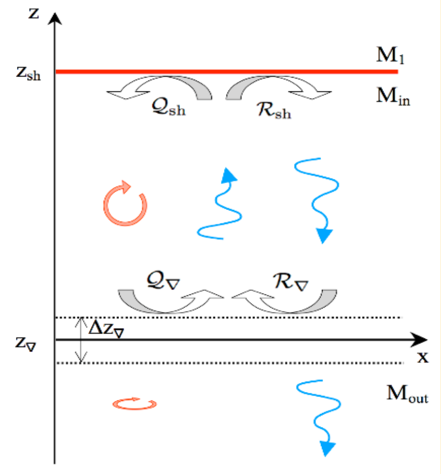
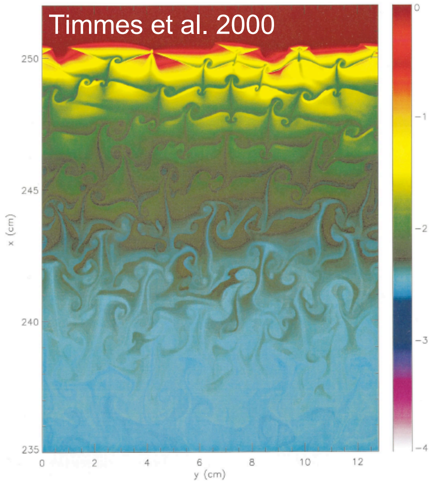
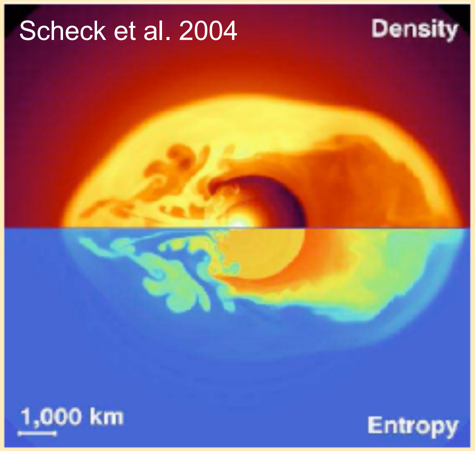
The Physics of Cellular Detonations and the Instability of Advective-acoustic Cycles

Thierry Foglizzo
CEA Saclay & KITP




Timmes et al. 2000


Scheck et al. 2004

OUTLINE

The basics of unstable detonations

The formalism of advective-acoustic cycles

The basics of stationary detonations

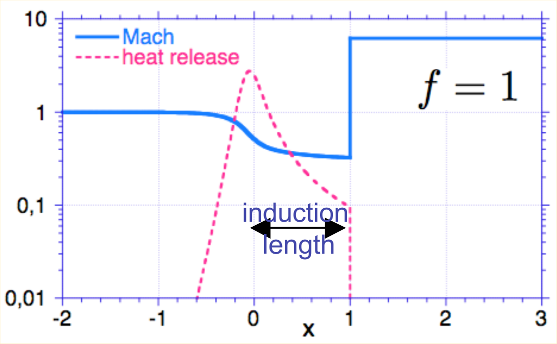
ZND: Zeldovich, von Neumann and Döring, 1940

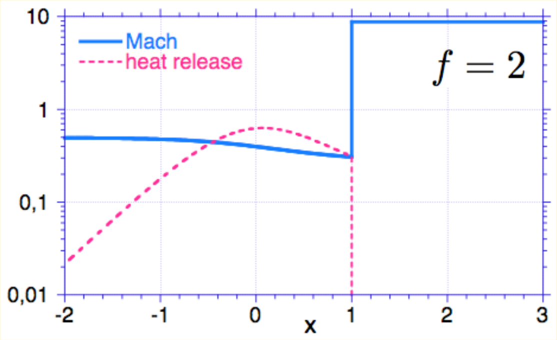
perfect gas, adiabatic index γ

-self-sustained (Chapman-Jouguet) detonation

$$\frac{v_{\mathrm{CJ}}^2}{c_0^2} = 1 + (\gamma^2 - 1)\frac{Q}{c_0^2} + \left\{\left[1 + (\gamma^2 - 1)\frac{Q}{c_0^2}\right]^2 - 1\right\}^{\frac{1}{2}}$$

-overdrive parameter f $\quad f \equiv \dfrac{v_{\mathrm{D}}^2}{v_{\mathrm{CJ}}^2}$





