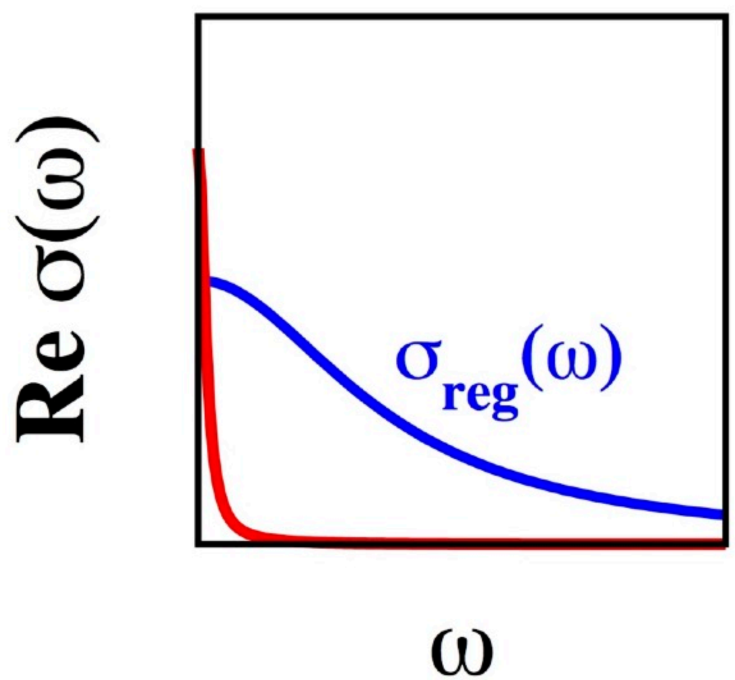
Finite-temperature transport in 1d quantum lattice systems: From quantum magnets to ultra-cold atomic gases



Fabian Heidrich-Meisner
Georg-August-Universität Göttingen

KITP, November 13, 2019

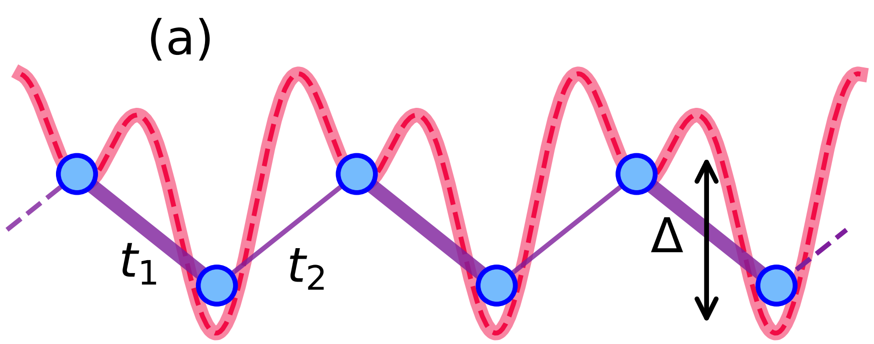
GEORG-AUGUST-UNIVERSITÄT GÖTTINGEN

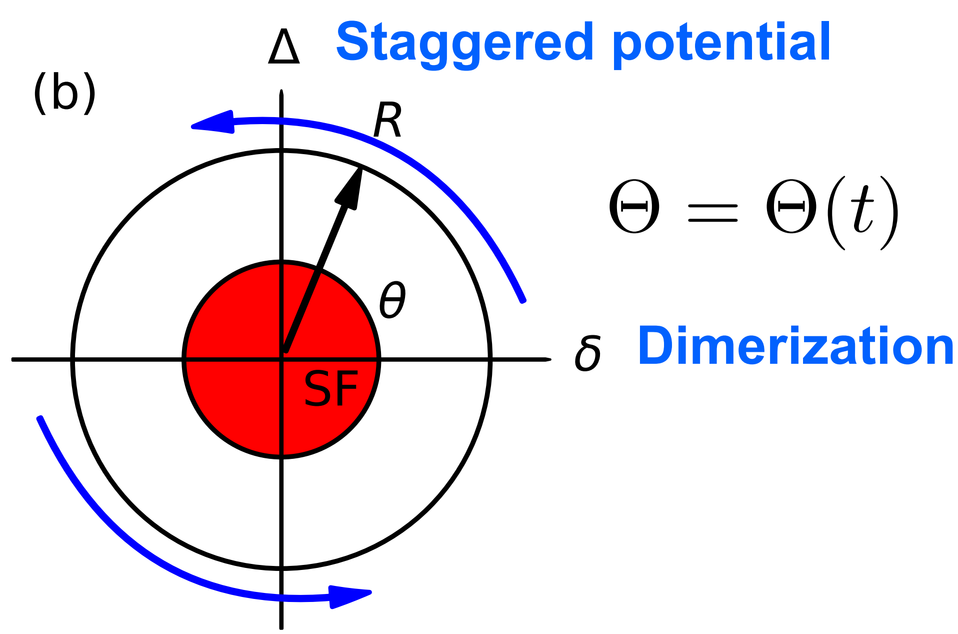
$[q,p]=i\hbar$

Teaser: Topological charge pumps

$$H = \sum_i \left[(-J_1 a^\dagger_{2i} a_{2i+1} - J_2 a^\dagger_{2i+1} a_{2i+2} + h.c.) + (-1)^i \frac{\Delta}{2} n_i + \frac{U}{2}(n_i - 1) n_i \right]$$

$$J_1 = (1+\delta)J \quad J_2 = J(1-\delta)J$$



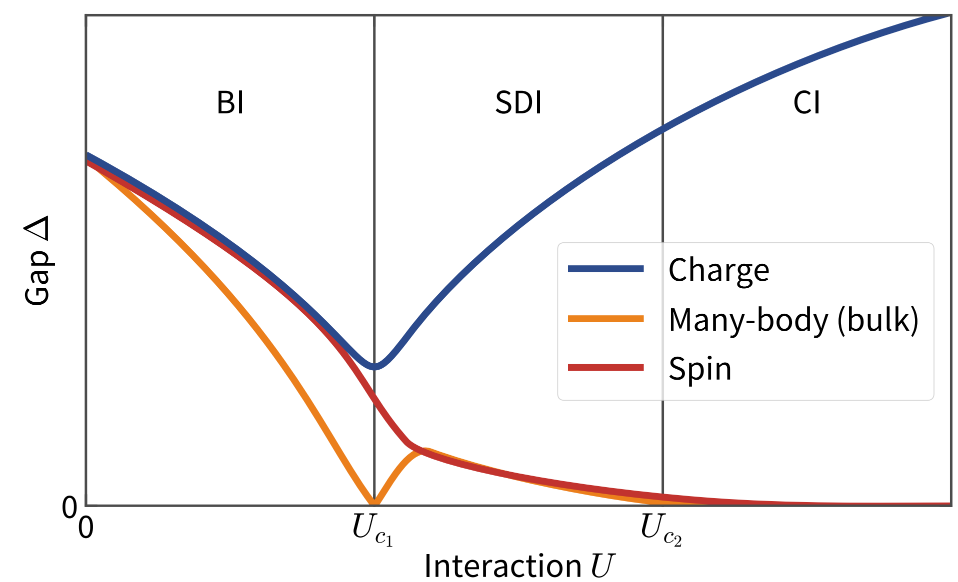


$$\Theta = \Theta(t)$$

Bosons

Hayward, Schweizer, Lohse, Aidelsburger, FHM
Phys. Rev. B 98, 245148 (2018)

Fermions



U-driven topological transition

Stenzel, Hayward, Hubig, Schollwöck, FHM
Phys. Rev. A 99, 053614 (2019)

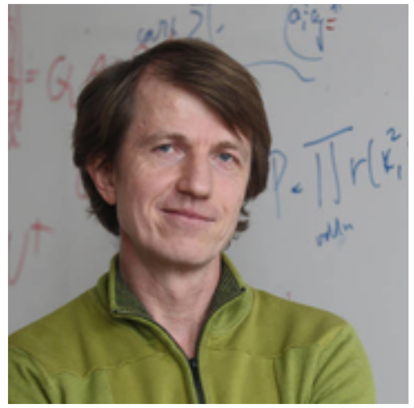
Experiments : Munich, Kyoto

*Lohse et al. Nature Phys. **12**, 350 (2016)*
Nakajima et al. Nature Phys 12 296 (2016)

Spin pump: : Schweizer et al. Phys. Rev. Lett. 117, 170405 (2016)

In collaboration with

Christoph Karrasch
TU Braunschweig



Tomaz Prosen
U Ljubljana

Joel Moore
Berkeley

Jan Stolpp
U Göttingen

Eric Jeckelmann
U Hannover

Optical-lattice experiments (LMU & MPQ):
P. Ronzheimer, S. Hodgman, M. Schreiber, S. Braun, I. Bloch, U. Schneider

Q-mag transport experiments (IFW Dresden):
C. Hess, B. Buechner

Phonons
F. Dorfner (LMU), C. Brockt (Hannover),
J. Herbrych (Wroclaw), E. Dagotto (UTK & ORNL)
L. Vidmar (Ljubljana)

Other related theory work with: B. Bertini, M. Znidaric (Ljubljana), S. Langer (), J. Hauschild (Berkeley), R. Steinigeweg, J. Gemmer (Osnabrueck), D. Kennes (Aachen), W. Brenig (Braunschweig), A. Honecker (Cergy-Pontoise), D. Cabra (La Plata)

DFG-FG 912

DFG-Forschergruppe 801

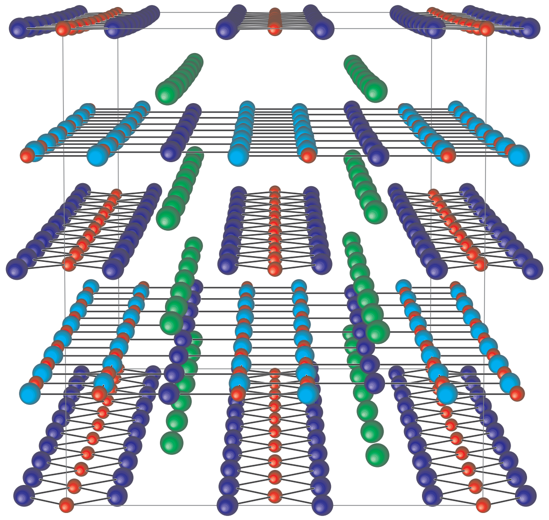
FOR 1807

SFB 1073

Outline

$$H = J \sum_i \vec{S}_i \cdot \vec{S}_{i+1}$$

Quantum magnets



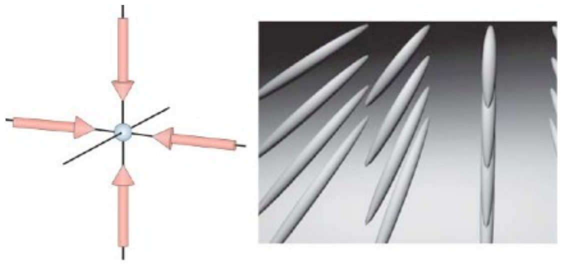
Strongly correlated systems

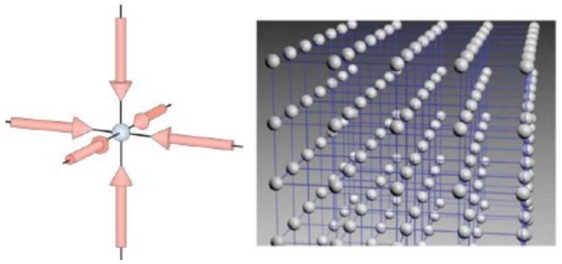
Anomalous conductivities in 1D integrable models
Reason: Non-trivial conservation laws in 1D

$$[H, Q] = 0 \rightarrow \sigma_{dc} = \infty$$

Ballistic, ..., diffusive dynamics

Optical lattices





1) Intro & Experimental context

2) Overview: Spin-1/2 XXZ chain
(a numerical DMRG/ED perspective)

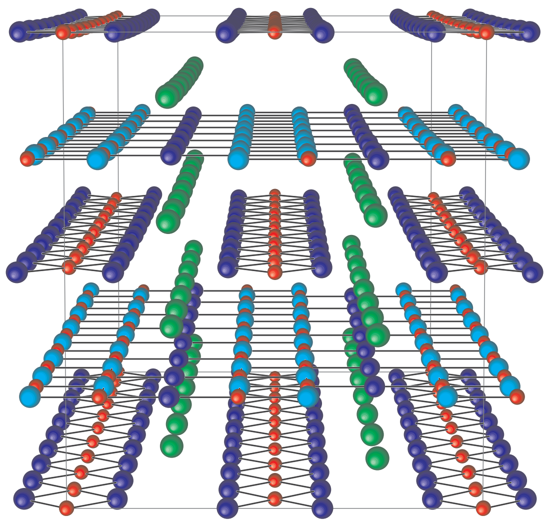
3) Proposal for optical lattice experiments:
Hubbard chains

4) Towards phonons: DMRG methods

Outline

$$H = J \sum_i \vec{S}_i \cdot \vec{S}_{i+1}$$

Quantum magnets



Mission statement:

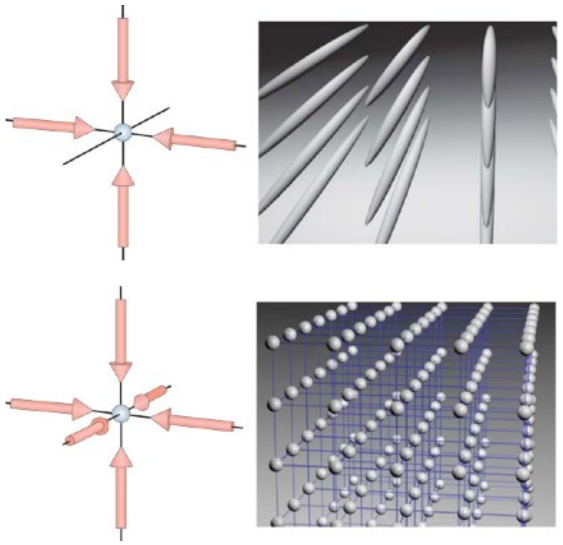
Microscopic models

T>0

Spin & heat conductivity from Kubo

Exact results

Optical lattices



Theoretical motivation (or obsession): Finite-temperature Drude weights

Linear response regime (Kubo): $C(t) = \langle j(t) j \rangle$

Drude weight & regular part

$$\mathrm{Re}\,\sigma(\omega) = D(T)\delta(\omega) + \sigma_{\mathrm{reg}}(\omega)$$

