Radboud Universiteit Nijmegen

IN·DEI·NOMINE·FELICITER

Anisotropy from spin-orbit interactions

new insights to chiral magnetism and magnetization dynamics

Mikhail Titov

KITP, UCSB
14 November 2019

Overview

DMI and its generalization to multispin interaction
for linear gradient classification of micromagnetic functionals

Chiral ferromagnetism beyond Lifshitz invariants

I. A. Ado, A. Qaiumzadeh, A. Brataas, and M. Titov, arXiv 1904.05337 (2019)

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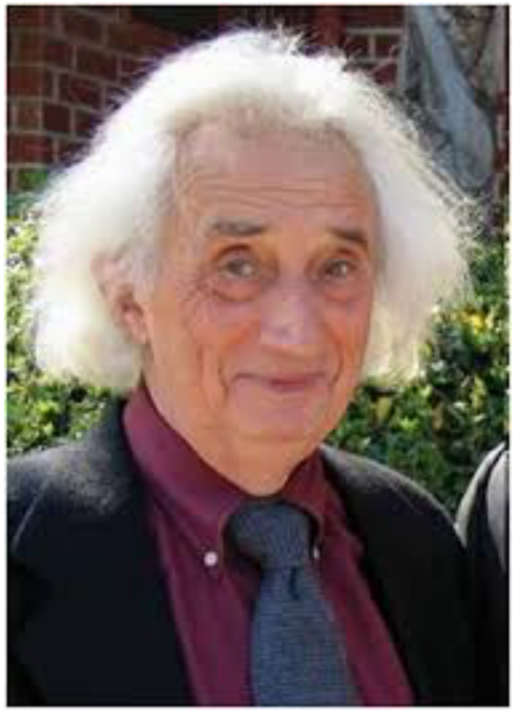
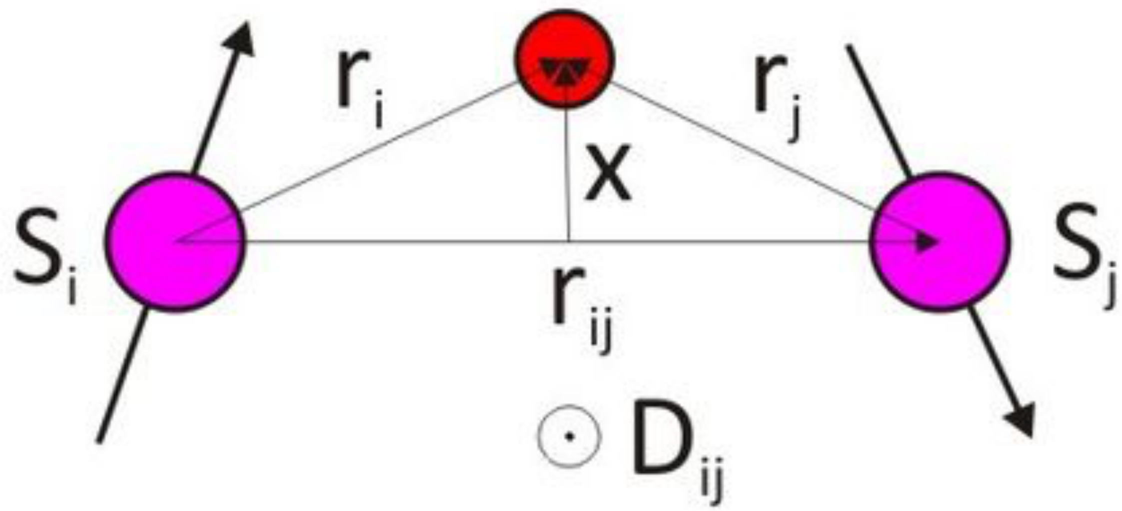
Giant anisotropy of Gilbert damping
in 2D Rashba antiferromagnet

Giant anisotropy of Gilbert damping in honeycomb Rashba antiferromagnet
M. Baglai *et al.*, arXiv:1911.03408 (2019)

Dzyaloshinskii-Moriya interaction



Igor Dzyaloshinskii (1958) and Tôru Moriya (1960)



lack of inversion symmetry

$$H = \sum_{<i,j>} J_{ij}\, \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \sum_{<i,j>} \boldsymbol{D}_{ij} \cdot \boldsymbol{S}_i \times \boldsymbol{S}_j$$

symmetric exchange

asymmetric exchange

A THERMODYNAMIC THEORY OF "WEAK" FERROMAGNETISM OF ANTIFERROMAGNETICS

I. DZYALOSHINSKY

Institute for Physical Problems, Academy of Sciences of the U.S.S.R., Moscow

(*Received* 19 *February* 1957)

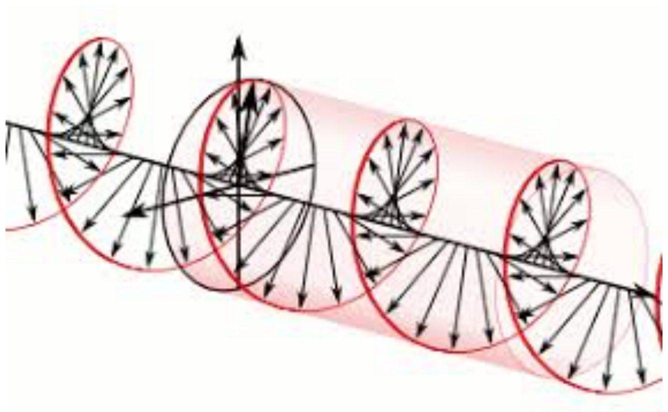
Anisotropic Superexchange Interaction and Weak Ferromagnetism

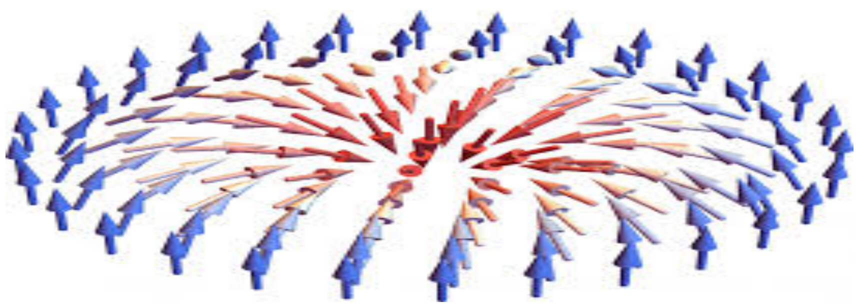
Tôru Moriya*

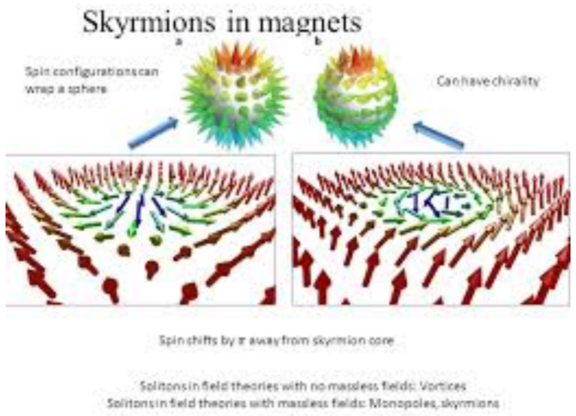
Bell Telephone Laboratories, Murray Hill, New Jersey

(Received May 25, 1960)

Chiral magnetism

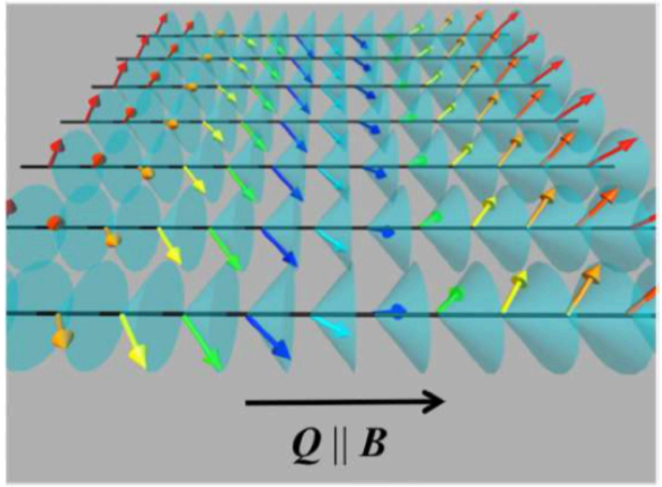






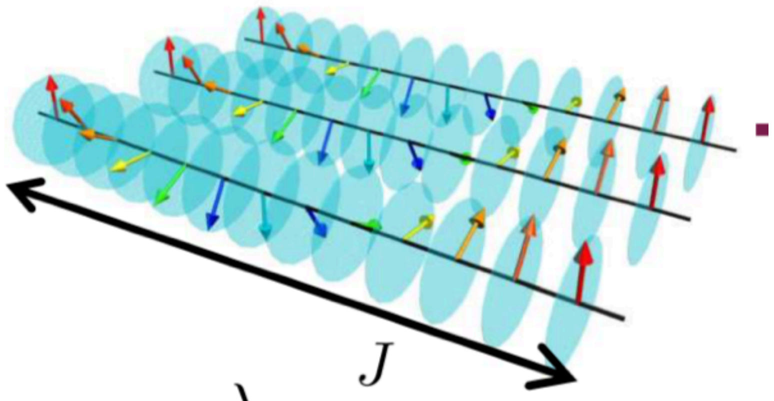
Micromagnetic energy (in a 2D magnet)

$$E = \int d^2\boldsymbol{r}\,\left[J\,(\boldsymbol{\nabla}\cdot\boldsymbol{S})^2 + \underline{D\,\boldsymbol{S}\cdot[(\hat{\boldsymbol{z}}\times\boldsymbol{\nabla})\times\boldsymbol{S}]} - K\,S_z^2\right]$$



$$\boldsymbol{D}_{ij} = D\,(\boldsymbol{d}_{ij}\times\boldsymbol{z})$$

$\boldsymbol{d}_{ij}$ **- lattice vectors**



$$\lambda = \frac{J}{D}$$

$$\sum_{\langle i,j\rangle}\boldsymbol{D}_{ij}\cdot[\boldsymbol{S}_i\times\boldsymbol{S}_j]\;\propto\;\int d^2\boldsymbol{r}\;D\,\boldsymbol{S}\cdot[(\hat{\boldsymbol{z}}\times\boldsymbol{\nabla})\times\boldsymbol{S}]$$

DMI and Lifshitz invariants

Lifshitz invariants:

$$\mathcal{L}_{xy}^{(s)} = n_x \frac{\partial n_y}{\partial s} - n_y \frac{\partial n_x}{\partial s}$$

$$\boldsymbol{n} = \boldsymbol{S}/S$$

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EXAMPLES:

bulk DMI energy density

$$w = D\left[\mathcal{L}^{(x)}_{zy} + \mathcal{L}^{(y)}_{xz} + \mathcal{L}^{(z)}_{yx}\right] = D\,\boldsymbol{n}\cdot(\boldsymbol{\nabla}\times\boldsymbol{n})$$

interfacial DMI energy density (class $C_{\infty v}$)

$$w = D\left[\mathcal{L}^{(x)}_{xz} + \mathcal{L}^{(y)}_{yz}\right] = D\left[(\boldsymbol{n}\cdot\boldsymbol{\nabla})n_z - n_z(\boldsymbol{\nabla}\cdot\boldsymbol{n})\right] = D\,\boldsymbol{n}\cdot(\hat{\boldsymbol{z}}\times\boldsymbol{\nabla})\times\boldsymbol{n}$$

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Thermodynamically stable "vortices" in magnetically ordered crystals. The mixed state of magnets

A. N. Bogdanov and D. A. Yablonskiĭ

Physicotechnical Institute, Donetsk, Academy of Sciences of the Ukrainian SSR

(Submitted 20 April 1988)
Zh. Eksp. Teor. Fiz. **95**, 178–182 (January 1989)

It is shown that in magnetically ordered crystals belonging to the crystallographic classes C_n, C_{nv}, D_n, D_{2d}, and S_4 ($n = 3, 4, 6$), in a certain range of fields, a thermodynamically stable system of magnetic vortices, analogous to the mixed state of superconductors, can be realized.