simplest chemistry: one-step Arrhenius kinetics

$$\frac{D\lambda}{Dt} = r \propto (1-\lambda)\exp\left(-\frac{E}{kT}\right)$$

reaction rate, activation energy E

$$\frac{DS}{Dt} \propto \frac{Q\rho r}{P}$$

entropy equation, total heat release Q

$$\frac{Dv}{Dt} = -\frac{\nabla P}{\rho}$$

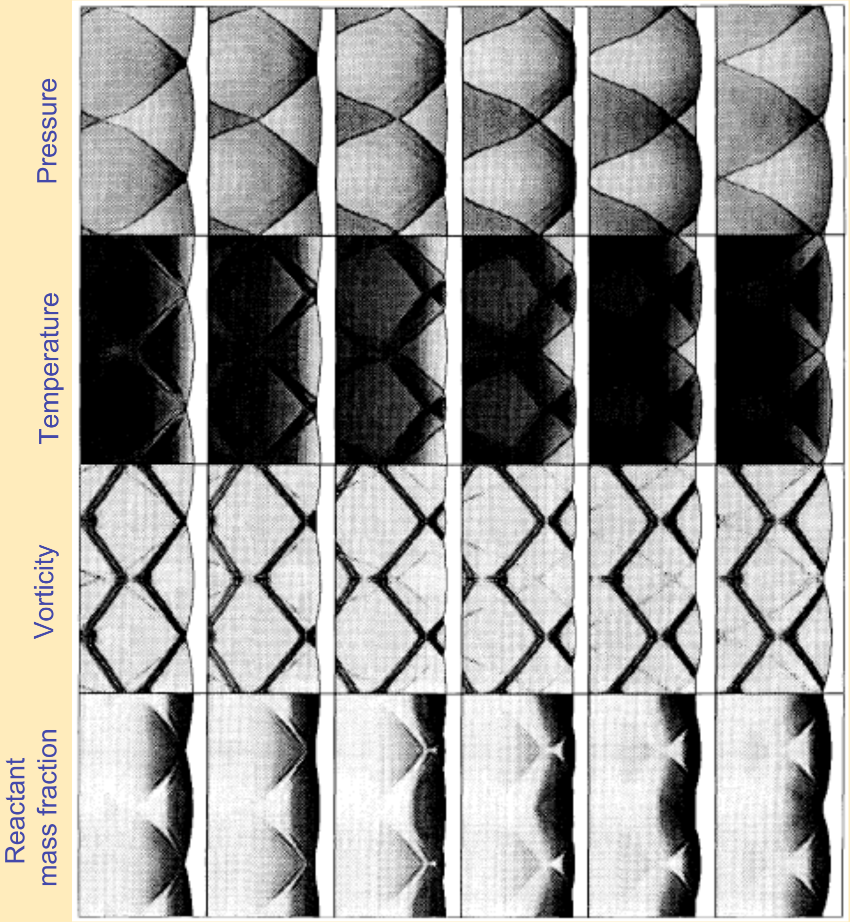
Euler equation

$$\frac{D\rho}{Dt} + \rho \nabla v = 0$$

mass conservation

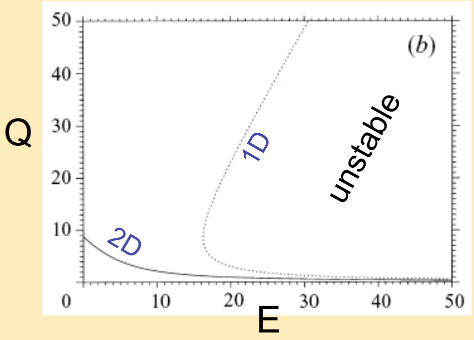
Instability of detonations

- experiments
- numerical simulations
- linear stability analysis



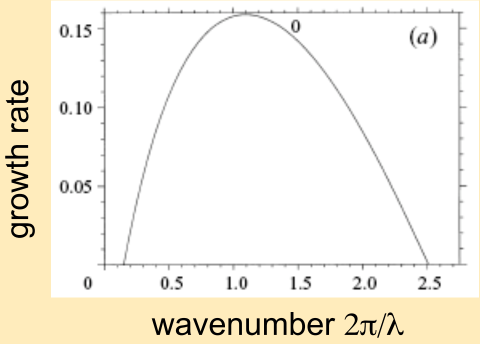
Bourlioux & Majda 1992

Linear stability analysis:
Most detonations are unstable in 2D (Erpenbeck 1962-1970)



f=1.2

Short & Stewart 1998



E=12.5
Q=50
f=1

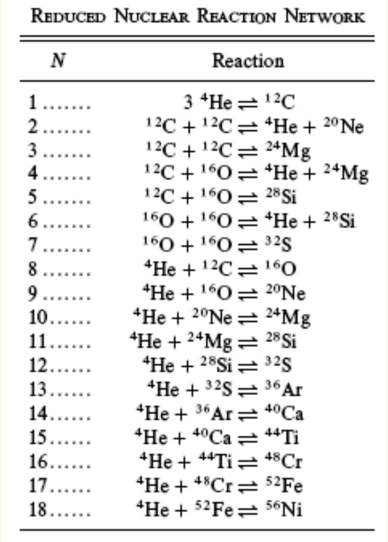
cell size ~ wavelength of the most unstable mode
~3-20 induction length

why ?

analytical asymptotic approach:
Clavin, He & Williams 1997: γ~1, E>>1, f>>1
Short & Stewart 1999: Q<<1

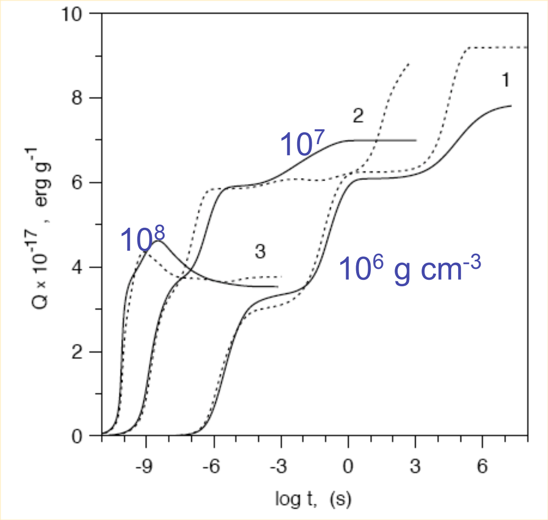
Cellular instability of detonations in SNIa

