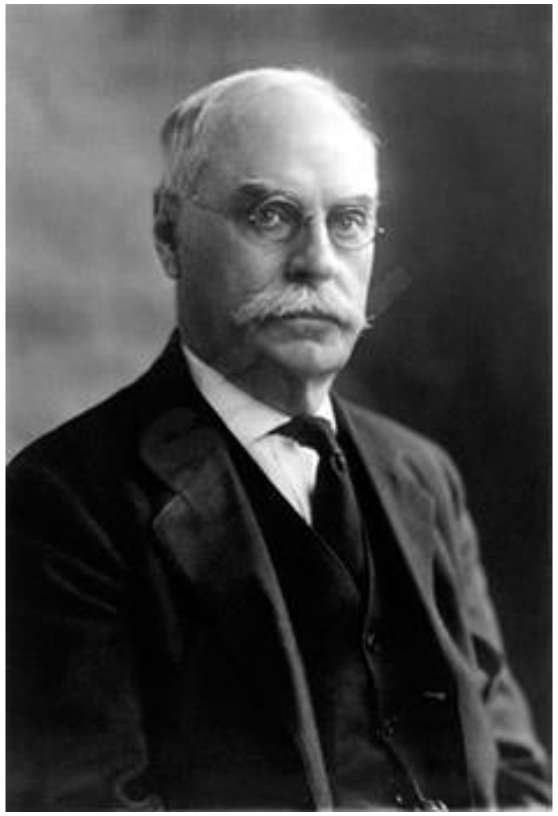
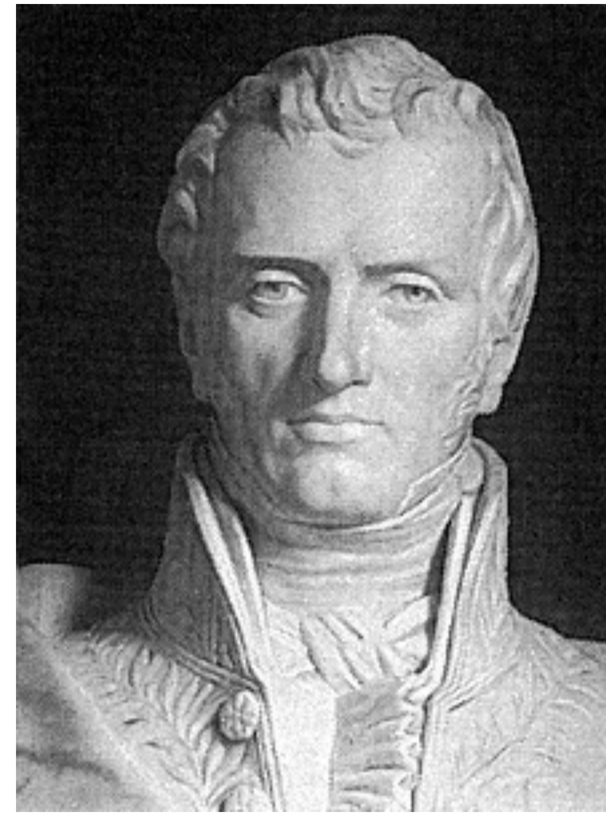
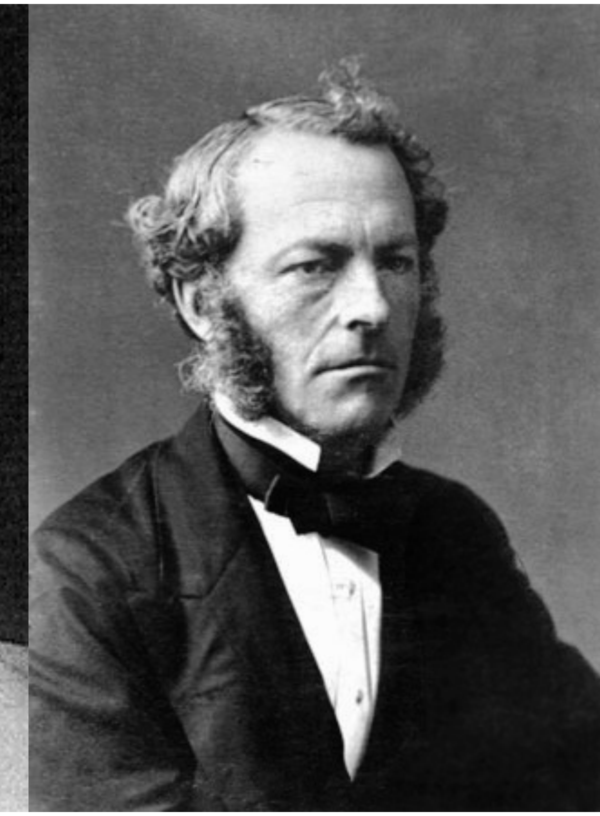
Hall effect in viscous electron fluids



Edwin Herbert Hall
1855-1938



Claude-Louis Navier
1785-1836



George Gabriel Stokes
1819-1903

KITP, Santa Barbara, November 13, 2019

from December 31, 2019

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Two postdoc openings (theory) - jobs starting on April 1, 2020 (marco.polini@icloud.com)

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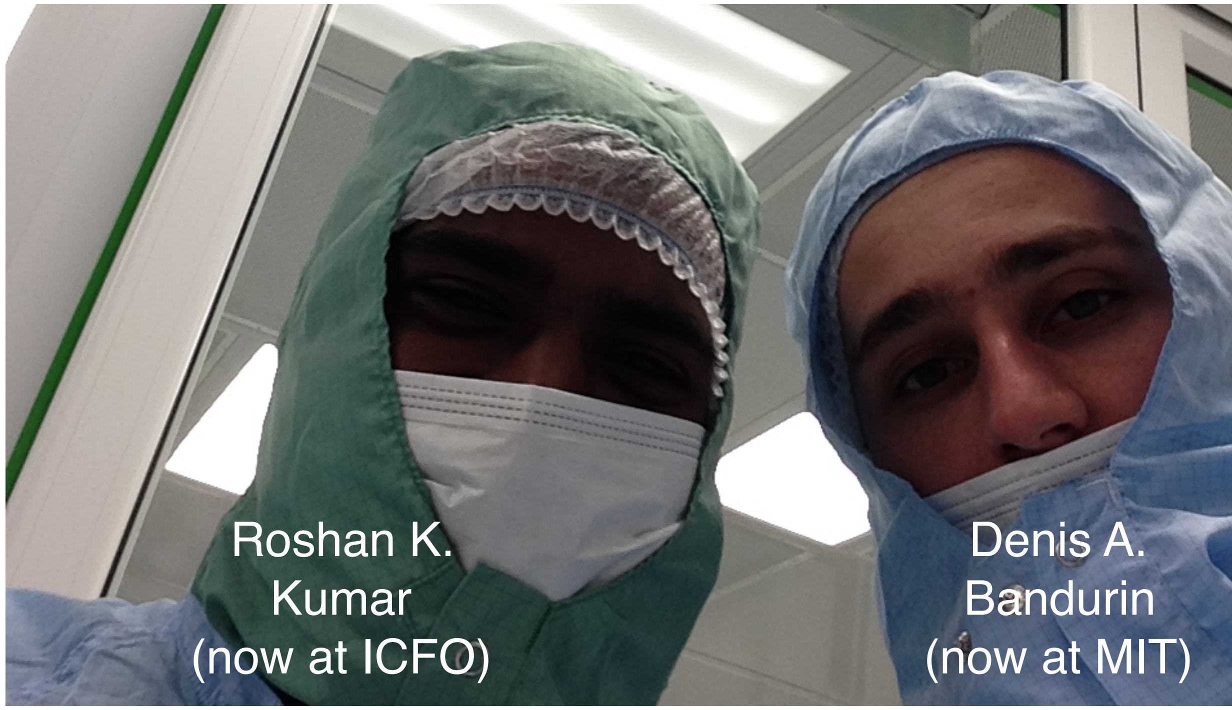
experiments



theory

For a recent popular review see
M. Polini and A.K. Geim, arXiv:1909.10615 (Physics Today)

Introduction

Solid-state hydrodynamics

- In a fluid or gas, hydrodynamics works because one has local conservation of momentum and energy

- All transport properties governed by just 3 quantities (when B=0): the shear viscosity (η), the bulk viscosity (ζ), and the thermal conductivity (κ); in this talk we will always use the kinematic viscosity ν=η/(nm)

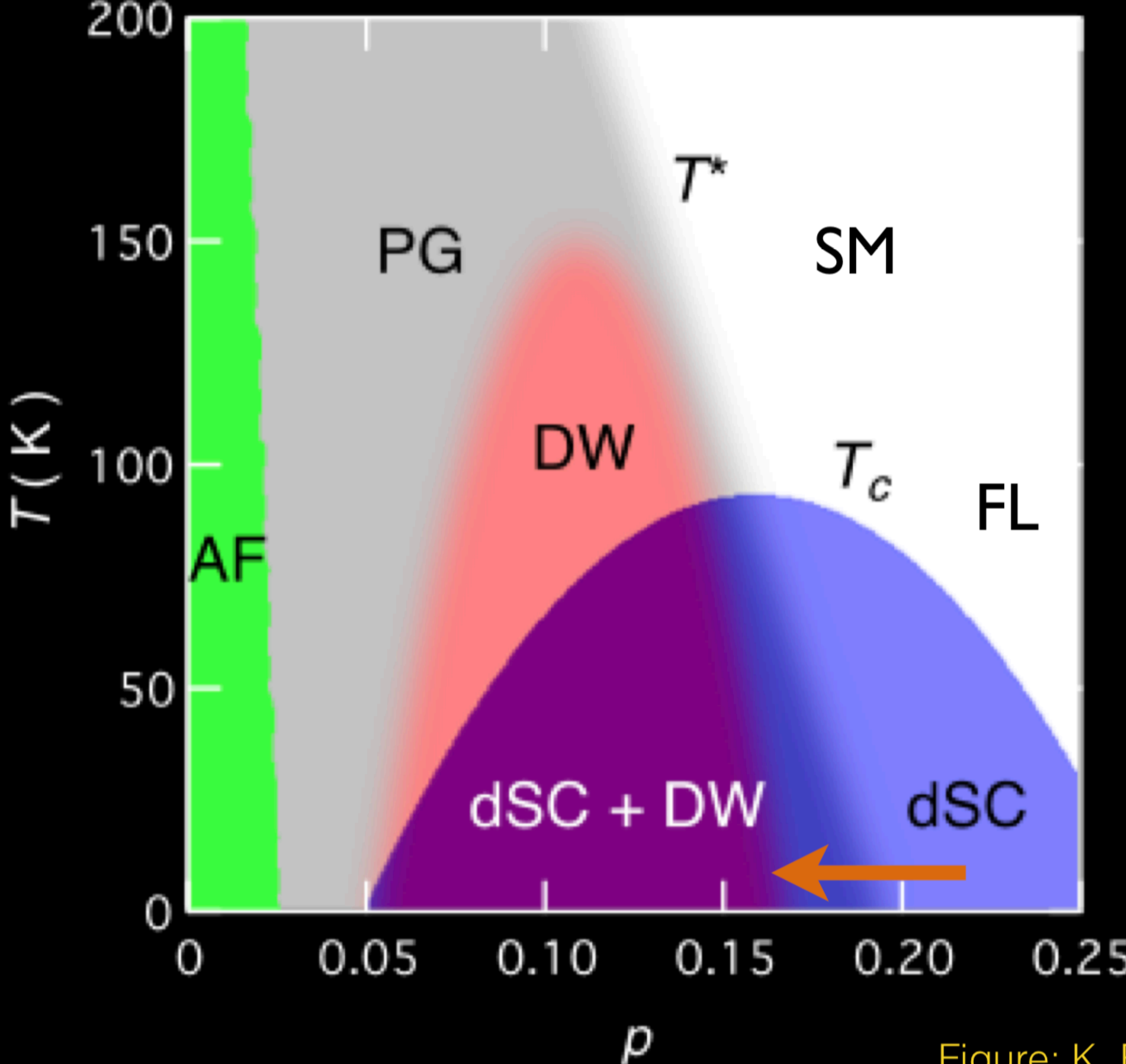
- Near-perfect fluids: measured (!!!) record-low shear viscosity in strongly-interacting quantum fluids like quark-gluon plasmas and ultracold Fermi gases at unitarity

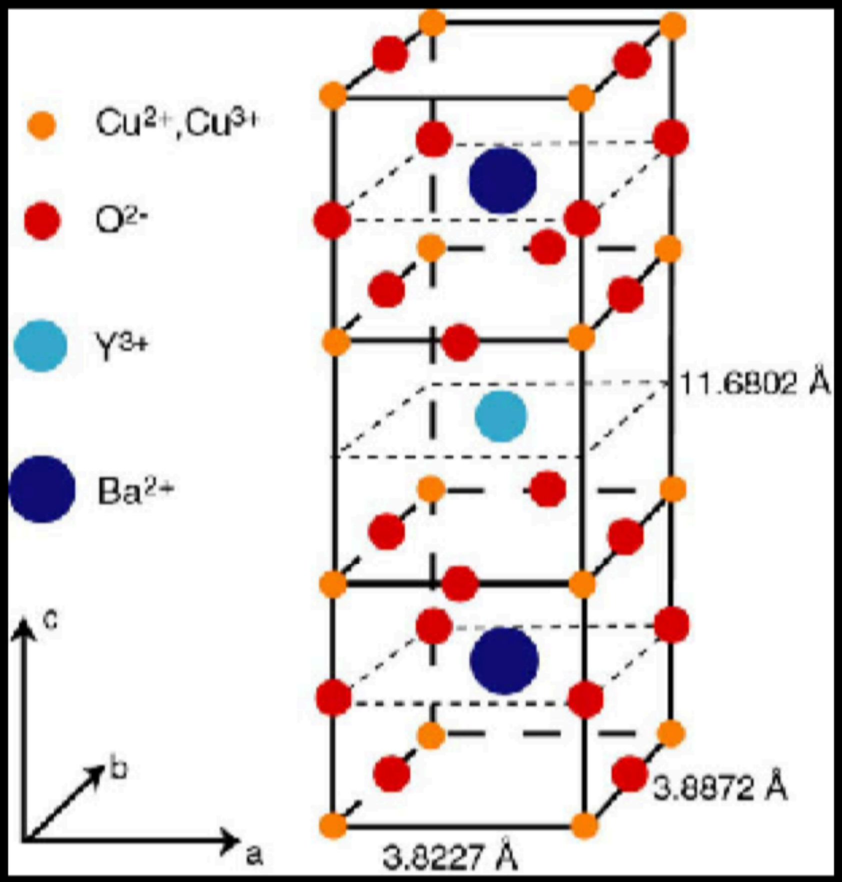
- What about electron systems in a solid? They can exchange energy and momentum with the lattice.

