Modeling Turbulent (Thermonuclear) Combustion

Wolfgang Hillebrandt
MPI für Astrophysik
Garching

KITP, UCSB,
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MAX-PLANCK-GESELLSCHAFT


MPI für Astrophysik
Simulation: W. Hillebrandt, F. Röpke
Visualisierung: R. Bruckschen
MAX-PLANCK-GESELLSCHAFT
Time(sec): 0.00 Size(km): 2029.9

The history of SuCCESs

(Supernova Combustion Code for Explosion Simulations)

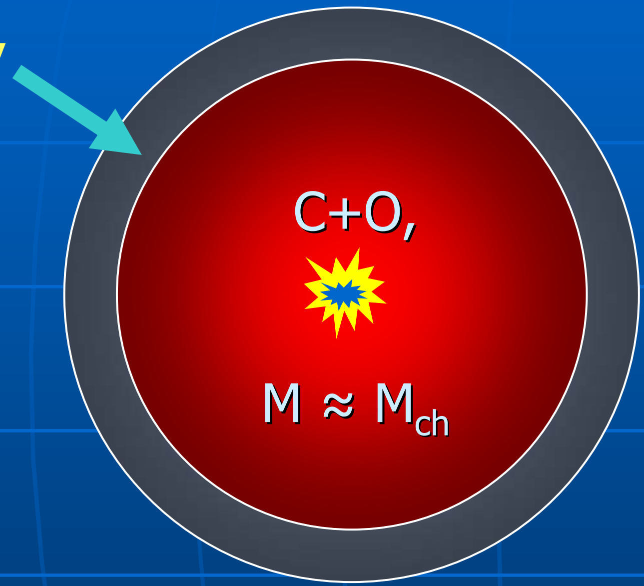
- *Jens Niemeyer (1994 -)*
- *Martin Reinecke (1996 - 2002)*
- *Wolfram Schmidt (2001 -)*
- *Fritz Röpke (2001 -)*
- *Michael Fink (2006 -)*

The “standard model”

- White dwarf in a binary system
- Growing to the Chandrasekhar mass by mass transfer

How does the model work?

He (+H) from binary companion



Density ~ 10^9 - 10^{10} g/cm

Temperature: a few 10^9 K

Radii: a few 1000 km

Explosion energy:

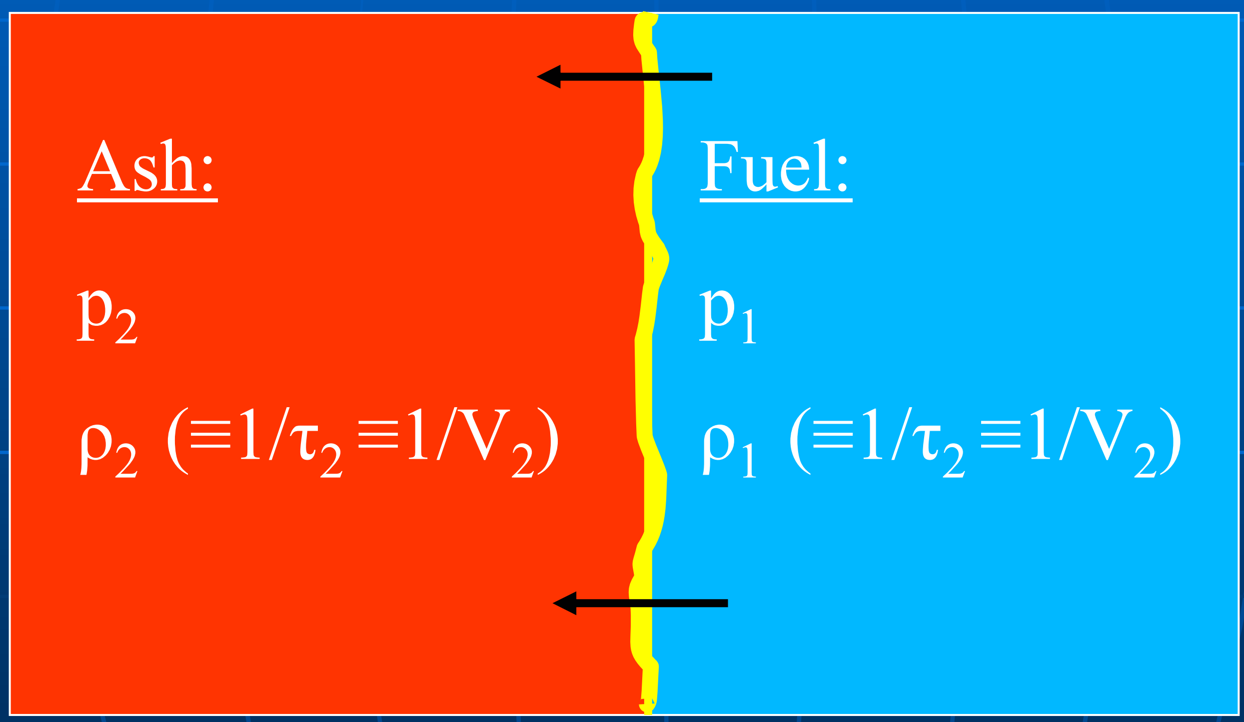
Fusion C+C, C+O, O+O → "Fe"

Laminar burning velocity:

$U_L \sim 100$ km/s $<< U_S$

Too little is burned!

Some fundamentals of combustion theory



The "Hugoniot-function" for the burned gas,

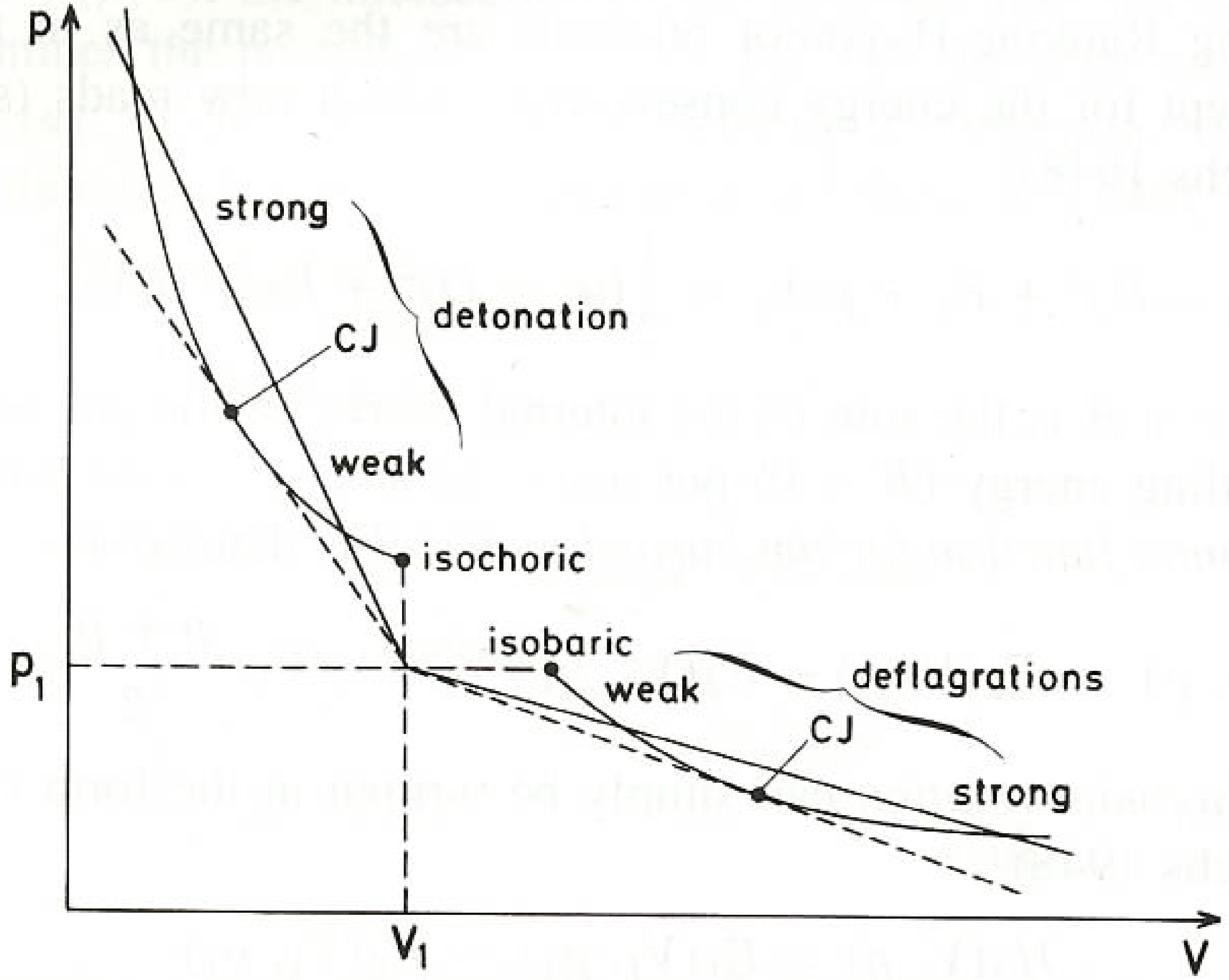
$$H_2(\tau,p) \equiv E_2(\tau,p) - E_2(\tau_1,p_1) + (\tau - \tau_1)(p + p_1)/2$$

and the "Rayleigh-condition"

$$v_B^2 = -(p_2 - p_1)/(\tau_2 - \tau_1); \quad p_2 - p_1 < \tau_2 - \tau_1$$

($\tau = 1/\rho$, "1" = unburned state, "2" = burned state)

("Jump conditions" from conservation laws; analogous to shock waves)



Observed in “real” combustion experiments:

Only weak deflagrations and Chapman-Jouguet detonations!

What is the mode of nuclear burning in SNe Ia?

- “Detonation”:

  (Super-) Sonic front;

  heating to ignition by a shock wave.

- “Deflagration”:

  Subsonic front;

  heating to ignition by heat diffusion.

Strong Si-lines at maximum light:

Pure detonations are excluded (Arnett, 1969)!

(But possibly at lower densities: DDT ???)

The physics of turbulent combustion

- Everydays experience: *Turbulence increases the burning velocity.*
- In a star: Reynoldsnumber ~ 10^{14} !
- In the limit of strong turbulence: $U_B \sim V_T$!
- Physics of thermonuclear burning is very similar to premixed chemical flames.

A couple of definitions:

Kolmogorov (length) scale

$\eta := (\nu^3/\varepsilon)^{1/4}$

(Turbulent) Reynolds number

$Re := v'/s_L \cdot l/l_F$

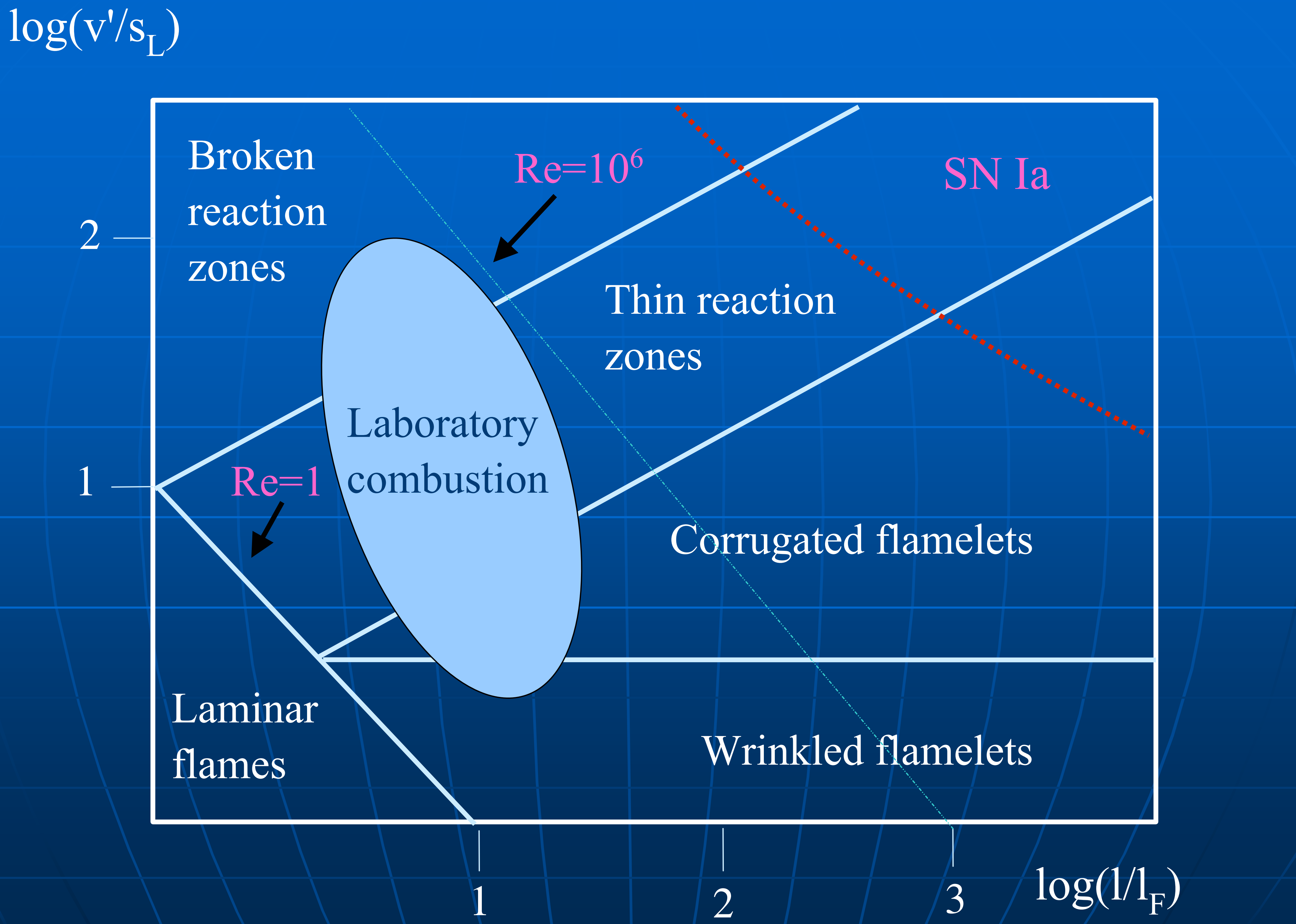
(Turbulent) Damköhler number

$Da := s_L/v' \cdot l/l_F$

(Turbulent) Karlovitz number

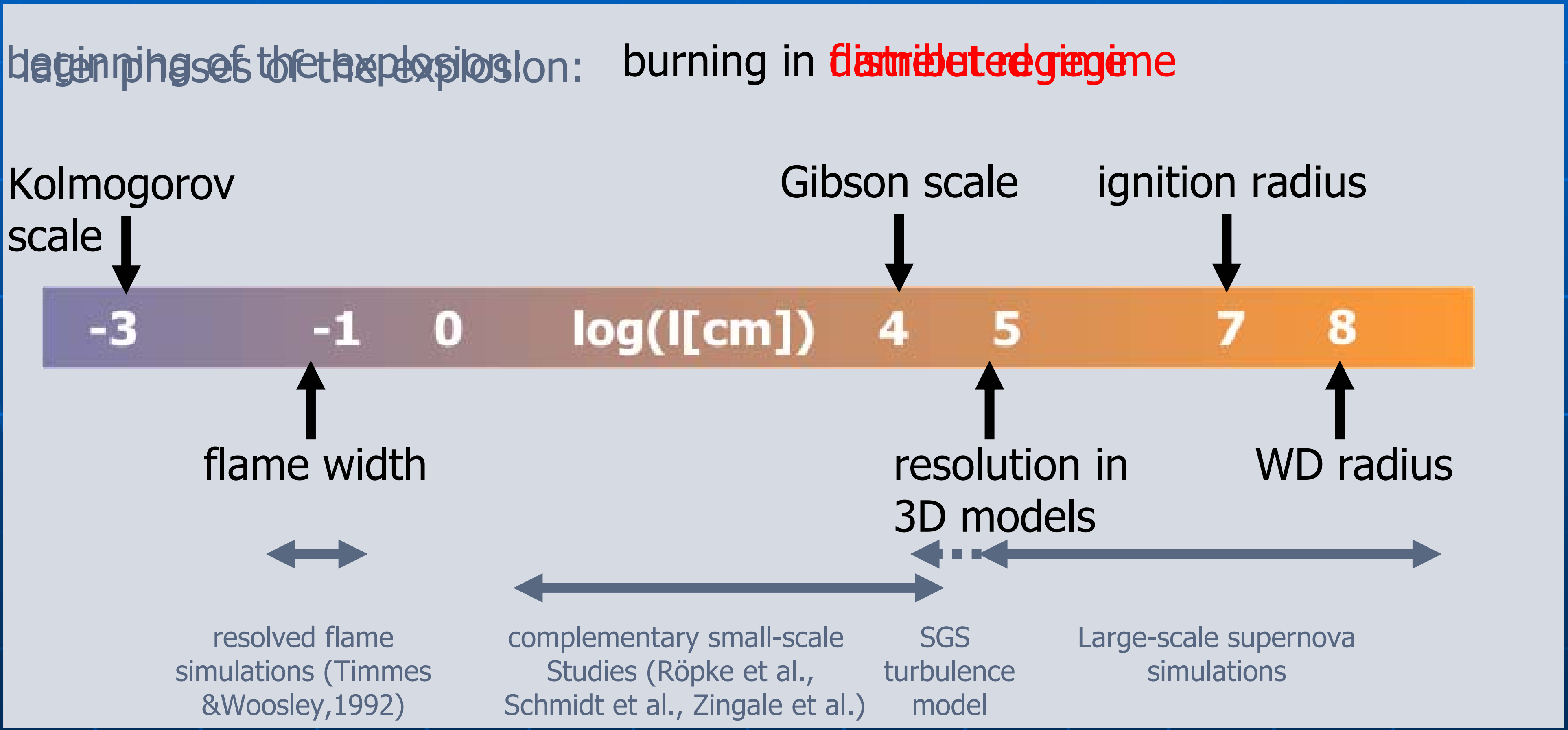
$Ka := l_F^2/\eta^2$

$\Rightarrow \quad Re = Da^2 \cdot Ka^2$


log(v'/s_L)
2
1
Broken
reaction
zones
Re=10^6
SN Ia
Thin reaction
zones
Laboratory
combustion
Re=1
Corrugated flamelets
Laminar
flames
Wrinkled flamelets
1
2
3
log(l/l_F)

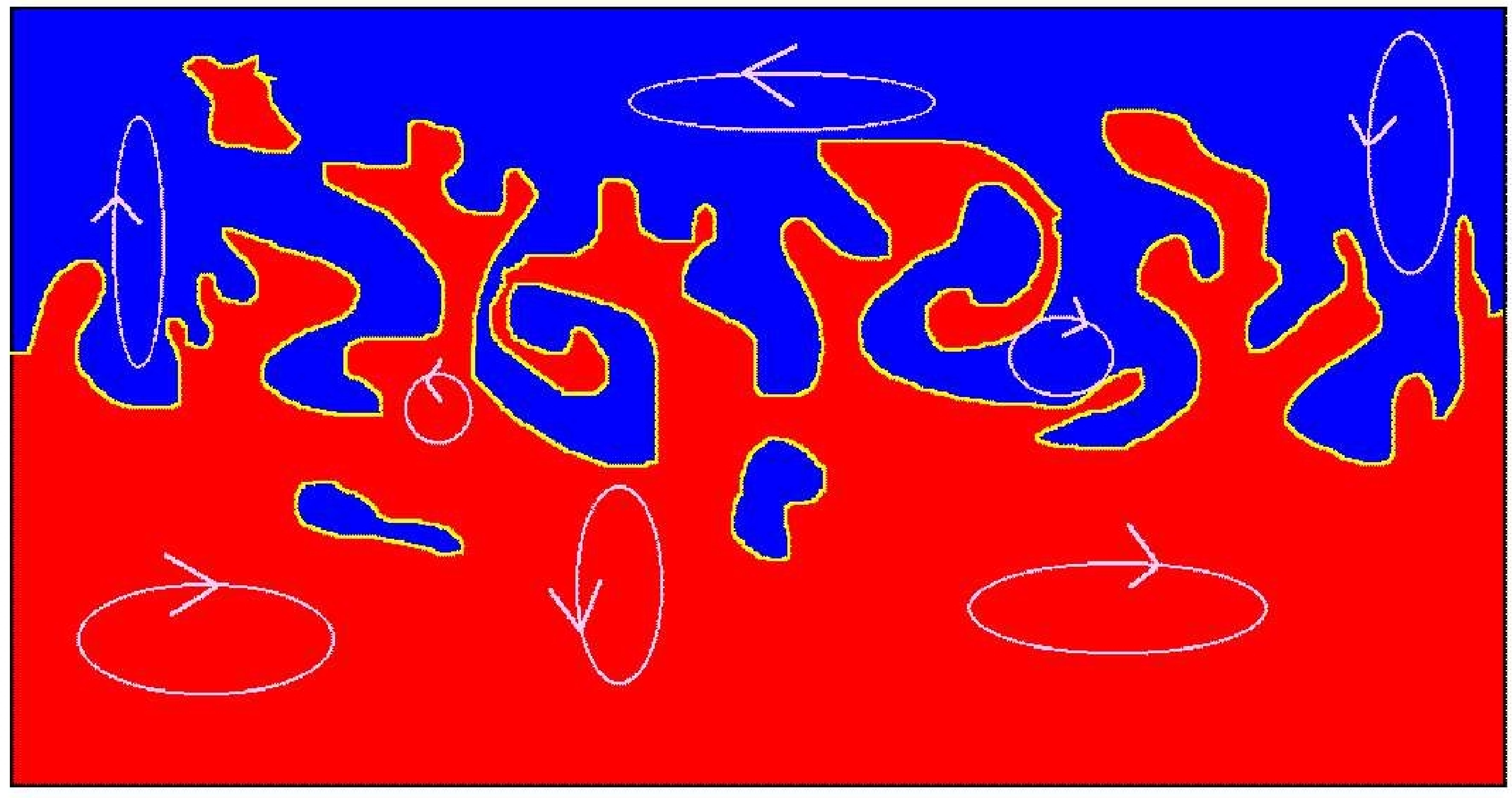
Simulating the relevant scales

- Gibson scale $s_L = v'$: below turbulence does not affect flame propagation



Burning regimes of pre-mixed flames

1. Cellular burning, wrinkled flamelets



Burning regimes of pre-mixed flames

1. Cellular burning, wrinkled flamelets

$u_{cell} = s_L\,[1+\varepsilon(\mu)]\ ;\ \mu = \rho_b/\rho_u\ ,$

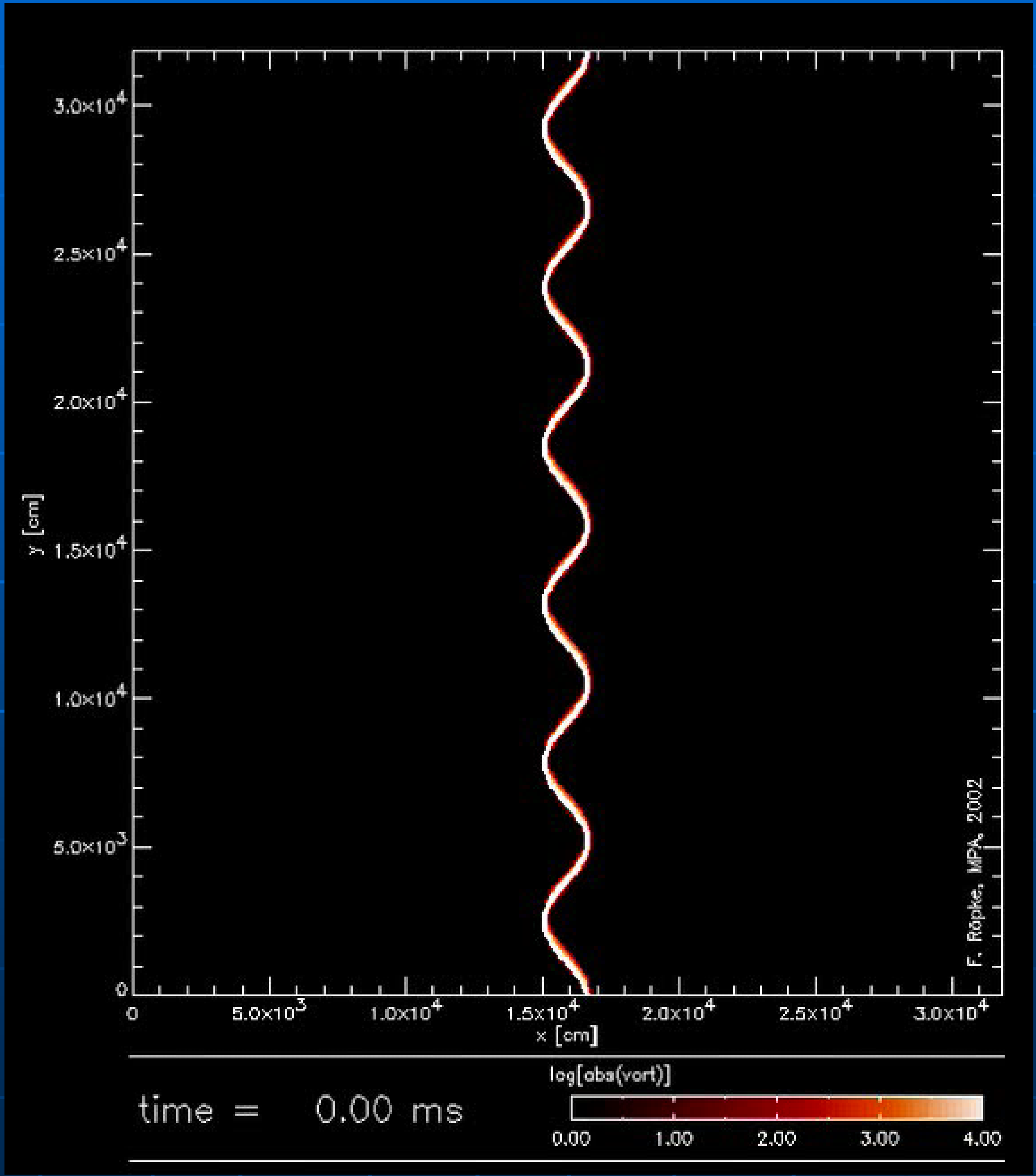
$\varepsilon(\mu) \approx 0.41\,(1 - \mu)^2$

Or: "Fractal model"

$u_{cell}(l) = s_L\,(l/l_{crit})^{D - 1}$

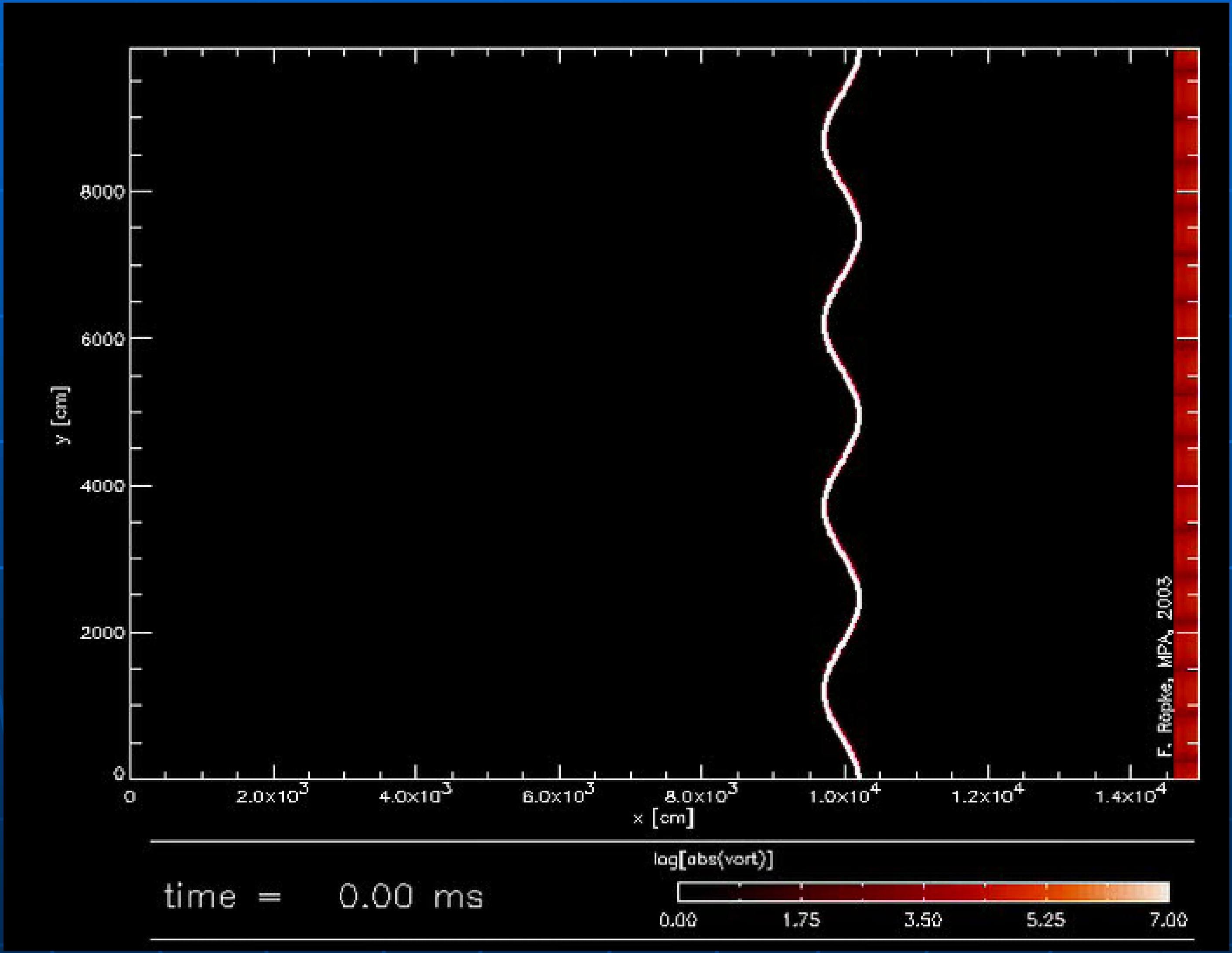
The Landau-Darrieus instability and its interaction with turbulence:

Quiescent fuel



(Röpke et al., 2003a)

The Landau-Darrieus instability and its interaction with turbulence:



Strong vortical flow

(Röpke et al., 2003b)

Burning regimes of pre-mixed flames

2. The corrugated flamelet regime

Transition at the "Gibson scale":

$v(l_{Gibs}) = u_{cell}(l_{Gibs})$

In the limit of strong turbulence:

$s_{turb}\,(l) \approx v'(l),\; l > l_{Gibs}$ (independent of s_L!!!)