Boisseau et al. 1996, Gamezzo et al. 1999, Timmes et al. 2000



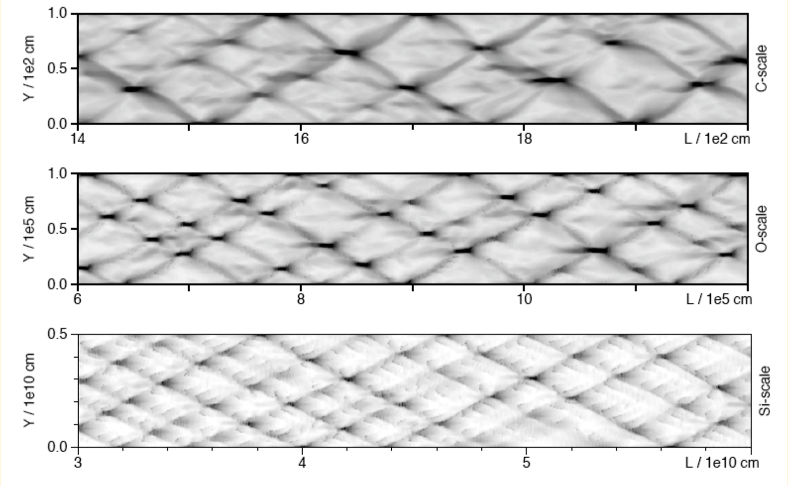
REDUCED NUCLEAR REACTION NETWORK

N	Reaction
1	$3\,^4\mathrm{He} \rightleftharpoons {}^{12}\mathrm{C}$
2	$^{12}\mathrm{C} + {}^{12}\mathrm{C} \rightleftharpoons {}^4\mathrm{He} + {}^{20}\mathrm{Ne}$
3	$^{12}\mathrm{C} + {}^{12}\mathrm{C} \rightleftharpoons {}^{24}\mathrm{Mg}$
4	$^{12}\mathrm{C} + {}^{16}\mathrm{O} \rightleftharpoons {}^4\mathrm{He} + {}^{24}\mathrm{Mg}$
5	$^{12}\mathrm{C} + {}^{16}\mathrm{O} \rightleftharpoons {}^{28}\mathrm{Si}$
6	$^{16}\mathrm{O} + {}^{16}\mathrm{O} \rightleftharpoons {}^4\mathrm{He} + {}^{28}\mathrm{Si}$
7	$^{16}\mathrm{O} + {}^{16}\mathrm{O} \rightleftharpoons {}^{32}\mathrm{S}$
8	$^4\mathrm{He} + {}^{12}\mathrm{C} \rightleftharpoons {}^{16}\mathrm{O}$
9	$^4\mathrm{He} + {}^{16}\mathrm{O} \rightleftharpoons {}^{20}\mathrm{Ne}$
10......	$^4\mathrm{He} + {}^{20}\mathrm{Ne} \rightleftharpoons {}^{24}\mathrm{Mg}$
11......	$^4\mathrm{He} + {}^{24}\mathrm{Mg} \rightleftharpoons {}^{28}\mathrm{Si}$
12......	$^4\mathrm{He} + {}^{28}\mathrm{Si} \rightleftharpoons {}^{32}\mathrm{S}$
13......	$^4\mathrm{He} + {}^{32}\mathrm{S} \rightleftharpoons {}^{36}\mathrm{Ar}$
14......	$^4\mathrm{He} + {}^{36}\mathrm{Ar} \rightleftharpoons {}^{40}\mathrm{Ca}$
15......	$^4\mathrm{He} + {}^{40}\mathrm{Ca} \rightleftharpoons {}^{44}\mathrm{Ti}$
16......	$^4\mathrm{He} + {}^{44}\mathrm{Ti} \rightleftharpoons {}^{48}\mathrm{Cr}$
17......	$^4\mathrm{He} + {}^{48}\mathrm{Cr} \rightleftharpoons {}^{52}\mathrm{Fe}$
18......	$^4\mathrm{He} + {}^{52}\mathrm{Fe} \rightleftharpoons {}^{56}\mathrm{Ni}$



Gamezo et al. 1999

3 scales: C, O, Si



m

km

50 000 km

Gamezo et al. 1999, (5 10^6 g cm^{-3})

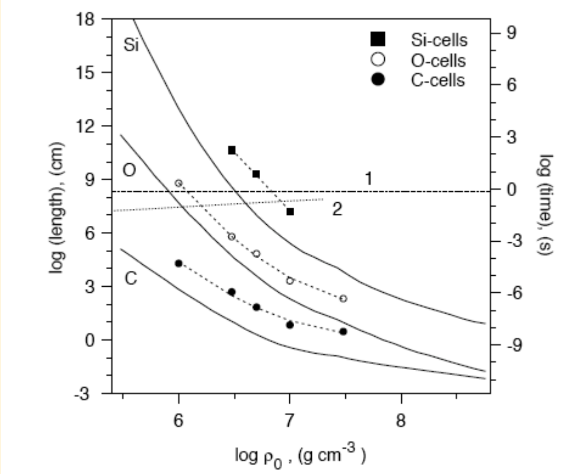
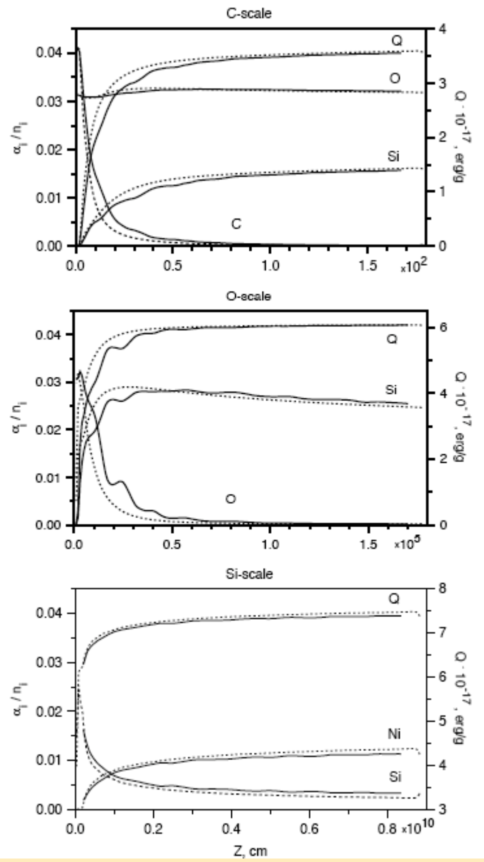


FIG. 5.—Key length scales and timescales as functions of initial density for self-sustained detonations. The solid lines are the half-reaction lengths and times for steady state one-dimensional detonations. The points are the detonation cell sizes for two-dimensional cellular detonations. The horizontal dotted line 1 shows the typical size of a Chandrasekhar mass white dwarf, ~2 × 10^8 cm. The dotted line 2 gives the scale of the density gradient, i.e., the length over which the density changes by a factor of 2 for a typical preexpanded white dwarf.

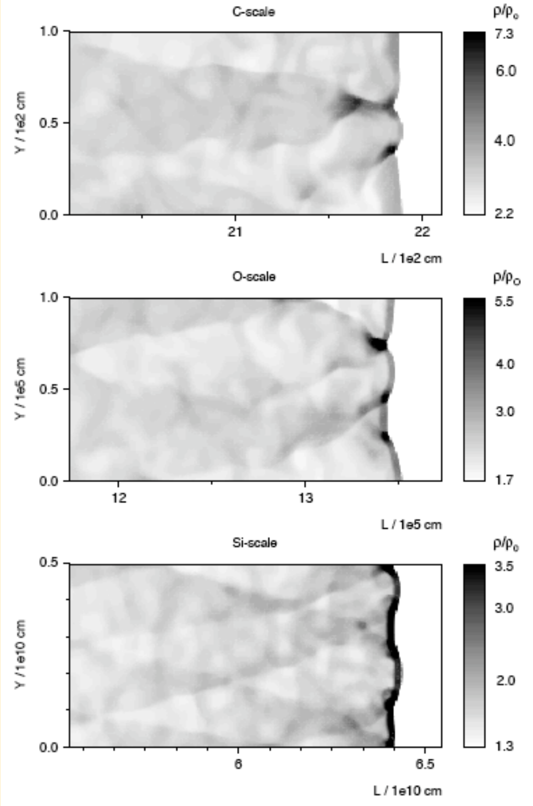
1D instability if $\rho > 2\ 10^7$ g cm^{-3}
(Khoklov 1993)