Exactly conserved current

$$[H, j] = 0 \;\rightarrow\; \mathrm{Re}\;\sigma(\omega) = D(T)\delta(\omega)$$

Finite Drude weight:
Divergent dc conductivity
at finite temperatures

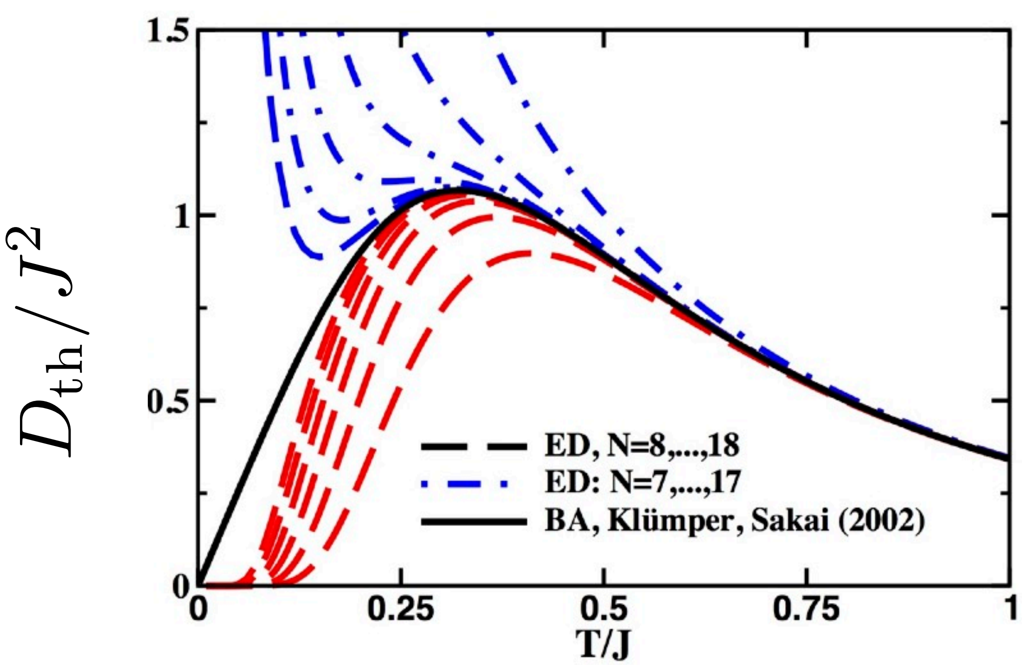
Same reasoning for
charge, particle, spin, thermal transport

Themal conductivity
in S=1/2 Heisenberg chain

$$H = J \sum_i \vec{S}_l \cdot \vec{S}_{l+1}$$

$$j_{\mathrm{th},l} \sim \vec{S}_l \cdot (\vec{S}_{l+1} \times \vec{S}_{l+2}) \quad [H, j_{\mathrm{th}}] = 0$$

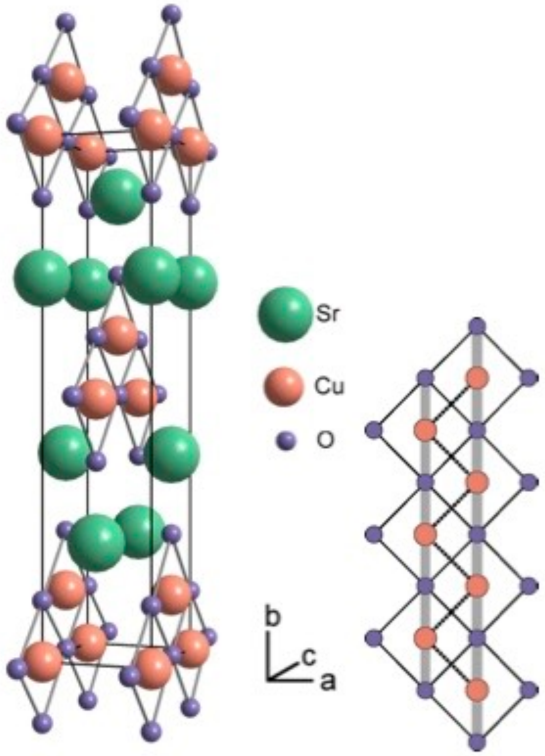
$$\mathrm{Re}\,\kappa(\omega) = D_{\mathrm{th}}(T)\delta(\omega)$$



Klümper, Sakai J. Phys. A 35, 2173 (2002)
Zotos, Naef, Prelovšek, Phys. Rev. B 55, 11029 (1997)
FHM, Honecker, Cabra, Brenig, Phys. Rev. B 66, 140406(R) (2002)

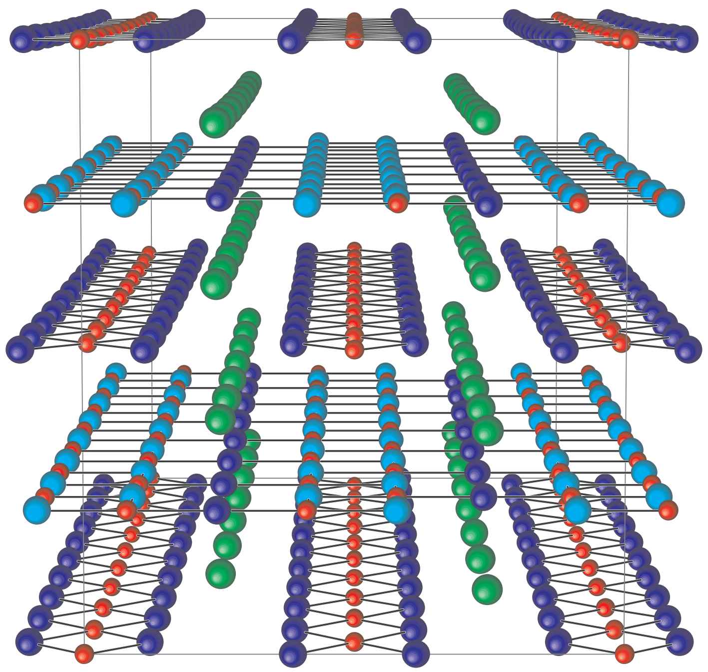
Thermal transport in (AFM) quantum magnets

1D



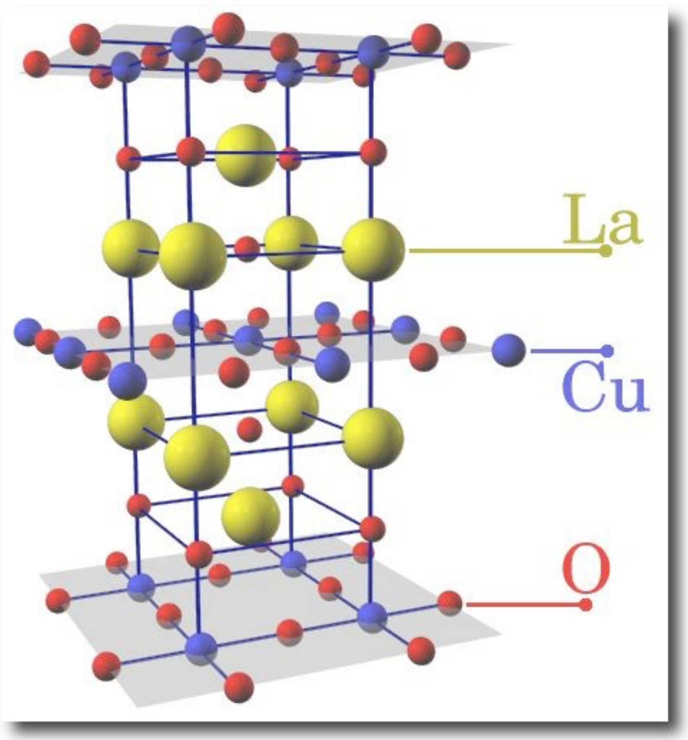
$SrCuO_2$

Ladders



$(Sr,Ca,La)_{14}Cu_{24}O_{41}$

2D

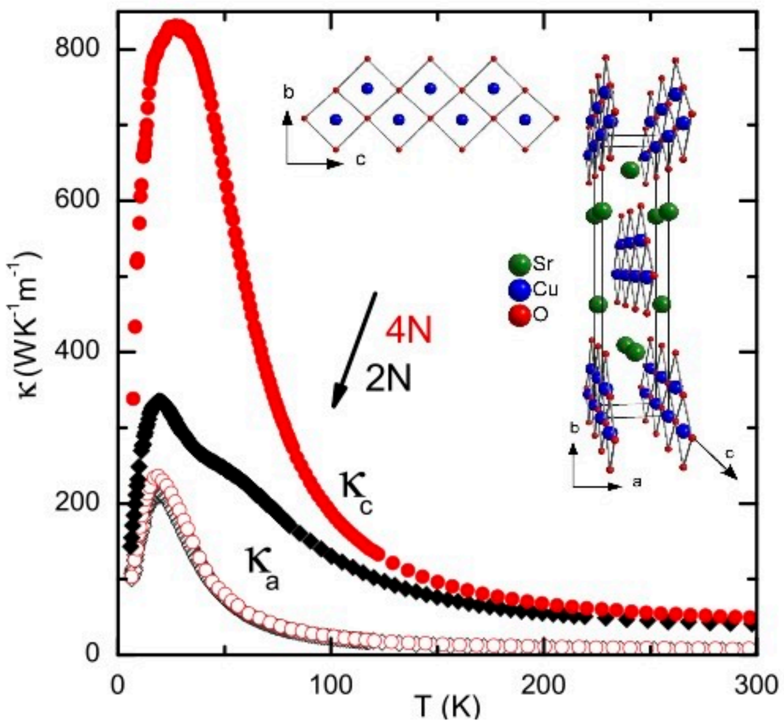


La_2CuO_4

See Christian Hess' talk

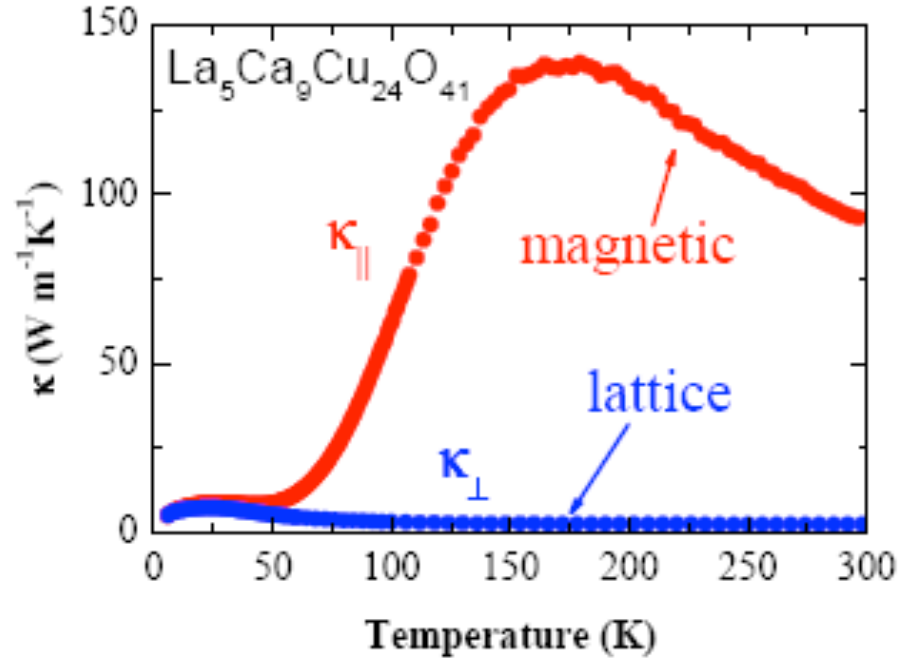
Thermal transport in (AFM) quantum magnets

1D - Spinons



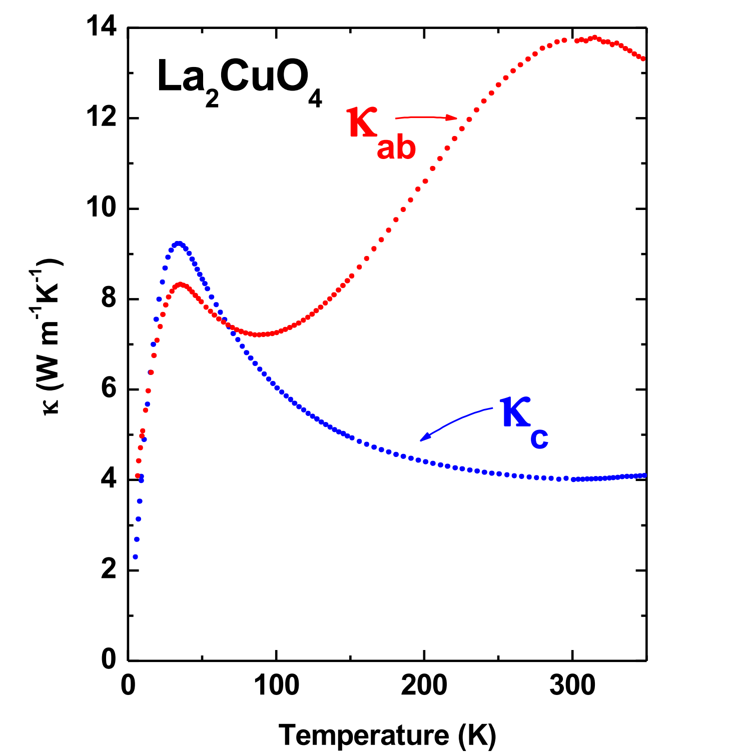
Hlubek, Büchner, Hess, et al., PRB 2010
Sologubenko et al. PRB 2001

Ladders - Triplet excitations



Hess, FHM, Brenig, Büchner, et al., PRB 2001
Solugubenko et al. PRL 2000

2D - Magnons



Hess, FHM, Brenig, Büchner et al., PRL 2003

Magnetic excitations contribute significantly to thermal conductivity κ
mean-free paths ~ 1μm

Many other thermal transport experiments: Lorenz, Sun, Sales, Mandrus, ...