$d_{turb} \approx l$ ("turbulent flame brush")

Fully developed turbulence?

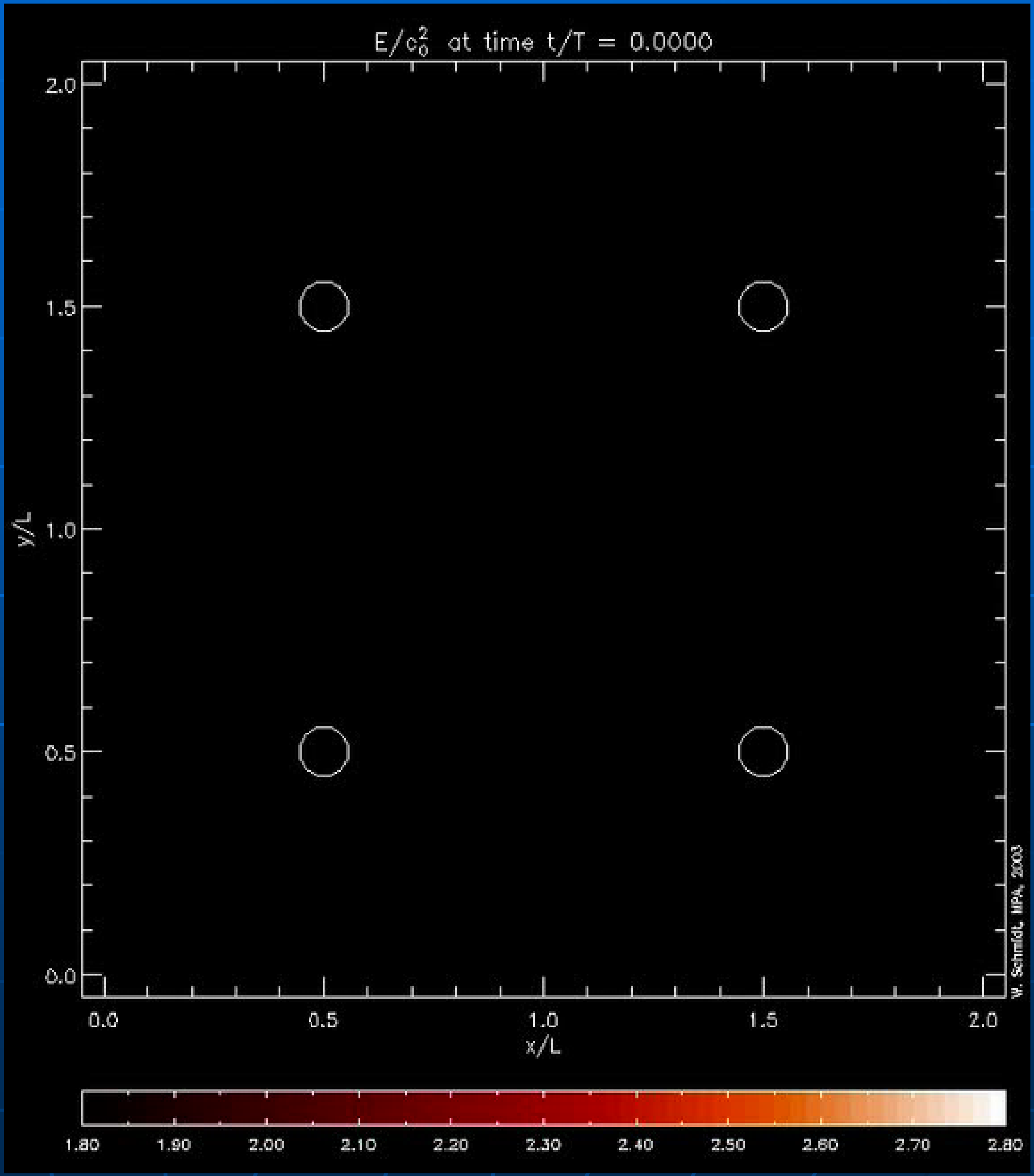
3-D "direct" numerical simulations of flames moving in white dwarf matter:
Energy

$\rho = 2.9 \cdot 10^9\ gcm^{-3}$

$V/s_{lam} = 4$

$V/c_0 = 0.043$

(Schmidt et al., 2004)



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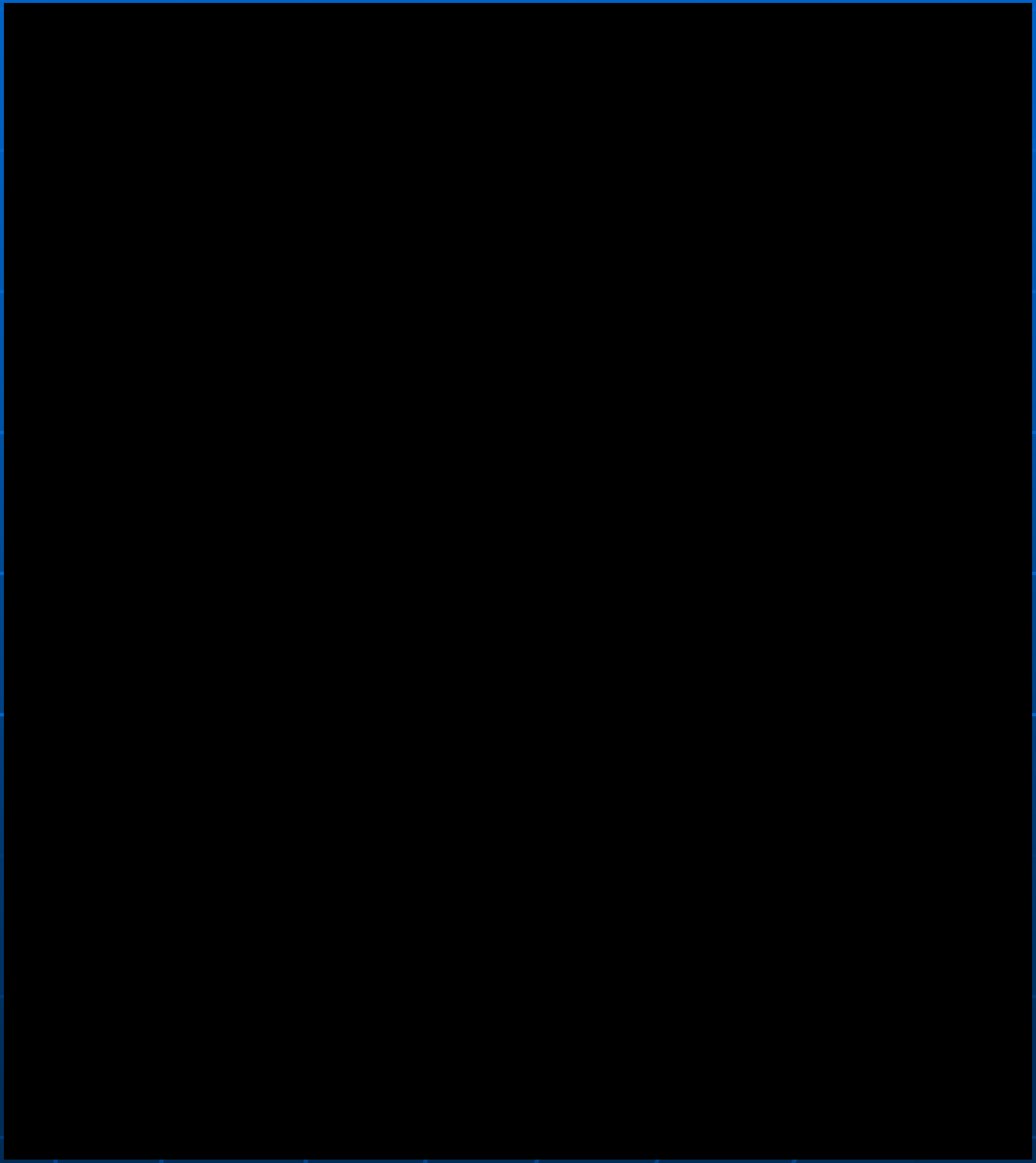
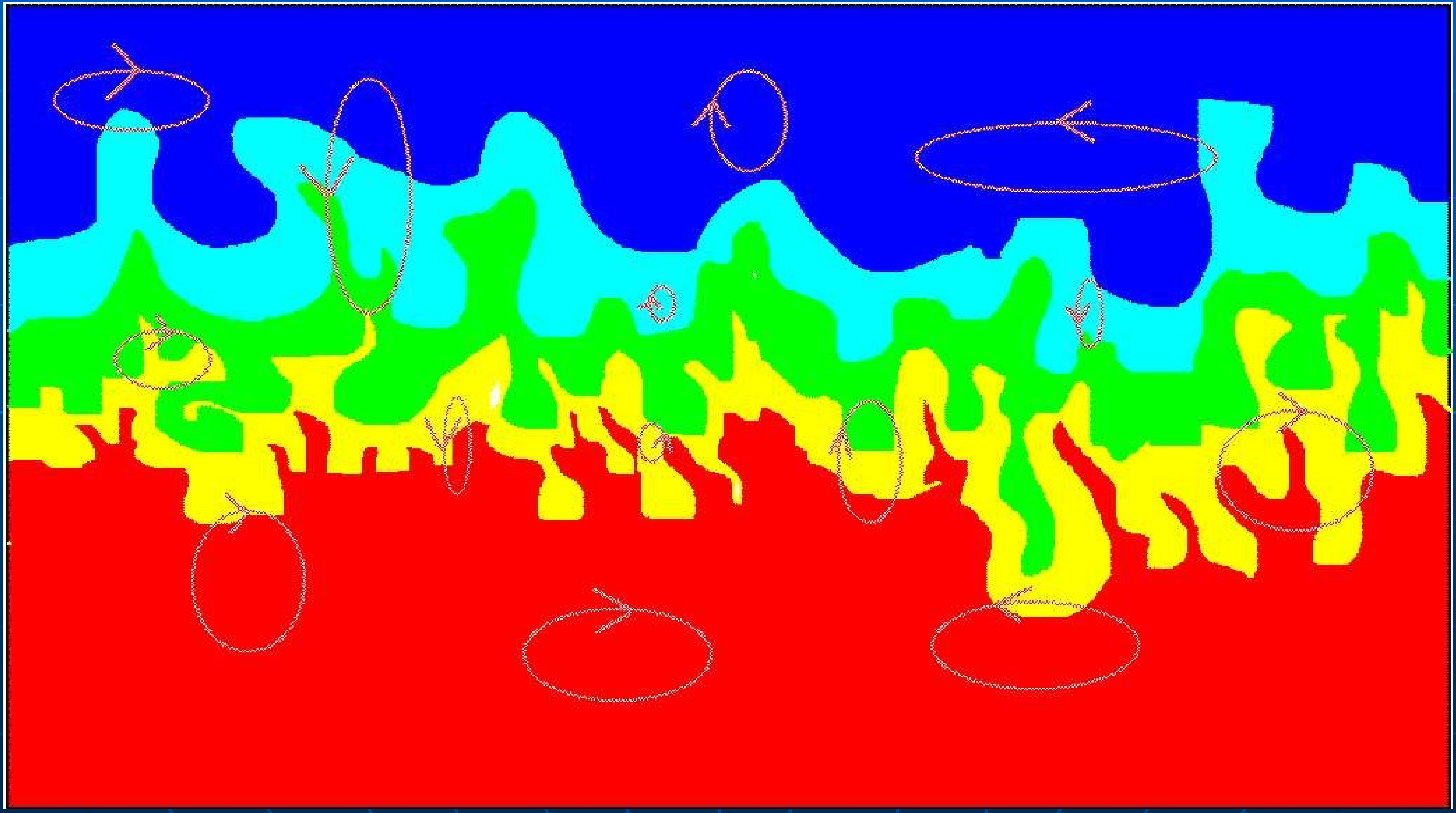


Burning regimes of pre-mixed flames

3. The distributed-burning



Burning regimes of pre-mixed flames

3. The distributed-burning

Turbulent eddies interact with the flame:

$l_F \geq l_{Gibs}$

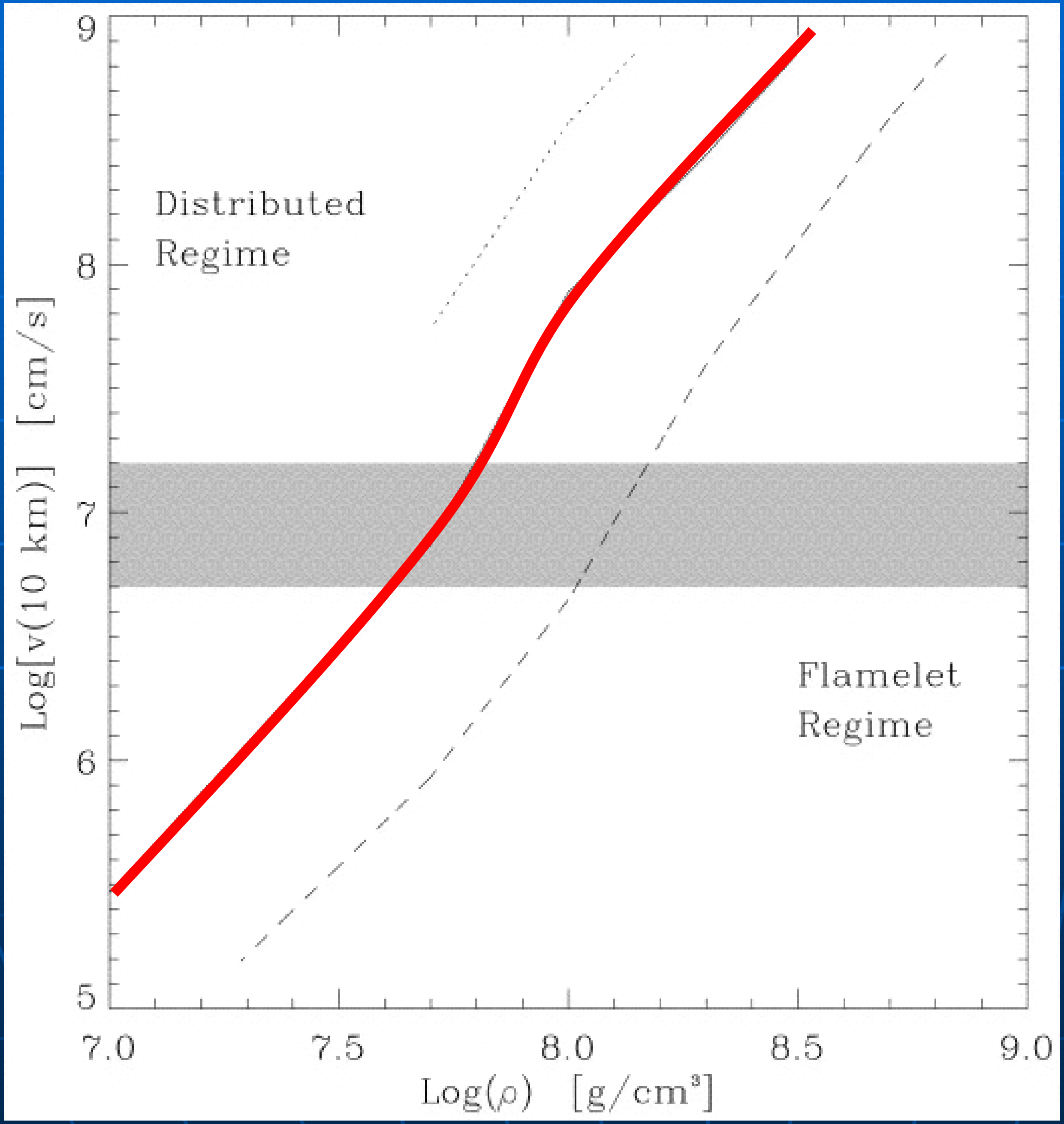
Rough estimate ("Damköhler scaling"):

$s_{turb}/s_L \approx \text{const}\,(D_t/D)^{1/2}$ (dependent on s_L !!!)

const = O(1)

Transition to detonation possible???

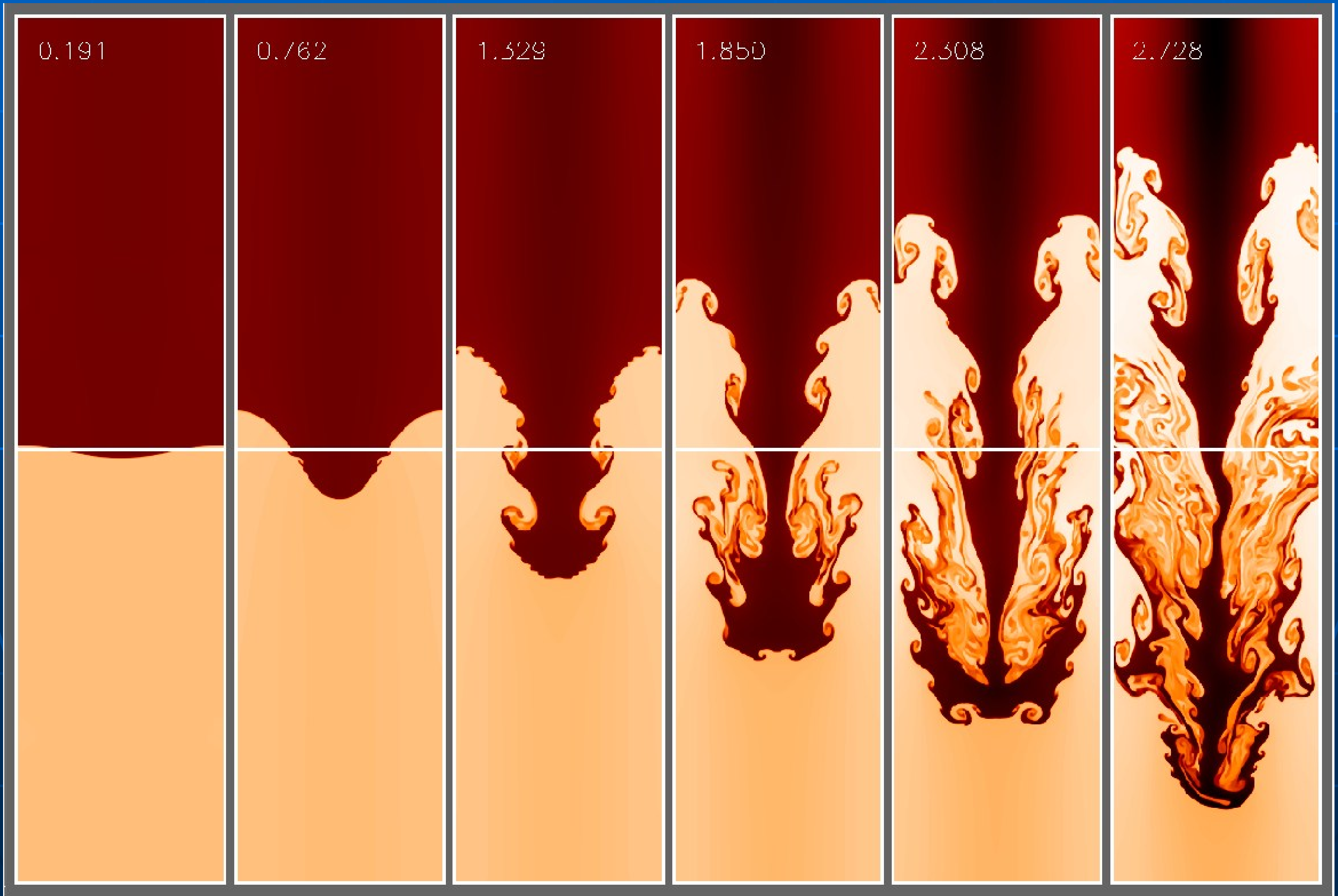
Application to type Ia supernova



Niemeyer & Woosley (1997)

Burning regimes of pre-mixed flames

4. The Rayleigh-Taylor regime



Burning regimes of pre-mixed flames

4. The Rayleigh-Taylor regime

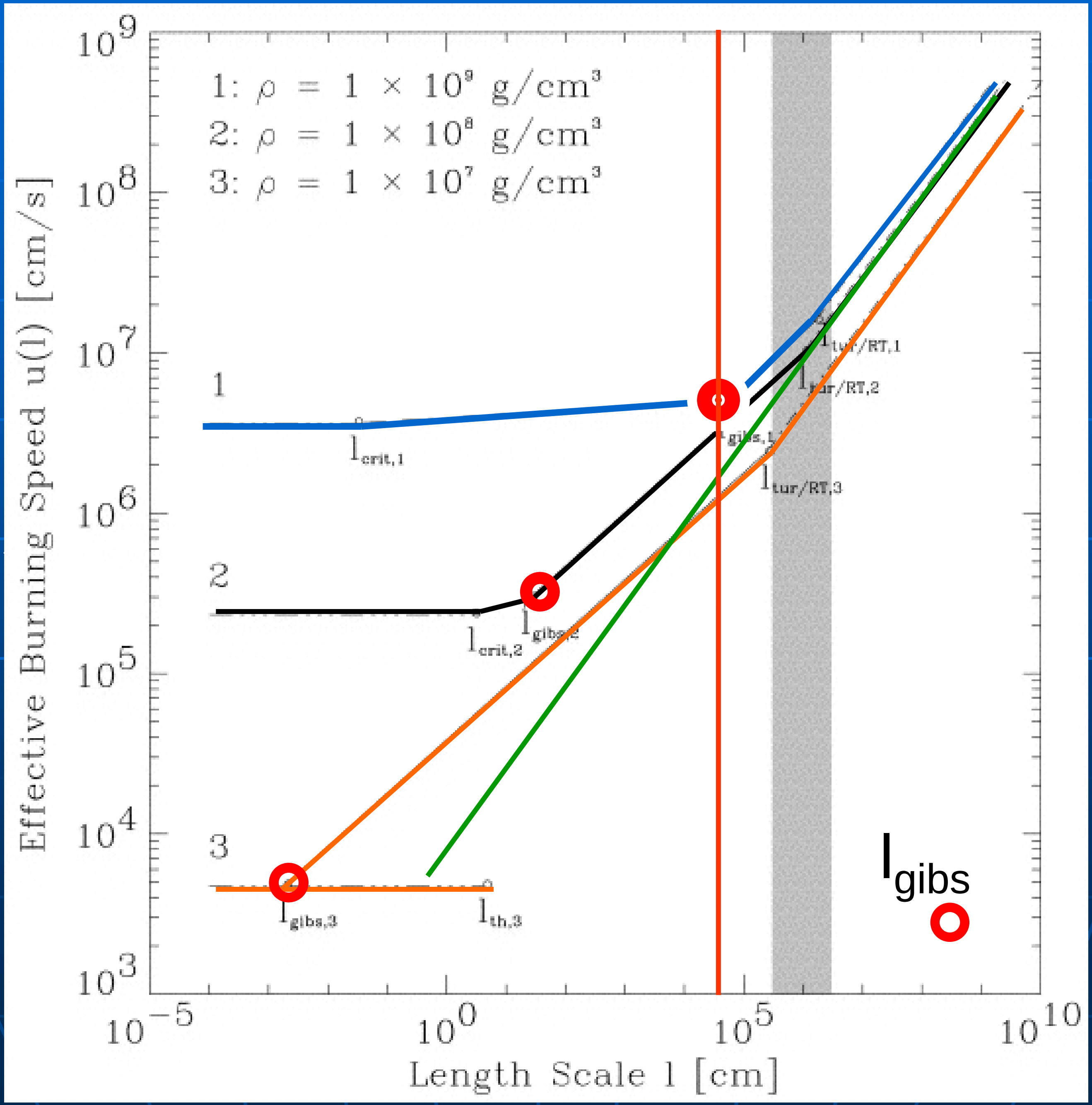
$v_{RT} = B\sqrt{(g_{eff}\, l)}$; $B \approx 0.5$; $g_{eff} = At \cdot g$

Sharp-Wheeler model:

$r_{sw} \approx 0.05\, g_{eff}\, t^2$; $v_{sw} \approx 0.1\, g_{eff}\, t$;

$l_{tur/RT} \approx 10^6$ cm

Effective burning velocities in SN Ia



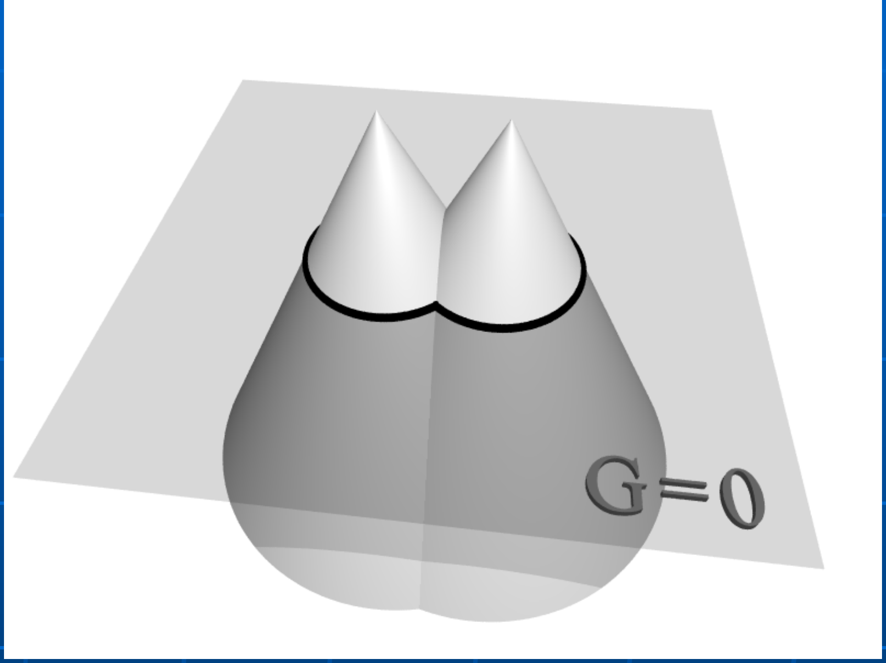
Niemeyer & Woosley (1997)

How to model thermonuclear flames?

- The "flames" cannot be resolved numerically.
- The amplitutes of turbulent velocity fluctuations in the length scale of the flame are determined on the integral scale.

→

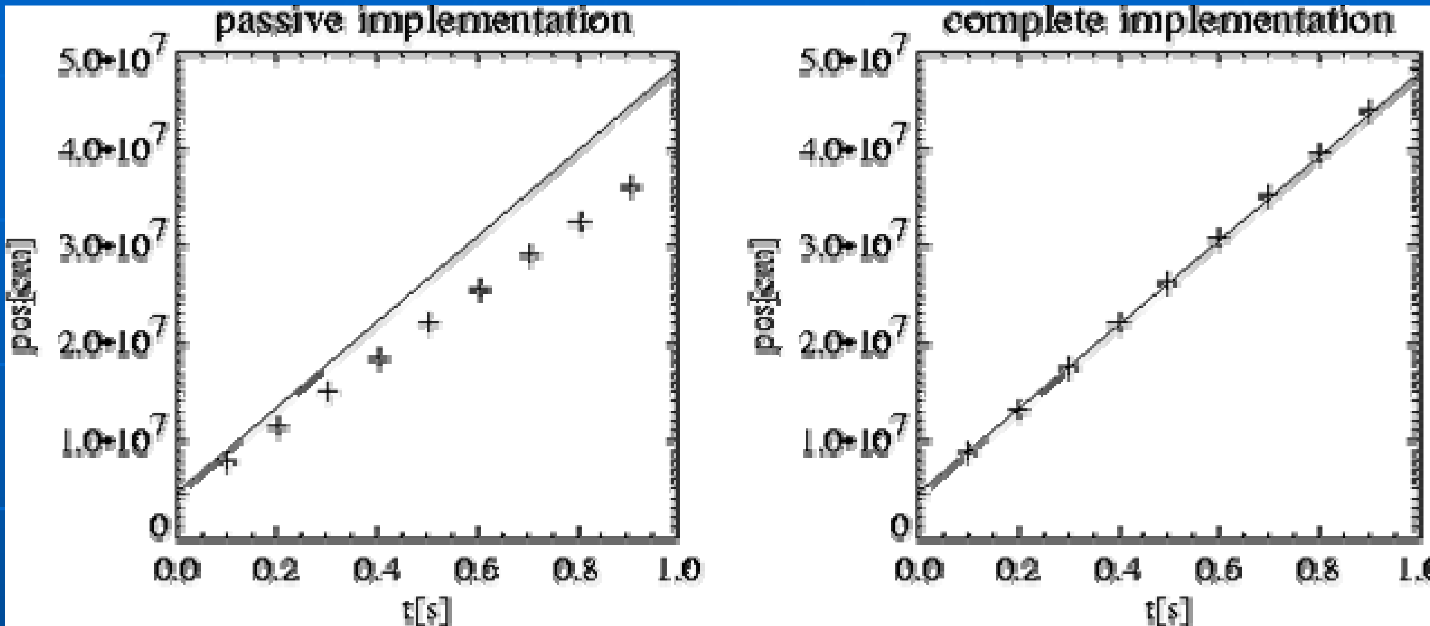
"LES" + "Level Set"



