האוניברסיטה העברית בירושלים
THE HEBREW UNIVERSITY OF JERUSALEM

SPIN SUPERFLUIDITY IN MAGNETICALLY ORDERED SOLIDS AND SPIN-1 BEC

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Spintronics Meets Topology in Quantum Materials
KITP, Santa Barbara, November 12, 2019

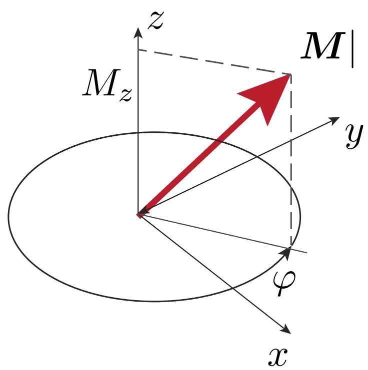
Content

- Superfluid spin current -> superfluid spin transport
- Experimental evidence of spin superfluidity
- Interplay of mass and spin superfluidity in spin-1 BEC

Analogy of superfluid hydrodynamics and magnetodynamics

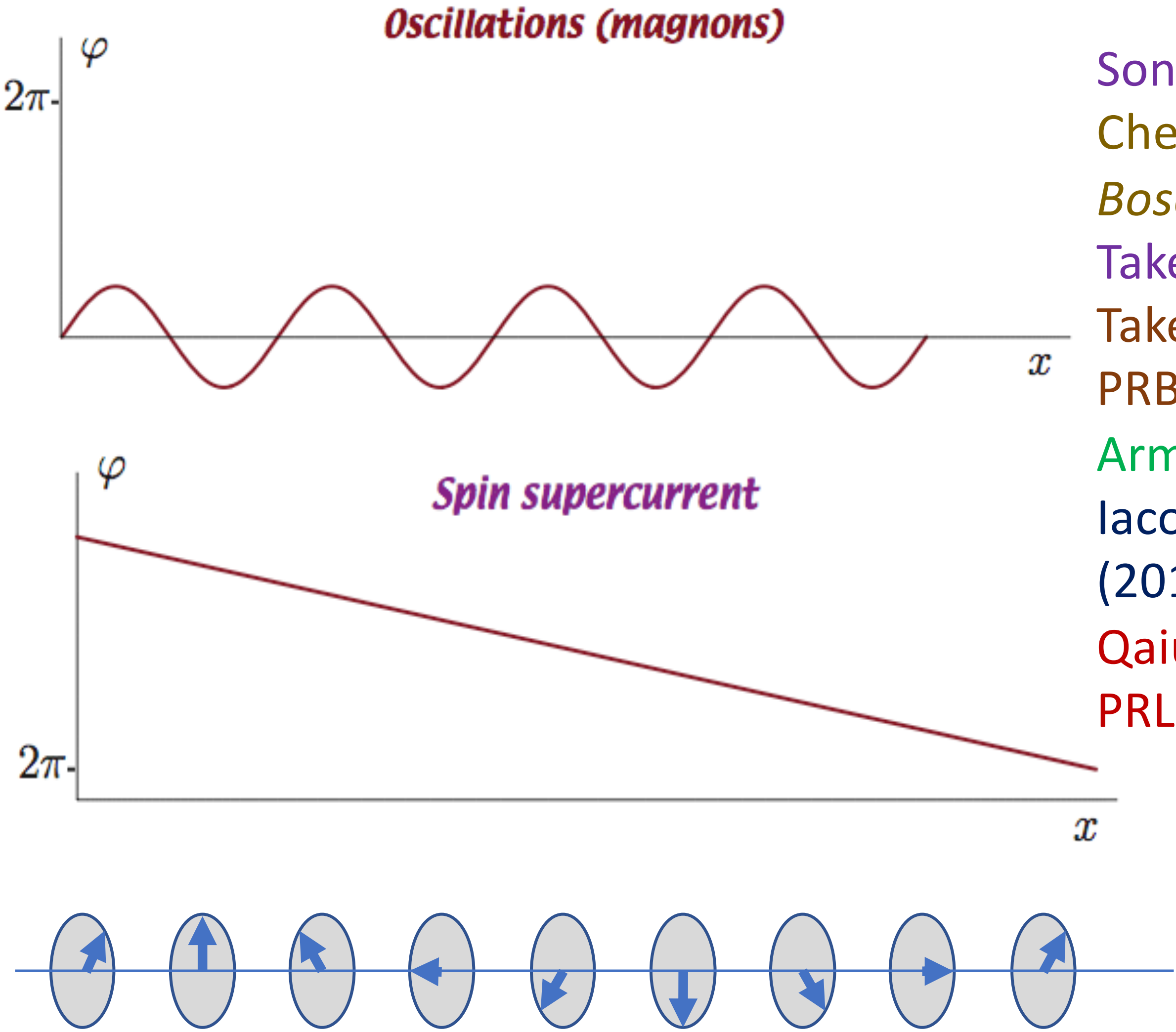
	Superfluid	Ferromagnet
Order parameter	$\psi = \psi_0 e^{i\varphi} = \sqrt{n} e^{i\varphi}$	$\boldsymbol{M}$
Pair of conjugated variables	Density n Phase φ	Magnetization M_z Rotation angle φ
Hamilton equations:	$\frac{d\varphi}{dt} = -\frac{\delta E}{\delta n},$ $\frac{dn}{dt} = \frac{\delta E}{\hbar \delta\varphi} = -\boldsymbol{\nabla} \cdot \boldsymbol{j}$	$\frac{d\varphi}{dt} = -\gamma \frac{M_z}{\chi},$ $-\frac{1}{\gamma}\frac{dM_z}{dt} + \boldsymbol{\nabla} \cdot \boldsymbol{J}^z = 0$

Current: $j \propto \nabla\varphi$ $\quad J^z \propto \nabla\varphi$



Halperin & Hohenberg, Phys. Rev. **188**, 898 (1969)

Hydrodynamic Theory of Spin Waves



Sonin, JETP (1978), Adv. Phys. **59**,181 (2010)

Chen & MacDonald, in: *Universal Themes of Bose-Einstein Condensation*, CUP, 2017

Takei and Tserkovnyak, PRL, **112**, 227201 (2014)

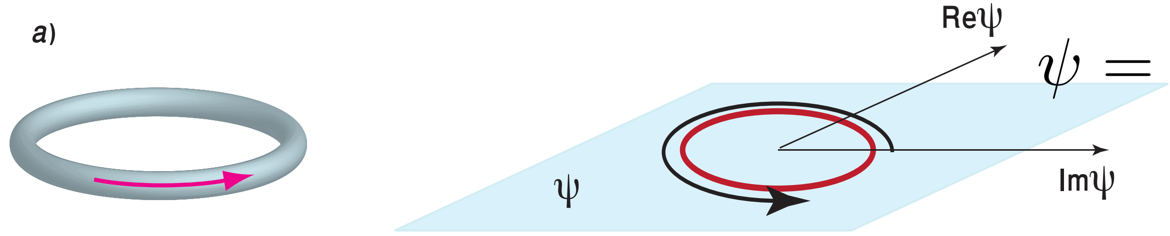
Takei, Halperin, Yacoby, and Tserkovnyak, PRB **90**, 094408 (2014)

