Ring Exchanges in a Perfect solids of ^4He

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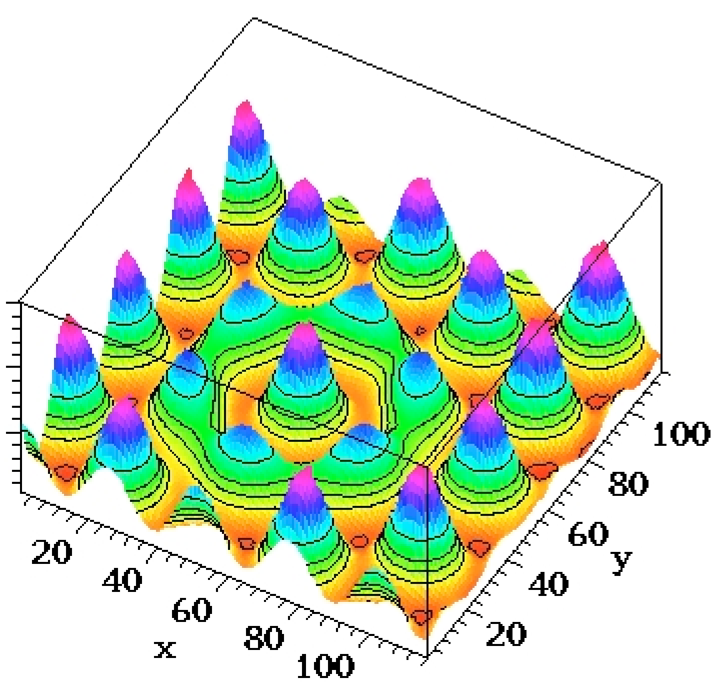


hexagonal exchange in 2D Wigner crystal

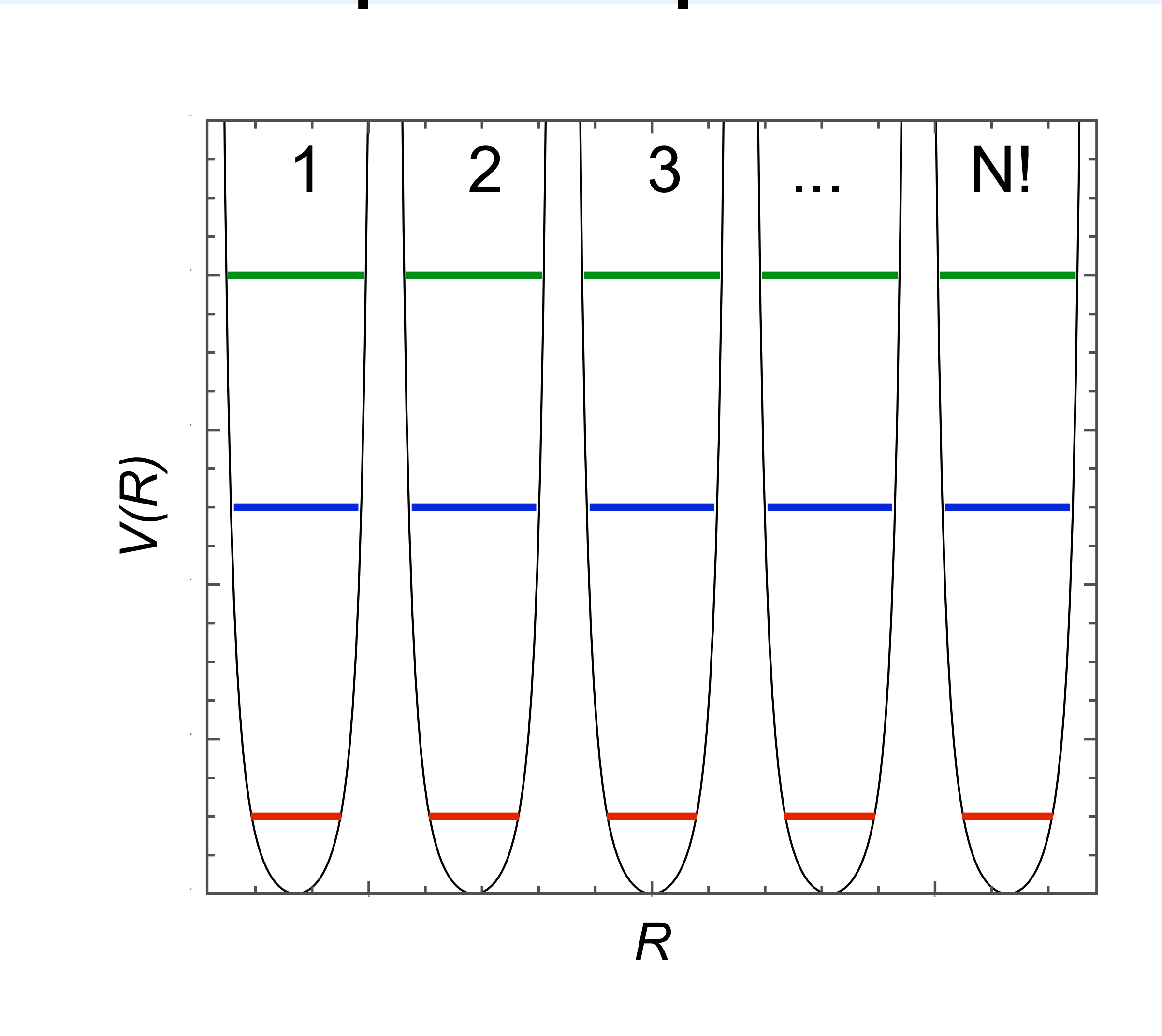
Ring Exchanges are Evaluated by PIMC

outline

- Thouless Theory:
  - For spin $1/2$ fermions
  - Extension to bosons
- Exchanges of n particles:
  - Energies decrease exponentially with n
  - Number increases exponentially with n
- Large n extrapolation

Thouless Theory (I)

