Nonperturbative Effects in D-brane Inflation

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Based on:

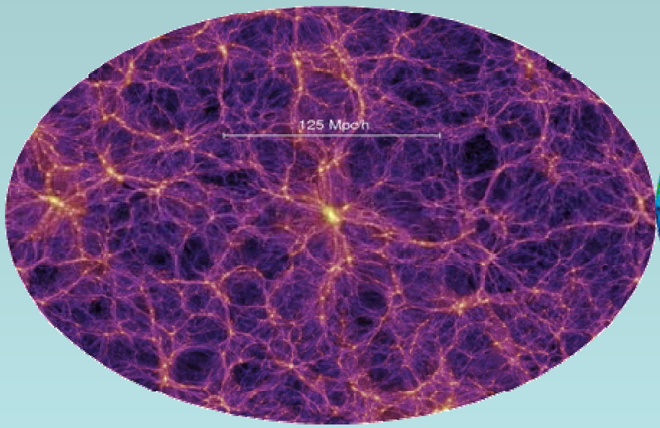
D. Baumann, A. Dymarsky, S. Kachru, I. Klebanov, L.M., 1001.5028

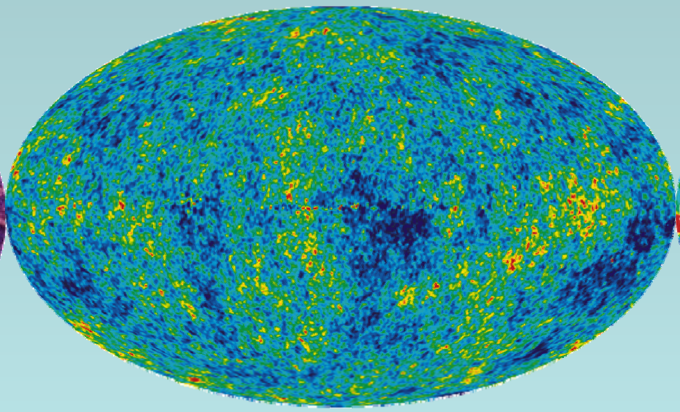
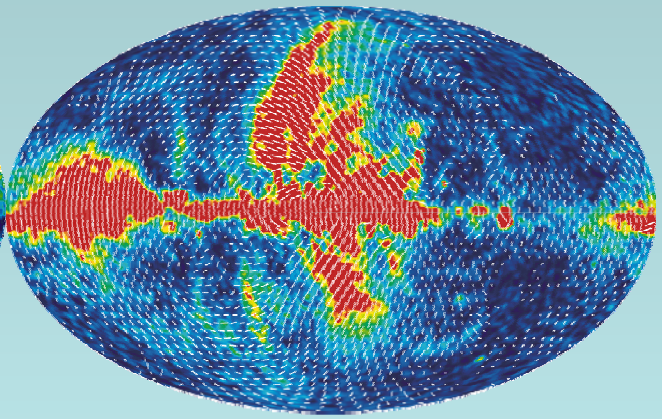
KITP

March 24, 2010

Motivation

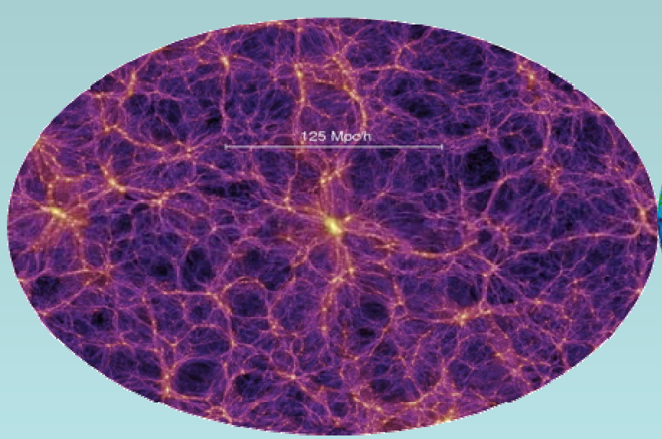
- Inflation provides a beautiful causal mechanism to generate the observed CMB anisotropies and distribution of large-scale structure.
- Inflation is sensitive to Planck-scale physics. Planck-suppressed operators generically make critical contributions to the dynamics.

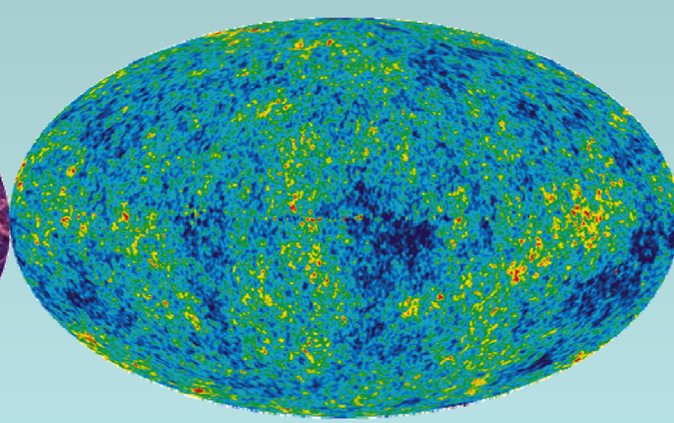


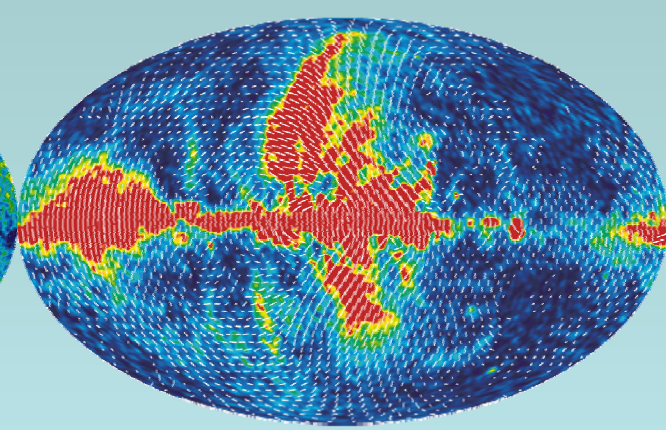



Motivation

- Inflation provides a beautiful causal mechanism to generate the observed CMB anisotropies and distribution of large-scale structure.
- Inflation is sensitive to Planck-scale physics.   Planck-suppressed operators generically make critical contributions to the dynamics.
- Therefore, we should understand inflation in string theory, and compute, or at least characterize, the Planck-suppressed contributions.







Physics of slow roll inflation

Scalar field with a potential,

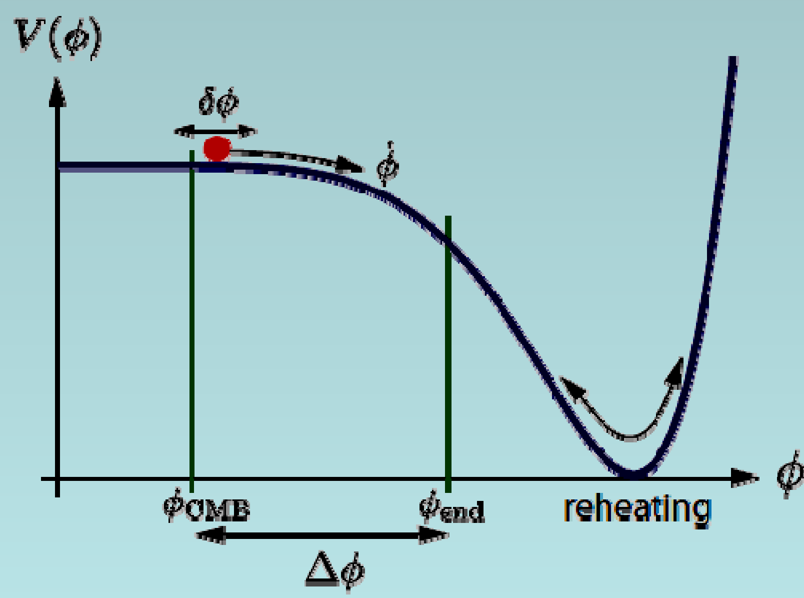
$$S = \int \mathrm{d}^4x \sqrt{-g} \left[\frac{1}{2} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi - V(\phi) \right]$$

Potential drives acceleration,

$$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2 \qquad H \approx const.$$

- Acceleration prolonged if V is flat in Planck units:

$$\eta \equiv M_p^2 \frac{V''}{V} \qquad \varepsilon \equiv M_p^2 \left(\frac{V'}{V} \right)^2$$



Planck-sensitivity of inflation

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V_0(\phi)\right]$$

$$\Delta V \equiv \mathcal{O}_6 = V_0 \frac{\phi^2}{M_p^2} \Longrightarrow \begin{matrix} \Delta\eta \sim 1 \\ N_e \sim 1 \end{matrix} \qquad \eta \equiv M_p^2 \frac{V''}{V}$$

For small inflaton excursions, $\Delta\phi \lesssim M_{pl}$, one must control corrections $\mathcal{O}_\Delta$ with $\Delta \lesssim 6$.

For large inflaton excursions, $\Delta\phi \gg M_{pl}$, one must control an infinite series of corrections, with arbitrarily large Δ.

Options for dealing with the sensitivity to Planck-scale physics.

I. Invoke a symmetry strong enough to forbid all such contributions.
   - i.e., forbid the inflaton from coupling to massive d.o.f.