In a crystal, hydrodynamics is relevant when

$$\ell_{\rm ee} \ll \ell, W$$

Strange metals (I)

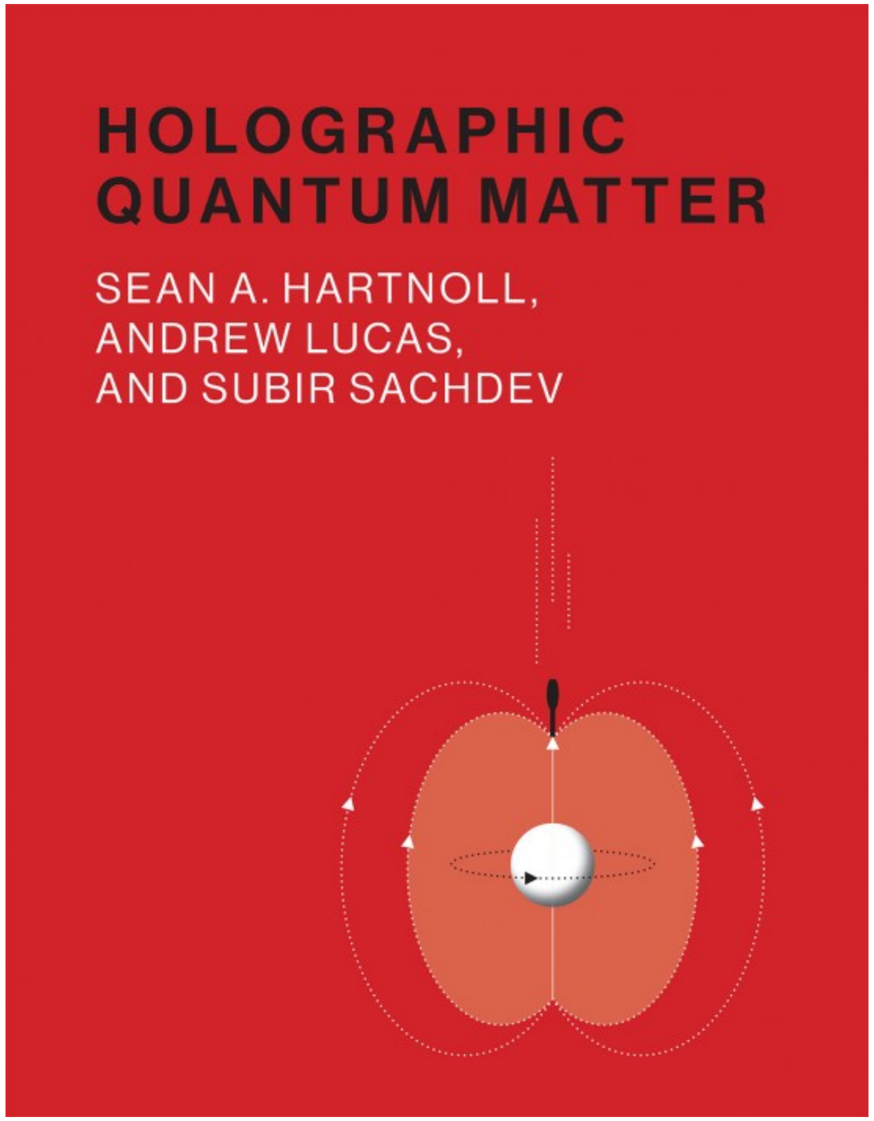


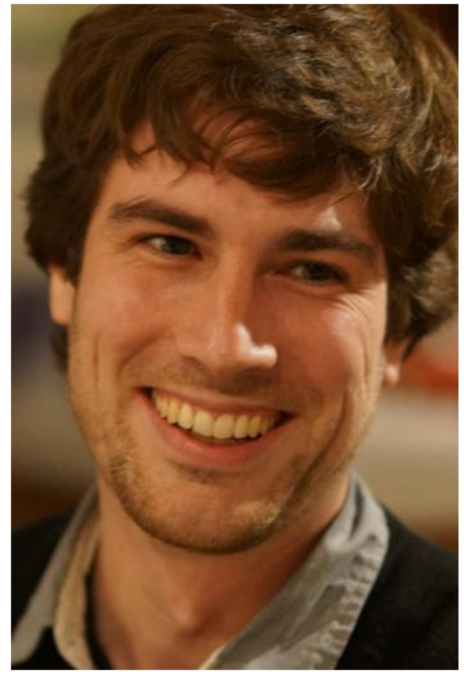


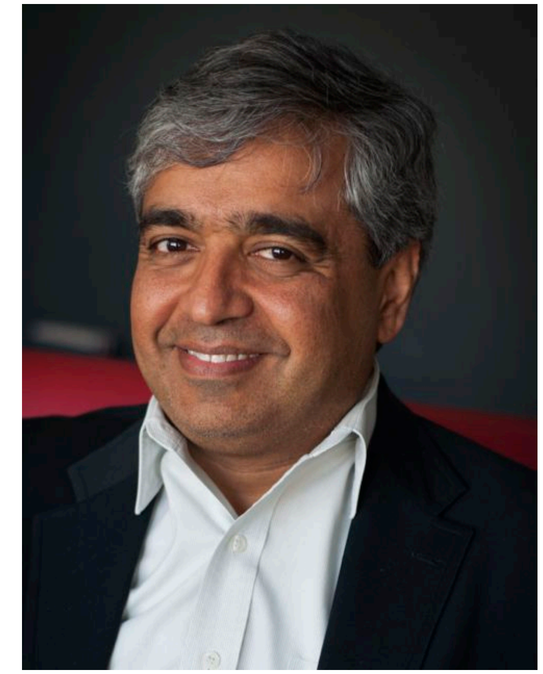
$YBa_2Cu_3O_{6+x}$

Figure: K. Fujita and J. C. Seamus Davis

Strange metals (II)







Sci|Post SciPost Phys. 6, 061 (2019)

Planckian dissipation, minimal viscosity and the transport in cuprate strange metals

Jan Zaanen

The Institute Lorentz for Theoretical Physics, Leiden University, Leiden, The Netherlands

Our humble approach:
focus on ultraclean Fermi liquids*

* Same approach taken by the MIT/Weizmann collaboration led by L. Levitov & G. Falkovich

Transport in ultraclean materials

Transport in ultraclean materials

$\mathcal{H}\Psi = E\Psi$

Ballistic regime:
Landauer-Büttiker theory (single-electron quantum mechanics)

$k_{\mathrm{B}}T/E_{\mathrm{F}}$

Transport in ultraclean materials

$\mathcal{H}\Psi = E\Psi$

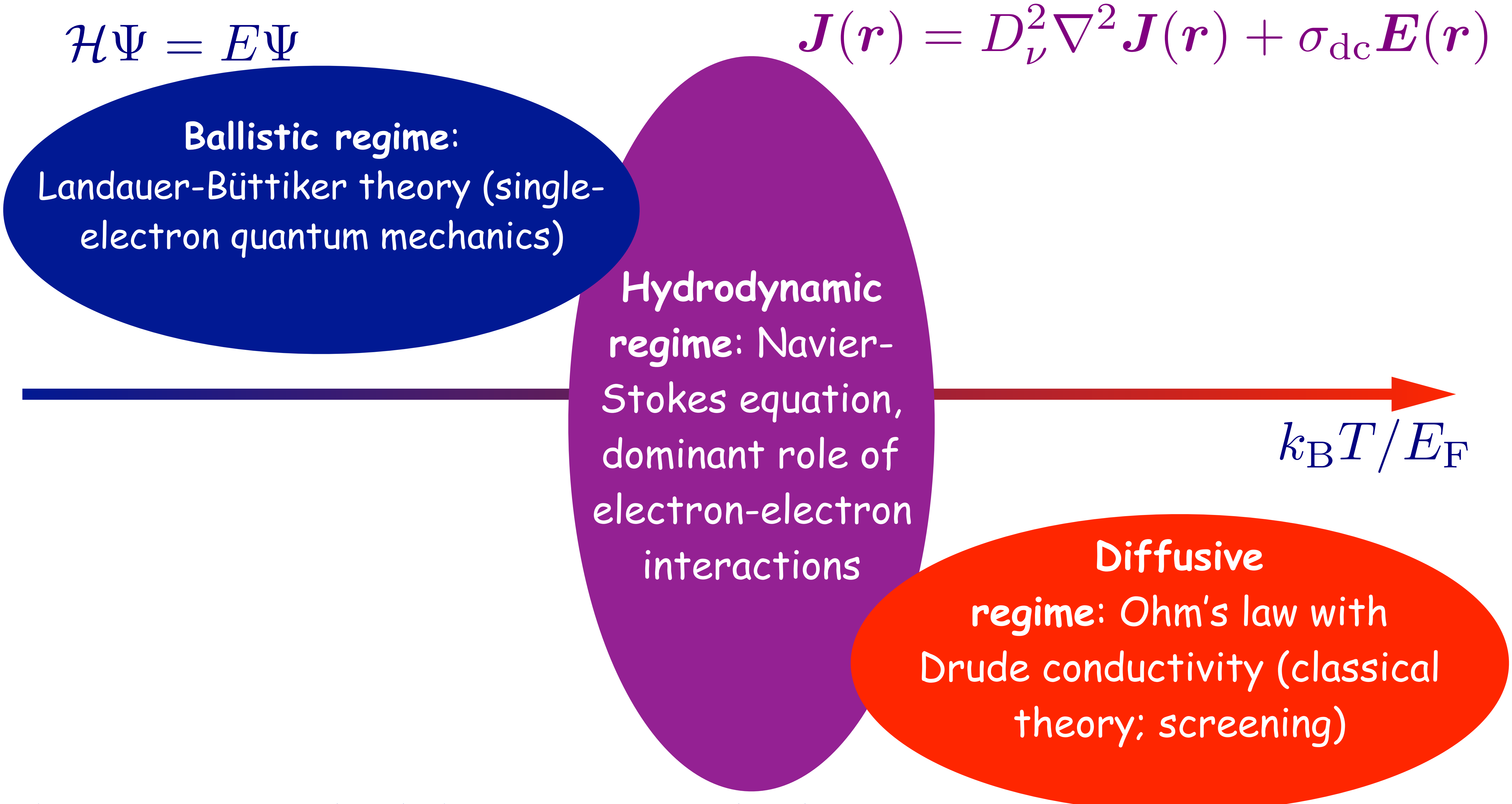
Ballistic regime: Landauer-Büttiker theory (single-electron quantum mechanics)

$k_{\rm B}T/E_{\rm F}$

Diffusive regime: Ohm's law with Drude conductivity (classical theory; screening)