Spin transport only probed indirectly via NMR, μsr or spin Seebeck effect

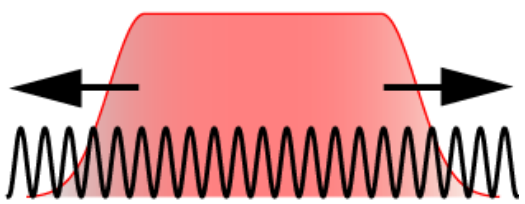
Thurber et al. PRL 2001, Maeter et al. 2013, Xiao et al. 2014

Hirobe et al. Nature Phys. 13, 30 (2017)

Nonequilibrium transport in optical lattice

^{39}K atoms

$$H = -J_{BH} \sum_{\langle i,j \rangle} a_i^\dagger a_j + \frac{U}{2} \sum_i n_i(n_i - 1) + V(t) \sum_i n_i \vec{r}_i^{\,2}$$



Remove trap V → 0,
Go to desired U/J_{BH}

Initial state:

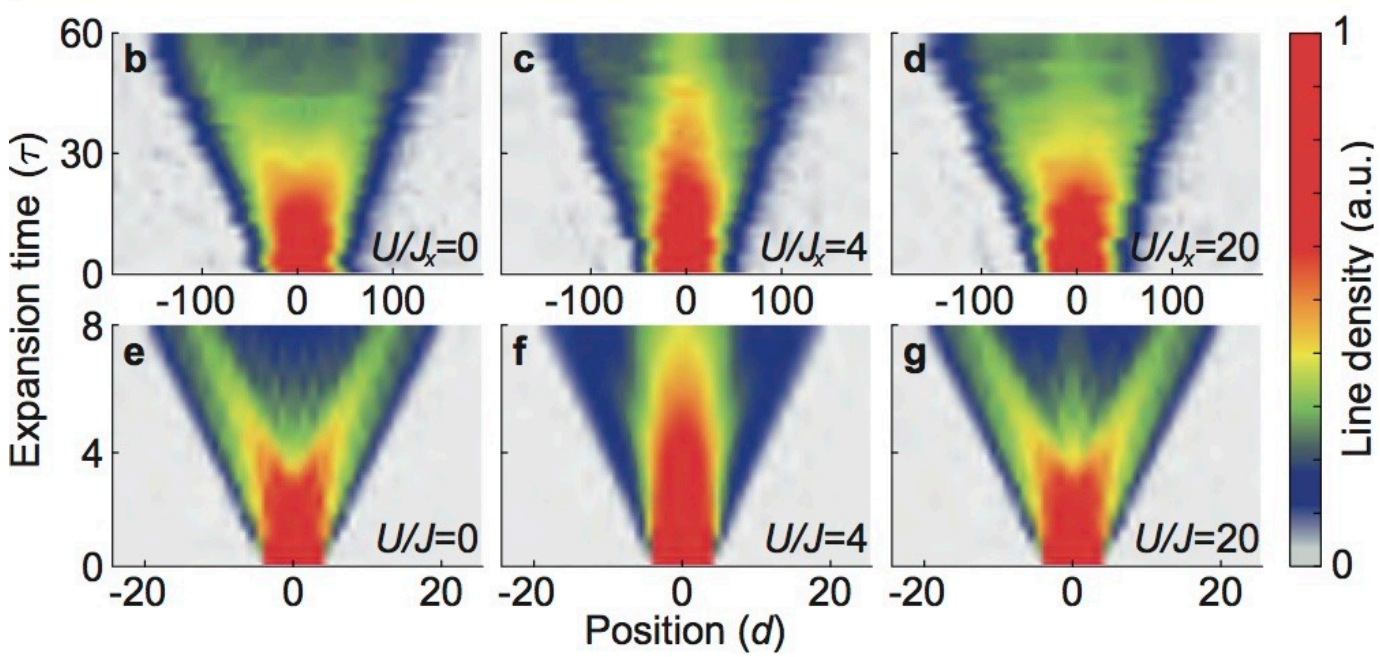
$$|\psi_{\text{initial}}\rangle = \prod_i a_i^\dagger |0\rangle$$

Spin-down - up - down

ballistic　　ballistic

Exp. data

DMRG

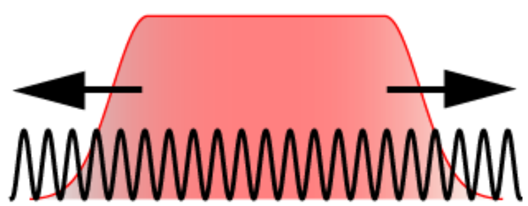


Identical Density Profiles for non-interacting & *strongly* interacting bosons: Ballistic nonequilibrium dynamics in integrable 1D model

Ronzheimer, Schreiber, Braun, Hodgman, Langer, McCulloch, FHM, Bloch, Schneider Phys. Rev. Lett. 110, 205301 (2013)
Nonequilibrium quasi-condensation: Vidmar, Ronzheimer, FHM, Bloch Schneider et al. PRL 115, 175301 (2015)

Nonequilibrium transport in optical lattice

$$U/J = \infty, n = 1: \quad H = -J_{\rm BH} \sum_i (S_i^+ S_{i+1}^- + h.c.)$$



Remove trap V → 0,
Go to desired U/J_{BH}

Initial state:

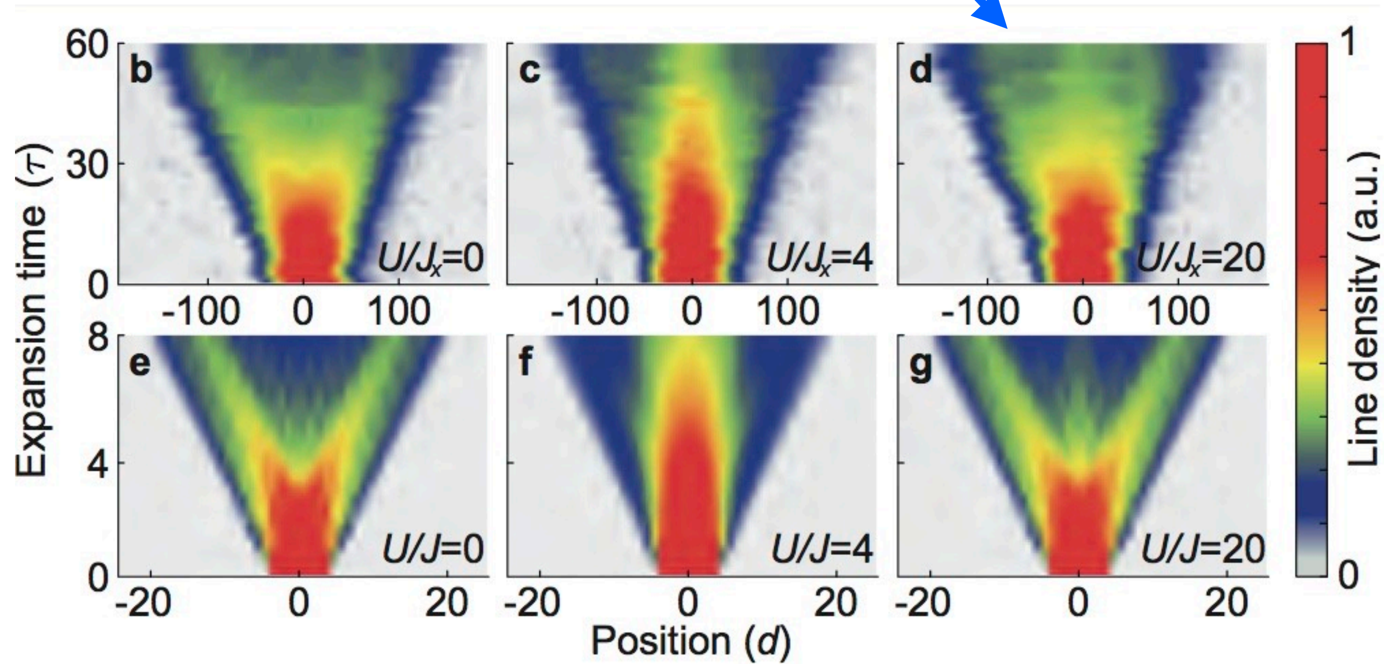
$$|\psi_{\rm initial}\rangle = \prod_i a_i^\dagger |0\rangle$$

Spin-down - up - down

ballistic **ballistic**

Exp. data

DMRG



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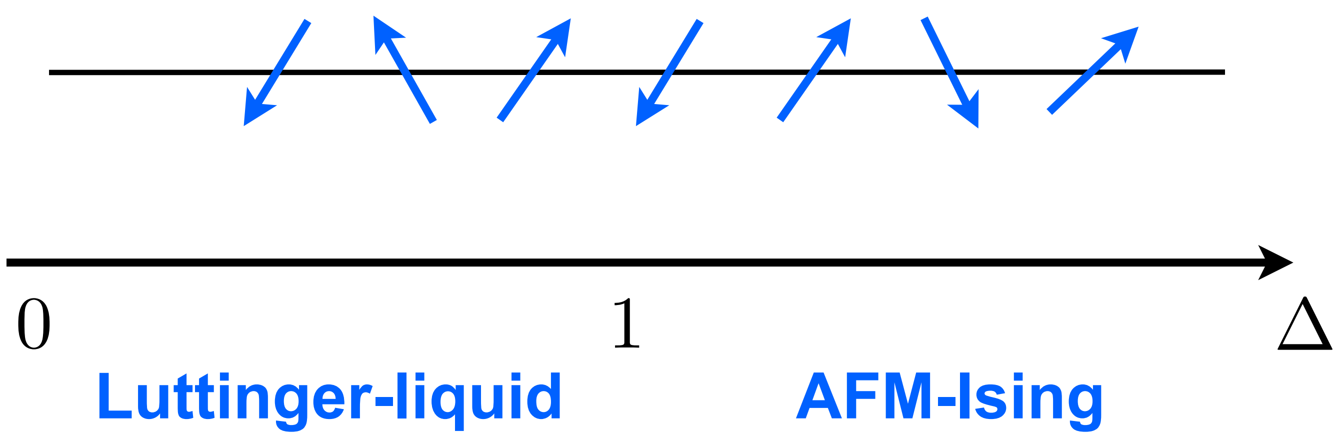
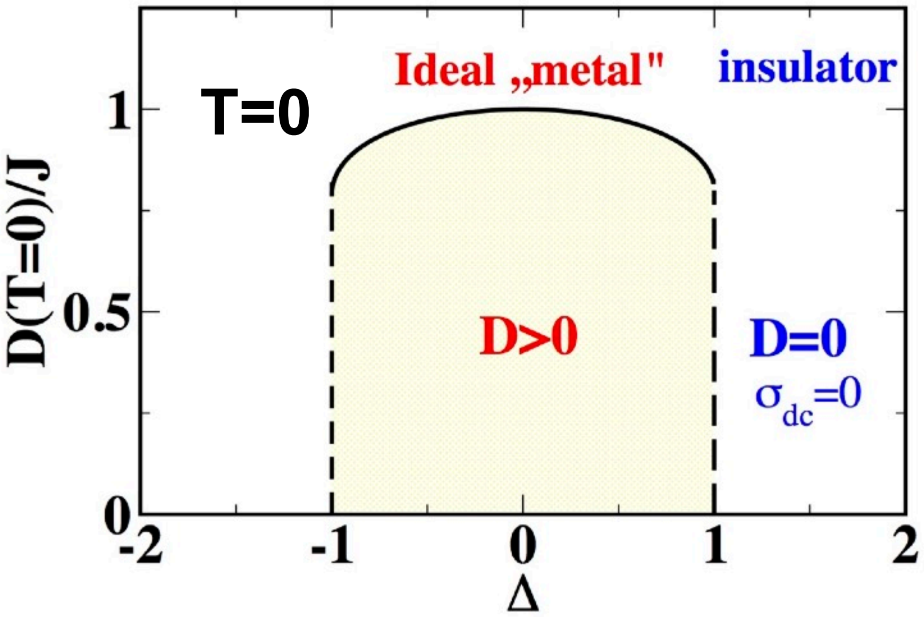
Nonequilibrium quasi-condensation: Vidmar, Ronzheimer, FHM, Bloch Schneider et al. PRL 115, 175301 (2015)

Spin transport in the spin-1/2 XXZ model

$$H = J \sum_{i=1}^{L} \left[\frac{1}{2} (S_i^+ S_{i+1}^- + h.c.) + \Delta S_i^z S_{i+1}^z \right]$$

$$\Delta \neq 0 : \quad [H, j_s] \neq 0; \quad j_{s,l} \sim S_l^+ S_{l+1}^- - h.c.$$





Shastry, Sutherland Phys. Rev. Lett. 65, 243 (1990)