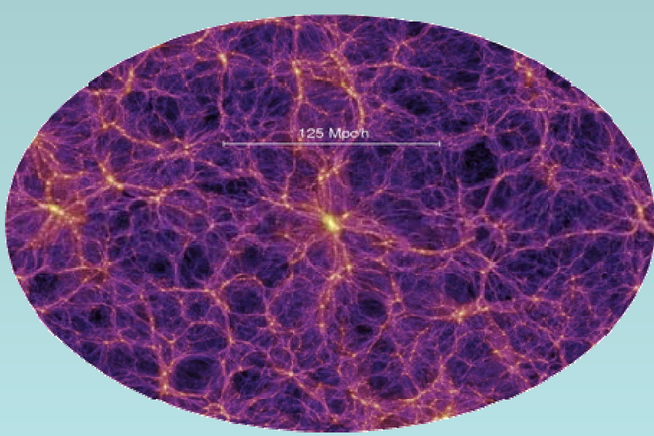
Freese, Frieman, Olinto 1990
Arkani-Hamed, Cheng, Creminelli, Randall 2003
Kallosh, Hsu, Prokushkin 2004
Dimopoulos, Kachru, McGreevy, Wacker 2005
Conlon & Quevedo 2005
Cicoli, Burgess, Quevedo 2008
L.M., Silverstein, Westphal 2008
Flauger, L.M., Pajer, Westphal, Xu 2009
Berg, Pajer, Sjörs 2009

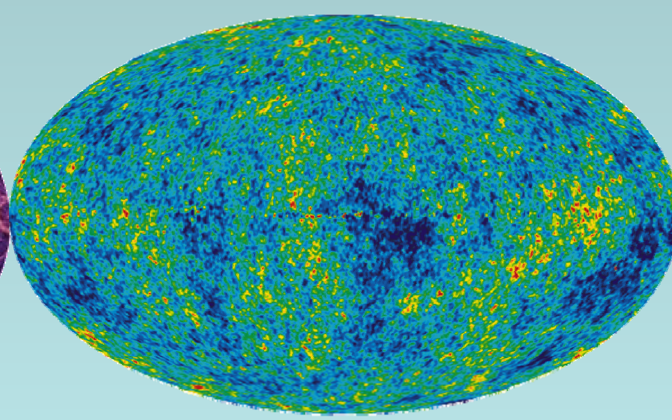
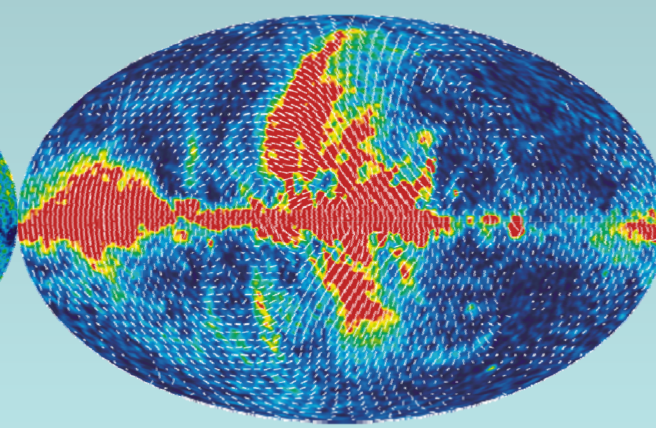
II. Enumerate all relevant contributions and determine whether fine-tuned inflation can occur.
   - i.e., arrange for cancellations.

Baumann, Dymarsky, Klebanov, L.M., 2007   Krause, Pajer, 2007
Haack, Kallosh, Krause, Linde, Lüst, Zagermann, 2008
Baumann, Dymarsky, Kachru, Klebanov, L.M., 2008, 2009, 2010

Status report

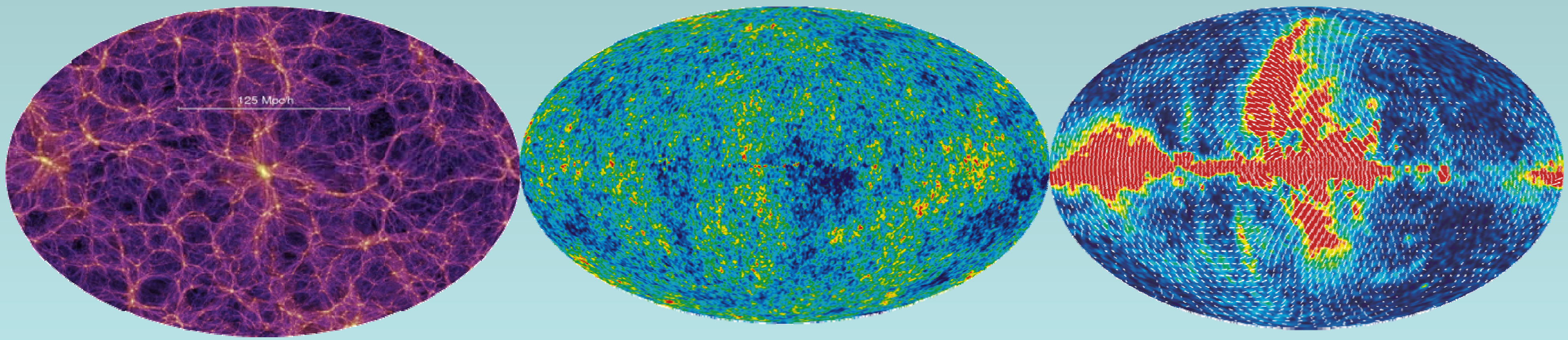
- Inflaton *candidates* continue to proliferate.
- Still very few examples where present techniques admit systematic study of the Planck-suppressed contributions.
- Why is it hard?



Motivation

- Inflationary solutions of string theory are only possible in vacua with stabilized moduli.
  - significant progress in the past decade.
- Integrating out the massive moduli induces interactions that are typically no more than Planck-suppressed.
- Thus, to characterize the inflaton action, we must work in a stabilized vacuum and carefully incorporate the couplings of the inflaton candidate to the moduli.



What's the problem?

N=0 supersymmetry.

Compactness crucial.

Effects that stabilize the moduli rarely decouple from inflation.

This talk

- Work out in full detail the structure of the effective action for a small-field model.

This talk

- Work in a carefully-chosen corner where powerful tools (noncompact approximation, AdS/CFT) are applicable.
- Goal: characterize the action for a spacetime-filling D3-brane in a type IIB flux compactification, including nonperturbative contributions.
- Nonperturbative effects
  - are crucial in some of the best-studied scenarios for Kähler moduli stabilization
  - make dominant contributions to the inflationary dynamics of a D3-brane in such vacua.
- Our analysis provides a toy model of inflation in quantum gravity.
  - A small step towards a more comprehensive and systematic understanding

Results presented here:

- Structure of the potential for a D3-brane in a conifold attached to a general compact space. All significant contributions to the D3-brane potential incorporated in 10D supergravity.

- Gaugino condensation on D7-branes wrapping a four-cycle sources IASD flux.

I.
D3-branes in flux compactifications

D3-branes in warped compactifications

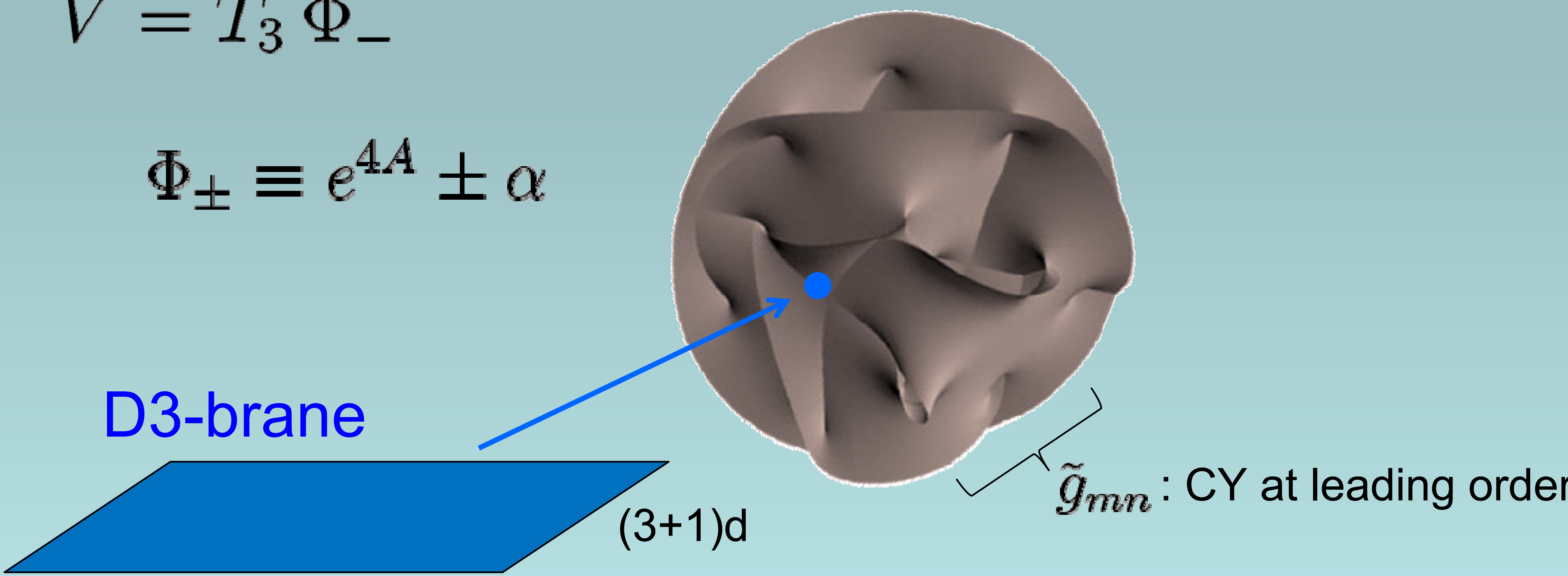
$$ds^2 = \underline{e^{2A(y)}} g_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} \tilde{g}_{mn}(y) dy^m dy^n$$

$$\tilde{F}_5 = (1 + \star_{10})\, d\underline{\alpha(y)} \wedge \sqrt{-\det g_{\mu\nu}}\, dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3$$

DBI+CS:

$$V = T_3\, \Phi_-$$

$$\Phi_\pm \equiv e^{4A} \pm \alpha$$



D3-branes in warped flux compactifications

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$$G_\pm \equiv (i \pm \star_6) G_3 \qquad \Phi_\pm \equiv e^{4A} \pm \alpha \qquad V = T_3\, \Phi_-$$

ISD solutions: $G_- = \Phi_- = 0$

GKP 2001

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GKP 2001

D3-branes feel no potential in ISD solutions (‘no-scale’), but nonperturbative stabilization of Kähler moduli spoils this.