$\boldsymbol{J}(\boldsymbol{r}) = \sigma_{\rm dc}\boldsymbol{E}(\boldsymbol{r})$

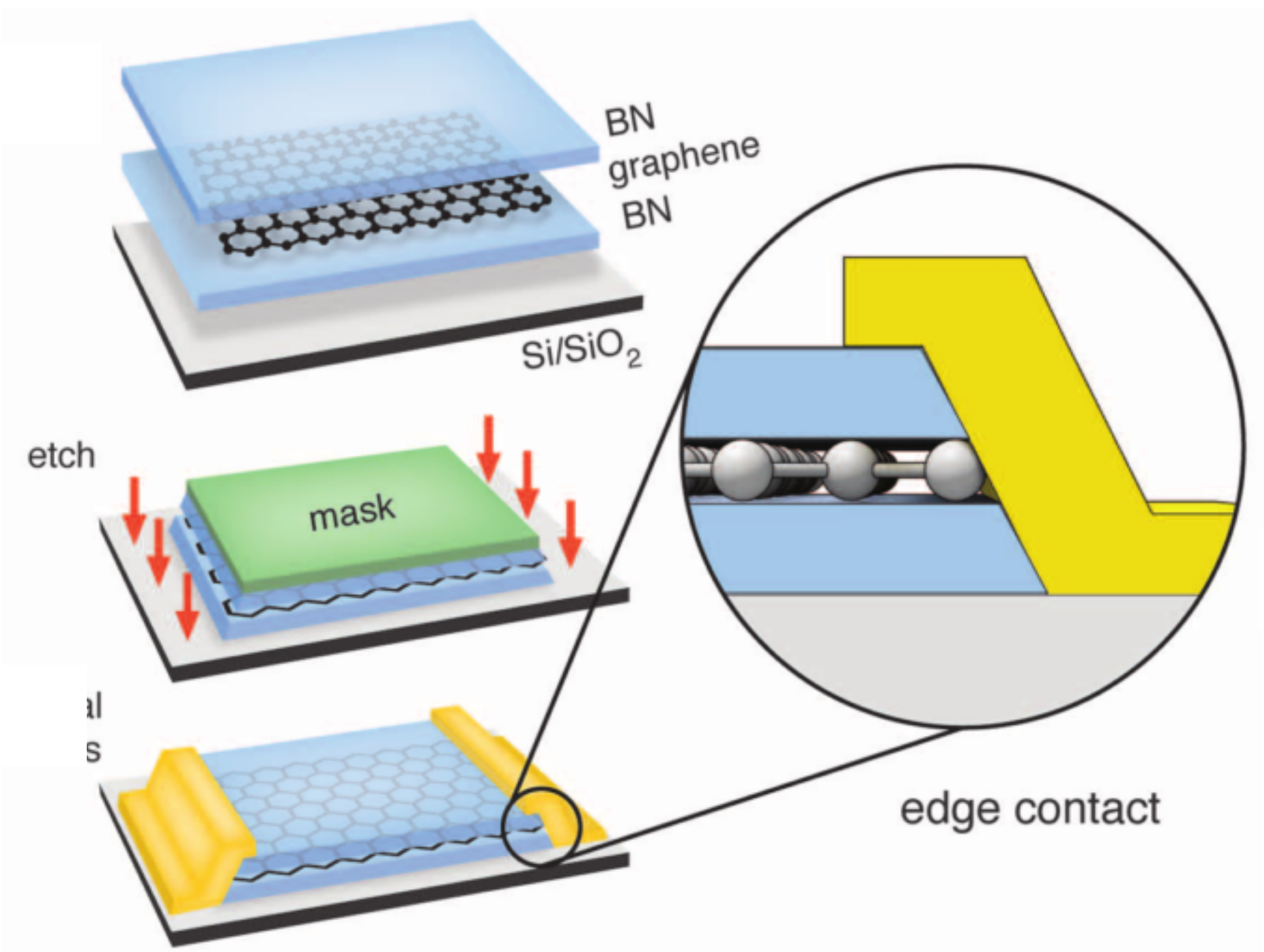
Transport in ultraclean materials

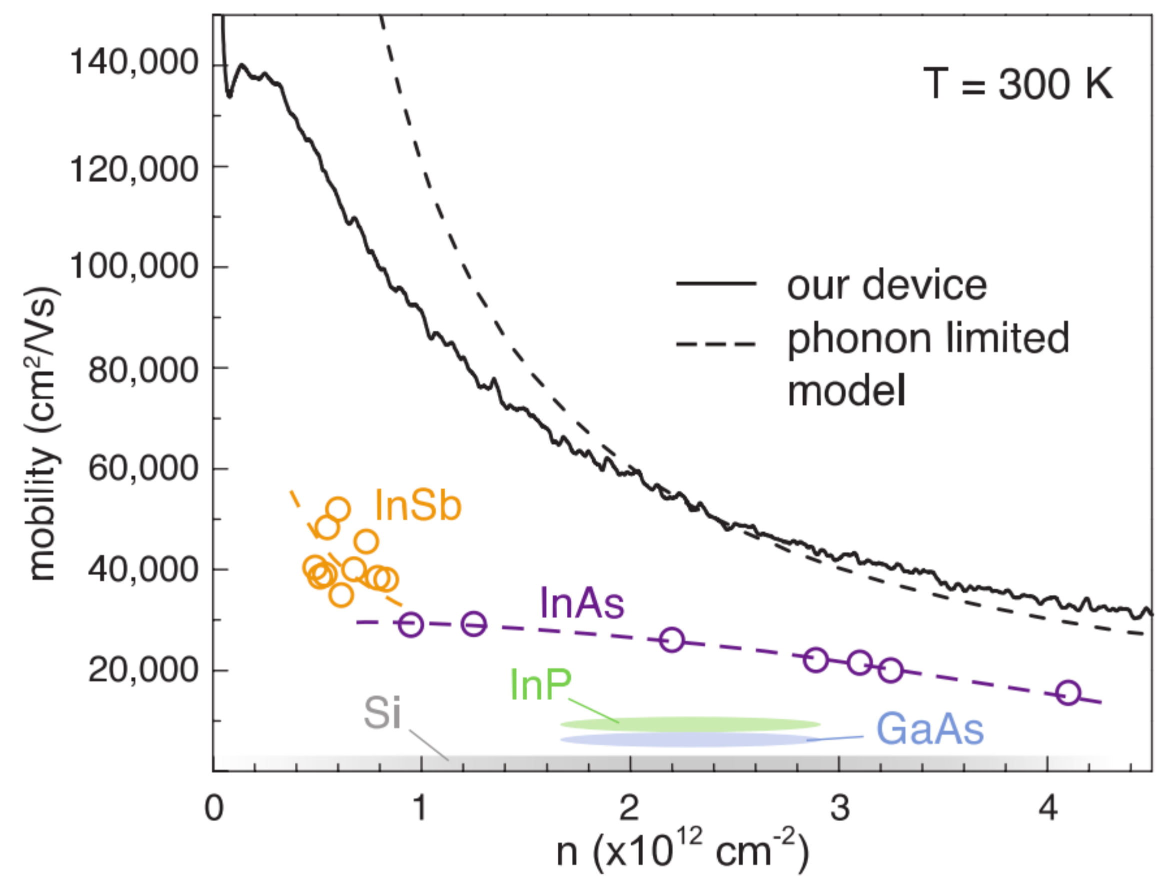


$$\boldsymbol{J}(\boldsymbol{r}) = \sigma_{\rm dc}\boldsymbol{E}(\boldsymbol{r})$$

Phys. Rev. B **92**, 165433 (2015); Phys. Rev. B **94**, 155414 (2016); Phys. Rev. B **96**, 195401 (2017); Phys. Rev. B **98**, 241304(R) (2018); Science **351**, 1055 (2016); Nature Phys. **13**, 1182 (2017); Phys. Rev. Lett. **121**, 236602 (2018); Science **364**, 162 (2019); Phys. Rev. Lett. **123**, 117203 (2019)

Great sandwiches: hBN/G/hBN

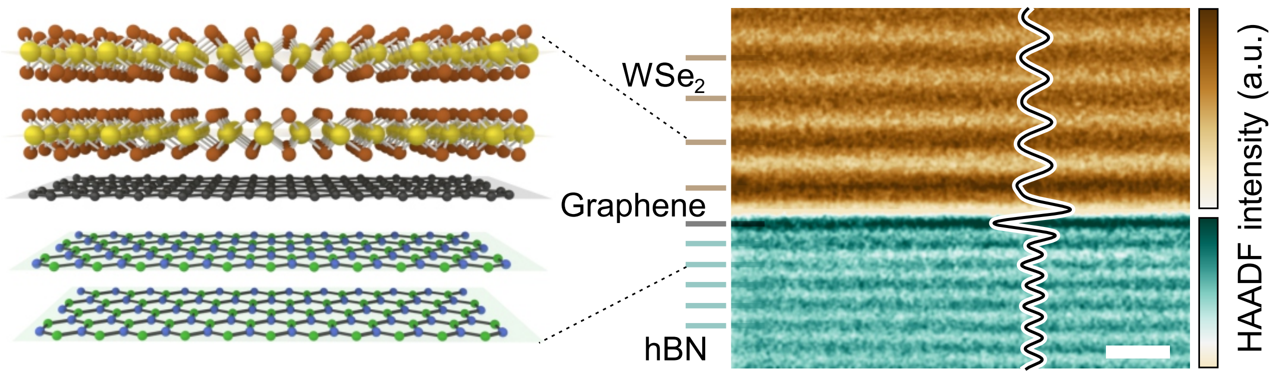


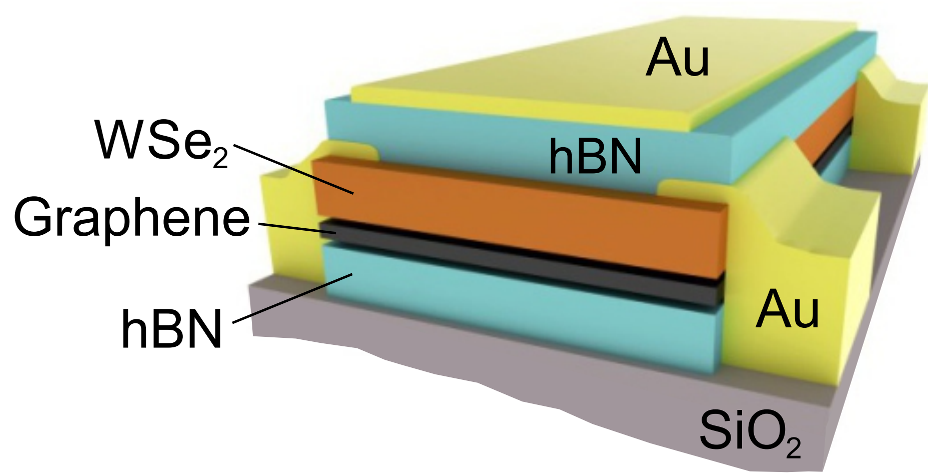


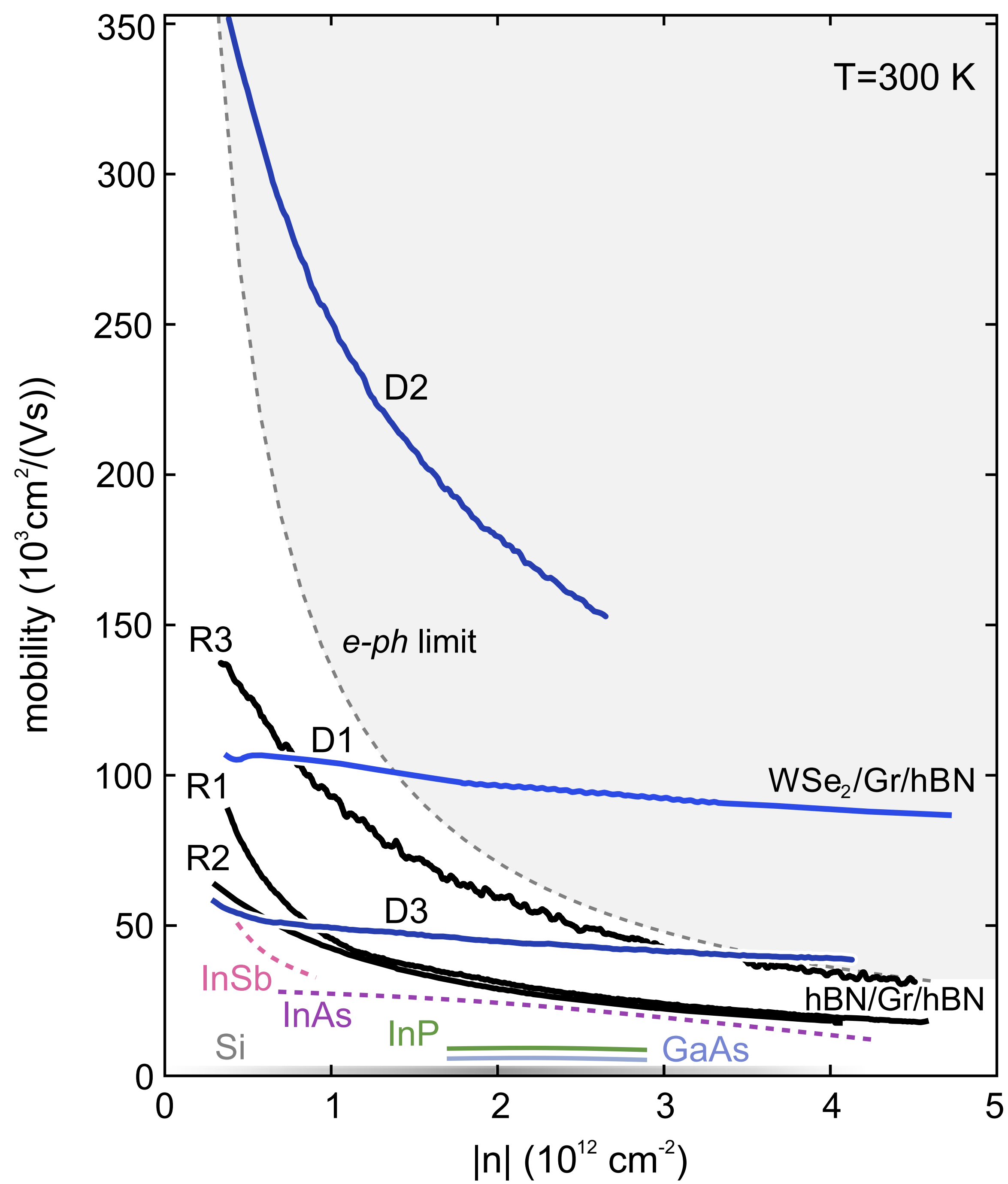
L. Wang et al., Science **342**, 614 (2013)

See also works by Manchester, MIT, Harvard, Zürich, Cambridge, Aachen, etc

Great sandwiches: WSe_2/G/hBN







Crucial heterostructures for the observation of non-linear hydro phenomena, such as preturbulence and other instabilities