- N! potential wells in phase space



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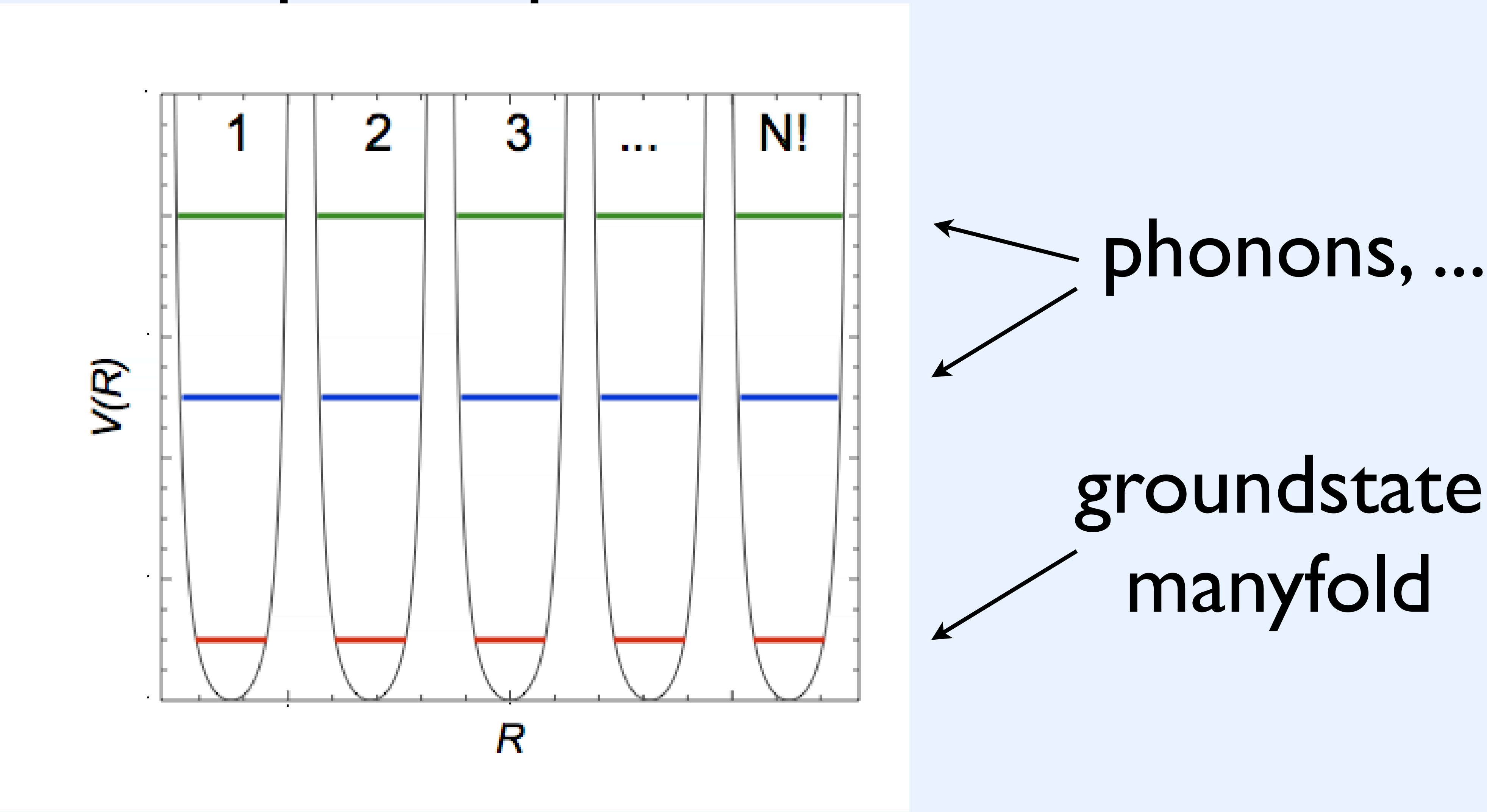

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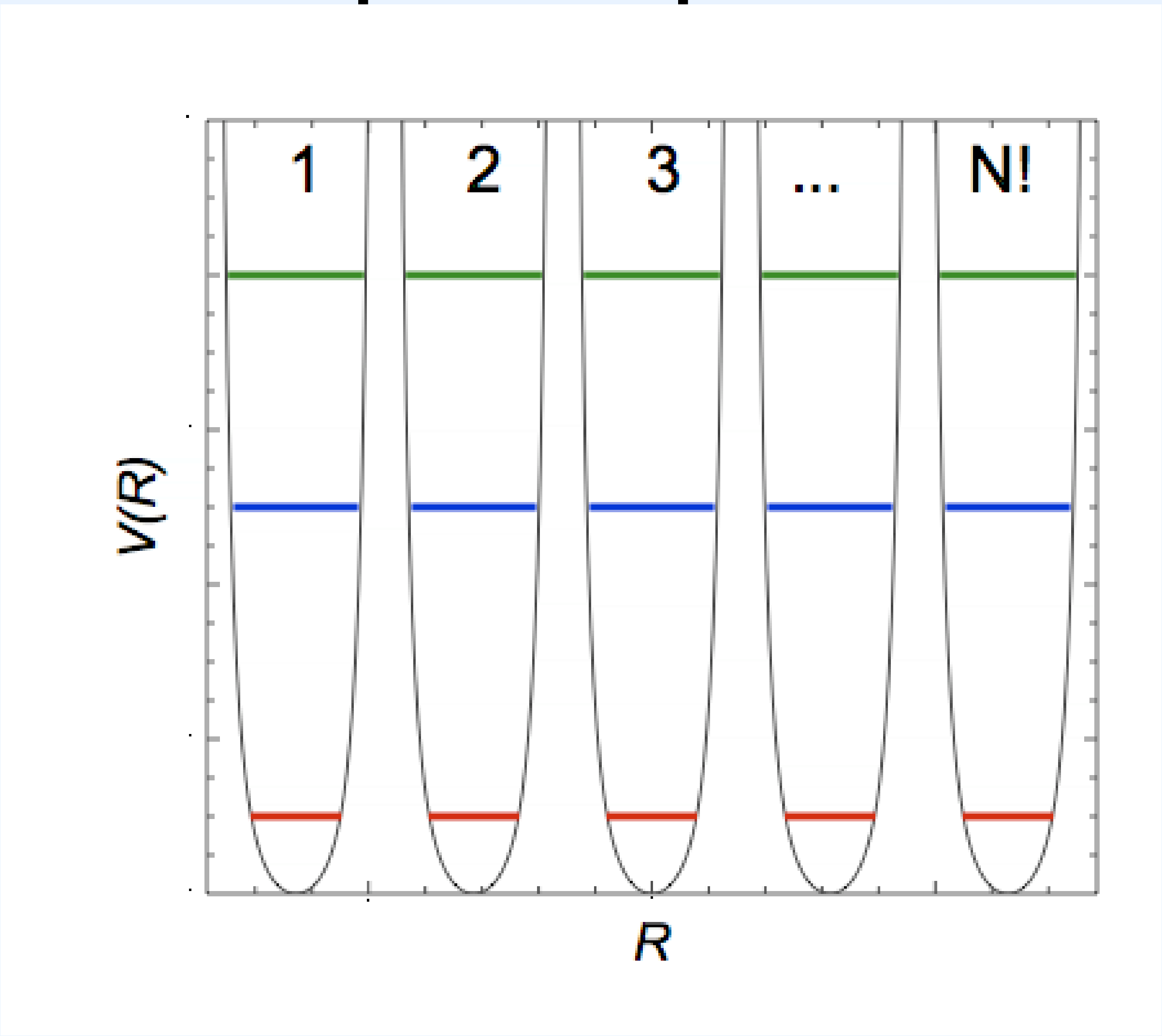


phonons, ...

groundstate manyfold

- Which states to keep depends on spin particles?