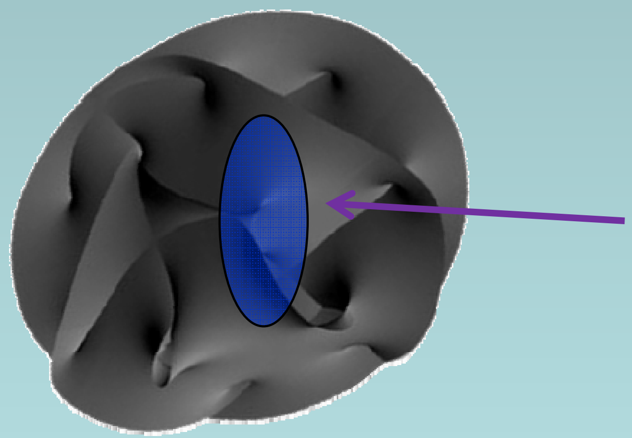
Nonperturbative contributions

No-scale symmetry $\rho \to \rho + const.$ is broken
by Euclidean D3-branes,
or gaugino condensation on N_c D7-branes,
wrapping suitable four-cycles Σ_i.

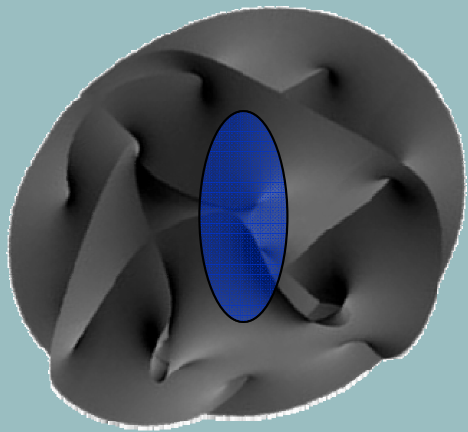
Witten, 1996 KKLT, 2003

$$W = \int G_3 \wedge \Omega + A(y) e^{-a\rho}, \qquad \mathcal{K} = -3 \log\Big(\rho + \bar{\rho} - k(y, \bar{y})\Big)$$

$$a = \frac{2\pi}{N_c}$$



Nonperturbative contributions



If divisor is defined by $h(z_\alpha) = 0$ in local coordinates, then

$$W_{\rm np}(z_\alpha) = \mathcal{A}_0\, h(z_\alpha)^{1/N_c}\, e^{-a\rho}$$

Ganor, 1996

Berg, Haack, Körs, 2004

Baumann, Dymarsky, Klebanov, Maldacena, L.M., Murugan, 2006

Koerber & Martucci, 2007

$$W = \int G_3 \wedge \Omega + W_{\rm np}(z_\alpha)$$

D3-brane vacua are generically isolated.
We want the potential in between.

DeWolfe, L.M.,
Shiu, Underwood, 2007

D3-branes in warped flux compactifications

$$ds^2 = e^{2A(y)} g_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} \tilde{g}_{mn}(y) dy^m dy^n$$

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$$G_\pm \equiv (i \pm \star_6) G_3 \qquad \Phi_\pm \equiv e^{4A} \pm \alpha \qquad V = T_3 \Phi_-$$

ISD solutions: $G_- = \Phi_- = 0$

GKP 2001

D3-branes feel no potential in ISD solutions (‘no-scale’), but nonperturbative stabilization of Kähler moduli spoils this.

We will expand around ISD solutions, $\Phi_-^{(0)} = G_-^{(0)} = 0$

and find that nonperturbative effects source IASD flux.

Equations of motion expanded around an ISD background

$$\nabla^2\Phi_- = \frac{g_s}{96}|\Lambda|^2 + \mathcal{R}_4$$

$$d\Lambda = 0 \qquad \star_6\Lambda = -i\Lambda \qquad \Lambda \equiv \Phi_+ G_-$$

Metric $\tilde{g}_{mn}$ and dilaton:

zeroth-order solutions suffice to determine leading contributions to D3-brane potential, Φ_-.

We only need to know the background metric.

Similarly, Bianchi identity for G must be solved, but is not relevant for determining leading contributions.

General compact model still intractable

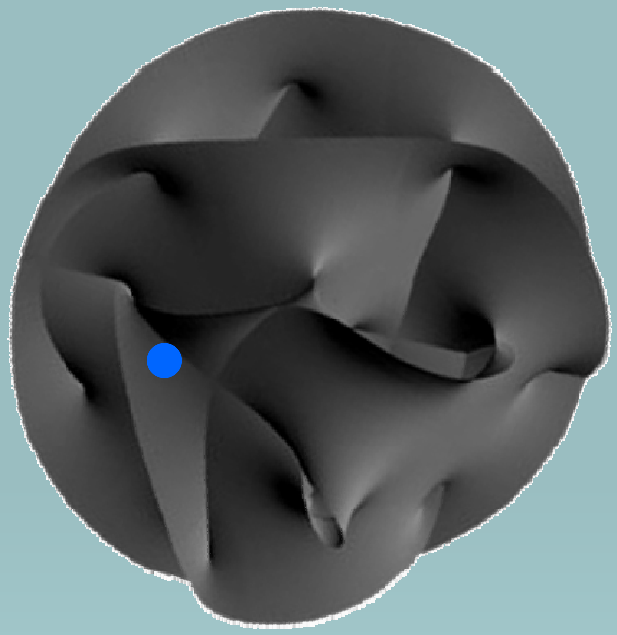
$$\nabla^2 \Phi_- = \frac{g_s}{96}|\Lambda|^2 + \mathcal{R}_4$$

$$\mathrm{d}\Lambda = 0 \qquad \star_6 \Lambda = -i\Lambda$$

We do need the internal background (zeroth-order) metric.

General structure of the D3-brane potential?

Hard to compute in full generality.



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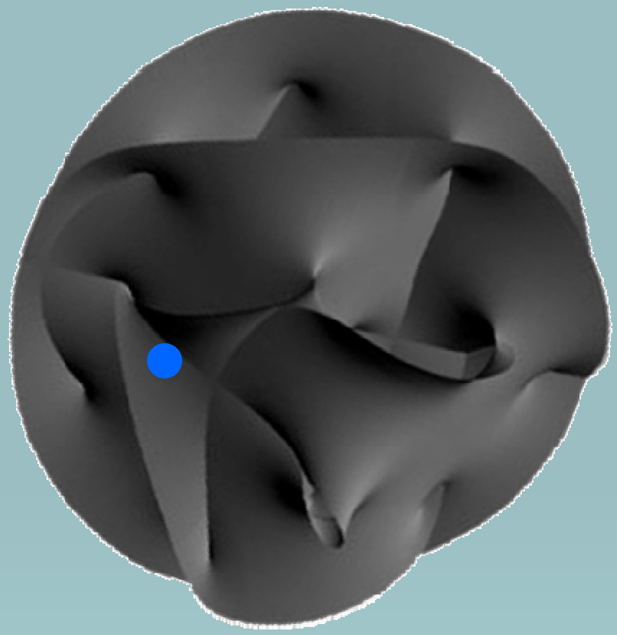

Idea: begin with a noncompact CY cone, and systematically incorporate compactification effects.