Effect of cellular detonations on SNIa

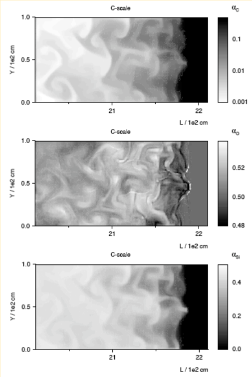
Gamezo et al. 1999, (5 10^6 g cm^{-3})



concentration & energy release



density

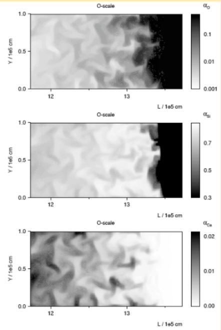


C

O

Si

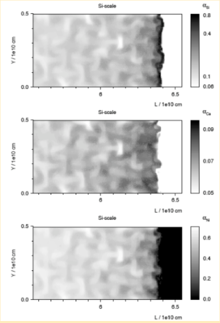
concentration



O

Si

Ca



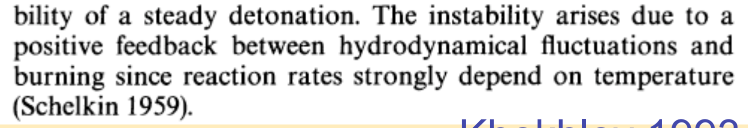
Si

Ca

Ni

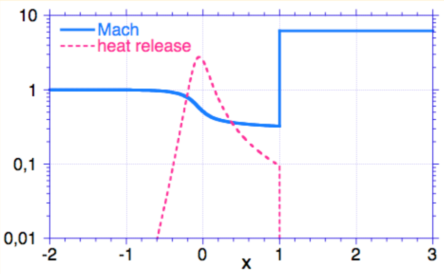
-increase of the half reaction lengths:
+30% for Si
+60% for C and O
-unreacted and overreacted regions: concentration inhomogeneities

Why should detonations be unstable ?



bility of a steady detonation. The instability arises due to a positive feedback between hydrodynamical fluctuations and burning since reaction rates strongly depend on temperature (Schelkin 1959).

Khokhlov 1993



The idea of a feedback loop
between the shock and the region of burning
is not new

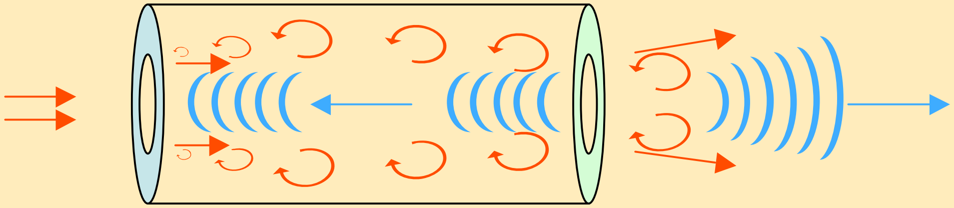
but it might be further developed,
quantitatively,
using a new formalism

« Aero-acoustic » instabilities

- advected perturbations
- acoustic feedback

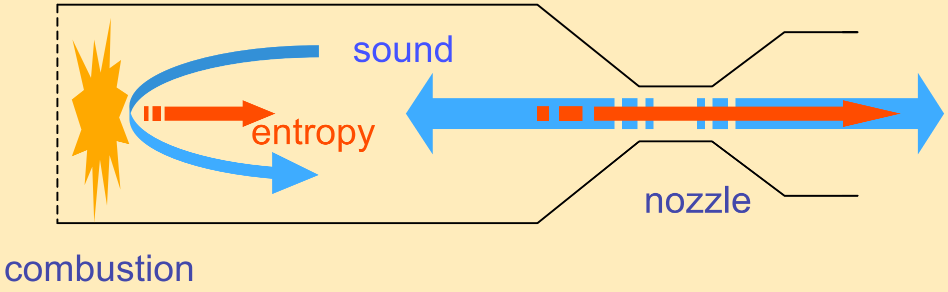
vibrations in Ariane 5:
segmented solid propergol
Mettenleiter et al. (2000)

- « vortical-acoustic » cycle



whistling kettle
Chanaud & Powell (1965)

- « entropic-acoustic » cycle



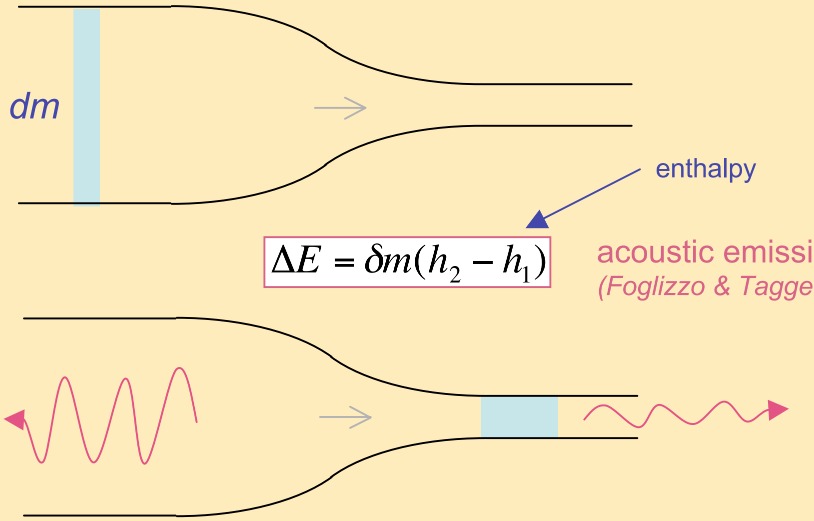
rumble instability of ramjet combustors
Abouseif, Keklak & Toong (1984)

Advective-acoustic coupling: 2 types of feedback

advection of entropy

↓

« entropic-acoustic » cycle



$$\Delta E = \delta m(h_2 - h_1)$$

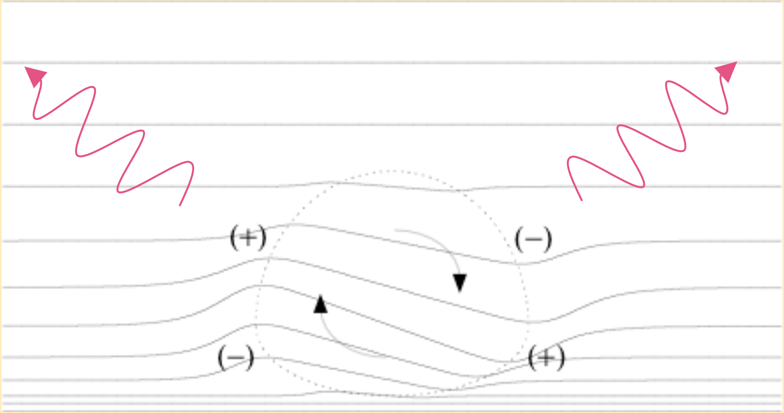
acoustic emission

(Foglizzo & Tagger 2000, adiabatic)

