The F-theory landscape in 6D

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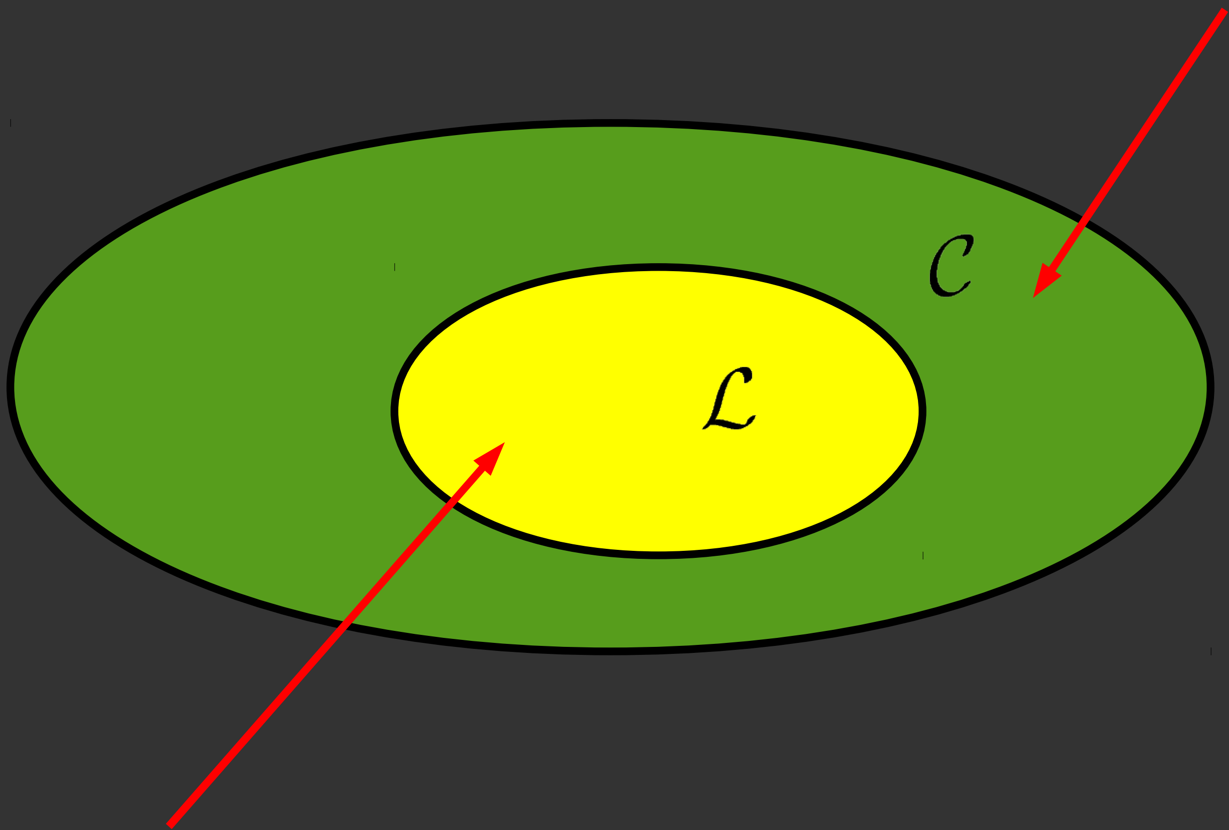
arXiv: 0911.3393, 1004.xxxx with Morrison, Taylor
0906.0987, 0910.1586 with Taylor

Outline

- Introduction
- Aspects of 6D supergravities
- Space of consistent 6D theories
- F-theory landscape
- Conclusions/Outlook

Introduction

C = EFTs with gravity obeying consistency conditions like anomaly cancellation, ...



L = Landscape: EFTs with gravity from string theory.

- C\L is called the swampland. [Vafa, Ooguri-Vafa]

  D=4: swampland >> landscape.

- Program: Take $x \in C \backslash L$. Find a string construction for x, or find an inconsistency. Expand set of string constructions, or new consistency conditions.

- In D=6 sugra, anomaly cancellation is very restrictive. Better control over C and L.

Aspects of 6D supergravities

6D, N=(1,0) SUSY (= 8 s.c.), chiral

Multiplet	Bosonic Content
sugra	$g_{\mu\nu}, \; B^-_{\mu\nu}$
tensor	$B^+_{\mu\nu}, \; \phi$
vector	A_μ
hyper	$\varphi_1, \cdots, \varphi_4$

Consider supergravities with T tensors, a nonabelian gauge group $G = \prod G_i$ and hypers in an arbitrary representation of G.