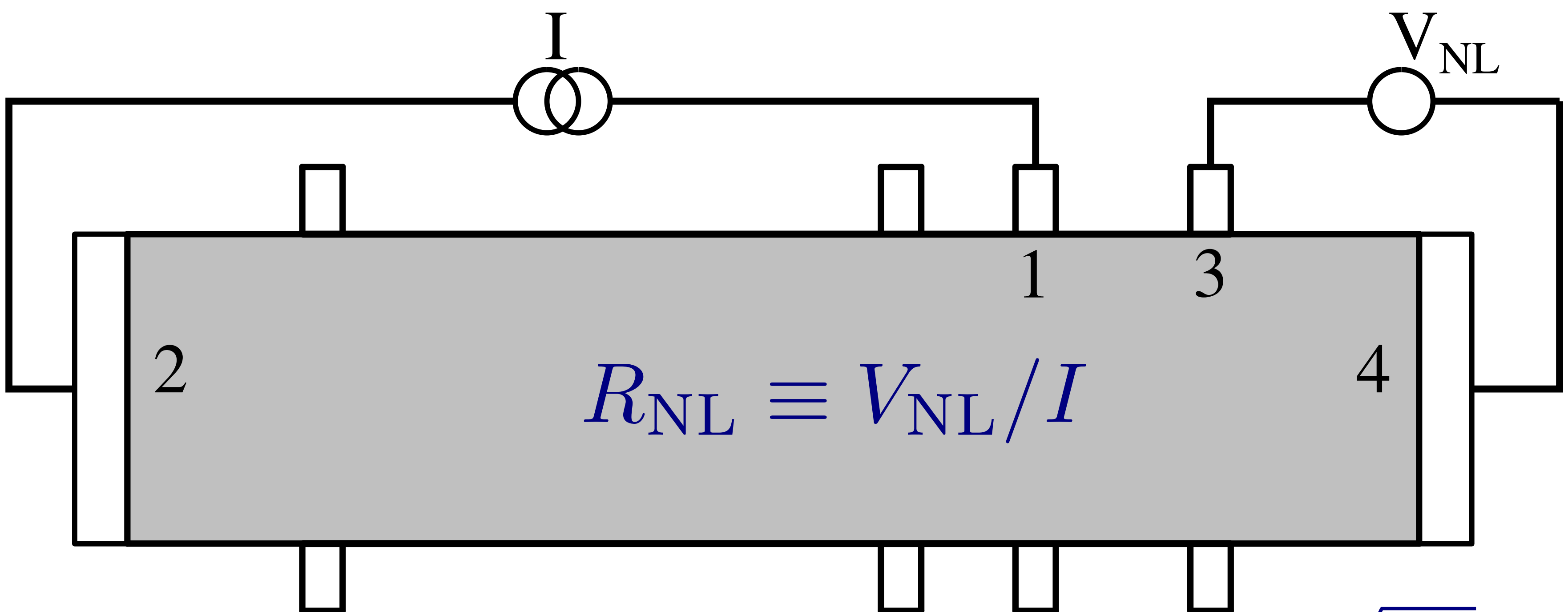
L. Banszerus, T. Sohier, A. Epping, F. Winkler, F. Libisch, F. Haupt, K. Watanabe, T. Taniguchi, K. Müller-Caspary4, N. Marzari, F. Mauri, B. Beschoten, and C. Stampfer, arXiv:1909.09523

How to observe hydro electron flow in these ultraclean materials (and also "old" ones such as GaAs*)

*B.A. Braem, F.M.D. Pellegrino, A. Principi, M. Röösli, S. Hennel, J.V. Koski, M. Berl, W. Dietsche, W. Wegscheider, M. Polini, T. Ihn, and K. Ensslin, Phys. Rev. B **98**, 241304(R) (2018)

The “vicinity” resistance geometry

Nonlocal transport: the "vicinity" resistance



$$D_\nu = \sqrt{\nu\tau}$$

Solve the following steady-state equations:

$$\nabla \cdot \boldsymbol{J}(\boldsymbol{r}) = 0$$

$$\boldsymbol{J}(\boldsymbol{r}) = D_\nu^2 \nabla^2 \boldsymbol{J}(\boldsymbol{r}) - \sigma_{\rm dc} \nabla \phi(\boldsymbol{r})$$

supplemented by boundary conditions:

$$[\partial_y J_x(x,y) + \partial_x J_y(x,y)]_{y=\pm W/2} = \mp \frac{J_x(x, y = \pm W/2)}{l_{\rm b}}$$

I. Torre, A. Tomadin, A.K. Geim, and M. Polini, Phys. Rev. B **92**, 165433 (2015)

Solution of the problem (I)

For a translationally-invariant geometry, the problem can be solved by employing FTs:

$$\boldsymbol{J}(x,y) = \int_{-\infty}^{\infty} \frac{dk_x}{2\pi} e^{ik_x x} \hat{\boldsymbol{J}}(k_x, y)$$

One can recast the problem into a system of 4×4 first-order differential equations

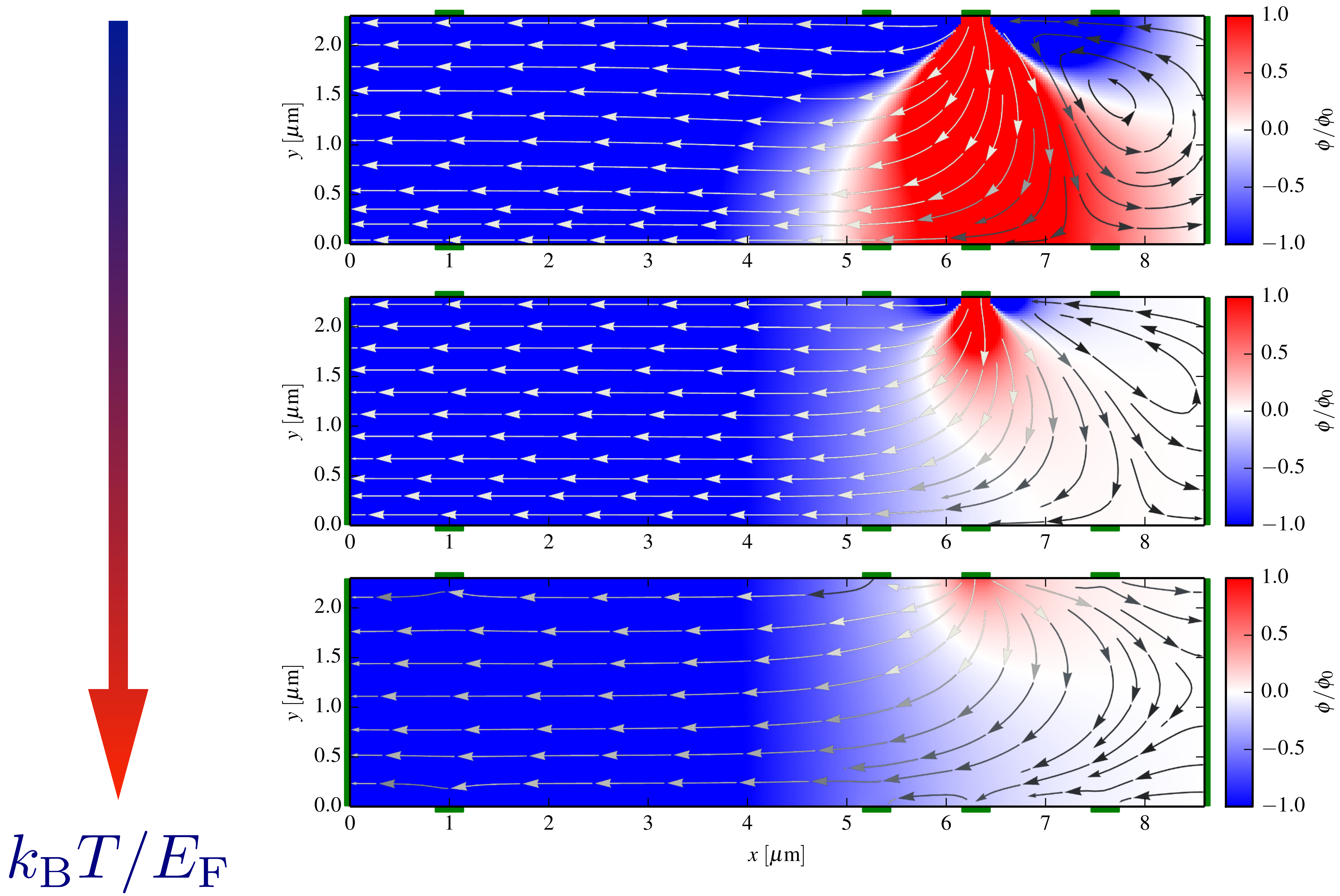
$$\partial_y \boldsymbol{w}(k_x, y) = \mathcal{M}(k_x) \boldsymbol{w}(k_x, y)$$

The general solution can be therefore written as:

$$\begin{aligned}
\boldsymbol{w}(k_x,y) &= C_1(k_x) e^{|k_x| y} \begin{pmatrix} i \\ +\mathrm{sgn}(k_x) \\ +i\mathrm{sgn}(k_x) \\ 1 \end{pmatrix} + C_2(k_x) e^{-|k_x| y} \begin{pmatrix} i \\ -\mathrm{sgn}(k_x) \\ -i\mathrm{sgn}(k_x) \\ 1 \end{pmatrix} \\
&+ C_3(k_x) e^{qy} \begin{pmatrix} +\frac{k_x}{q} \\ -i\frac{k_x^2}{q^2} \\ 1 \\ 0 \end{pmatrix} + C_4(k_x) e^{-qy} \begin{pmatrix} -\frac{k_x}{q} \\ -i\frac{k_x^2}{q^2} \\ 1 \\ 0 \end{pmatrix}
\end{aligned}$$

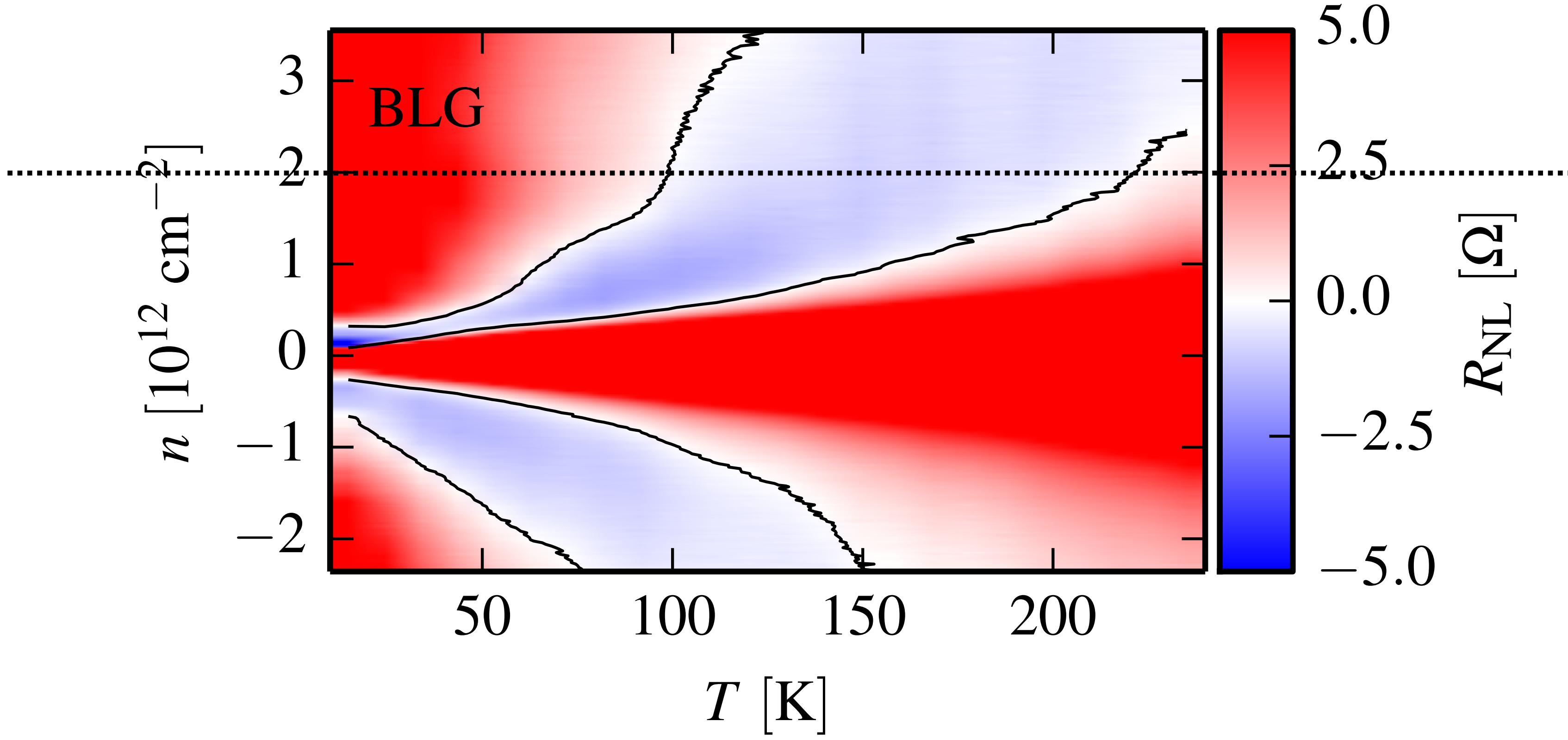
$$q \equiv \sqrt{k_x^2 + 1/D_\nu^2}$$

Solution of the problem (II)



$$\phi(r,\theta) \approx -\frac{I}{\pi\sigma_{\rm dc}}\left[\ln\left(\frac{r}{D_\nu}\right) + 2D_\nu^2\frac{\cos(2\theta)}{r^2}\right]$$