Wonder why

one does not consider contributions to the energy density like

$$w \propto n_x \frac{\partial n_y}{\partial x} + n_y \frac{\partial n_x}{\partial x} \quad ?$$

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one does not consider contributions to the energy density like

$$w \propto n_x \frac{\partial n_y}{\partial x} + n_y \frac{\partial n_x}{\partial x} \quad ?$$

BECAUSE

$$\delta E \propto \int d^3\boldsymbol{r} \left(n_x \frac{\partial n_y}{\partial x} + n_y \frac{\partial n_x}{\partial x} \right) = \int d^3\boldsymbol{r} \frac{\partial}{\partial x} \left(n_x n_y \right) = \text{Boundary Term}$$

Boundary terms

PRL **119,** 127203 (2017) PHYSICAL REVIEW LETTERS week ending 22 SEPTEMBER 2017

New Boundary-Driven Twist States in Systems with Broken Spatial Inversion Symmetry

Kjetil M. D. Hals[1,2] and Karin Everschor-Sitte[1]

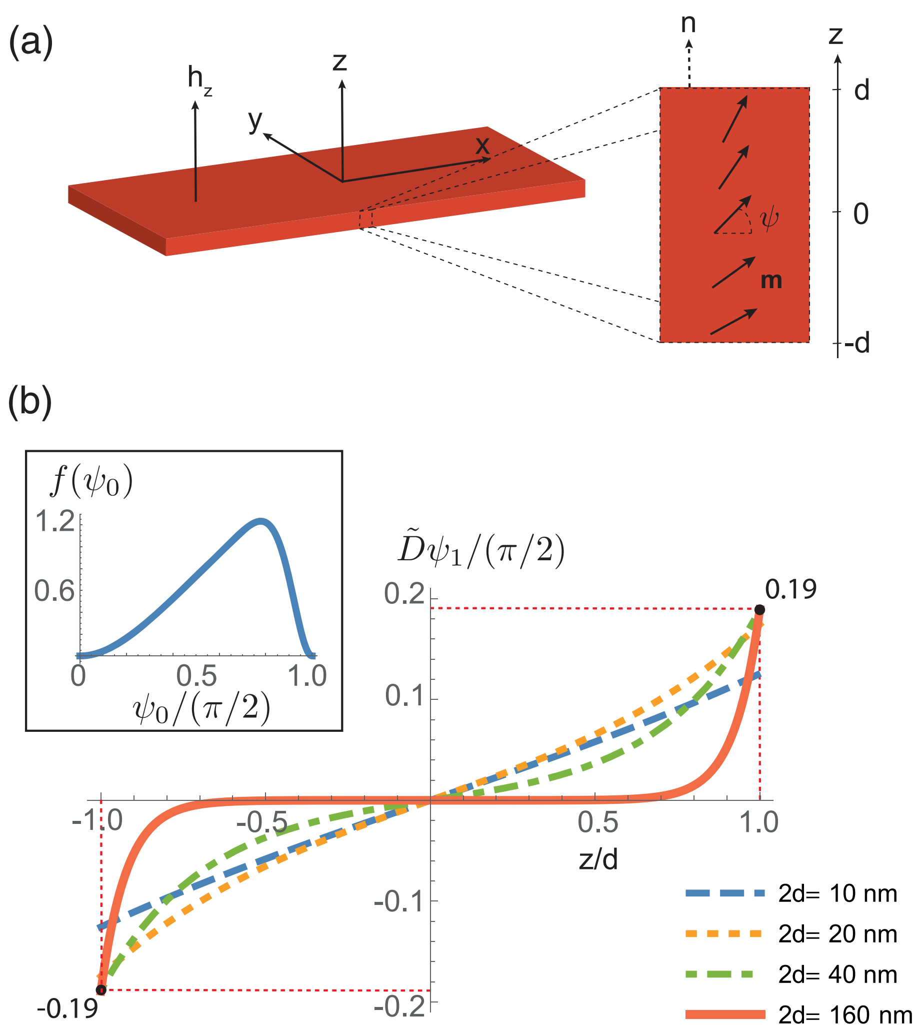

FIG. 1. (a) Illustration of the boundary-driven twist state. The black arrows in the close-up view of the sample indicate the canting of the magnetization across the ferromagnetic thin film.

PHYSICAL REVIEW B **98**, 064429 (2018)

Editors' Suggestion

Effect of boundary-induced chirality on magnetic textures in thin films

Jeroen Mulkers,[1,2,*] Kjetil M. D. Hals,[3,4] Jonathan Leliaert,[2] Milorad V. Milošević,[1,†] Bartel Van Waeyenberge,[2] and Karin Everschor-Sitte[3]

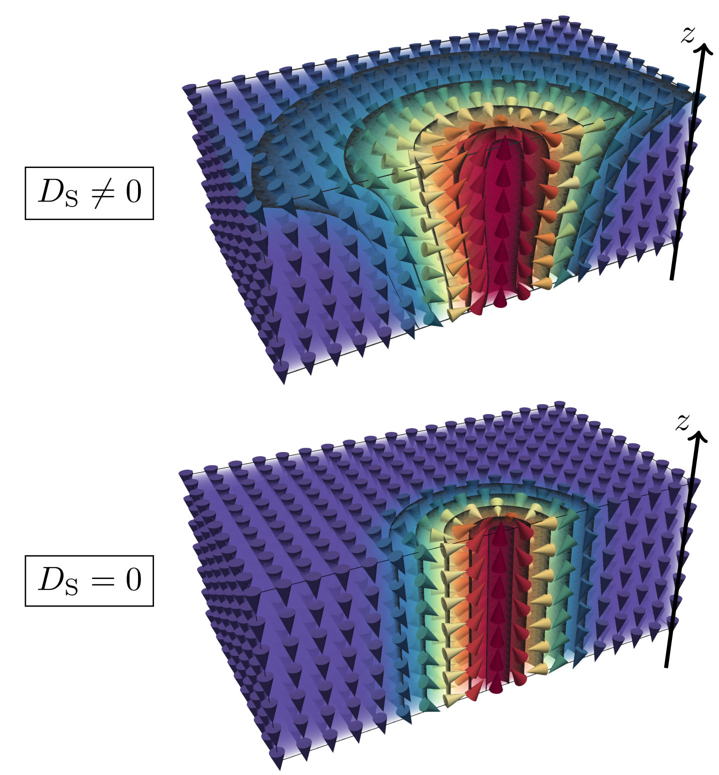

FIG. 1. Cross section of a skyrmion in an extended film with boundary-induced DMI ($D_S \neq 0$) at the top and bottom surface, and without boundary-induced DMI ($D_S = 0$) . Both systems are shown on the same scale. The dark contours represent isomagnetizations.

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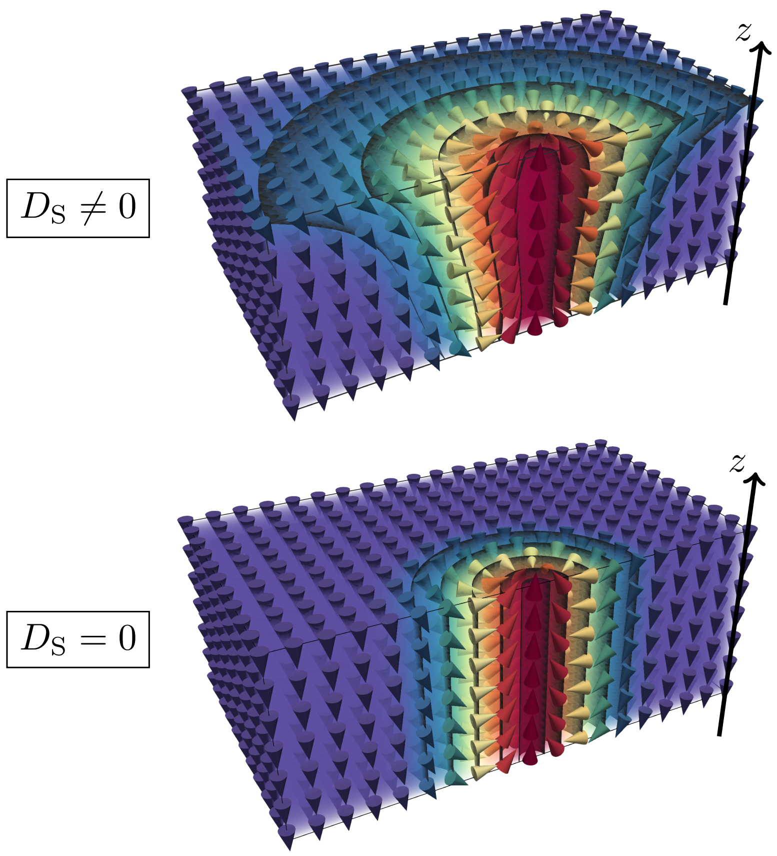

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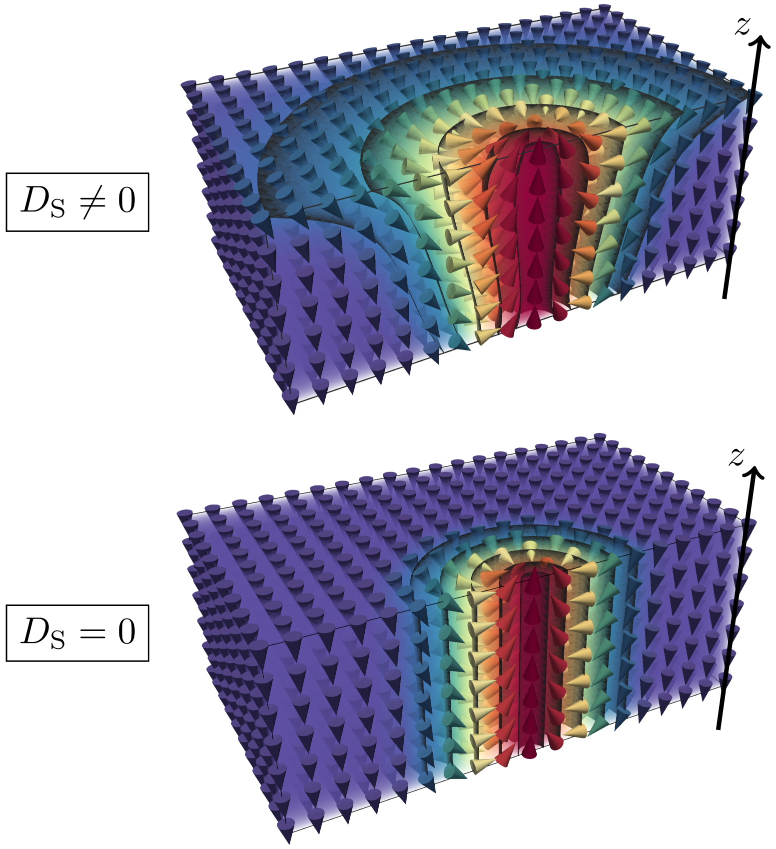

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PHYSICAL REVIEW B **96**, 054403 (2017)

Relation of the Dzyaloshinskii-Moriya interaction to spin currents and to the spin-orbit field

Frank Freimuth,[*] Stefan Blügel, and Yuriy Mokrousov

II. FIRST-ORDER CONTRIBUTION OF SOI TO DMI

Due to DMI, the free energy density $F(\boldsymbol{r})$ at position $\boldsymbol{r}$ contains a term linear in the gradients of magnetization [22]:

$$F^{\mathrm{DMI}}(\boldsymbol{r}) = \sum_j \boldsymbol{D}_j(\hat{\boldsymbol{n}}(\boldsymbol{r})) \cdot \left[\hat{\boldsymbol{n}}(\boldsymbol{r}) \times \frac{\partial \hat{\boldsymbol{n}}(\boldsymbol{r})}{\partial r_j}\right], \qquad (1)$$

where $\boldsymbol{D}_j(\hat{\boldsymbol{n}})$ are the DMI coefficient vectors, which depend on the magnetization direction $\hat{\boldsymbol{n}}(\boldsymbol{r})$ in systems where DMI is anisotropic. The index j runs over the three cartesian directions x, y, and z.