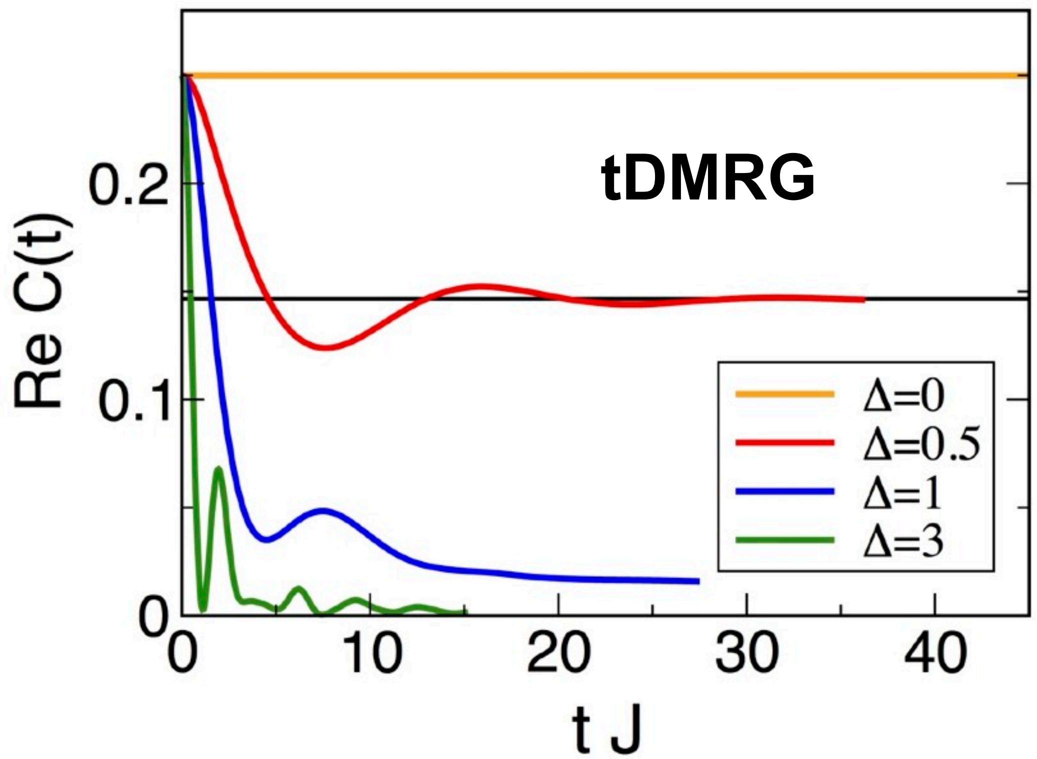
Spin Drude weight in spin-1/2 XXZ chain

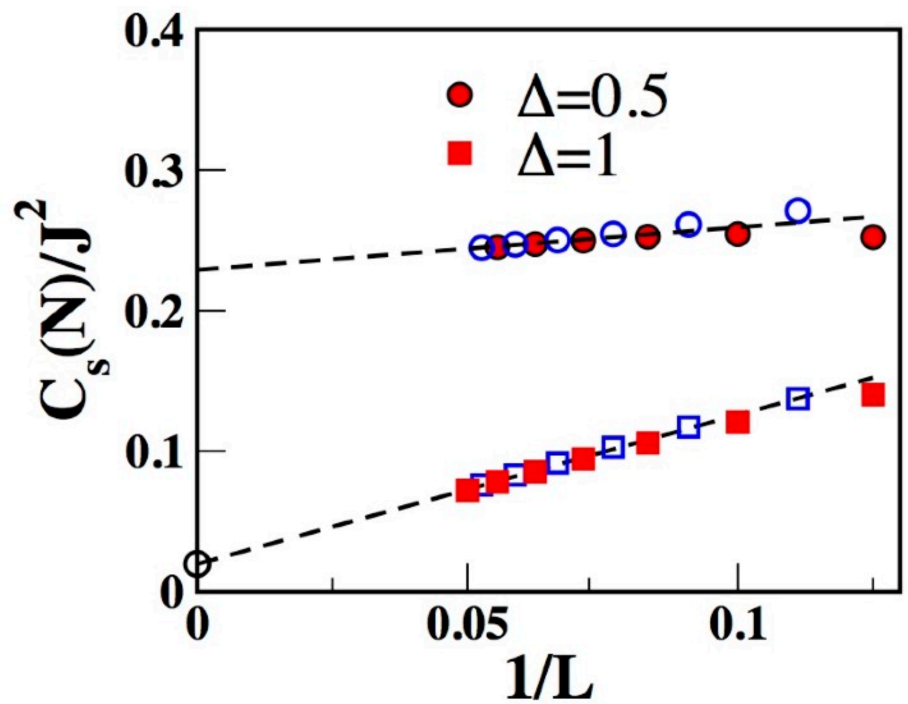
$$H = J\sum_{i=1}^{L}\left[\frac{1}{2}(S_i^+ S_{i+1}^- + h.c.) + \Delta S_i^z S_{i+1}^z\right]$$

Spin-current autocorrelations $T = \infty$ **Exact diagonalization**

$$C(t) = \langle j_s(t) j_s \rangle / L$$

$$D(T) \approx C_s / T$$





Karrasch, Bardarson, Moore PRL 2012
Karrasch, Kennes, FHM PRB 2015
Karrasch, Moore, FHM PRB 2014
Karrasch, Kennes, Moore PRB 2014

HM et al., PRB 2003, EPJST 2007; Prelovsek, Zotos PRB 1996; Narozhny, MIllis, Andrei PRB 1998; Rigol, Shastry PRB 2008, …

Dynamical typicality:
Steinigeweg, Gemmer, Brenig Phys. Rev. Lett. 112, 120601 (2014)

Spin Drude weight in spin-1/2 XXZ chain

$$H = J \sum_{i=1}^{L} \left[\frac{1}{2} (S_i^+ S_{i+1}^- + h.c.) + \Delta S_i^z S_{i+1}^z \right]$$

Spin-current autocorrelations $T = \infty$ **Exact diagonalization**

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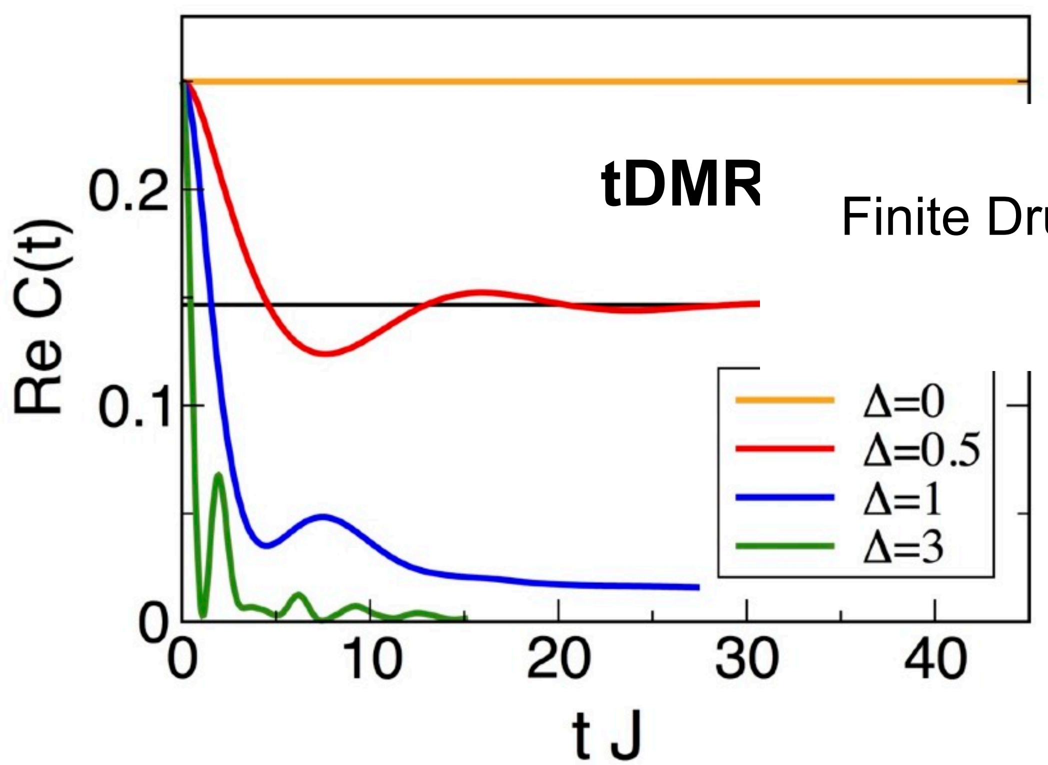
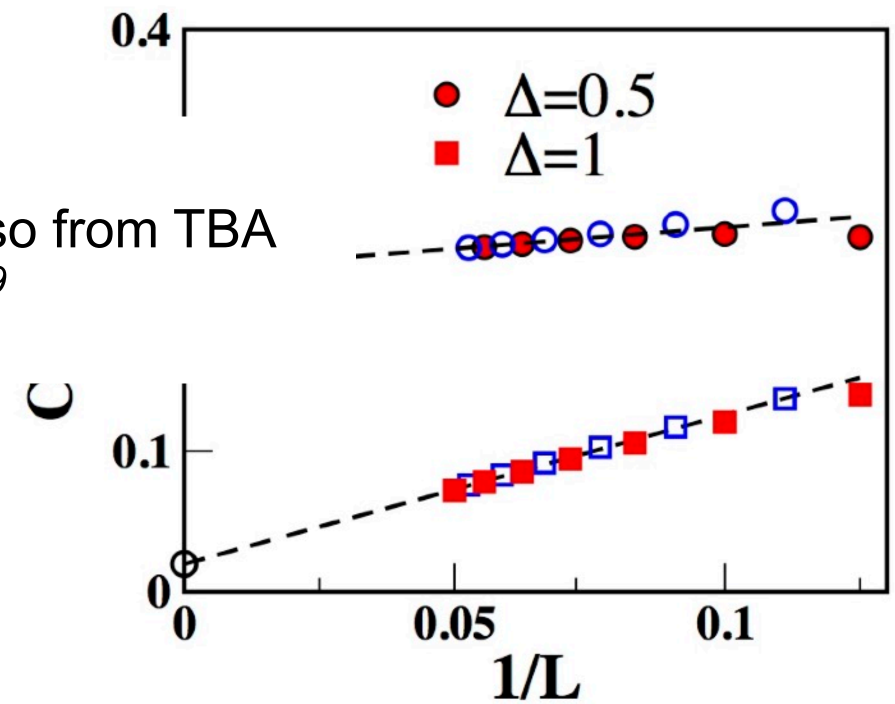


Finite Drude weight also from TBA
Zotos PRL 1999



Karrasch, Bardarson, Moore PRL 2012
Karrasch, Kennes, FHM PRB 2015
Karrasch, Moore, FHM PRB 2014
Karrasch, Kennes, Moore PRB 2014

HM et al., PRB 2003, EPJST 2007; Prelovsek, Zotos PRB 1996; Narozhny, MIllis, Andrei PRB 1998; Rigol, Shastry PRB 2008, …

Dynamical typicality:
Steinigeweg, Gemmer, Brenig Phys. Rev. Lett. 112, 120601 (2014)

Spin-1/2 XXZ chains: Finite T spin transport

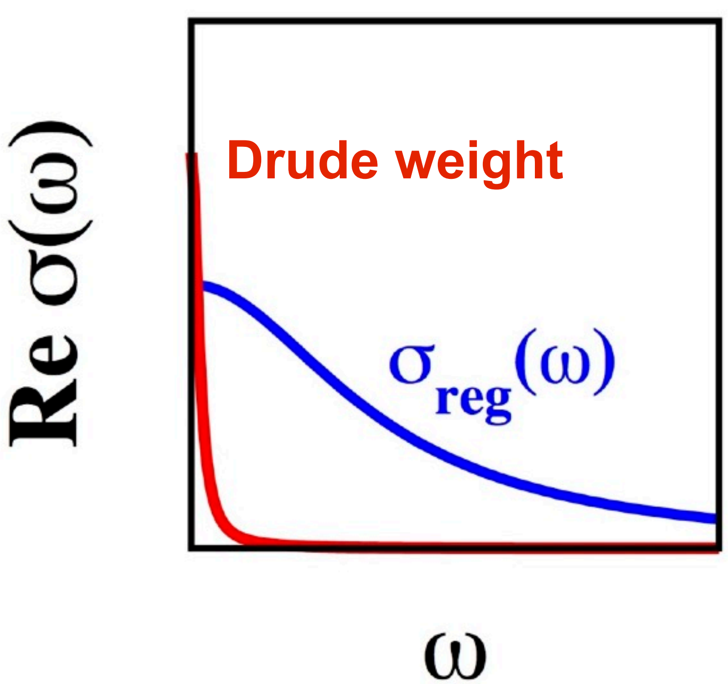
Ballistic ✓

$0 \quad 1 \quad \Delta$

$$D(T) > 0$$

$$\mathrm{Re}\sigma(\omega) = D(T)\delta(\omega)$$

$$\sigma_{\mathrm{reg}} \neq 0; \sigma_{\mathrm{dc}} > 0$$



$$D(T) \geq \mathrm{const}\frac{|\langle j_s Q_\alpha\rangle|^2}{\langle Q_\alpha^2\rangle} > 0$$

Mazur inequality

Zotos, Naef, Prelovsek, Phys. Rev. B 55, 11029 (1997)

Quasilocal charges discovered by Prosen

Prosen PRL 2011, Ilievski & Prosen PRL 2013,
Pereira, Pasquier, Sirker Affleck J. Stat. Mech. (2014) P09037

Zotos, Naef, Prelovsek PRB 1997, Zotos PRL 1999
Sirker, Pereira, Affleck PRL 2009,
GHD: Ilievski, De Nardis PRL 2017
Urichuk, et al. SciPost Phys. 6, 005 (2019)
… many more …

