Compulsory Deep Mixing of ^{3}He and CNO Isotopes in the Envelopes of low-mass Red Giants.

Peter P. Eggleton, David S. P. Dearborn,

and John C. Lattanzio.

A serendipitous discovery made during 3D modeling of the He flash.

The Big Bang is supposed to produce ^{3}He at $\sim 10^{-5}$ by mass fraction.

Low-mass stars ($1-2\,M_\odot$) produce further ^{3}He, $\sim 10^{-3}$, in their interiors, and mix it into their convective envelopes on the FGB.

They eject $\gtrsim 0.3\,M_\odot$ of envelope into the ISM further up on the FGB.

So the ISM ought to be richer in ^{3}He than the Big-Bang value, but it isn't (Hata et al. 1995).

We have discovered that an important and unexpected mixing process takes place on the FGB, once the surface convective envelope has reached its maximum depth and then started to retreat.

The mechanism is a Rayleigh-Taylor instability, driven by a small local minimum in the mean molecular weight.

The minimum occurs because the ^{3}He left behind by the retreating SCZ is burnt primarily by

$$^3\mathrm{He} + {}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + {}^1\mathrm{H} + {}^1\mathrm{H}.$$

This reaction *lowers* the mean molecular weight, unusually for a stellar nuclear reaction.

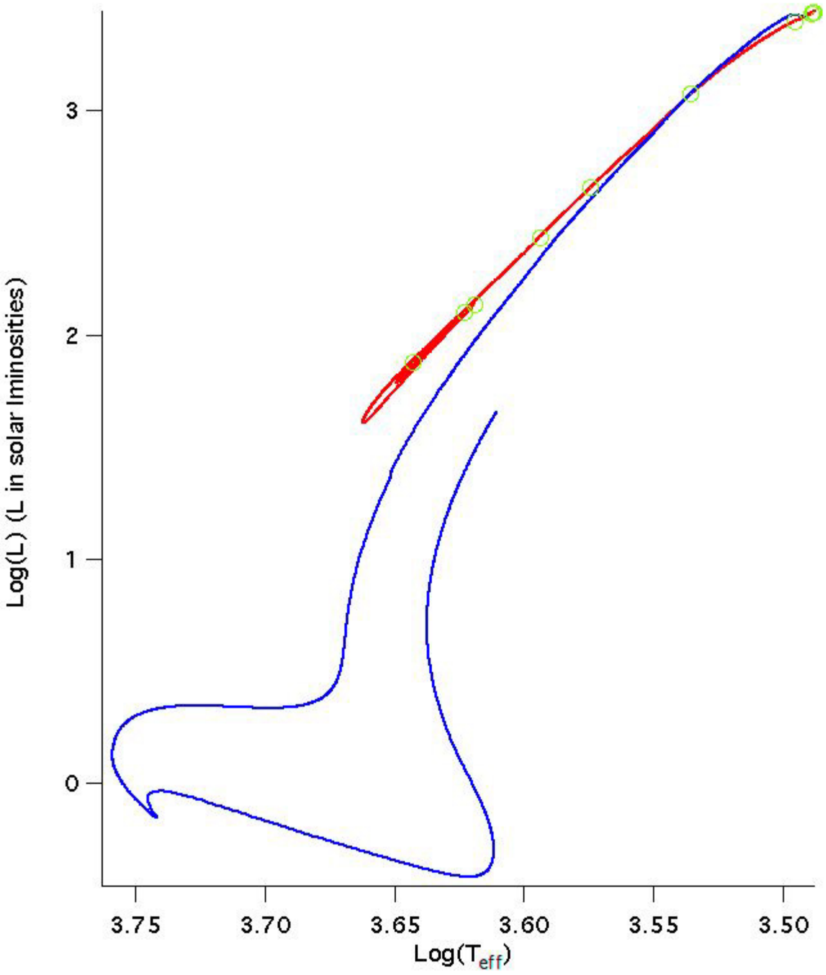
The ^{3}He-burning occurs somewhat above the main H-burning shell, but well below the classical base of the SCZ.