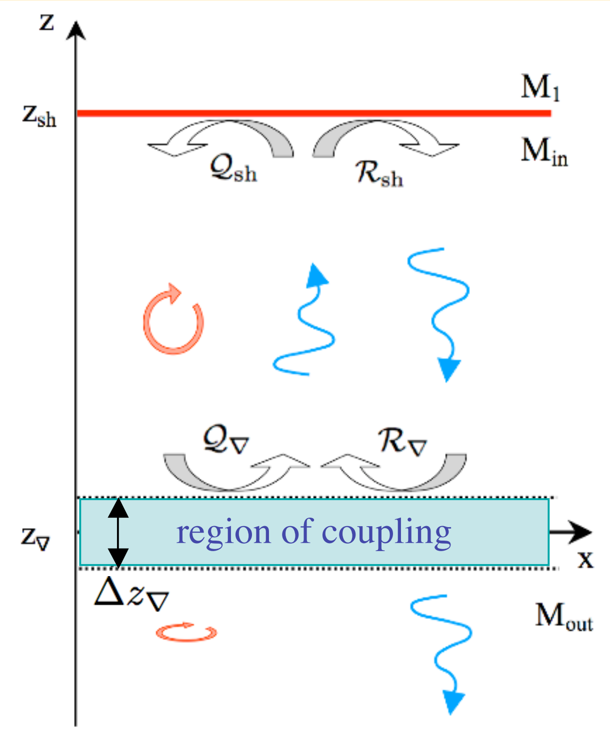
advection of vorticity

↓

« vortical-acoustic » cycle



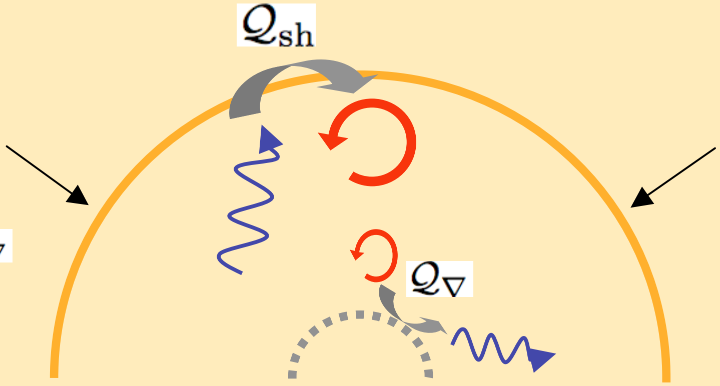
The formalism Q, R, τ_Q, τ_R of advective-acoustic instabilities



$$\mathcal{Q}e^{i\omega\tau_\mathcal{Q}}+\mathcal{R}e^{i\omega\tau_\mathcal{R}}=1$$

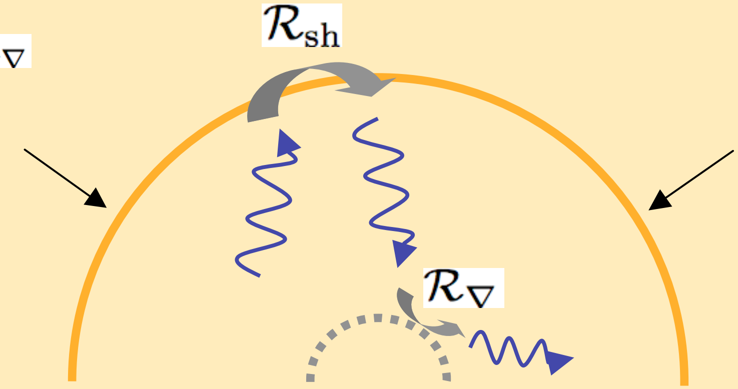
advective-acoustic cycle
efficiency $\mathcal{Q}\equiv\mathcal{Q}_{sh}\mathcal{Q}_\nabla$

timescale $\tau_\mathcal{Q}$

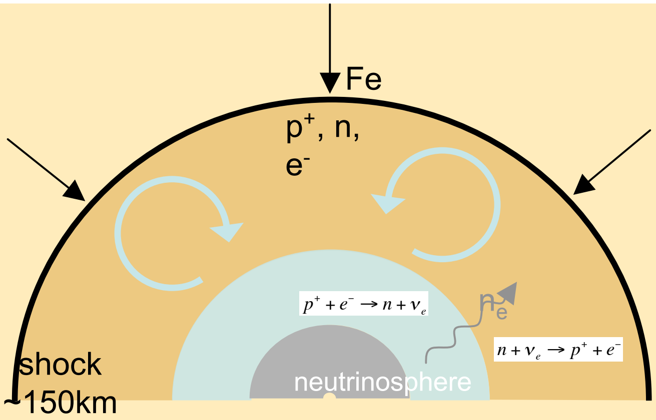


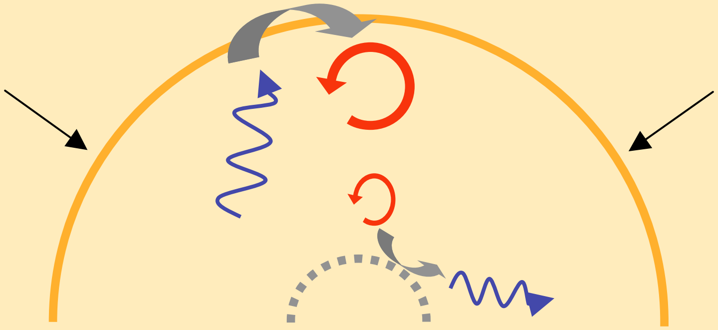
purely acoustic cycle
efficiency $\mathcal{R}\equiv\mathcal{R}_{sh}\mathcal{R}_\nabla$