Thouless Theory (II)

- wave function : $\Psi(R,S) = \Psi(R)\Psi(S)$
- Ψ(R) ground state : symmetric
  - spin 1/2 : Ψ(S) antisymmetric

$$N! \longrightarrow 2^N$$

$$H(S) = -\sum_P (-1)^P J_P \hat{P}(S)$$

Exchange in fermionic solids

- Thouless Theory: spin degrees of freedom decoupled from space degrees of freedom
  - Free spins : phonons ($\sim T^D$)
  - "Frozen" phonons spin exchanges

$$H^{\text{eff}} = -\sum_P (-1)^P J_P P$$

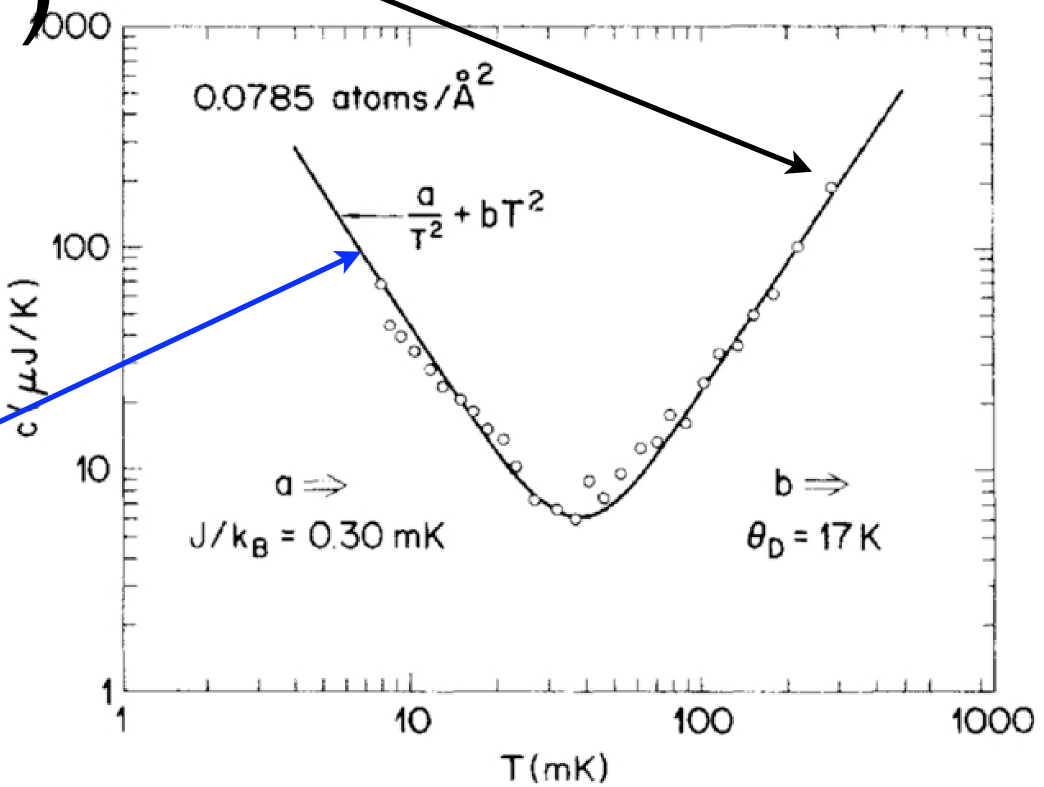


FIG. 8. Log-log plot of the heat capacity of an incommensurate solid ^{3}He monolayer. Below 30 mK the heat capacity is dominated by the nuclear spins and at higher temperatures by the phonons.

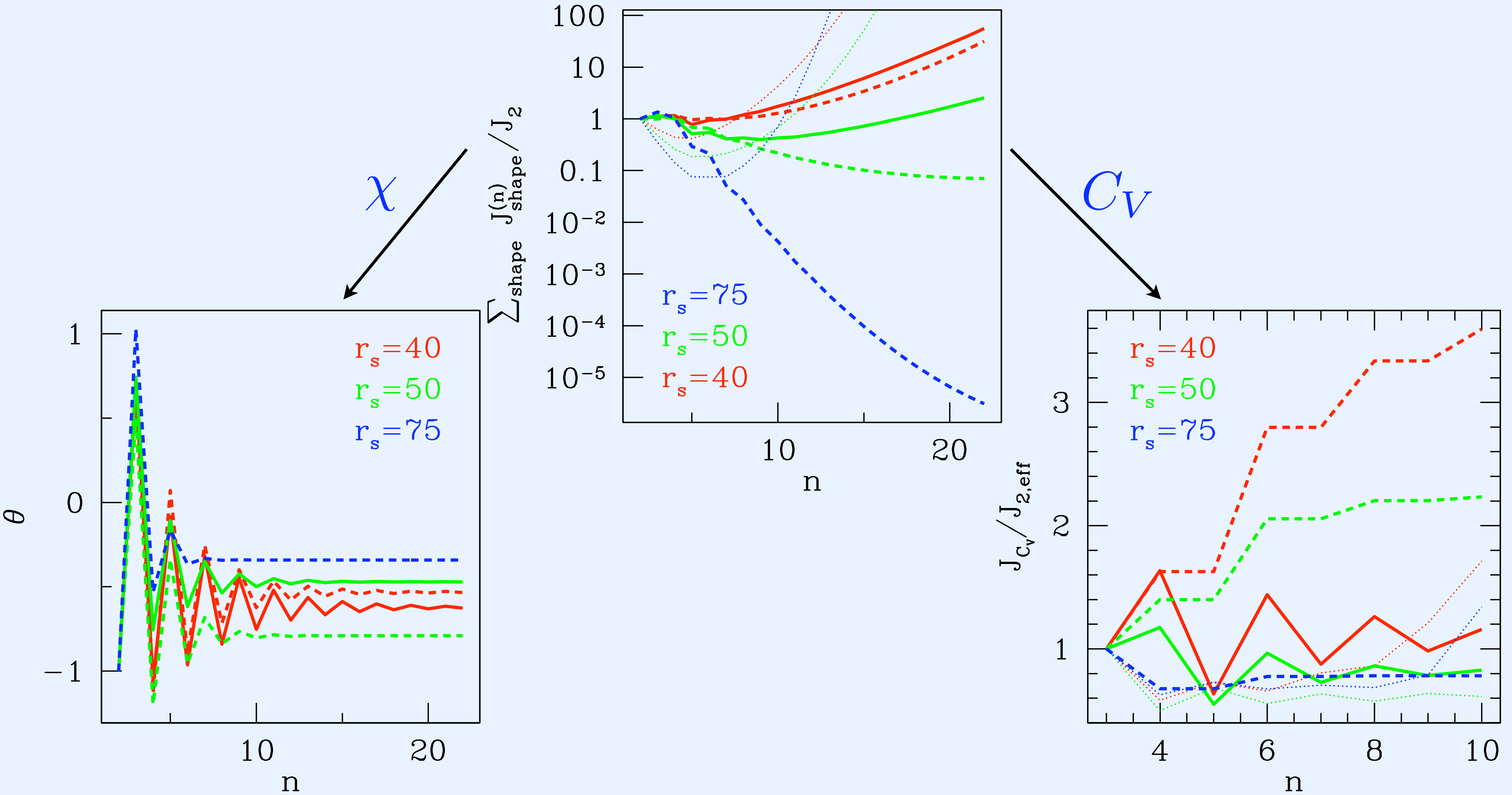
^{3}He on graphite *(D. Greywall, PRL 1990)*