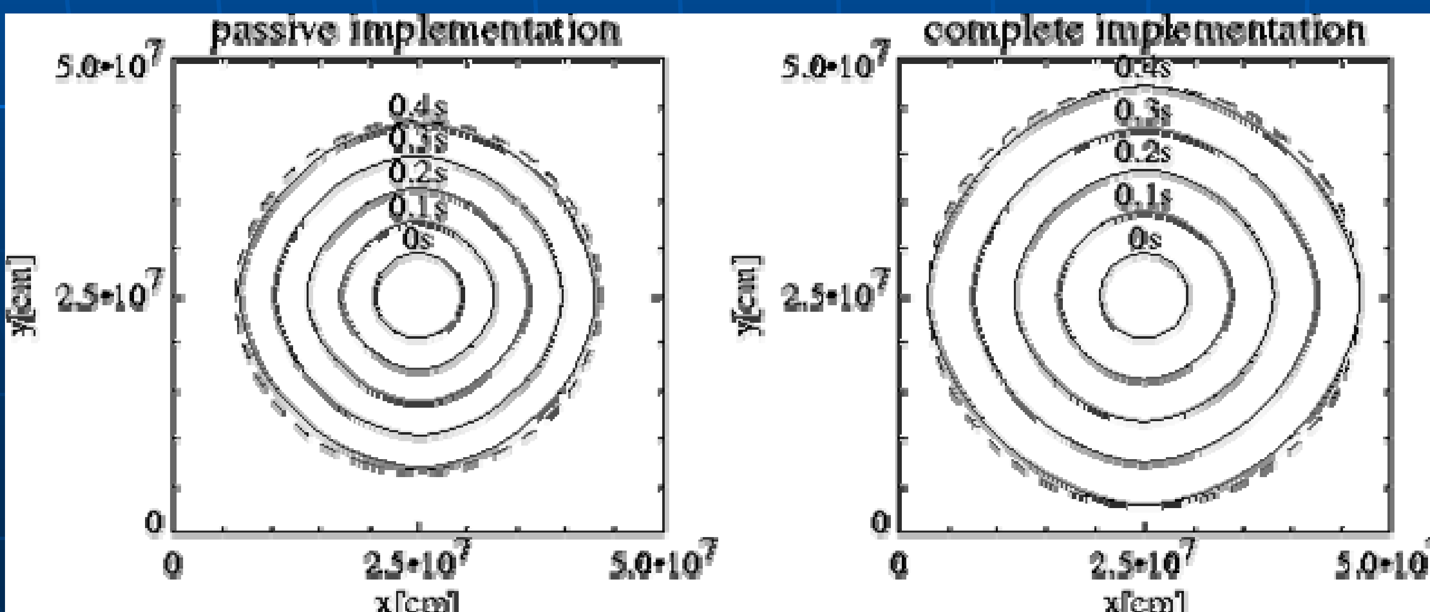
$$\partial G/\partial t = -\mathcal{D}_f \nabla G$$

$$\mathcal{D}_f = \mathbf{v}_u + s_{tur}\,\mathbf{n}; \quad |\nabla G| = 1$$

Some test of the code



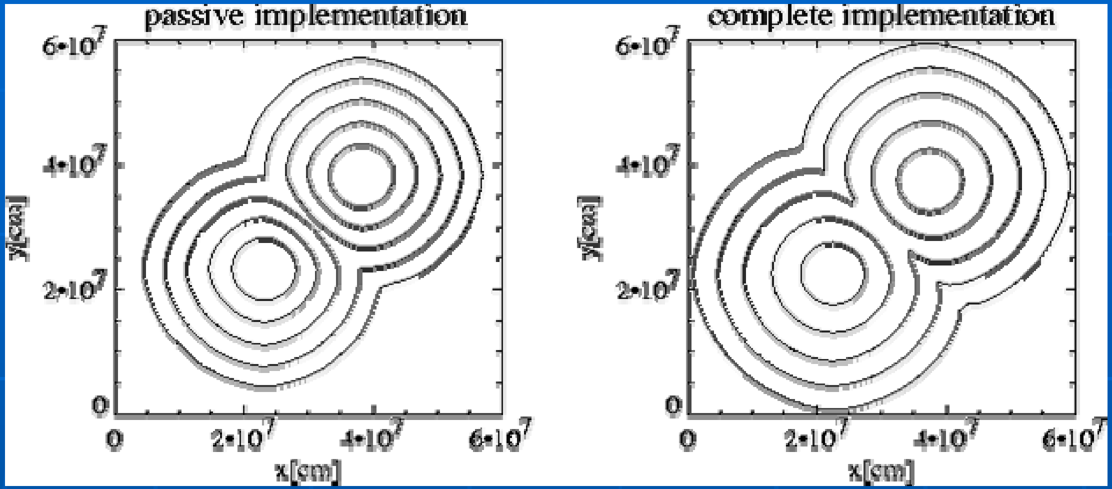
Planar flame



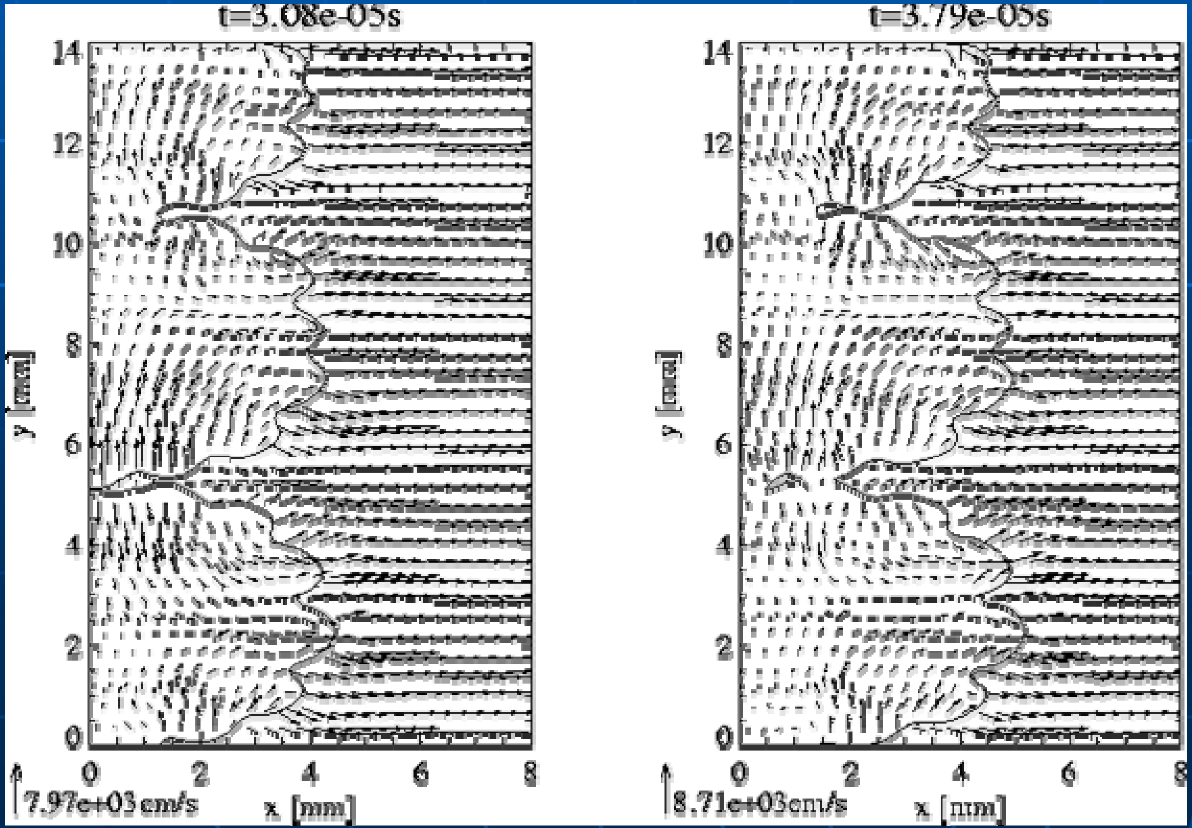
Circular flame

Reinecke et al. (1999)

Some test of the code (ctn.)



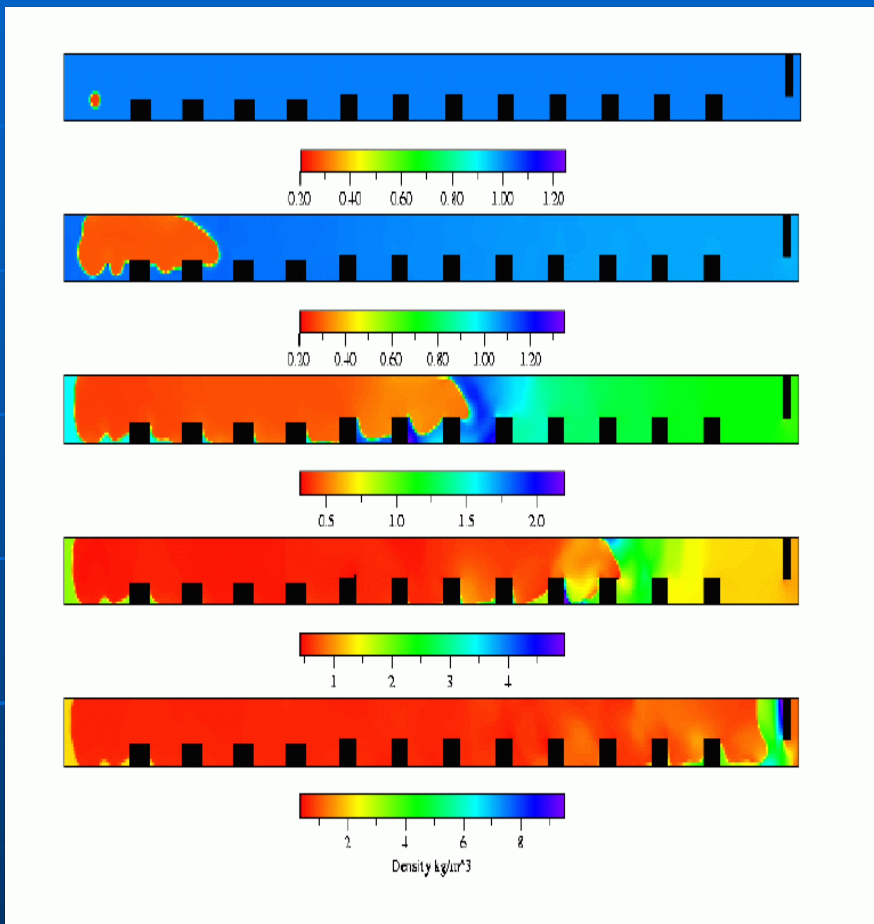
Merging circular flames

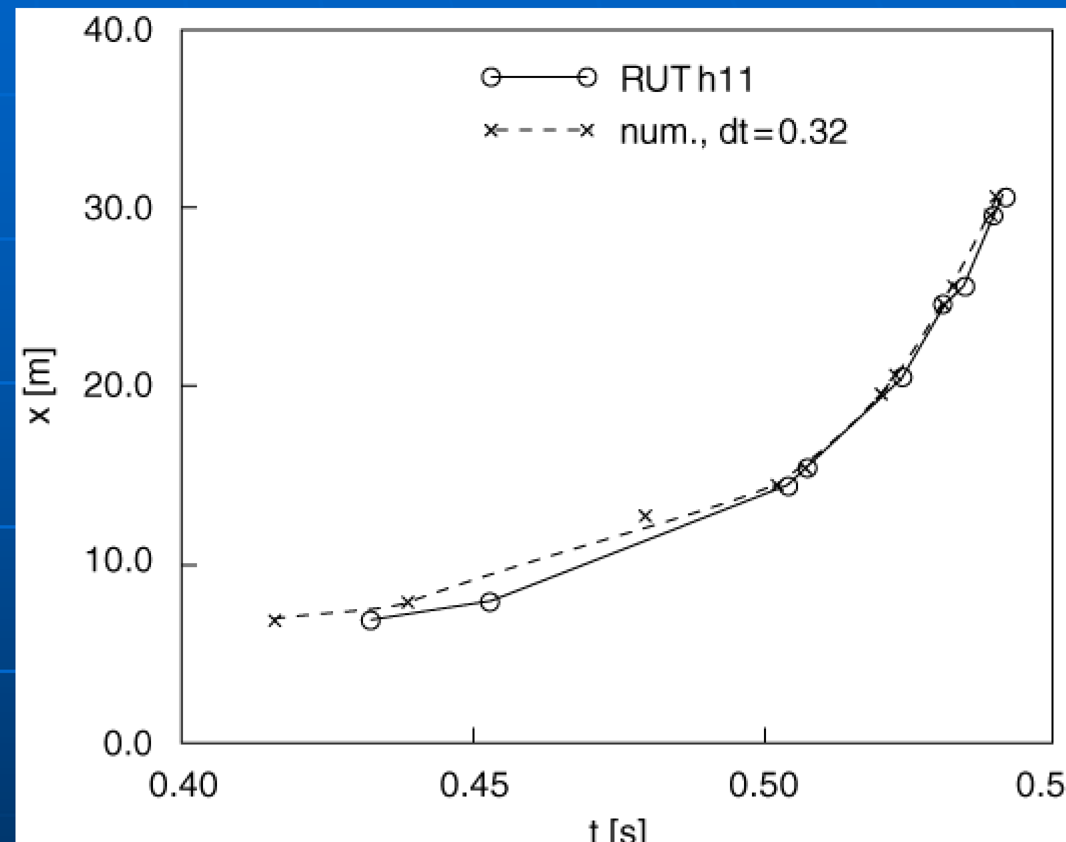


Hydrogen-in-air flames

Reinecke et al. (1999)