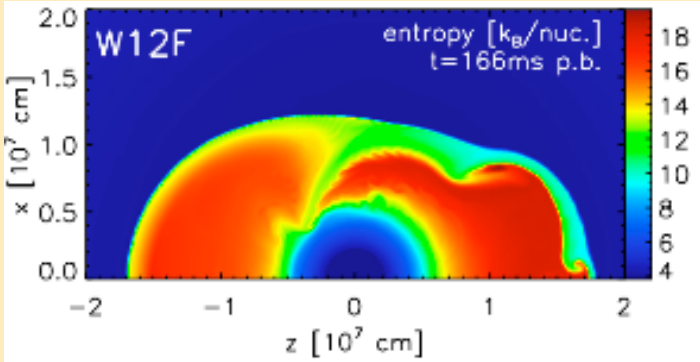
timescale $\tau_\mathcal{R}$

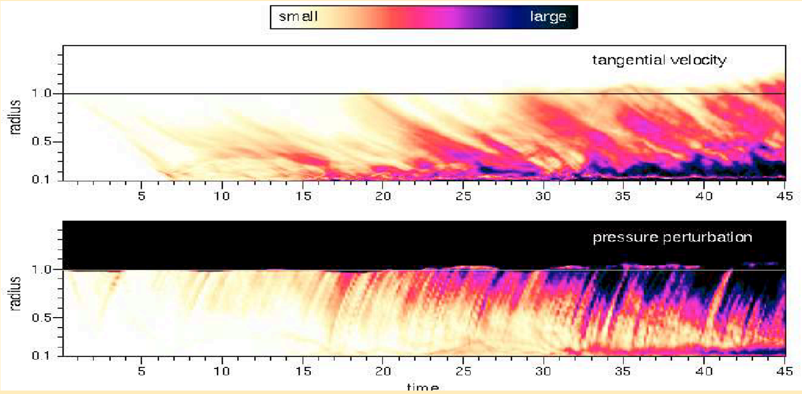


Core-collapse supernova: advective-acoustic cycle of a stalled accretion shock
(SASI)







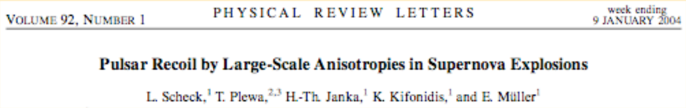


Evidence for a vortical-acoustic cycle *(Blondin et al. 2003)*

Blondin et al. ‘03, Ohnishi et al. ‘06, Blondin & Mezzacappa ‘06
Foglizzo et al. ‘07, Yamasaki & Yamada ‘07

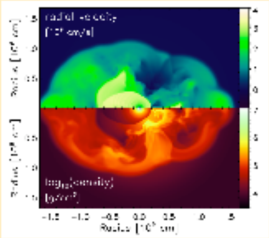
Some beautiful (possible) consequences of SASI

-Neutron star kicks *(Scheck et al. 2004, 2006)*

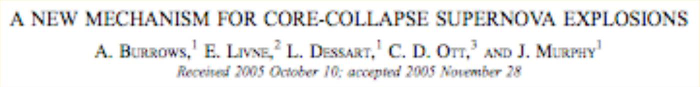

VOLUME 92, NUMBER 1 PHYSICAL REVIEW LETTERS week ending 9 JANUARY 2004

Pulsar Recoil by Large-Scale Anisotropies in Supernova Explosions

L. Scheck,[1] T. Plewa,[2,3] H.-Th. Janka,[1] K. Kifonidis,[1] and E. Müller[1]

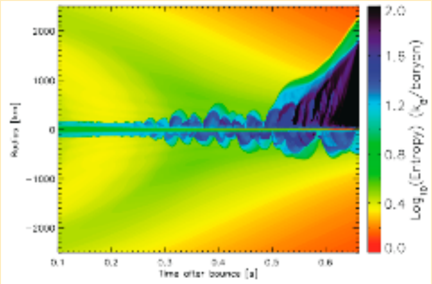


-New explosion mechanism driven by acoustic waves, initiated by the advective-acoustic cycle *(Burrows et al. 2005, 2006)*


A NEW MECHANISM FOR CORE-COLLAPSE SUPERNOVA EXPLOSIONS

A. BURROWS,[1] E. LIVNE,[2] L. DESSART,[1] C. D. OTT,[3] AND J. MURPHY[1]

Received 2005 October 10; accepted 2005 November 28

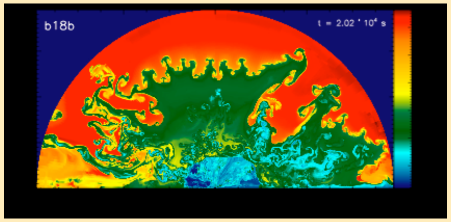


-Seed the H/He mixing in the neutrino-driven explosion of 1987A, $1 s < t < 10^4 s$ *(Kifonidis et al. 2006)*

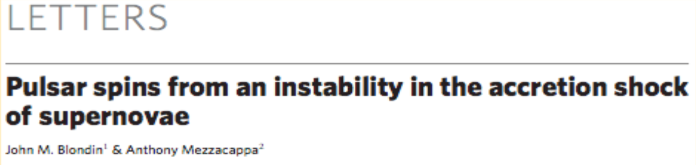

Non-spherical core collapse supernovae

II. The late-time evolution of globally anisotropic neutrino-driven explosions and their implications for SN 1987 A*

K. Kifonidis[1], T. Plewa[2], L. Scheck[1], H.-Th. Janka[1], and E. Müller[1]

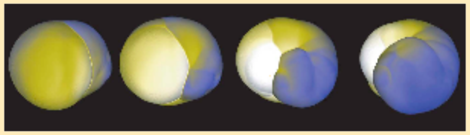


-Spin up of the neutron star *(Blondin & Mezzacappa 2007)*


LETTERS

Pulsar spins from an instability in the accretion shock of supernovae

John M. Blondin[1] & Anthony Mezzacappa[2]