Exchange in fermionic solids

- Wigner crystal in two dimension:

Bernu, B. and D. M. Ceperley *cond-mat/0310309, J. Phys.: Condens. Matter 16 (2004) p701-707*

$$H^{\mathrm{eff}} = -\sum_P (-1)^P J_P P$$



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$$N! \longrightarrow 2^N \qquad\qquad H(S) = -\sum_P (-1)^P J_P \hat{P}(S)$$

  - spin 0 : $\Psi(S)$ symmetric (one single state)

$$\Psi_N(R) = \frac{1}{N!}\sum_P \Psi_{Z_0}(PR)$$

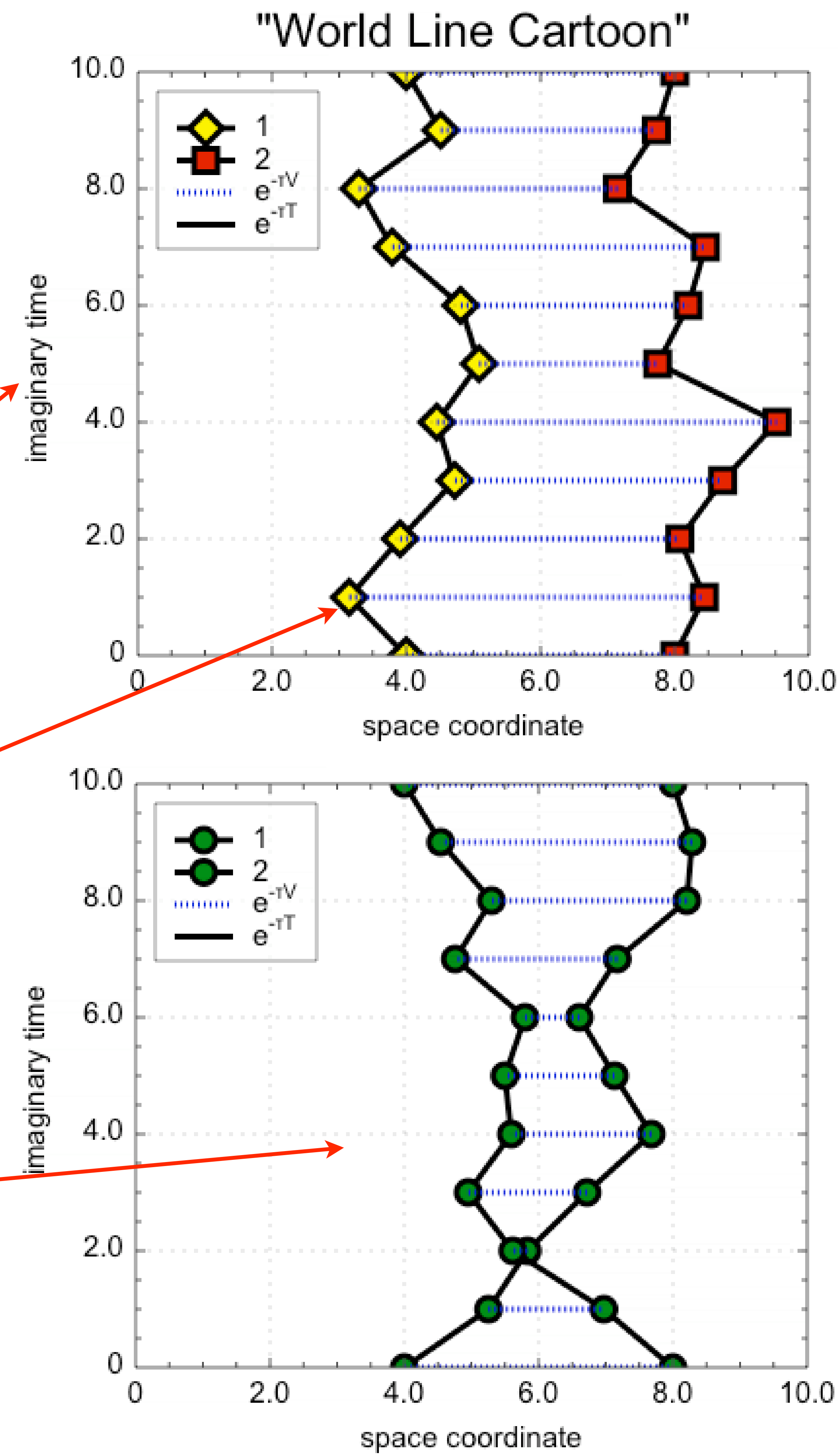
$$\delta E \simeq \sum_{P\neq 1} J_p$$

Path Integral