Anomaly Cancellation

- D=4k+2, lots of anomalies...
  Gauge, Gravitational, Grav+Gauge
- Green-Schwarz: Use 2-form $B_{\mu\nu}$ to cancel anomalies by adding a local counterterm.
- 1-loop anomaly $\Longleftrightarrow$ tree-level counterterm
- Sagnotti generalized mechanism in 6D. Multiple 2-forms available.

[GSW, Alvarez-Gaume-Witten, Sagnotti, Sadov]

Details: Anomaly Polynomial

Given matter content – hyper, vector, tensor, the anomaly polynomial can be written.

$$I_8 = \frac{9-T}{8}(\mathrm{tr}R^2)^2 + \frac{1}{6}\mathrm{tr}R^2 \sum_i Y^i_{(2)} - \frac{2}{3}\sum_i Y^i_{(4)} + 4\sum_{i\neq j} Y^{ij}_{(4)}$$

i,j index gauge factors.

depend on hyper matter content.

Explicit formulas for anomalies in GSW.

Anomalies cancel if ...

$$I_8 = \frac{1}{2}\Omega_{\alpha\beta} X^{\alpha}_{(4)} X^{\beta}_{(4)}$$

- $\Omega_{\alpha\beta}$ is a signature (1,T) metric in $\mathbb{R}^{1,T}$

$$X^{\alpha}_{(4)} = \frac{1}{2} a^{\alpha} \mathrm{tr} R^2 + 2 \sum_i b_i^{\alpha} \frac{1}{\lambda_i} \mathrm{tr} F_i^2$$

vectors in $\mathbb{R}^{1,T}$

- For each G_i, $\mathrm{tr} F_i^4$ should cancel.
- Gravitational anomaly: $\boxed{H - V + 29T = 273}$

$SO(1,T)$ structure

- Tensor moduli space – $SO(1,T)/SO(T)$
  parameterized by $j^\alpha \in \mathbb{R}^{1,T}$, $j \cdot j = 1$ [Romans]
- Gauge kinetic terms positive if
  $\boxed{j \cdot b_i > 0}$ for each factor G_i [Sagnotti]
- 6D EFT $\longrightarrow$ vectors $j,\ a,\ b_i$
- Anomaly polynomial fixes SO(1,T) inner products.

Lagrangian

$$\boxed{\begin{aligned}\mathcal{L} \sim -R - G_{\alpha\beta}(j)H^{\alpha}H^{\beta} - \sum_i (j \cdot b_i)\mathrm{tr}F^2 - (\partial j)^2 \\ +\Omega_{\alpha\beta}B^{\alpha} \wedge X^{\beta}_{(4)} + \ldots\end{aligned}}$$

$$H^{\alpha} = dB^{\alpha} + \frac{1}{2}a^{\alpha}\omega_{3L} + 2\sum_i b_i^{\alpha}\omega^i_{3YM}$$

[Romans, Sagnotti, Nishino-Sezgin, Riccioni-Sagnotti, ...]

Integral Lattice Structure

[Kumar, Morrison, Taylor, to appear]

$$a \cdot a = 9 - T$$

$$a \cdot b_i = \frac{1}{6}\lambda_i (A^i_{adj} - \sum_R x^i_R A^i_R)$$

$$b_i \cdot b_i = -\frac{1}{3}\lambda_i^2 (C^i_{adj} - \sum_R x^i_R C^i_R)$$

$$b_i \cdot b_j = \lambda_i \lambda_j \sum_{RS} x^{ij}_{RS} A^i_R A^j_S$$

$A_R,\ B_R,\ C_R$ are group theory coeffs

$$\mathrm{tr}_R F^2 = A_R \mathrm{tr} F^2$$

$$\mathrm{tr}_R F^4 = B_R \mathrm{tr} F^4 + C_R (\mathrm{tr} F^2)^2$$

Inner products $a \cdot a, \ a \cdot b_i, \ b_i \cdot b_j$ quantized in integers!