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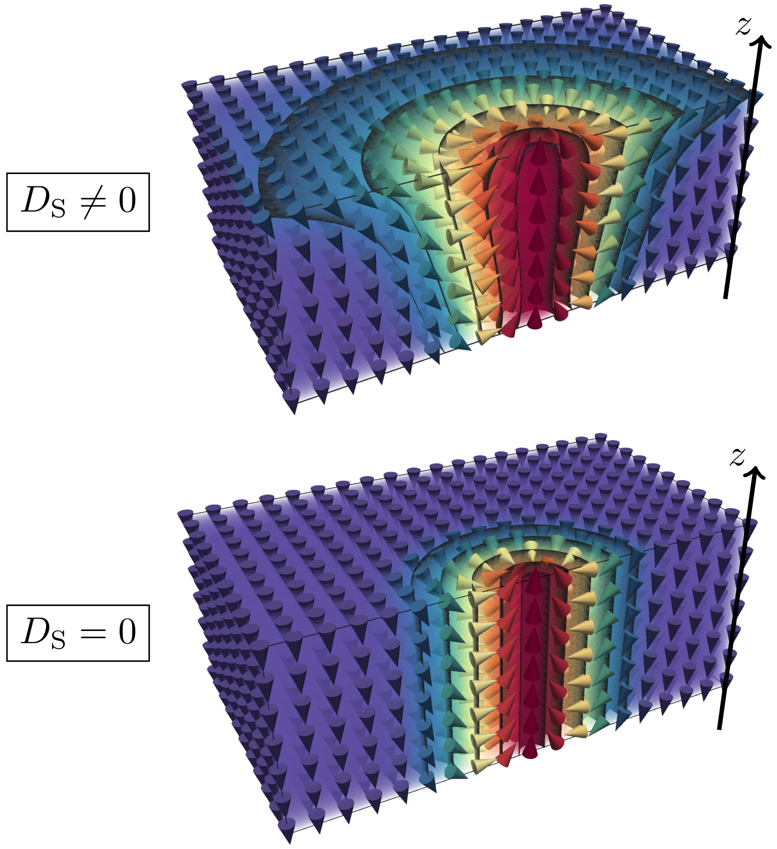

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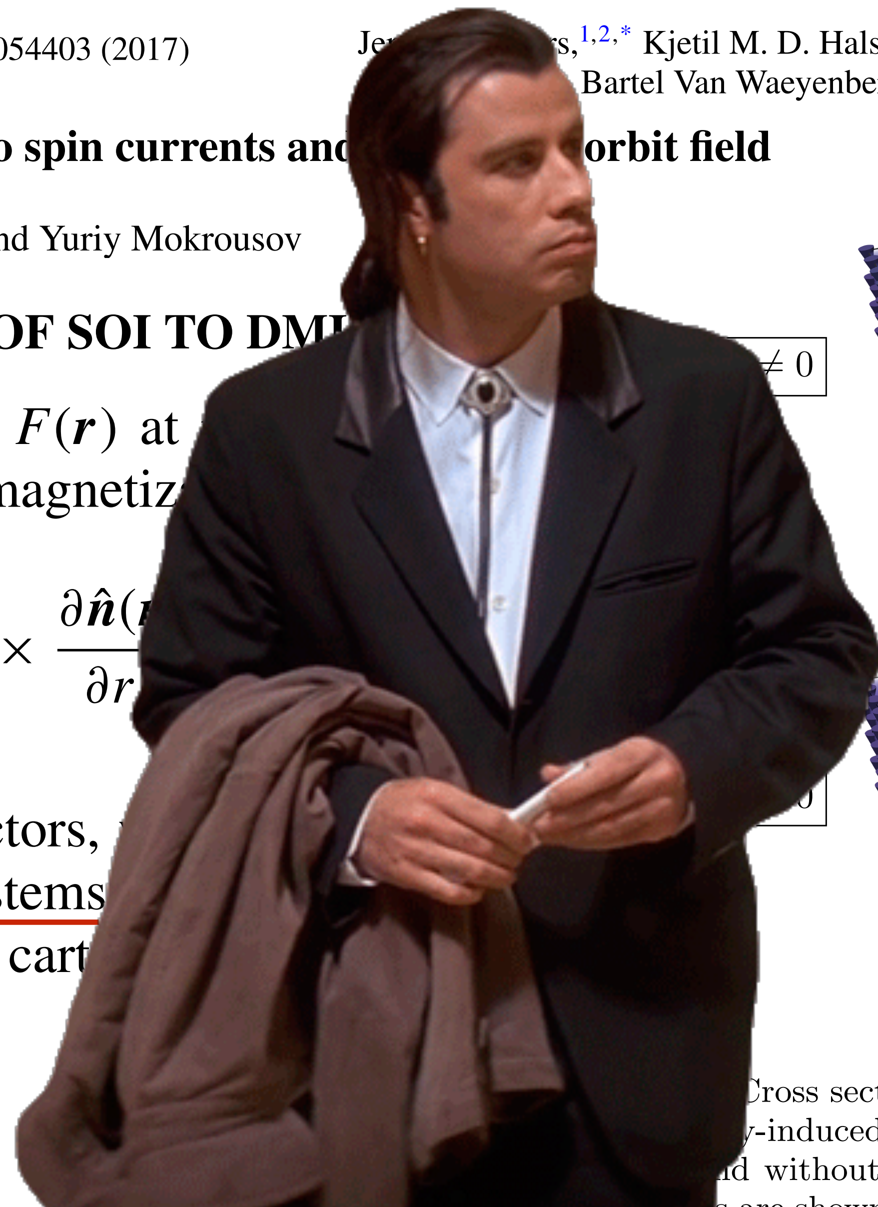

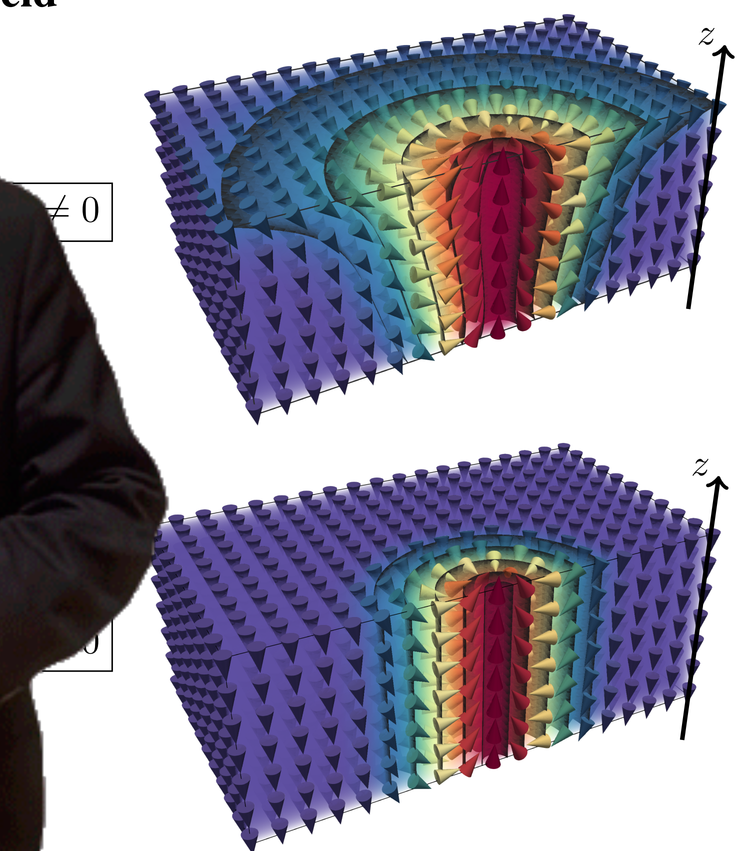

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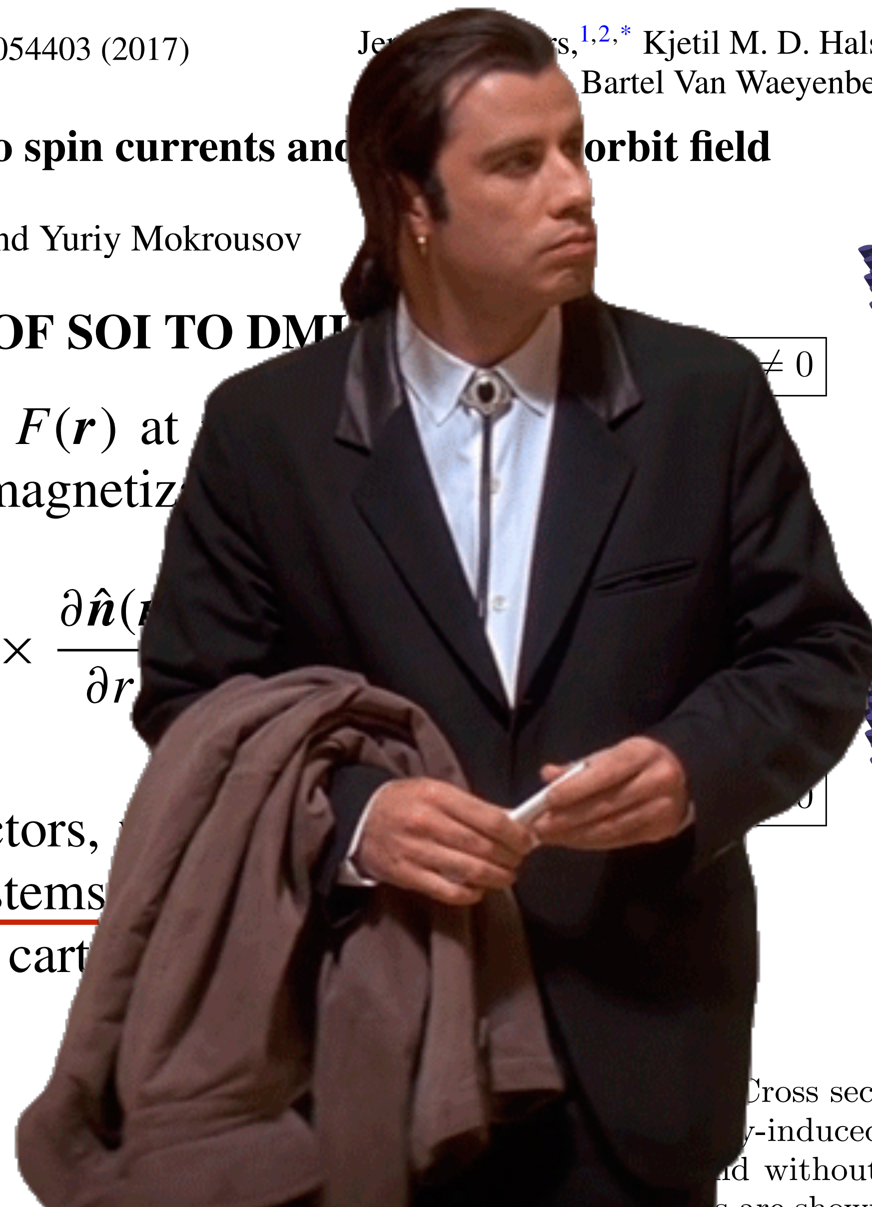

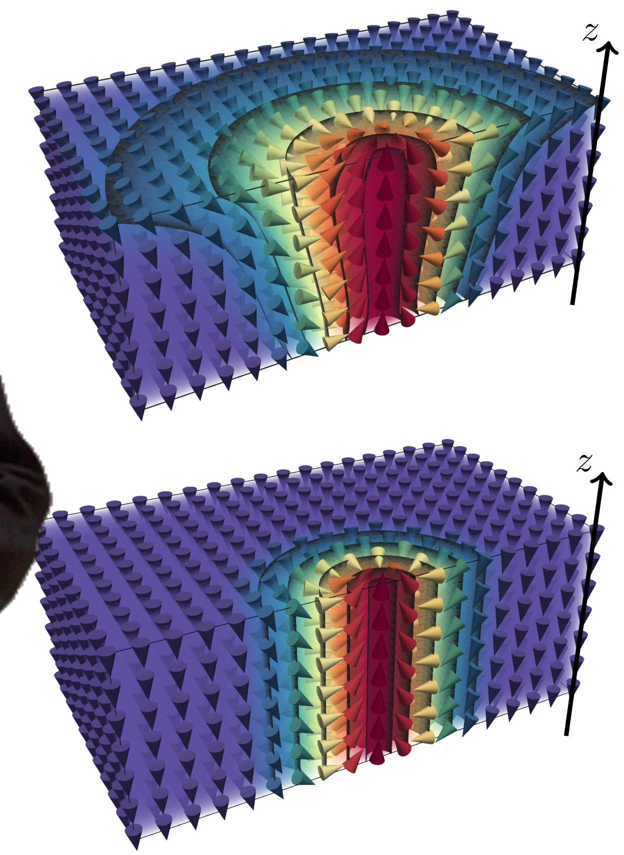

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How to see if new bulk terms exist?