Measurement of the vicinity resistance

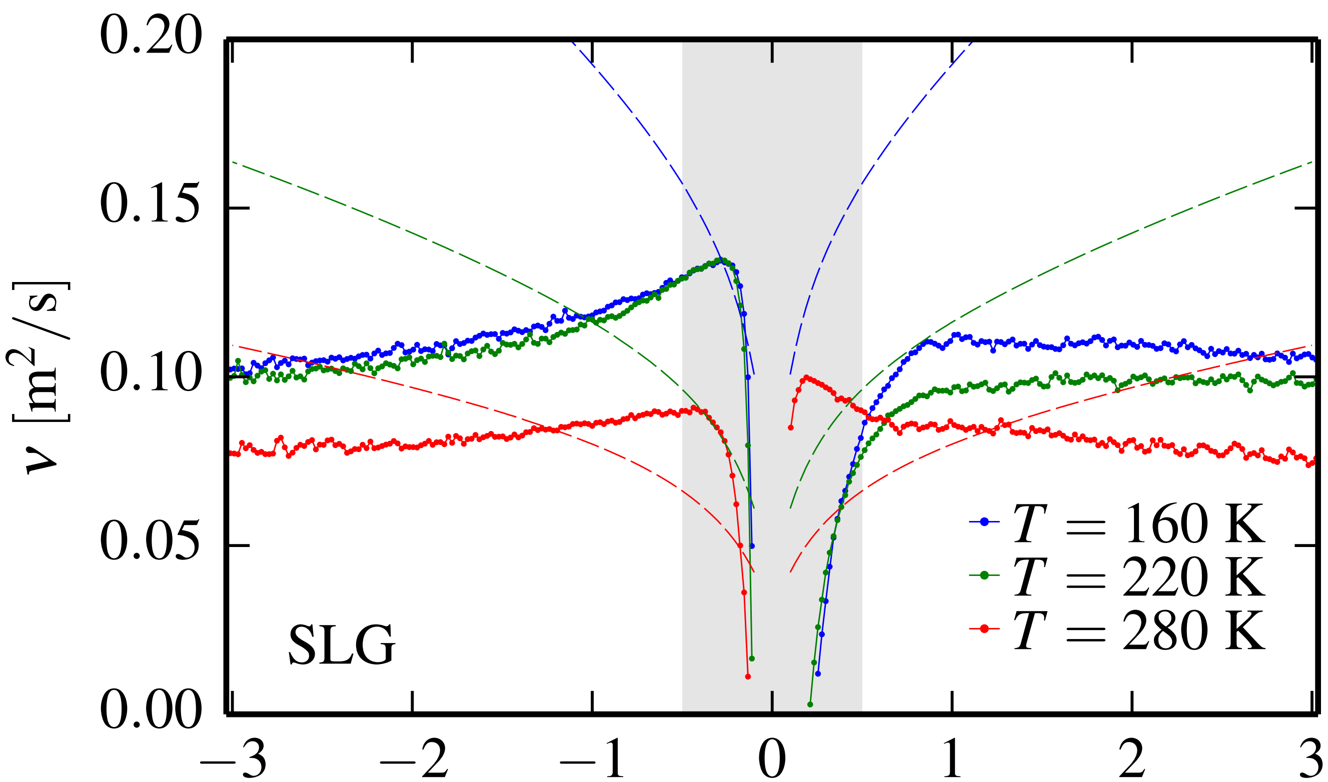


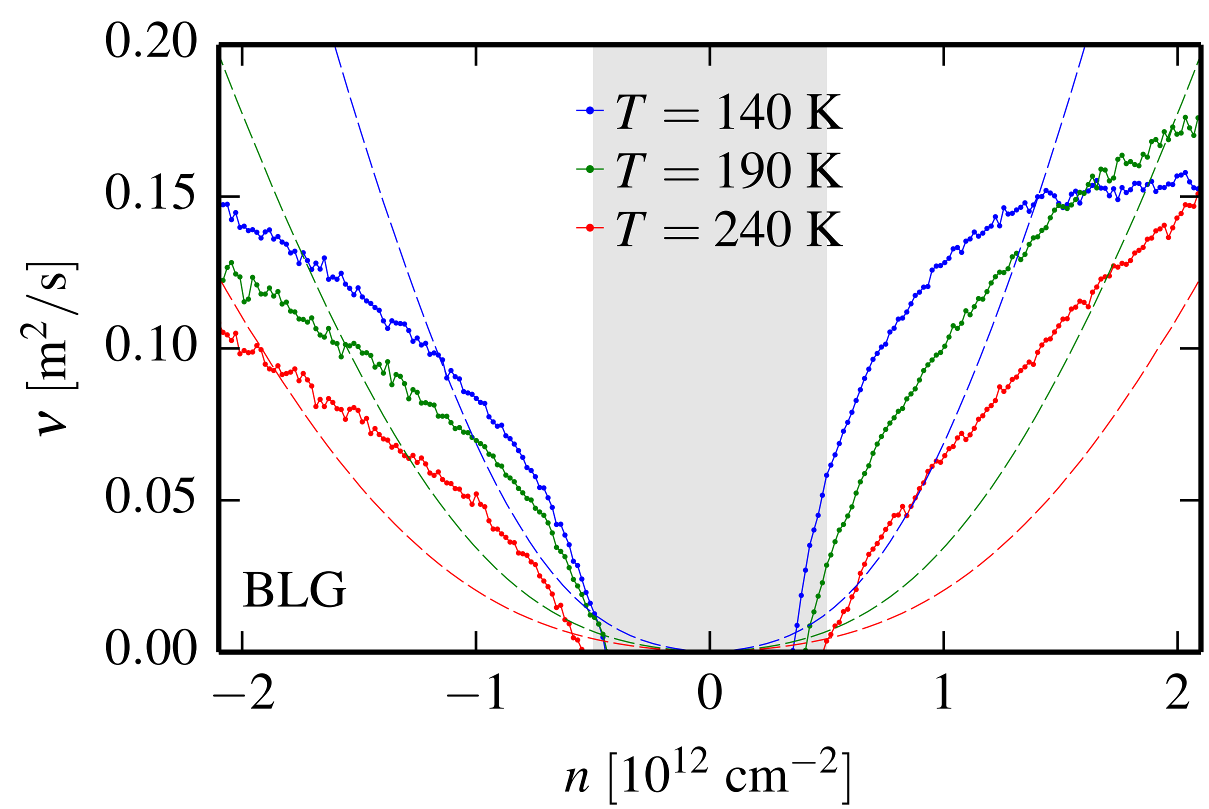
- The first sign change signals the crossover from the ballistic regime (governed by single-particle quantum mechanics) to an interaction-dominated regime and then the hydrodynamic regime (where the anomalous contribution to the potential dominates)

- The second sign change signals the crossover from the hydrodynamic regime to the ordinary diffusive regime governed by local Ohm's law

D. Bandurin, I. Torre, R.K. Kumar, M. Ben Shalom, A. Tomadin, A. Principi, G.H. Auton, E. Khestanova, K.S. Novoselov, I.V. Grigorieva, L.A. Ponomarenko, A.K. Geim, and M. Polini, Science **351**, 1055 (2016)

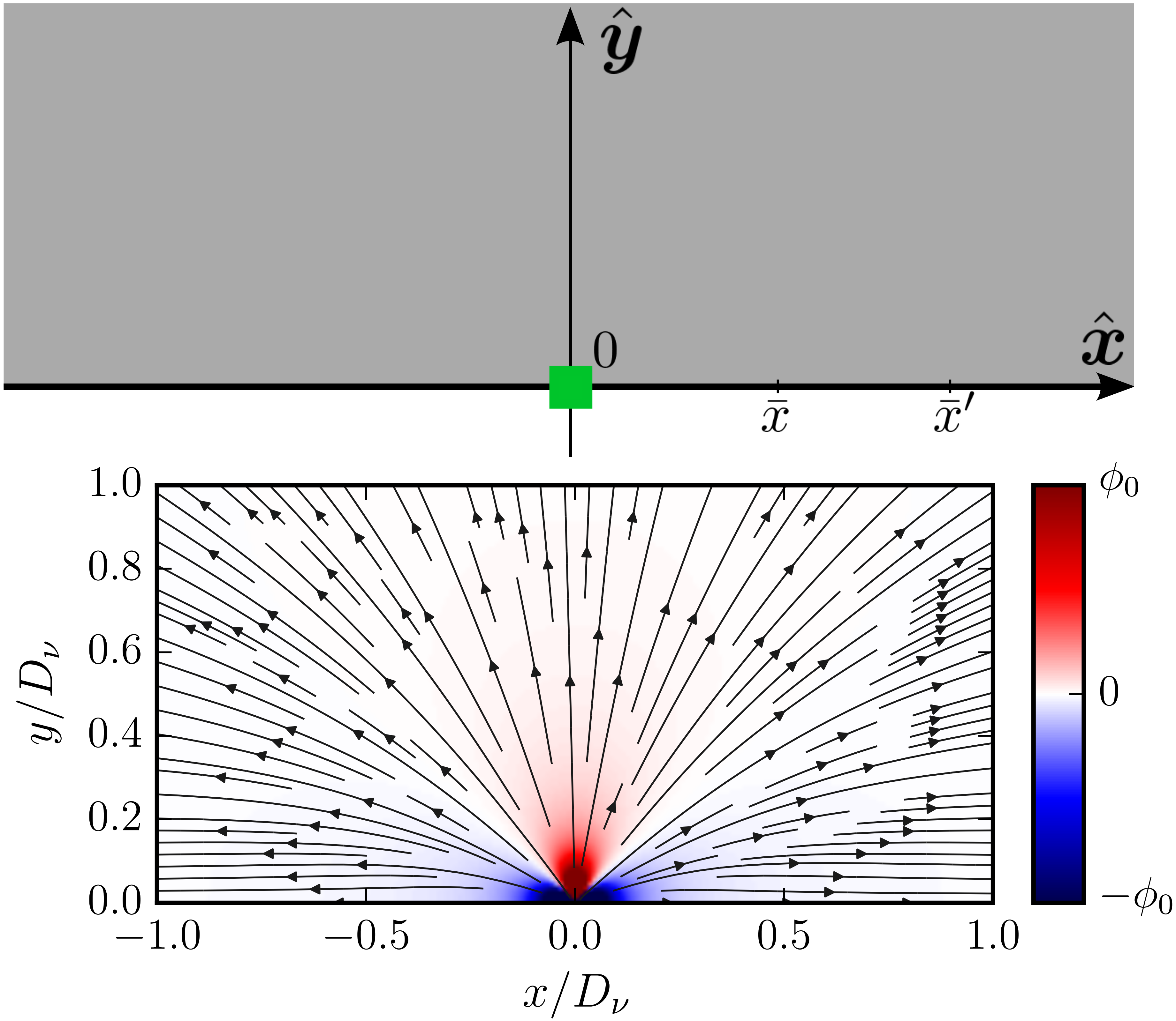
Kinematic viscosity





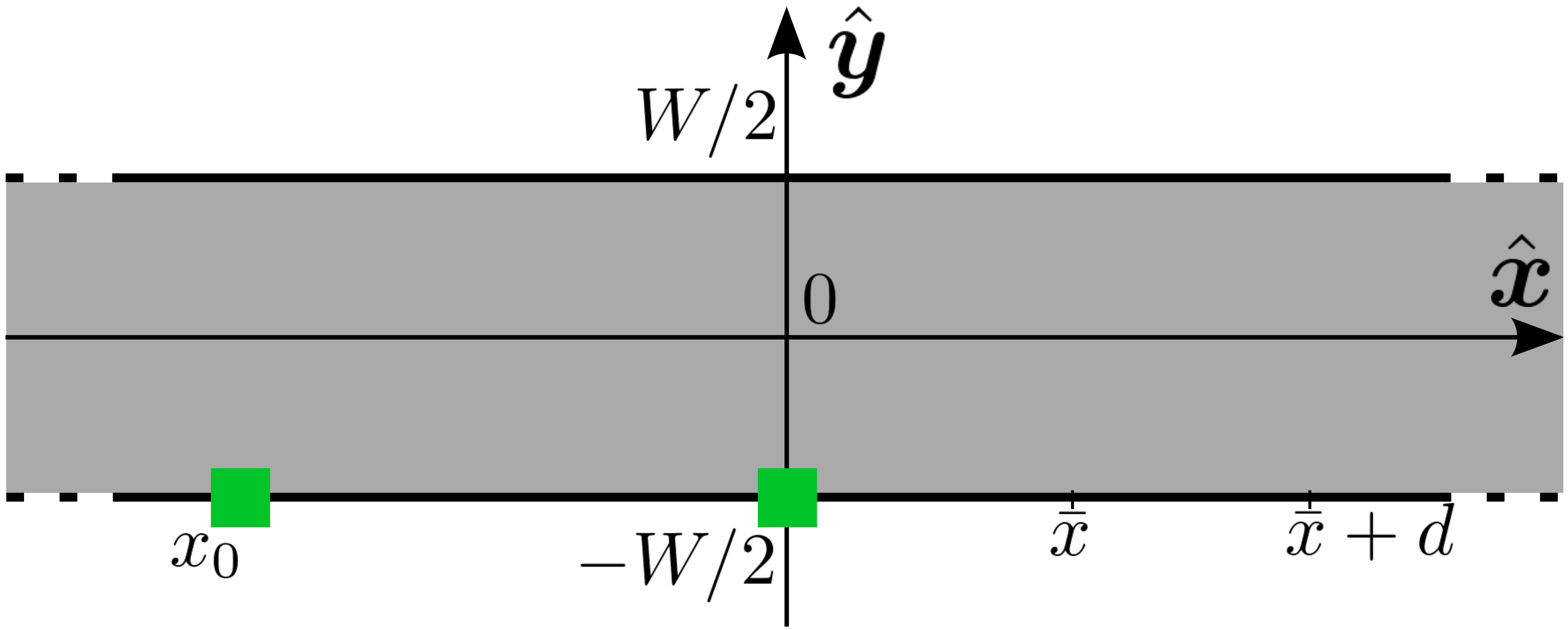
D. Bandurin, I. Torre, R.K. Kumar, M. Ben Shalom, A. Tomadin, A. Principi, G.H. Auton, E. Khestanova, K.S. Novoselov, I.V. Grigorieva, L.A. Ponomarenko, A.K. Geim, and M. Polini, Science **351**, 1055 (2016)

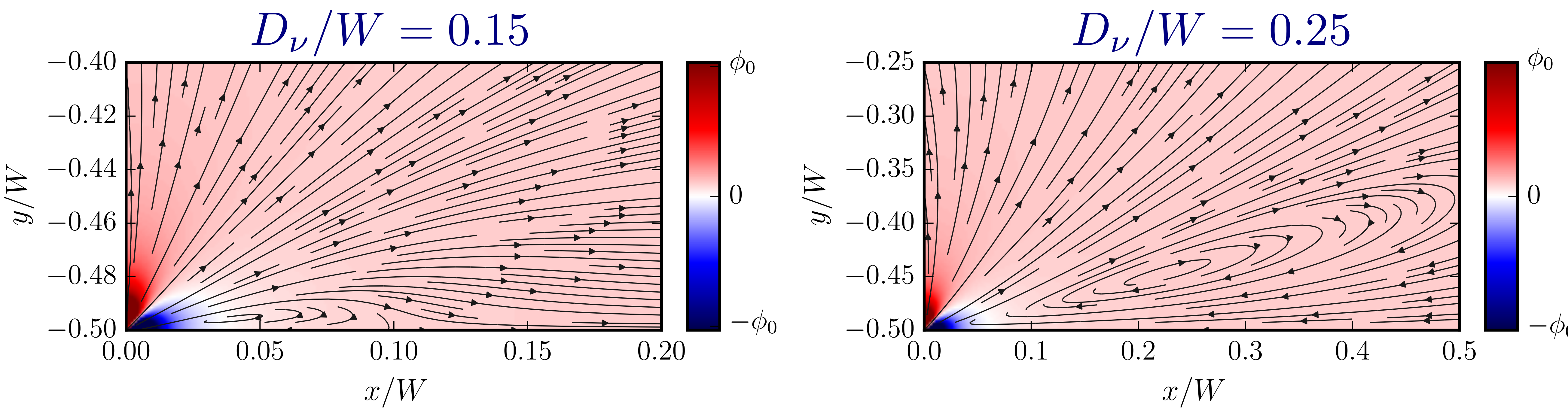
Whirlpools or no whirlpools?



