Differential Rotation in the Sun

(Modeling with the ASH Code)

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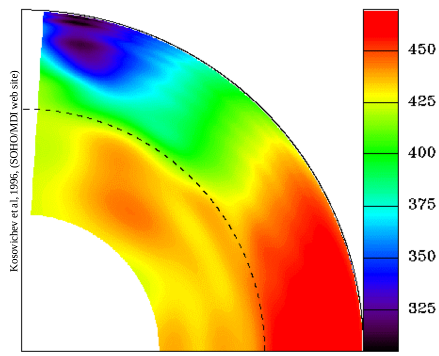
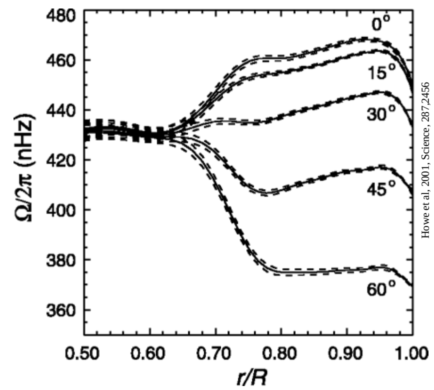
Outline

- The Solar Rotation
- Modeling with ASH
- The Deep Convection Zone
- The Upper Shear Layer
- The Tachocline Rotation

- (MASH)
- (CASH)
- (SLASH)

The Solar Rotation





- Latitudinal shear in the envelope but little in the interior
- Vertical shear near the top and bottom of the convection zone
- Angular velocity increasing outward (with slow poles!)
- Smooth and Steady

Where does the Differential Rotation come from?

Assume Lorentz forces and viscous dissipation are negligible:

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \mathbf{v}) = 0$$

$$\rho \frac{\partial \mathbf{v}}{\partial t} = -\rho\,(\mathbf{v} \cdot \boldsymbol{\nabla})\,\mathbf{v} - \boldsymbol{\nabla} P + \rho \boldsymbol{\nabla} \Phi + 2\rho \mathbf{v} \times \boldsymbol{\Omega}$$

Average the zonal component over longitude and time

(Assume a statistically steady state)

$$\mathcal{L} = r \sin\theta\,(\Omega r \sin\theta + \langle v_\phi \rangle)$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{F} = 0$$

$$F_r = \langle \rho v_r \rangle\, \mathcal{L} \quad + \quad r \sin\theta \left\langle\, (\rho v_r - \langle \rho v_r \rangle)\,(v_\phi - \langle v_\phi \rangle)\, \right\rangle$$

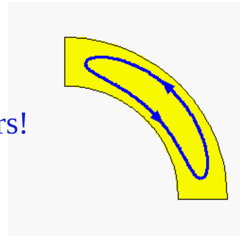
$$F_\theta = \langle \rho v_\theta \rangle\, \mathcal{L} \quad + \quad r \sin\theta \left\langle\, (\rho v_\theta - \langle \rho v_\theta \rangle)\,(v_\phi - \langle v_\phi \rangle)\, \right\rangle$$

Where does the Differential Rotation come from?

- Reynolds stresses vs Meridional Circulation
- Meridional Circulation contribution can also be written as:

$$\nabla \cdot (\langle \rho \mathbf{v}_M \rangle \mathcal{L}) = \langle \rho \mathbf{v}_M \rangle \cdot \nabla \mathcal{L}$$

Streamlines = angular momentum contours!
Not like the Sun!



- Reynolds stresses (no mystery here!)

Rotation induces systematic velocity correlations in the convection!

What Else Influences the Rotation Profile?

$$\rho \frac{\partial \mathbf{v}}{\partial t} = -\rho (\mathbf{v} \cdot \nabla) \mathbf{v} - \nabla P + \rho \nabla \Phi + 2 \rho \mathbf{v} \times \mathbf{\Omega}$$

Take the curl, average over longitude and time (assume steady state)

$$\nabla \times \langle \mathbf{v} \times (2\mathbf{\Omega} + \boldsymbol{\omega}) \rangle = \left\langle \frac{\nabla \rho \times \nabla P}{\rho^2} \right\rangle$$

Now make the following approximations:

$$R_o = \frac{\omega_{rms}}{2\Omega} \ll 1 \qquad S = C_P \ln \left(\frac{P^{1/\gamma}}{\rho} \right) \qquad \nabla P \approx -\rho g \hat{\mathbf{r}}$$

And you come up with:
Thermal Wind

$$\mathbf{\Omega} \cdot \nabla \langle v_\phi \rangle = \frac{g}{2 r C_P} \left\langle \frac{\partial S}{\partial \theta} \right\rangle$$

Modeling Strategy = Brute Force!

