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Josephson-type dynamics in magnetic domain walls

- inertial effects from the conduction electrons

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Academy of Finland
Research funding and expertise

arXiv:1908.02299

Collaborators

See also Hilary's poster!

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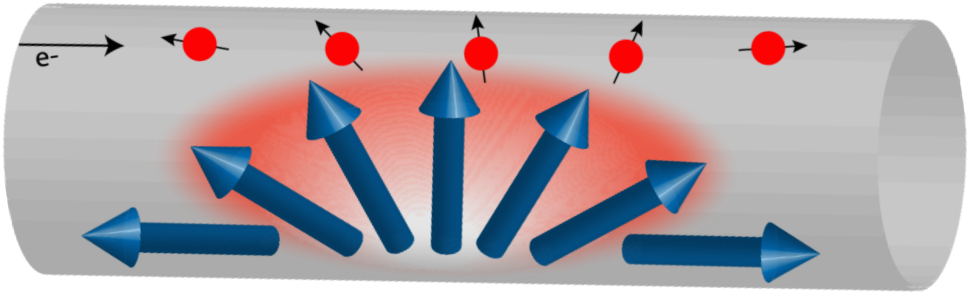
University of Maryland

Real-space topological defects in magnets, coupled to electrons

Current →



This talk: Néel domain wall

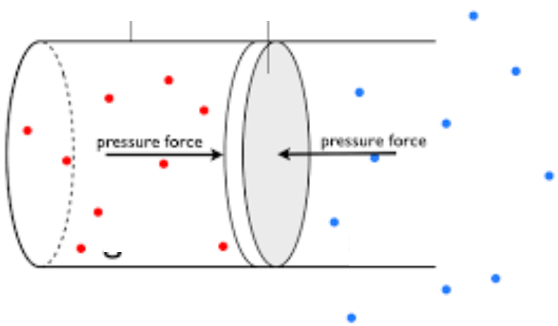
Effect on the dynamics?
- Torque, force
- Damping
- "Inertia" (mass)
- Fluctuations (Langevin terms)

arXiv:1908.02299

This talk

- Domain wall dynamics
- Electrons as an external bath to the domain wall: “inertia”
- Analogy to Josephson junctions
- (If time allows:) spin diffusion equation approach to DW dynamics

Friction from coupling to a bath



A "large" object at X moving in a 'soup' of excitations

Newton's equations of motion

$$M\ddot{X} = -\frac{dV}{dx} - \eta\dot{X} + \xi(t)$$

$$\langle \xi(t)\xi(t')\rangle = 2\eta T\,\delta(t-t')$$

- Generalized Newton-Langevin equation

$$M\ddot{X} + \int_0^t \eta(t-t')\dot{X}(t') = -\frac{dV}{dx} + \xi(t) \quad ; \quad \langle \xi(t)\xi(t')\rangle = \mathcal{C}(t-t')$$

Quantum tunnelling in a dissipative system
A. O. Caldiera, A. J. Leggett (1983)

- Equilibrium: kernel & noise correlator are related via fluctuation-dissipation theorem

$$\eta(t) \sim \int d\omega\, \frac{J(\omega)}{\omega} \cos(\omega t)$$

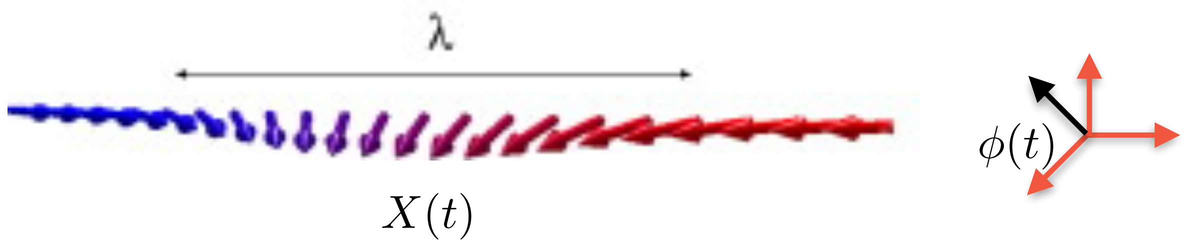
$$\mathcal{C}(t) \sim \int d\omega\; J(\omega) \coth\left(\frac{\hbar\omega}{2T}\right) \cos(\omega t)$$

Spectral function $J(\omega)$ is the central quantity.
$J(\omega) \sim \omega$ leads to Ohmic friction

Here: also mass (reactive component)

DW collective coordinates

Assumption: length of the spin stays constant, direction can change



Typical Ansatz that minimises DW anisotropy energy:

$$\Theta = 2\arctan\left[\exp\left(\frac{X - x}{\lambda}\right)\right]$$

$$\phi = \text{const.}$$

$\chi = X/\lambda$ and ϕ as collective coordinates. Dynamics?