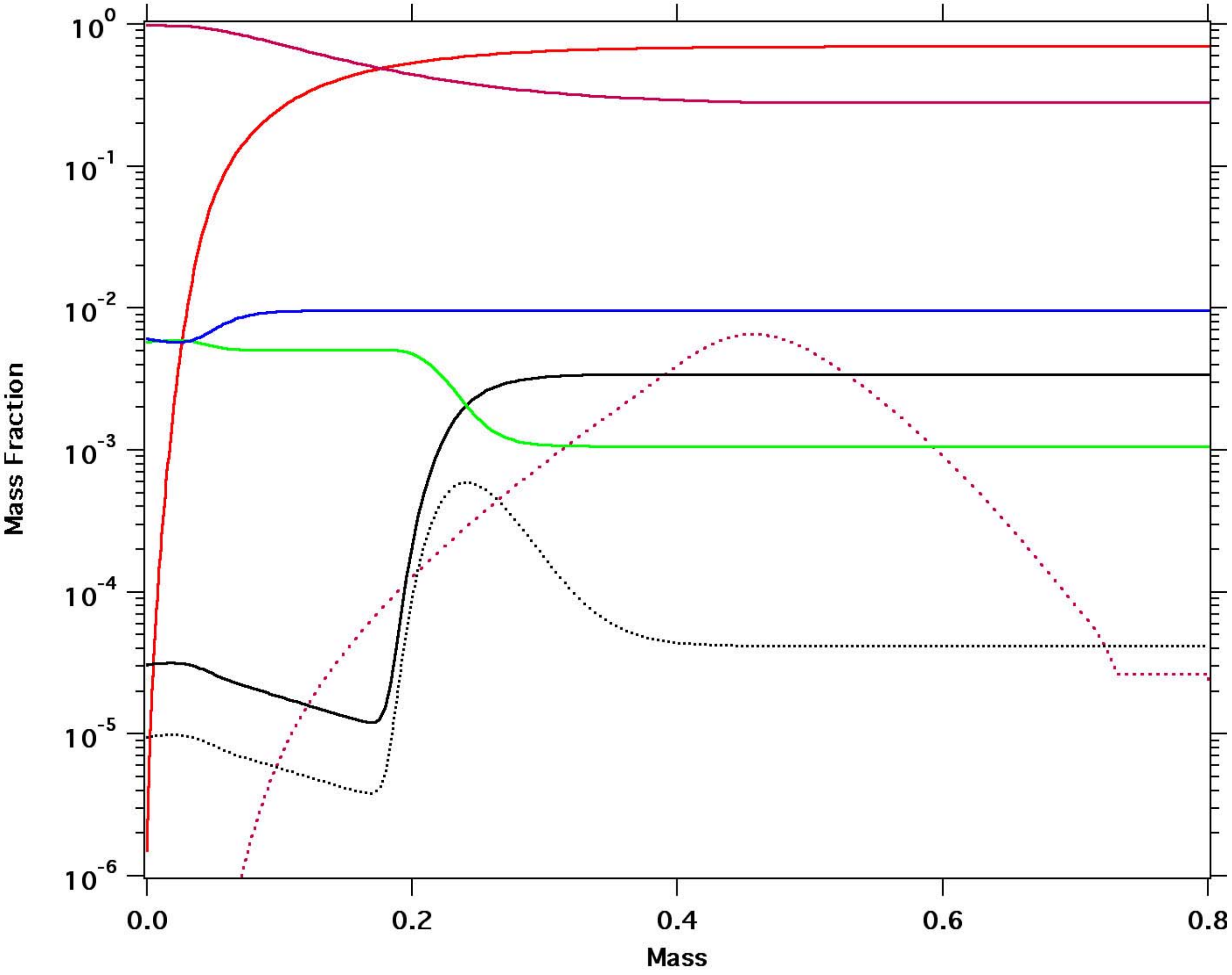
Although first noticed during a helium-flash calculation, the phenomenon shows up earlier, as soon as the classical SCZ, having penetrated to maximum depth inwards, starts moving outwards again.