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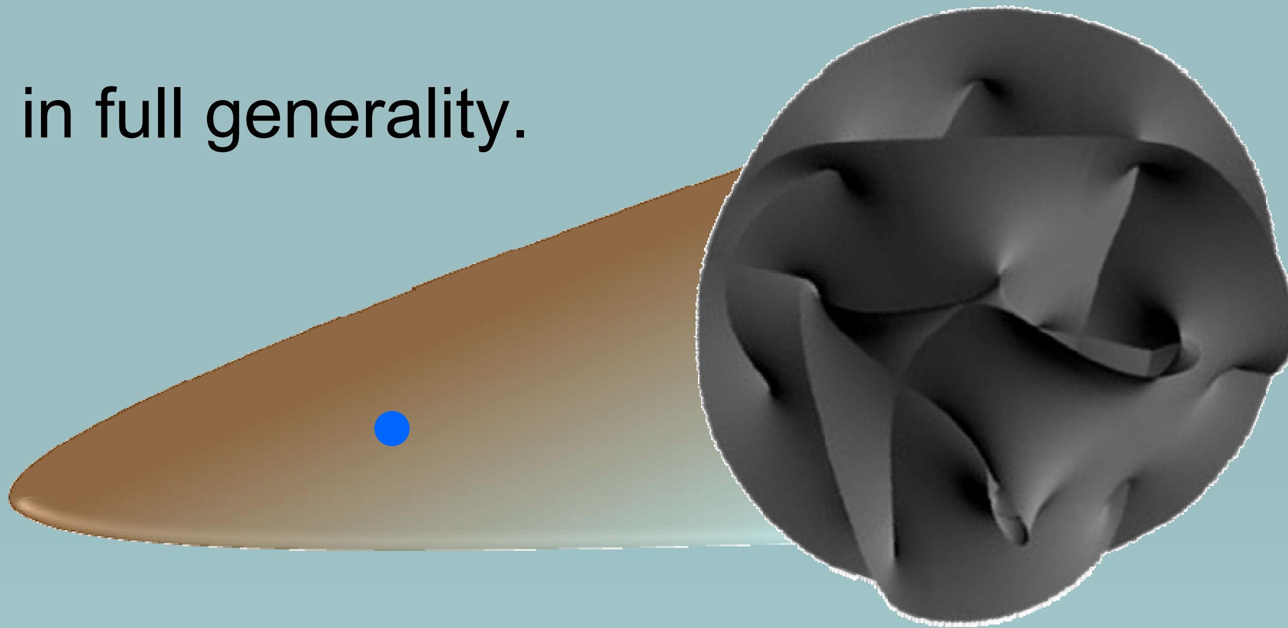
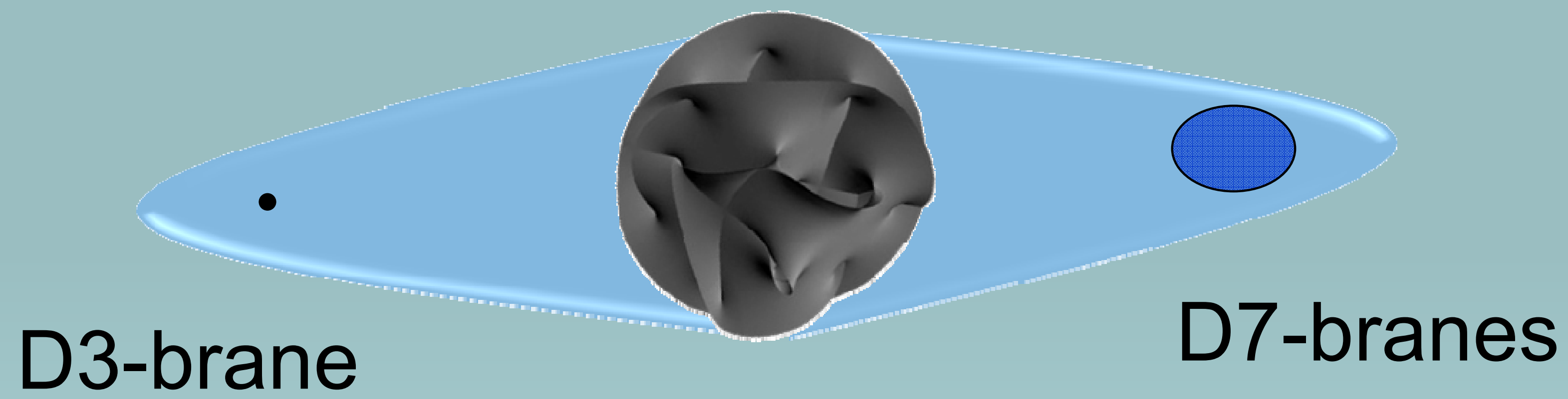


Idea: begin with a noncompact CY cone, and systematically incorporate compactification effects.

Compactification effects

e.g.:



Warped throats in compact spaces

We will obtain a further handle on the problem by taking the cone to be warped, such as the warped conifold or a more general warped CY cone.

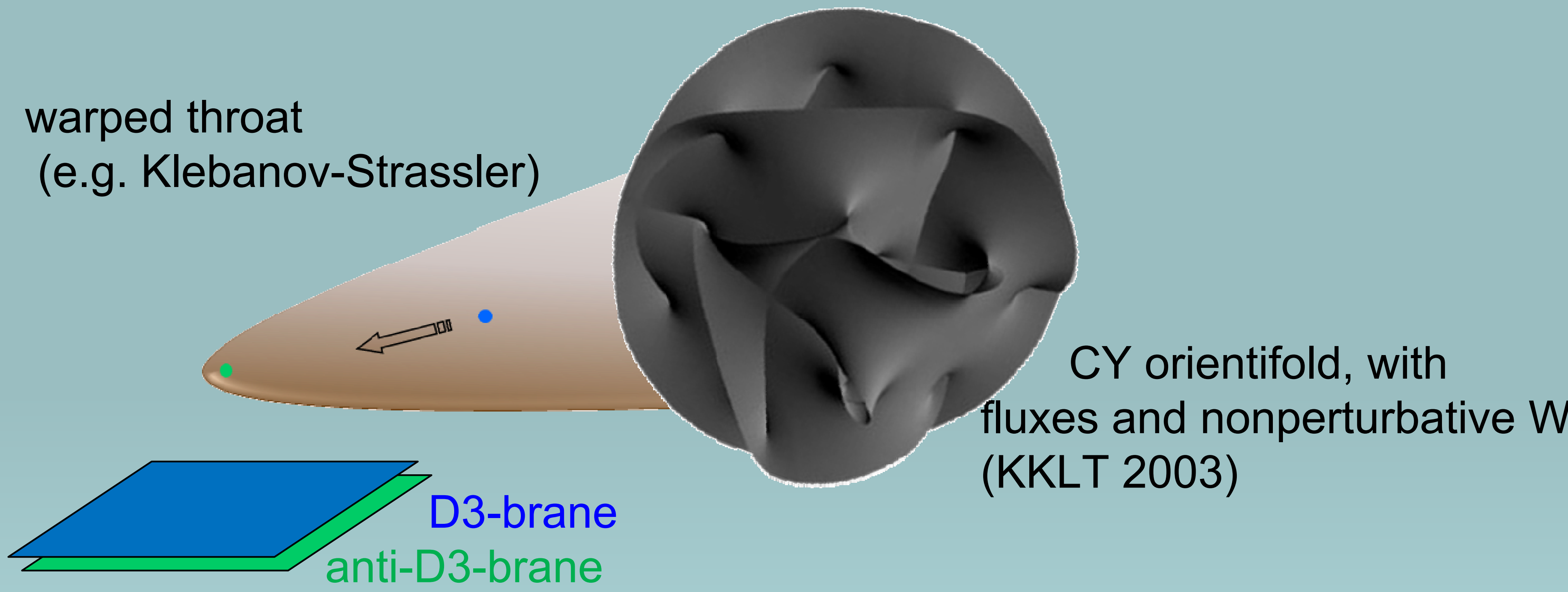
Concrete example: a finite-length KS throat, which we approximate by AdS_5 x $T^{1,1}$.

$$ds^2 = e^{2A_{(0)}(r)}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + e^{-2A_{(0)}(r)}(\mathrm{d}r^2 + r^2 \mathrm{d}\Omega^2_{T^{1,1}})$$

$$e^{-4A_{(0)}(r)} = \frac{L^4}{r^4} \qquad L^4 \equiv \frac{27\pi}{4} g_s N(\alpha')^2$$

A warped CY cone attached to a stabilized compactification is precisely the configuration of interest in warped D-brane inflation.

Warped D-brane inflation



Dvali&Tye 1998

Dvali,Shafi,Solganik 2001

Burgess,Majumdar,Nolte,Quevedo,Rajesh,Zhang 2001

Kachru, Kallosh, Linde, Maldacena, L.M., Trivedi, 2003

Filtering in the throat

The warped geometry filters the compactification effects: the dominant effects in the IR are those with the smallest dimensions Δ_i.

$$V = \sum_i c_i \phi^{\Delta_i} h_i(\Psi)$$

By determining the spectrum of Δ_i we can extract the leading terms in the potential.

Double expansion:

around ISD backgrounds $\quad \Phi_-, G_- \ll 1$

and in distance from the UV (hierarchy of scales) $\quad r_{D3} \ll r_{UV}$

Noncompact approximation

The D3-brane potential comes from Φ_- alone. At leading order in an expansion around ISD backgrounds, the only relevant 10d source for Φ_- is IASD flux Λ.

$$\nabla^2\Phi_- = \frac{g_s}{96}|\Lambda|^2$$

Arbitrary compactification effects can be represented by specifying boundary conditions for Φ_- and Λ in the UV of the throat, i.e. by allowing arbitrary non-normalizable profiles.