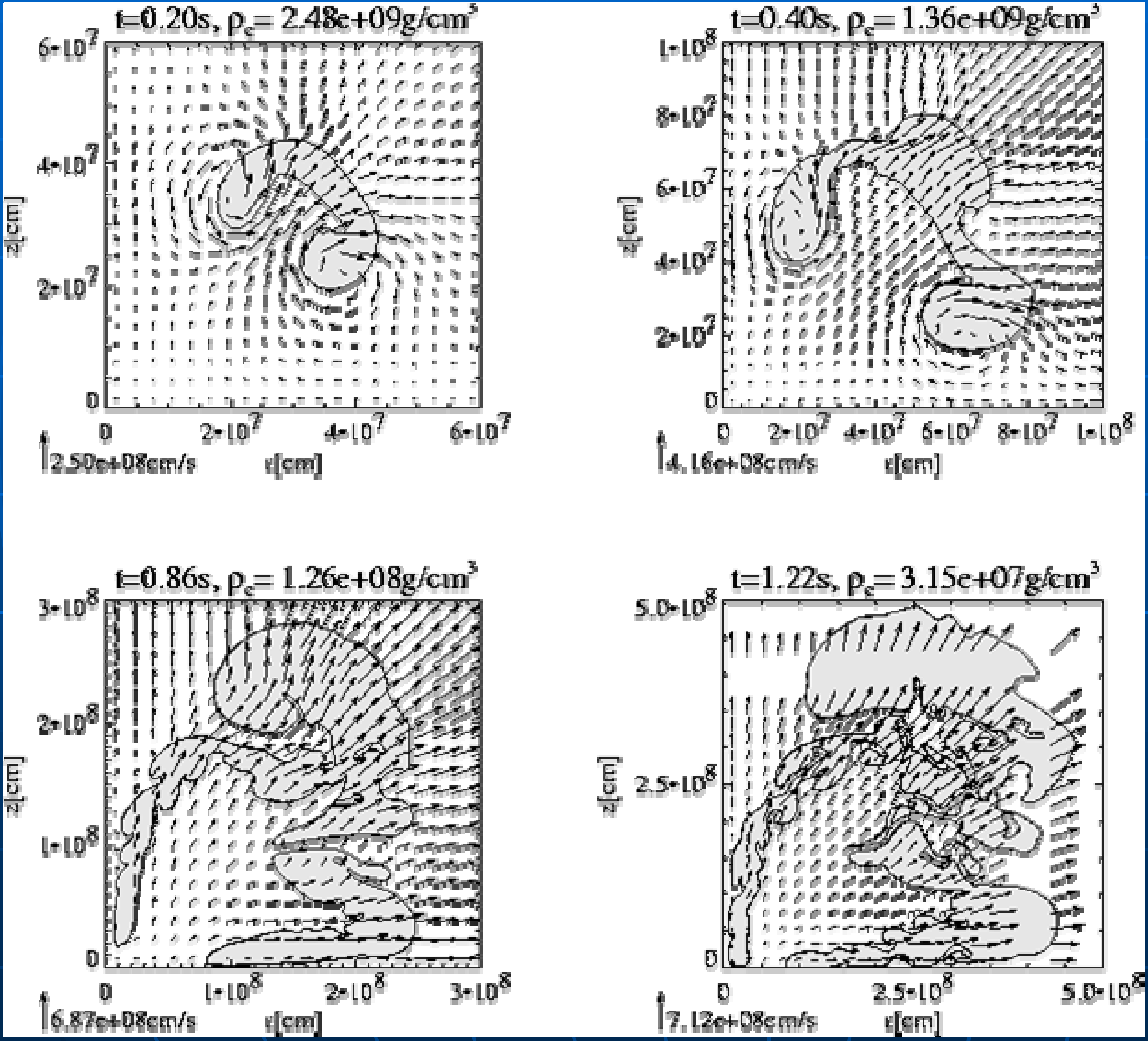
Application to laboratory flames (hydrogen in air)





The method can reproduce terrestrial experiments well!
(Smiljanowski et al. 1997)

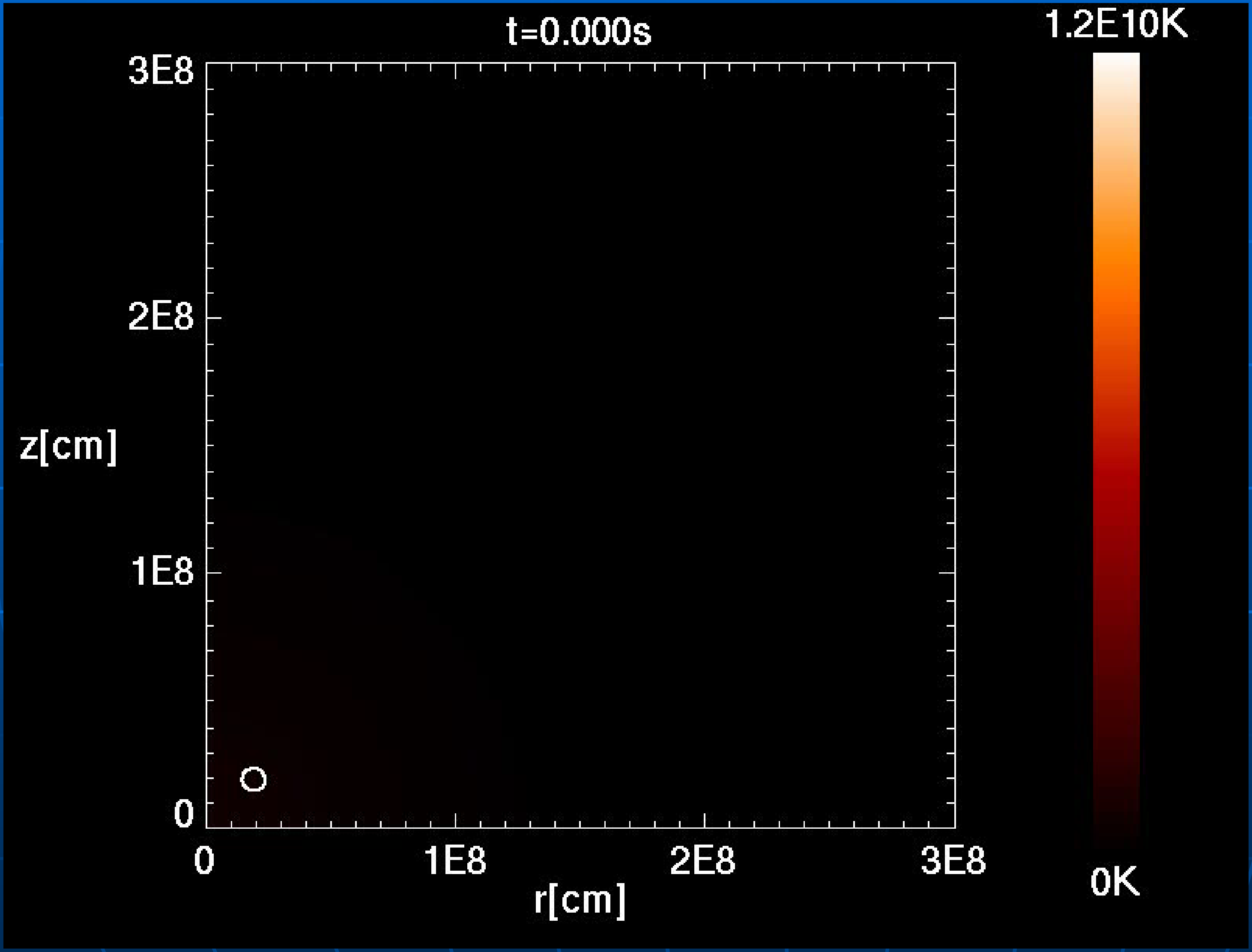
Application to the SN Ia problem



One rising blob (in 2D)

Reinecke et al. (1997)

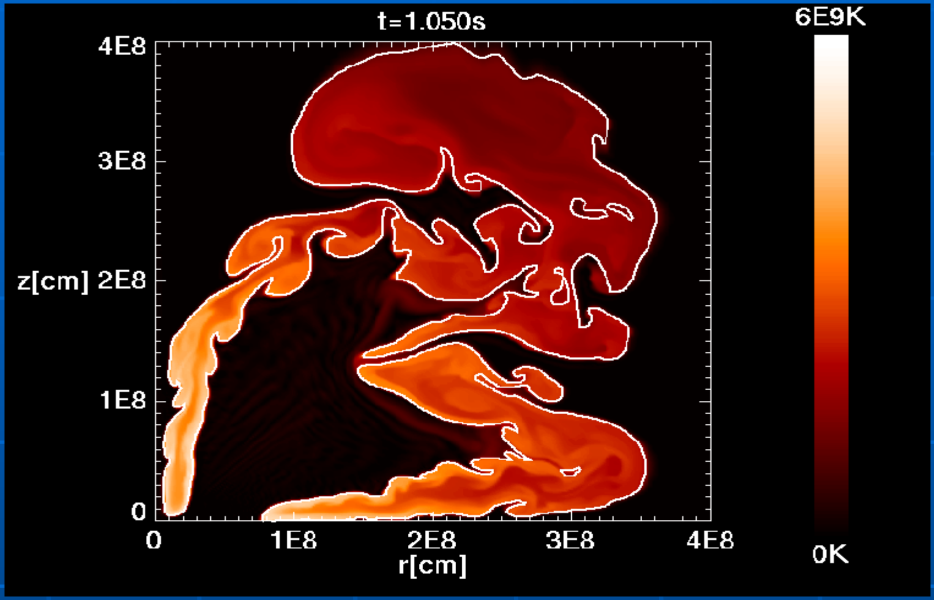
Application to the SN Ia problem

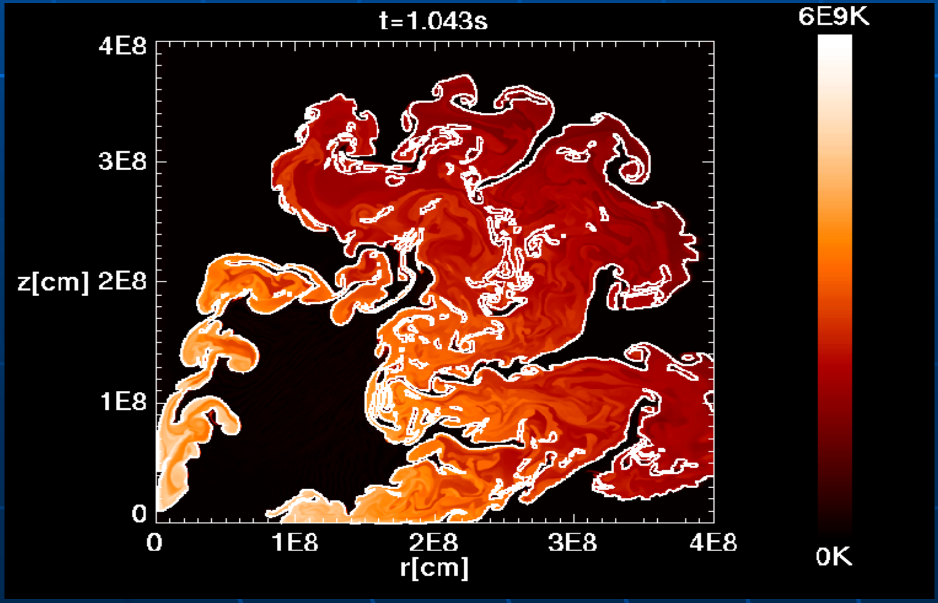


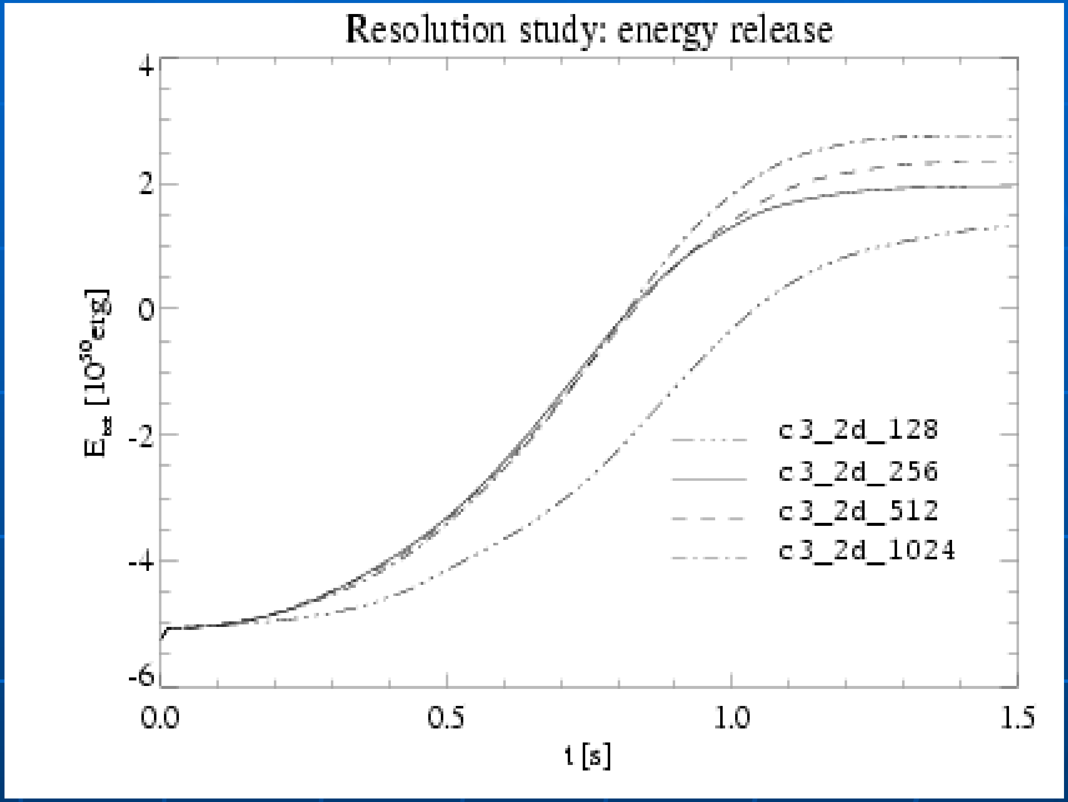
One rising blob (in 2D)

Reinecke et al. (1997)