Spin-1/2 XXZ chains: Finite T spin transport

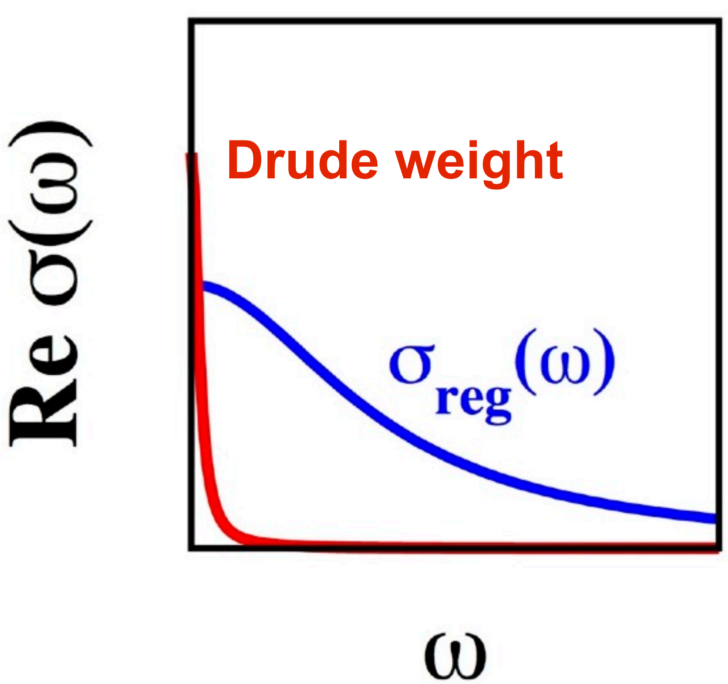
Ballistic ✓

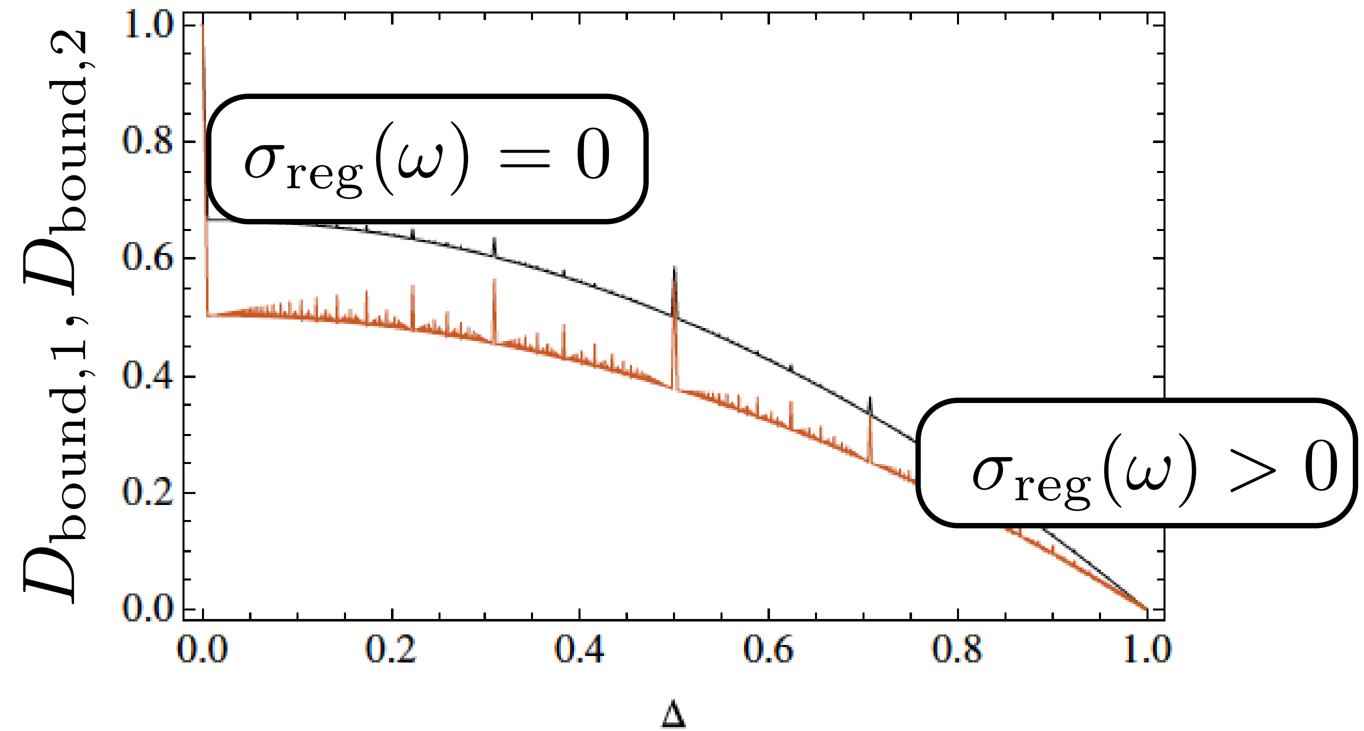
0 — 1 — Δ

$D(T) > 0$

$\mathrm{Re}\sigma(\omega) = D(T)\delta(\omega)$

$\sigma_{\mathrm{reg}} \neq 0; \sigma_{\mathrm{dc}} > 0$





Fractal structure !?

Sum rule: subleading correction
superdiffusive
almost everywhere

Zotos, Naef, Prelovsek PRB 1997, Zotos PRL 1999
Sirker, Pereira, Affleck PRL 2009,
GHD: Ilievski, De Nardis PRL 2017
Urichuk, et al. SciPost Phys. 6, 005 (2019)
… many more …

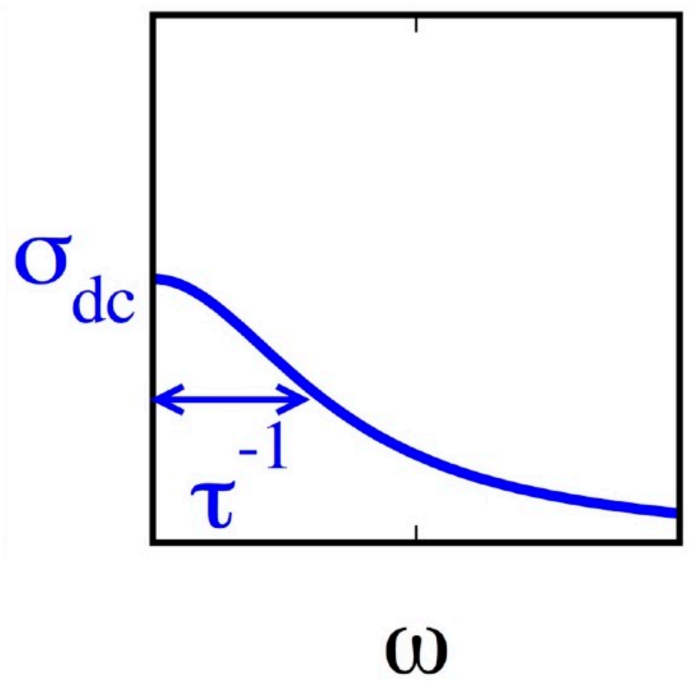
Agrawal, Gopalakrishnan, Vasseur, Ware, arXiv:1909.05263
Ilievski et al. Phys. Rev. Lett. 121, 230602 (2018)

Spin-1/2 XXZ chains: Finite T spin transport

Ballistic ✓ **Diffusive ✓**

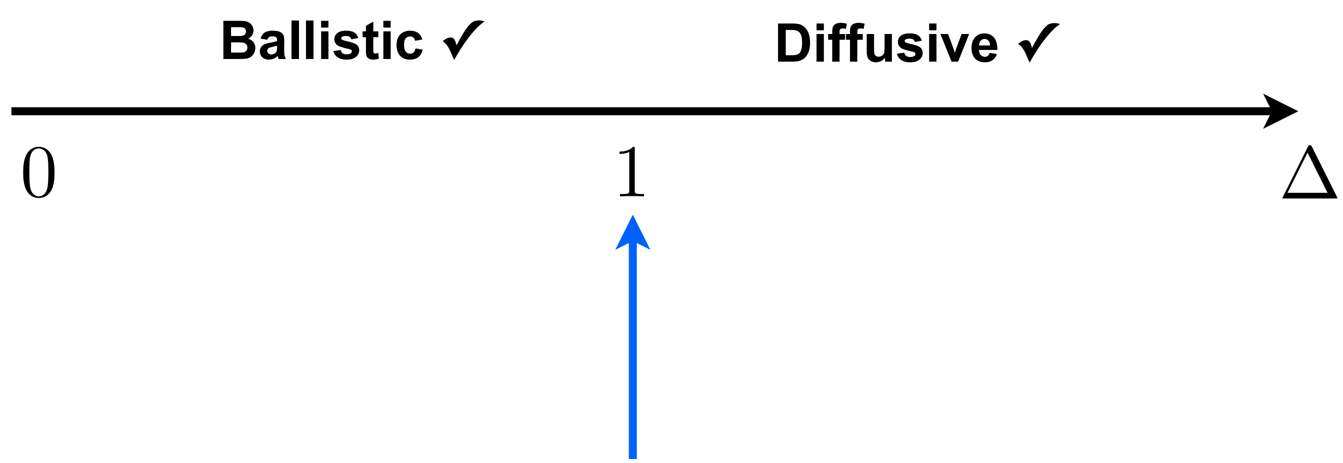
0 1 Δ

$$D(T) = 0; \mathcal{D}_s < \infty$$



Znidaric PRL 2011, Steinigeweg, Brenig, PRL 2011, Karrasch, Moore, FHM, PRB 2014, Ilievski et al PRL 2018, Gopalakrishnan, Vasseur PRL 2019, De Nardis et al. SciPost. 2019

Spin-1/2 XXZ chains: Finite T spin transport



$$D(T) = 0?$$

Heisenberg chains:
Superdiffusive !

Znidaric PRL 2011,
Ljubotina et al. Nat. Comm. 2017, PRL 2019,
Gopalakrishnan, Vasseur PRL 2019
De Nardis et al. PRL 2019
Dupont, Moore arXiv:1907.12115

Spin-1/2 XXZ chains: Finite T spin transport

Ballistic ✓ **Diffusive ✓**

$0 \qquad 1 \qquad \Delta$

$D(T) > 0$

$D(T) = 0; \mathcal{D}_s < \infty$

$\mathrm{Re}\sigma(\omega) = D(T)\delta(\omega)$

$\sigma_{\mathrm{reg}} \neq 0; \sigma_{\mathrm{dc}} > 0$

$D(T) = 0?$

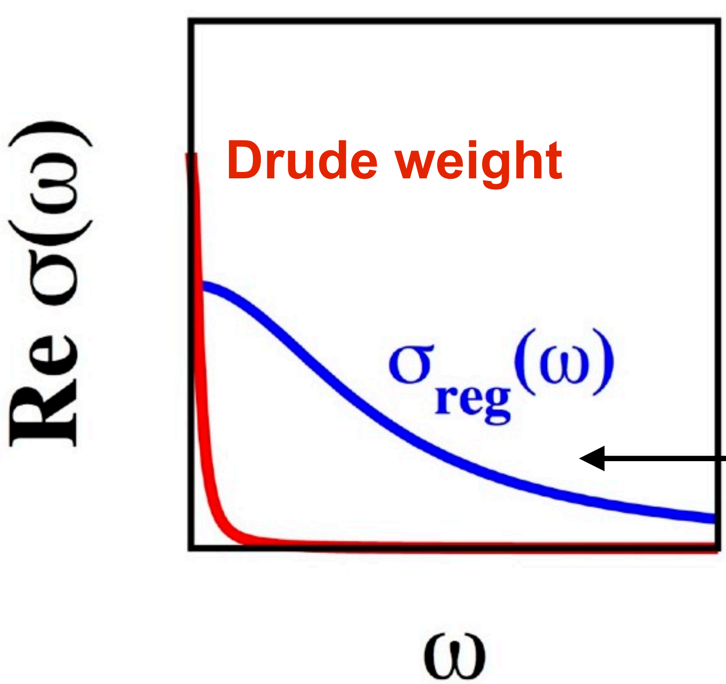
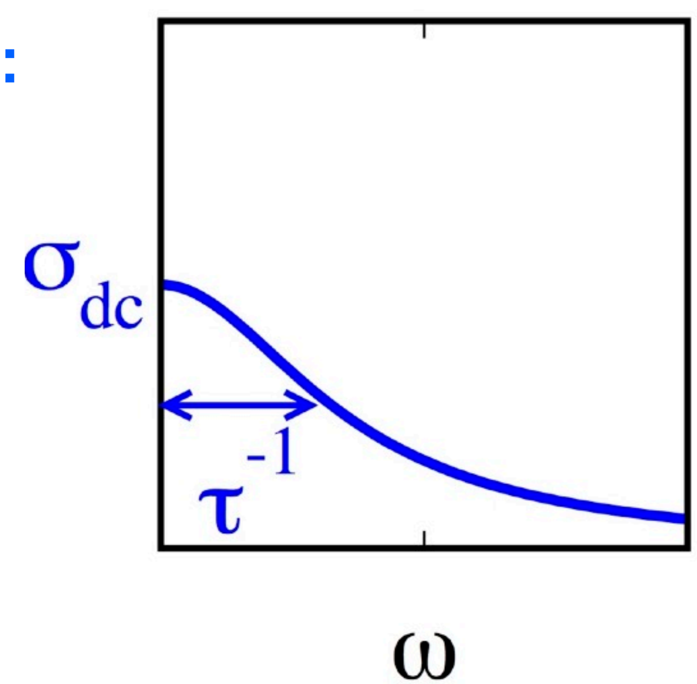


