"Hydrodynamics" of Supersolids:

Andreev and Lifshitz (1969)
Saslow (1977)
Liu (1978)

Supersolids Miniconference

Host: Kavli Institute of Theoretical Physics

Organizers: Moses Chan and David Ceperley

Chester (PRA, 1969)

Speculations on Bose-Einstein Condensation and Quantum Crystals*

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It is shown, by almost rigorous arguments, that there exist many-body states of a system of interacting bosons which exhibit both crystalline order and Bose-Einstein condensation into the zero-momentum eigenstate of the single-particle density matrix. The implications of this result are discussed in relation to theories of superfluidity and the nature of quantum crystals.

Invokes proof by Reatto on Bose-Einstein condensation for Jastrow wavefunction.

Notes that equivalent classical Monte Carlo theories yield periodic solids.

Proposes that Quantum Monte Carlo can also yield a periodic solid with Bose-Einstein condensation.

Chester, continued

vacancies. We may therefore add one final speculation, namely, that a quantum crystal can only have a Bose-Einstein condensate if it has a finite fraction of vacancies. We see no reason, whatsoever, to suppose that a quantum crystal cannot have a finite fraction of vacancies at absolute zero. Liquid helium exists at absolute zero and this suggests that a crystal with a finite amount of spatial disorder could exist at absolute zero. If, on the other hand, a finite fraction of vacancies can only exist at elevated temperatures, then it might be impossible to have a Bose-Einstein condensate because of the high temperature. We pointed out in Sec. IV that it is almost impossible to predict the temperature at which such a condensation might occur.

Also proposes possibility of vacancy-induced Bose-Einstein condensation.

A.F.Andreev and I.M.Lifshitz (1969)

Quantum Theory of Defects in Crystals

Zh. Eksp. Teor. Fiz. 56, 205(1969); JETP 29, 1107 (1969).

- From Abstract: "At sufficiently low temperatures localized defects or impurities change into excitations that move practically freely through a crystal. As a result instead of the ordinary diffusion of defects, there arises a flow of a liquid consisting of 'defectons' and 'impuritons'…Such a crystal is neither a solid nor a liquid. Two kinds of motion are possible in it; one possesses the properties of motion as an elastic solid, the second possesses the properties of motion in a liquid. Under certain conditions the 'liquid' type of crystal motion possesses the property of superfluidity…"
- From Text: "A change in the number of defectons may only take place on the surface of the crystal. The number of defectons in the bulk changes as a consequence only of their diffusion to the surface.''

Andreev and Lifshitz, II

- "As already noted, in a crystal containing zero-point defectons the number of lattice sites, i.e., the number of maxima in the density function, does not coincide with the number of atoms. For this reason there are two possible types of motion in the crystal. The first of these is associated with the displacement of the lattice sites and is characteristic of an elastic solid. The second is associated with mass transport by means of the motion of the zero-point defectons while the lattice sites remain essentially fixed. This kind of motion possesses the properties of motion of a liquid.[1] Thanks to this a crystal is able to flow through a capillary in a gravitational field.

  These two types of motion are related to each other by the conditions at the surface of the crystal. Namely, a single-valued relation exists between the stresses normal to the surface and the concentration of defects near the surface. Upon deformation of the crystal there arises therefore a self-consistent motion of the lattice sites and a flow of defectons, accompanied by a transport of mass and therefore causing quantum fluidity of the crystal."

1) We note that a similar situation occurs in metals, which are crystals containing an electronic liquid. In metals, however, the situation is complicated due to the condition of electrical neutrality.

Andreev and Lifshitz, III

- "Since a non-ideal Bose gas possesses the property of superfluidity, the liquid type of motion of a crystal containing Bose zero-point defectons will also possess this property (superfluid flow of the crystal through a capillary). The same also holds under well-known conditions in the case of Fermi defectons.
- It is of interest to note that defects of a fundamentally new type may exist in a superfluid crystal. The question concerns vortex lines, i.e., linear defects for which the phase of the condensate's wavefunction changes by 2π upon going around the defect."

A.F.Andreev and I.M.Lifshitz, IV

- Considerations on scattering of impurities and vacancies.
- Hydrodynamics has three velocities, two densities:
  (1) lattice ($v_L = du/dt$), with u the lattice coordinate
  (2) normal fluid (excitations) v_n and density ρ_s
  (3) superfluid v_s and density ρ_s ($\rho_0 = \rho_n + \rho_s$)
- Derived equations of "hydrodynamics" by analogy with derivation for superfluid HeII.
- As usual, superfluid accelerates in response to gradient in chemical potential.
- Get $v_L = v_n$, and lattice moves in response to stress.
- Two longitudinal modes: for small ρ_s, elastic waves not mass-loaded by ρ_s, and 4th sound-like wave loaded only by ρ_s (not second sound).
- Transverse elastic waves (Suzuki et al, 2005) not mass-loaded by ρ_s.
- No Landau-like, excitation-based theory for ρ_n or for ρ_s.

Saslow PRB 15, 173 (1977)
Micro & Macro Theory of Supersolid

- Developed Landau-like theory for normal fluid density and theory for long-wavelength fourth sound that was based on all-scale response (like Born's microscopic theory of long-wavelength phonons).
- Re-did A&L hydrodynamics with view to building in more microscopics. Took v_s to be defined relative to the lattice, so it was not a fully Galilean velocity, contrary to A&L. Only v_n and lattice velocity *du/dt* are Galilean.
- Results very similar to Andreev and Lifshitz; same normal mode frequencies, but different v_n/v_s ratio. Incomplete analysis of mode-generation problem.

Mario Liu - Macroscopics
PRB 18, 1165 (1978)

- Re-examined hydrodynamics of A&L and of Saslow.
- Concluded that A&L made inappropriate approximation for elastic properties. When corrected, Liu showed how, for small ρ_s/ρ, the theory gives A&L's 4th sound-like mode for the supersolid but 2nd sound for the superfluid.
- Concluded that Saslow should have used Galilean v_s for a consistent hydrodynamics.
- Showed that proper non-Galilean velocity is like the "thermal velocity" $dv_T/dt \sim dT/dx$, first noted by Halperin and Hohenberg (1968). "superthermality" involves entropy flow. Discussed relation to Enz's theory of 2nd sound in crystals.

Hydrodynamics: Puzzles and Problems?

- At T=0, expect that ρ_s/ρ_0 is small and that $\rho_n/\rho_0 = 0$. What takes up the difference? Surely the lattice, but how?
- At $T=T_c$, $\rho_s/\rho_0 = 0$ but ρ_n/ρ_0 is small. What takes up the difference? The lattice, but how? Not until $\rho_n/\rho_0 = 1$, $T=T^*$, should ordinary lattice dynamics apply.
- Is there a separate hydrodynamics for $T_c < T < T^*$?
- How does this connect to theories of 2nd sound in solids?

Saving the Phenomenon?

- Moses Chan told us that NCRIF disappears for really pure solid ^{4}He (1ppb of ^{3}He) that was not annealed - but probably not glassy.
- John Reppy told us that NCRIF disappears for annealed samples of moderately pure solid ^{4}He (1ppm of ^{3}He).
- Numerous Monte Carlo theories tell us that pure solid ^{4}He either isn't supersolid or is barely supersolid, whereas adding vacancies is good for ρ_s.
- Possibilities: (1) We need vacancies and ^{3}He helps make vacancies and annealing removes vacancies; (2) perhaps the critical velocity is very low in very "clean" samples (why?); (3) ?….