Take $H = H_{\text{Heisenberg}} + H_{\text{tight-binding}} + J_{\text{sd}} S\, \boldsymbol{\sigma} \cdot \boldsymbol{n}(\boldsymbol{r})$

classical magnet **Fermi liquid** **local coupling**

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classical magnet | **Fermi liquid** | **local coupling**

Compute microscopic "chiral energy density"

$$w^{\text{ch}} = \sum_{\alpha\beta} \Omega^{\text{ch}}_{\alpha\beta}[\boldsymbol{n}] \nabla_\alpha n_\beta$$

collects all terms in micromagnetic energy density that are linear in gradients of magnetization direction

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Note that the tensor $\Omega^{\text{ch}}_{\alpha\beta}$ has an essential GAUGE FREEDOM

due to the constraint $|\boldsymbol{n}| = 1$ $\qquad \sum_\beta n_\beta \nabla_\alpha n_\beta = 0$

Microscopic theory

$$H = H_{\text{tight-binding}} + H_{\text{Heisenberg}} + J_{\text{sd}} S \,\boldsymbol{\sigma} \cdot \boldsymbol{n}(\boldsymbol{r})$$

Grand-potential density of conduction electrons

$$\Omega[\boldsymbol{n}(\boldsymbol{r})] = \frac{T}{2\pi i} \int d\varepsilon \, g(\varepsilon) \, \left[\mathcal{G}^R(\boldsymbol{r}, \boldsymbol{r}) - \mathcal{G}^A(\boldsymbol{r}, \boldsymbol{r})\right], \qquad g(\varepsilon) = \ln\left[1 + e^{-(\varepsilon-\mu)/T}\right]$$

is expanded in the gradients of magnetization $\Omega[\boldsymbol{n}] = w^{\text{ch}}[\boldsymbol{n}] + w^{\text{ex}}[\boldsymbol{n}] + \ldots$

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First gradients define the chiral tensor $w^{\text{ch}} = \sum_{\alpha\beta} \Omega^{\text{ch}}_{\alpha\beta} \nabla_\alpha n_\beta$

Second gradients contribute to the symmetric exchange (exchange stiffness)

$$w^{\text{ex}} = \sum_{\alpha\alpha'\beta\beta'} \Omega^{\text{ex-I}}_{\alpha\alpha'\beta\beta'} (\nabla_\alpha n_\beta)(\nabla_{\alpha'} n_{\beta'}) + \sum_{\alpha\alpha'\beta} \Omega^{\text{ex-II}}_{\alpha\alpha'\beta\beta'} \nabla_\alpha \nabla_{\alpha'} n_\beta$$

General formula

The problem is reduced to the analysis of the following formula

$$\Omega^{\mathrm{ch}}_{\alpha\beta} = T\frac{J_{\mathrm{sd}}S}{2\pi\hbar}\,\mathrm{Im}\int d\varepsilon\; g(\varepsilon)\,\mathrm{Tr}[G^R\sigma_\beta G^R v_\alpha G^R - G^R v_\alpha G^R \sigma_\beta G^R]$$

$$g(\varepsilon) = \ln\left[1 + e^{-(\varepsilon-\mu)/T}\right]$$

This is similar but more general than the formula proposed before

Relation of the Dzyaloshinskii-Moriya interaction to spin currents and to the spin-orbit field

Frank Freimuth, Stefan Blügel, and Yuriy Mokrousov
Phys. Rev. B **96**, 054403 – Published 2 August 2017

The formula above is obtained by the expansion in the gradients of *n*

$$\mathcal{G}(\boldsymbol{r},\boldsymbol{r}) = G(\boldsymbol{r}-\boldsymbol{r}) + J_{\mathrm{sd}}S\int d\boldsymbol{r}'\; G(\boldsymbol{r}-\boldsymbol{r}')\sum_{\alpha\beta}\left[(\boldsymbol{r}'-\boldsymbol{r})_\alpha \nabla_\alpha n_\beta(\boldsymbol{r})\sigma_\beta\right] G(\boldsymbol{r}'-\boldsymbol{r})$$

An example from the class $C_{\infty \mathrm{v}}$

Generalized Rashba ferromagnet in 2D:

$$H = \xi(p) + \alpha_{\mathrm{R}}\zeta(p)\,[\boldsymbol{p}\times\boldsymbol{\sigma}]_z + J_{\mathrm{sd}}S\,\boldsymbol{n}\cdot\boldsymbol{\sigma} \qquad \Delta_{\mathrm{sd}} = J_{\mathrm{sd}}S$$

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Very general symmetry-based result:

$$\Omega^{\mathrm{ch}}_{\alpha\beta} = D_{\perp}(n_z^2) \, \delta_{\beta z} n_\gamma \, \epsilon_{\gamma\eta\beta} \, \epsilon_{\eta z \alpha} + D_{||}(n_z^2) \, (1 - \delta_{\beta z}) n_\gamma \, \epsilon_{\gamma\eta\beta} \, \epsilon_{\eta z \alpha} + U(n_z^2) \, n_\alpha \, n_\beta$$

U-term does not contribute to *w* due to gauge invariance

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$$H = \xi(p) + \alpha_{\mathrm{R}}\zeta(p)\,[\boldsymbol{p}\times\boldsymbol{\sigma}]_z + J_{\mathrm{sd}}S\,\boldsymbol{n}\cdot\boldsymbol{\sigma} \qquad \Delta_{\mathrm{sd}} = J_{\mathrm{sd}}S$$

Very general symmetry-based result:

$$\Omega^{\mathrm{ch}}_{\alpha\beta} = D_{\perp}(n_z^2)\,\delta_{\beta z}n_{\gamma}\,\epsilon_{\gamma\eta\beta}\,\epsilon_{\eta z\alpha} + D_{||}(n_z^2)\,(1-\delta_{\beta z})n_{\gamma}\,\epsilon_{\gamma\eta\beta}\,\epsilon_{\eta z\alpha} + U(n_z^2)\,n_{\alpha}\,n_{\beta}$$

U-term does not contribute to *w* due to gauge invariance

An example from the class $C_{\infty \mathrm{v}}$

Generalized Rashba ferromagnet in 2D:

$$H = \xi(p) + \alpha_{\mathrm{R}} \zeta(p) \, [\boldsymbol{p} \times \boldsymbol{\sigma}]_z + J_{\mathrm{sd}} S \, \boldsymbol{n} \cdot \boldsymbol{\sigma} \qquad \Delta_{\mathrm{sd}} = J_{\mathrm{sd}} S$$

Very general symmetry-based result:

$$\Omega^{\mathrm{ch}}_{\alpha\beta} = D_{\perp}(n_z^2) \, \delta_{\beta z} n_\gamma \, \epsilon_{\gamma\eta\beta} \, \epsilon_{\eta z \alpha} + D_{||}(n_z^2) \, (1 - \delta_{\beta z}) n_\gamma \, \epsilon_{\gamma\eta\beta} \, \epsilon_{\eta z \alpha} + U(n_z^2) \, n_\alpha \, n_\beta$$

U-term does not contribute to *w* due to gauge invariance

RESULT: $w = D_{\perp} \, \boldsymbol{n} \cdot [[\hat{\boldsymbol{z}} \times \boldsymbol{\nabla}] \times \boldsymbol{n}_{\perp}] + D_{||} \, \boldsymbol{n} \cdot [[\hat{\boldsymbol{z}} \times \boldsymbol{\nabla}] \times \boldsymbol{n}_{||}]$

OR: $w = (D_{\perp} \, n_x \nabla_x n_z - D_{||} \, n_z \nabla_x n_x) + (D_{\perp} \, n_y \nabla_x n_z - D_{||} \, n_z \nabla_x n_y)$

Expected Lifshitz invariant form $w = D \, \boldsymbol{n} \cdot (\hat{\boldsymbol{z}} \times \boldsymbol{\nabla}) \times \boldsymbol{n}$

is restored ONLY if $D_{\perp} = D_{||} = D$ **does not depend on** n_z^2

First order in spin-orbit

The limit of vanishing spin-orbit coupling $\alpha_{\mathrm{R}} \to 0$

$$H = \xi(p) + \alpha_{\mathrm{R}}\zeta(p)\,[\boldsymbol{p} \times \boldsymbol{\sigma}]_z + J_{\mathrm{sd}}S\,\boldsymbol{n} \cdot \boldsymbol{\sigma} \qquad \Delta_{\mathrm{sd}} = J_{\mathrm{sd}}S$$

RESULT: $w = D\,\boldsymbol{n} \cdot (\hat{\boldsymbol{z}} \times \boldsymbol{\nabla}) \times \boldsymbol{n}$ **expected Lifshitz invariant**

The DMI strength *D* does not depend on n_z^2

Ado *et al.*, Phys. Rev. Lett. 121, 086802 (2018)

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RESULT: $w = D\,\boldsymbol{n}\cdot(\hat{\boldsymbol{z}}\times\boldsymbol{\nabla})\times\boldsymbol{n}$ **expected Lifshitz invariant**

The DMI strength *D* does not depend on n_z^2

General expression

$$D = \frac{\alpha_{\rm R}\Delta_{\rm sd}}{8\pi\hbar}\frac{\partial}{\partial\Delta_{\rm sd}}\left[\int_0^\infty dp\,\frac{p^2\,\zeta(p)\xi'(p)}{\Delta_{\rm sd}}(f_- - f_+)\right]$$

Ado *et al.*, Phys. Rev. Lett. 121, 086802 (2018)

Benchmarking

The limit of zero spin-orbit coupling $\alpha_{\mathrm{R}} = 0$

$$H = \xi(p) + J_{\mathrm{sd}} S\, \boldsymbol{n} \cdot \boldsymbol{\sigma} \qquad \Delta_{\mathrm{sd}} = J_{\mathrm{sd}} S$$

By-product: the same style expression for exchange stiffness

$$\Omega^{\mathrm{ex}}[\boldsymbol{n}] = A\left[(\nabla_x \boldsymbol{n})^2 + (\nabla_y \boldsymbol{n})^2\right]$$

$$A = \frac{\Delta_{\mathrm{sd}}}{32\pi} \frac{\partial}{\partial \Delta_{\mathrm{sd}}} \left[\int_0^\infty dp\, \frac{p[\xi'(p)]^2}{\Delta_{\mathrm{sd}}} (f_- - f_+)\right]$$

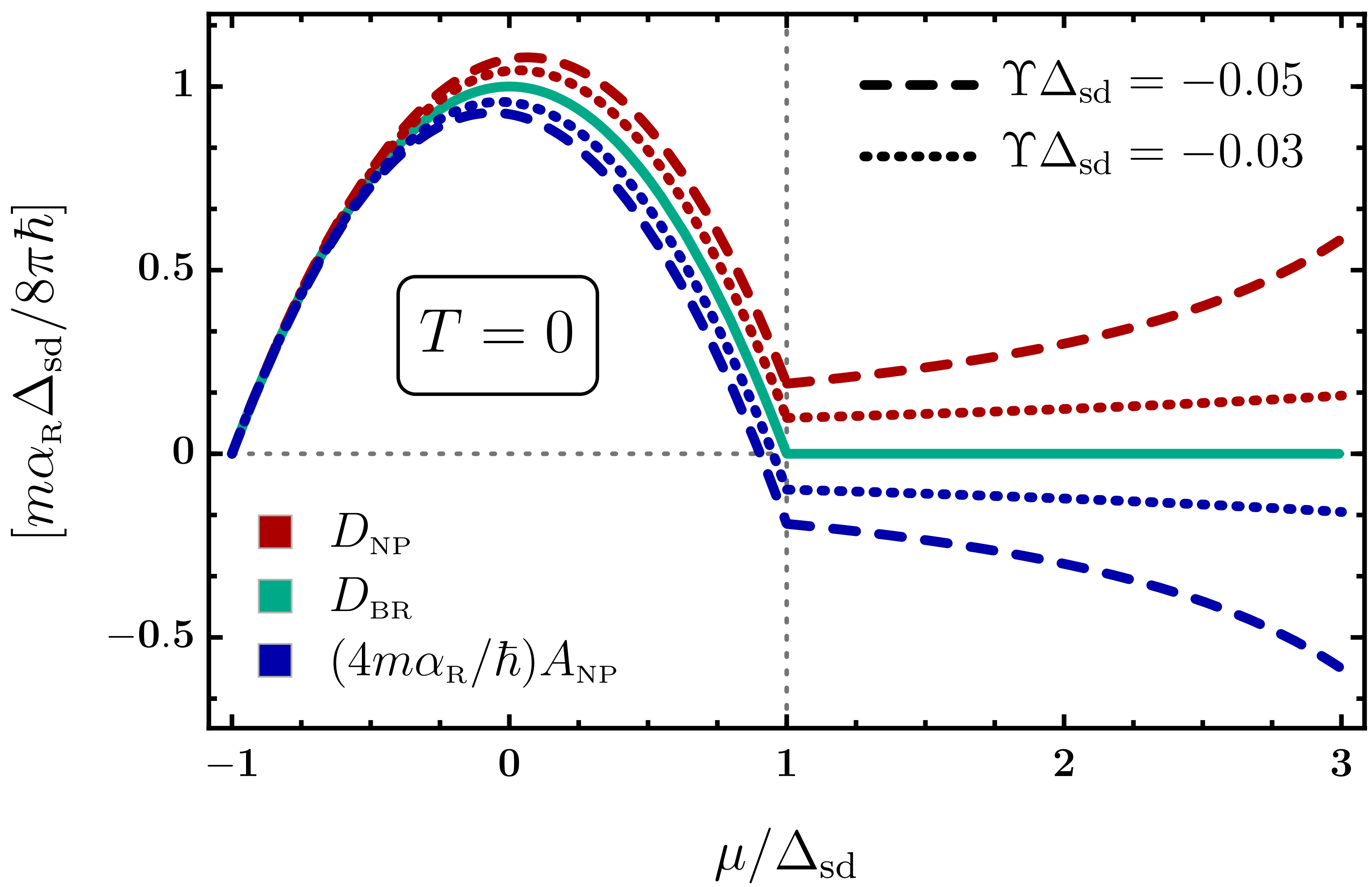
Coincides with well-established theoretical results!