figure not quite accurate

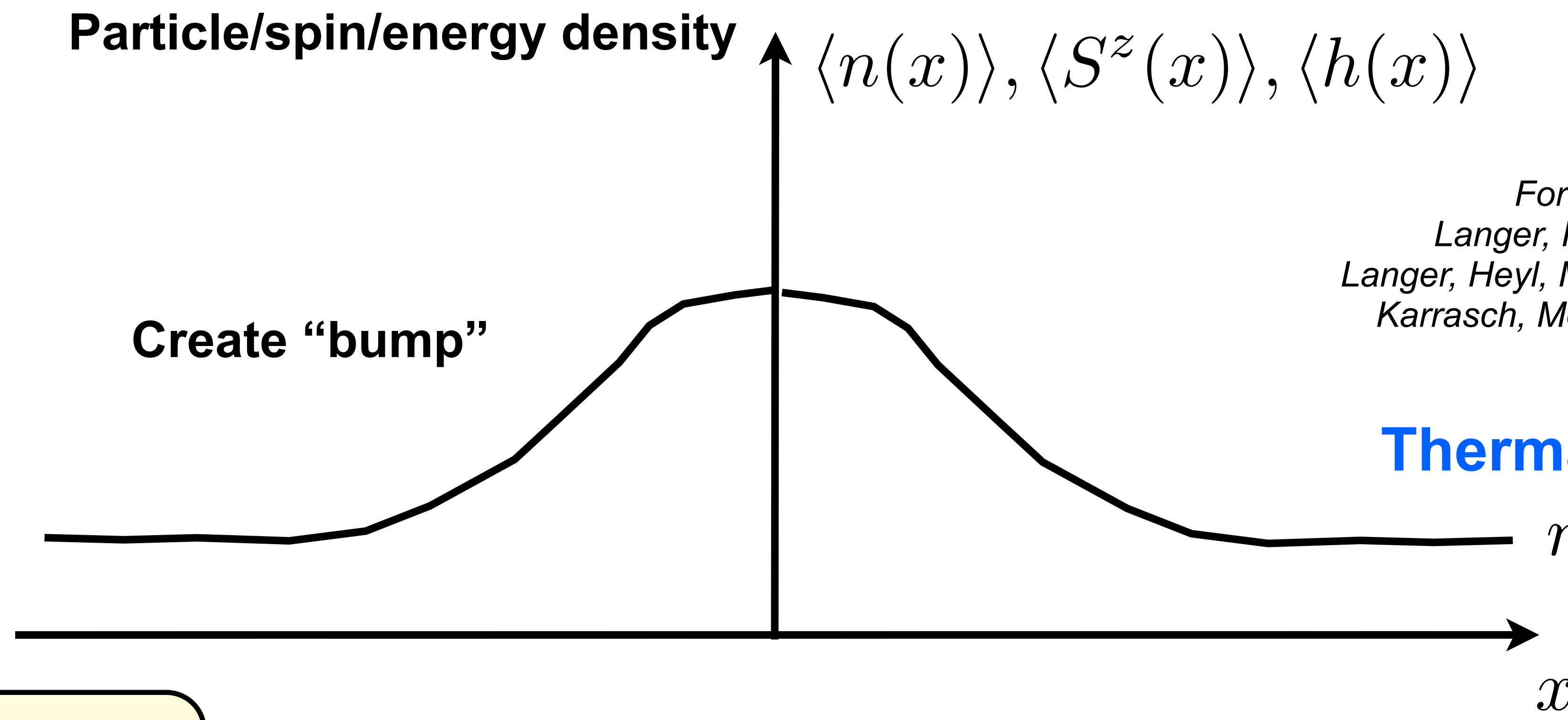
Heisenberg chains: Superdiffusive



dissipationless heat & spin transport (+ diffusion/superdiffusion)

dissipationless heat but diffusive spin transport

Signatures in local quenches



For 1D spin systems:
Langer, FHM, et al. PRB (2009)
Langer, Heyl, McCulloch, FHM, PRB (2011)
Karrasch, Moore, FHM, PRB (2014), ...

Study width:

$$\sigma_\nu(t) \propto t^\alpha$$

$$\sigma_\nu^2(t) \sim \sum_i (i - i_0)^2 \langle S_i^z(t)\rangle$$

Steinigeweg, Wichterich, Gemmer, EPL (2009)

Generalized Einstein relation: $T = \infty$

$$\delta\sigma_\nu^2(t) = \frac{2}{L\chi_\nu}\int_0^t dt_1 \int_0^{t_1} dt_2 \langle j_\nu(t_2) j_\nu(0)\rangle_{\mathrm{eq}}$$

Diffusive case:

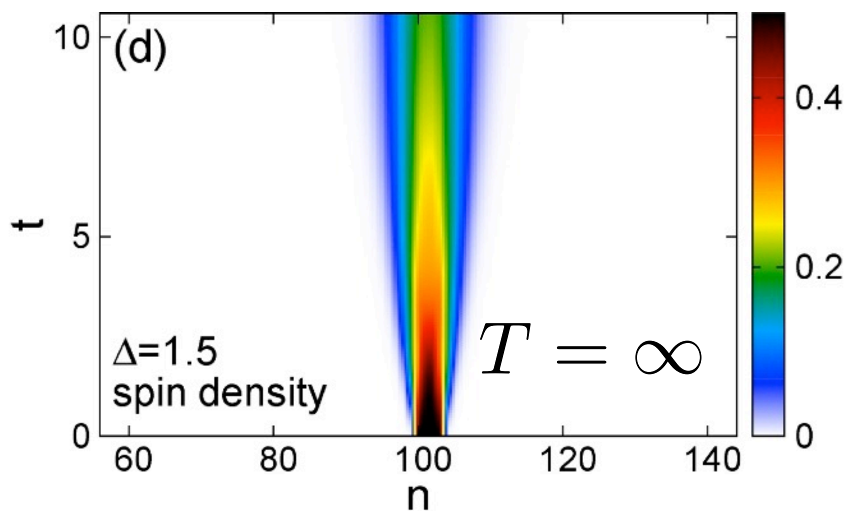
$$\delta\sigma_\nu^2(t) = 2\mathcal{D}_\nu t; \quad \mathcal{D}_\nu = \frac{\sigma_{dc,\nu}}{\chi_\nu}$$

Ballistic case: Drude weight!

$$\delta\sigma_\nu^2(t) \propto \frac{D_\nu t^2}{\chi_\nu}$$

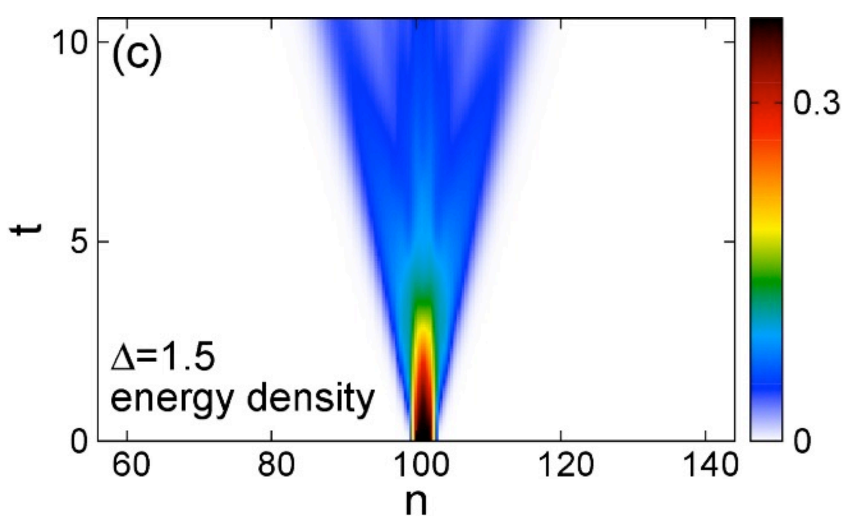
Diffusive spin dynamics in local quenches at finite T

Spin density: Diffusive

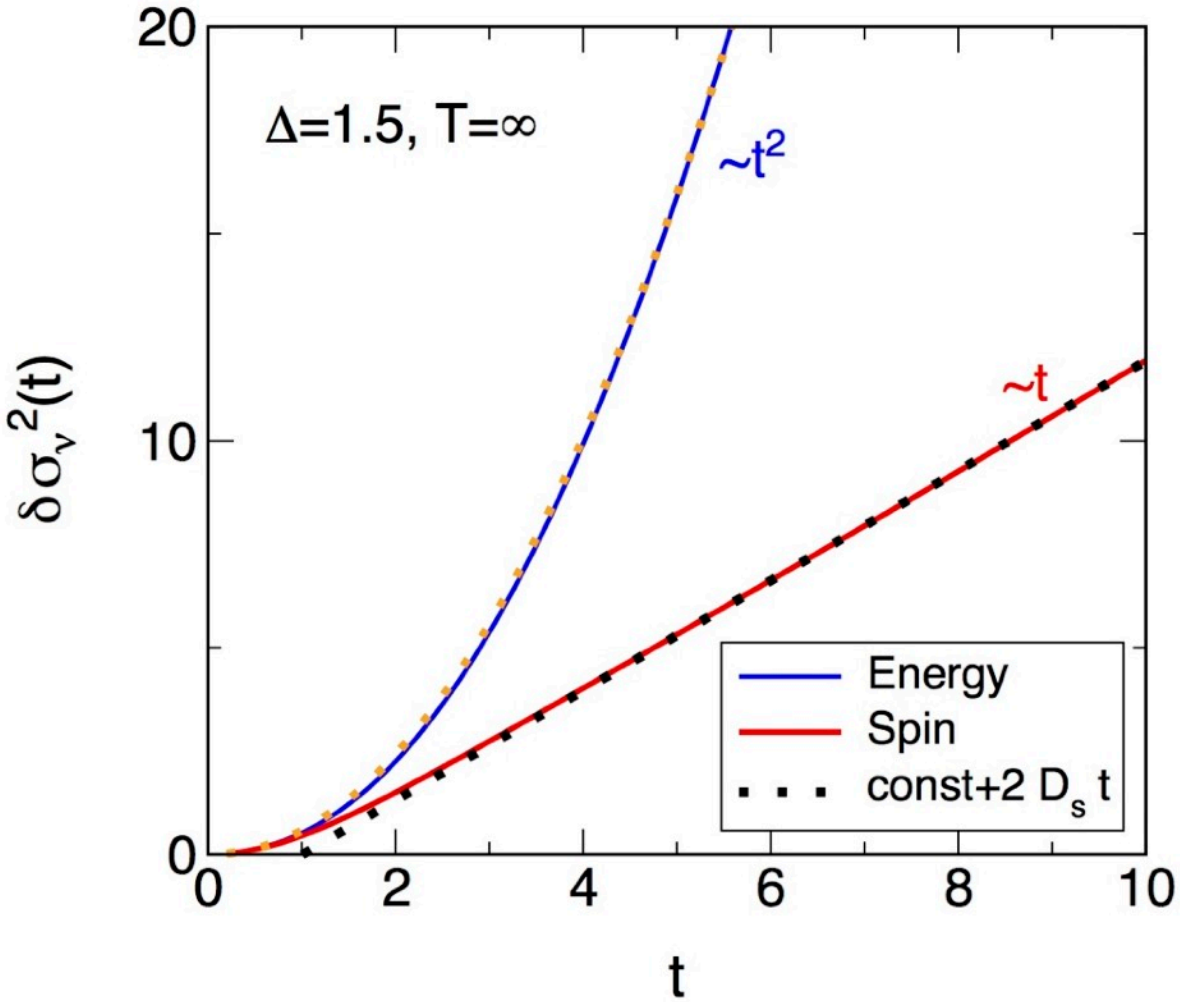


$$\sigma(t) \sim \sqrt{t}$$

Energy density: Ballistic



$$\sigma_e(t) \sim t$$



spin diffusion constant

$$\sigma(t) \sim \sqrt{\mathcal{D}_s t}$$

(agrees with Kubo!)

Karrasch, Moore, FHM, Phys. Rev. B 89, 075139 (2014)

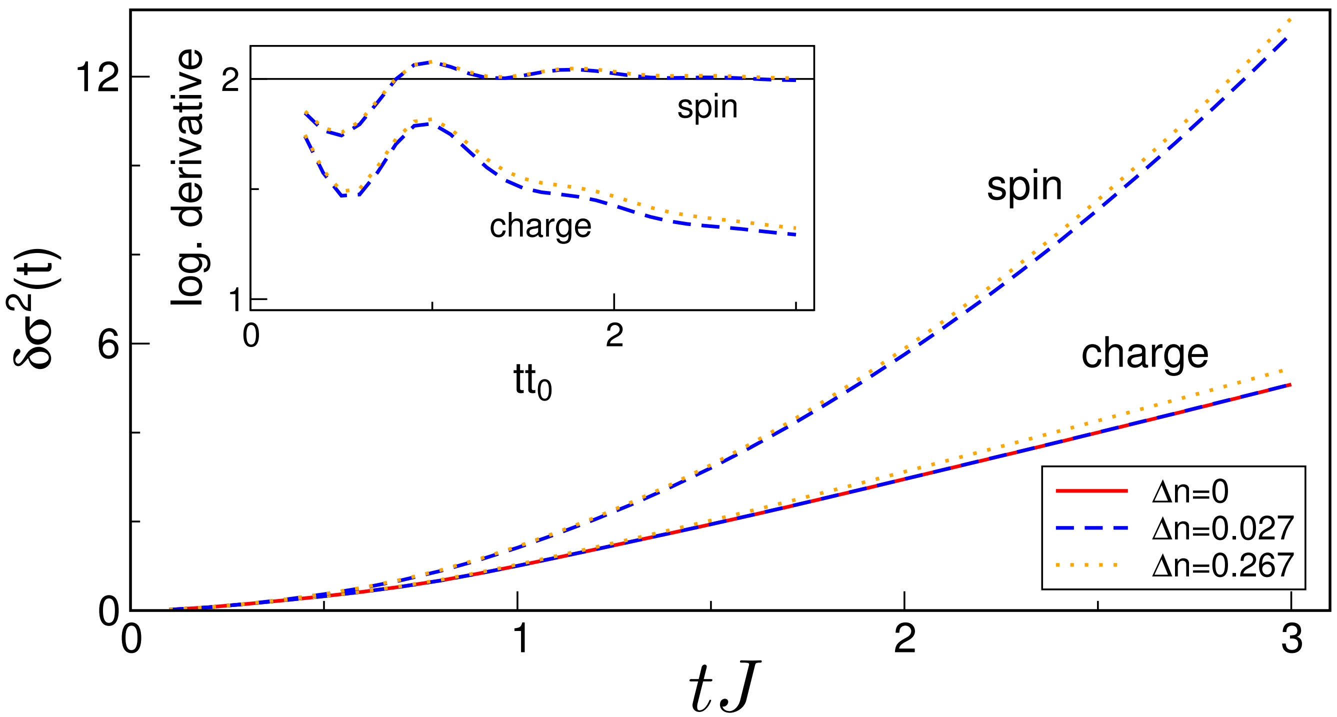
Experiments: Use integrable 1D Hubbard!

$$H = -J \sum_{\langle i,j \rangle} (c_{i\sigma}^\dagger c_{j\sigma} + h.c.) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

$$N_\uparrow \neq N_\downarrow$$

$$\Delta n = (N_\uparrow - N_\downarrow)/N$$

Spreading of density perturbation



ballistic

"diffusive"

Optical-lattice experiments do better by a factor of 2-3 !