DW dynamics

Thiele (PRL 1973):
DW shape stays undeformed, dynamical variables are $X(t)$ and $\phi(t)$

Berger (1978-1984): hypothesis on the effect of electrons ("force" and "torque")

Microscopic derivation by Tatara and Kohno, PRL 2004

(see also Tatara, et al. Phys. Rep. 2008)

localised spin

damping

# of spins in the wall

force (momentum transfer)

$$\frac{\hbar N S}{\lambda}\left(\dot{\phi}_0 + \alpha \frac{\dot{X}}{\lambda}\right) = F_{\mathrm{pin}} + F_{\mathrm{el}},$$

transverse magnetic anisotropy

$$\frac{\hbar N S}{\lambda}(\dot{X} - \alpha \lambda \dot{\phi}_0) = \frac{N S^2 K_\perp}{2} \sin 2\phi_0 + T_{\mathrm{el},z}$$

spin torque

Our result: Mass Due to an Electronic ‘Bath’

arXiv:1908.02299

$$m\ddot{\phi} + \alpha\dot{\phi} - \dot{\chi} + j_t + \sin(2\phi) = \xi_\phi(t)$$

$$m\ddot{\chi} + \dot{\phi} + \alpha\dot{\chi} + f_x + k_\mathrm{p}\chi = \xi_\chi(t)$$

$$m = \frac{K_\perp}{2\Delta}\frac{s}{N} \;\; ; \;\; s = \frac{(k_{F\downarrow} - k_{F\uparrow})\lambda}{2\pi}$$

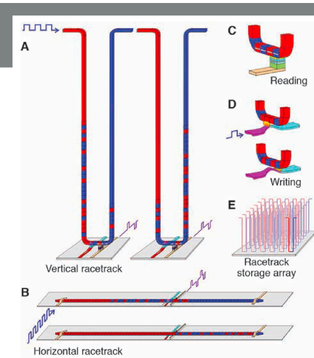
$$\langle \xi_i(t)\xi_j(t')\rangle = \mathcal{C}^{ij}(t-t')$$

- Effective domain wall mass for both coordinates (in contrast to pinning induced mass)
- New way to describe electron-DW interaction
- The effect of the mass is determined by both magnetic and electronic system properties.

REVIEW

Magnetic Domain-Wall Racetrack Memory

Stuart S. P. Parkin,* Masamitsu Hayashi, Luc Thomas



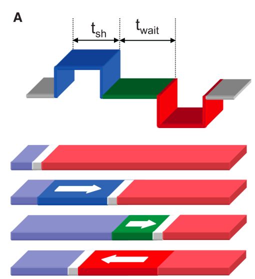
Science (2008)

Many proposals for DW devices based on fast manipulation via electric current.

However, their mass (inertia) ultimately determines their operation speed.

Dynamics of Magnetic Domain Walls Under Their Own Inertia

Luc Thomas,* Rai Moriya, Charles Rettner, Stuart S. P. Parkin*



Science (2010)

Mass is thought to arise from *pinning* and/or *wall deformation*. It appears to be highly material dependent.

ARTICLE

Received 4 Mar 2016 | Accepted 7 Oct 2016 | Published 24 Nov 2016 DOI: 10.1038/ncomms13533 OPEN

Tunable inertia of chiral magnetic domain walls

Jacob Torrejon[1,2], Eduardo Martinez[3] & Masamitsu Hayashi[1,4]

Current-induced resonance and mass determination of a single magnetic domain wall

Eiji Saitoh[1], Hideki Miyajima[1], Takehiro Yamaoka[2] & Gen Tatara[3]

PRL 108, 247202 (2012) PHYSICAL REVIEW LETTERS week ending 15 JUNE 2012

Direct Observation of Massless Domain Wall Dynamics in Nanostripes with Perpendicular Magnetic Anisotropy

Electrons as a bath

$$\mathcal{S}_{dw}(\chi,\phi) = NS\int dt\ \hbar\dot{\chi}\phi - \frac{K_\perp S}{2}\sin^2\phi$$

Overall aim:

$$\mathcal{S}_e = \int dtdx\bar{\psi}(x,t)[i\hbar\partial_t - \hat{H}\left[\mathbf{\Omega}[X(t),\phi(t)]\right]\psi(x,t)$$

- Write $\mathcal{S} = \mathcal{S}_{dw} + \mathcal{S}_e$
- Identify terms due to dynamic fields ($\dot{X}$ and $\dot{\phi}$)
- Write the action in Keldysh contour

$$\int_C dt\ \mathcal{L}^{\mathrm{K}}(t) = \int_{-\infty}^{\infty} dt\ \mathcal{L}^+(t) + \int_{\infty}^{-\infty} dt\ \mathcal{L}^-(t)$$

- Integrate out electrons
- Expand in the dynamic fields
- Result: a new magnetic action with a dissipation kernel (polarization operator) describing the ensuing dynamics

Spin torque and force

Action for the magnet $\mathcal{S}_{\rm m} = \int dt \frac{dx}{a}\, \hbar S \mathbf{A}[\mathbf{\Omega}] \cdot \dot{\mathbf{\Omega}} - \frac{S^2}{2} \int \frac{dx}{a}\, J(\nabla \mathbf{\Omega})^2 - K_z \Omega_z^2 + K_\perp \Omega_y^2$

Rigid domain wall $\mathcal{S}_{dw}(\chi, \phi) = NS \int dt\, \hbar \dot{\chi} \phi - \frac{K_\perp S}{2} \sin^2 \phi + V_{\rm pin}[\chi]$