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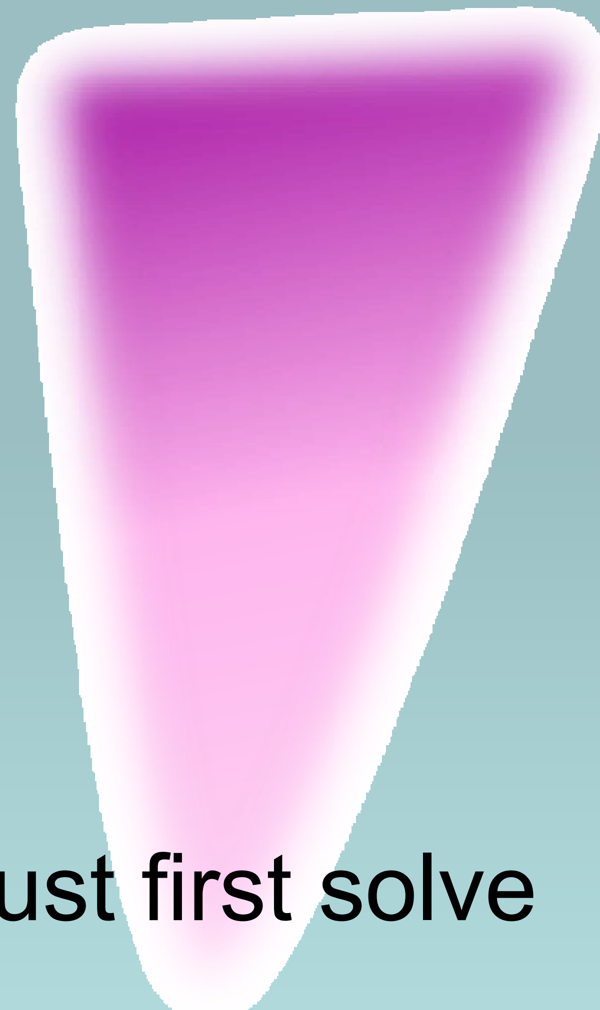
Arbitrary compactification effects can be represented by specifying boundary conditions for Φ_- and Λ in the UV of the throat, i.e. by allowing arbitrary non-normalizable profiles.

Fluxes are the leading source

$$\mathrm{d}\Lambda = 0 \qquad \star_6 \Lambda = -i\Lambda$$

$$\Lambda \equiv \Phi_+ G_-$$

$$\nabla^2 \Phi_- = \frac{g_s}{96}|\Lambda|^2 + \mathcal{R}_4$$



To solve for the potential, we must first solve for the IASD flux.

Three classes of IASD flux solutions in Calabi-Yau cones

$$\boxed{\mathrm{d}\Lambda = 0}$$

$$\boxed{\star_6 \Lambda = -i\Lambda}$$

$$\Lambda_{\rm III} = f_3\,\Omega$$

(3,0)

Baumann, Dymarsky, Kachru, Klebanov, L.M., 1001.5028

Three classes of IASD flux solutions in Calabi-Yau cones

$$\boxed{\mathrm{d}\Lambda = 0}$$

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$$\Lambda_{\mathrm{II}} = \partial f_2 \wedge J$$

$(2,1)_{\mathrm{NP}}$

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Three classes of IASD flux solutions in Calabi-Yau cones

$$\boxed{\mathrm{d}\Lambda = 0}$$

$$\boxed{\star_6 \Lambda = -i\Lambda}$$

$$(\Lambda_{\rm I})_{\alpha\bar\beta\bar\gamma} \;=\; \nabla_\alpha \nabla_\sigma f_1\, g^{\sigma\bar\zeta}\, \bar\Omega_{\bar\zeta\bar\beta\bar\gamma}$$ (1,2)

$$\Lambda_{\rm II} = \partial f_2 \wedge J$$ $(2,1)_{NP}$

$$\Lambda_{\rm III} = f_3\, \Omega$$ (3,0)

Here f_i are holomorphic functions. Easy to generalize to harmonic functions.

Baumann, Dymarsky, Kachru, Klebanov, L.M., 1001.5028

D3-brane potential from holomorphic fluxes

$$\nabla^2 \Phi_- = \frac{g_s}{96} |\Lambda|^2$$

$$\Lambda_{\mathrm{I}} = \nabla \nabla f_1 \cdot \bar{\Omega}$$
$$\Lambda_{\mathrm{II}} = \partial f_2 \wedge J$$
$$\Lambda_{\mathrm{III}} = f_3 \, \Omega$$

$$\Phi_- = \frac{g_s}{32} \left[g^{\alpha \bar{\beta}} \nabla_\alpha f_1 \, \overline{\nabla_\beta f_1} + 2 |f_2|^2 + 2 \, \nabla^{-2} |f_3|^2 \right]$$

Simple when fluxes are holomorphic.

Harmonics on AdS$_5$ x T^{1,1}

$$\nabla^2 f = 0 \qquad\qquad f = r^{\Delta[j_1,j_2,R]} Y_{[j_1,j_2,R]}(\Psi)$$

$$\begin{aligned}\Delta_f &\equiv -2 + \sqrt{H(j_1,j_2,R_f)+4} \\ &= \frac{3}{2}\ ,\ 2\ ,\ 3\ ,\ \sqrt{28}-2\ ,\ \cdots\end{aligned} \qquad H(j_1,j_2,R_f) \equiv 6\left[j_1(j_1+1)+j_2(j_2+1)-R_f^2/8\right]$$

Ceresole, Dall'Agata, D'Auria, Ferrara 1999

$$(\Lambda_{\rm I})_{\alpha\bar\beta\bar\gamma} = \nabla_\alpha \nabla_\sigma f_1\, g^{\sigma\bar\zeta}\, \bar\Omega_{\bar\zeta\bar\beta\bar\gamma} \qquad \Lambda_{\rm I} \sim r^4 G_- \sim r^{\delta_{\rm I}}$$

$$\delta_{\rm I} = 1 + \Delta_f = -1 + \sqrt{H(j_1,j_2,R+2)+4}$$

D3-brane potential from non-holomorphic fluxes

$$\nabla^2\Phi_- = \frac{g_s}{96}|\Lambda|^2$$

$$\Phi_- = \sum_{\delta_i,\delta_j} r^{\Delta(\delta_i,\delta_j)} h_{(\delta_i,\delta_j)}(\Psi) \qquad \Delta \equiv \delta_i + \delta_j - 4$$

$$\delta_{\mathrm{I}} = 1 + \Delta_f = -1 + \sqrt{H(j_1, j_2, R+2) + 4}$$

Four-dimensional curvature

$$\nabla^2 \Phi_- = \mathcal{R}_4$$

$$\mathcal{R}_4 = 12H^2 \approx \frac{4}{M_{\rm pl}^2} V = \frac{4}{M_{\rm pl}^2}\Big(V_0 + T_3 \Phi_-\Big)$$

Four-dimensional curvature

$$\nabla^2 \Phi_- = \mathcal{R}_4 \qquad \mathcal{R}_4 = 12H^2 \approx \frac{4}{M_{\rm pl}^2} V = \frac{4}{M_{\rm pl}^2}\left(V_0 + T_3 \Phi_-\right)$$

$$V(\phi) \;=\; V_0 + T_3\Phi_- \;=\; 2V_0 \frac{M_{\rm pl}^2}{\phi^2} \sum_L I_{n(L)}\left(2\frac{\phi}{M_{\rm pl}}\right) h_L(\Psi)$$

$$n^2(L) \equiv H(L) + 4 \qquad I_n(x) \;=\; i^{-n} J_n(ix)$$

cf. Kachru, Kallosh, Linde, Maldacena, L.M., Trivedi, 2003
Buchel & Roiban, 2003

Four-dimensional curvature

$$\nabla^2 \Phi_- = \mathcal{R}_4 \qquad \mathcal{R}_4 = 12H^2 \approx \frac{4}{M_{\rm pl}^2} V = \frac{4}{M_{\rm pl}^2}\left(V_0 + T_3\Phi_-\right)$$

$$V(\phi) \;=\; V_0 + T_3\Phi_- \;=\; 2V_0\,\frac{M_{\rm pl}^2}{\phi^2}\sum_L I_{n(L)}\left(2\frac{\phi}{M_{\rm pl}}\right)h_L(\Psi)$$

$$n^2(L) \equiv H(L) + 4 \qquad I_n(x) \;=\; i^{-n} J_n(ix)$$

$$\eta \;=\; M_{\rm pl}^2 \frac{V''}{V} \;=\; \frac{2}{3} \;+\; \cdots$$

cf. Kachru, Kallosh, Linde, Maldacena, L.M., Trivedi, 2003
Buchel & Roiban, 2003

Spectrum of the D3-brane potential

$$\nabla^2 \Phi_- = \frac{g_s}{96} |\Lambda|^2 + \mathcal{R}_4$$

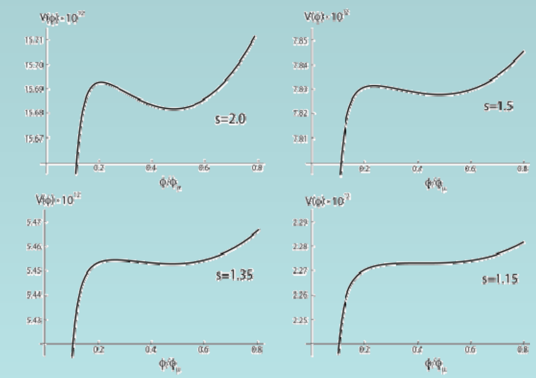
$$\Delta_{\mathcal{H}} = \frac{3}{2}\,,\ 2\,,\ 3\,,\ \ldots$$

$$\Delta_{\Lambda} = 1\,,\ 2\,,\ \frac{5}{2}\,,\ \sqrt{28} - \frac{5}{2}$$

$$\Delta_{\mathcal{R}} = 2_s\,,\ 3\,,\ \frac{7}{2}\,,\ 4\,,\ \ldots$$

$$V = \sum_i c_i \phi^{\Delta_i} h_i(\Psi)$$

$$V(\phi) = V_0 + b_1\, j_1(\Psi)\, \phi^1 + a_{3/2}\, h_{3/2}(\Psi)\, \phi^{3/2} + \Big(c_2 + a_2\, h_2(\Psi) + b_2\, j_2(\Psi)\Big)\, \phi^2$$
$$+ b_{5/2}\, j_{5/2}(\Psi)\, \phi^{5/2} + b_{2.79}\, j_{2.79}(\Psi)\, \phi^{2.79} + \ldots$$