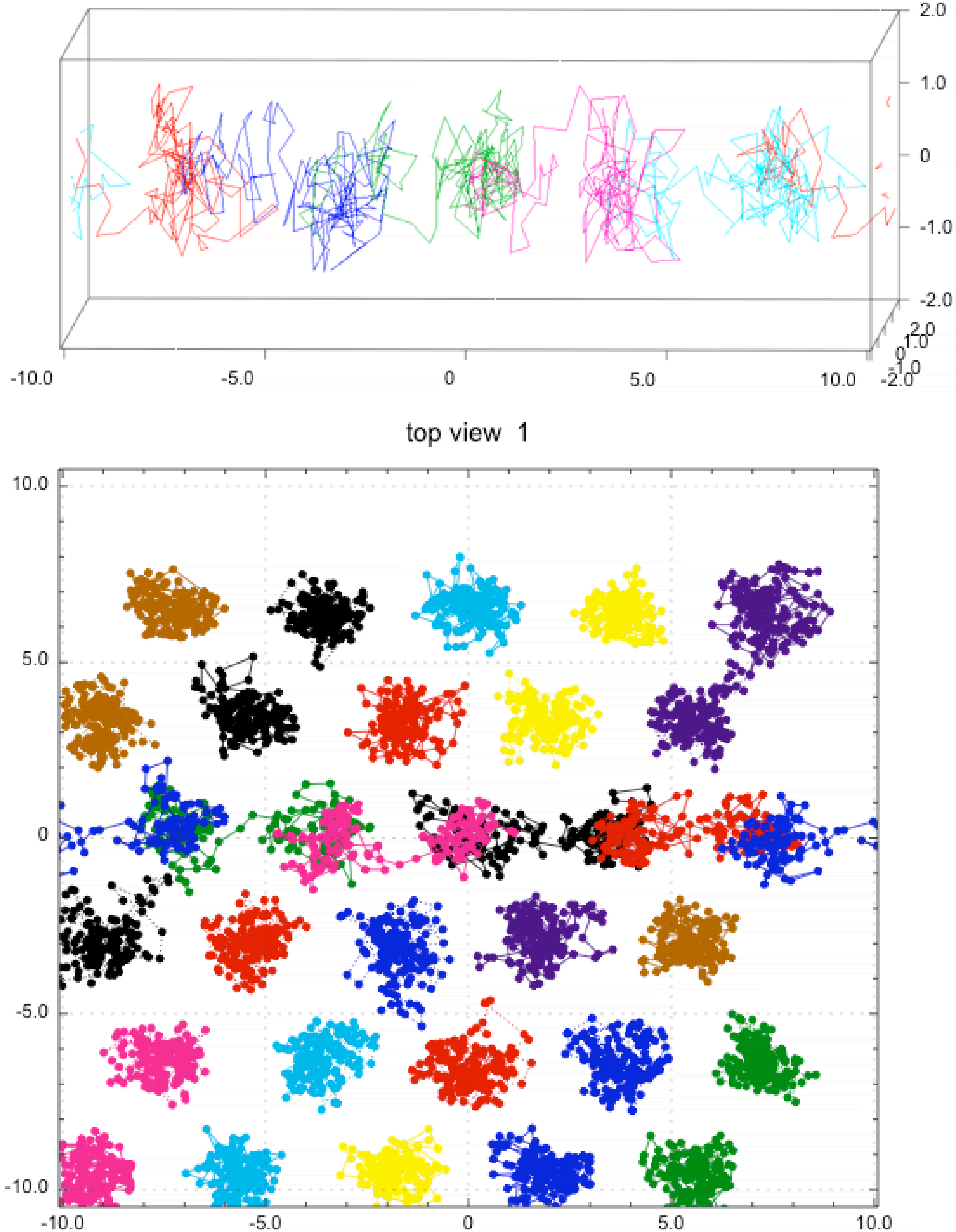
- Trotter Formula

$$e^{-\beta(T+V)} = \lim_{M\to\infty}\left[e^{\beta T/M}e^{\beta V/M}\right]^{M}$$

- Exact mapping to a classical system (also true for bosons and fermions)
- a particle : a polymer
- an exchange of p particles : one single bigger polymer from p-polymers
- equilibrium : closed polymers

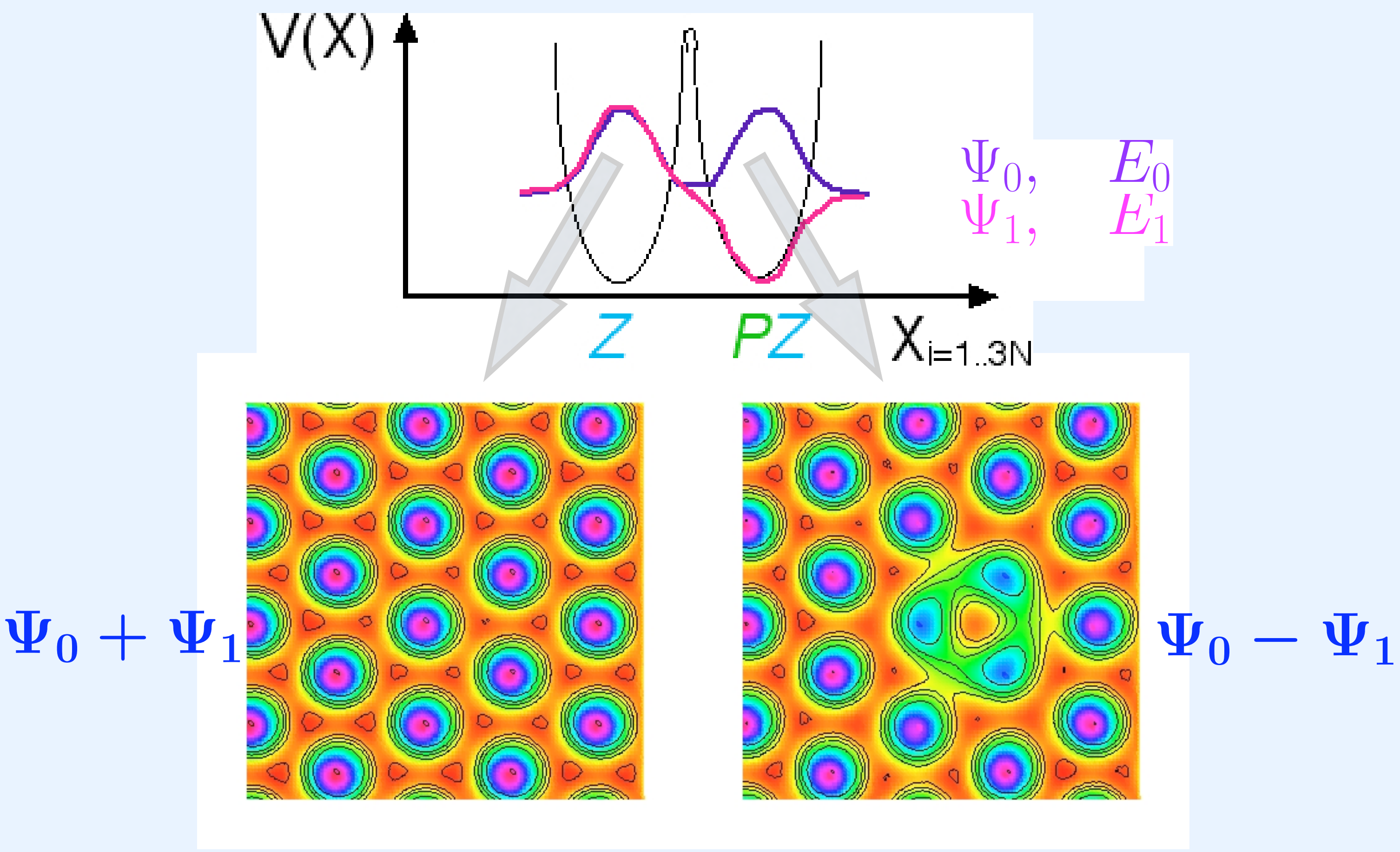


basal plan of HCP



Thouless theory : from $V(r)$ to P_{ij}

- N! wells in configuration space
- For a given P : 2 wells in configuration space



