Electron hydrodynamics with a polygon Fermi surface

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Caleb Cook
Stanford

arXiv: 1903.05652

Strong Interactions

- goal: transport/dynamics of electrons in solids

$$H = H_{\mathrm{electron}} + H_{\mathrm{phonon}} + H_{\mathrm{impurity}} + H_{\mathrm{el-el}} + H_{\mathrm{el-ph}}$$

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  - Hilbert space $\dim(\mathcal{H}) \propto \exp[N]$
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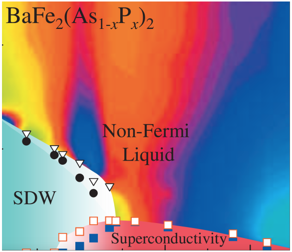
- real systems can be hard:



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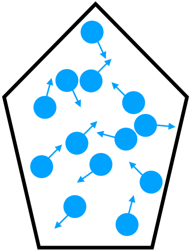

$\longrightarrow$

- hydrodynamics is valid on long time and length scales:

$$\omega\tau_{\mathrm{ee}} \ll 1, \qquad k\ell_{\mathrm{ee}} \ll 1$$

where $\ell_{\mathrm{ee}} = v_{\mathrm{F}}\tau_{\mathrm{ee}} =$ e-e collision mean free path

- slow modes are locally conserved quantities: in typical solid

$$\frac{\partial}{\partial t}\begin{pmatrix} \epsilon \\ \rho \end{pmatrix} = \begin{pmatrix} D_{11} & D_{12} \\ D_{12} & D_{22} \end{pmatrix} \nabla^2 \begin{pmatrix} \epsilon \\ \rho \end{pmatrix}$$

(up to Coulomb interactions...can ignore if $\partial_t = 0$)

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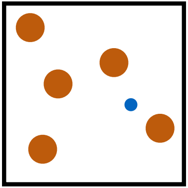

ordinary metal

no hydro

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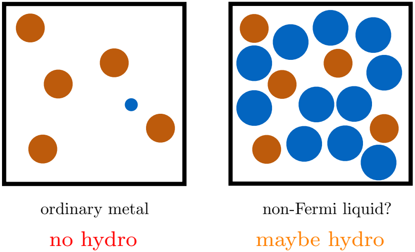

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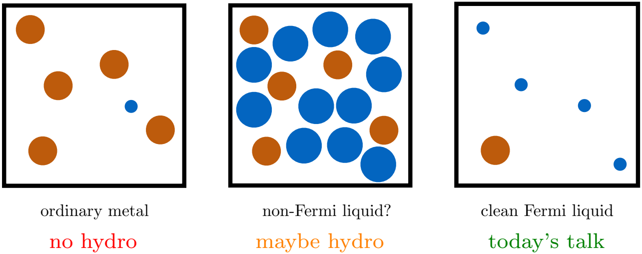
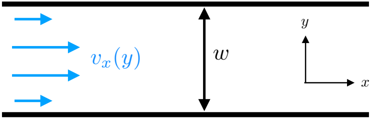


- planar Poiseuille flow (called Gurzhi flow here):



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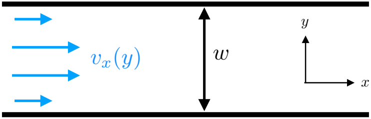

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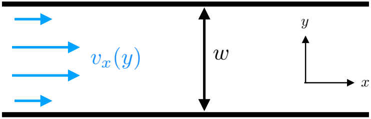

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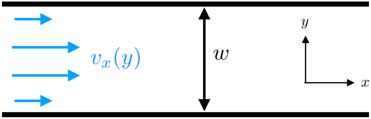

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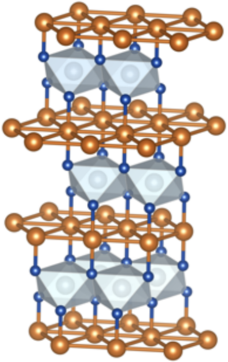
- ballistic limit ($\ell_{\mathrm{ee}} \gg w$): $\mathcal{R}_{\mathrm{ballistic}} \propto \frac{1}{w} \times \frac{v_{\mathrm{F}}}{w}$

delafossite $PdCoO_2$:



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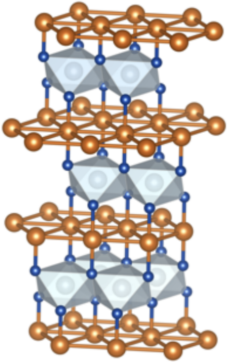