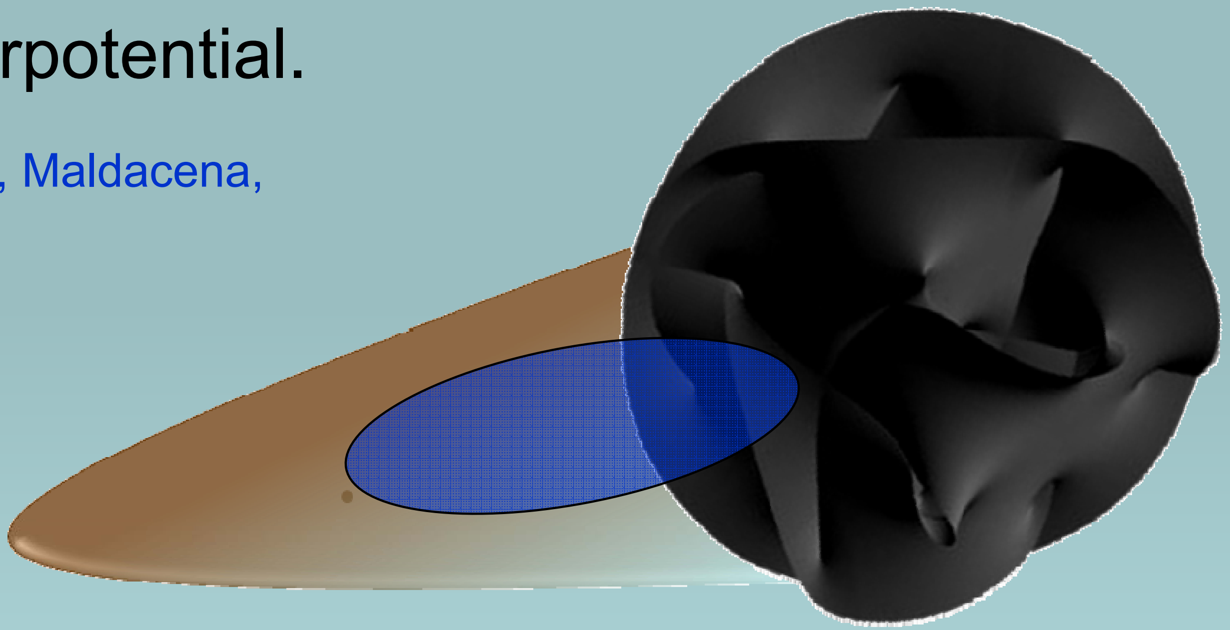
Baumann, Dymarsky, Kachru, Klebanov, L.M., 1001.5028

General structure attested in examples

Special case is explicitly computable: assume the moduli-stabilizing D7-branes hang into the throat region

Can then compute superpotential.

Baumann, Dymarsky, Klebanov, Maldacena, L.M., & Murugan, 2006.



Resulting potential:

$$V(\phi) \;=\; V_0 + b_1\, j_1(\Psi)\,\phi^1 + a_{3/2}\, h_{3/2}(\Psi)\,\phi^{3/2} + \Big(c_2 + a_2\, h_2(\Psi) + b_2\, j_2(\Psi)\Big)\,\phi^2 + \ldots$$

Identical structure!

II.
Gauge theory description

Gauge theory version

Arbitrary compactification effects can be represented by incorporating arbitrary perturbations of the CFT Lagrangian, including coupling it to 4D gravity and to hidden sector degrees of freedom.

$$\mathcal{L}_0 + \delta\mathcal{L} = \int d^2\theta d^2\bar{\theta}\,(K_0 + \delta K) + \int d^2\theta\,(W_0 + \delta W) + h.c.$$

We must first classify all operators in the CFT that are dual to the IASD flux modes of interest.

Gauge theory version

Some IASD flux perturbations are dual to supersymmetric perturbations of the CFT Lagrangian,

$$\int d^2\theta \, \mathcal{O}_{CFT}$$

while others have supersymmetry broken by a hidden sector spurion,

$$\int d^2\theta \, \mathcal{O}_{CFT} X \qquad X = \theta^2 F_X$$

Analogous formulae for non-chiral contributions.

Gauge theory version

Klebanov-Witten SCFT:

SU(N) x SU(N) gauge group $$W^{\pm}_{\alpha} \equiv W^{(1)}_{\alpha} \pm W^{(2)}_{\alpha}$$

SU(2) x SU(2) x $U(1)_R$ global symmetry [j_1, j_2, R]

bifundamentals A_i, B_i

Flux	**Operator**	$\boldsymbol{\Delta}$	R	j_1	j_2
$\nabla\nabla \mathrm{f}_1 \cdot \bar{\Omega}$	$[\mathrm{Tr}(AB)^k]_{\theta^2}$	$\frac{3}{2}k+1$	$k-2$	$\frac{1}{2}k$	$\frac{1}{2}k$
$\partial \mathrm{f}_2 \wedge J$	$[\mathrm{Tr}[W^{\alpha}_{+}(AB)^k]]_{\theta}$	$\frac{3}{2}k+2$	k	$\frac{1}{2}k$	$\frac{1}{2}k$
$\mathrm{f}_3\Omega$	$[\mathrm{Tr}[(W^2_{+}(AB)^k]]_{\mathrm{b}}$	$\frac{3}{2}k+3$	$k+2$	$\frac{1}{2}k$	$\frac{1}{2}k$

cf. Ceresole, Dall'Agata, D'Auria, Ferrara 1999, Ceresole, Dall'Agata, D'Auria 1999

Table 7: *Matching between supergravity G_- flux modes and CFT operators.*

δ	j_1	j_2	R	Operator		Multiplet	Type	Flux Series
$\frac{5}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	-1	$[S^1]_{\theta^2}$	$[\mathrm{Tr}(AB)]_{\theta^2}$	V.I	chiral	I
3	0	0	2	$[\Phi^0_+]_\mathrm{b}$	$[\mathrm{Tr}(W^2_{(1)} + W^2_{(2)})]_\mathrm{b}$	V.IV	chiral	III
$\frac{7}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$[T^1_\alpha]_\theta$	$[\mathrm{Tr}(W_\alpha(AB))]_\theta$	G.I	chiral	II
4	0	0	0	$[\Phi^0_-]_{\theta^2}$	$[\mathrm{Tr}(W^2_{(1)} - W^2_{(2)})]_{\theta^2}$	V.III	chiral	$\star$
4	0	1	0	$[{}_aL^{2,0}_\alpha]_\theta$	$[\mathrm{Tr}(W_\alpha J_a)]_\theta$	G.I+G.III	semi-long	II
4	1	0	0	$[{}_bL^{2,0}_\alpha]_\theta$	$[\mathrm{Tr}(W_\alpha J_b)]_\theta$	G.I+G.III	semi-long	II
4	1	1	0	$[S^2]_{\theta^2}$	$[\mathrm{Tr}(AB)^2]_{\theta^2}$	V.I	chiral	I
$\sqrt{28}-1$	1	1	-2	–	$[\mathrm{Tr}(f)]_{\theta^2}$	V.I	long	I
$\frac{9}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	3	$[\Phi^1_+]_\mathrm{b}$	$[\mathrm{Tr}(W^2_{(1)} + W^2_{(2)})(AB)]_\mathrm{b}$	V.IV	chiral	III
$\frac{9}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$[\bar{\Phi}^1_+]_\mathrm{b}$	$[\mathrm{Tr}(W^2_{(1)} + W^2_{(2)})(\overline{AB})]_\mathrm{b}$	V.IV	–	III
$\frac{9}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	-1	$[{}_aJ^1]_{\theta^2}$	$[\mathrm{Tr}(J_a(AB))]_{\theta^2}$	V.I	semi-long	I
$\frac{9}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	-1	$[{}_bJ^1]_{\theta^2}$	$[\mathrm{Tr}(J_b(AB))]_{\theta^2}$	V.I	semi-long	I
5	1	1	2	$[T^2_\alpha]_\theta$	$[\mathrm{Tr}(W_\alpha(AB)^2)]_\theta$	G.I	chiral	II
5	0	1	2	$[{}_aI^0]_\mathrm{b}$	$[\mathrm{Tr}((W^2_{(1)} + W^2_{(2)})J_a)]_\mathrm{b}$	V.IV	semi-long	III
5	1	0	2	$[{}_bI^0]_\mathrm{b}$	$[\mathrm{Tr}((W^2_{(1)} + W^2_{(2)})J_b)]_\mathrm{b}$	V.IV	semi-long	III
$\sqrt{28}$	1	1	0	–	$[\mathrm{Tr}(W_\alpha f)]_\theta$	G.I+G.III	long	II
$\sqrt{40}-1$	0	2	-2	–	$[\mathrm{Tr}(f_a)]_{\theta^2}$	V.I	long	I
$\sqrt{40}-1$	2	0	-2	–	$[\mathrm{Tr}(f_b)]_{\theta^2}$			

cf. Ceresole, Dall'Agata, D'Auria 1999

Gauge theory version

The leading (r^1) term comes from a superpotential perturbation by the lowest-dimension gauge invariant operator in the Klebanov-Witten SCFT,

$$\int d^2\theta \, \mathrm{Tr}(AB)$$

exactly as one would expect.

(relation to G-flux: cf. Graña & Polchinski 2000.)