We estimate that mixing driven by this instability will allow most of the ^{3}He produced during the MS phase to be destroyed again, on the upper half of the FGB. This solves the problem mentioned on page 1.

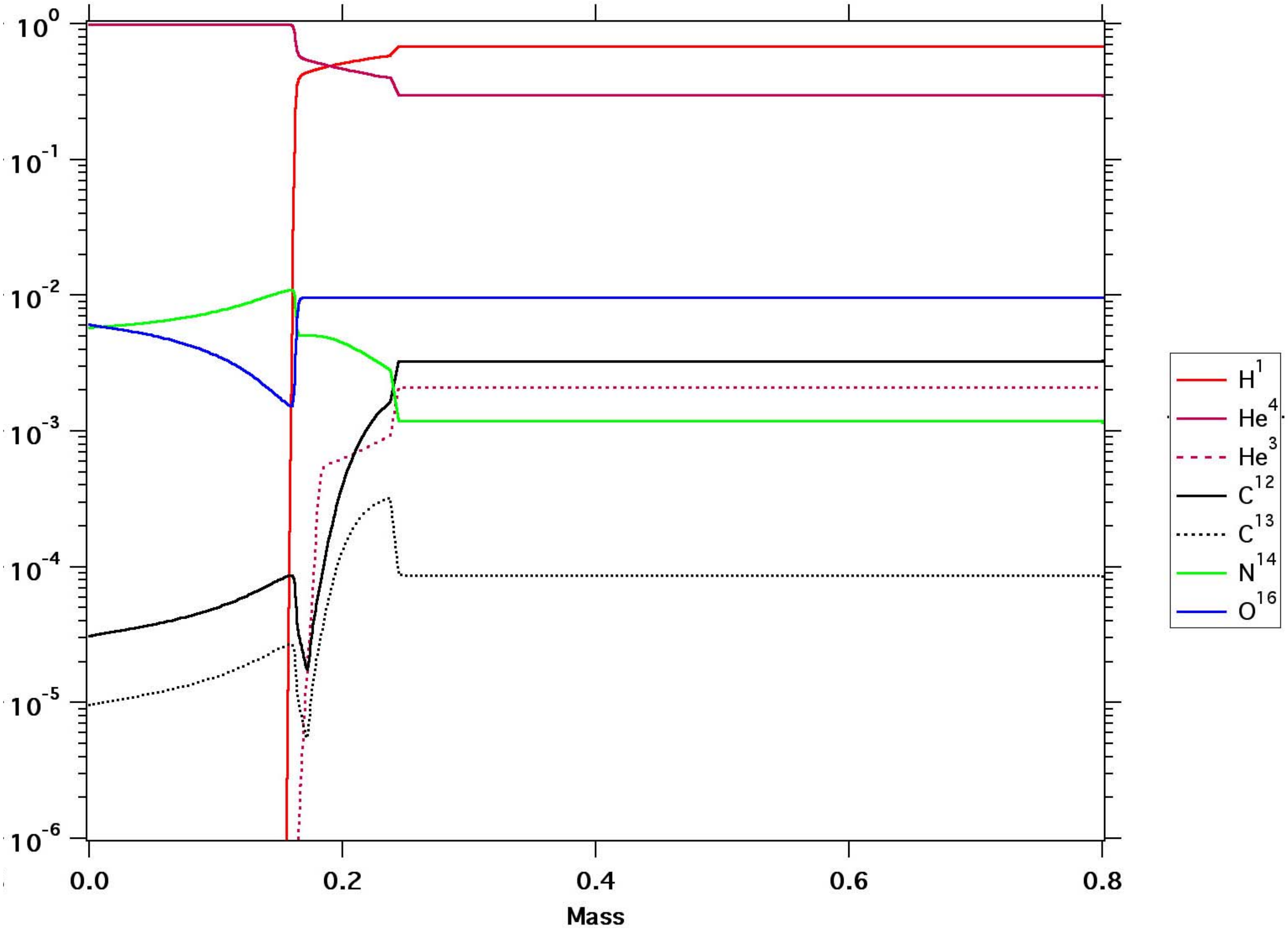
At the same time, the mixing will also enhance the abundance of ^{13}C relative to ^{12}C on the second half of the FGB.