Can prove using group theory for

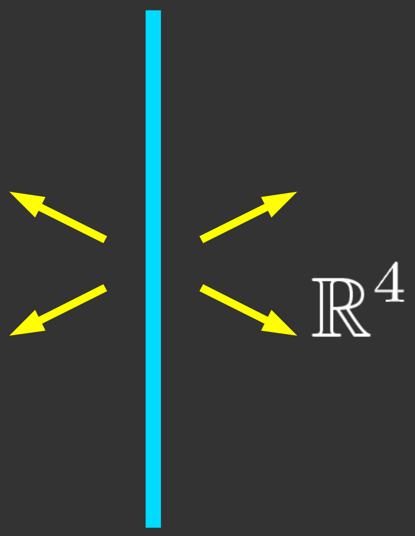
$$SU(N \geq 4), SO(N), Sp(N), E_{6,7,8}, F_4$$

For $SU(2), \ SU(3), \ G_2$ global anomalies when inner products are non-integral.

[Analysis of global anomalies: Bershadsky-Vafa, Suzuki-Tachikawa]

Another argument – BPS strings

For EFTs that come from string theory,



Charge lattice of 2-form fields with inner product

$$e_1 g_2 + e_2 g_1 \in 2\pi\mathbb{Z}$$

BPS strings with charge $b_i^\alpha n_i$ [Seiberg-Witten]

Dirac quantization implies that $b_i \cdot b_j \in \mathbb{Z}$

Summary so far...

- 6D EFTs with anomaly cancellation (local+global).
- Vectors $j^\alpha,\ a^\alpha,\ b_i^\alpha \in \mathbb{R}^{1,T}$ with inner products fixed by matter, gauge group.
- $a,\ b_i$ define an integral lattice.
- Positive kinetic terms $j \cdot b_i > 0\ \forall i$
- Constraints from anomalies – no net $\mathrm{tr} F_i^4$, $H - V + 29T = 273$ etc.

Space of consistent 6D theories

A simple example

$\boxed{SO(N),\ N_f \text{ fund hypers, T=1}}$

WLOG for T=1, $\Omega = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $a = (-2, -2)$

$I_8 = (\mathrm{tr}R^2)^2 + \mathrm{tr}R^2\mathrm{tr}F^2 - 2(\mathrm{tr}F^2)^2$ factorizes

Constraints –
$$\begin{aligned} \mathrm{tr}F^4 &: \quad N_f = N - 8 \\ \mathrm{tr}R^4 &: \quad N \leq 30 \end{aligned}$$

$\boxed{SO(8 \leq N \leq 30),\ N - 8 \text{ fund hypers.}}$

Anomaly cancellation restricts us to a finite set of theories in this class. Is this true more generally?

$\boxed{\text{Answer: Yes, for } T < 9.}$

[Kumar-Taylor, Kumar-Morrison-Taylor]

Statement of finiteness

$\boxed{\text{There are only finitely many combinations of nonabelian gauge group, matter with anomaly cancellation for supergravities with T<9.}}$

Infinite families of $T = 9$ theories, and theories with $T \to \infty$ with anomaly cancellation.

* U(1) case will be considered [Park-Taylor]

Sketch of proof

[Kumar-Morrison-Taylor]

Let $\mathcal{M}$ denote a family of supergravities with anomaly cancellation and distinct gauge groups and matter hypers with fixed T.

$$H - V + 29T = 273 \;\Rightarrow\; V \to \infty$$

1. $G = \prod_{i=1}^{N} G_i, \; |G_i| < M$

2. $G \supset SU(N)$ (or $SO(N), \; Sp(N)$)

$N \to \infty$

Case 1: Bifundamental hypers forced.

$\Rightarrow H \sim \mathcal{O}(N^2)$. Bounded $|G_i| \Rightarrow V \sim \mathcal{O}(N)$.

Contradiction for any T.

Case 2: We find infinite families, e.g.

$$SU(N) \times SU(N),\ (N, \bar{N}) + (\bar{N}, N)$$

For T$<$ 9 in every such case, a gauge kinetic term is negative everywhere in moduli space.

Complete proof is subtle. To appear in [KMT].

Infinite families

If $T \geq 9$ we have infinite families in case 2. Can find $j \in \mathbb{R}^{1,T}$ such that all gauge kinetic terms are positive.

If we allow $T \to \infty$, we have infinite families in case 1, e.g. $E_8^k,\ T = 9 + 8k$

Block Structure of 6D EFTs

Block – gauge group factor + all matter charged under that factor.

T=1 case: b_i determined by i-th block alone. Can combine compatible blocks.

Can enumerate SU(N) blocks with fund, antisym hypers.

[Kumar-Morrison-Taylor]

A systematic enumeration of $SU(N \geq 3)$ multi-block models with fund, antisym, and bifund matter – 68,997 theories.