F.M.D. Pellegrino, I. Torre, A.K. Geim, and M. Polini, Phys. Rev. B **94**, 155414 (2016)

Whirlpools or no whirlpools?



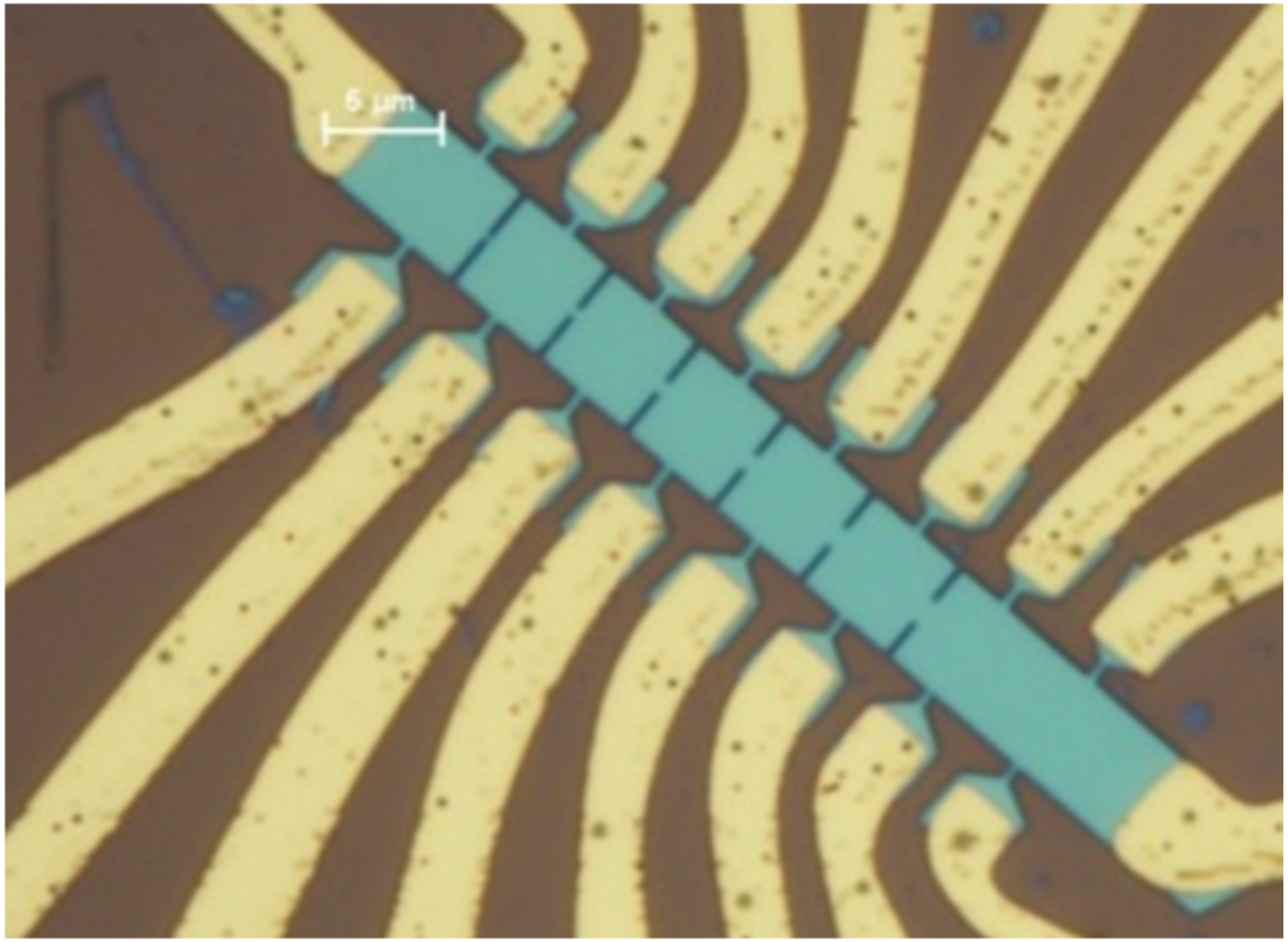


$$D_\nu/W = \frac{1}{\sqrt{2\pi}} \simeq 0.225$$

F.M.D. Pellegrino, I. Torre, A.K. Geim, and M. Polini, Phys. Rev. B **94**, 155414 (2016)

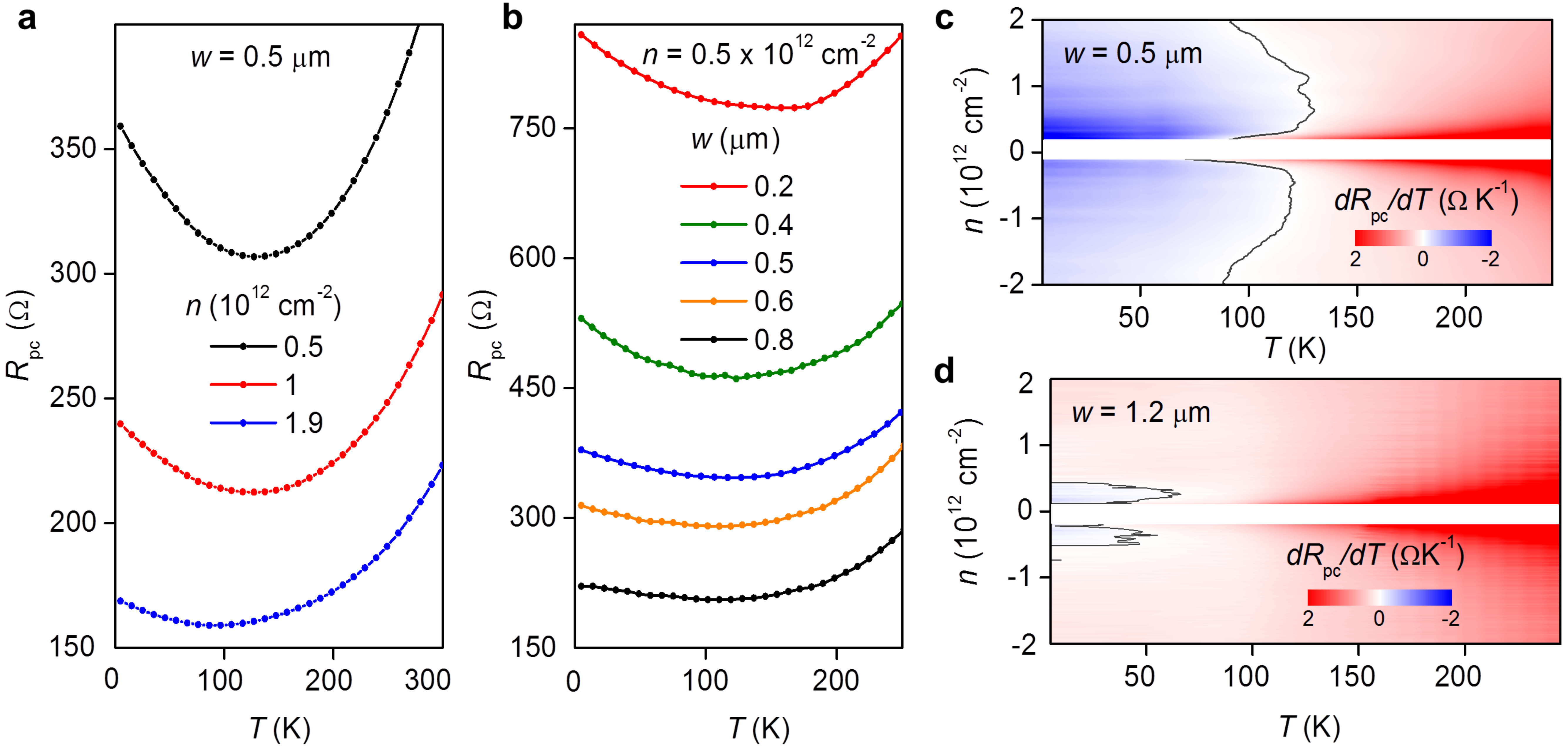
The "point contact" geometry

Super-flow through point contacts



R.K. Kumar, D.A. Bandurin, F.M.D. Pellegrino, Y. Cao, A. Principi, H. Guo, G.H. Auton, M. Ben Shalom, L.A. Ponomarenko, G. Falkovich, I.V. Grigorieva, L.S. Levitov, M. Polini, and A.K. Geim, Nature Phys. **13**, 1182 (2017)

Super-flow through point contacts



R.K. Kumar, D.A. Bandurin, F.M.D. Pellegrino, Y. Cao, A. Principi, H. Guo, G.H. Auton, M. Ben Shalom, L.A. Ponomarenko, G. Falkovich, I.V. Grigorieva, L.S. Levitov, M. Polini, and A.K. Geim, Nature Phys. **13**, 1182 (2017)

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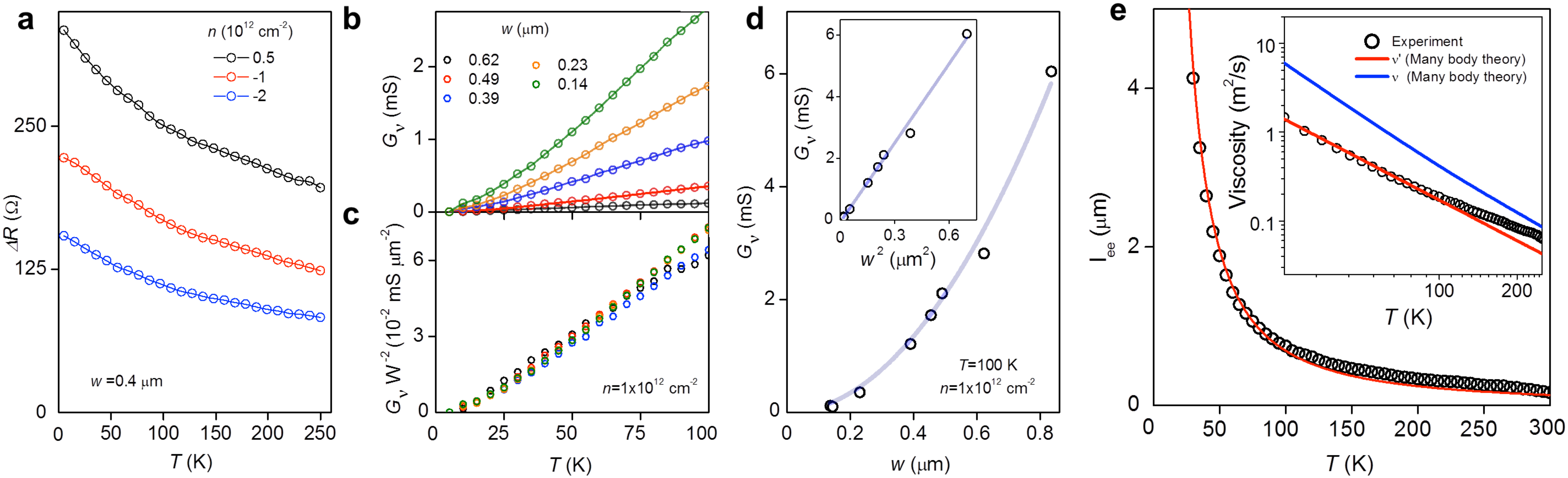
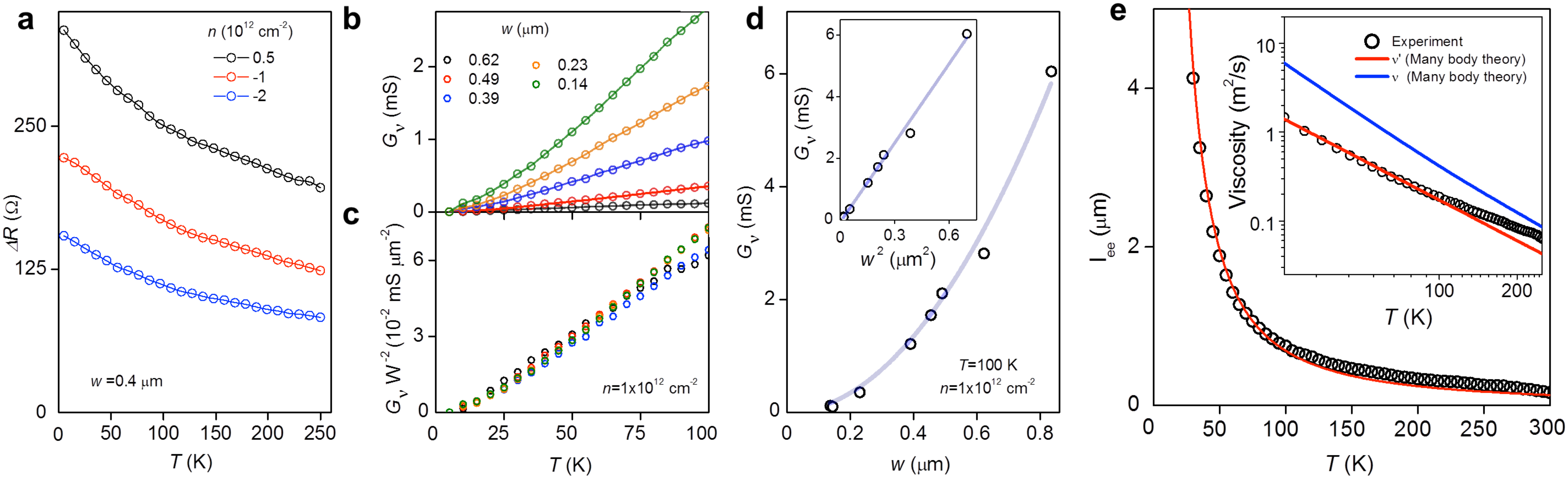