Coexistence of *ballistic* spin & (super) *diffusive* charge transport

Potentially better numerical approach:
Time-dep. variational principle

Leviatan, Pollmann, Bardarson, Huse, Altman arXiv:1702.08894
Haegeman et al. PRL 107, 070601 (2011)

For details: See Karrasch, Prosen, FHM Phys. Rev. B 95, 060406(R) (2017)

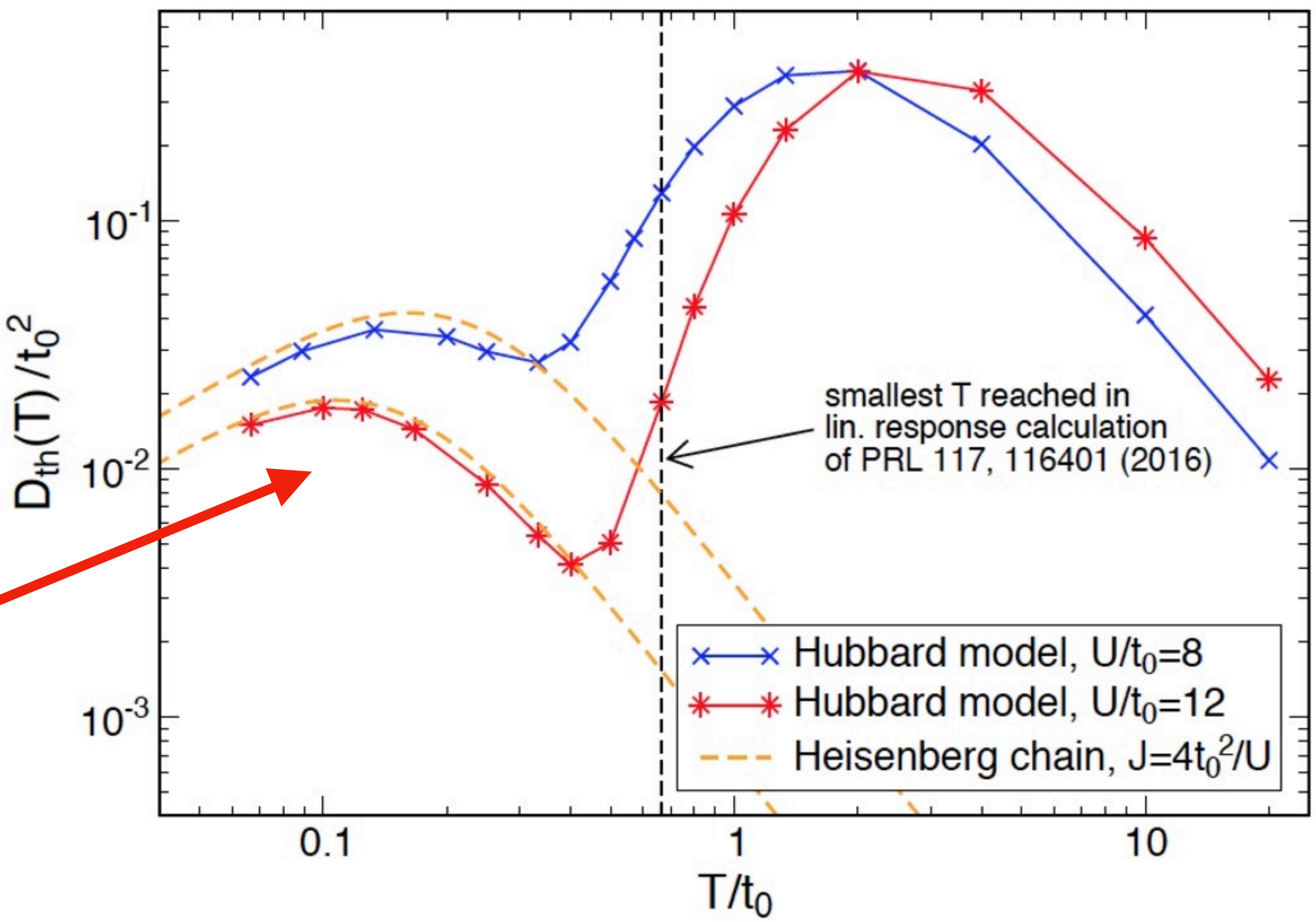
Experiments: Use integrable 1D Hubbard!

$$H = -J \sum_{\langle i,j \rangle} (c^\dagger_{i\sigma} c_{j\sigma} + h.c.) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

$$\langle j_E Q_3 \rangle \neq 0$$

Zotos, Naef, Prelovšek, PRB (1997)

Ballistic thermal conductor



Spin dominates: Quantum magnets

Charge dominates: Optical lattices

$$t_0 \rightarrow J$$

Karrasch New J. Phys. 19, 033027 (2017) (using Vasseur, Karrasch, Moore PRL 115, 267201) , Karrasch, Kennes, FHM PRL 117, 116401 (2016), Ilievski, De Nardis Phys. Rev. B 96, 081118(R) (2017)

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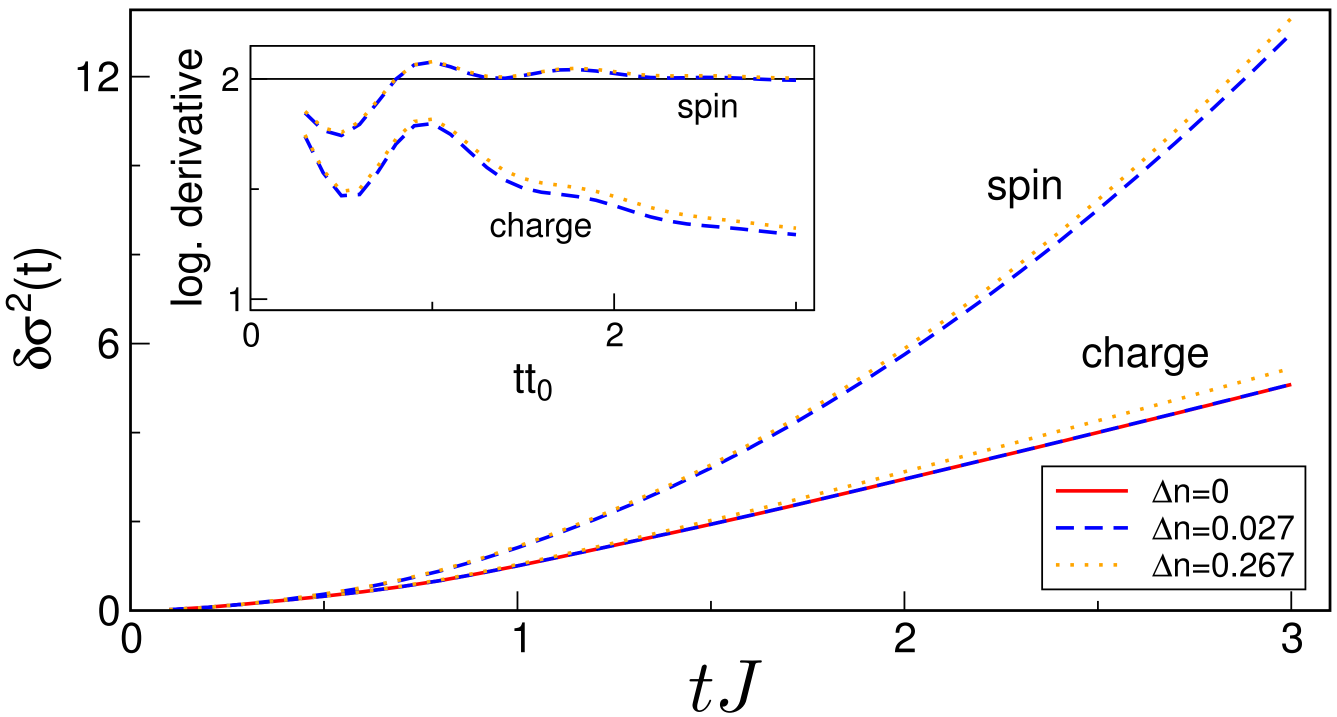
$$H = -J \sum_{\langle i,j \rangle} (c^\dagger_{i\sigma} c_{j\sigma} + h.c.) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

$$N_\uparrow \neq N_\downarrow$$

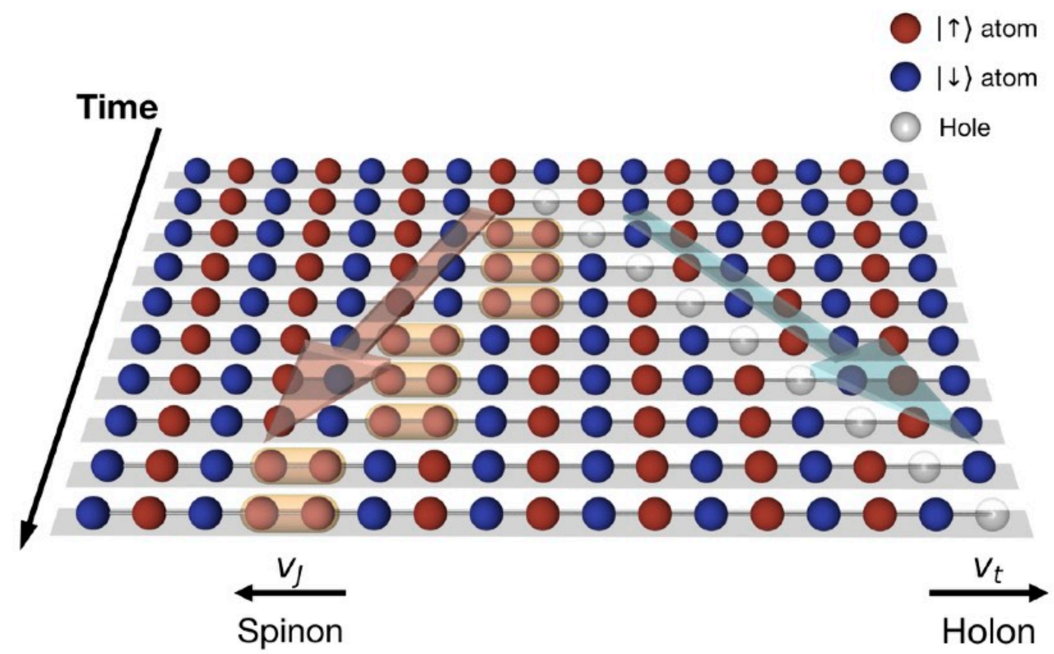
Spreading of density perturbation



Coexistence of *ballistic* spin & super-*diffusive* charge transport

Ilievski, De Nardis, Medenjak, Prosen Phys. Rev. Lett. 121, 230602 (2018)

Fermionic quantum gas microscopes!



Viljayan et al. arXiv:1905.13638 (MPQ)
Greiner (Harvard), Bloch/Gross (MPQ), Zwierlein (MIT), Kuhr (Strathclyde), Thywissen (Toronto), Bakr (Princeton), ...
1D: Boll et al, Science 353, 1257 (2016)

For details: See Karrasch, Prosen, FHM Phys. Rev. B 95, 060406(R) (2017)

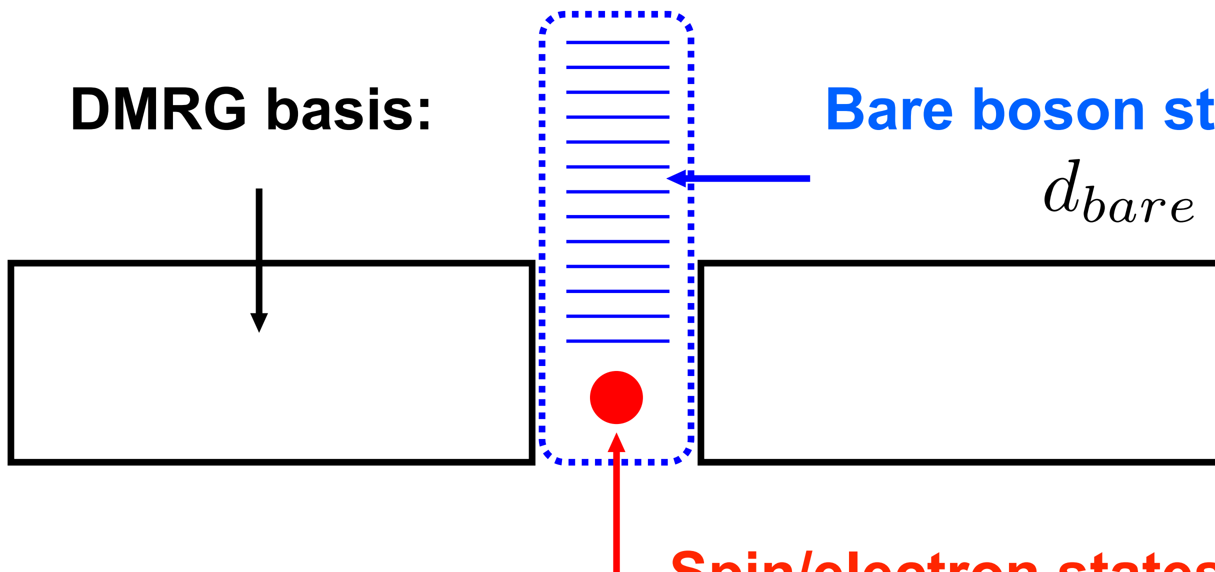
Finally ... towards phonons !