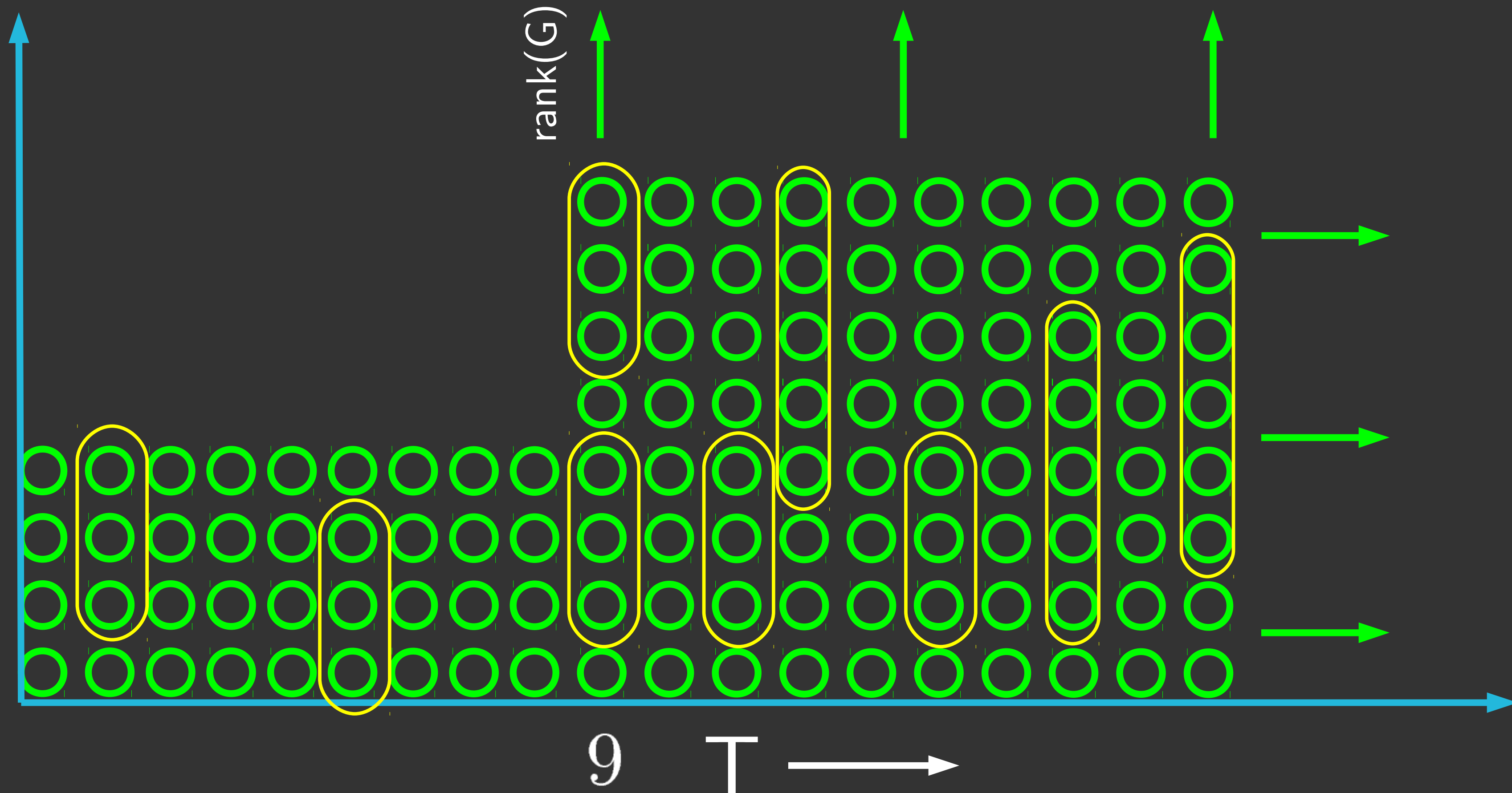
Maximal rank gauge group: $SU(16) \times SU(32)$

Maximal factors: $SU(18) \times SU(3)^{12}$

In the T>1 case, this is a little tricky due to SO(1,T) freedom.