Armaitis and Duine, PRA **95**, 053607 (2017)

Iacocca, Silva, and Hoefer, PRL, **118**, 017203 (2017)

Qaiumzadeh, Skarsvag, Holmqvist, and Brataas, PRL, **118**, 137201 (2017)

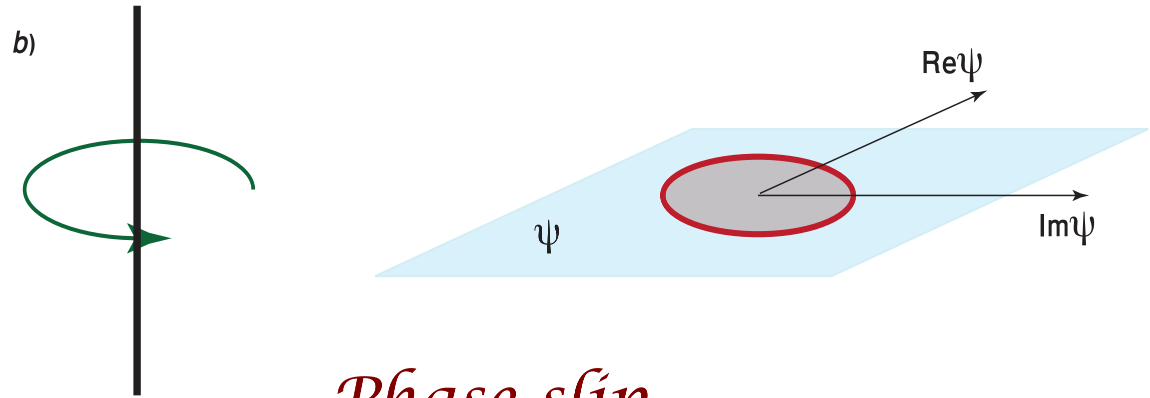
Topological stability of supercurrents (persistent currents)



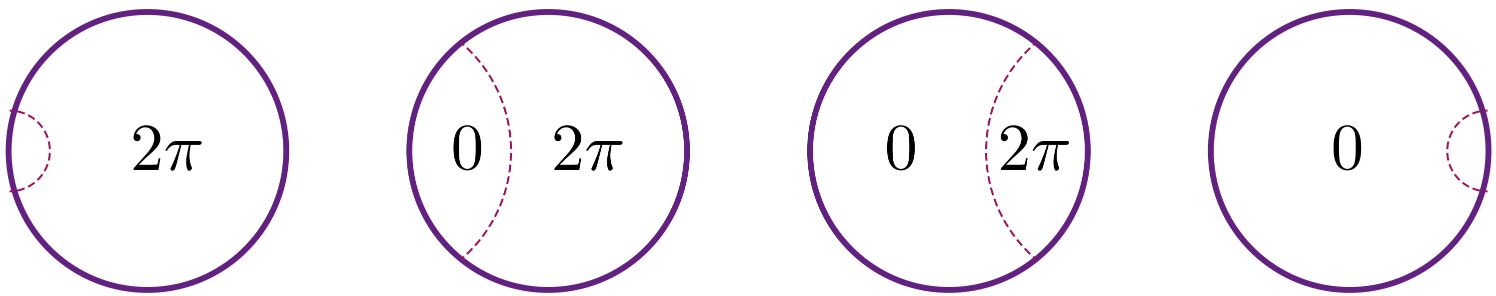
$$\psi = \psi_0 e^{i\varphi} = \sqrt{n}e^{i\varphi}$$

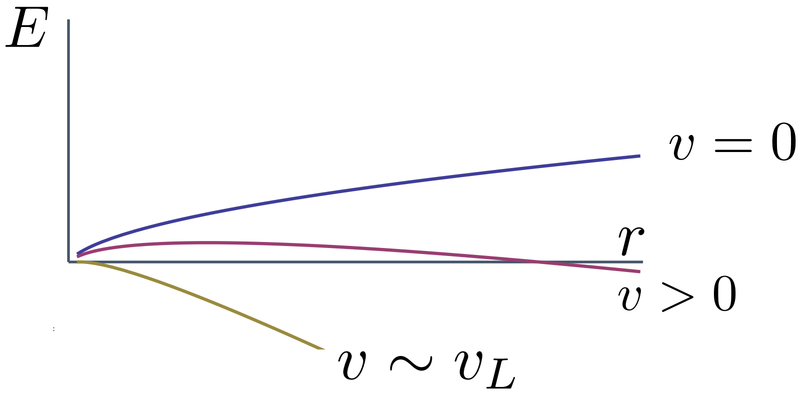
Phase variation around the ring: $\delta\varphi = 2\pi n$ $\quad n$ is a topological charge

Vortex:



Phase slip





Barrier height:

$$E_b = \frac{\rho \hbar^2}{4\pi m^2} \ln \frac{\hbar}{m v r_c}$$

Core radius: $r_c \sim \dfrac{\hbar}{m c_s}$

Barriers for phase slips vanish when the superfluid velocity becomes of the order of the sound velocity (phase gradient of the order of the inverse core radius).

Landau criterion: any elementary excitation increases the energy of the current state.

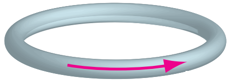
$$\varepsilon(\boldsymbol{p}) = \varepsilon_0(\boldsymbol{p}) - \boldsymbol{p}\cdot\boldsymbol{v}_s > 0$$

$$\omega(\boldsymbol{k}) = c_s k - \boldsymbol{v}_s \boldsymbol{k} > 0$$

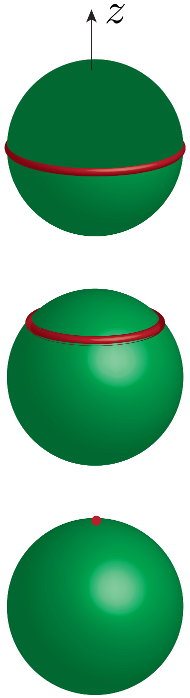
$$\boldsymbol{p} = \hbar \boldsymbol{k}, \quad \varepsilon(\boldsymbol{p}) = \hbar\omega(\boldsymbol{k})$$

Landau critical velocity: $v_L = c_s$

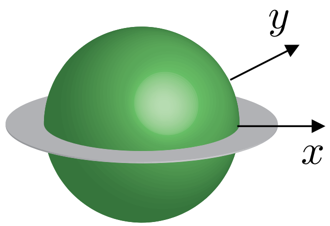
Topological stability of spin supercurrents in ferromagnets