Convergence tests in 2D



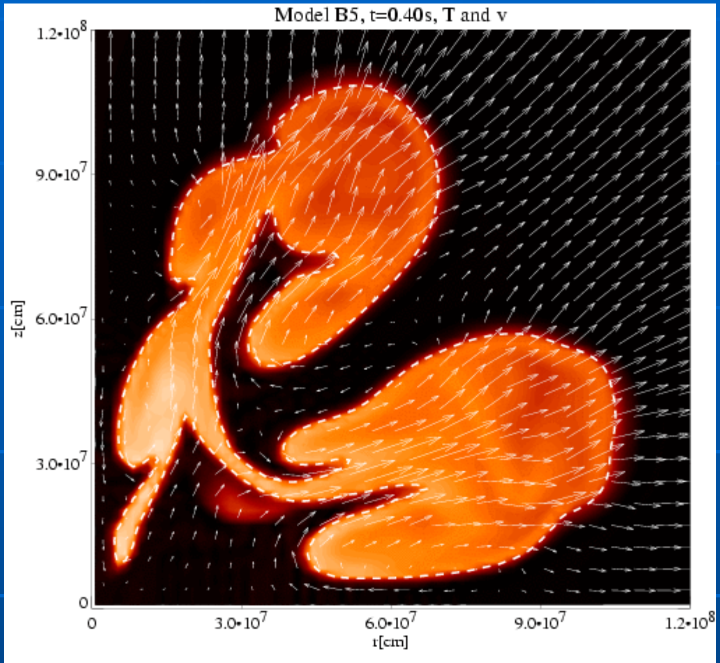


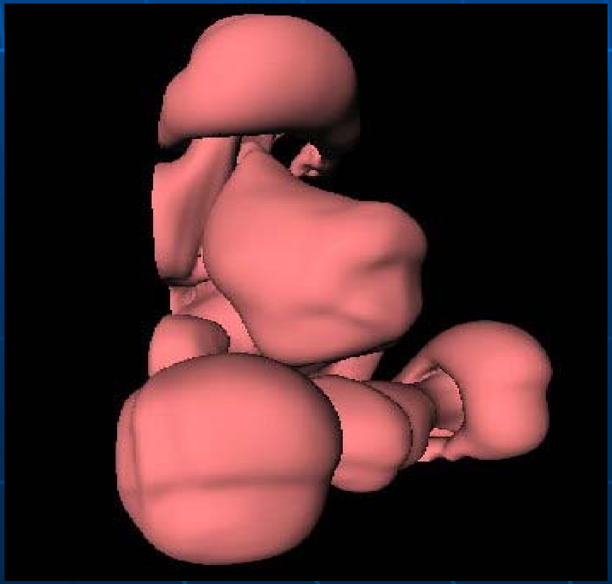


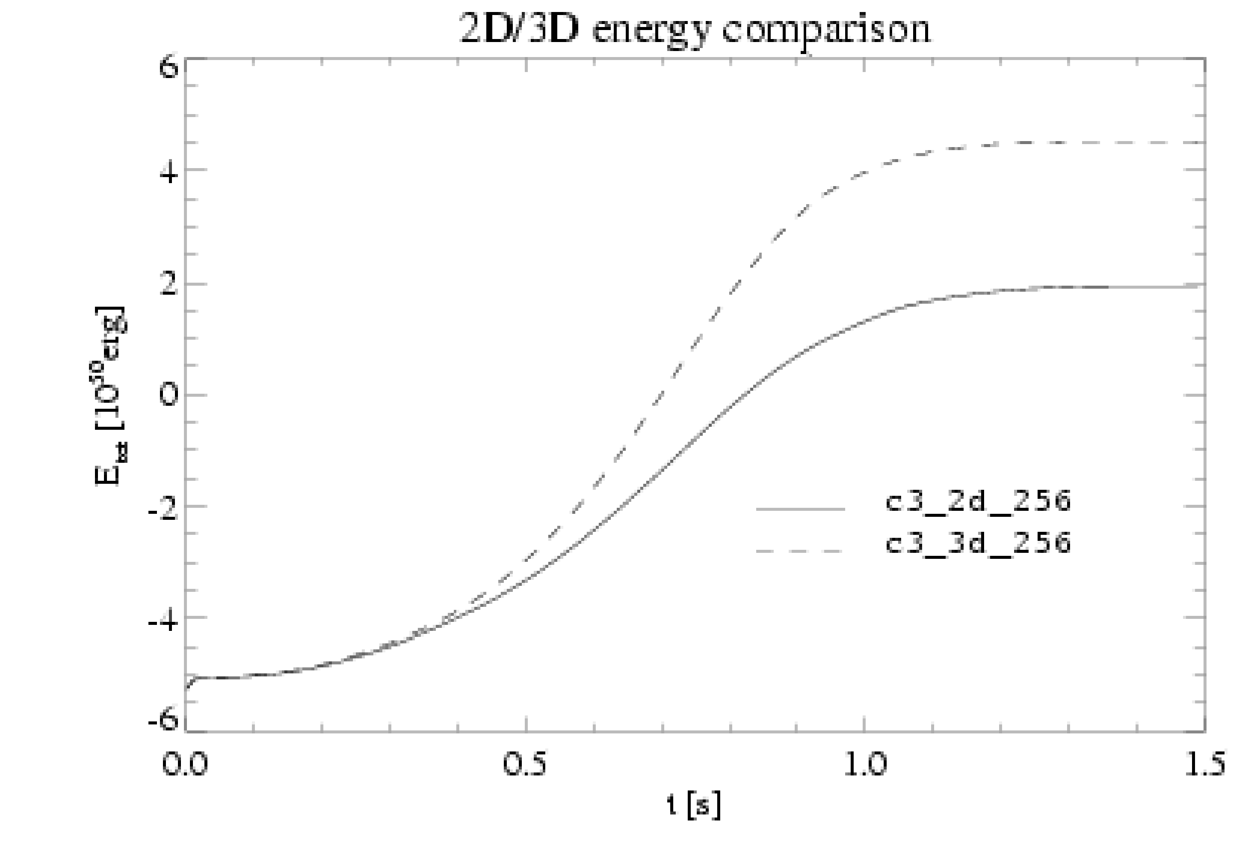
Global results are independent of the numerical resolution!

Reinecke et al. (1999, 2002)

2D ⇒ 3D



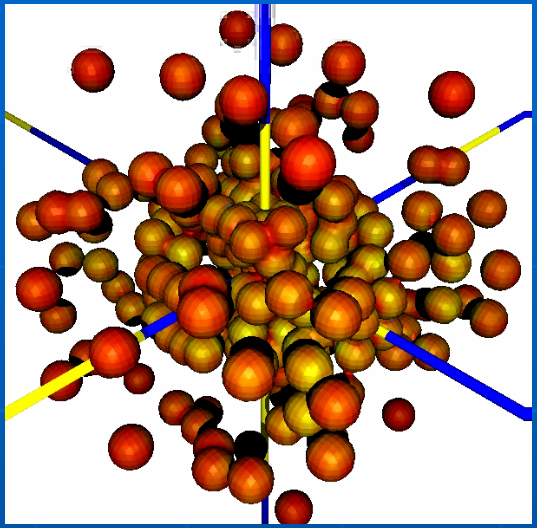


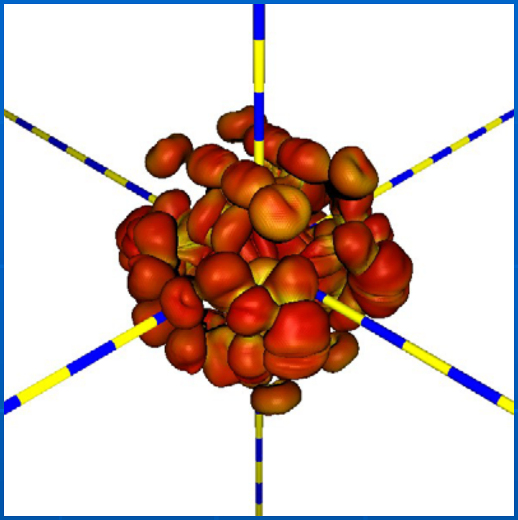


Because of larger surface area:
<u>More energy is produced!</u>

Reinecke et al. (2001)
(See also Gamezo et al., 2003)

3D models: The best we could do (until recently):





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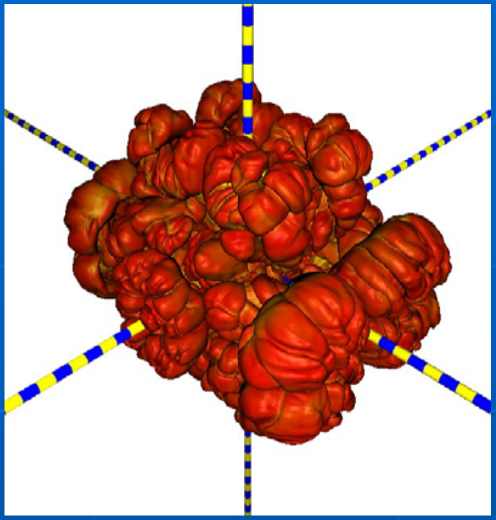

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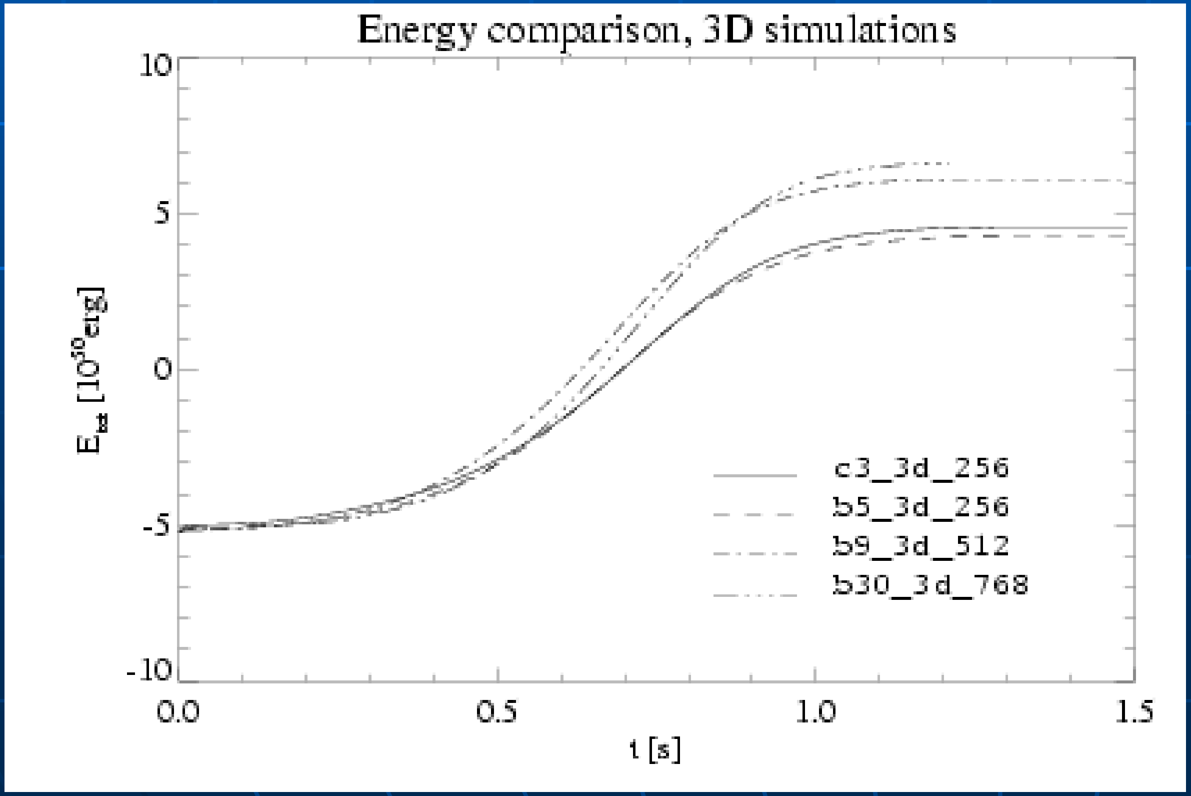
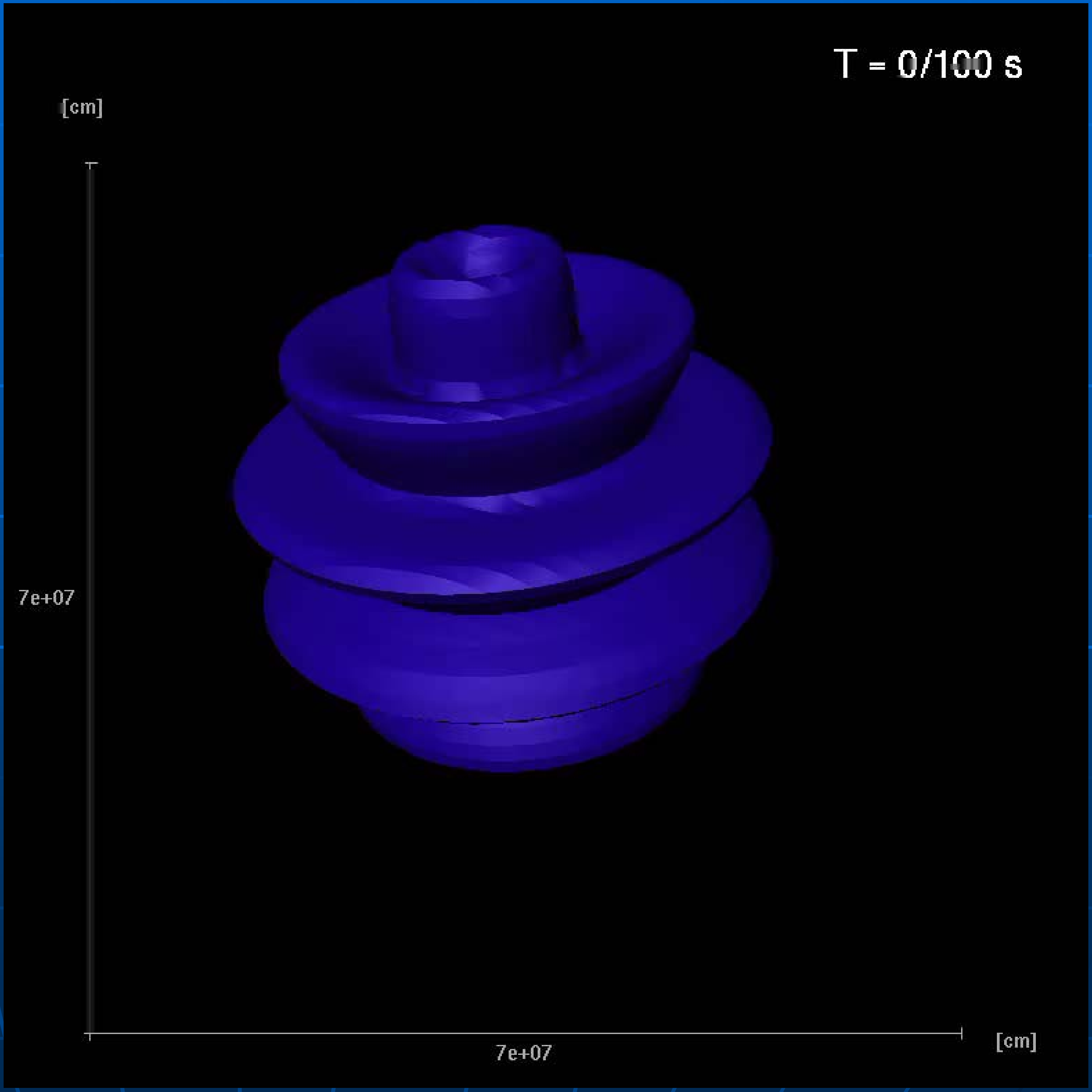


Mod b30_3d

(Reinecke et al., 2003)