Pinning

Exchange coupling to electrons $H_{sd} = -\Delta \int dx\, \mathbf{\Omega}(x - \chi, \phi, t) \cdot \bar{\boldsymbol{\psi}}(x) \hat{\boldsymbol{\tau}} \boldsymbol{\psi}(x)$

Assume electrons are very fast, include charge and spin currents acting on the DW:

$$\alpha \dot{\phi} - \dot{\chi} + j_t + \sin(2\phi) = 0 \qquad j_t \sim P_s j \qquad \text{“torque”}$$

$$\dot{\phi} + \alpha \dot{\chi} + f_x + k_{\rm p} \chi = 0 \qquad f_x \sim \rho_w j \qquad \text{“force”}$$

L. Berger, Journal of Applied Physics (1984)
Tatara & Kohno, PRL (2004) (microscopic derivation)

α: damping

”Non-adiabaticity parameter” $\beta = f_x / j_t \sim \lambda_F / \lambda$ (clean case)

Integrating out the electrons

Magnet + electrons $$\mathcal{S} = \mathcal{S}_{dw} + \int dt dx\, \bar{\boldsymbol{\psi}}(x,t)[i\hbar\partial_t - h_0 + \Delta\boldsymbol{\Omega}(x - X(t), \phi(t)) \cdot \hat{\boldsymbol{\tau}}]\boldsymbol{\psi}(x,t)$$

Redefine the fermion fields to formally "undo" the DW dynamics

$$\tilde{\boldsymbol{\psi}}(x,t) = \exp\left[i\frac{\hat{\tau}^3\phi(t)}{2}\right]\boldsymbol{\psi}(x - \lambda\chi(t), t), \qquad \bar{\tilde{\boldsymbol{\psi}}}(x,t) = \exp\left[-i\frac{\hat{\tau}^3\phi(t)}{2}\right]\bar{\boldsymbol{\psi}}(x - \lambda\chi(t), t).$$

Resulting action $$\mathcal{S} = \mathcal{S}_{dw} + \int dt dx\, \bar{\boldsymbol{\psi}}'(x,t)[i\hbar\partial_t - \hat{H}[\boldsymbol{\Omega}_0(x)] - i\hbar\lambda\dot{\chi}\partial_x + \frac{\hbar}{2}\dot{\phi}\hat{\tau}^z]\boldsymbol{\psi}'(x,t)$$

Change to the eigenspace of the Hamiltonian: $\hat{H}[\boldsymbol{\Omega}_0(x)]\varphi_{\sigma k}(x) = \varepsilon_{\sigma k}\varphi_{\sigma k}(x)$

$$\psi(x,t) = \sum_{\sigma k}\varphi_{\sigma k}(x)c_{\sigma k}(t) \qquad \bar{\psi}(x,t) = \sum_{\sigma k}\varphi^*_{\sigma k}(x)\bar{c}_{\sigma k}(t),$$

Results into $$\mathcal{S}_e = \int dt \sum_{\sigma\sigma' k k'} \bar{c}_{\sigma k}(t)\left[i\hbar\partial_t - \varepsilon_{\sigma k}\right]c_{\sigma k}(t) - \dot{\chi}\bar{c}_{\sigma k}(t)\, {}^{\chi}V^{\sigma\sigma'}_{kk'} c_{\sigma' k'}(t) - \dot{\phi}\bar{c}_{\sigma k}(t)\, {}^{\phi}V^{\sigma\sigma'}_{kk'} c_{\sigma' k'}(t)$$

Perturbation $${}^{\chi}V^{\sigma\sigma'}_{kk'} \sim \int dx\, \varphi^*_{\sigma k}\partial_x\varphi_{\sigma' k'} \qquad {}^{\phi}V^{\sigma\sigma'}_{kk'} \sim \int dx\, \varphi^*_{\sigma k}\hat{\tau}^z\varphi_{\sigma' k'}$$

Rewrite S on Keldysh contour, expand into second order, perform the Grassman integral

Effective Keldysh Domain Wall Action

After integrating out the electrons

$$\mathcal{S}'_{dw} = \mathcal{S}_{dw} - \int dt dt' \; Q^i_q \eta^{ij}(t-t') \dot{Q}^j_c(t') + \frac{i}{2} Q^i_q(t) \mathcal{C}^{ij}(t-t') Q^j_q(t')$$

$$\vec{Q} = \begin{pmatrix} \chi \\ \phi \end{pmatrix}$$

- Noise Correlation Function
- Response kernel

Equations of Motion:

$$\hbar N \dot{\phi} + \int dt' \; \eta^{\chi i}(t-t') \dot{Q}^i(t') = \xi_\chi(t)$$

$$-\hbar N \dot{\chi} + \frac{K_\perp N}{2} \sin(2\phi) + \int dt' \; \eta^{\phi i}(t-t') \dot{Q}^i(t') = \xi_\phi(t)$$

$$\langle \xi_i(t) \xi_j(t') \rangle = \mathcal{C}^{ij}(t-t')$$

$$\eta^{ij}(\omega) = \frac{1}{\omega}[J^{ij}(\omega) + i f^{ij}(\omega)]$$

$$\mathcal{C}^{ij}(\omega) = \coth\left(\frac{\hbar\omega}{2T}\right) J^{ij}(\omega)$$