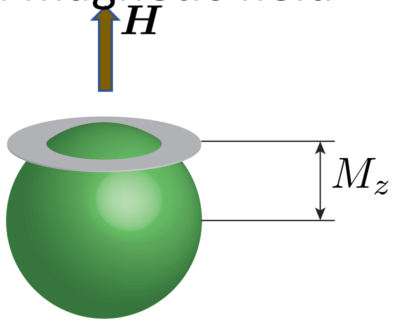
Isotropic ferromagnet



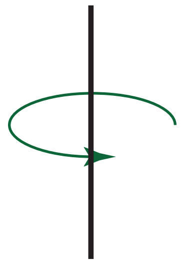
Easy-plane ferromagnet

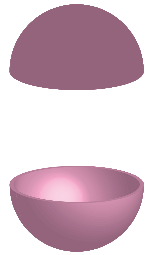


Easy-plane ferromagnet
In a magnetic field



Magnetic vortex:





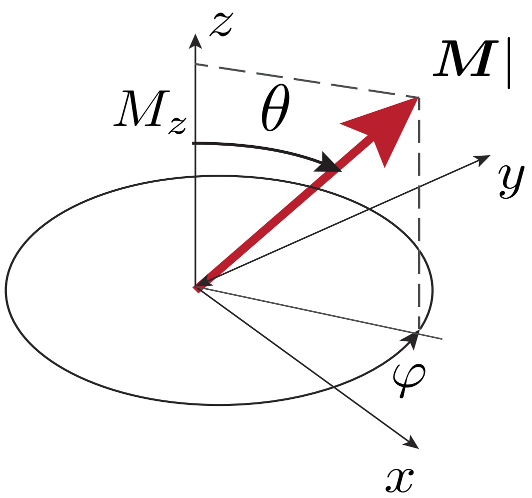
Pumping-supported precession

Bose-Einstein condensation of quasi-equilibrium magnons at room temperature under pumping

Demokritov et al., Nature **443**. 430 (2006)

Spin waves and the Landau criterion in ferromagnets

$$M_x = M\cos\theta\cos\varphi$$
$$M_y = M\cos\theta\sin\varphi$$
$$M_z = M\sin\theta$$



$$\frac{d\boldsymbol{M}}{dt} = \gamma\,[\boldsymbol{H}_{eff}\times\boldsymbol{M}] \quad \boldsymbol{H}_{eff} = -\frac{\partial\mathcal{H}}{\partial\boldsymbol{M}} + \nabla_j\frac{\partial\mathcal{H}}{\partial\nabla_j\boldsymbol{M}}$$

$$\mathcal{H} = \frac{M_z^2}{2\chi} + A\nabla_i\boldsymbol{M}\cdot\nabla_i\boldsymbol{M} - M_zH_0 \cdot$$

$$\frac{M\cos\theta\dot{\theta}}{\gamma} = -\boldsymbol{\nabla}\cdot\boldsymbol{J}_s,$$

$$\frac{\dot{\varphi}}{\gamma} = -\frac{M\sin\theta}{\chi}[1-\chi A(\nabla\varphi)^2] + H_0 + \frac{AM\nabla^2\theta}{\cos\theta}$$

Current state: $M_z = M\sin\theta_0 = const, \quad \nabla\varphi = K = const$

Spectrum of plane spin waves $\propto e^{i\boldsymbol{k}\cdot\boldsymbol{r}-i\omega t}$:

$$(\omega + 2\gamma M_z A\boldsymbol{K}\cdot\boldsymbol{k})^2 = \tilde{c}_s^2 k^2$$

Pseudo-Doppler effect

Landau critical gradient:

$$K_c = \frac{1}{\sqrt{\chi A}} = \frac{\gamma M_\perp}{\chi c_s}$$

Spin wave velocity in the ground state:

$$c_s = \gamma M_\perp \sqrt{\frac{A}{\chi}}$$

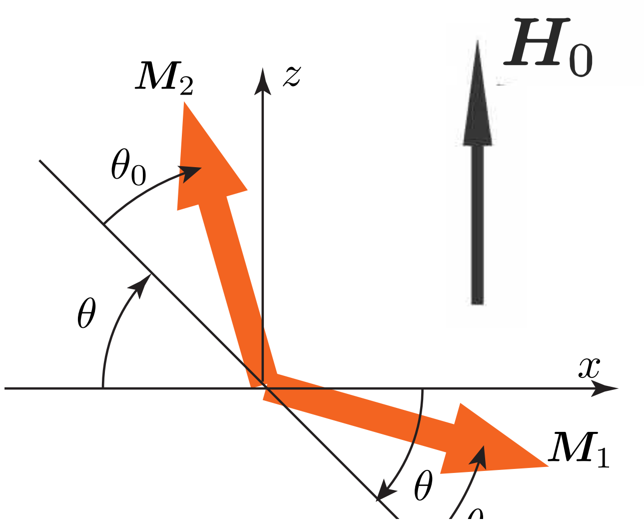
Spin wave velocity in the current state:

$$\tilde{c}_s = c_s\sqrt{1-\chi A K^2}$$

Spin waves and the Landau criterion in bipartite antiferromagnets

$$\frac{d\boldsymbol{M}_i}{dt} = \gamma\,[\boldsymbol{M}_i \times \boldsymbol{H}_i] \qquad \boldsymbol{H}_i = \frac{\delta\mathcal{H}}{\delta\boldsymbol{M}_i}$$

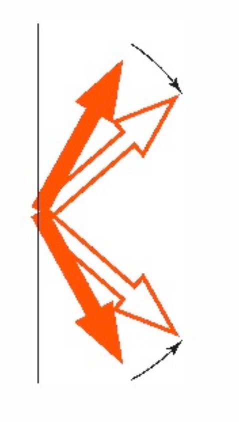
$$\mathcal{H} = \frac{\boldsymbol{M}_1\cdot\boldsymbol{M}_2}{\chi} + \frac{A(\nabla_i\boldsymbol{M}_1\cdot\nabla_i\boldsymbol{M}_1+\nabla_i\boldsymbol{M}_2\cdot\nabla_i\boldsymbol{M}_2)}{2} + A_{12}\nabla_j\boldsymbol{M}_1\cdot\nabla_j\boldsymbol{M}_2 - \boldsymbol{H}_0\cdot(\boldsymbol{M}_1+\boldsymbol{M}_2)$$



$$\theta_0 = \frac{\pi+\theta_1-\theta_2}{2}, \quad \theta = \frac{\theta_1+\theta_2-\pi}{2},$$

$$\varphi_0 = \frac{\varphi_1+\varphi_2}{2}, \quad \varphi = \frac{\varphi_1-\varphi_2}{2}$$