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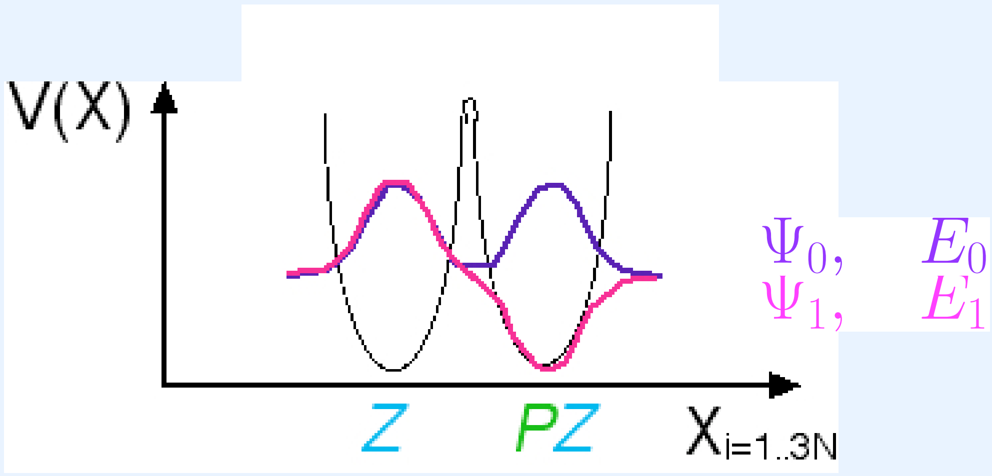


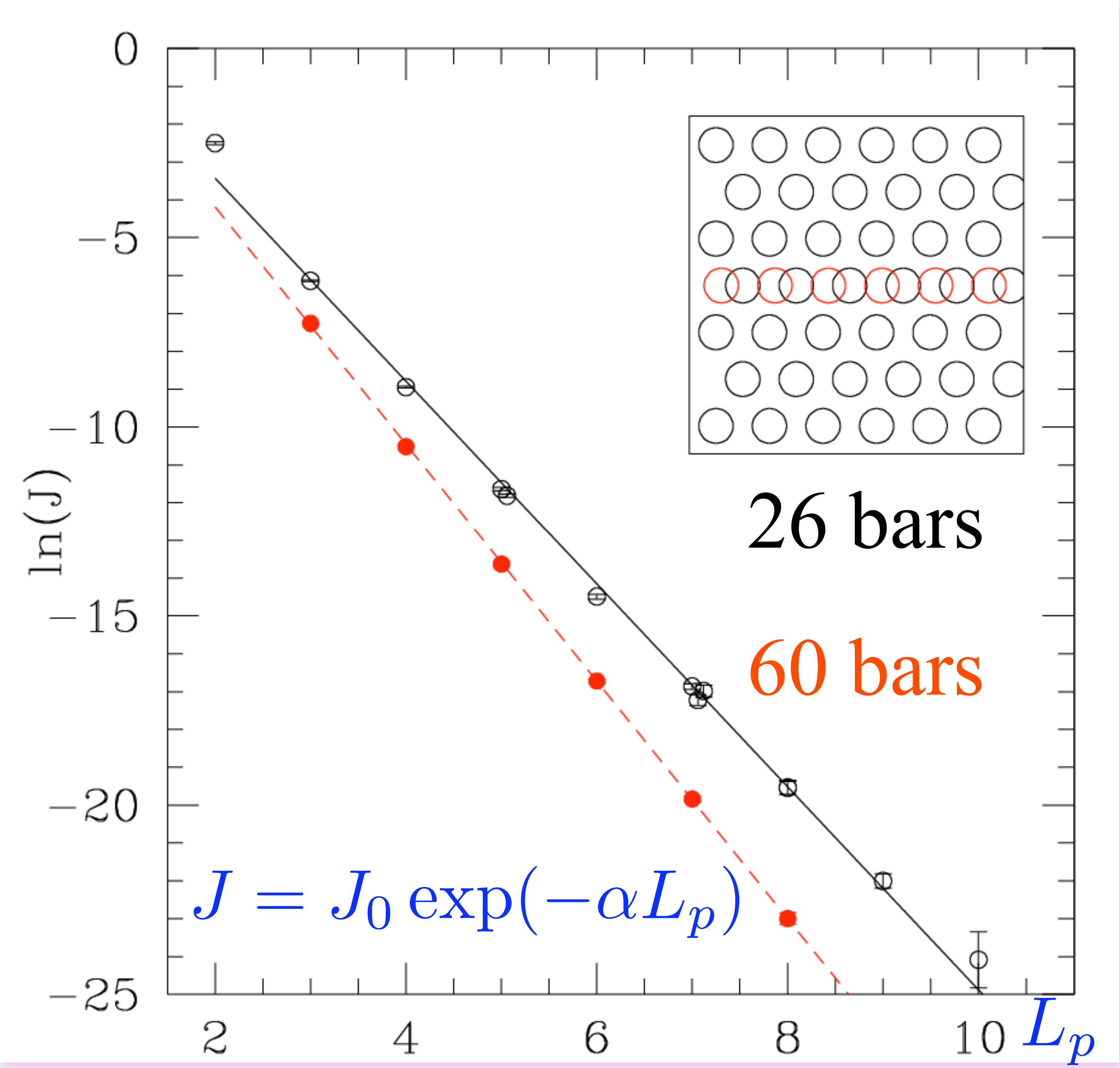
$$F_P(\beta) = \frac{< Z|\exp(-\beta H)|PZ >}{< Z|\exp(-\beta H)|Z >} = \tanh(J_P(\beta - \beta_0))$$

$$J_P = (E_1 - E_0)/2 \qquad \beta_0 = \ln|\psi_1(Z)/\psi_0(Z)|$$

D. Ceperley, G. Jacucci, Phys. Rev. Lett. 58 (1987) 1648

Feynman-Kikuchi Model

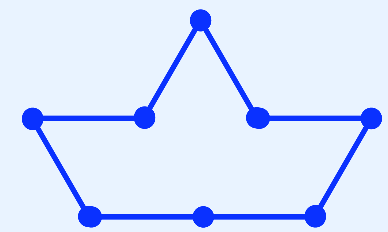
- J decreases exponentially with the cycle length L_p
- linear exchange :