Novel DMRG/TEBD algorithm using local basis optimisation

Adaptive update & truncation
DMRG & local state space
Diagonalize reduced
single-site density matrix

$$\rho^{(1)}|\varphi_\alpha\rangle = \omega_\alpha|\varphi_\alpha\rangle$$

Zhang, Jeckelmann, White PRL 1998
Guo et al. PRL 2012



Theory for transport in spin-phonon systems

Chernyshev , Rozhkov Phys. Rev. Lett. 116, 017204 (2016)
Boulat, Mehta, Andrei, Shimshoni, Rosch Phys. Rev. B 76, 214411 (2007)
Rozhkov Chernyshev Phys. Rev. Lett. 94, 087201 (2005)

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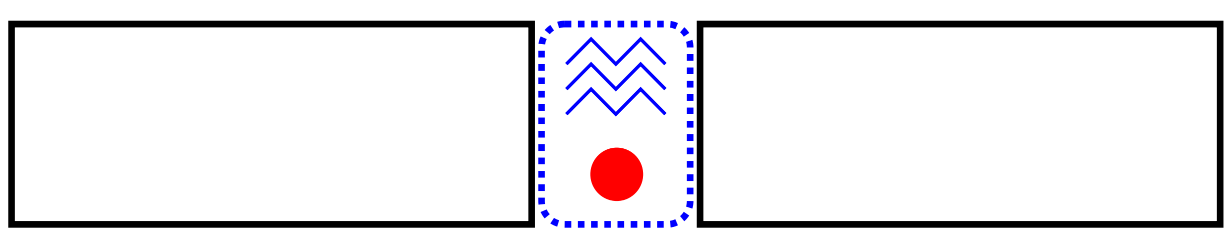
Guo et al. PRL 2012

Local basis optimization:

$$d_O \ll d$$



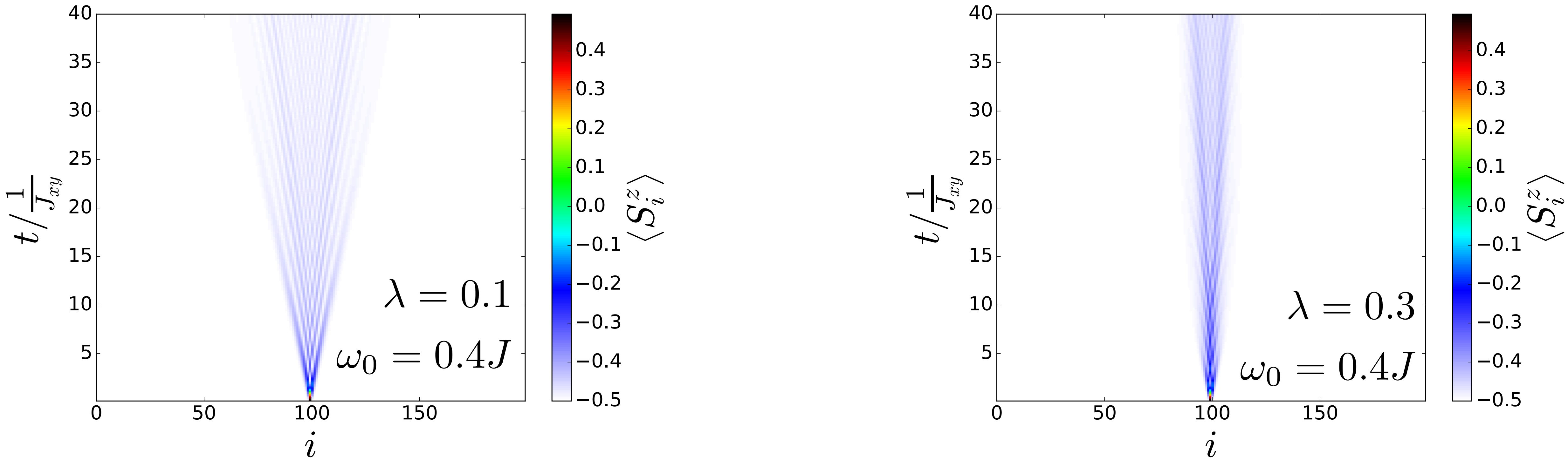
Time-dependent version !

Brockt, Dorfner, Vidmar, FHM, Jeckelmann Phys. Rev. B 92, 241106(R) (2015)

First steps: Dynamics in spin-phonon chains

$$H = J \sum_l \left(1 - \lambda(b_{l+1}^\dagger + b_{l+1} - b_l^\dagger - b_l)\right) \vec{S}_l \cdot \vec{S}_{l+1} + \omega_0 \sum_{l=0}^{L-1} b_l^\dagger b_l$$

Initial state: $|\psi_0\rangle = |\ldots\downarrow\downarrow\downarrow\downarrow\uparrow\downarrow\downarrow\downarrow\downarrow\ldots\rangle$

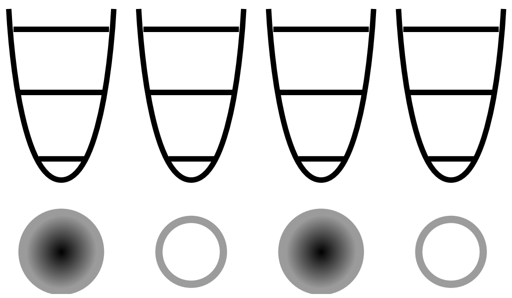


Spin polaron formation? Dispersive phonons?
Adiabatic limit: Diffusion?

Stolpp, Jeckelmann, FHM, work in progress

Relaxation from bare-electron CDW state

(a) $|\mathrm{BCDW}\rangle$



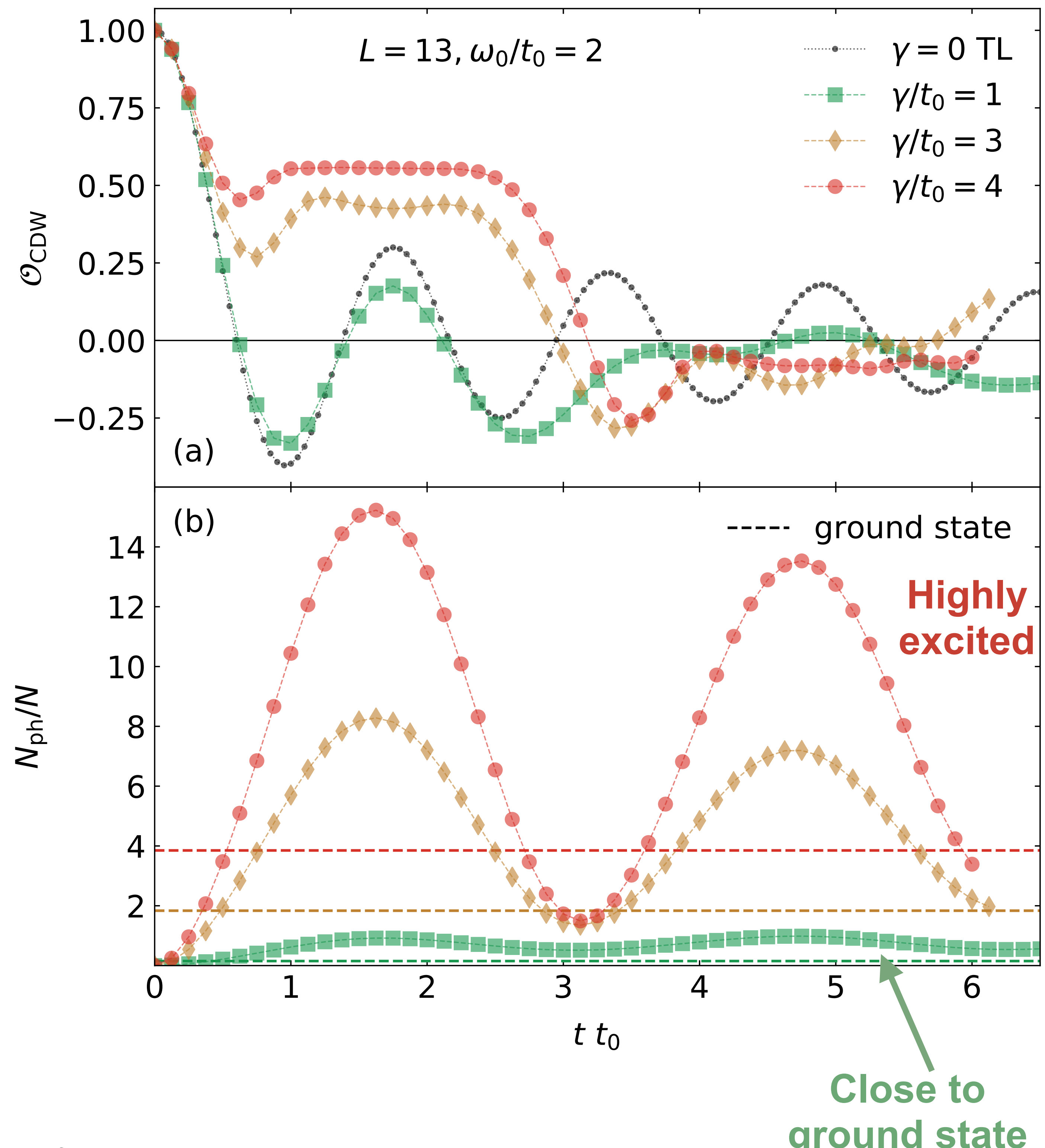
$$|\mathrm{BCDW}\rangle = \left[\prod_{l=1}^{(L-1)/2} c_{2l}^{\dagger}\right] |\emptyset\rangle_{\mathrm{el}} |\emptyset\rangle_{\mathrm{ph}} \, .$$

Order parameter:

$$\mathcal{O}_{\mathrm{CDW}} = \frac{1}{N} \sum_{l=1}^{L} (-1)^l \langle n_l \rangle$$

Step-like relaxation

$$M_{ph}/L = 40; \; N = 6$$

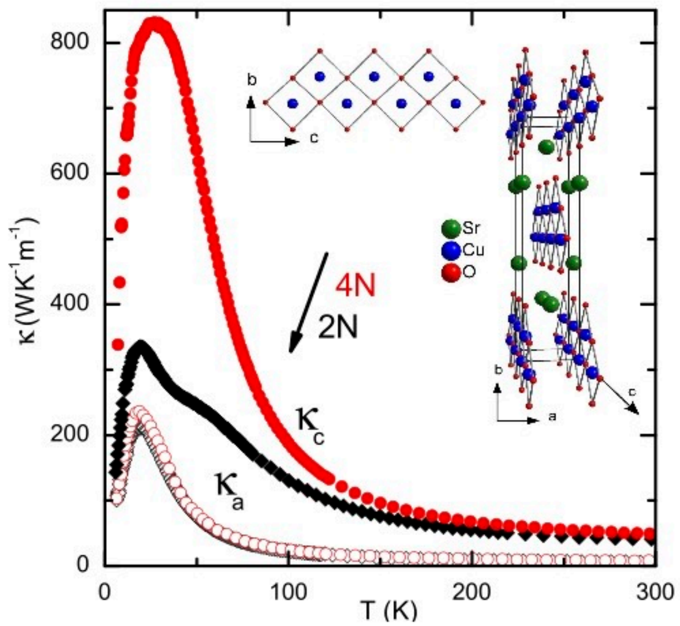


Stolpp, Herbrych, Dorfner, Dagotto, FHM arXiv:1911.01718

Hashimoto , Ishihara, Phys. Rev. B 96, 035154 (2017)

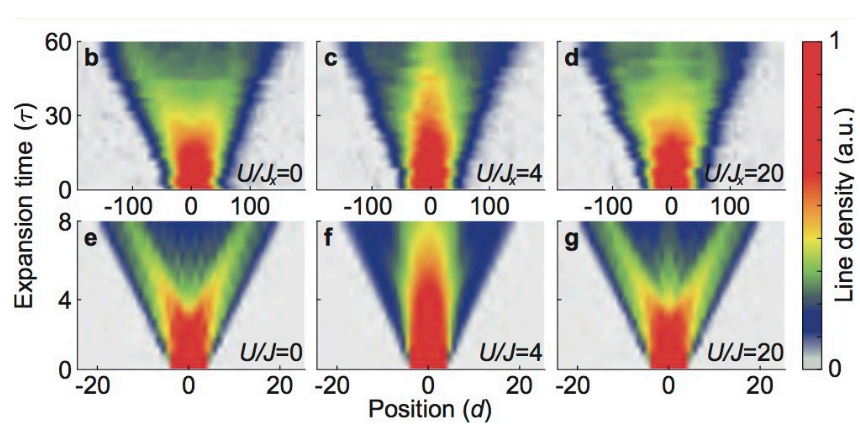
Summary

Large “magnon” heat transport in AFM quantum magnets



Hlubek, Büchner, Hess et al., PRB 2010

Ballistic nonequilibrium transport in optical lattice



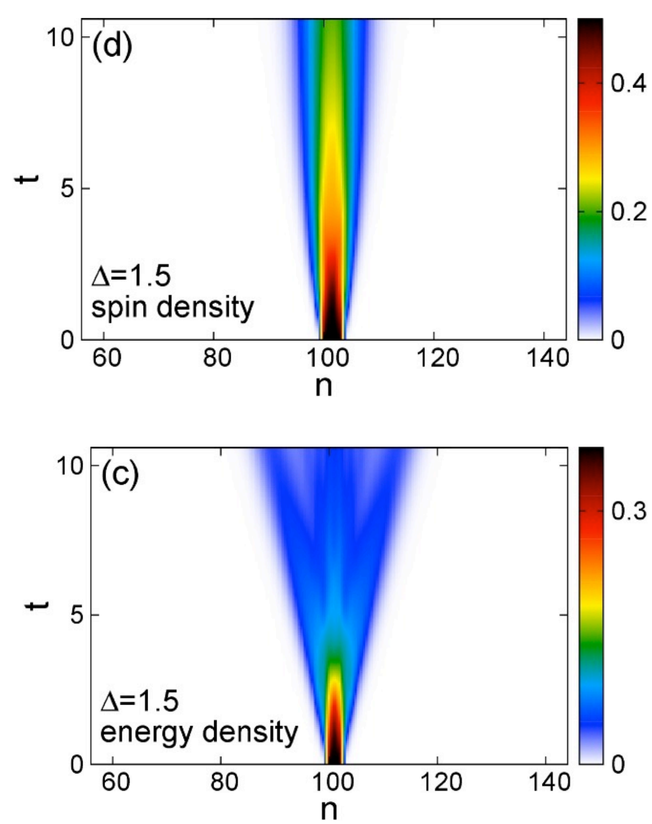
Ronzheimer, FHM, Bloch, Schneider et al. Phys. Rev. Lett. 110, 205301 (2013)

Theory: Spin-1/2 chains

Dissipationless heat & spin transport possible (integrability)

Coexistence of ballistic heat with diffusive spin transport!

Optical lattice expts!



Karrasch, Moore, FHM, PRB 89, 075139 (2014)
Karrasch, Prosen, FHM PRB 95, 060406(R) (2017)

DMRG for e-phonon

Local basis Optimisation

CDW melting

Brockt et al. PRB 92, 241106(R) (2015)
Stolpp et al. arXiv:1911.01718

Future goals

Transport in spin/electron-phonon systems

Thank you!