Gapeless Goldstone mode

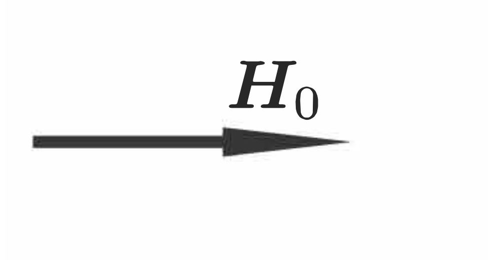


$$(\omega + \gamma m_z A_- \boldsymbol{K} \cdot \boldsymbol{k})^2 = \tilde{c}_s^2 k^2$$

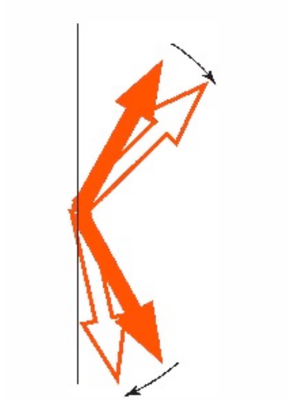
$$\tilde{c}_s = c_s \sqrt{1 - \frac{\chi A_- K^2}{2}}, \quad c_s = \gamma M_\perp \sqrt{\frac{2A_-}{\chi}}$$

$$A_- = A - A_{12}$$

$$K_c = \sqrt{\frac{2}{\chi A_-}} = \frac{\gamma M_\perp}{\chi c_s}$$



Gapped mode



$$(\omega + \gamma m_z A_- \boldsymbol{K} \cdot \boldsymbol{k})^2$$
$$= (\omega_0^2 + c_s^2 k^2)(1 + \chi A_{12} K^2)$$

Gap:

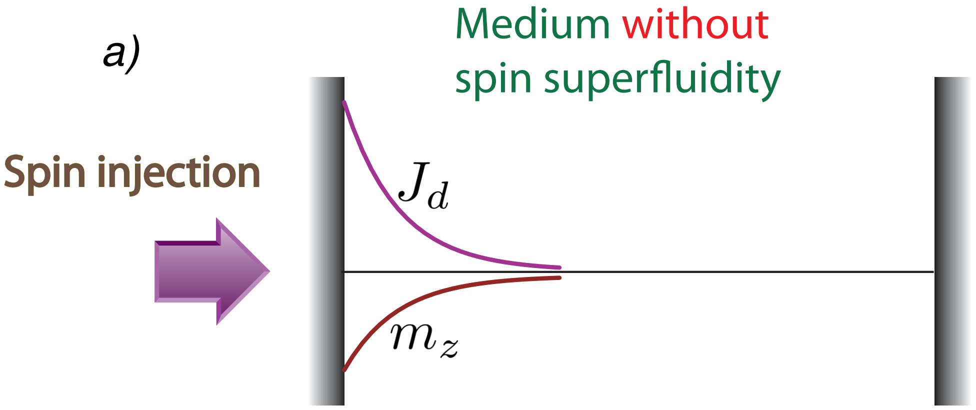
$$\omega_0 = \sqrt{\frac{\gamma^2 m_z^2}{\chi^2} - c_s^2 K^2} = \sqrt{\gamma^2 H_0^2 - c_s^2 K^2}$$

Landau critical gradient:

$$K_c = \frac{\gamma m_z}{\chi c_s} = \frac{\gamma H_0}{c_s}$$

Observable consequences of spin supercurrents

Sonin, JETP (1978)
Takei & Tserkovnyak, PRL (2014)

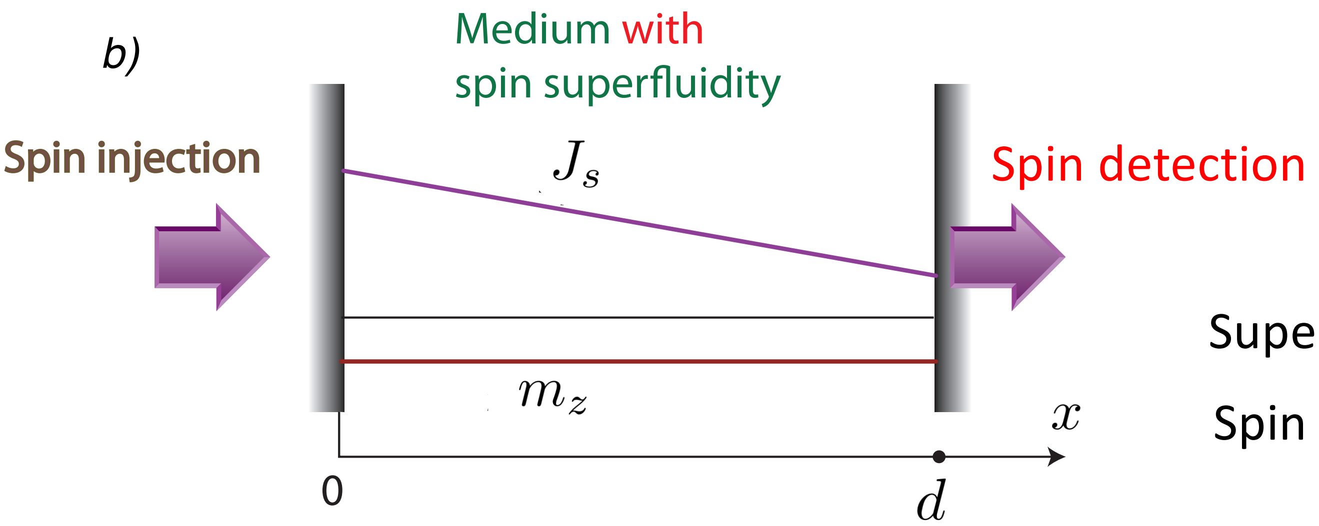


$$\frac{dm_z}{dt} + \nabla \cdot \boldsymbol{J}_d + \frac{m_z}{T_1} = 0$$

Spin diffusion current: $\boldsymbol{J}_d = -D\nabla m_z$

Spin diffusion length: $L_d = \sqrt{DT_1}$

Spin accumulation: $m_z(d) \propto e^{-x/L_d}$



$$\frac{d\varphi}{dt} = -\frac{\gamma m_z}{\chi}.$$

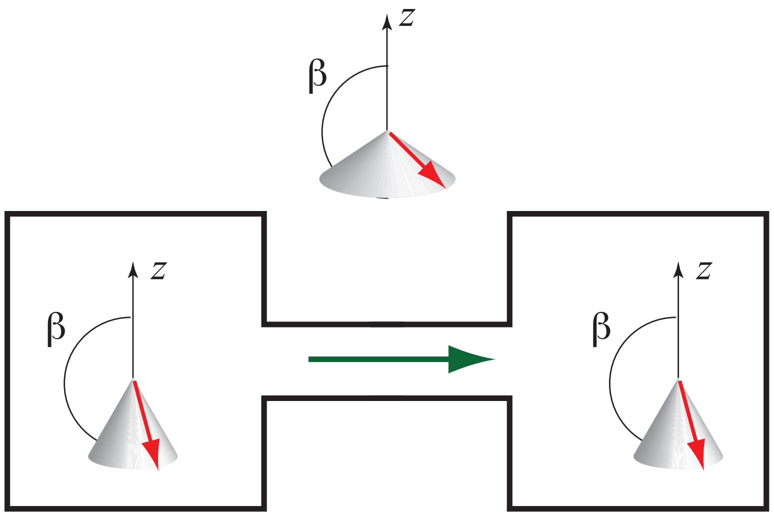
$$\frac{dm_z}{dt} + \nabla \cdot \boldsymbol{J}_s + \frac{m_z}{T_1} = 0$$