$$f^{ij}(\omega) = \frac{\hbar^2 \omega^2}{2} \sum_{\substack{\sigma\sigma' \\ kk'}} {}^{i}V^{\sigma\sigma'}_{kk'} \; {}^{j}V^{\sigma'\sigma}_{k'k} \left[\frac{h_{\sigma' k'} - h_{\sigma k}}{\hbar\omega - (\varepsilon_{\sigma' k'} - \varepsilon_{\sigma k})} \right]$$

$$J^{ij}(\omega) = \frac{\pi \hbar^2}{2} \sum_{\substack{\sigma\sigma' \\ kk'}} {}^{i}V^{\sigma\sigma'}_{kk'} \; {}^{j}V^{\sigma'\sigma}_{k'k} \left[h_{\sigma' k'} - h_{\sigma k}\right] (\varepsilon_{\sigma' k'} - \varepsilon_{\sigma k})^2 \delta\left[\hbar\omega - (\varepsilon_{\sigma' k'} - \varepsilon_{\sigma k})\right]$$

Fluctuation-Dissipation Theorem

$$\vec{Q} = \begin{pmatrix} \chi \\ \phi \end{pmatrix}$$

$$\mathcal{S}'_{dw} = \mathcal{S}_{dw} - \int dtdt' \; Q_q^i \eta^{ij}(t-t')\dot{Q}_c^j(t') + \frac{i}{2} Q_q^i(t)\mathcal{C}^{ij}(t-t')Q_q^j(t')$$

dissipation → J^{ij}, reactive → f^{ij}

$$\eta^{ij}(\omega) = \frac{1}{\omega}[J^{ij}(\omega) + i f^{ij}(\omega)]$$

$$\mathcal{C}^{ij}(\omega) = \coth\left(\frac{\hbar\omega}{2T}\right) J^{ij}(\omega)$$ FD theorem

$$\langle \xi_i(t)\xi_j(t')\rangle = \mathcal{C}^{ij}(t-t')$$

Correlated noise occurs naturally in this model

Here: (DW, no spin relaxation)

$J(\omega = 0) \to 0$

f determines low-frequency DW dynamics and the effective mass

Spectral function *J* is only relevant for $\omega \approx 2\Delta$

No ohmic friction for $\omega \to 0$

$$f^{ii} \approx \frac{4\Delta\hbar^2\omega^2 s}{(\hbar\omega)^2 - 4\Delta^2} \qquad s = \frac{(k_{F\downarrow} - k_{F\uparrow})\lambda}{2\pi}$$

$$f^{\phi\chi} \approx \frac{2i\hbar^3\omega^3 s}{(\hbar\omega)^2 - 4\Delta^2}$$

New Dynamical System Description

$$M_{\rm dw}\ddot{\phi} - \hbar N\dot{\chi} + \frac{NK_\perp}{2}\sin(2\phi) + F^\phi = \xi_\phi(t)$$
$$M_{\rm dw}\ddot{\chi} + \hbar N\dot{\phi} + F^\chi = \xi_\chi(t).$$

$$M_{\rm dw} = \frac{s}{\Delta}$$

$$\hbar\omega \ll 2\Delta$$

Make it dimensionless:

$$m\ddot{\phi} + \alpha\dot{\phi} - \dot{\chi} + j_t + \sin(2\phi) = \xi_\phi(t)$$
$$m\ddot{\chi} + \dot{\phi} + \alpha\dot{\chi} + f_x + k_{\rm p}\chi = \xi_\chi(t)$$

Dimensionless mass parameter
Recover previous theory for
$m \ll 1$

$$m = \frac{K_\perp}{2\Delta}\frac{s}{N} \quad ; \quad s = \frac{(k_{F\downarrow} - k_{F\uparrow})\lambda}{2\pi}$$

Interpretation of the "mass"

$$M_{\mathrm{dw}} = \frac{s}{\Delta} \qquad s = \frac{(k_{F\downarrow} - k_{F\uparrow})\lambda}{2\pi} \qquad k_{F\sigma} = \sqrt{2m_e(\mu - \sigma\Delta)/\hbar^2}$$

Expand for small $\Delta \Rightarrow \quad M_{\mathrm{dw}} \approx \dfrac{\lambda}{\pi v_F}$

$\sim$ time it takes for the electrons to traverse λ

$$m = \frac{K_\perp}{2\Delta} \frac{s}{N}$$

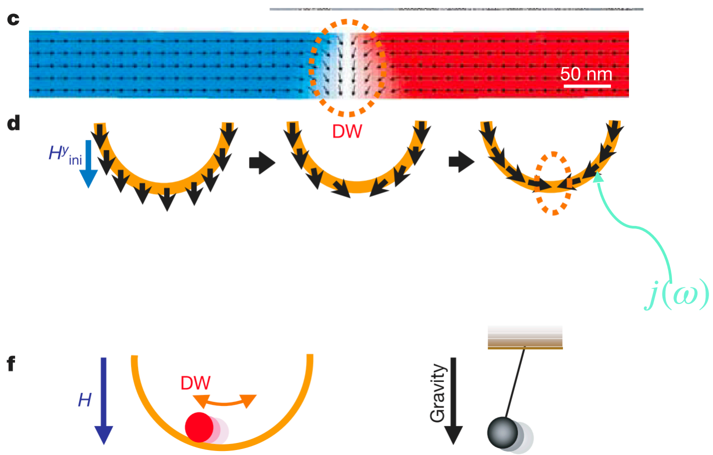
Example: Co/Ni nanowires:

$K_\perp = K_u \lambda w^2$
$v_F \sim 10^6$ m/s
$\lambda \sim 100$ nm
$w \sim 50$ nm
$a \sim 1$ Å

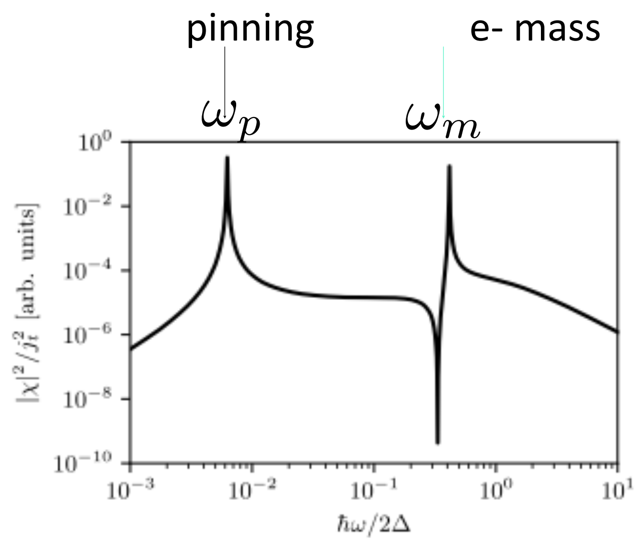
$$m \sim 10$$

Observable Effects I: Resonant Excitation

$$\omega_p = \sqrt{k_p/m_D} \quad m_D\text{: Döring mass}$$



Saitoh, E. at al Nature (2004)



$$m\ddot{\phi} + \alpha\dot{\phi} - \dot{\chi} + j_t + \sin(2\phi) = 0$$
$$m\ddot{\chi} + \dot{\phi} + \alpha\dot{\chi} + k_p\chi = 0$$

$$\hbar\omega_m \approx \frac{2\Delta}{N+2s}\sqrt{N^2 + \frac{K_\perp N s}{\Delta}}$$

Usually quite large (THz)

Effective Lagrangian/Hamiltonian

The dynamics is produced from the Lagrangian

$$\mathcal{L} = \frac{1}{2M_{\mathrm{dw}}}(\dot{\phi}^2 + \dot{\chi}^2) + \frac{2+\epsilon}{2}\dot{\chi}\phi + \frac{\epsilon}{2}\chi\dot{\phi} + \frac{1}{2}\cos 2\phi - j_t\phi - f_x\chi$$

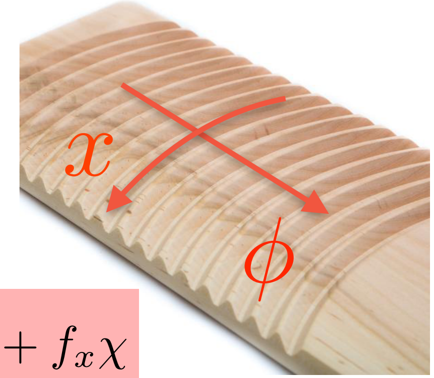
$$M_{\mathrm{dw}} = \frac{\hbar^2 s}{\Delta}$$

(ϵ is a free parameter)

Conjugate momenta:

$$p_\phi = \frac{d\mathcal{L}}{d\dot{\phi}} = \dot{\phi} + \frac{1}{2}\epsilon\beta^2 x$$

$$p_x = \frac{d\mathcal{L}}{d\dot{x}} = \dot{x} + \frac{2+\epsilon}{2}\beta^2\phi.$$



Hamiltonian: "gauge coupling"

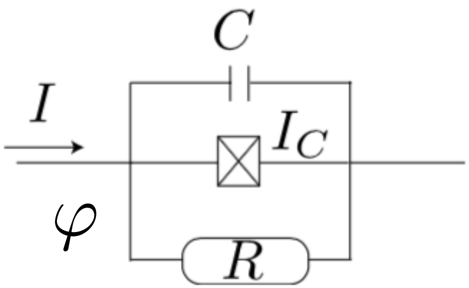
$$H = \frac{1}{2M_{\mathrm{dw}}}\left(p_\phi - \frac{1}{2}\epsilon\chi\right)^2 + \frac{1}{2M_{\mathrm{dw}}}\left(p_x - \frac{2+\epsilon}{2}\phi\right)^2 - \frac{1}{2}\cos 2\phi + j_t\phi + f_x\chi$$

"tilted washboard potential"