- 3D, Nonlinear, Anelastic fluid equations
  + biggest computers we can find
  = high resolution, low dissipation = turbulence!
- Shave off granulation layer and deep interior for practical reasons
- Investigate turbulent transport
  - Reynolds Stresses
  - Heat Flux

The Anelastic Spherical Harmonic Code

- Anelastic Approximation:

$$\nabla\cdot(\hat{\rho}\mathbf{v}) = 0$$

$$\hat{\rho}\frac{D\mathbf{v}}{Dt} = -\nabla P + \rho\mathbf{g} + 2\hat{\rho}(\mathbf{v}\times\Omega) - \nabla\cdot\mathcal{D} - \left[\nabla\hat{P} - \hat{\rho}\mathbf{g}\right]$$

$$\hat{\rho}\hat{T}\frac{D}{Dt}\left(\hat{S}+S\right) = \nabla\cdot\left[\kappa\hat{\rho}\hat{T}\nabla(\hat{S}+S) + \kappa_r\hat{\rho}C_P\nabla(\hat{T}+T)\right] + \Psi$$

- Pseudospectral: spherical harmonics and stacked Chebyshevs (or compact FD)
- Poloidal/Toroidal: $\hat{\rho}\mathbf{v} = \nabla\times\nabla\times(W\hat{\mathbf{r}}) + \nabla\times(Z\hat{\mathbf{r}})$
- Adams-Bashforth/Crank-Nicholson
- FORTRAN 90/MPI

Deep (Shell) Questions

- Can we reproduce the mean flows inferred from helioseismology?
- What should we expect the fluctuating flows to be like?
- What structures dominate the transport?
- How long do they live?
- Can we detect them?
- How important are the boundary layers?
- How are they influenced by rotation, stratification, magnetic field, ionization, etc
- How can all this mess produce a cyclic, large-scale magnetic field?

Differential Rotation

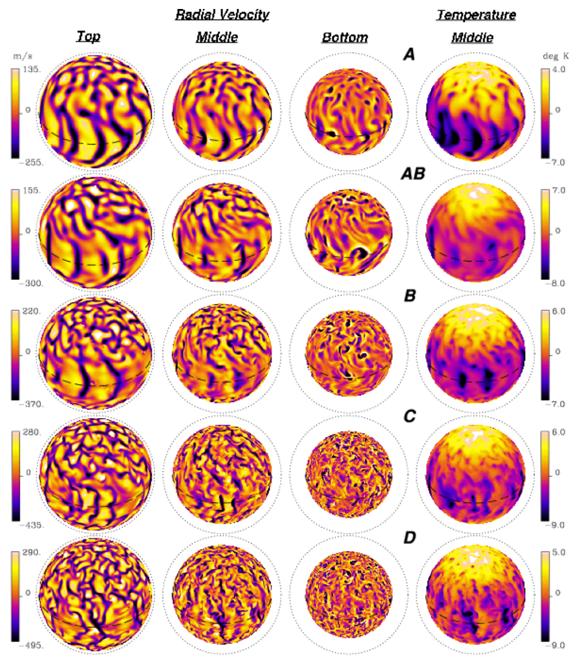
Three Challenges from helioseismology

- Nearly radial angular velocity contours at mid-latitudes (not cylindrical)
- Monotonic decrease in angular velocity from equator to pole (no polar spin-up)
- 30% contrast from equator-to-pole

Rogue's Gallery

(Brun & Toomre 2002)

- Complexity increases as the viscosity and diffusivity are decreased
- Flows evolve on timescales of days and weeks
- Patterns propagate and are advected by the differential rotation
- The more turbulent simulations are dominated by intermittent downflow plumes and lanes



Brun & Toomre (2002)