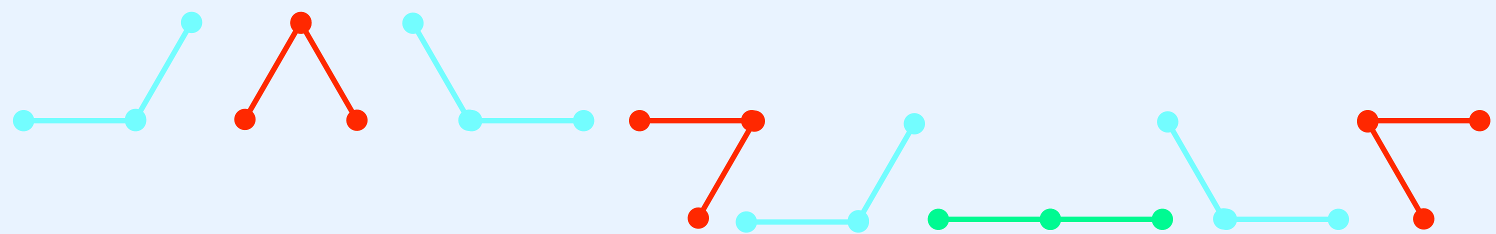
Geometrical effects:

$$J_{n,s} = e^{\alpha_0 + \alpha_n n + \alpha_p p_s + \sum_v \alpha_v N_{s,v}}$$

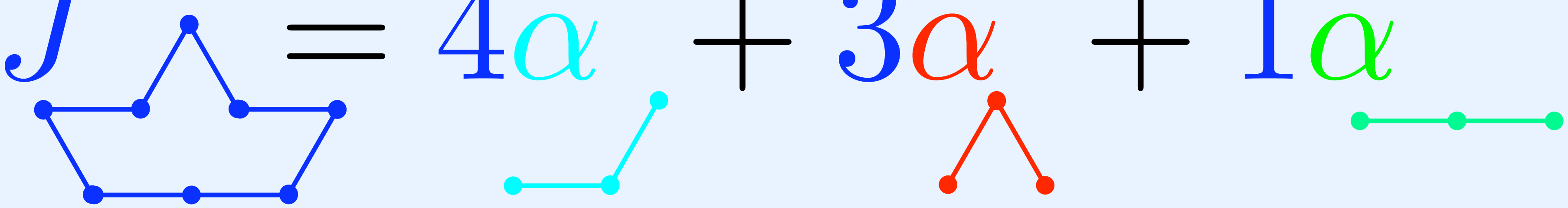
- Exchanges do depend on geometry :