$$f_\pm = \frac{1}{1 + e^{(\xi(p) \pm \Delta_{\mathrm{sd}} - \mu)/T}}$$

see *e.g.* Half-metallic ferromagnets: From band structure to many-body effects

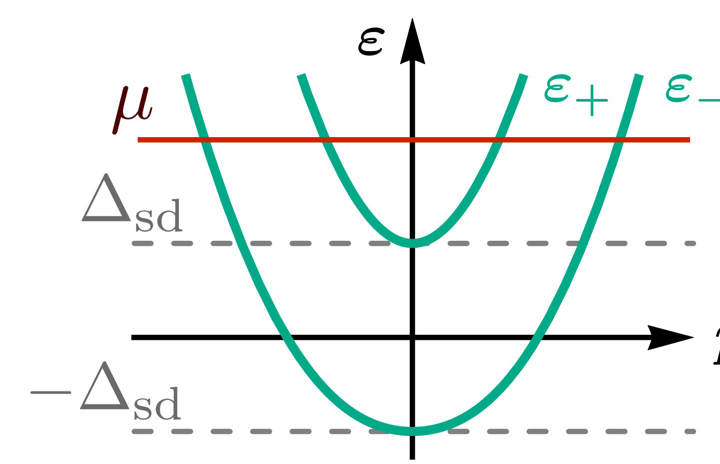
M. I. Katsnelson, V. Yu. Irkhin, L. Chioncel, A. I. Lichtenstein, and R. A. de Groot
Rev. Mod. Phys. **80**, 315 – Published 1 April 2008

Non-parabolic dispersion



$$\xi(p) = \frac{p^2}{2m}\left(1 + \Upsilon\frac{p^2}{2m}\right)$$

$$\zeta(p) = 1$$



The relation $D = \dfrac{4m\alpha_{\rm R}}{\hbar}A$ (Kim, Lee, Lee, Stiles) **breaks down beyond the BR model!**

***E*.g. for non-parabolic kinetic energy it is replaced by** $D = -\dfrac{4m\alpha_{\rm R}}{\hbar}A$ **in the metal regime**

Broken Lifshitz invariants beyond weak SOC

$$H = \xi(p) + \alpha_{\rm R}\zeta(p)\,[\boldsymbol{p}\times\boldsymbol{\sigma}]_z + J_{\rm sd}S\,\boldsymbol{n}\cdot\boldsymbol{\sigma} \qquad \Delta_{\rm sd} = J_{\rm sd}S$$

$$w = D_\perp\,\boldsymbol{n}\cdot[[\hat{\boldsymbol{z}}\times\boldsymbol{\nabla}]\times\boldsymbol{n}_\perp] + D_{||}\,\boldsymbol{n}\cdot[[\hat{\boldsymbol{z}}\times\boldsymbol{\nabla}]\times\boldsymbol{n}_{||}]$$

most importantly: $D_\perp = D_\perp(n_z^2)$ $\qquad$ $D_{||} = D_{||}(n_z^2)$

Total free energy (integration by parts):

$$E = \int d^2\boldsymbol{r}\{D_{\rm tot}\,\boldsymbol{n}\cdot[[\hat{\boldsymbol{z}}\times\boldsymbol{\nabla}]\times\boldsymbol{n}] + D_{\rm diff}\,n_z(\boldsymbol{n}\cdot\boldsymbol{\nabla})\theta\} + \partial\Omega$$

$$n_z = \cos\theta$$

Broken Lifshitz invariants beyond weak SOC

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new bulk term linear in gradients

$$n_z = \cos\theta$$

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new bulk term linear in gradients

$$n_z = \cos\theta$$

$$D_{\rm tot} = \frac{D_{||}+D_\perp}{2} = D_{\rm tot}^{(0)} + D_{\rm tot}^{(2)}\cos(2\theta) + D_{\rm tot}^{(4)}\cos(4\theta) + \dots$$

$$D_{\rm diff} = \frac{\partial}{\partial\theta}\frac{D_{||}-D_\perp}{2} = D_{\rm diff}^{(2)}\sin(2\theta) + D_{\rm dif}^{(4)}\sin(4\theta) + \dots$$

Domain wall energy

$$E = \int d^2\boldsymbol{r} \{ D_{\mathrm{tot}}\, \boldsymbol{n} \cdot [[\hat{\boldsymbol{z}} \times \boldsymbol{\nabla}] \times \boldsymbol{n}] + D_{\mathrm{diff}}\, n_z (\boldsymbol{n} \cdot \boldsymbol{\nabla})\theta \}$$

Domain wall parameterization

$$\boldsymbol{n}(x) = \hat{\boldsymbol{x}} \sin\theta(x)\, \cos\phi + \hat{\boldsymbol{y}}\, \sin\phi + \hat{\boldsymbol{z}} \cos\theta(x)$$

$$\theta(x) = \arccos\left[\tanh(x/\ell)\right]$$

Domain wall energy

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$$E_{\text{DW}} = \pi W\left(D_{\text{tot}}^{(0)} - \frac{1}{4}D_{\text{diff}}^{(2)}\right)\cos\phi$$

W - domain wall length in y direction

Domain wall energy

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W - domain wall length in y direction

$$w^{\rm ch} = D_{\rm tot}^{(0)}\,\boldsymbol{n}\cdot[(\hat{\boldsymbol{z}}\times\boldsymbol{\nabla})\times\boldsymbol{n}] + 2D_{\rm diff}^{(2)}\,n_z^2(\boldsymbol{n}\cdot\boldsymbol{\nabla})n_z + \ldots$$

Domain wall energy

$$E = \int d^2\boldsymbol{r} \{ D_{\text{tot}}\, \boldsymbol{n} \cdot [[\hat{\boldsymbol{z}} \times \boldsymbol{\nabla}] \times \boldsymbol{n}] + D_{\text{diff}}\, n_z (\boldsymbol{n} \cdot \boldsymbol{\nabla})\theta \}$$

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$$w^{\text{ch}} = D_{\text{tot}}^{(0)}\, \boldsymbol{n} \cdot [(\hat{\boldsymbol{z}} \times \boldsymbol{\nabla}) \times \boldsymbol{n}] + 2 D_{\text{diff}}^{(2)}\, n_z^2 (\boldsymbol{n} \cdot \boldsymbol{\nabla}) n_z + \ldots$$

$$\sum_{\langle i,j \rangle} (\hat{\boldsymbol{z}} \times \boldsymbol{d}_{ij}) \cdot (\boldsymbol{S}_i \times \boldsymbol{S}_j)$$

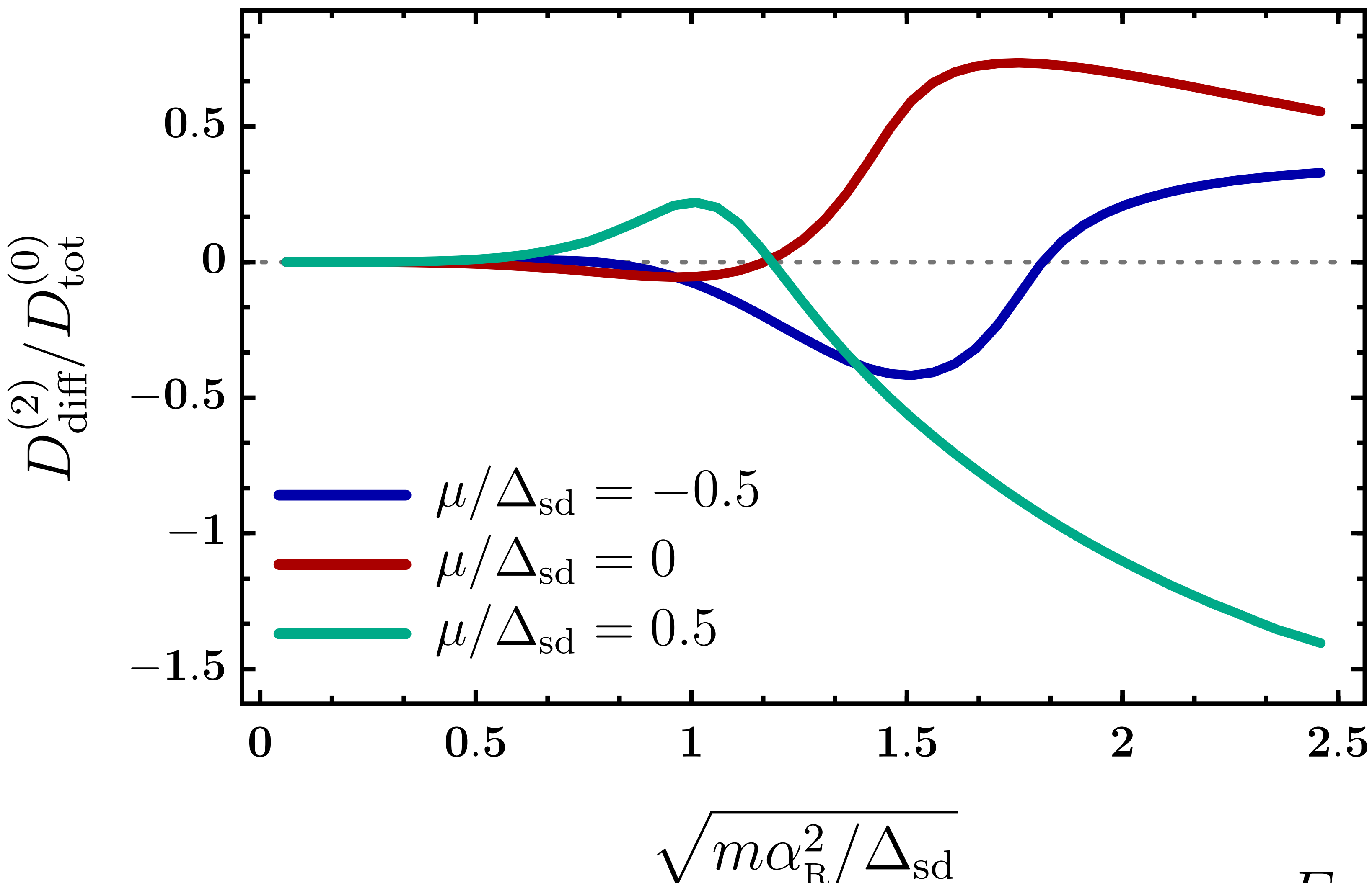
$$\sum_{\langle ij,kl \rangle} S_k^z S_l^z (\boldsymbol{S}_i \cdot \boldsymbol{d}_{ij}) S_j^z$$