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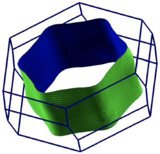

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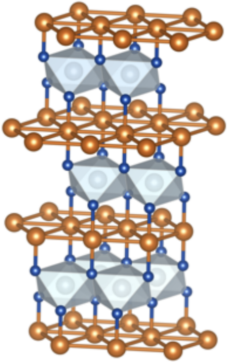

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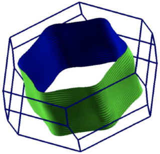
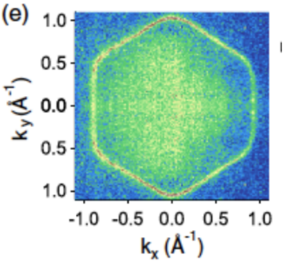


- big hexagonal FS:



[Mackenzie; 1612.04948]

Gurzhi Effect?

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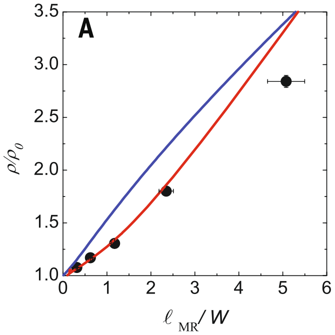

[Moll *et al*; `1509.05691`]

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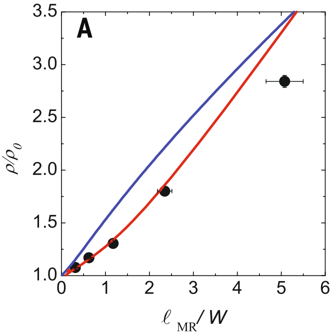

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$$\rho \sim w^{-1} \quad \text{(ballistic)}$$
$$\rho \sim w^{-2} \quad \text{(hydro)}$$

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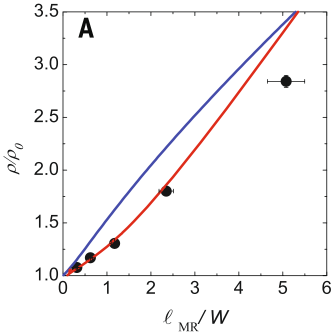

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- T-dependence?

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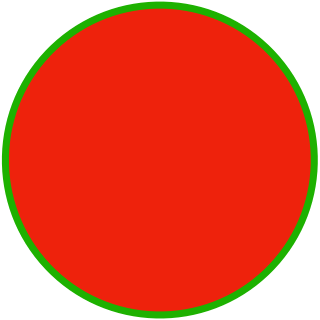
- so far, models of ballistic-to-hydro crossover for this FS...



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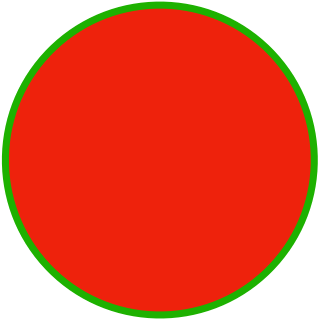

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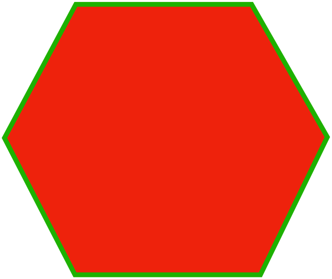

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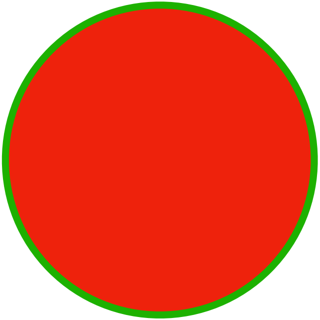

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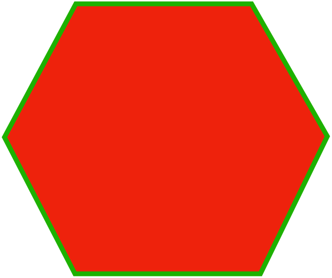

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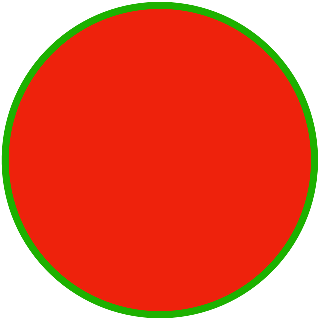