The space of *consistent* 6D supergravities

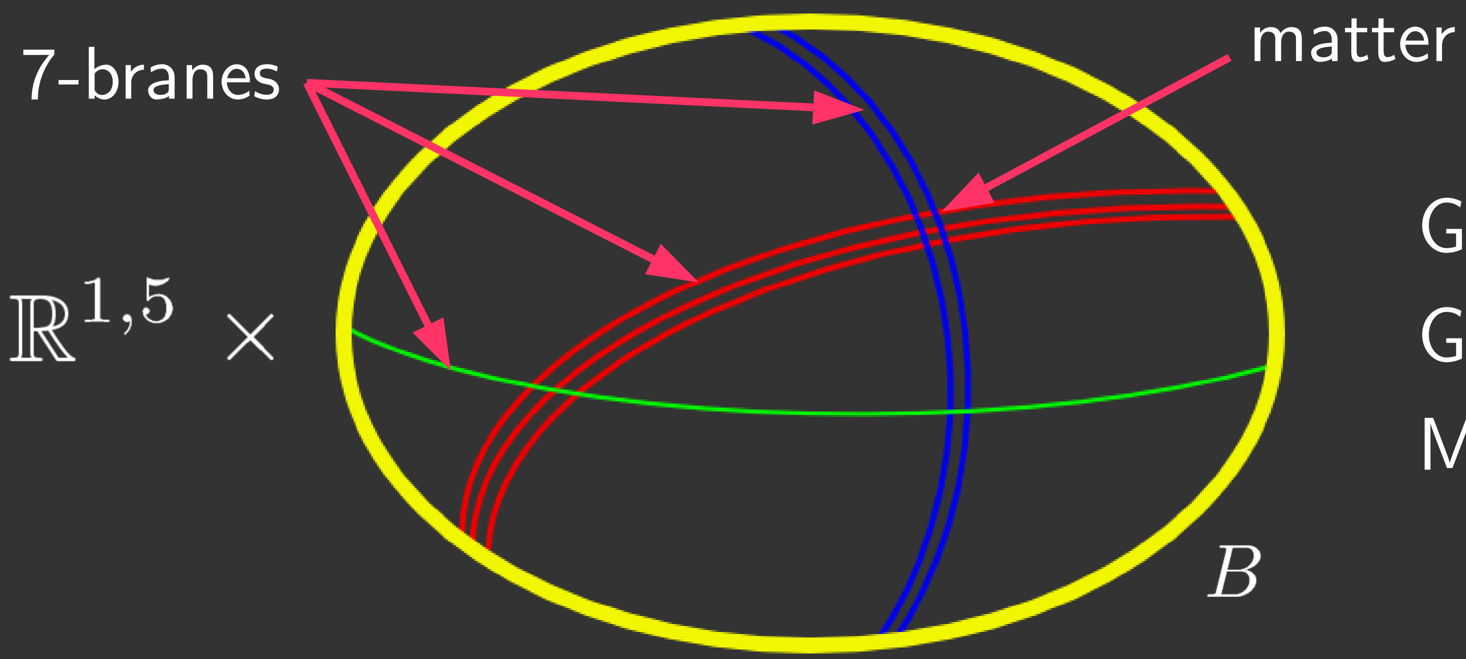
consistent = no local or global anomalies,
positive gauge kinetic terms.



6D F-theory landscape

F-theory

We will consider F-theory on a compact, Calabi-Yau 3-fold elliptically fibered over a complex surface B.



Gravity	10D
Gauge theory	8D
Matter	6D

[Vafa, Morrison-Vafa, Denef's lecture notes]

Gauge group

- Elliptic fiber degenerates on codim 1 loci on B (curves). Tate's algorithm $\Longrightarrow$ gauge grp.

- Kodaira: $$-12K = \sum_i N_i \xi_i + Y$$

- Components ξ_i give nonabelian gauge groups. Y is the residual locus.

[Morrison-Vafa, Bershadsky-etal]

Matter content - Hypers

- Matter localized at intersections of 7-branes.

  Higgsing mnemonic: derive matter reps.

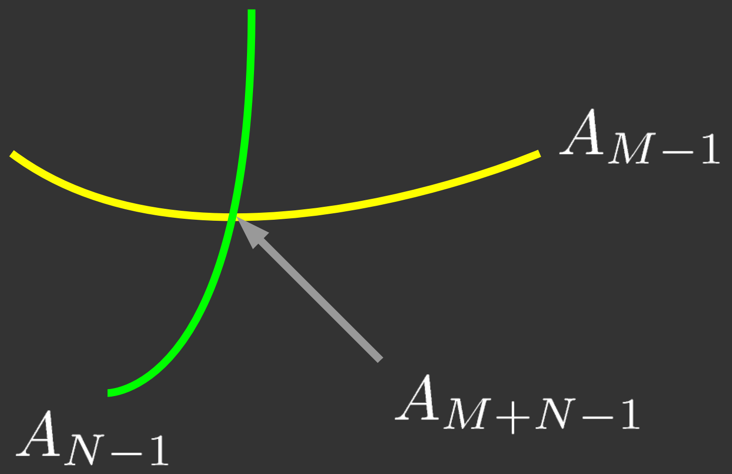
- $\#$ of hypers computed by intersection theory.