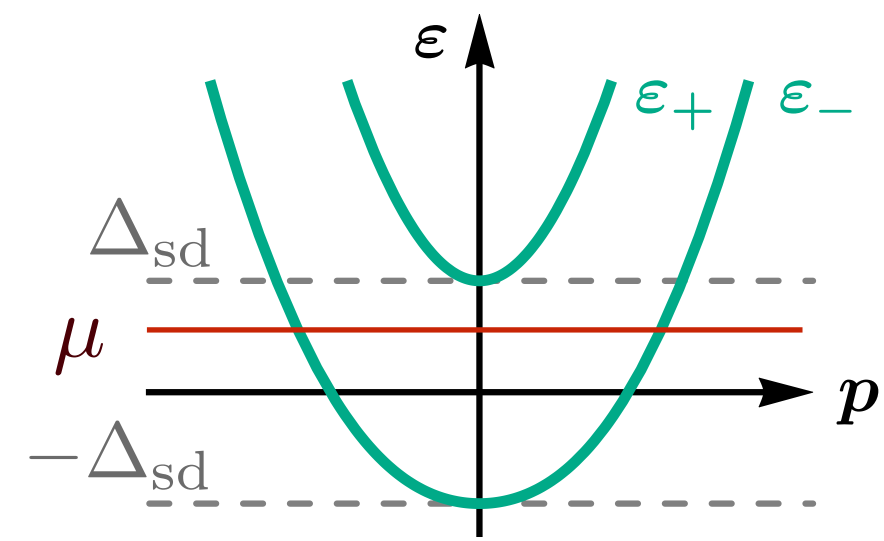
new four-spin interaction

Importance of the four-spin interaction

$$2D^{(2)}_{\rm diff}\int d^2\boldsymbol{r}\; n_z^2(\boldsymbol{n}\cdot\boldsymbol{\nabla})n_z \propto \sum_{\langle ij,kl\rangle} S^z_k S^z_l(\boldsymbol{S}_i\cdot\boldsymbol{d}_{ij})S^z_j$$

ij and kl are links in the lattice plaquette





$$E_{\rm DW} = \pi W\left(D^{(0)}_{\rm tot} - \frac{1}{4}D^{(2)}_{\rm diff}\right)\cos\phi$$

Arbitrary ferromagnet

$$w = D_{ijk}\, n_i \nabla_k n_j \implies w = \frac{1}{2}\left[D^{\text{as}}_{ijk}\mathcal{L}^{(k)}_{ij} + D^{\text{sym}}_{ijk}\nabla_k\left(n_i n_j\right)\right]$$

$$D^{\text{as(sym)}}_{ijk} = (D_{ijk} \mp D_{jik})/2$$

$$\text{after partial integration:}$$

$$w_{\text{ch}} = \frac{1}{2}\left[D^{\text{as}}_{ijk}\mathcal{L}^{(k)}_{ij} - \frac{\partial D^{\text{sym}}_{ijk}}{\partial\theta}\Theta^{(k)}_{ij} - \frac{\partial D^{\text{sym}}_{ijk}}{\partial\phi}\Phi^{(k)}_{ij}\right]$$

$$\mathcal{A}^{(k)}_{ij} = \qquad \Theta^{(k)}_{ij} = n_i n_j \nabla_k \theta, \quad \Phi^{(k)}_{ij} = n_i n_j \nabla_k \phi$$

[I. Ado *et al.*, arXiv:1904.05337 (2019)]

Generalization of Bogdanov–Yablonskii classification

symmetry	LI-type terms	non-LI-type terms
$C_2\ (D_1)$	$\mathcal{L}^{(x)}_{zx};\ \mathcal{L}^{(y)}_{yz};\ \mathcal{L}^{(x)}_{yz};\ \mathcal{L}^{(y)}_{zx};\ \mathcal{L}^{(z)}_{xy}$	$\mathcal{A}^{(x)}_{zx};\ \mathcal{A}^{(y)}_{yz};\ \mathcal{A}^{(x)}_{yz};\ \mathcal{A}^{(y)}_{zx};\ \mathcal{A}^{(z)}_{xy};\ \mathcal{A}^{(z)}_{xx};\ \mathcal{A}^{(z)}_{yy}$
$C_{2v}\ (D_{1h})$	$\mathcal{L}^{(x)}_{zx};\ \mathcal{L}^{(y)}_{yz}$	$\mathcal{A}^{(x)}_{zx};\ \mathcal{A}^{(y)}_{yz};\ \mathcal{A}^{(z)}_{xx};\ \mathcal{A}^{(z)}_{yy}$
D_2	$\mathcal{L}^{(x)}_{yz};\ \mathcal{L}^{(y)}_{zx};\ \mathcal{L}^{(z)}_{xy}$	$\mathcal{A}^{(x)}_{yz};\ \mathcal{A}^{(y)}_{zx};\ \mathcal{A}^{(z)}_{xy}$
D_{2d}	$\mathcal{L}^{(x)}_{yz} - \mathcal{L}^{(y)}_{zx}$	$\mathcal{A}^{(x)}_{yz} + \mathcal{A}^{(y)}_{zx};\ \mathcal{A}^{(z)}_{xy}$
C_3	$\mathcal{L}^{(x)}_{zx} - \mathcal{L}^{(y)}_{yz};\ \mathcal{L}^{(x)}_{yz} + \mathcal{L}^{(y)}_{zx};\ \mathcal{L}^{(z)}_{xy}$	$\mathcal{Q};\ \mathcal{S};\ \mathcal{A}^{(x)}_{zx} + \mathcal{A}^{(y)}_{yz};\ \mathcal{A}^{(x)}_{yz} - \mathcal{A}^{(y)}_{zx};\ \mathcal{A}^{(z)}_{xx} + \mathcal{A}^{(z)}_{yy}$
C_{3v}	$\mathcal{L}^{(x)}_{zx} - \mathcal{L}^{(y)}_{yz}$	$\mathcal{Q};\ \mathcal{A}^{(x)}_{zx} + \mathcal{A}^{(y)}_{yz};\ \mathcal{A}^{(z)}_{xx} + \mathcal{A}^{(z)}_{yy}$
C_{3h}		$\mathcal{Q};\ \mathcal{S}$
D_3	$\mathcal{L}^{(x)}_{yz} + \mathcal{L}^{(y)}_{zx};\ \mathcal{L}^{(z)}_{xy}$	$\mathcal{Q};\ \mathcal{A}^{(x)}_{yz} - \mathcal{A}^{(y)}_{zx}$
D_{3h}		$\mathcal{Q}$
$C_n,\ n>3$	$\mathcal{L}^{(x)}_{zx} - \mathcal{L}^{(y)}_{yz};\ \mathcal{L}^{(x)}_{yz} + \mathcal{L}^{(y)}_{zx};\ \mathcal{L}^{(z)}_{xy}$	$\mathcal{A}^{(x)}_{zx} + \mathcal{A}^{(y)}_{yz};\ \mathcal{A}^{(x)}_{yz} - \mathcal{A}^{(y)}_{zx};\ \mathcal{A}^{(z)}_{xx} + \mathcal{A}^{(z)}_{yy}$
$C_{nv},\ n>3$	$\mathcal{L}^{(x)}_{zx} - \mathcal{L}^{(y)}_{yz}$	$\mathcal{A}^{(x)}_{zx} + \mathcal{A}^{(y)}_{yz};\ \mathcal{A}^{(z)}_{xx} + \mathcal{A}^{(z)}_{yy}$
$D_n,\ n>3$	$\mathcal{L}^{(x)}_{yz} + \mathcal{L}^{(y)}_{zx};\ \mathcal{L}^{(z)}_{xy}$	$\mathcal{A}^{(x)}_{yz} - \mathcal{A}^{(y)}_{zx}$
S_4	$\mathcal{L}^{(x)}_{zx} + \mathcal{L}^{(y)}_{yz};\ \mathcal{L}^{(x)}_{yz} - \mathcal{L}^{(y)}_{zx}$	$\mathcal{A}^{(x)}_{zx} - \mathcal{A}^{(y)}_{yz};\ \mathcal{A}^{(x)}_{yz} + \mathcal{A}^{(y)}_{zx};\ \mathcal{A}^{(z)}_{xy};\ \mathcal{A}^{(z)}_{xx} - \mathcal{A}^{(z)}_{yy}$
T	$\mathcal{L}^{(x)}_{yz} + \mathcal{L}^{(y)}_{zx} + \mathcal{L}^{(z)}_{xy}$	$\mathcal{A}^{(x)}_{yz} + \mathcal{A}^{(y)}_{zx} + \mathcal{A}^{(z)}_{xy}$
T_d		$\mathcal{A}^{(x)}_{yz} + \mathcal{A}^{(y)}_{zx} + \mathcal{A}^{(z)}_{xy}$
O	$\mathcal{L}^{(x)}_{yz} + \mathcal{L}^{(y)}_{zx} + \mathcal{L}^{(z)}_{xy}$	

$$\mathcal{A}^{(k)}_{ij} = \Theta^{(k)}_{ij},\, \Phi^{(k)}_{ij} \qquad \mathcal{Q} = \mathcal{A}^{(x)}_{xx} - \mathcal{A}^{(x)}_{yy} - 2\mathcal{A}^{(y)}_{xy}, \quad \mathcal{S} = \mathcal{A}^{(y)}_{yy} - \mathcal{A}^{(y)}_{xx} - 2\mathcal{A}^{(x)}_{xy}$$

[I. Ado *et al.*, arXiv:1904.05337 (2019)]

Special Classes: C_{3h}, D_{3h}, T_d

Livshitz invariants (LI) are forbidden by symmetry

non-LI multi-spin terms are the only linear-in-gradient terms in micromagnetic energy

sd-like model for magnetization dynamics

$$H = -J_{\rm ex} \sum_{\langle nm \rangle} \boldsymbol{S}_n \cdot \boldsymbol{S}_m + D \sum_{\langle nm \rangle} \boldsymbol{S}_n \times \boldsymbol{S}_m + K \sum_{\langle nm \rangle} S^z_n S^z_m$$

$$-t \sum_{\langle nm \rangle} (c^\dagger_n c_m + \text{h.c.}) + \lambda_{\rm R} \sum_{\langle nm \rangle} c^\dagger_n (\boldsymbol{\sigma} \times \boldsymbol{d}_{nm})_z c_m + \sum_n V_n c^\dagger_n c_n$$

$$-J_{\rm sd} \sum_n \boldsymbol{S}_n \cdot c^\dagger_n \boldsymbol{\sigma} c_n$$

consists of

classical Heisenberg model for localized spins

+

tight-binding model for conduction electrons
with spin-orbit interaction & DISORDER (electron bath)

+

local exchange coupling *a la* s-d

Equation of motion (microscopic LLG)

$$\frac{\partial \boldsymbol{n}}{\partial t} = \boldsymbol{H}_{\mathrm{eff}} \times \boldsymbol{n} + \frac{J_{\mathrm{sd}}\mathcal{A}}{\hbar}\, \boldsymbol{n} \times \boldsymbol{s}$$

$\mathcal{A}$ - area of the unit cell

$$\boldsymbol{s}(\boldsymbol{r},t) = \tfrac{1}{2}\langle \Psi^\dagger(\boldsymbol{r},t)\boldsymbol{\sigma}\Psi(\boldsymbol{r},t)\rangle = -i\,\mathrm{Tr}_\sigma\big[\tfrac{1}{2}\boldsymbol{\sigma}\, G^<(\boldsymbol{r},t;\boldsymbol{r},t)\big]$$

- non-equilibrium electron spin density (polarization)
- sensitive to electric current and electric field
- defines all kinds of torques, Gilbert dampings

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$$H = \frac{(p - e\boldsymbol{A}/c)^2}{2m} + \alpha_{\mathrm{R}}[\boldsymbol{p} \times \boldsymbol{\sigma}]_z + \Delta_{\mathrm{sd}}\, \boldsymbol{n} \cdot \boldsymbol{\sigma} + V(\boldsymbol{r})$$

spin-orbit   s-d exchange   disorder

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$$H = \frac{(p - e\boldsymbol{A}/c)^2}{2m} + \alpha_{\text{R}}[\boldsymbol{p} \times \boldsymbol{\sigma}]_z + \Delta_{\text{sd}}\, \boldsymbol{n} \cdot \boldsymbol{\sigma} + V(\boldsymbol{r})$$

spin-orbit — s-d exchange — disorder

$$s_\alpha = T^{\text{SOT}}_{\alpha\beta} E_\beta + T^{\text{STT}}_{\alpha\beta\gamma\delta} E_\beta \nabla_\delta n_\gamma + T^{\text{GD}}_{\alpha\beta}\frac{\partial n_\beta}{\partial t} + T^{\text{ch}}_{\alpha\beta\gamma}\nabla_\beta n_\gamma + \ldots$$

spin-orbit torque — in-plane spin-transfer torque — Gilbert damping — DMI + linear grads

