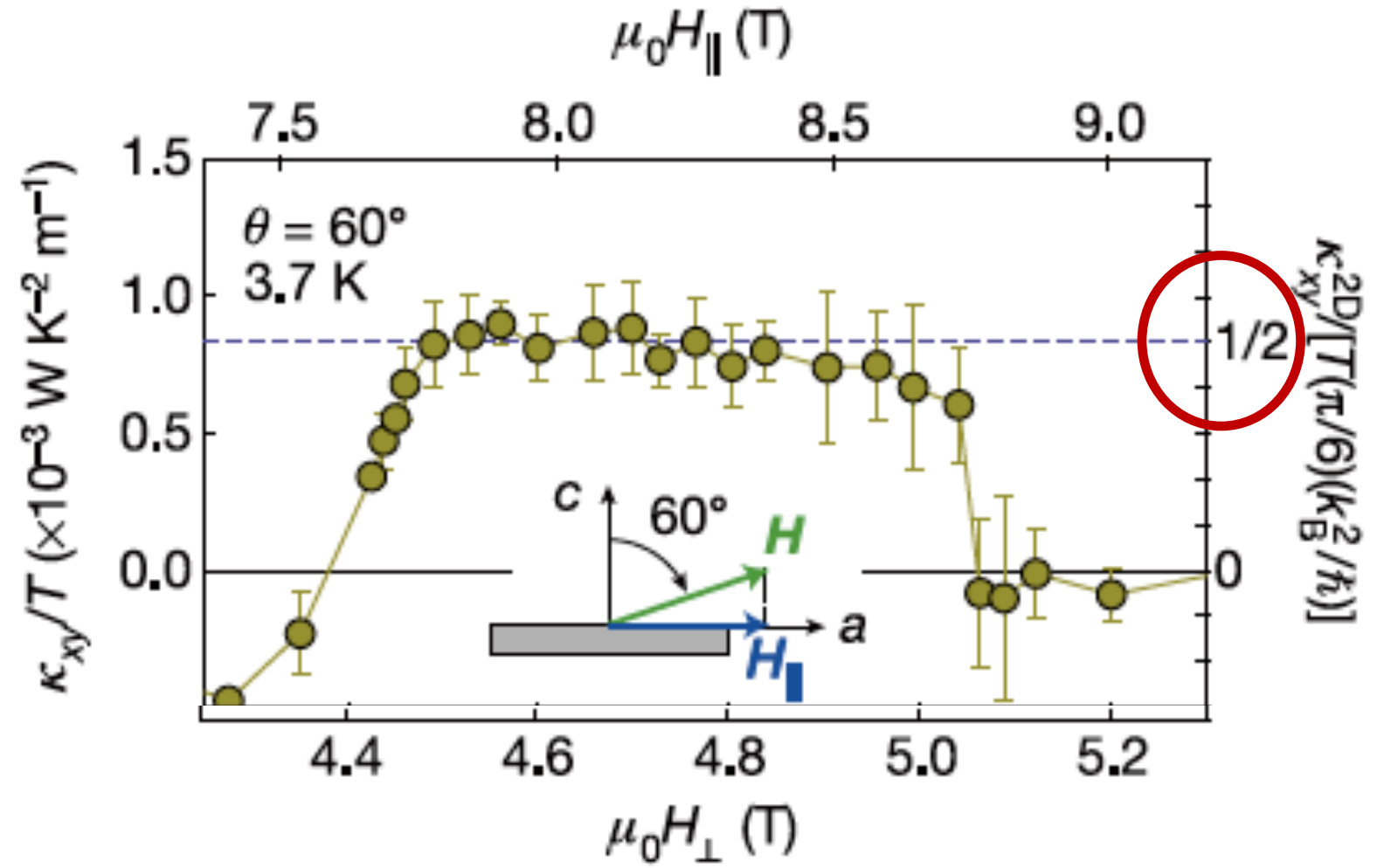
$$I(\omega) = \frac{1}{N} \int_{-\infty}^{\infty} dt e^{i\omega t} \langle \mathcal{R}(t) \mathcal{R} \rangle$$

Signatures of a QSL: *quantized* thermal Hall conductance



$$\kappa_{xy}^{2D} = \left[\frac{1}{2} \right] \frac{\pi}{6} \frac{k_B^2}{\hbar} T$$