A 1D calculation of a star from the pre-MS Hayashi track to the First Giant Branch, through the He flash, on to the Horizontal Branch and back to the Asymptotic Giant Branch.

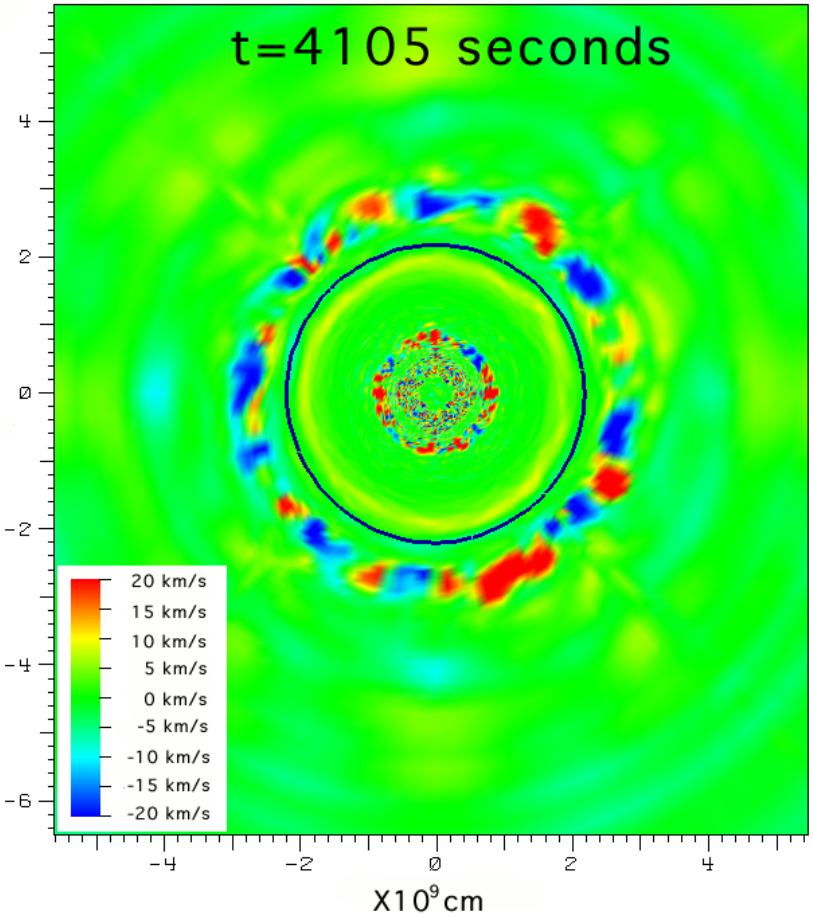




End of MS. Notice ^{3}He peak (red dots) and ^{13}C peak (black dots).



Halfway up the FGB, the surface convection zone (SCZ) reaches its deepest extent. Surface ^{3}He is increased by $\sim$ 100, ^{13}C by $\sim$3.



3D model of He flash, in 2D cross-section. Blue circle is H-burning shell.