- "Non-local" matter – adjoint (genus), symmetric tensor (no. of double points).

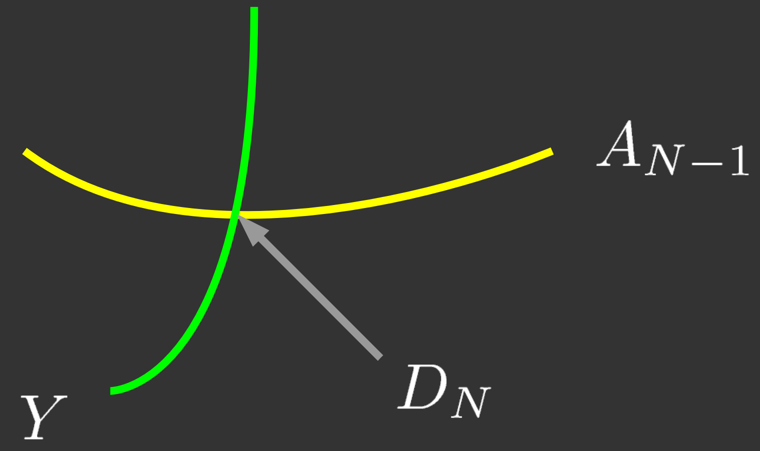
- Complete dictionary unknown.

[Katz-Vafa,Katz-Morrison-Plesser, Sadov, Morrison-Vafa, Bershadsky-etal, Aspinwall-Katz-Morrison, ...]

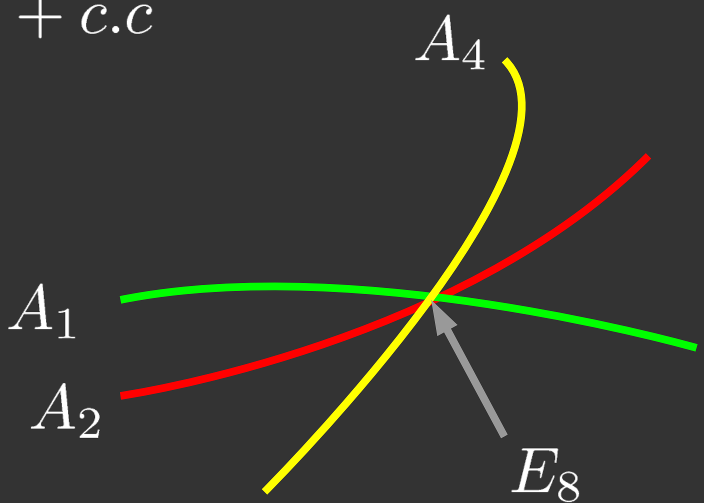
Some examples



$(N, \bar{M}) + c.c$



Antisymmetric + c.c



Tri-fundamental + c.c

[Katz-Vafa, Grassi-Morrison]

Tensor multiplets

$$T = h^{1,1}(B) - 1$$

Tensor multiplet scalars $=$ Kahler moduli

$\boxed{j^\alpha \in \mathbb{R}^{1,T} \text{ in EFT} \Longleftrightarrow \text{Kahler form of B}}$

SUGRA: $SO(1,T)/SO(T)$ structure of MS.

F-theory: $SO(1,T)/SO(T)$ away from Kahler cone walls.

Anomaly data and Divisors

Anomaly conditions define vectors $a^\alpha,\ b_i^\alpha$.

We make the identification

$$
\begin{aligned}
a &\leftrightarrow K_B \\
b_i &\leftrightarrow \xi_i \\
\Omega_{\alpha\beta} x^\alpha y^\beta &\leftrightarrow \#(x, y)
\end{aligned}
$$

[Sadov, Grassi-Morrison]