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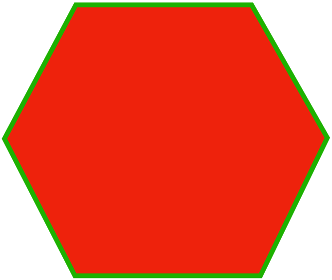

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  - less obvious: ≥ 2 scattering times

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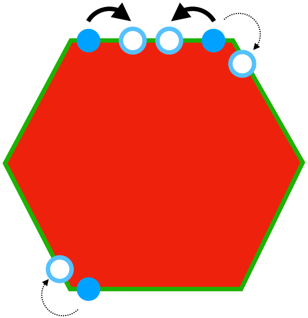

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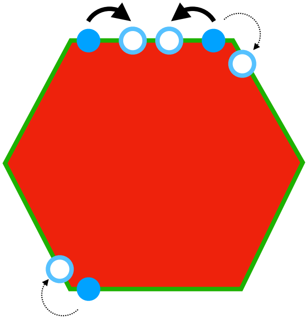

- (at least) two scattering rates (up to logs):

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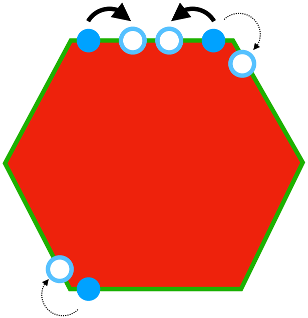

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  $\alpha \to 0$ if branches are perfectly flat...
- similar, but not equivalent, to discrepancy in parity-even/odd scattering rates [Ledwith *et al*; 1708.01815]

The True Hydrodynamic Limit for the Hexagon

- on time scales $\omega \ll \gamma_{\mathrm{s}}$...**hydrodynamics:**

$$\partial_t n + \partial_i (n_0 v_i - D_0 \partial_i n) = 0,$$
$$m n_0 \partial_t v_i + m v_{\mathrm{s}}^2 \partial_i n - \eta_{jikl} \partial_j \partial_k v_l = 0$$

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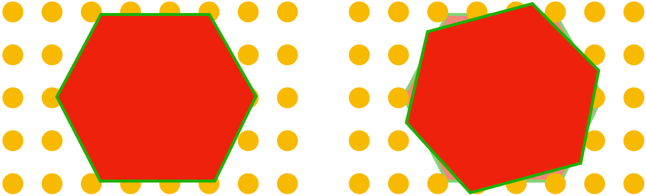
- $\tilde{\eta} > 0$ is **dissipative** – not Hall viscosity!

- $\tilde{\eta} > 0$ allowed: ionic lattice **explicitly breaks rotational invariance** for electrons ($\tau_{ij} \neq \tau_{ji}$)



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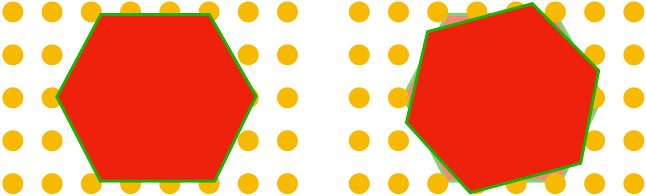
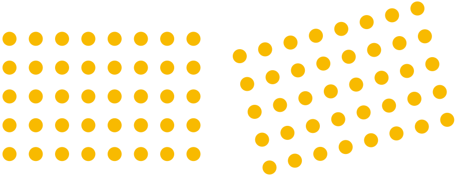
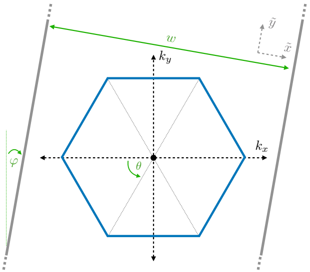


- contrast: the ionic lattice **spontaneously breaks isotropy**: it (w/ electrons) must have local angular momentum conservation ($\tau_{ij}^{\text{universe}} = \tau_{ji}^{\text{universe}}$)



- Gurzhi effect with hexagonal FS?