Superfluid spin current: $\boldsymbol{J}_s = -\mathcal{A}\nabla\varphi$

Spin accumulation:

$$m_z = \frac{T_1}{d + v_d T_1} J_0 \to \frac{J_0 T_1}{d}$$

Superfluid ³He-B



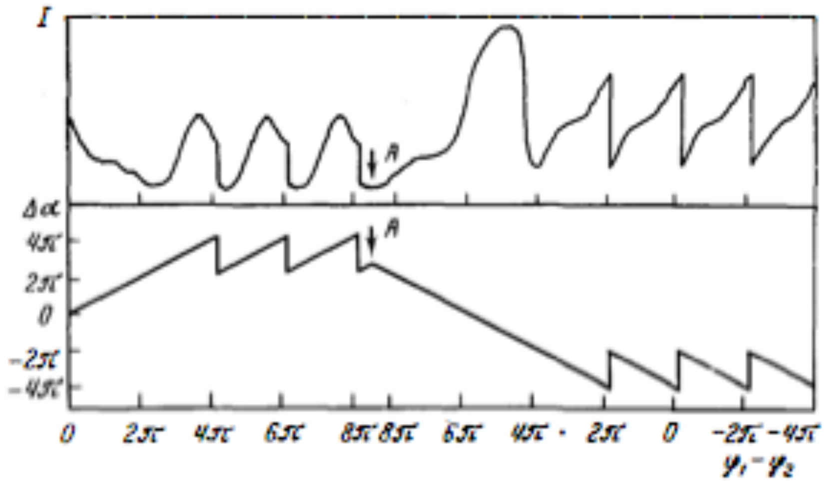


FIG. 2. Signal from the receiving coil and proposed profile of the precession phase difference along the channel. $P = 11$ bar, $\gamma H/2\pi = 460$ kHZ, $T = 0.584\ T_c$, $\omega_d/2\pi = 460.40$ kHz.

A.S. Borovik-Romanov, Yu.M. Bunkov, V.V. Dmitriev, and Yu.M. Mukharskiy, JETP Lett. **45**, 124 (1987)

Bozhko , Serga, Clausen, Vasyuchka, Heussner, Melkov, Pomyalov, L'vov, and Hillebrands,
Nat. Phys. **12,** 1057 (2016) Supercurrent in a room-temperature Bose-Einstein magnon condensate

the x-axis (see Fig. 2d).

There are two reasons for the x-dependence of the BEC phase φ in our experiment. The first is the already mentioned temperature dependence of ω_c. Within the hot spot of radius R centred at $x=0$ (that is, for $|x|<R$) the temperature $T(x)$ is higher than the temperature T_0 of the rest of the film (see Fig. 2d). Since in an in-plane magnetized YIG film $d\omega_c(T)/dT<0$, the BEC frequency in the spot is smaller than outside: $\delta\omega_c(x)=\omega_c(T(x))-\omega_c(T_0)<0$. Correspondingly, the phase accumulation $\delta\varphi(x)=\delta\omega_c(x)t$ inside of the spot is smaller than in the surrounding cold film. Therefore, the phase gradient $\partial\delta\varphi(x)/\partial x$ is positive for $x>0$ and negative for $x<0$. It means that a thermally induced supercurrent flows out from the spot (mostly in x-direction), as is shown by the red arrows in Fig. 2d:

$$J_T = N_c D_x \frac{\partial(\delta\omega_c t)}{\partial x} \qquad (3)$$

This outflow decreases the magnon BEC density $N_c(x)$ in the spot, $|x|<R$, with respect to that in the cold film, where $N_c(x \gg R)=N_c^0$.

Spatial deviations in the density $N_c(x)$ of the magnon condensate constitute the second reason for the variation of its phase $\partial\varphi/\partial x \neq 0$.