Zonal velocity	Cylindrical Gradient	Thermal Wind Component	Residual

Case AB

$R_a = 3.4\times10^4$

$R_e = 85$

$P_e = 21$

$R_o = 0.16$

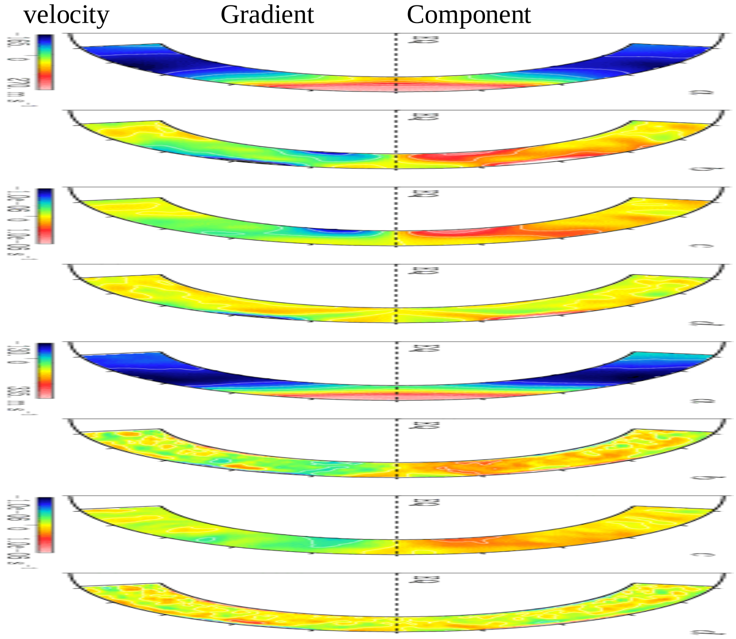
Case D

$R_a = 6.5\times10^5$

$R_e = 410$

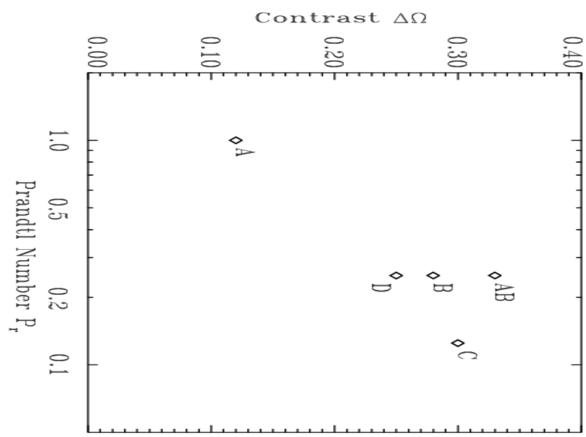
$P_e = 103$

$R_o = 0.16$



Keep the thermal diffusivity constant as you decrease the viscosity or you'll lose your differential rotation!

(Brun & Toomre 2002)



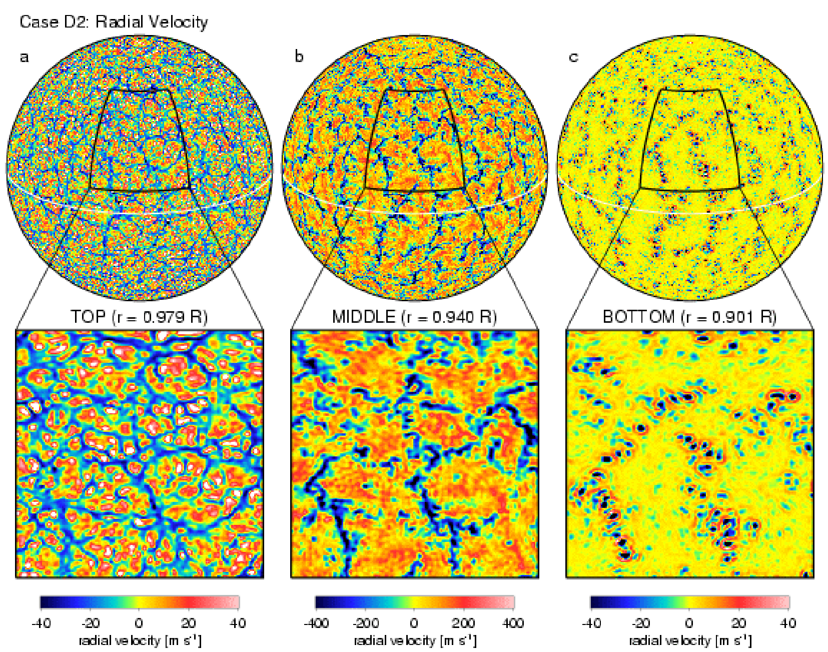
Summary of Deep Shell Results

- Approaching consistency with helioseismic data: definite improvement over the pioneering (laminar) simulations of Gilman and Glatzmaier
- The most turbulent cases generally don't give the best agreement with helioseismic inversions
- Thermal wind (dS/dtheta) important but not the whole story
- Flows are dominated by strong downflow lanes and plumes which exhibit substantial variation on timescales of weeks and even days
- Still not in the low-dissipation limit: results are sensitive to Reynolds and Prandtl numbers