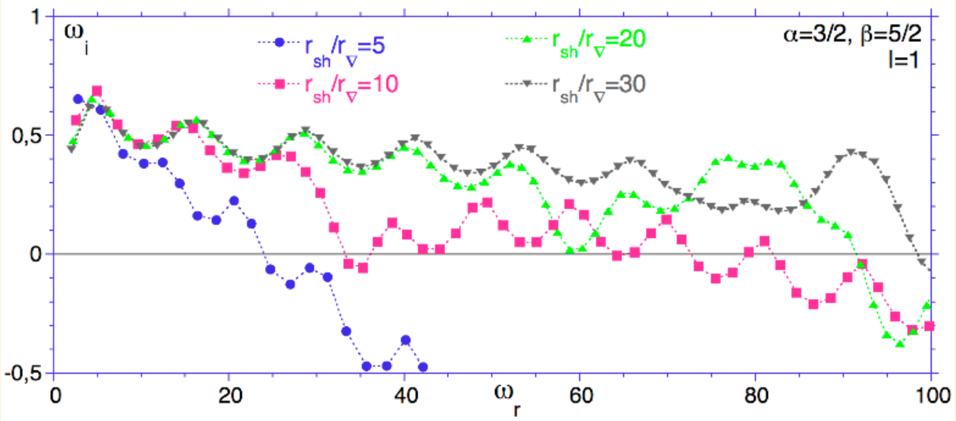


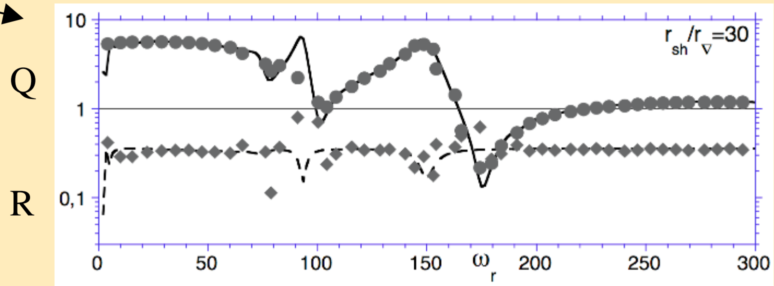
- Onset of a neutrino-driven explosion, 15 M star *(T. Janka, Aspen 2007)*

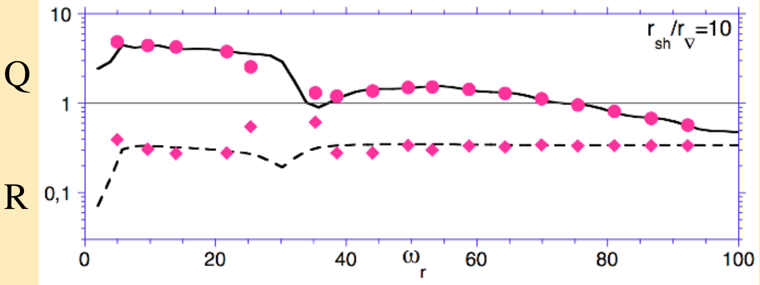
NEW

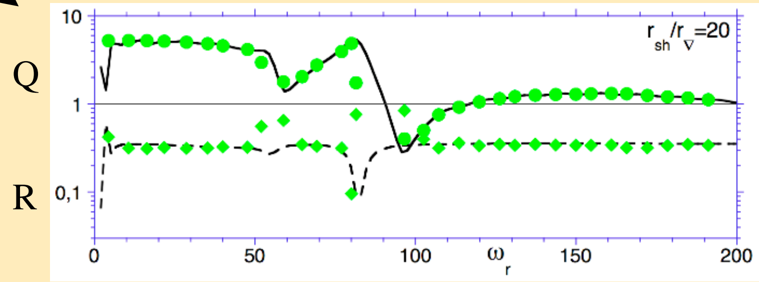
Core-collapse SN:
determination of the instability mechanism behind SASI

(Foglizzo, Galletti, Scheck & Janka 2007)







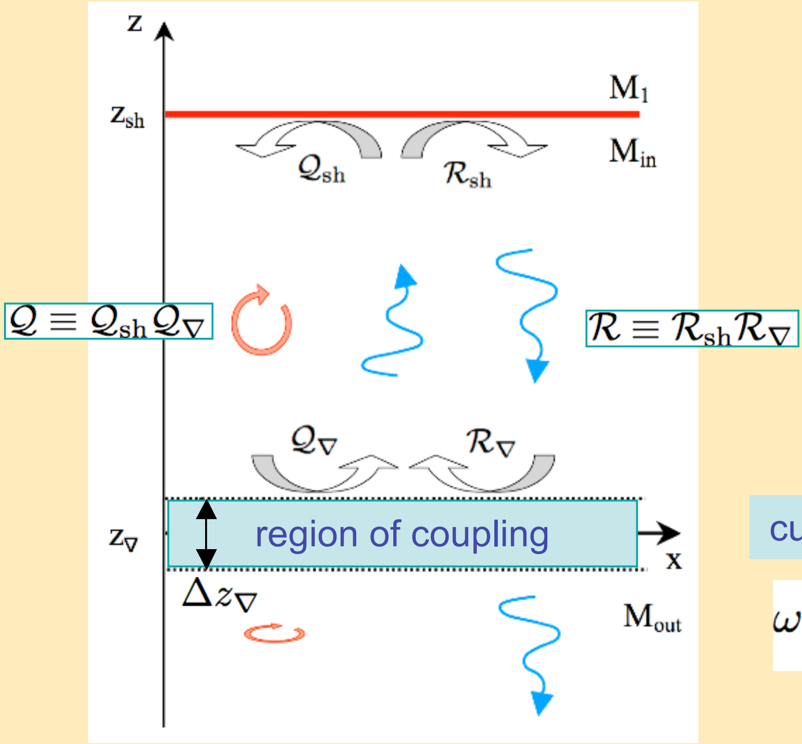


-The advective-acoustic cycle is unstable: Q>1
-The purely acoustic cycle is stable: R<1

SASI is due to an advective-acoustic instability

A generic example of advective-acoustic instabilities

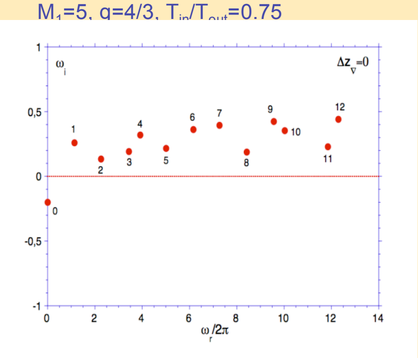
$$\mathcal{Q}e^{i\omega\tau_{\mathcal{Q}}}+\mathcal{R}e^{i\omega\tau_{\mathcal{R}}}=1$$



-parallel flow, localized coupling (γ, M_1, Δz_∇)
-2-D perturbations $\omega, k_\perp$

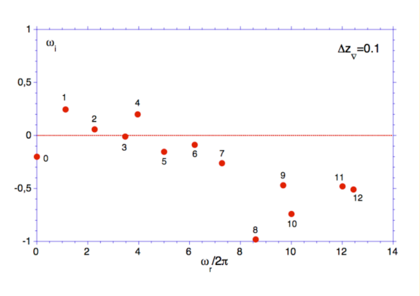
cut-off frequency

$$\omega_\nabla \sim \frac{v_\nabla}{\Delta z_\nabla}$$



$$\Delta z_\nabla = 0$$

(fully analytic)