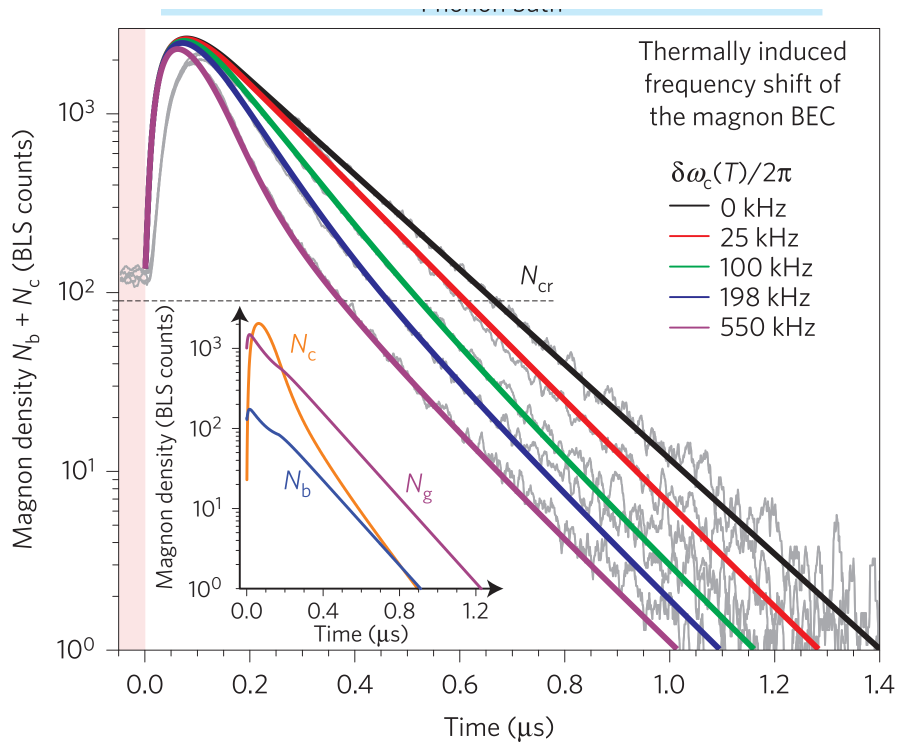
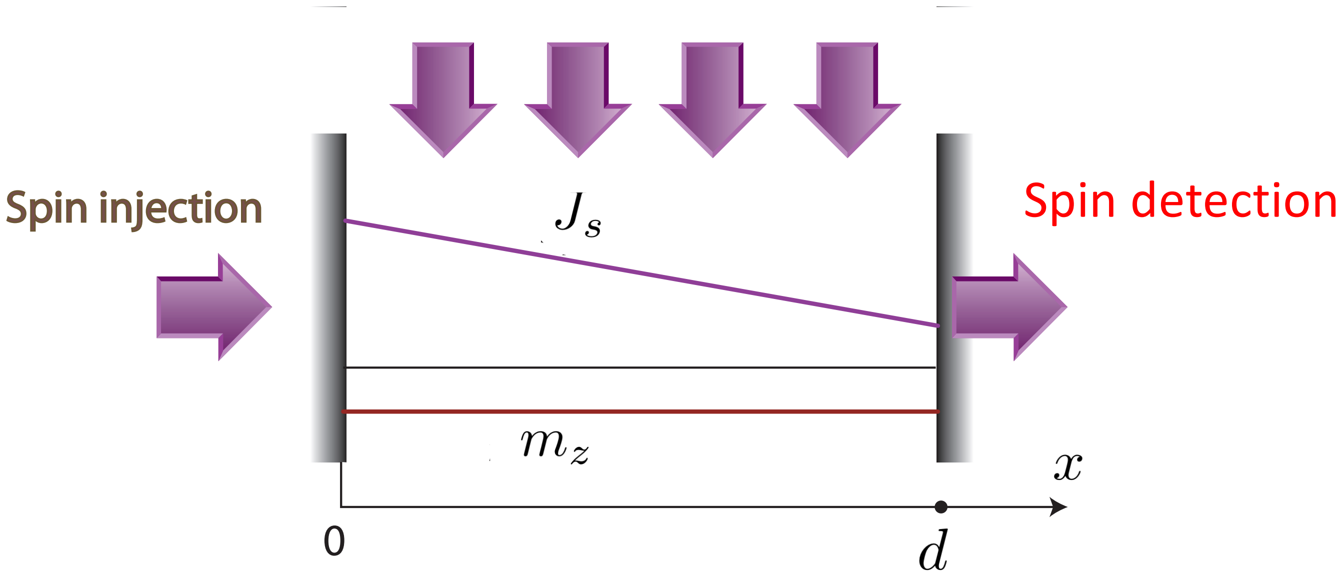


Figure 5 | Theoretically calculated magnon dynamics in a thermal gradient. Theoretical dependencies (coloured lines) of the observable

$$\text{Phase accumulation } \delta\varphi = \delta\omega_c t < \frac{2\pi}{3}!$$

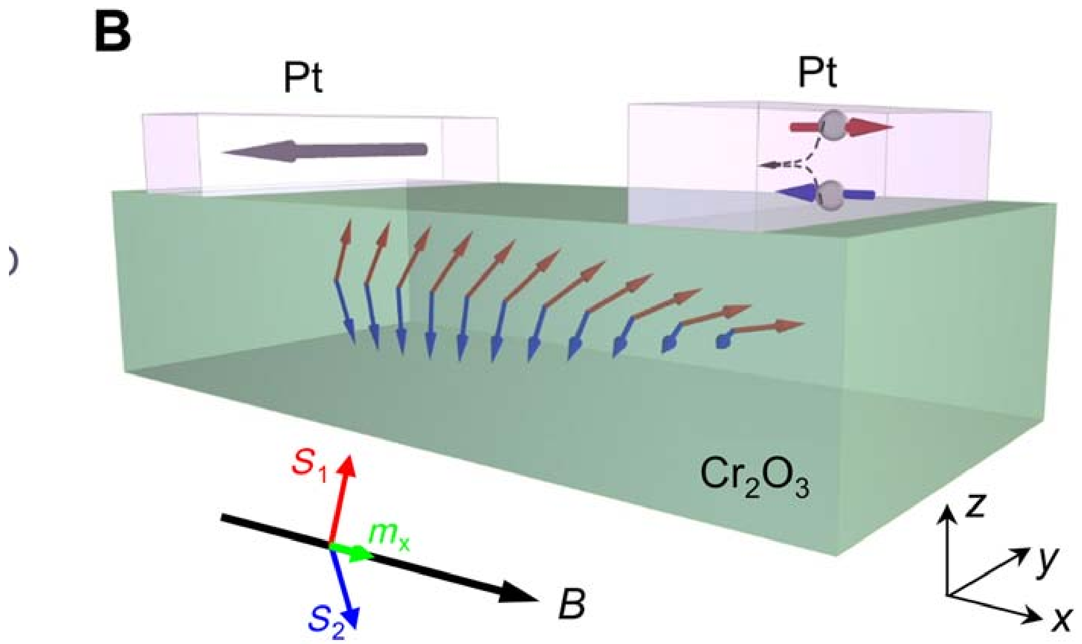
Superfluid spin transport in a non-equilibrium magnon BEC Supported by magnon pumping???



PHYSICS

Experimental signatures of spin superfluid ground state in canted antiferromagnet Cr_2O_3 via nonlocal spin transport

Wei Yuan,[1,2] Qiong Zhu,[1,2] Tang Su,[1,2] Yunyan Yao,[1,2] Wenyu Xing,[1,2] Yangyang Chen,[1,2] Yang Ma,[1,2] Xi Lin,[1,2] Jing Shi,[3]* Ryuichi Shindou,[1,2] X. C. Xie,[1,2]* Wei Han[1,2]*



Yuan et al. (2018), Cr_2O_3

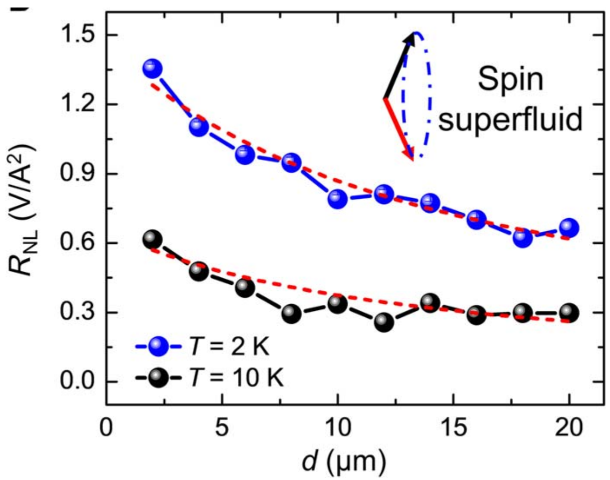


Fig. 3. Spacing dependence of the nonlocal spin transport in spin superfluid ground state. (**A**) The nonlocal spin signal as a function of $1/T$ for the spacing between the two Pt strips (d) of 2, 8, 14, and 20 μm. These results are obtained under the in-plane magnetic field of 9 T. (**B**) The nonlocal spin signal at 2 and 10 K in the spin superfluid ground state as a function of the spacing between the two Pt strips. The red dashed lines are the fitting curves based on spin superfluid model using the Eq. 2.