The Upper Shear Layer

- Why does the radial angular velocity gradient become negative?
- What happens with the meridional circulation?
- What role do supergranules play?
- What other scales of motion are present?
- How do the convective patterns evolve over time and how might they be detected?
- How does this layer couple to the deep convection zone?
- Is this where poloidal field regeneration occurs? (the “alpha-effect”)

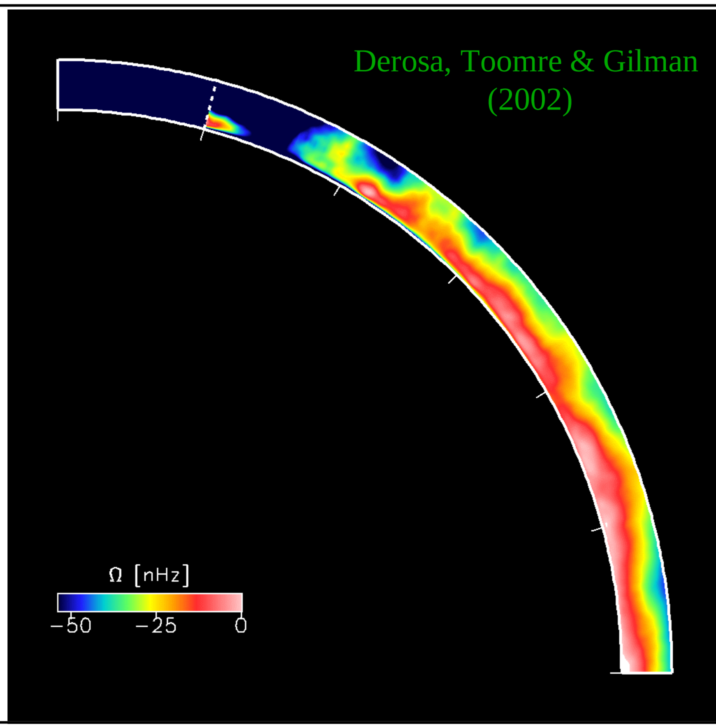
Derosa, Toomre & Gilman (2002)



DeRosa, Toomre,
& Gilman
(2002)

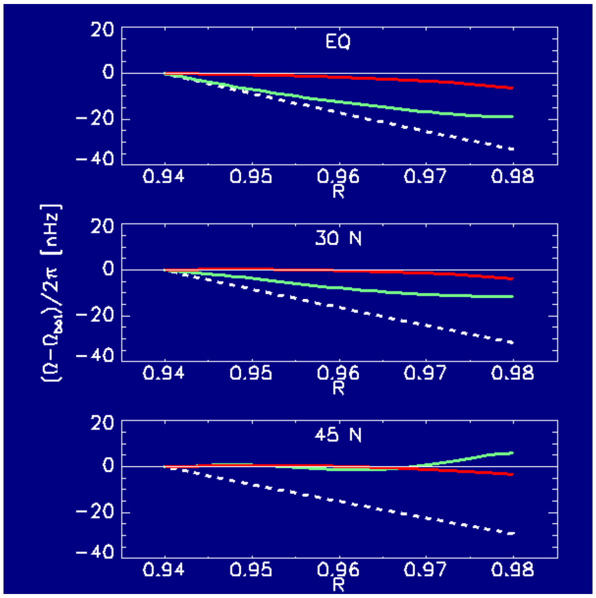
QuickTime™ and a
Video decompressor
are needed to see this picture.

Solar-like
differential
rotation
imposed
on the
inner
boundary
with a
stress-free
top



Radial Angular Velocity Profiles

DeRosa, Toomre & Gilman (2002)



Red: uniformly rotating Inner boundary

Blue: Differentially Rotating Inner boundary

➔ Negative radial gradient but smaller than what you'd expect from angular momentum conservation

Summary of results from the upper shear layer

- First global simulations to resolve super-granular scale motions
- Larger-scale (100-200 Mm) cells also present which advect and distort "supergranules"
- Flow structure dominated at the top by a rapidly evolving network of downflow lanes and at greater depths by intermittent plumes
- Negative radial angular velocity gradients maintained through an inward angular momentum flux by Reynolds stresses

Tachocline Questions

- Why is it so thin?
- Is turbulence generated by either shear instabilities or penetrative convection?
- If so, how does this turbulence feed back on the mean rotation profile?
- What is the dynamical importance of the magnetic field?
- Can we account for the inferred temporal variations?
- How does the tachocline couple to the convection zone?
- What role does it play in the solar dynamo?