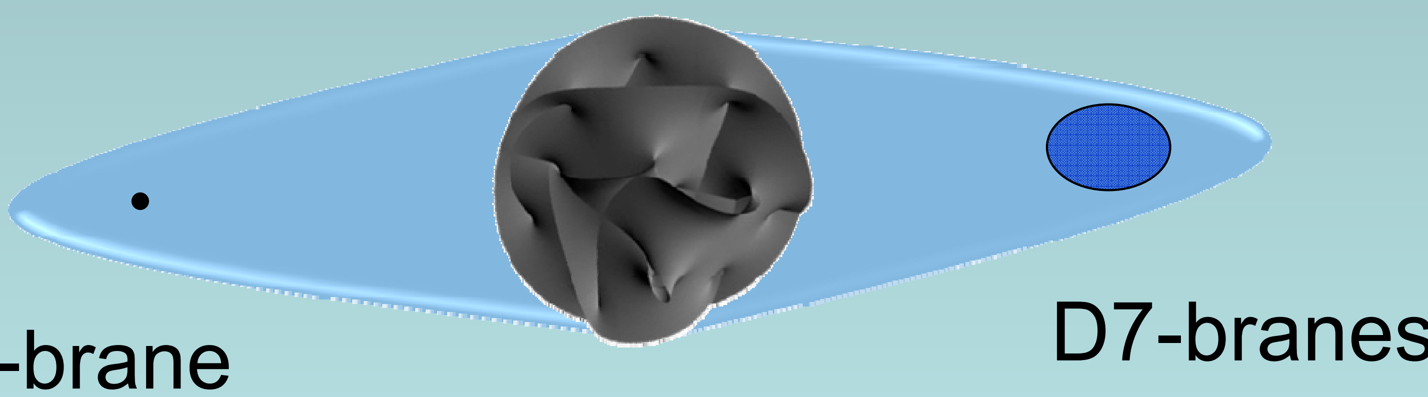
For general perturbations by chiral operators, we reproduce the gravity-side potential.

We study perturbations by non-chiral operators exclusively in the dual gravity description, as modes of IASD flux (with Δ irrational in general).

III.
IASD fluxes sourced by nonperturbative effects

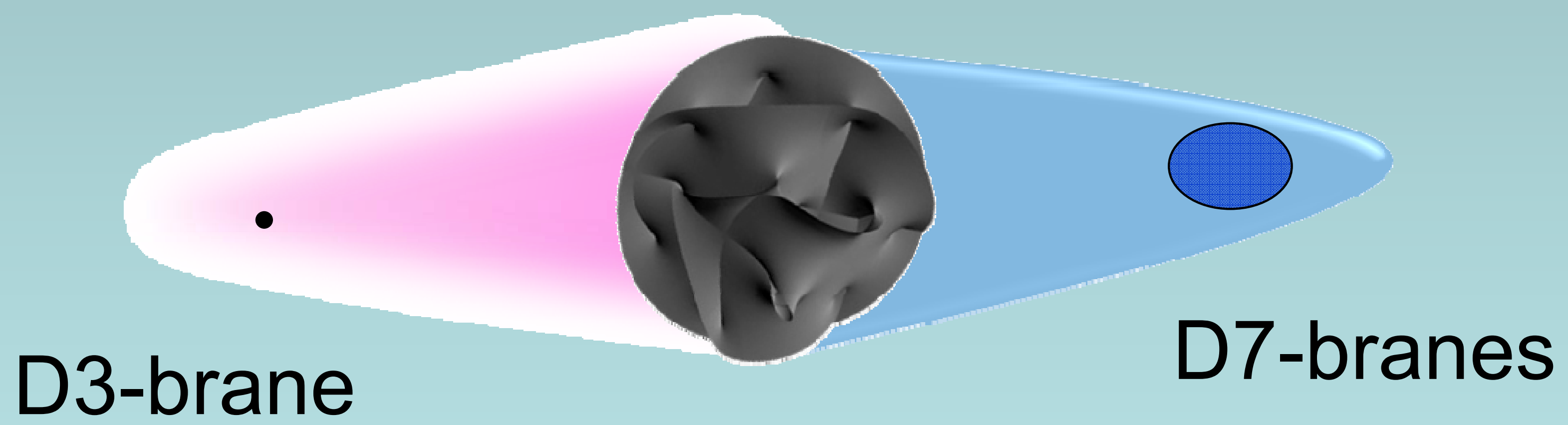
Four-dimensional effects in ten dimensions?

cf. Frey and Lippert 2005, Koerber and Martucci 2007

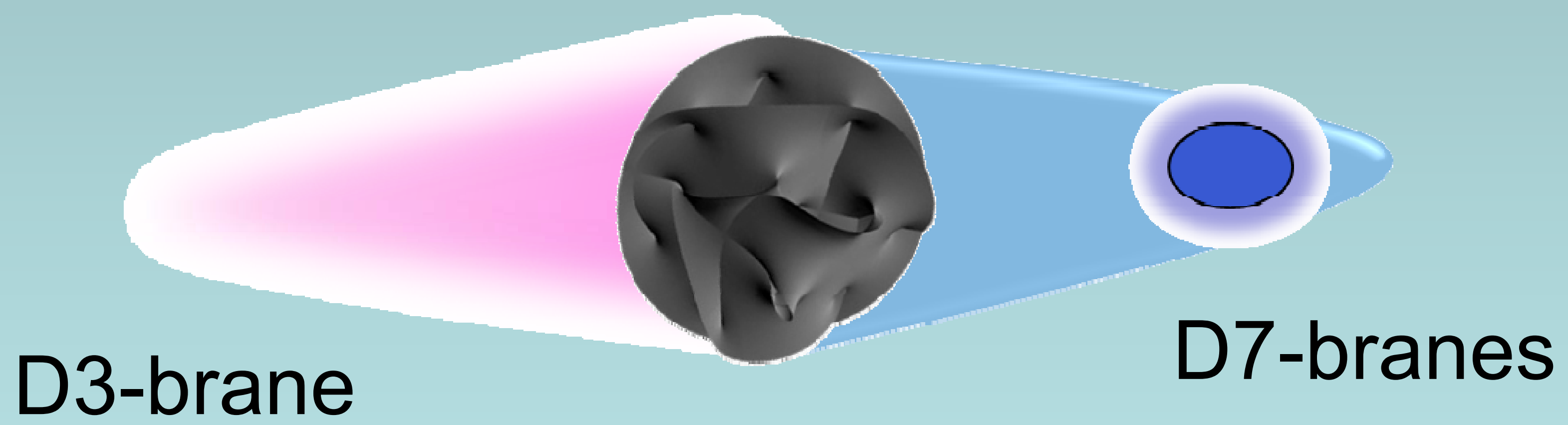


Baumann & L.M., 2006

Four-dimensional effects in ten dimensions?

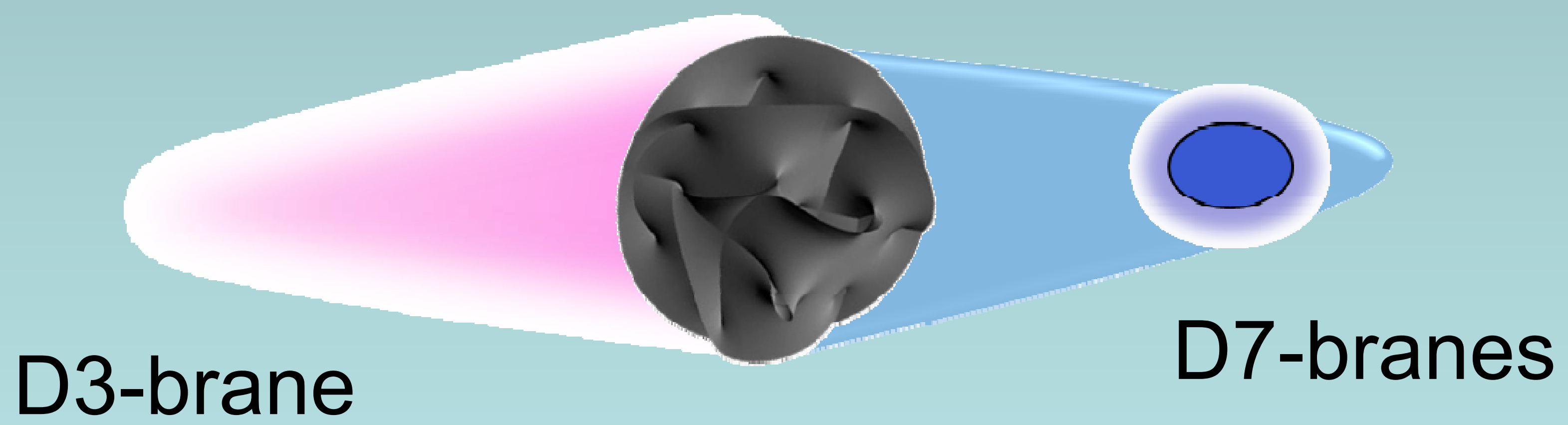


Four-dimensional effects in ten dimensions?



Four-dimensional effects in ten dimensions?