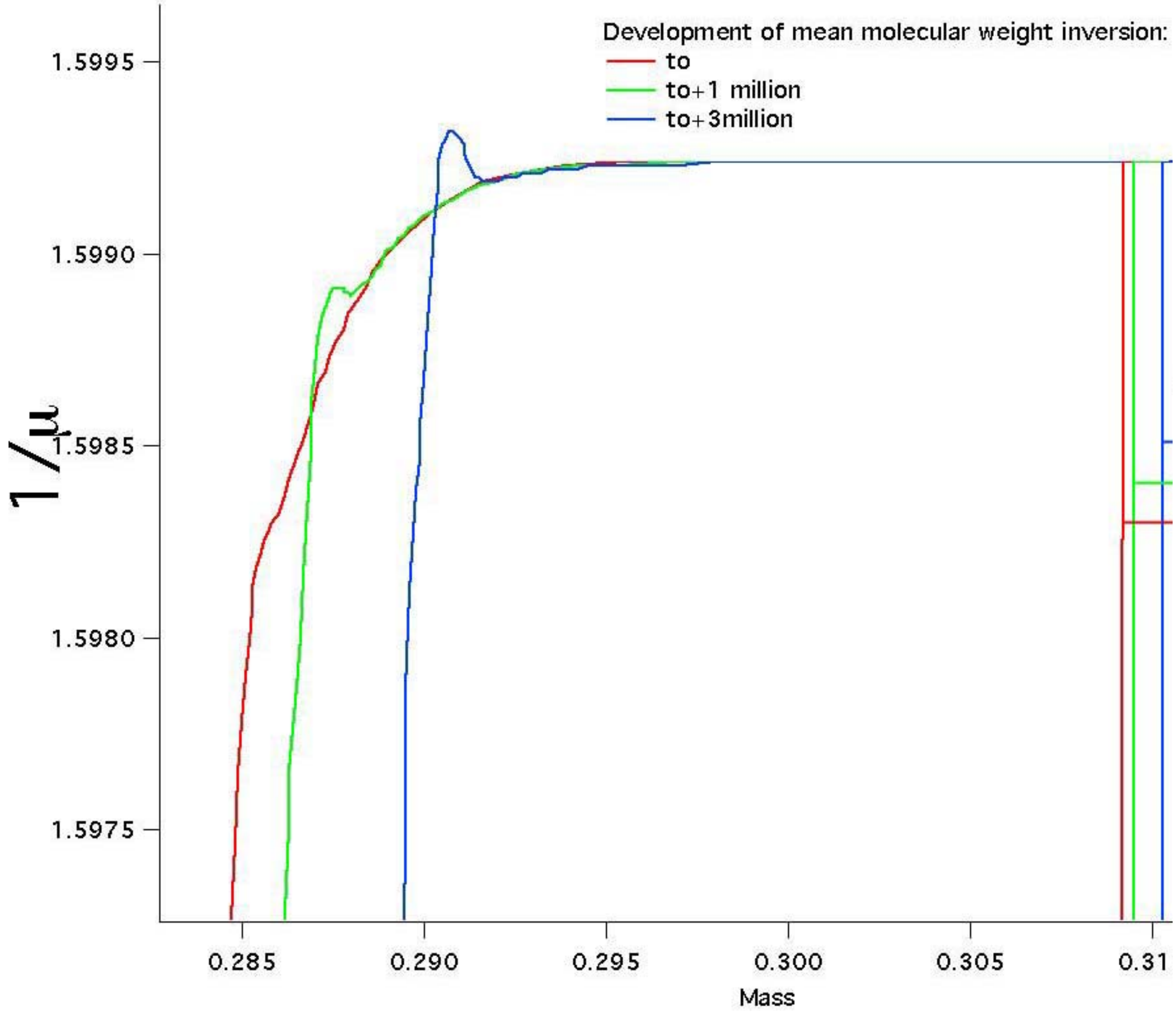
SCZ is far outside frame. Turbulent motion occurs in He-burning shell.

(He-flash movie here)

The previous Fig. showed a ring of turbulent motion *outside* the H-burning shell, but well below the SCZ.

It was unexpected: the structure should be stable to motion there.

Close inspection showed a slight mol. wt (μ) inversion there.


Development of mean molecular weight inversion:
to
to+1 million
to+3million
1.5995
1.5990
1.5985
1.5980
1.5975
1/μ
0.285
0.290
0.295
0.300
0.305
0.31
Mass