The Solar Tachocline

- Stably stratified, rapidly rotating
  - Rossby modes (vertical vorticity)
  - Gravity modes (horizontal divergence)
- Differential rotation is maintained primarily by stresses from the overlying convective envelope.
- Why doesn’t the differential rotation spread to the interior?
  - Does turbulence in the tachocline wipe out the latitudinal gradient?

YES - Spiegel & Zahn 1992

NO - Gough & McIntyre 1998
Kitchatinov & Rudiger 1996

ASH Tachocline Model (Boussinesq, Thin-Shell)

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\; v) + \frac{1}{\sin\theta}\frac{\partial u}{\partial\phi} + \frac{\partial w}{\partial z} = 0$$

$$\frac{D\zeta}{Dt} = \left(\zeta + \frac{\cos\theta}{R_o}\right)\frac{\partial w}{\partial z} + \frac{\sin\theta}{R_o}v - \frac{\partial u}{\partial z}\frac{\partial w}{\partial\theta} + \frac{\partial v}{\partial z}\left(\frac{1}{\sin\theta}\frac{\partial w}{\partial\phi}\right) + \mathcal{F}^\zeta + \frac{1}{R_e}\nabla^2\zeta$$

$$\frac{\partial\Delta}{\partial t} = -\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\;\mathbf{v}\cdot\boldsymbol{\nabla}v - u^2\cos\theta\right] - \frac{1}{\sin\theta}\frac{\partial}{\partial\phi}\left[\mathbf{v}\cdot\boldsymbol{\nabla}u + uv\cot\theta\right] - \nabla_H^2 P + \frac{1}{R_o}(\zeta\cos\theta - u\sin\theta) + \mathcal{F}^\Delta + \frac{1}{R_e}\nabla^2\Delta$$

$$\delta\frac{Dw}{Dt} - \frac{u^2+v^2}{1+\delta z} = -\frac{1}{\delta}\frac{\partial P}{\partial z} + \frac{1}{\delta}\frac{T}{F_r^2} + \frac{1}{R_o}u\sin\theta + \frac{\delta}{R_e}\nabla^2 w$$

$$\frac{DT}{Dt} + w = \frac{1}{\sigma R_e}\nabla^2 T$$

Decaying Turbulence

What happens if we put in a spectrum of random velocity fluctuations and let it go?

Consider both vortex modes (Rossby waves) and horizontally divergent modes (gravity waves)

Vertical Vorticity

Unforced, random vortex initial conditions

Non-Rotating

QuickTime™ and a
GIF decompressor
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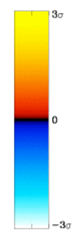

Vertical Vorticity

Unforced, random vortex initial conditions

Rapidly Rotating (R_o = 0.1)

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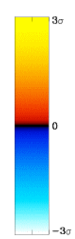

Horizontal Divergence

Unforced, random wave initial conditions

Rapidly Rotating (R_o = 0.1)

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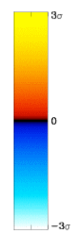

Randomly-Forced Simulations

What happens when you stir things up with random, high-wavenumber external forcing?

(intended to represent penetrative convection)

Consider forcing either the Rossby wave or the gravity wave component of the flow

Vertical Vorticity

Random vortex forcing l=10-12

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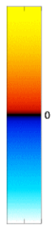

Vertical Vorticity

Random vortex forcing l=30-35

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Video decompressor
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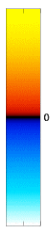
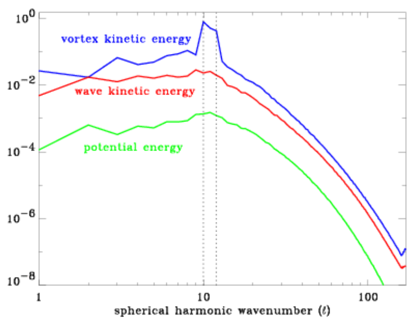