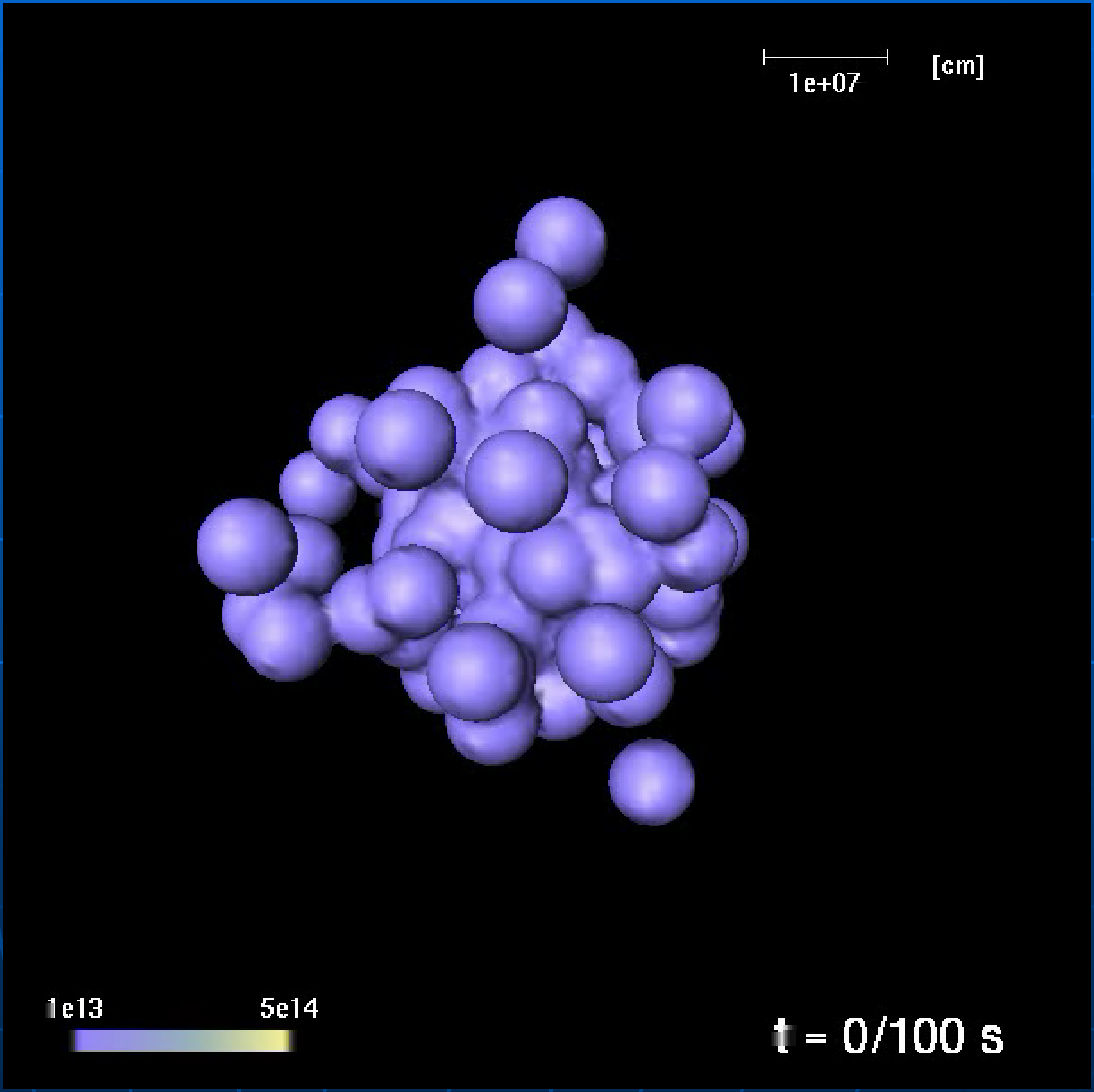
Recent modifications of the code:

1. Moving grid



Röpke (2004)

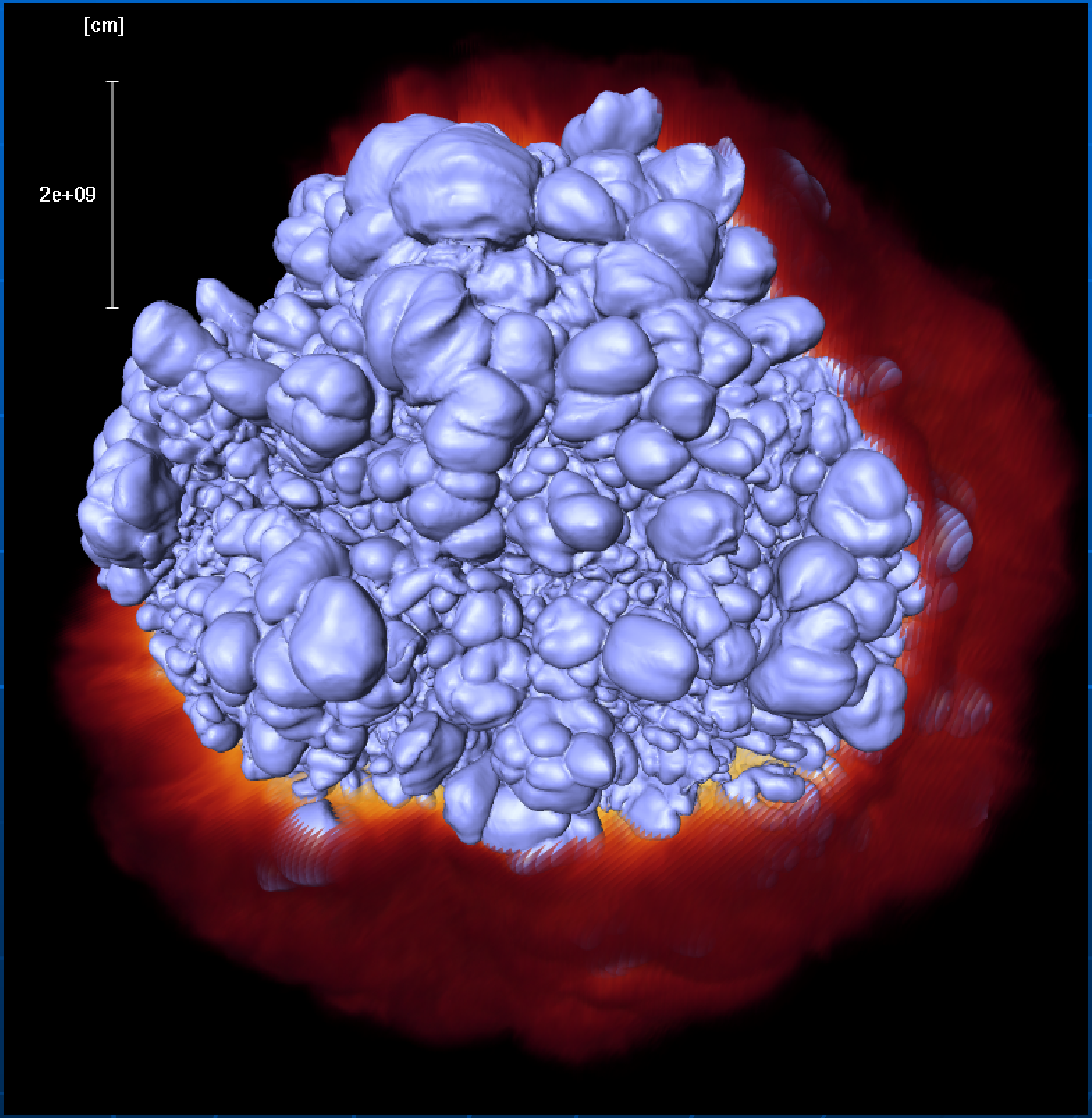
2. Full star ("4π")



Röpke & Hillebrandt (2004)

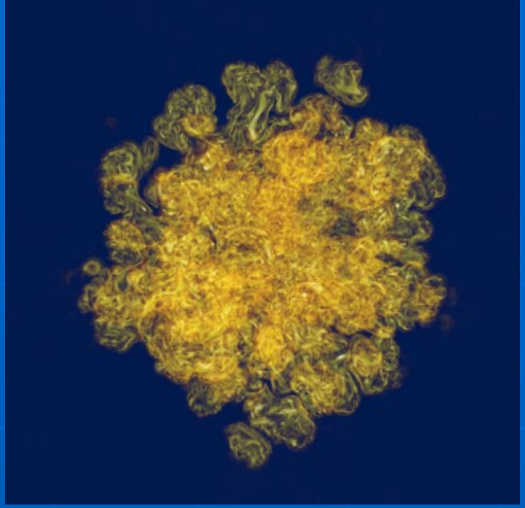
A high-resolution model ("the SNOB run")

- "4π"
- 1024^3 grid
- initial resolution near the center ≈ 800m
- moving grid
- Local & dynamical sgs-model
- ~ 1000 h on 512 processors, IBM/Power4, at RZG



Röpke et al. (2006)

Turbulence?



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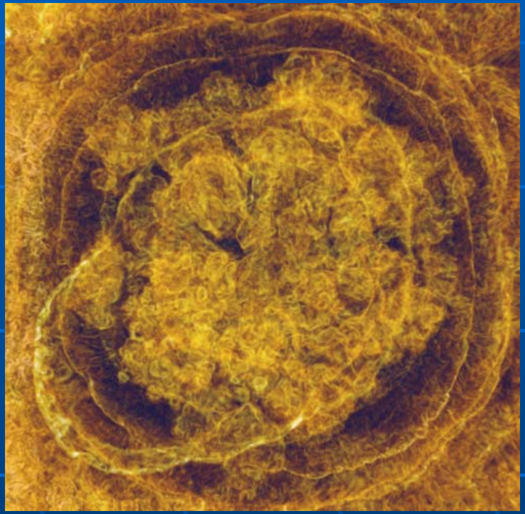

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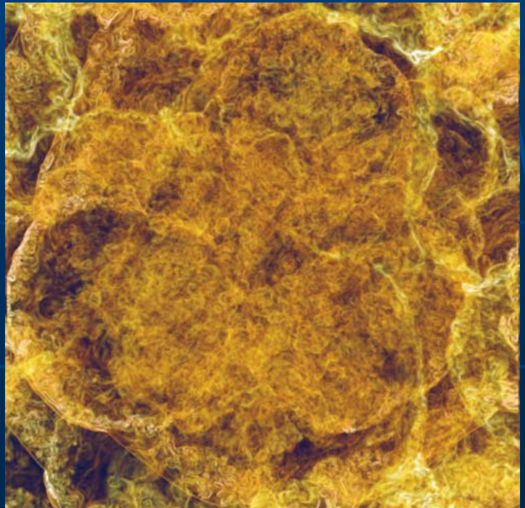

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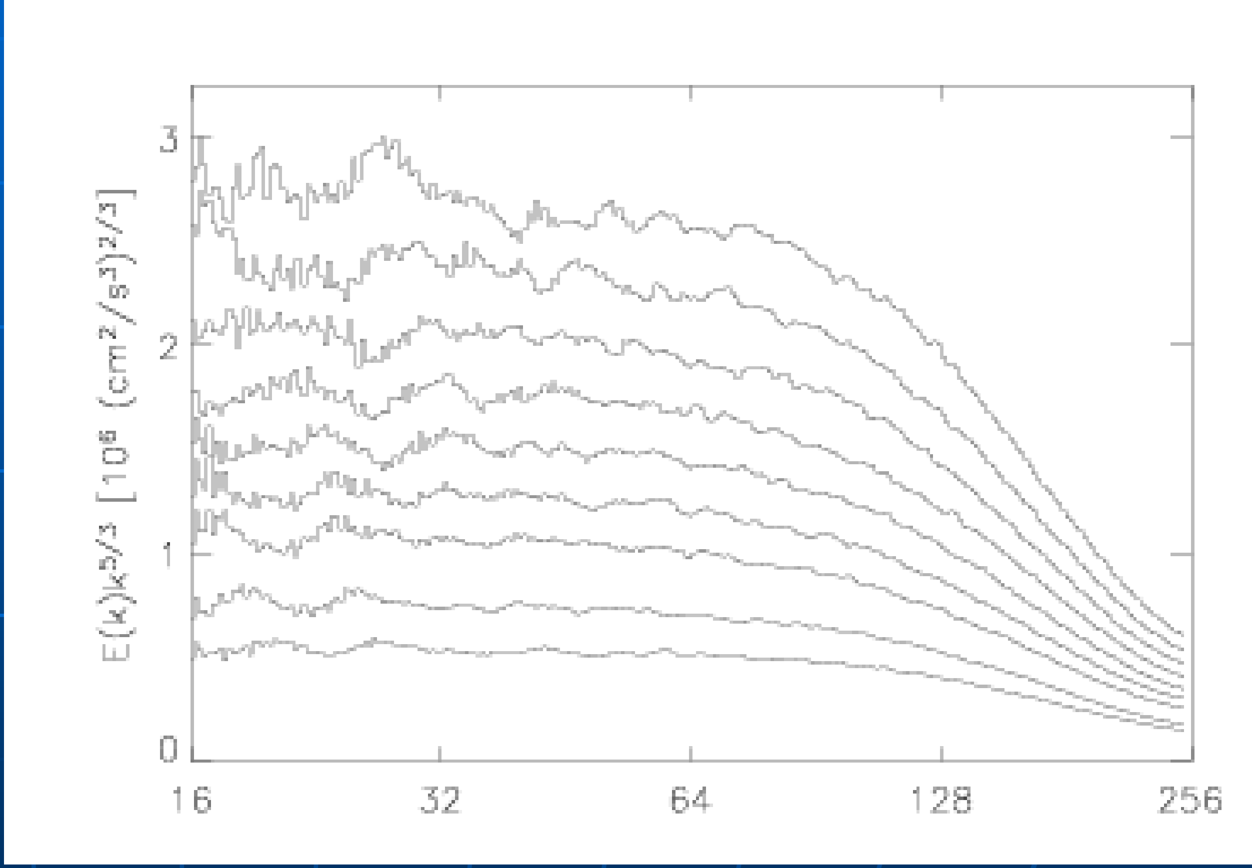

Schmidt et al. (in preparation)

Some (preliminary) results:

- $E_{kin} = 8.1 \cdot 10^{50}$ erg
- Iron-group nuclei: 0.61 M_{sun} (~ 0.41 M_{sun} ^{56}Ni)
- Intermediate-mass nuclei: 0.43 M_{sun} (from hydro)
- Unburnt C+O: 0.37 M_{sun} (from hydro)
  (less than 0.08 M_{sun} at v<8000km/s)
- Vmax ≈ 17,000 km/s

Good agreement with observations!

Röpke et al. (in preparation)

Questions and challenges (to theory)

- Ignition conditions:

  How do WDs reach M_{Ch} ? Center/off-center ignition? One/multiple "points"?

- Combustion modeling:

  Interaction of nuclear flames with turbulence;

  "distributed burning"; "active turbulent combustion" ?

  Deflagration/detonation transition: Does it happen? Is it "needed"?

- New generation of "full-star" models:

  Light curves? Spectra?

- Other progenitors:

  Mergers? Sub-Chandrasekhars?