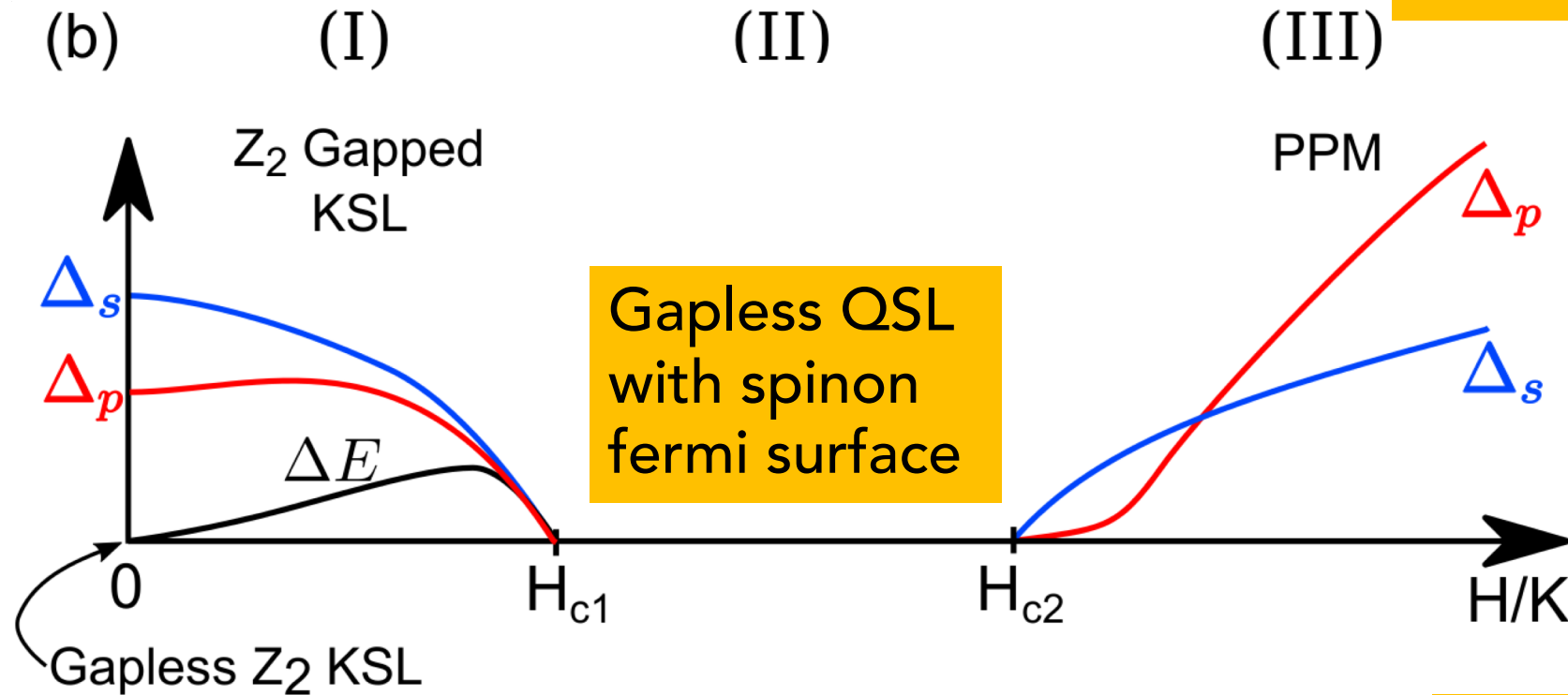
$$\kappa_{xy}^{2D} = \kappa_{xy} d$$



Y. Kasahara, T. Ohnishi, Y. Mizukami, O. Tanaka, Sixiao Ma, K. Sugii, N. Kurita, H. Tanaka, J. Nasu, Y. Motome, T. Shibauchi & Y. Matsuda, Nature 559, 227 (2018).

Our Main Results: Kitaev Model in a Magnetic Field

K: AF exchange



Δ_s : single spin flip energy
 Δ_p : 2-spin flip energy

PNAS 201821406 (2019)
 Phys. Rev. B 99, 140413(R) (2019)
[arXiv:1908.10877](https://arxiv.org/abs/1908.10877)

K: F exchange
 No gapless QSL

Theory

Spinon fermi surface

Materials

F and AF Kitaev materials

Phenomena

Spin fractionalization in
pump-probe dynamics