Universal magnetization dynamics

$$H = \frac{p^2}{2m} + \alpha_{\rm R}[\boldsymbol{\sigma} \times \boldsymbol{p}]_z - \Delta_{\rm sd}\, \boldsymbol{n} \cdot \boldsymbol{\sigma} + V(\boldsymbol{r})$$

$$\boldsymbol{n} = \boldsymbol{S}/S$$

Microscopic LLG up to the first gradients:

$$\frac{\partial \boldsymbol{n}}{\partial t} = -\,\boldsymbol{n} \times \boldsymbol{H}_{\rm eff} - a\, \boldsymbol{n} \times (\hat{z} \times \boldsymbol{J})$$
$$-\,\xi_0(n_z^2)\, D_t \boldsymbol{n} + \xi_{||}(n_z^2)\, \boldsymbol{n} \times D_t \boldsymbol{n}_{||} + \xi_{\perp}(n_z^2)\, \boldsymbol{n} \times D_t \boldsymbol{n}_{\perp}$$

Long derivative:

$$D_t = \frac{\partial}{\partial t} - \boldsymbol{v}_d \cdot \boldsymbol{\nabla}$$

Classical drift velocity:

$$\boldsymbol{v}_d = \boldsymbol{J}/en$$

Microscopic theory of spin-orbit torque
I. A. Ado, O. A. Tretiakov, and M. Titov, PRB 95, 094401 (2017)

Anisotropy of spin-transfer torques and Gilbert damping induced by Rashba coupling
I. A. Ado, P. Ostrovsky and M. Titov, arXiv 1907.0241 (2019)

Universality of torques

$$\frac{\partial \boldsymbol{n}}{\partial t} = -\,\boldsymbol{n} \times \boldsymbol{H}_{\rm eff} - a\,\boldsymbol{n} \times (\hat{z} \times \boldsymbol{J})$$
$$-\,\xi_0(n_z^2)\,D_t\boldsymbol{n} + \xi_{||}(n_z^2)\,\boldsymbol{n} \times D_t\boldsymbol{n}_{||} + \xi_{\perp}(n_z^2)\,\boldsymbol{n} \times D_t\boldsymbol{n}_{\perp}$$

Automodel solutions DO NOT DECAY $\boldsymbol{n} = \boldsymbol{n}(\boldsymbol{r} - \boldsymbol{v}_d t)$

$$\frac{\partial \boldsymbol{n}}{\partial t} = -\boldsymbol{n} \times \boldsymbol{H}_{\rm eff} - a\,\boldsymbol{n} \times (\hat{\boldsymbol{z}} \times \boldsymbol{J})$$

Can this phenomenon be observed beyond Rashba model?

Vanishing spin-orbit limit

$$D_t = \frac{\partial}{\partial t} - \boldsymbol{v}_d \cdot \boldsymbol{\nabla}$$

$$\frac{\partial \boldsymbol{n}}{\partial t} = \gamma \boldsymbol{H}_{\text{eff}} \times \boldsymbol{n} - \xi_0 D_t \boldsymbol{n} + \xi_{\|}\, \boldsymbol{n} \times D_t \boldsymbol{n}_{\|} + \xi_{\perp}\, \boldsymbol{n} \times D_t \boldsymbol{n}_{\perp}$$

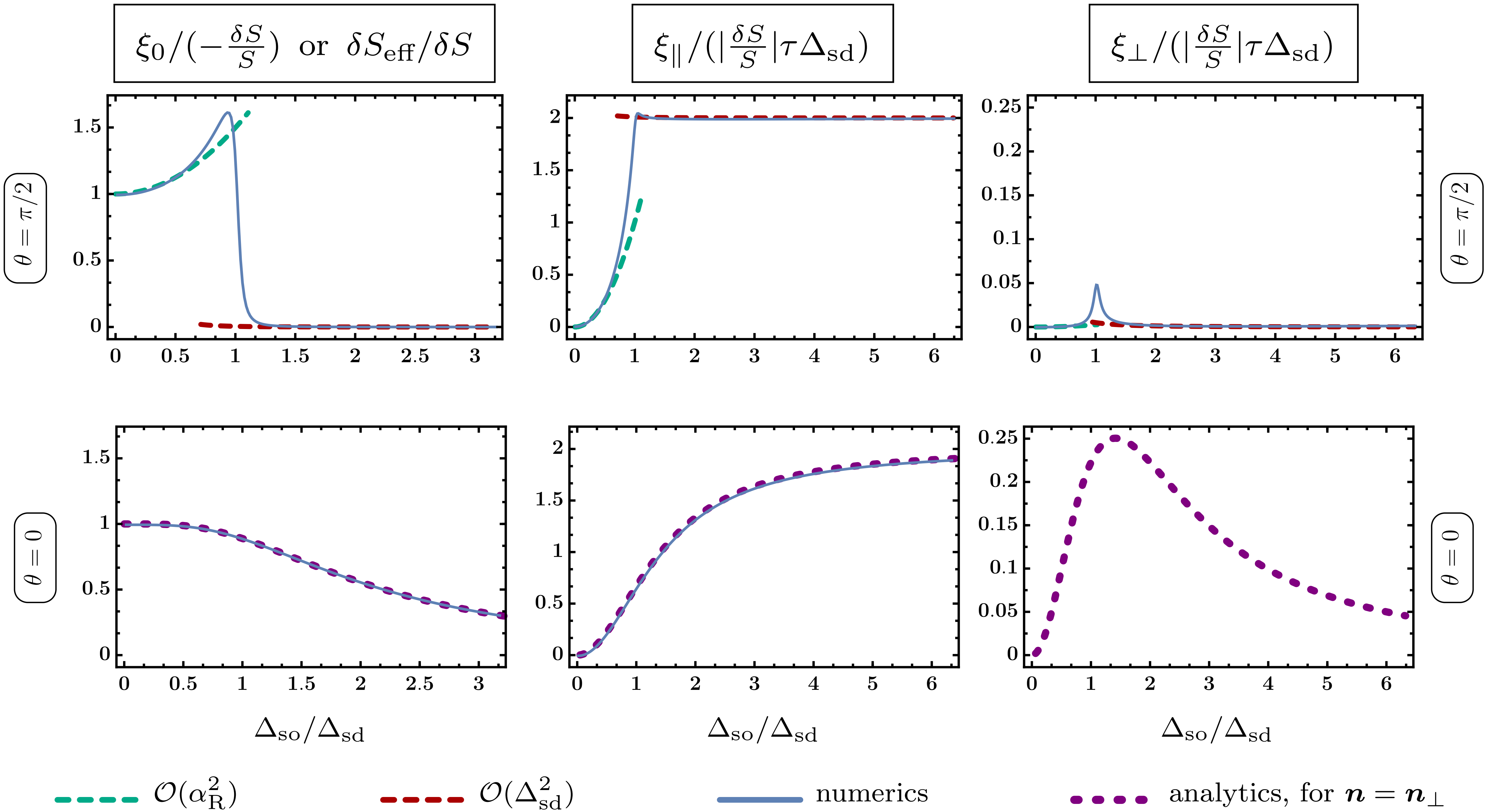
For vanishing spin-orbit: $\xi_0 = \delta S / S \qquad \xi_{\perp,\|} = 0$

where δS **is polarization of conduction electrons per unit cell**

hence, the LLG equation is reduced to angular momentum conservation

$$(S + \delta S)\partial_t \boldsymbol{n} + \delta S\, (\boldsymbol{v}_{\text{d}} \cdot \boldsymbol{\nabla})\boldsymbol{n} = 0$$

Large anisotropy of torques!



Traditional formulation

$$\frac{\partial \boldsymbol{n}}{\partial t} = \gamma \boldsymbol{H}_{\text{eff}} \times \boldsymbol{n} - \xi_0 D_t \boldsymbol{n} + \xi_{\|}\, \boldsymbol{n} \times D_t \boldsymbol{n}_{\|} + \xi_{\perp}\, \boldsymbol{n} \times D_t \boldsymbol{n}_{\perp}$$

For finite spin-orbit: $\xi_0 = \delta S_{\text{eff}}/S$

$$\delta S_{\text{eff}} \neq \delta S$$

$$\frac{\partial \boldsymbol{n}}{\partial t} = \bar{\gamma} \boldsymbol{H}_{\text{eff}} \times \boldsymbol{n} + (\boldsymbol{j}_s \cdot \boldsymbol{\nabla})\boldsymbol{n} + \alpha_{\|}\, \boldsymbol{n} \times \partial_t \boldsymbol{n}_{\|} + \alpha_{\perp}\, \boldsymbol{n} \times \partial_t \boldsymbol{n}_{\perp} + \beta_{\|}\, \boldsymbol{n} \times (\boldsymbol{j}_s \cdot \boldsymbol{\nabla})\boldsymbol{n}_{\|} + \beta_{\perp}\, \boldsymbol{n} \times (\boldsymbol{j}_s \cdot \boldsymbol{\nabla})\boldsymbol{n}_{\perp}$$

$$\boldsymbol{j}_s = \boldsymbol{v}_d \frac{\xi_0}{1+\xi_0} = \boldsymbol{v}_d \frac{\delta S_{\text{eff}}}{S + \delta S_{\text{eff}}}$$

- *a la* polarization current

$\boldsymbol{v}_d = \boldsymbol{J}/en$

$$\bar{\gamma} = \frac{\gamma}{1+\xi_0} \qquad \alpha_{\|,\perp} = \frac{\xi_{\|,\perp}}{1+\xi_0} \qquad \beta_{\|,\perp} = -\frac{\xi_{\|,\perp}}{\xi_0}$$

Giant anisotropy of Gilbert damping

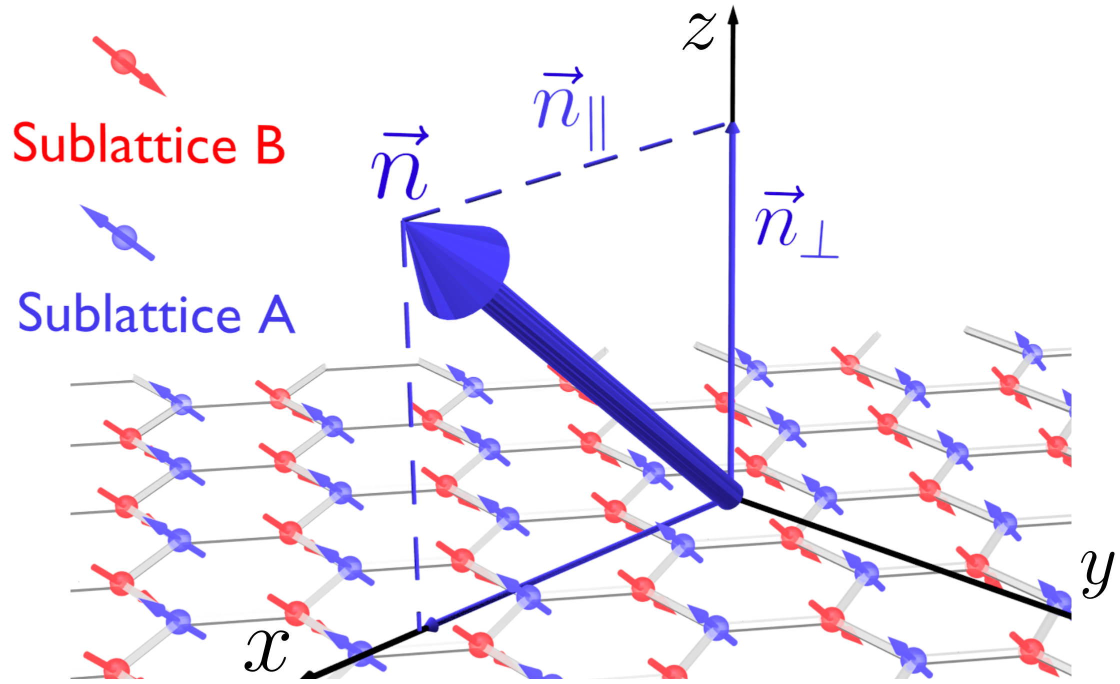
in 2D Rashba AFM

$\boldsymbol{n}$ - Neel vector $\boldsymbol{m}$ - magnetization vector

$$\boldsymbol{n} \cdot \boldsymbol{m} = 0, \qquad n^2 + m^2 = 1$$

$$m \ll n$$

$\boldsymbol{s}_\pm = (\boldsymbol{s}_A \pm \boldsymbol{s}_B)/2$ - electron spin densities