$$R_{\rm pc} = \frac{1}{G_{\rm b} + G_{\nu}} + R_{\rm C}$$

R.K. Kumar et al., Nature Phys. **13**, 1182 (2017)
H. Guo, E. Ilseven, G. Falkovich, and L. Levitov, PNAS **114**, 3068 (2017)
A. Principi, G. Vignale, M. Carrega, and M. Polini, Phys. Rev. B **93**, 125410 (2016)

Super-flow through point contacts



$$R_{\rm pc} = \frac{1}{G_{\rm b} + G_{\nu}} + R_{\rm C}$$

"contact resistance" (diffusive contribution)

Sharvin (ballistic)

e-e interaction contribution

$$G_{\nu} = \frac{\sqrt{\pi |n|} e^2 w^2 v_{\rm F}}{32 \hbar \nu}$$

R.K. Kumar et al., Nature Phys. **13**, 1182 (2017)
H. Guo, E. Ilseven, G. Falkovich, and L. Levitov, PNAS **114**, 3068 (2017)
A. Principi, G. Vignale, M. Carrega, and M. Polini, Phys. Rev. B **93**, 125410 (2016)

Recent developments

Spin-vorticity coupling and viscous magnon fluids

Rembert Duine (Utrecht)

J. Phys.: Mater. **2** (2019) 015006 https://doi.org/10.1088/2515-7639/aaf8fb

JPhys Materials

PAPER

Spin–vorticity coupling in viscous electron fluids

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[2] Istituto Italiano di Tecnologia, Graphene Labs, Via Morego 30, I-16163 Genova, Italy
[3] Department of Applied Physics, Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands
[4] Author to whom any correspondence should be addressed.

E-mail: R.A.Duine@uu.nl

Keywords: spintronics, spin–orbit coupling, viscous electron systems

PHYSICAL REVIEW LETTERS **123,** 117203 (2019)

Nonlocal Spin Transport as a Probe of Viscous Magnon Fluids

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[1]*Institute for Theoretical Physics, Utrecht University, Princetonplein 5, 3584 CC Utrecht, Netherlands*
[2]*Department of Physics, Lancaster University, Lancaster LA1 4YB, United Kingdom*
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[4]*Istituto Italiano di Tecnologia, Graphene Labs, Via Morego 30, I-16163 Genova, Italy*
[5]*Department of Applied Physics, Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, Netherlands*

(Received 27 March 2019; published 13 September 2019)

Viscous Hall effect*

Many other papers on measurement protocols to access the Hall viscosity of electron liquids:
Gromov et al., Scaffidi et al., A. Stern et al., E.M. Hankiewicz et al.

Hall viscosity

2D viscous stress tensor

$$\sigma'_{ij} = \frac{1}{2} \sum_{k,\ell} \eta_{ij,k\ell} (\partial_k v_\ell + \partial_\ell v_k)$$

where the rank-4 tensor introduced above is given by

Bulk viscosity

Shear viscosity

$$\eta_{ij,k\ell} \equiv \zeta \delta_{ij} \delta_{k\ell} + \eta(\delta_{ik}\delta_{j\ell} + \delta_{i\ell}\delta_{jk} - \delta_{ij}\delta_{k\ell}) + \eta_{\mathrm{H}}(\delta_{jk}\epsilon_{i\ell} - \delta_{i\ell}\epsilon_{kj})$$

Hall viscosity

$$\boldsymbol{\sigma}' = (\eta + i\eta_{\mathrm{H}}\tau_y)[(\partial_x v_x - \partial_y v_y)\tau_z + (\partial_x v_y + \partial_y v_x)\tau_x] + \zeta \nabla \cdot \boldsymbol{v}$$

See e.g. B. Bradlyn, M. Goldstein, and N. Read, Phys. Rev. B **86**, 245309 (2012)

Navier-Stokes-Lorentz Eq (I)

We start from the Navier-Stokes equation in the presence of a magnetic field:

$$-\nabla P(\boldsymbol{r}) + \nabla \cdot \boldsymbol{\sigma}'(\boldsymbol{r}) + e\bar{n}\nabla\varphi(\boldsymbol{r}) - \frac{e}{c}\boldsymbol{J}(\boldsymbol{r}) \times \boldsymbol{B} = \frac{m}{\tau}\boldsymbol{J}(\boldsymbol{r})$$

Using the previous expression for the viscous stress tensor and introducing the electrochemical potential, we finally get:

$$\frac{\sigma_0}{e}\nabla\phi(\boldsymbol{r}) = (1 - D_\nu^2\nabla^2)\boldsymbol{J}(\boldsymbol{r}) + \omega_{\rm c}\tau\left(1 + D_{\rm H}^2\nabla^2\right)\boldsymbol{J}(\boldsymbol{r}) \times \hat{\boldsymbol{z}}$$

which, as usual, needs to be solved together with the continuity equation and suitable BCs (such as no-slip, free surface, or interpolations between the two) on the components of the current orthogonal and tangential to the device boundary

$$D_{\rm H} = \sqrt{\nu_{\rm H}/\omega_{\rm c}} \qquad \nu_{\rm H} \approx \nu_0 \frac{B B_0}{B_0^2 + B^2}$$

F.M.D. Pellegrino, I. Torre, and M. Polini, Phys. Rev. B **96**, 195401 (2017)

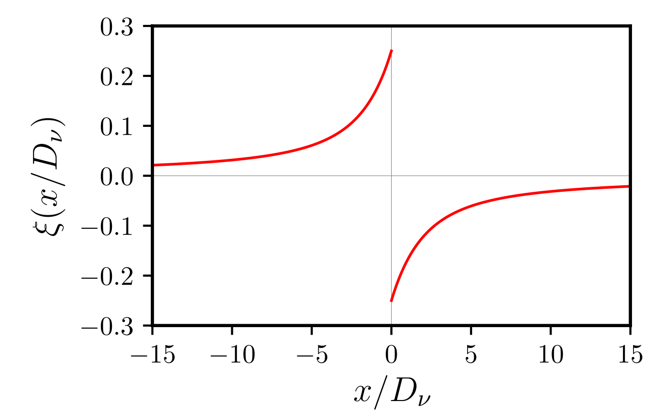
Navier-Stokes-Lorentz Eq (II)

Solutions can be found for both free-surface and no-slip BCs, and devices of finite geometry such as Hall bars with finite width.

The solution for a single injector into a half space with no-slip BCs is however enlightening:

$$\phi(x) = I[r_{\rm even}(x) + r_{\rm odd}(x)]$$

$$r_{\rm even}(x) = -\rho_0 \left[\frac{1}{\pi}\ln\left(\frac{|x|}{D_\nu}\right) + \frac{D_\nu^2}{\pi x^2} + \frac{D_\nu}{\pi |x|} K_1\left(\frac{|x|}{D_\nu}\right)\right]$$



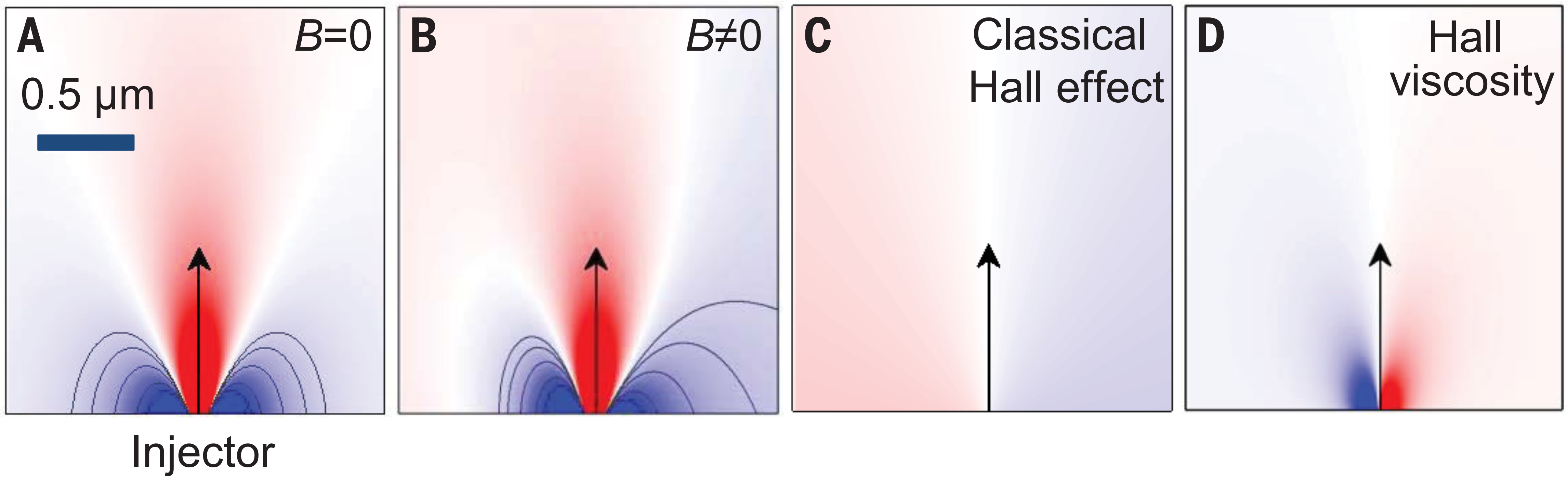
Hall resistivity

$$\rho_{\rm H} = B/(-e\bar{n}c)$$

$$r_{\rm odd}(x) = \frac{\rho_{\rm H}}{2}{\rm sgn}(x) - \rho_0 \frac{\nu_{\rm H}}{\nu}\frac{D_\nu}{2x}\left[\mathbf{L}_1\left(\frac{|x|}{D_\nu}\right) - I_1\left(\frac{|x|}{D_\nu}\right)\right]$$

F.M.D. Pellegrino, I. Torre, and M. Polini, Phys. Rev. B **96**, 195401 (2017)

Navier-Stokes-Lorentz Eq (III)



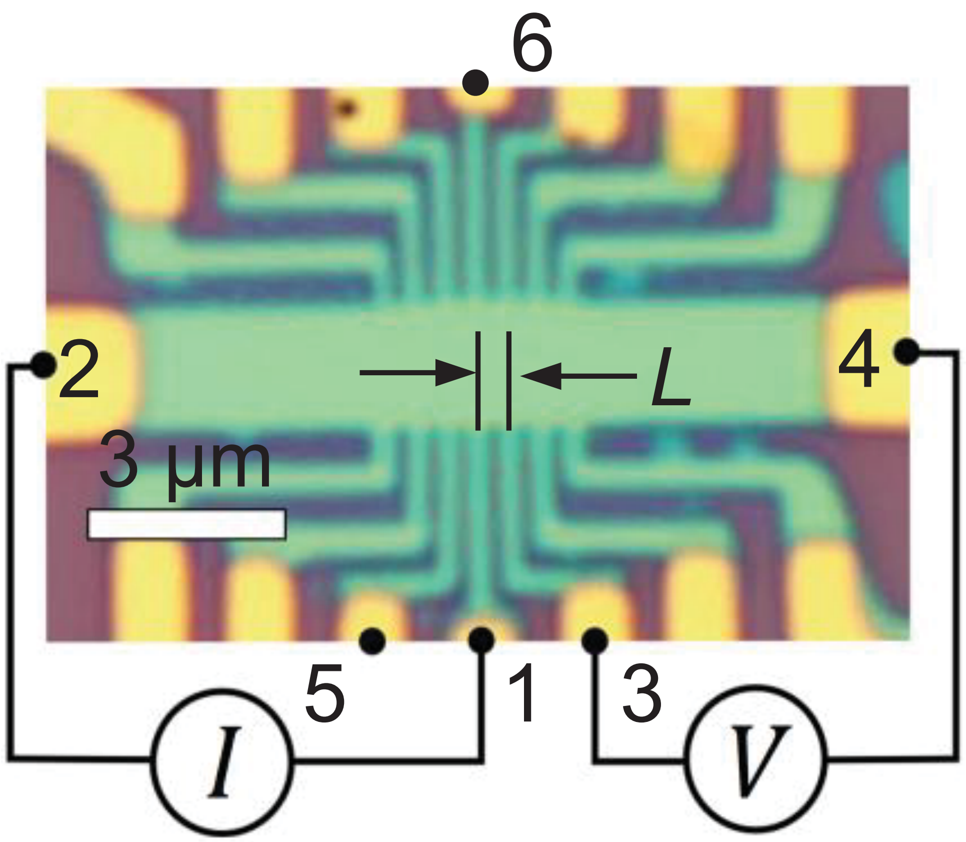
For experimental convenience, in order to extract the Hall viscosity we introduce the viscous Hall resistance:

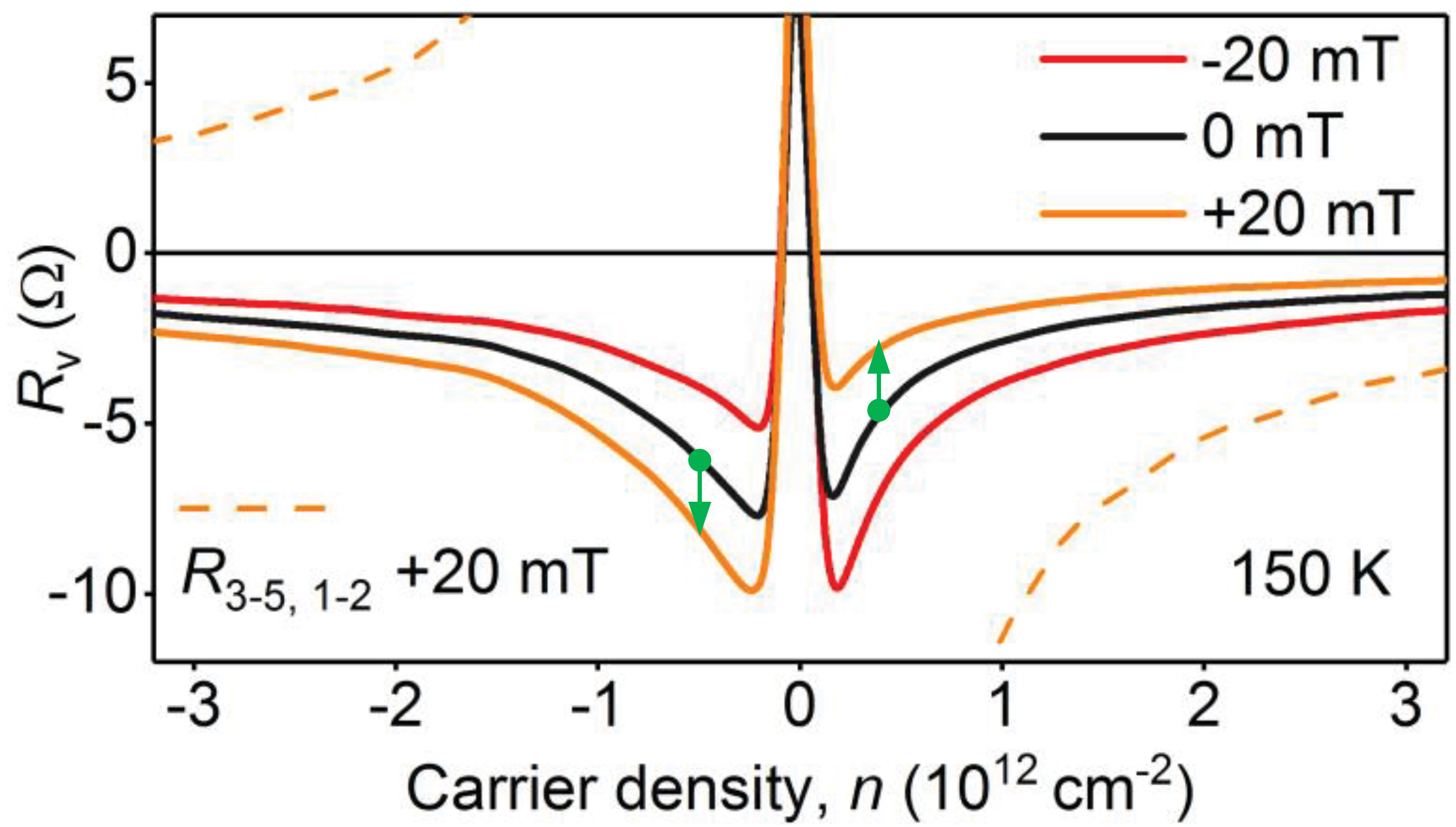
$$R_{\rm A}(x) \equiv \frac{R_{\rm V}(x,-B) - R_{\rm V}(x,B)}{2}$$

$$= \rho_0 \frac{\nu_{\rm H}}{\nu} \frac{D_\nu}{2x} \left[\mathbf{L}_1 \left(\frac{|x|}{D_\nu} \right) - I_1 \left(\frac{|x|}{D_\nu} \right) \right]$$

F.M.D. Pellegrino, I. Torre, and M. Polini, Phys. Rev. B **96**, 195401 (2017)

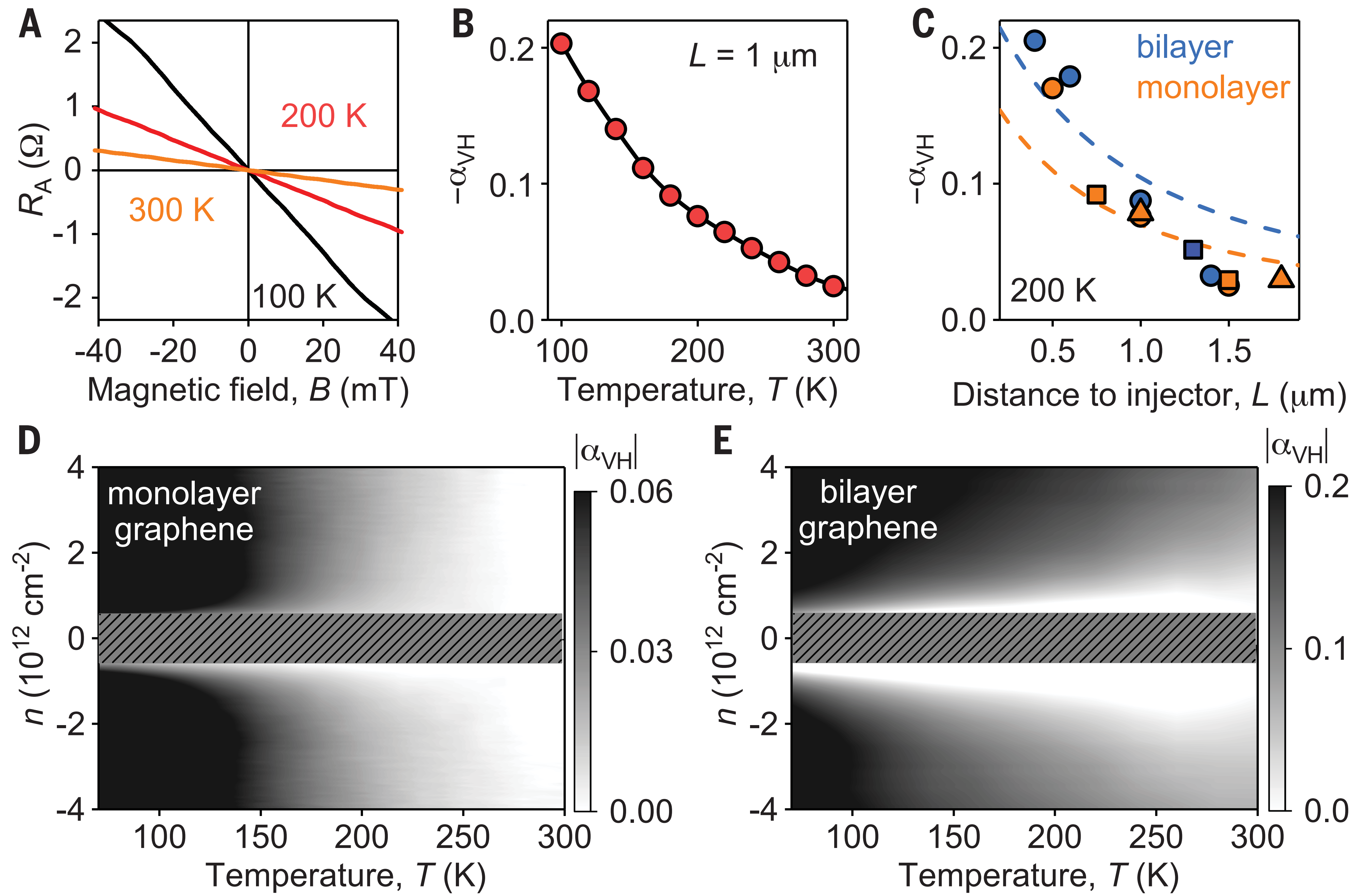
Measuring the Hall viscosity





A. I. Berdyugin, S. G. Xu, F. M. D. Pellegrino, R. Krishna Kumar, A. Principi, I. Torre, M. Ben Shalom
T. Taniguchi, K. Watanabe, I. V. Grigorieva, M. Polini, A. K. Geim, and D.A. Bandurin,
Science **364**, 162 (2019)

Measuring the Hall viscosity



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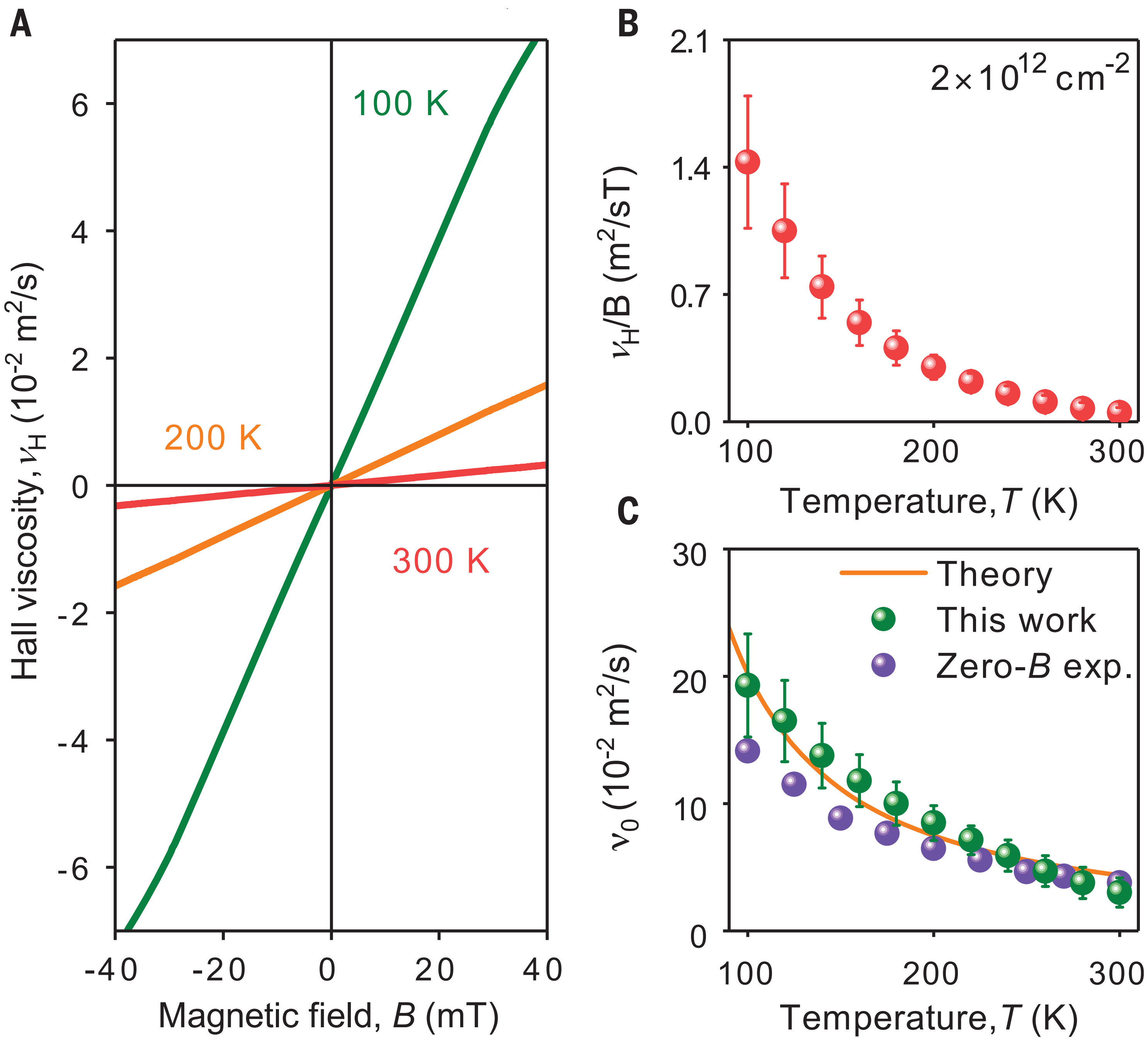
Measuring the Hall viscosity



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Take-home messages

- Vicinity and point-contact geometries have enabled the observation of viscous phenomena in graphene; they maybe soon observed in "old" systems including graphite, bismuth, etc. but also strongly correlated and topological materials and, maybe, even "strange metals" such as high-temperature superconductors in the normal state, which are expected to be (minimally) viscous

- The Hall viscosity has also been measured in the semiclassical regime; it is certainly very interesting to think about (possibly all-electrical) protocols to measure it in the quantum Hall regime

- Particularly enticing is to extend the existing hydrodynamic studies into the regime where nonlinear terms in the Navier-Stokes equation could no longer be ignored. The observation of preturbulence, for example, requires materials with smaller ν and longer τ as compared to the 2DESs studied so far

Thank you for your attention!

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