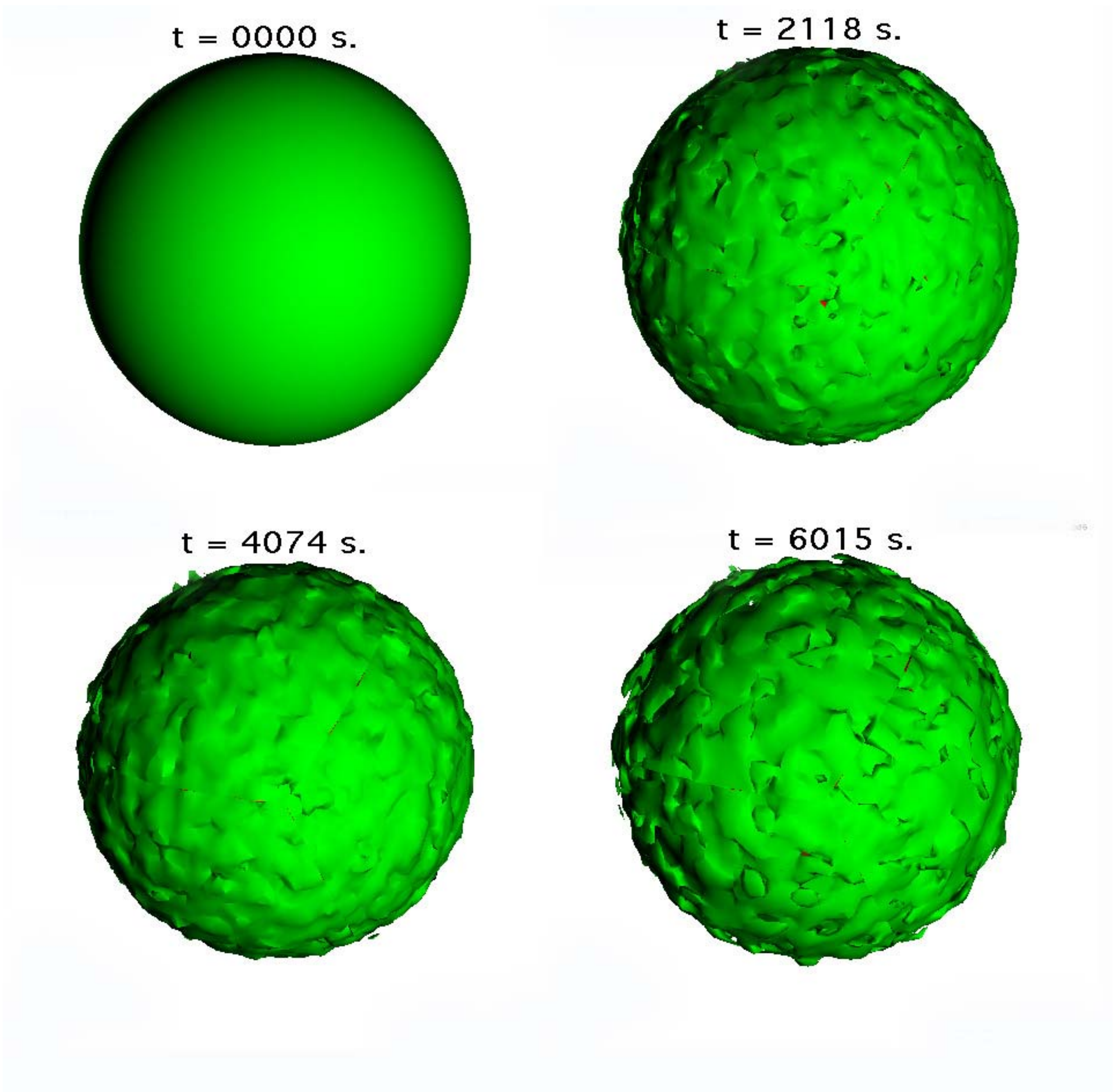
We went back down the 1D Giant Branch to a point not long after the SCZ bottomed out.

We mapped a 1D model that had developed this μ-inversion on to a 3D model, and evolved the latter using the 3D hydrodynamics code ‘DJEHUTY’ of the Lawrence Livermore National Laboratory. This code can in principle handle a complete star; but to economise on meshpoints we placed our outer boundary just below the SCZ.

(shell movie here)

The code is discussed in some detail by Dearborn, Lattanzio, & Eggleton, *Astrophys. J.* **639**, 405 (2006).