$$m_z(d) \propto \frac{1}{d + const}$$

Yuan et al. (2018), Cr_2O_3

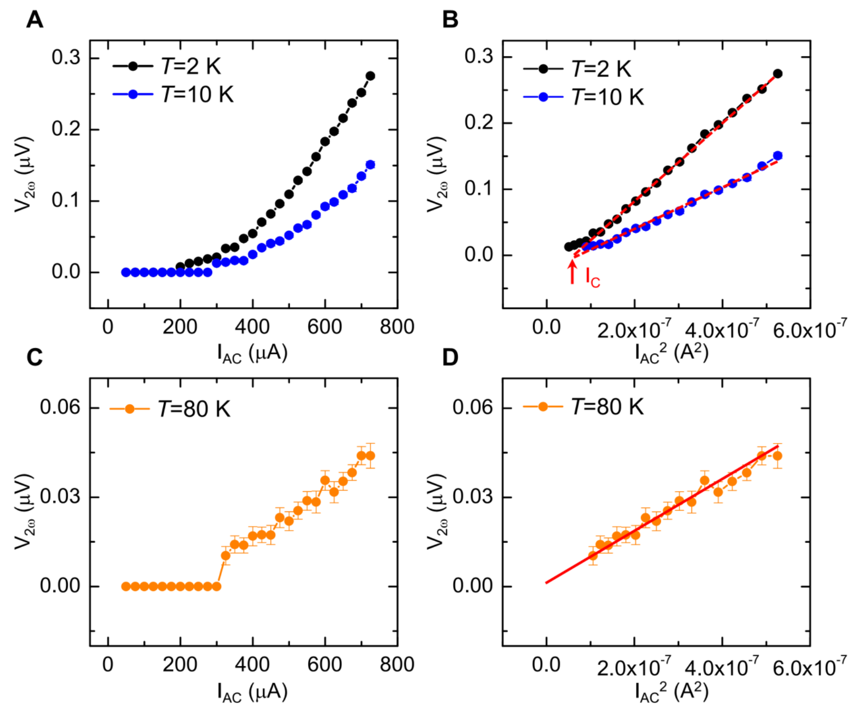


fig. S6. Current dependence of the nonlocal spin transport on the ~19-nm (0001)-oriented Cr_2O_3 film. (**A-B**) The second harmonic spin voltage vs. I and I^2 at $T = 2$ and 10 K and $B = 9$ T on the device with $d = 10$ μm. A critical current (I_C) is observed, which is needed to overcome uniaxial anisotropy to induce the spin superfluid transport. (**C-D**) The second harmonic spin voltage vs. I and I^2 at $T = 80$ K and $B = 9$ T on the device with $d = 10$ μm. The second harmonic voltage is proportional to I^2 without a critical current.

Hydrodynamics of spin-1 BEC

Irreducible basis

$$\boldsymbol{\psi} = \begin{pmatrix} \psi_+ \\ \psi_0 \\ \psi_{-1} \end{pmatrix}$$

Cartesian basis

$$\boldsymbol{\psi} = \begin{pmatrix} \psi_x \\ \psi_y \\ \psi_z \end{pmatrix}$$

$$\psi_x = \frac{\psi_+ - \psi_-}{\sqrt{2}}, \quad \psi_y = \frac{i(\psi_+ + \psi_-)}{\sqrt{2}}, \quad \psi_z = -\psi_0$$

Gross-Pitaevskii equations:

$$i\hbar\frac{\partial \boldsymbol{\psi}}{\partial t} = -\frac{\hbar^2}{2m}\nabla_j^2\boldsymbol{\psi}$$
$$+V|\boldsymbol{\psi}|^2\boldsymbol{\psi} + V_s(|\boldsymbol{\psi}|^2\boldsymbol{\psi} - \boldsymbol{\psi}^2\boldsymbol{\psi}^*) - \gamma\boldsymbol{H}\cdot\boldsymbol{S}|\boldsymbol{\psi}|^2$$

$$j_i = -\frac{i\hbar}{2}(\psi_j^*\nabla_i\psi_j - \psi_j\nabla_i\psi_j^*)$$

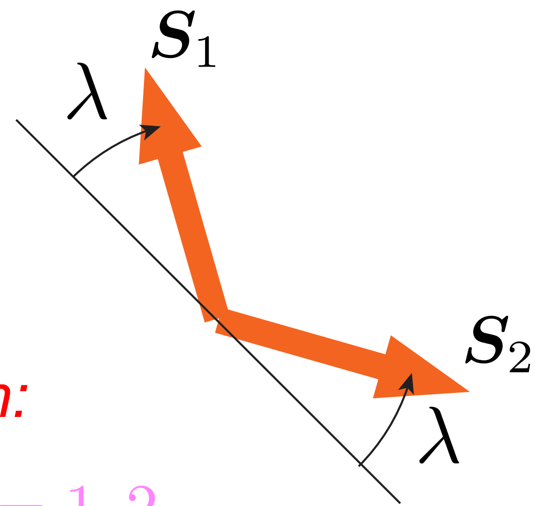
$$\rho = m\boldsymbol{\psi}^*\cdot\boldsymbol{\psi}$$

$$\boldsymbol{S} = i\hbar[\boldsymbol{\psi}\times\boldsymbol{\psi}^*] \quad \boldsymbol{s} = \frac{\boldsymbol{S}}{S} \quad S = \frac{\hbar\rho}{m}$$

Madelung transformation ➡ Hydrodynamics

Hydrodynamical variable: $n, \boldsymbol{v}_s, \boldsymbol{S}_1, \boldsymbol{S}_2$

Sonin, arXiv: 1908.1063



Two subspin vectors $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$:

Total spin: $\boldsymbol{S} = \boldsymbol{S}_1 + \boldsymbol{S}_2$

Antiferromagnetic vector (staggered magnetization): $\boldsymbol{L} = \boldsymbol{S}_1 - \boldsymbol{S}_2$

Extended Landau-Lifshitz-Gilbert equation:

$$n[\boldsymbol{S}_i + (\boldsymbol{v}_s \cdot \boldsymbol{\nabla})\boldsymbol{S}_i] = -\left[\boldsymbol{S}_i \times \frac{\delta \mathcal{H}_0}{\delta \boldsymbol{S}_i}\right], \quad i = 1, 2$$

Ferromagnetic spin-1 BEC

$$\lambda = \frac{\pi}{2}, \quad n[\boldsymbol{S} + (\boldsymbol{v}_s \cdot \boldsymbol{\nabla})\boldsymbol{S}] = -\left[\boldsymbol{S} \times \frac{\delta \mathcal{H}_0}{\delta \boldsymbol{S}}\right]$$