Low-energy anomaly data

Divisors in F-theory UV completion

Consistency checks

- 6D gauge coupling $\sim j \cdot b_i$. In F-theory, this is the volume of the curve $j \cdot \xi_i$ [Morrison-Vafa]
- # of bifund $= b_i \cdot b_j \leftrightarrow \xi_i \cdot \xi_j$
- For an SU(N) block along ξ_i , for example # antisym $= -a \cdot b_i \leftrightarrow -K_B \cdot \xi_i$, and the number of fund also works out.
- T=1 case: For SU(N) blocks with fund, antisym matter,

$$\boxed{\text{anomaly cancellation} \Longrightarrow \text{Kodaira}}$$

T=1 case: Explicit Map

Possible bases: Hirzebruch surfaces $B = \mathbb{F}_m$

$$H^2(B, \mathbb{Z}) \cong \begin{pmatrix} 0 & 1 \\ 1 & -m \end{pmatrix} \text{ in a basis } \{D_s, D_v\}$$

$$\boxed{b_i = \frac{1}{2}(\alpha, \tilde{\alpha}) \mapsto \frac{\alpha}{2}(D_v + \frac{m}{2} D_s) + \frac{\tilde{\alpha}}{2} D_s}$$

Works for all 68,997 theories built from SU(N) blocks (at the topological level)!

Map also works for SO, exceptional grps.

Weierstrass models?

If we prescribe singularities along divisors from Kodaira's list and satisfying all the topological conditions and the anomaly conditions, is there a Weierstrass model?

Conjecture: Yes.

Some evidence in $T=1$ case: d.o.f count.

Example: SU(N) with fundamentals

$\boxed{\text{T=1,SU(N), 2N fund, } N \leq 15}$

Map: Realize SU(N) on divisor D_v in $\mathbb{F}_2$

We constructed explicit Weierstrass models in this case.

Cmplx Struct. d.o.f fixed $= 243 - N^2$

Neutral hypers in EFT $= 243 - N^2$

D.O.F match nicely! General proof?

Exotic Representations

- Discover new matter reps through 6D anomalies.
- T=1 examples:

  $\boxed{\begin{array}{l}\text{SU(8) with 4-index antisym} \\ \text{SU(2)xSU(3)xSU(6) tri-fundamental}\end{array}}$

- Can you find them in F-theory?

Summary so far...

- Discussed map from EFT $\longrightarrow$ F-theory.
- T$=$1 case: Map specifies the right divisor. Swampland reduced to a puddle!
- Topological data $\longrightarrow$ Weierstrass?
- A simple way to find new singularity structures in F-theory.

Coming up: Does the map fail? Is there a swampland in 6D? How big?

Mapping EFTs to F-theory

A necessary condition for the existence of a UV-completion in F-theory is the existence of a lattice embedding

$$\Lambda \hookrightarrow H^2(B, \mathbb{Z})$$

Lattice defined by $a, \ b_i$

(1,T) Unimodular lattice

Can one see the unimodular lattice from the low-energy theory?

Some interesting examples

- No lattice embedding

$\boxed{\text{SU(N), 1 adj + 10 anti + 40 fund, T=1}}$

$$\Lambda = \begin{pmatrix} 8 & 10 \\ 10 & 10 \end{pmatrix}$$

Cannot be embedded into either of the two possible (1,1) unimodular lattices.

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

EFT with no conventional F-theory realization!

Some interesting examples

- Divisor not effective

$$\boxed{\text{SU(N), 1 sym + (N-8) fund, T=1}}$$

$$\Lambda = \begin{pmatrix} 8 & -1 \\ -1 & -1 \end{pmatrix}$$

There is an embedding into $\mathbb{F}_{odd}$ but the divisor is not effective ($-D_v$ on $\mathbb{F}_1$).
(Higgs to SO(N)+(N-8) fund $\hookrightarrow$ heterotic)

Some interesting examples

- Infinite families
  SU(N) x SU(N), 2 bifund, T=9 family violates the Kodaira condition.
  (N=8 case realized by Dabholkar-Park.)

- E_8^k case is more subtle, but also ruled out.
  As $T \to \infty$ we need to blow-up lots of points and we only have finitely many dof.
  k=2 case from heterotic M-theory.