We used $\sim 10^7$ meshpoints. The timestep is Courant-limited, to about 0.1 sec, and we ran it on 351 processors for $\sim$ 10,000 secs, taking a few days of cpu time.


t = 0000 s.
t = 2118 s.
t = 4074 s.
t = 6015 s.

Doesn't the motion due to the Rayleigh-Taylor instability soon stop, as the molecular-weight inversion gets diluted by the mixing it causes?

No, because the mixing advects fresh ^{3}He into the ^{3}He-burning zone at much the same rate as it advects the products out. We conclude that the RT-mixing zone will keep growing, until it unites with the normal SCZ.

Isn't the RT motion very small compared to the motion in the normal SCZ, because $\Delta\mu/\mu$ is small ($\sim 10^{-4}$)?

No, because the buoyancy term that drives the normal convection, $\Delta T/T$, is also small. ΔT is the *excess* temperature of an element that has been raised adiabatically (by about one pressure scaleheight) over the ambient temperature. Since, in the deeper parts of an SCZ, the ambient temperature gradient is very nearly adiabatic, $\Delta T/T$ is quite small ($\sim 10^{-4}$!).

Our 3D model produced rapid motion, comparable to the normal convective motion, because the motion generated by the RT-instability was artificially suppressed in the 1D model, allowing the μ-inversion to reach its maximum size.

In reality, motion will start sooner, almost as soon as the μ-inversion begins. The actual velocities produced will be smaller, but we estimate that they will still be quite enough to mix the entire zone between the ^{3}He-burning shell and the classical SCZ. The velocities would have to be less than $\sim 10^{-4}$ cm/s in order *not* to do this.

By various back-of-the-envelope calculations we estimate that the velocities are likely to be in the range $0.2 - 2$ cm/s. Our 3D model, starting from an artificially large inversion, produced velocities of ~ 200 m/s.

Over the last 30 years or so, observation has suggested that some mixing process must connect the SCZs of FGB stars to the H-burning shell. Giants often show ^{13}C abundances elevated above the value expected at and beyond the first dredge-up. Also ^{14}N/^{12}C is often elevated. We expect the SCZ to be processed to a substantially deeper level, to at least the ^{3}He-burning shell, where some at least of the ^{12}C can burn to ^{13}C; and perhaps there is sufficient 'undershooting' to process significant amounts of ^{12}C to ^{14}N.

We have incorporated in our 1D model a crude diffusive-mixing model where the diffusion coefficient is proportional to the difference between the local μ and the minimal μ, provided that the minimal μ is further in. The magnitude of the mixing rate that we use is chosen to give roughly the velocity of rising material that we expect between the μ-inversion peak and ths SCZ. The results, fortunately but not surprisingly, do not depend very strongly on the value chosen.