Energy loss due to damping: $\dot{H} = -\alpha(\dot{\phi}^2 + \dot{\chi}^2)$

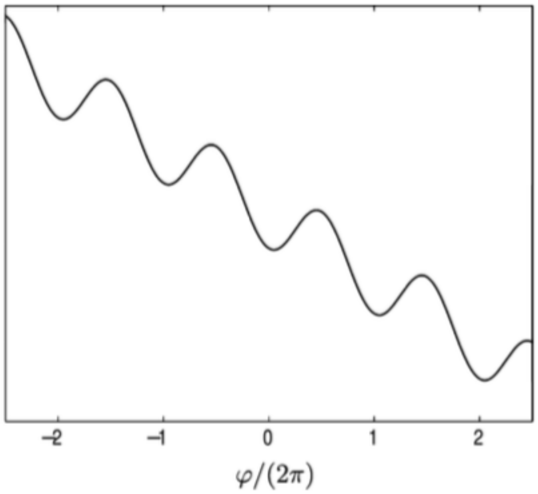
RCSJ model of Josephson junctions

RCSJ: $$I = I_c \sin(\varphi) + \frac{\hbar}{2e}\dot{\varphi}/R + C\frac{\hbar}{2e}\ddot{\varphi}.$$



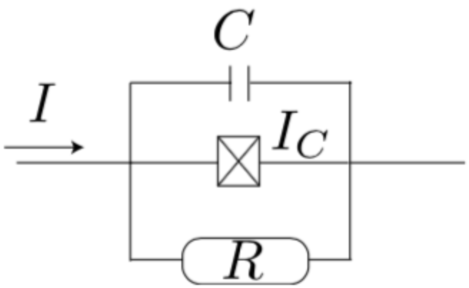
Tilted washboard potential:

$$V(\varphi) = E_J \cos(\varphi) - I\varphi$$

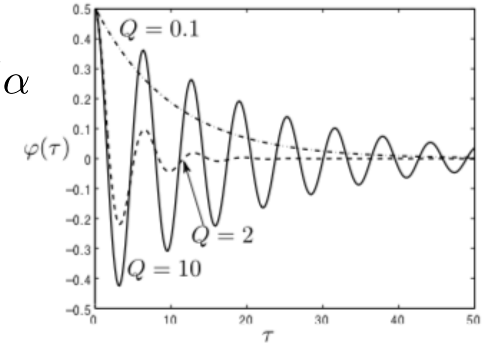


RCSJ model of Josephson junctions

RCSJ: $\ddot{\varphi} + \alpha\dot{\phi} + \sin\varphi - i_b = 0$



$Q = 1/\alpha$



Overdamped/underdamped limits

$\alpha > 2 \qquad \alpha < 2$

Domain wall without force or pinning:

$$m\ddot{\phi} + \alpha\dot{\phi} - \dot{\chi} + \sin 2\phi + j_t = 0$$

$$m\ddot{\chi} + \dot{\phi} + \alpha\dot{\chi} = 0$$

$m \to 0$: solve $\dot{\chi} = -\dot{\phi}/\alpha$

$$\frac{\alpha^2 + 1}{\alpha}\dot{\phi} + \sin 2\phi + j_t = 0$$

Analogous to over damped RSJ!

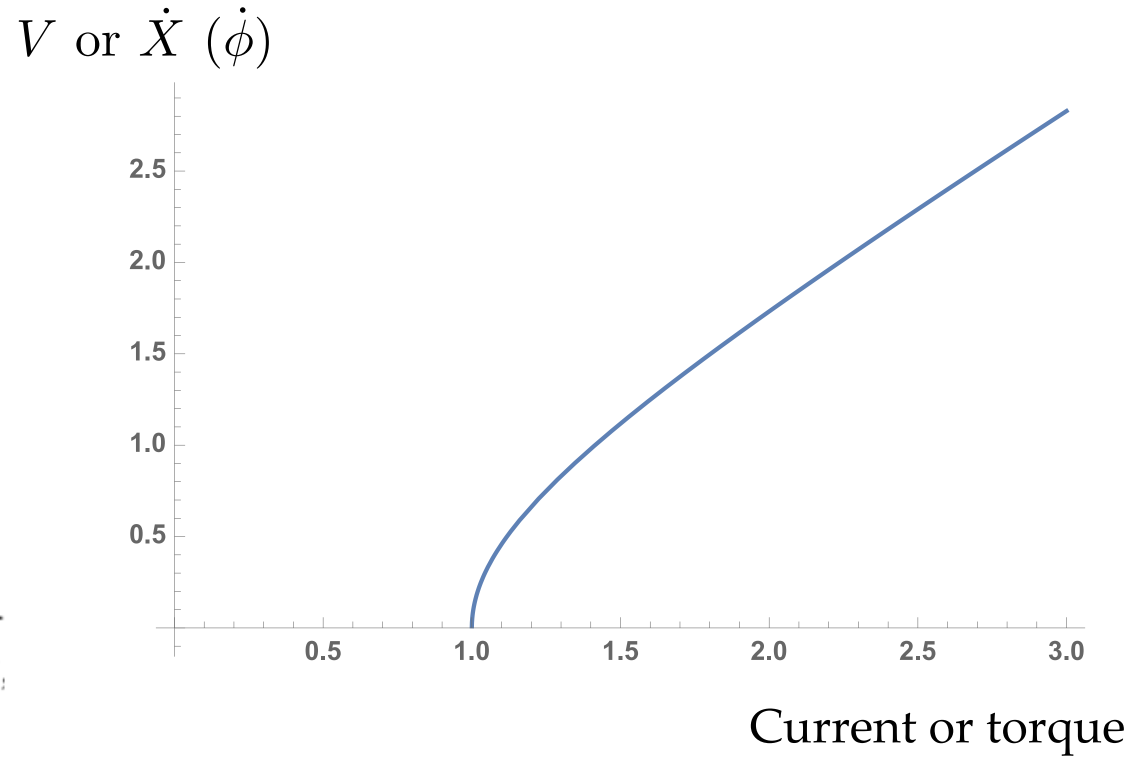
Threshold current…

Overdamped RSCJ:

$$\frac{\hbar\langle\dot{\varphi}\rangle}{2e} = V = R_N\sqrt{I^2 - I_c^2}$$

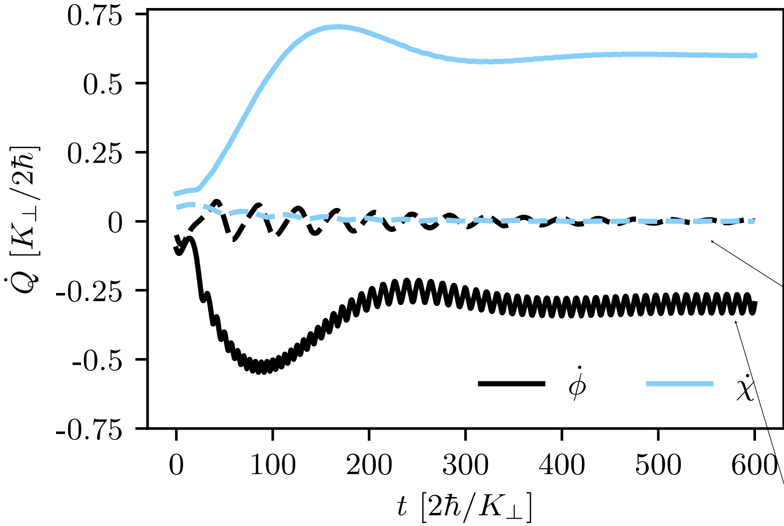
Domain wall speed:

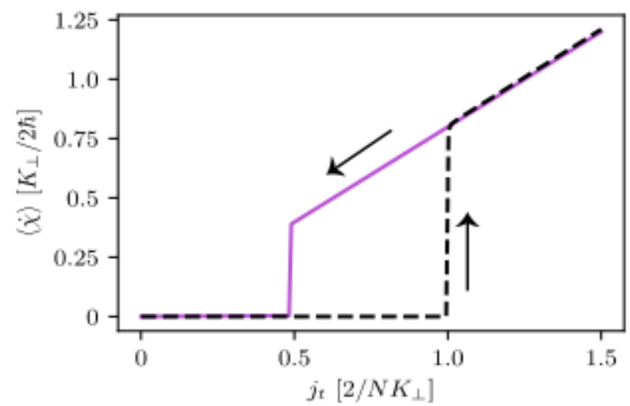
$$\langle\dot{X}\rangle = \frac{1}{1+\alpha^2}\frac{1}{2S}\frac{a^3}{e}\sqrt{j_s^2 - (j_s^{cr(1)})^2}$$



Include m: hysteretic dynamics

(Exact analogue to RCSJ no longer valid)





damped state

$\dot{\chi}\ ,\ \dot{\phi} = 0$

'running' state

$\dot{\chi} \sim -\dot{\phi}/\alpha$

Width of hysteresis loop $\sim m$

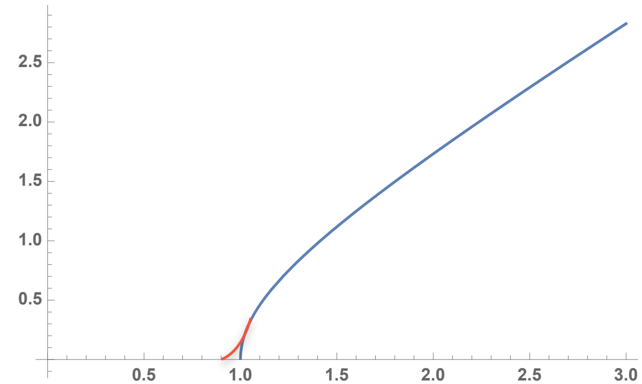
$$m\dot{\phi} + \alpha\dot{\phi} - \dot{\chi} + j_t + \sin(2\phi) = 0$$