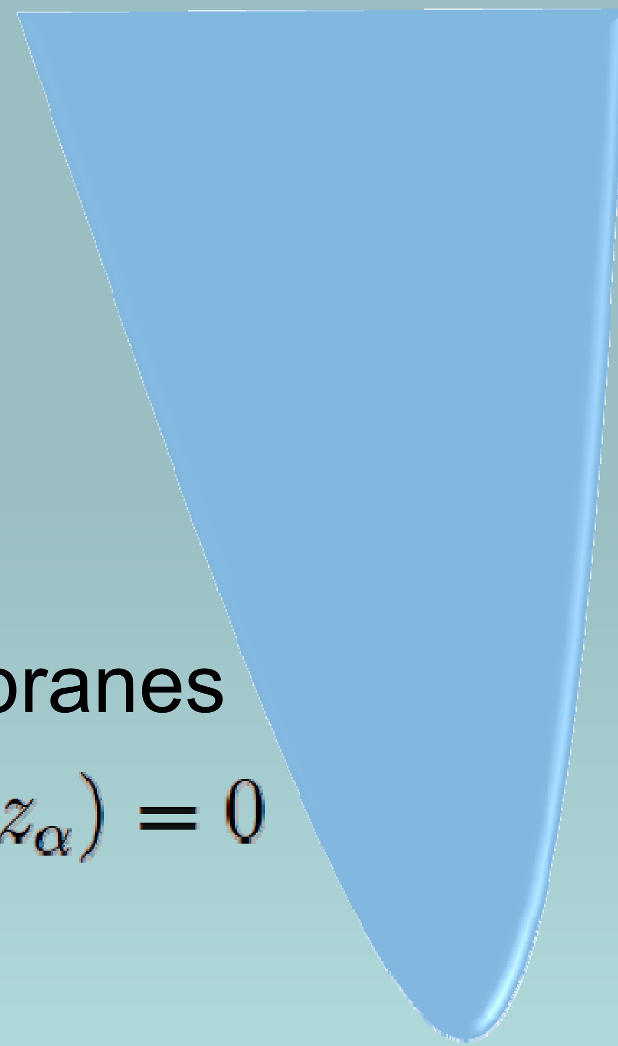
- Gaugino condensate is a 4d IR effect, but:
- In a suitably anisotropic compactification, can have 10d description valid locally (near D3-brane) even as D7-brane theory undergoes gaugino condensation.
- Equivalently, non-locality due to 4d nonperturbative effects is localized near the corresponding four-cycle.



Noncompact example

$$W_{\rm np}(z_\alpha) = \mathcal{A}_0\, h(z_\alpha)^{1/N_c}\, e^{-a\rho}$$

$$V = g^{\alpha\bar\beta}\nabla_\alpha W\,\overline{\nabla_\beta W}$$



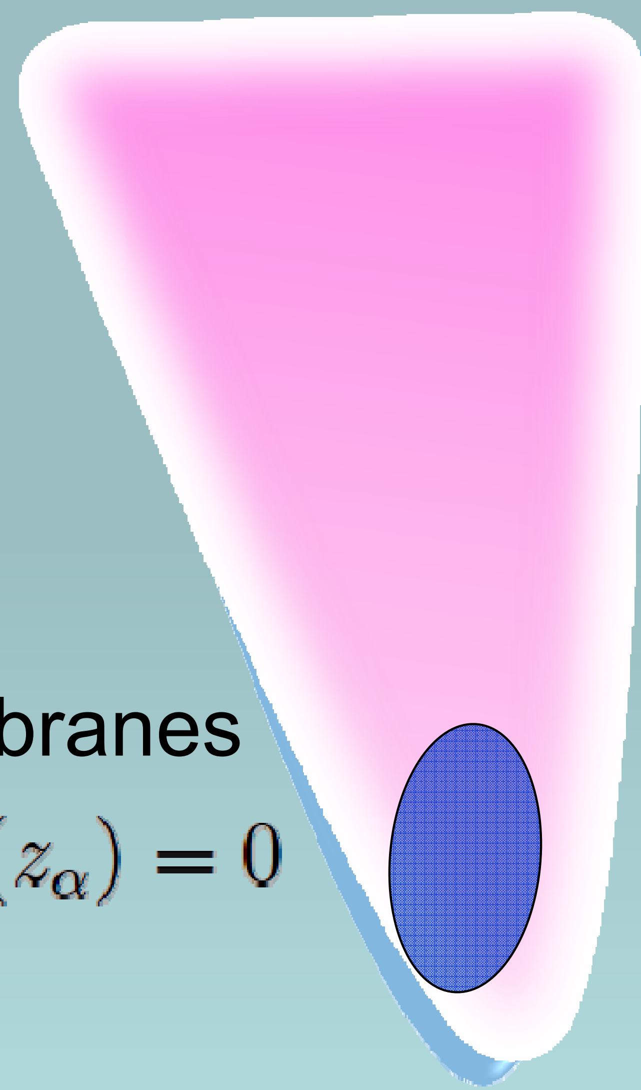
N_c D7-branes

$$\Sigma:\ h(z_\alpha) = 0$$

Noncompact example

$$W_{\rm np}(z_\alpha) = \mathcal{A}_0\, h(z_\alpha)^{1/N_c}\, e^{-a\rho}$$

$$V = g^{\alpha\bar{\beta}} \nabla_\alpha W \overline{\nabla_\beta W}$$



N_c D7-branes

$$\Sigma :\ h(z_\alpha) = 0$$

Gaugino condensation sources IASD flux

First, work out complete 10d EOM for fluxes.
The D7-brane gaugino mass comes from the coupling

$$\mathcal{L} = 16\, c_0\, \zeta \int_\Sigma \sqrt{g}\, G_3 \cdot \Omega\, \bar{\lambda}\bar{\lambda} \; + \; c.c. \qquad \zeta \equiv T_3 \sqrt{\frac{g_s}{32}}$$

Cámara, Ibáñez, Uranga 2004

$$m_\lambda \propto G_{0,3} \Leftrightarrow F_\rho$$

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Cámara, Ibáñez, Uranga 2004

This yields a local source term in the 10d EOM,

$$\mathrm{d}\Lambda = \mathrm{d}\left(\frac{2\pi}{\zeta}\, \bar{\Omega}\, \lambda\lambda\, \delta^{(0)} \right)$$

$$\delta^{(0)} = \frac{1}{2\pi} \nabla^2 \mathrm{Re}(\log h)$$

$$\star_6\, \Lambda = -i\Lambda$$

Heretofore omitted because $\langle \mathrm{Tr} \lambda\lambda \rangle = 0$ in classical solutions. But we must include this source term!

Gaugino condensation sources IASD flux

To solve

$$\mathrm{d}\Lambda = \mathrm{d}\Big(\frac{2\pi}{\zeta}\, \bar{\Omega}\, \lambda\lambda\, \delta^{(0)} \Big)$$

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we note that for $(\Lambda_{\mathrm{I}})_{\alpha\bar{\beta}\bar{\gamma}} = \nabla_\alpha \nabla_\sigma f_1\, g^{\sigma\bar{\zeta}}\, \bar{\Omega}_{\bar{\zeta}\bar{\beta}\bar{\gamma}}$,

$$\mathrm{d}\Lambda = \mathrm{d}\Big(\frac{1}{2} \nabla^2 f_1 \wedge \bar{\Omega} \Big)$$

Gaugino condensation sources IASD flux

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$$\mathrm{d}\Lambda = \mathrm{d}\Big(\frac{2\pi}{\zeta}\, \bar{\Omega}\, \lambda\lambda\, \delta^{(0)} \Big)$$

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so the solution is $f_1 = 2\zeta^{-1}\lambda\lambda\ \mathrm{Re}(\log h(z_\alpha))$

Gaugino condensation sources IASD flux

- With (1,2) flux given by this solution,

$$(\Lambda_{\rm I})_{\alpha\bar{\beta}\bar{\gamma}} \;=\; \nabla_{\alpha}\nabla_{\sigma} f_1\, g^{\sigma\bar{\zeta}}\, \bar{\Omega}_{\bar{\zeta}\bar{\beta}\bar{\gamma}} \qquad f_1 = 2\zeta^{-1}\lambda\lambda\ \mathrm{Re}(\log h(z_\alpha))$$

Gaugino condensation sources IASD flux

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$$(\Lambda_{\rm I})_{\alpha\bar{\beta}\bar{\gamma}} \;=\; \nabla_\alpha \nabla_\sigma f_1 \, g^{\sigma\bar{\zeta}} \, \bar{\Omega}_{\bar{\zeta}\bar{\beta}\bar{\gamma}} \qquad f_1 = 2\zeta^{-1}\lambda\lambda \; \mathrm{Re}(\log h(z_\alpha))$$

we use $$\Phi_- = \frac{g_s}{32}\left[g^{\alpha\bar{\beta}}\nabla_\alpha f_1 \, \overline{\nabla_\beta f_1}\right]$$

to find a 10d DBI+CS potential

Gaugino condensation sources IASD flux

- With (1,2) flux given by this solution,

$$(\Lambda_{\rm I})_{\alpha\bar\beta\bar\gamma} = \nabla_\alpha \nabla_\sigma f_1\, g^{\sigma\bar\zeta}\, \bar\Omega_{\bar\zeta\bar\beta\bar\gamma} \qquad f_1 = 2\zeta^{-1}\lambda\lambda\ {\rm Re}(\log h(z_\alpha))$$

we use $\Phi_- = \frac{g_s}{32}\left[g^{\alpha\bar\beta}\nabla_\alpha f_1\, \overline{\nabla_\beta f_1}\right]$

to find a 10d DBI+CS potential

that precisely coincides with

the 4d F-term potential

computed with W = $\mathcal{A}_0\, h(z_\alpha)^{1/N_c}\, e^{-a\rho}$

The sourced IASD flux 'geometrizes' the gaugino condensate superpotential.

Conclusions

- Obtained structure of the potential for a D3-brane in a conifold attached to a general compact space.
- All significant contributions to the D3-brane potential captured in 10d supergravity.
- Results consistent with computation in the dual gauge theory and in 4d supergravity.
- Gaugino condensation on D7-branes sources IASD flux.
  - The flux ‘geometrizes’ the nonperturbative superpotential.