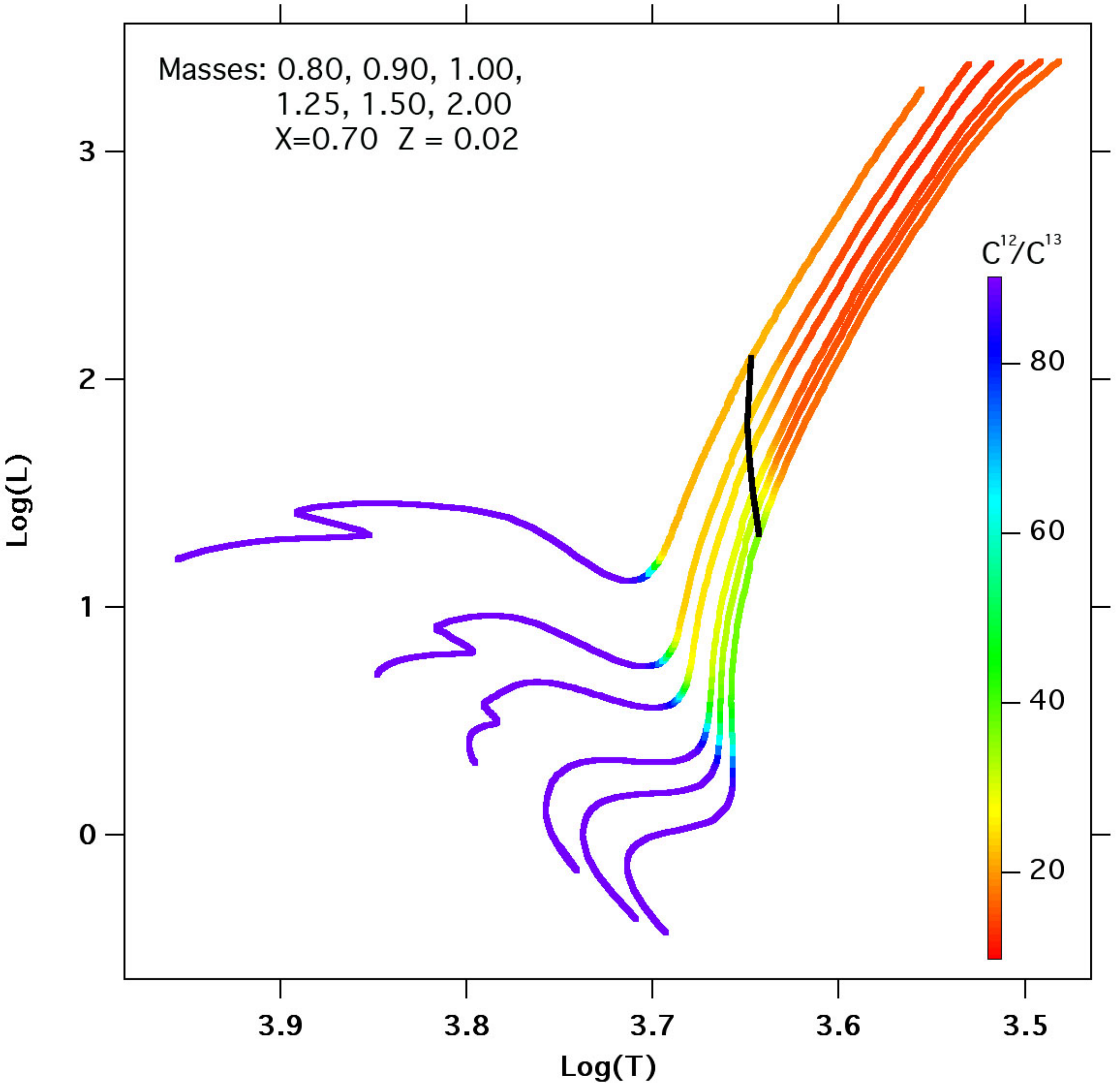

Masses: 0.80, 0.90, 1.00,
1.25, 1.50, 2.00
X=0.70 Z = 0.02
C^{12}/C^{13}
80
60
40
20
3
2
1
0
Log(L)
3.9
3.8
3.7
3.6
3.5
Log(T)

Table 6. ^{3}He/^{3}He (original)

	X=0.70, Z=0.02		X=0.738, Z=0.001		X=0.74, Z=0.0004	
M	Peak	Mixed	Peak	Mixed	Peak	Mixed
0.80	76.6	2.7	54.7	0.86	53.0	0.62
0.85	68.2	2.9	46.5	0.90	45.1	0.66
0.90	61.1	3.1	40.0	0.95	38.7	0.73
1.00	49.7	3.7	32.9	1.12	30.0	0.83
1.25	31.7	4.5	21.8	1.60	19.9	1.21
1.50	21.8	5.5	15.7	2.08	14.4	1.91
2.00	12.8	7.0	9.5	6.70	8.7	8.39

The Table shows the extent of ^{3}He enrichment, over the original value, in the SCZ if (a) our process does *not* take place ('peak'), and (b) if it does ('mixed').

This project exemplifies two principles:

(1) Interesting processes are often discovered by accident, while one is looking for something else. So much for the bean-counters who insist that a project should identify what it will ‘deliver’ in year 1, year 2 and year 3.

(2) 3D computation, though expensive, can produce something useful and unexpected. With the aid of a short run in 3D, one can hope to refine a simplistic 1D model which will reach results quickly.

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