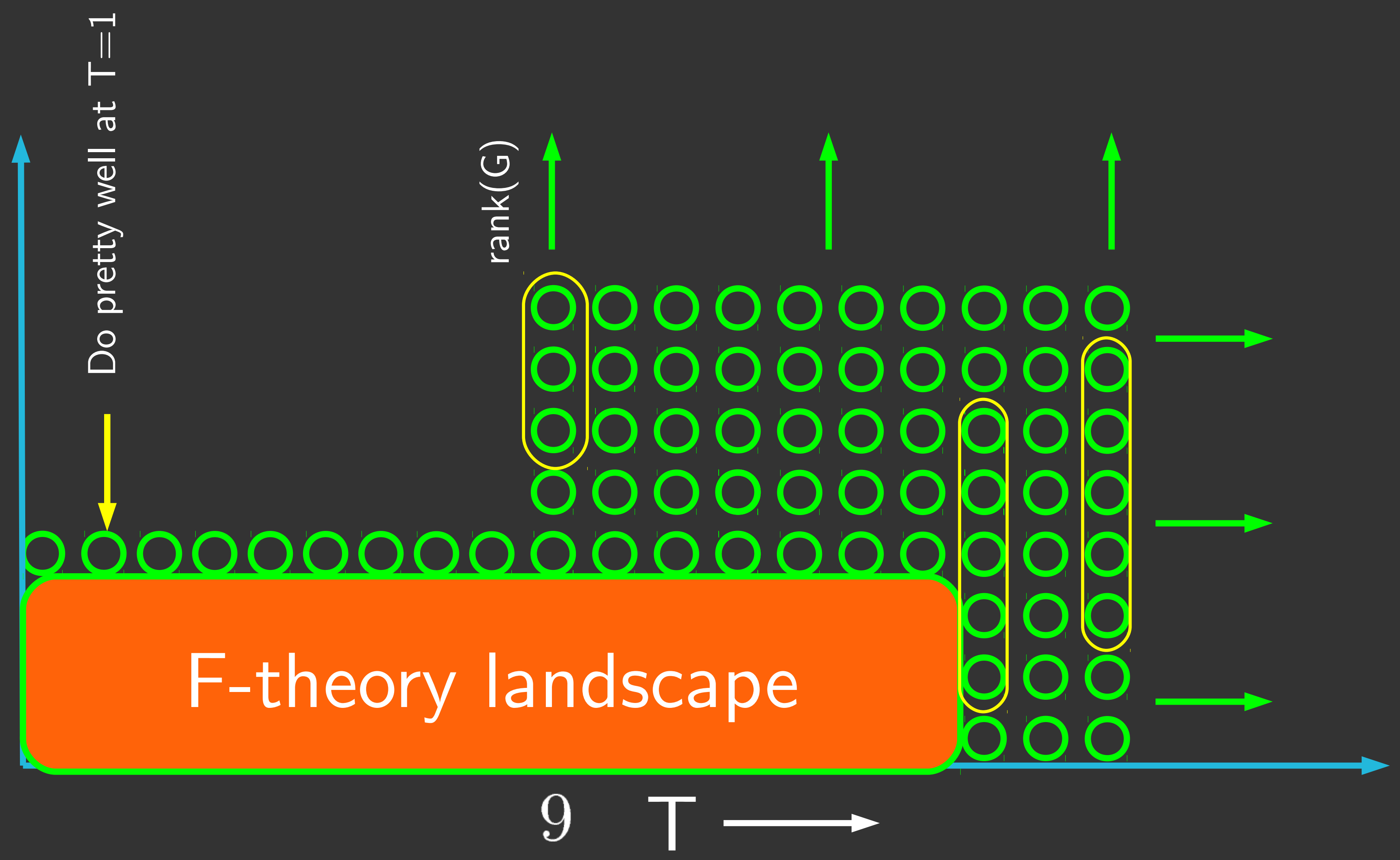
Exotic transitions

Can trade 29 hypers for one tensor multiplet.
Consistent with $H - V + 29T = 273$

Tune coeffs in Weierstrass to non-minimal type (29 dof), then blow-up the base (extra tensor), then resolve to get smooth model.
Related to small E8 instantons. Connect all T$=$1, perturbative heterotic vacua.

[Duff-Minasian-Witten, Seiberg-Witten, Morrison-Vafa, ...]

Ordinary higgsing and these exotic transitions connect F-theory vacua.



Conclusions/Outlook

- Anomaly cancellation in 6D drastically cuts down the space of theories. Yet, there is an infinite swampland, and a (no proof) finite landscape.
- Every 6D sugra has an associated lattice. Unimodular lattice condition from the low-energy theory?

- T=1 case: Map from low-energy anomaly data $\Longrightarrow$ divisor data describing an F-theory compactification. Swampland tiny.

- Exotic matter reps from anomalies. Is there a corresponding F-theory singularity? Map would be useful.

- Understand transitions to piece together the 6D F-theory landscape [in progress].

- We have examples of C\L. 3 options
  1. Discover new low-energy consistency conditions
  2. Expand known string constructions
  3. Sickness cannot be seen from the low-energy theory

Thanks for your attention, and to the organizers for a wonderful workshop.

Thank You