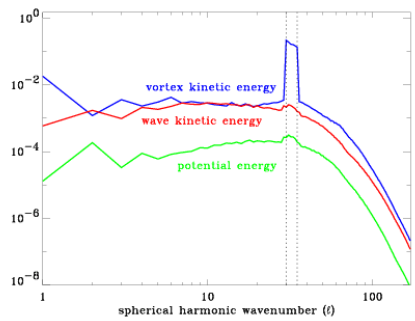
Average Energy Spectra

Vortex Forcing

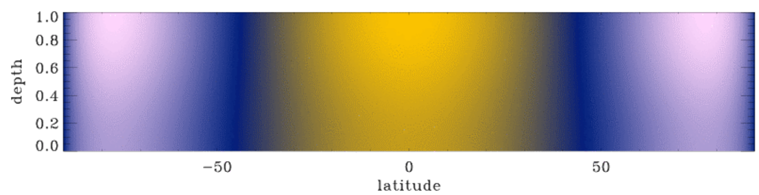
l=10-12



l=30-35

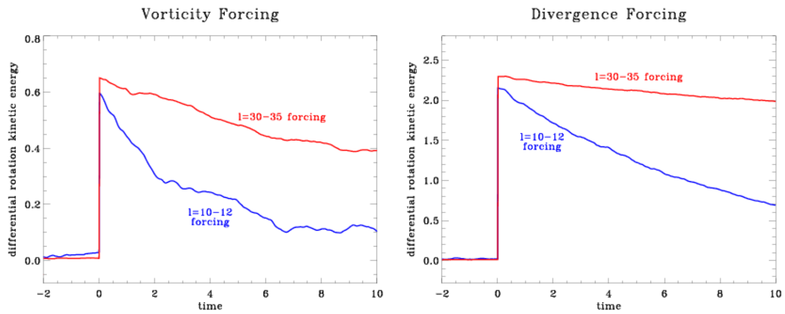


How would this turbulence interact with a background shear flow?



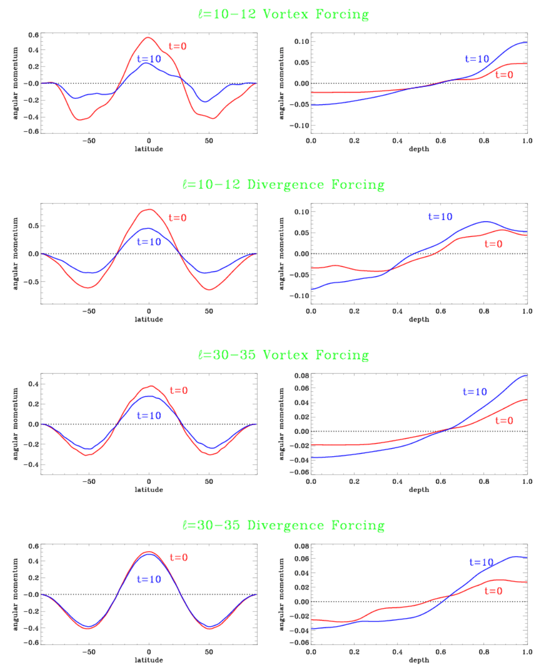
- Continue the randomly-forced simulations but now introduce a zonal shear flow
- Maintain this shear flow against viscous dissipation by also introducing a steady, axisymmetric forcing term to the vertical vorticity equation
- The imposed differential rotation is primarily latitudinal but the vertical shear is actually a bit larger due to the thin-shell geometry
- Shear flow kinetic energy comparable to turbulent kinetic energy
- Initially in hydrostatic and geostrophic balance (thermal wind)

Evolution of Differential Rotation Kinetic Energy



- ➔ Differential rotation is reduced by the turbulence
- ➔ Reduction is most efficient for the larger-scale forcing

Evolution of Angular Momentum Profiles



Summary of Tachocline Results

- Strong coupling between Rossby and gravity wave components when the rotation is strong with equatorward-propagating wave modes
- Nonlinear interactions exhibit both upscale and downscale transfer and the upscale transfer is most efficient when the rotation and stratification are strong
- Randomly forced simulations with imposed shear produce angular momentum transport which is:

Down-gradient (diffusive) in latitude and
Counter-gradient (antidiffusive) in radius

Conclusion

- Where do we stand?
  - Simulations are beginning to look more realistic
  - Helioseismic comparisons are promising but questions remain
  - Tachocline simulations are still in preliminary stages
- Where do we go from here?
  - Still searching for more highly turbulent cases which produce mean flows like the Sun
  - Coupling between the bulk of the convection zone, the upper shear layer, and the tachocline requires much more investigation
  - What role does each play in the solar dynamo?
    - MHD shear instabilities in the tachocline