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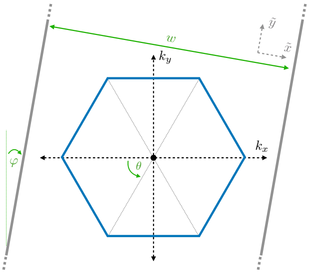
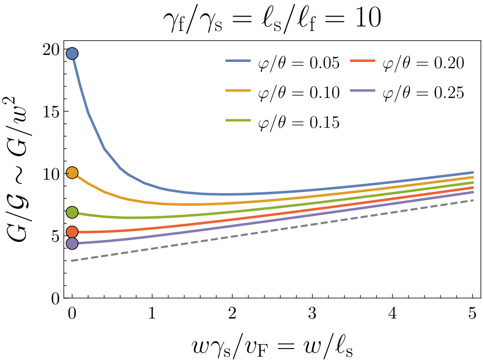
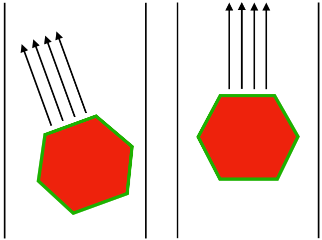




- extreme angular dependence as $\gamma_s \to 0$:



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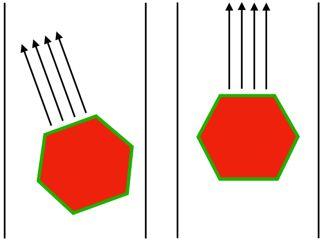

- hydrodynamic limit (no slip boundary conditions):

$$G = \frac{e^2 n^2 w^3}{12\eta_{xyxy}(\phi)}$$

for the hexagon, $\eta_{xyxy}(\phi) = \eta + \tilde{\eta}$.

Replotting Data

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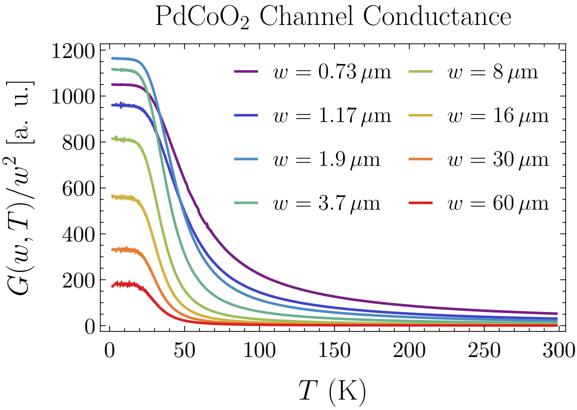

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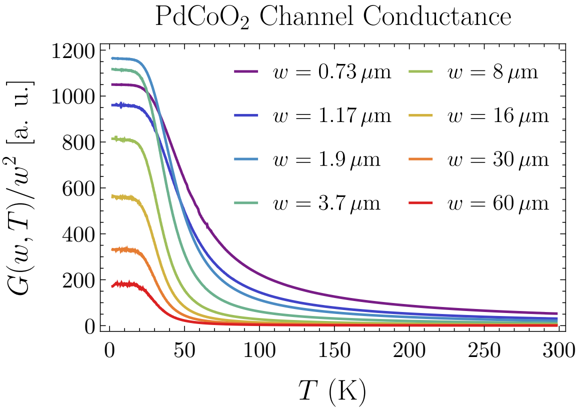


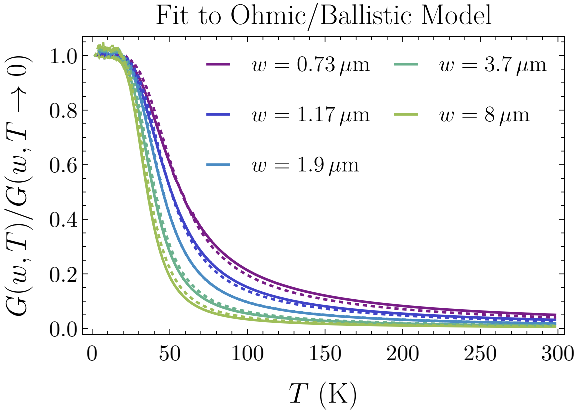
- why small w curves don't collapse as $T \to 0$?

Ohmic Fit?

- we fit these curves to the model

$$\frac{G(w,T)}{G(w,0)} = \frac{A_{\text{ballistic}} + \rho_{\text{bulk}}(0)w}{A_{\text{ballistic}} + \rho_{\text{bulk}}(T)w}$$

with $A_{\text{ballistic}}$ only fit parameter, same for all w



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  - non-monotonic T and w dependence in Gurzhi effect: viscous transport not necessarily "superballistic"
- $PdCoO_2$ – still some open questions...?