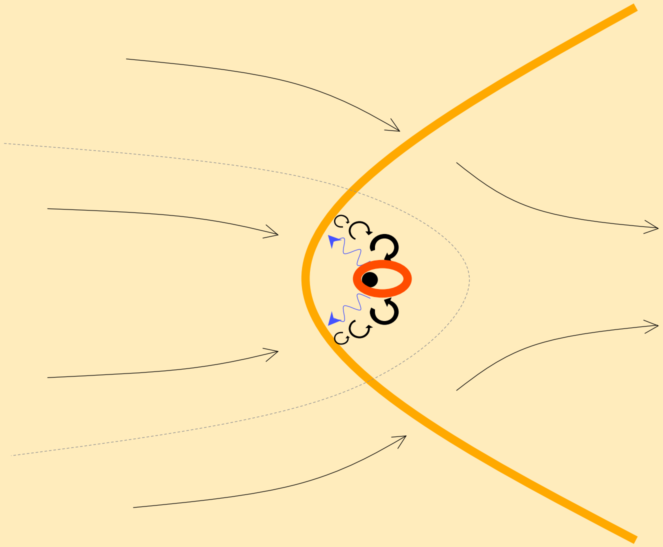
$$\Delta z_\nabla = 0.1$$

cut-off frequency :
-> SASI is a low frequency instability
-> Pathologies of the square wave model for detonations and importance of the size of the region of heat release (Zaidel 1961, Clavin et al. 1997, Short & Sharpe 2003)

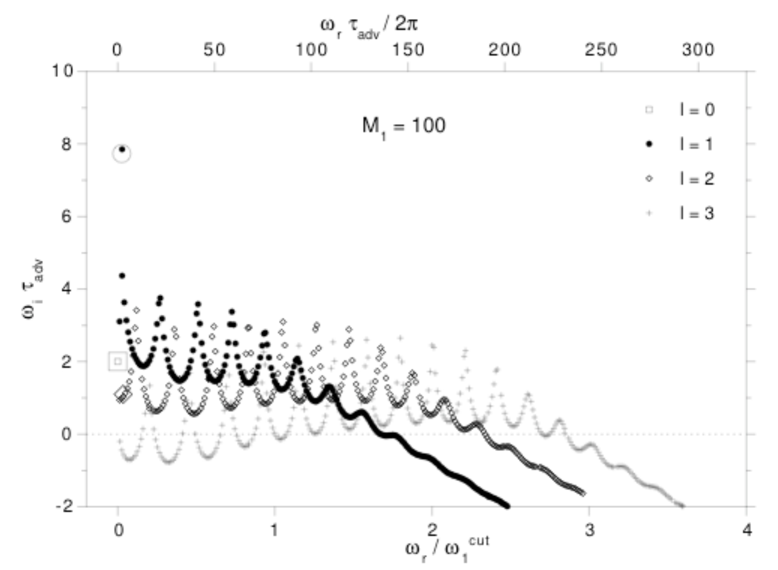
Advective-acoustic cycle in a transonic flow

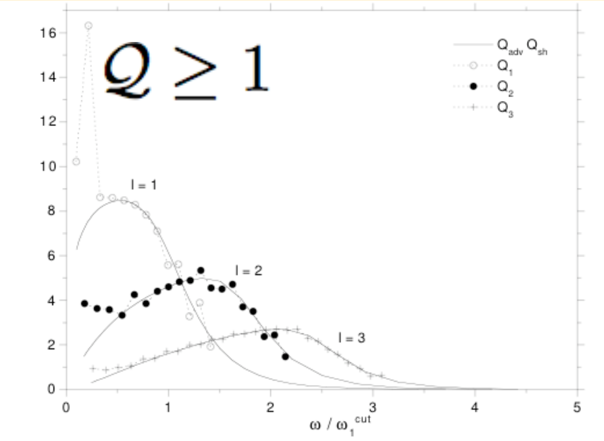
Foglizzo (2001, 2002), Foglizzo, Galletti & Ruffert (2005)

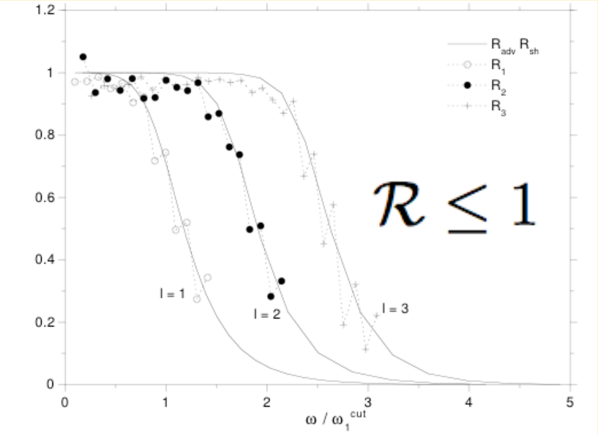
identification of the 2 cycles (Q, τ_Q) and (R, τ_R)



Bondi-Hoyle-Lyttleton accretion





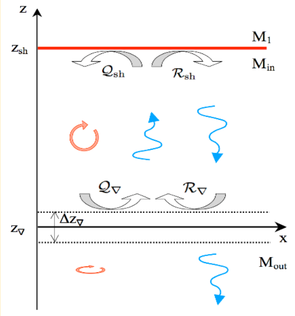


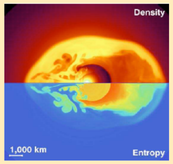
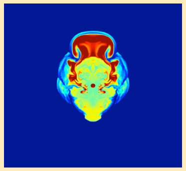
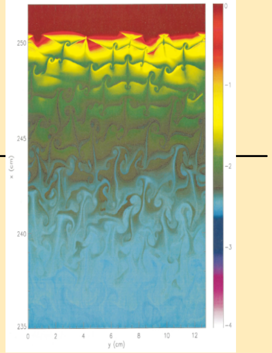
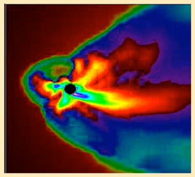
Conclusions

The instability of detonations can be viewed as an advective-acoustic cycle between the shock and the region of heat release

The Q,τ_Q R,τ_R formalism will be applied to better understand unstable detonations

- Entropic-acoustic instability of pulsating detonations (1D)
- Vortical-acoustic instability of cellular detonations (2D)
- sensitivity to the size of the region of heat release



	cooling	adiabatic	heating
decelerated, subsonic			
accelerated, subsonic			
accelerated, transonic			

Scheck et al. '04, Blondin et al. '03, Ruffert '95, Timmes et al. '00