Coexistence of mass and spin superfluidity

Mass superfluidity alone:

Landau critical velocity v_L is equal to the sound wave velocity c_s

Spin superfluidity alone:

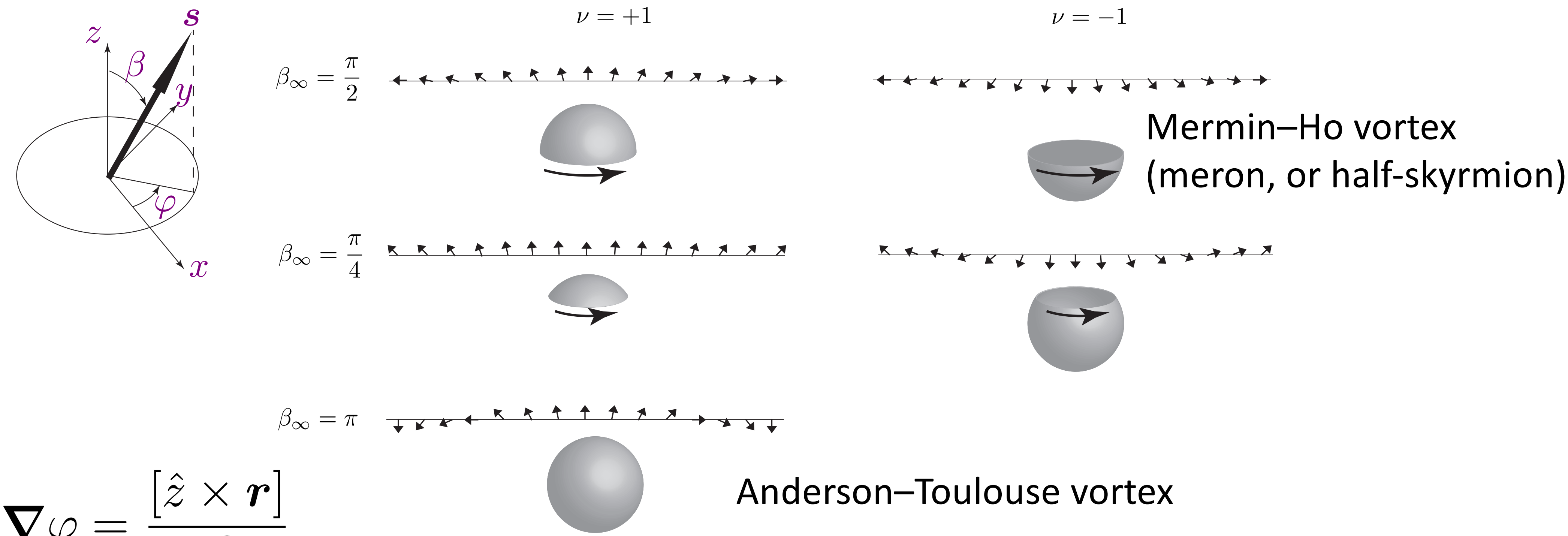
Landau critical velocity v_L is equal to the spin wave velocity c_{sp}

Spin and mass superfluidity coexist: $v_L = \min(c_s, c_{sp})$

Beattle, Moulder, Fletcher, and Hadzibabic, PRL, **110**, 025301 (2013)

Spin-wave velocity: $c_{sp} = s_\perp \sqrt{\frac{G}{2}}, \quad s_\perp = \sin\beta$ $\qquad \boldsymbol{j}^z = -\frac{\hbar^2 \rho}{2m^2} \sin^2\beta \boldsymbol{\nabla}\varphi$

Incompressible superfluids: $c_s \gg c_{sp}$



$$\boldsymbol{\nabla}\varphi = \frac{[\hat{z}\times \boldsymbol{r}]}{r^2}$$

$$\boldsymbol{v}_s = \frac{\hbar(1-\cos\beta_\infty)[\hat{z}\times\boldsymbol{r}]}{mr^2} \qquad \boldsymbol{v}_s = -\frac{\hbar(1+\cos\beta_\infty)[\hat{z}\times\boldsymbol{r}]}{mr^2}$$

$$\text{Circulation: } \frac{\hbar(1-\cos\beta)}{m} \to \frac{\hbar\beta_\infty^2}{2m} \qquad \text{Circulation: } -\frac{\hbar(1+\cos\beta)}{m}$$

Bicirculation vortices (N_Φ, N_φ)
with two topological charges (winding numbers)

$$\boldsymbol{v}_s = \frac{\hbar}{m}(\boldsymbol{\nabla}\Phi - \sin\theta_0 \cos\theta \boldsymbol{\nabla}\varphi_0) \qquad N_\Phi = \frac{1}{2\pi}\oint \boldsymbol{\nabla}\Phi \cdot d\boldsymbol{l}, \quad N_\varphi = \frac{1}{2\pi}\oint \boldsymbol{\nabla}\varphi_0 \cdot d\boldsymbol{l}$$

Circulation of velocity: $$\Gamma = \oint \boldsymbol{v}_s \cdot d\boldsymbol{l} = \frac{h}{m}[N_\Phi - \sin\theta_0(\infty) N_\varphi]$$

Nonsingular vortices

Vortex $(0, N)$:

$$\theta_0(0) = 0, \quad \theta(0) = \pm\frac{\pi}{2}$$

$$\Gamma = -\frac{Nh}{m}\sin\theta_0(\infty)$$

N is integer

Vortex $(N, \pm N)$:

$$\theta_0(0) = \pm\frac{\pi}{2}, \quad \theta(0) = 0$$

$$\Gamma = \frac{Nh}{m}[1 \mp \sin\theta_0(\infty)]$$

N is integer or half-integer

Half-integer vortices

$$\Gamma = \frac{Nh}{m}[1 \mp \sin\theta_0(\infty)] \qquad N = \frac{n}{2}$$

Circulation quantum: $\Gamma = n\kappa \quad \kappa = \dfrac{h}{2m}[1 \mp \sin(\theta_0(\infty)]$

U. Leonhardt and G. E. Volovik, JETP Lett. **72,** 46 (2000)]:

Half-quantum vortex: $\theta_0(\infty) = 0, \quad \kappa = \dfrac{h}{2m}$

In general: $0 < \kappa < \dfrac{h}{m}$

Velocity circulation quantum is tuned by magnetic field

Conclusions:

- The experiment in the easy-plane antiferromagnet shows evidence of long-distance spin superfluid transport.

- In spin-1 BEC mass and spin superfluidity coexist and mutually affect one another. As a result of interplay of two types of superfluidity, metastability of both mass and spin supercurrents is always determined softer modes, which are spin waves in our case.

- In spin-1 BEC vortices are characterized by two topological charges (winding numbers). The velocity circulation is not a topological charge! Its quantum can be tuned continuously by a magnetic field

Thanks