$$m\ddot{\chi} + \dot{\phi} + \alpha\dot{\chi} = 0$$

$m \gtrsim \frac{2}{\alpha^2}$ Condition for hysteresis

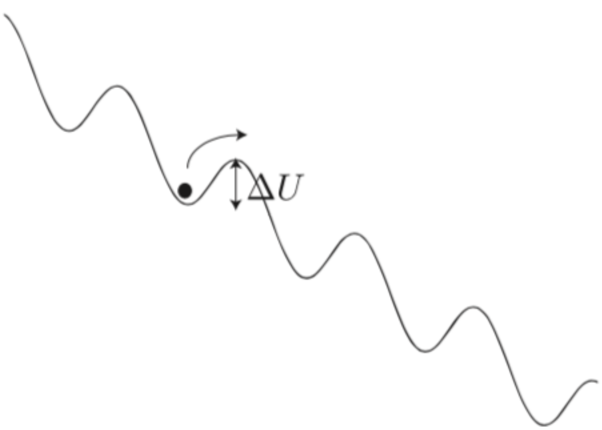
Including fluctuations: speculations

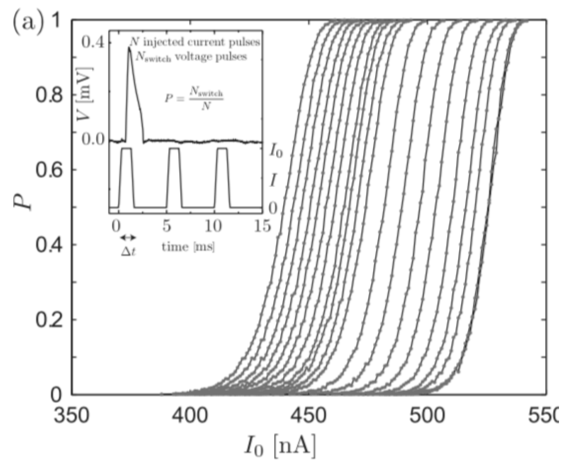
Thermal fluctuations: broadening of the threshold for critical dynamics
(relevant temperature ~ anisotropy energy)



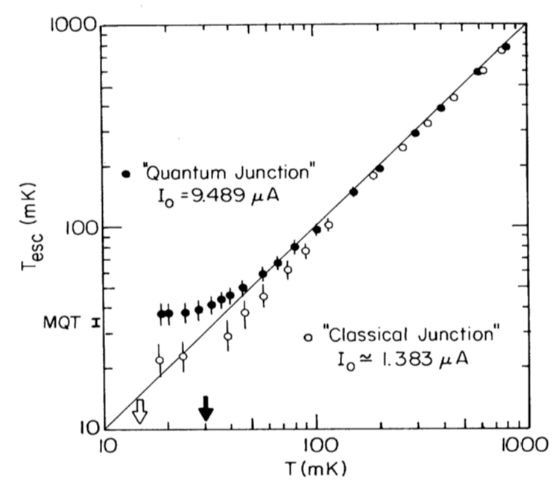
“Escape dynamics:”

(studied already for DWs, see Thomas, *et al.*, Science **330**, 1810 (2010))





J. Peltonen, dissertation (2011)

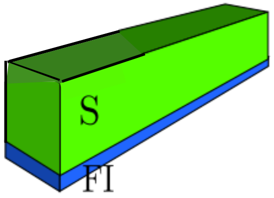


M. Devoret, J. Martinis & J. Clarke, PRL 1985

Thermal activation vs. macroscopic quantum tunneling?

Other topics of interest for KITP

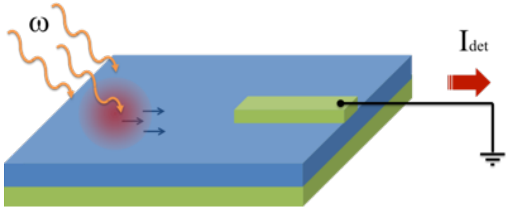
Giant thermoelectric effect and long-range spin transport in super-ferro systems



Ozaeta, Virtanen, Bergeret, TTH, PRL 2014

Bergeret, Silaev, Virtanen, TTH, Rev. Mod. Phys. **90**, 041001 (2018)

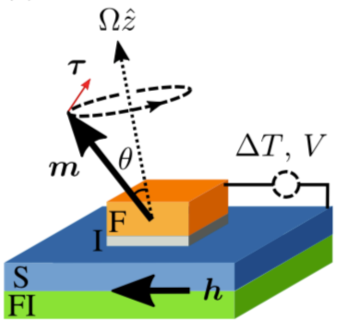
SUPERconducting ThermoElectric Detector



SUPERTED

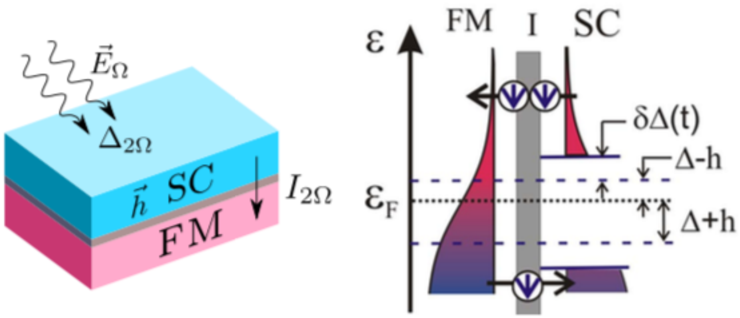
FUNDING OPPORTUNITIES
FUTURE & EMERGING TECHNOLOGIES

Magnetization dynamics coupled with superconductivity: thermoelectric torques and precession induced cooling



R. Ojajärvi, J. Manninen, TTH, and P. Virtanen, arXiv:1907.00424

Higgs mode coupling to spin currents



Silaev, Ojajärvi, TTH, arXiv:1907.00539

Conclusions on the mass

- New formalism to describe electrons coupled to rigid magnetic textures
- New equation of motion for domain walls with correlated noise
- Intrinsic, electron-induced mass *unrelated to dissipation*
- Extensions:
  - disordered systems
  - Nonzero fluctuations
  - Including spin-waves
- Further reading: **arXiv:1908.02299**