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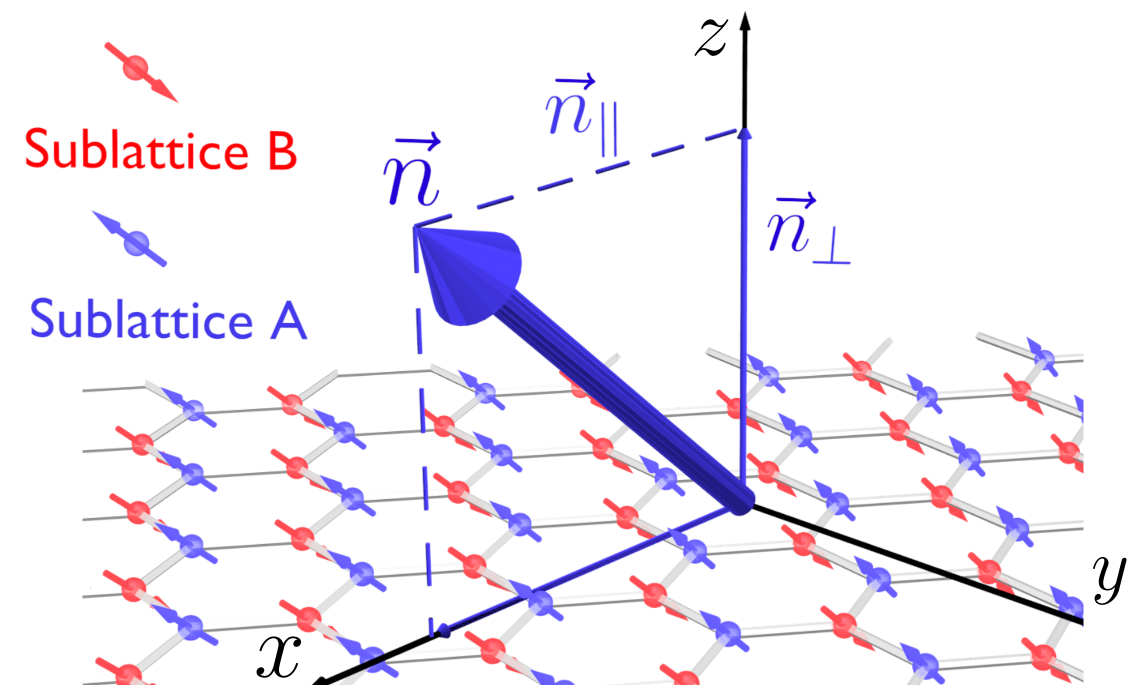

$\boldsymbol{s}_{\pm} = (\boldsymbol{s}_A \pm \boldsymbol{s}_B)/2$ - electron spin densities

equations of motion:

$$\dot{\boldsymbol{n}} = -\Omega\, \boldsymbol{n} \times \boldsymbol{m} + \boldsymbol{n} \times \boldsymbol{s}^{+} + \boldsymbol{m} \times \boldsymbol{s}^{-} + \ldots,$$

$$\dot{\boldsymbol{m}} = \boldsymbol{m} \times \boldsymbol{s}^{+} + \boldsymbol{n} \times \boldsymbol{s}^{-} + \ldots$$

Giant anisotropy of Gilbert damping

in 2D Rashba AFM

$\boldsymbol{n}$ - Neel vector $\quad\quad$ $\boldsymbol{m}$ - magnetization vector

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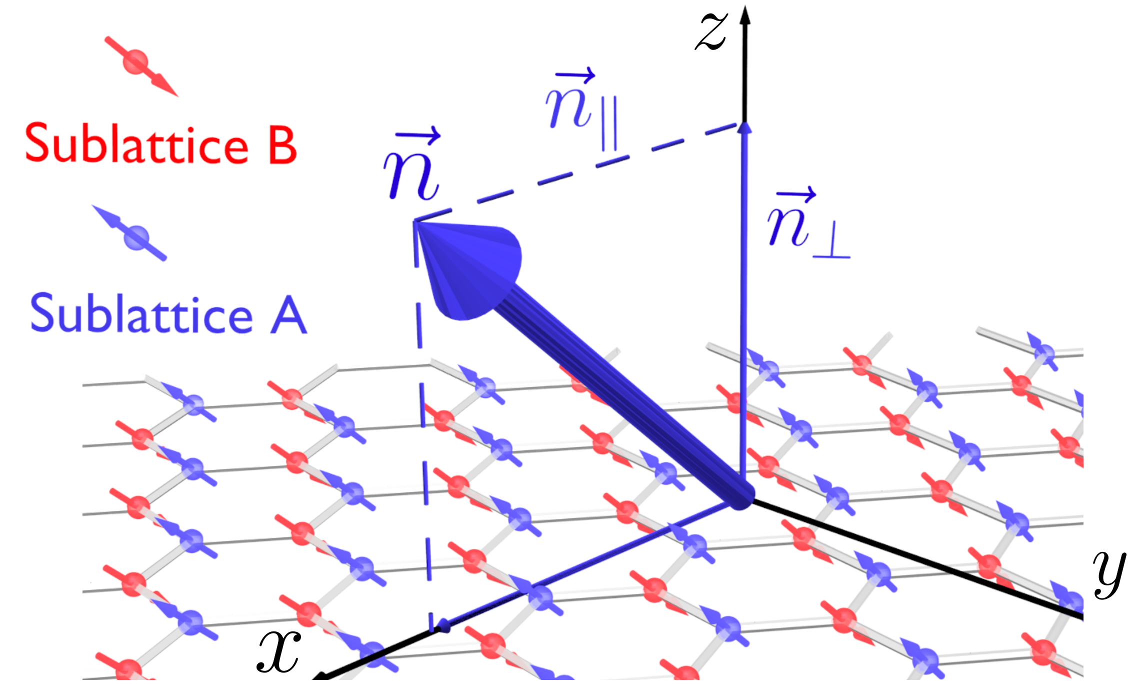


$\boldsymbol{s}_{\pm} = (\boldsymbol{s}_A \pm \boldsymbol{s}_B)/2$ - electron spin densities

equations of motion:

$$\dot{\boldsymbol{n}} = -\Omega\,\boldsymbol{n}\times\boldsymbol{m} + \boldsymbol{n}\times\boldsymbol{s}^{+} + \boldsymbol{m}\times\boldsymbol{s}^{-} + \dots,$$

$$\dot{\boldsymbol{m}} = \boldsymbol{m}\times\boldsymbol{s}^{+} + \boldsymbol{n}\times\boldsymbol{s}^{-} + \dots$$

$\lambda_{\rm so} \ll \hbar/\tau$

$$\delta\boldsymbol{s}^{+} = \alpha_m \dot{\boldsymbol{m}}$$

$$\delta\boldsymbol{s}^{-} = \alpha_n \dot{\boldsymbol{n}}$$

α_m - very very large

α_n - vanishingly small

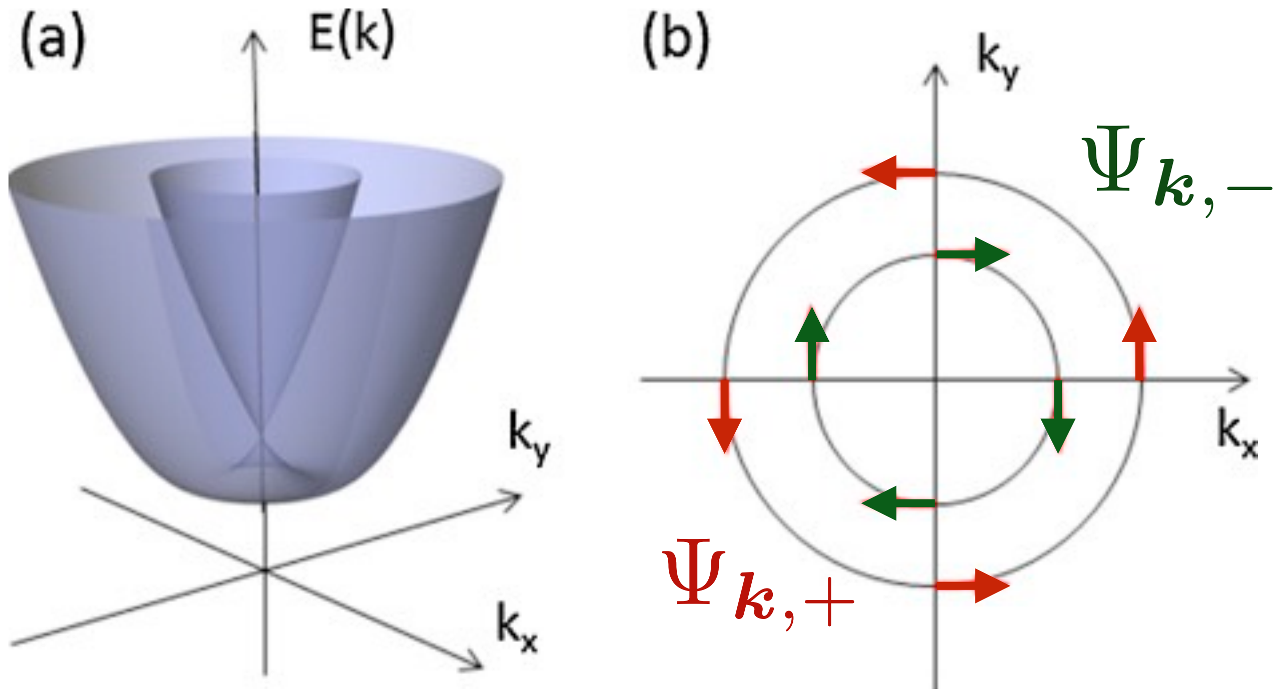
$\lambda_{\rm so} \gg \hbar/\tau$

$$\delta\boldsymbol{s}^{+} = \alpha_m^{\parallel} \dot{\boldsymbol{m}}_{\parallel}$$

$$\delta\boldsymbol{s}^{-} = \alpha_n^{\perp} \dot{\boldsymbol{n}}_{\perp}$$

$\alpha_m^{\parallel} \ll \alpha_m \qquad \alpha_n^{\perp} \gg \alpha_n$

Giant anisotropy
from spin-orbit split Fermi surfaces



$\lambda_{\rm so} \ll \hbar/\tau$

$$\langle \Psi_{\boldsymbol{k},+} | \sigma_z | \Psi_{\boldsymbol{k},+} \rangle = 0$$
$$\langle \Psi_{\boldsymbol{k},-} | \sigma_z | \Psi_{\boldsymbol{k},-} \rangle = 0$$

$$\langle \Psi_{\boldsymbol{k},+} | \sigma_z | \Psi_{\boldsymbol{k},-} \rangle = 1$$
$$\langle \Psi_{\boldsymbol{k},-} | \sigma_z | \Psi_{\boldsymbol{k},+} \rangle = 1$$

$\lambda_{\rm so} \gg \hbar/\tau$

$$\langle \Psi_{\boldsymbol{k},+} | \sigma_z | \Psi_{\boldsymbol{k},+} \rangle = 0$$
$$\langle \Psi_{\boldsymbol{k},-} | \sigma_z | \Psi_{\boldsymbol{k},-} \rangle = 0$$

~~$\langle \Psi_{\boldsymbol{k},-} | \sigma_z | \Psi_{\boldsymbol{k},+} \rangle$~~
~~$\langle \Psi_{\boldsymbol{k},+} | \sigma_z | \Psi_{\boldsymbol{k},-} \rangle$~~

Undamped non-equilibrium precession

$$\dot{\boldsymbol{n}} = -\Omega\, \boldsymbol{n} \times \boldsymbol{m} + \alpha_m^{\|}\, \boldsymbol{n} \times \dot{\boldsymbol{m}}_{\|} + \alpha_n^{\perp}\, \boldsymbol{m} \times \dot{\boldsymbol{n}}_{\perp} + \ldots,$$

$$\dot{\boldsymbol{m}} = \alpha_n^{\perp}\, \boldsymbol{n} \times \dot{\boldsymbol{n}}_{\perp} + \alpha_m^{\|}\, \boldsymbol{m} \times \dot{\boldsymbol{m}}_{\|} + \ldots.$$

For Neel vector in plane $\boldsymbol{n} = \boldsymbol{n}_{\|}$ and
non-equilibrium magnetization perpendicular to the plane $\boldsymbol{m} = \boldsymbol{m}_{\perp}$

one finds an undamped precession

$$\dot{\boldsymbol{n}} = -\Omega\, \boldsymbol{n} \times \boldsymbol{m},$$

$$\dot{\boldsymbol{m}} = 0$$

with the frequency: Ωm

Ivan Ado
Radboud University, Nijmegen

Robert Sokolewicz
Radboud University, Nijmegen

Misha Baglai
Uppsala University, Sweden

Too many open questions to be listed

Thank you for your attention