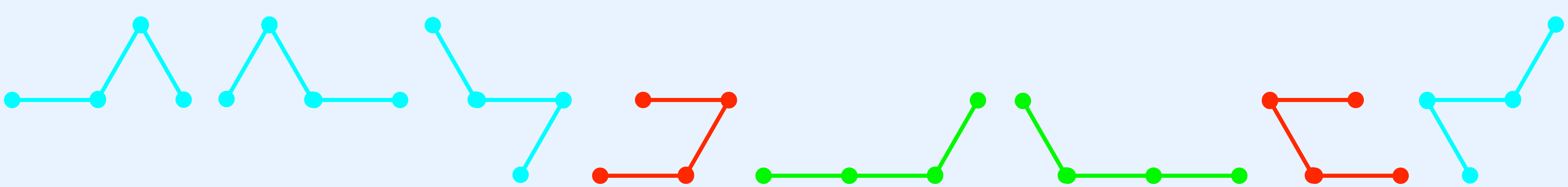
2 links pattern



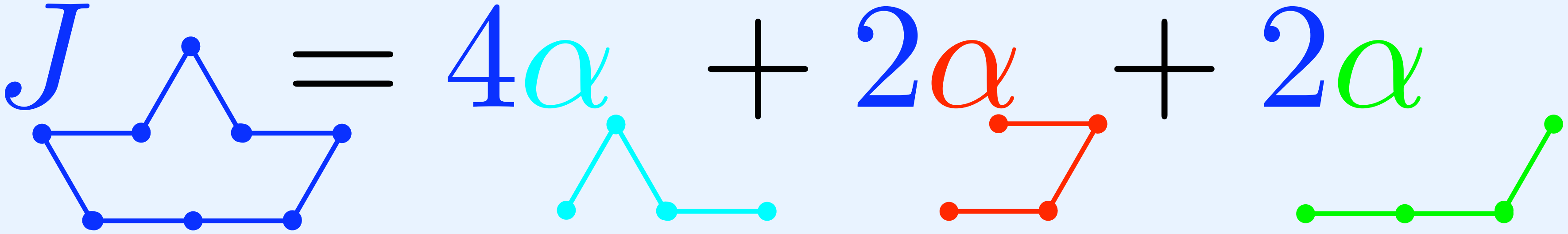
$$\ln J = 4\alpha + 3\alpha + 1\alpha$$



3 links pattern



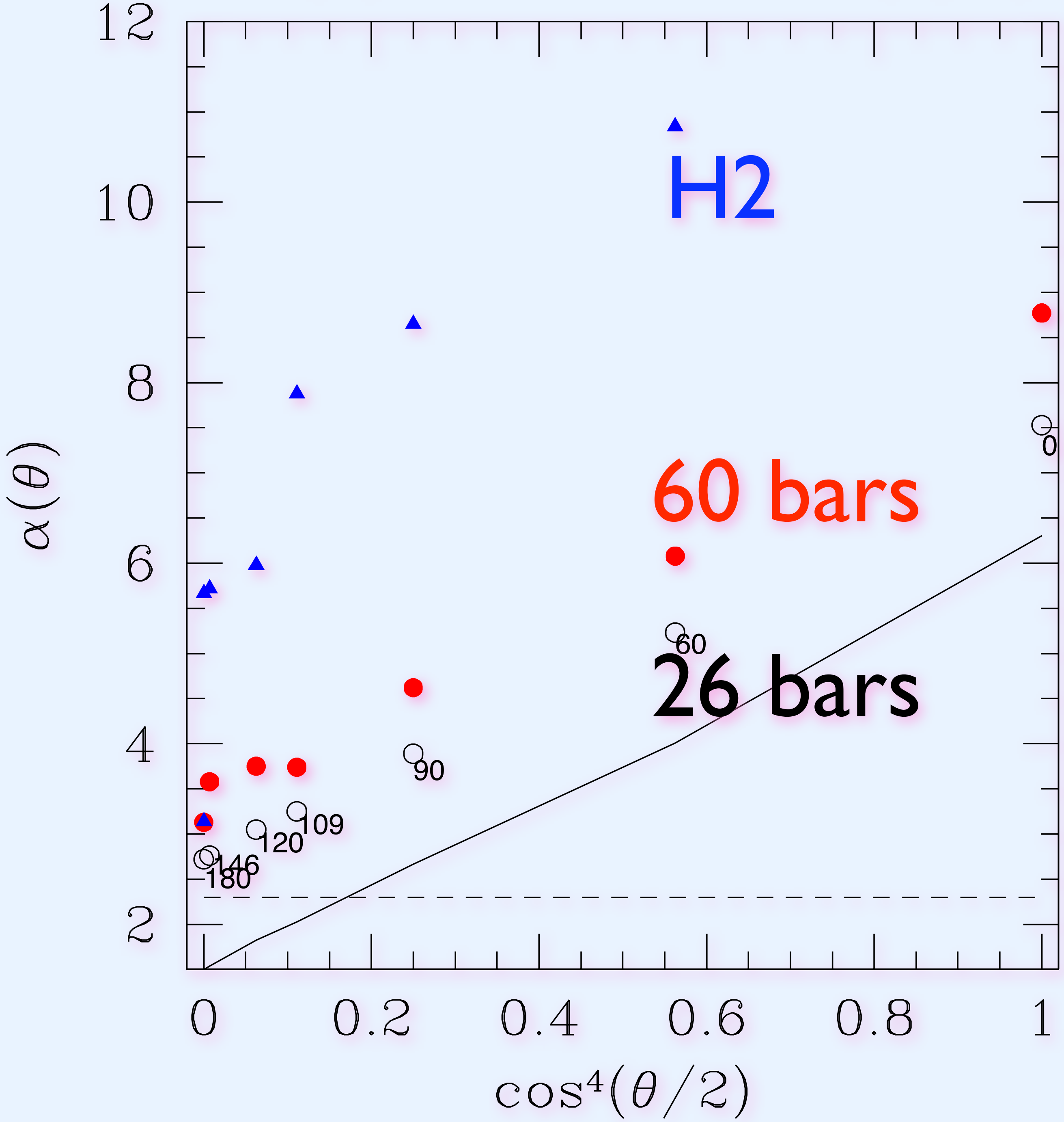
$$\ln J = 4\alpha + 2\alpha + 2\alpha$$



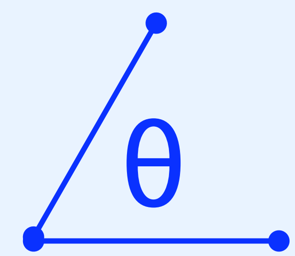
- **Fit:**

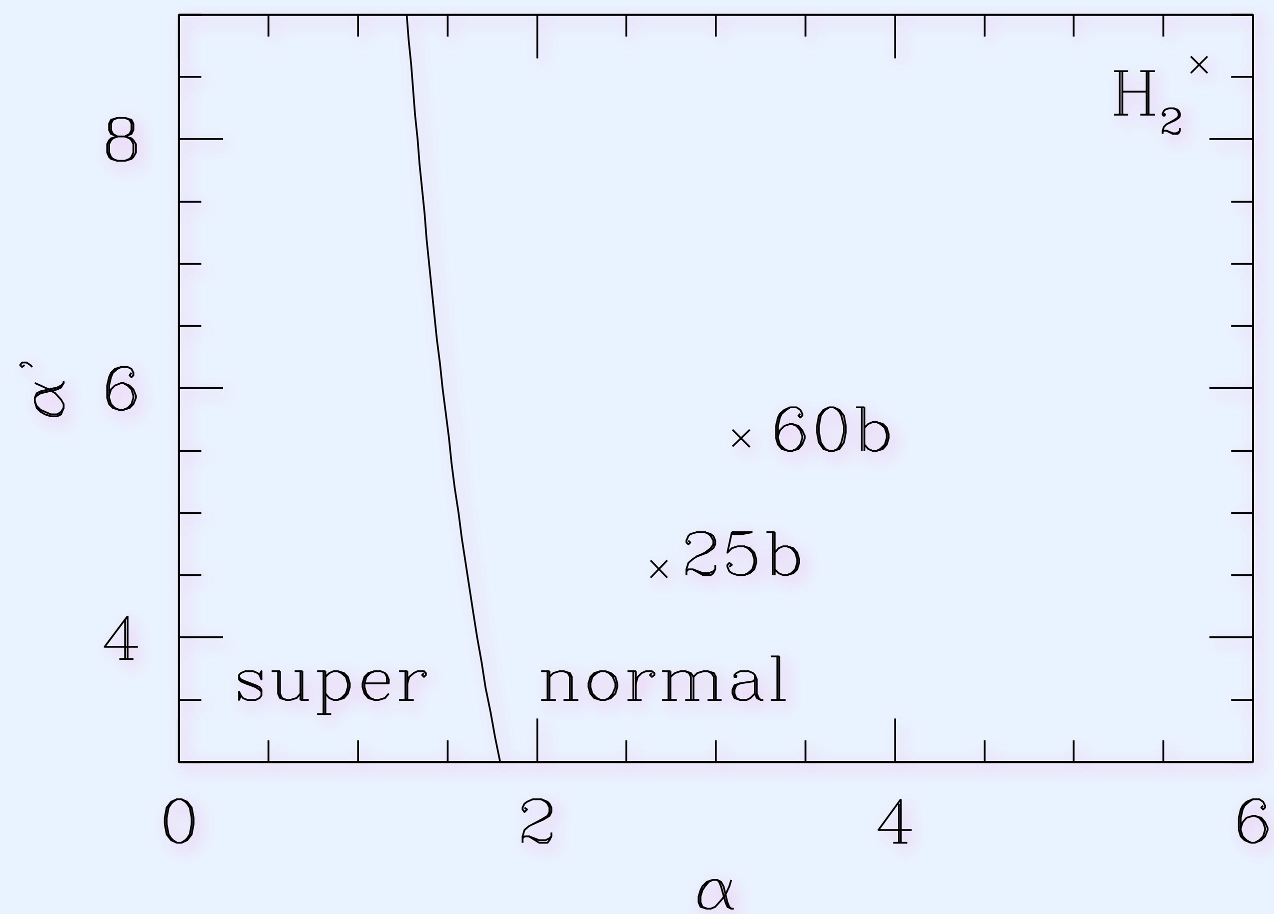
$$J = J_0 \exp\left[-\sum_{k=1}^{L_p} \alpha(\theta_k)\right]$$

$$\alpha(\theta) = \alpha + \alpha' \cos^4(\theta/2)$$



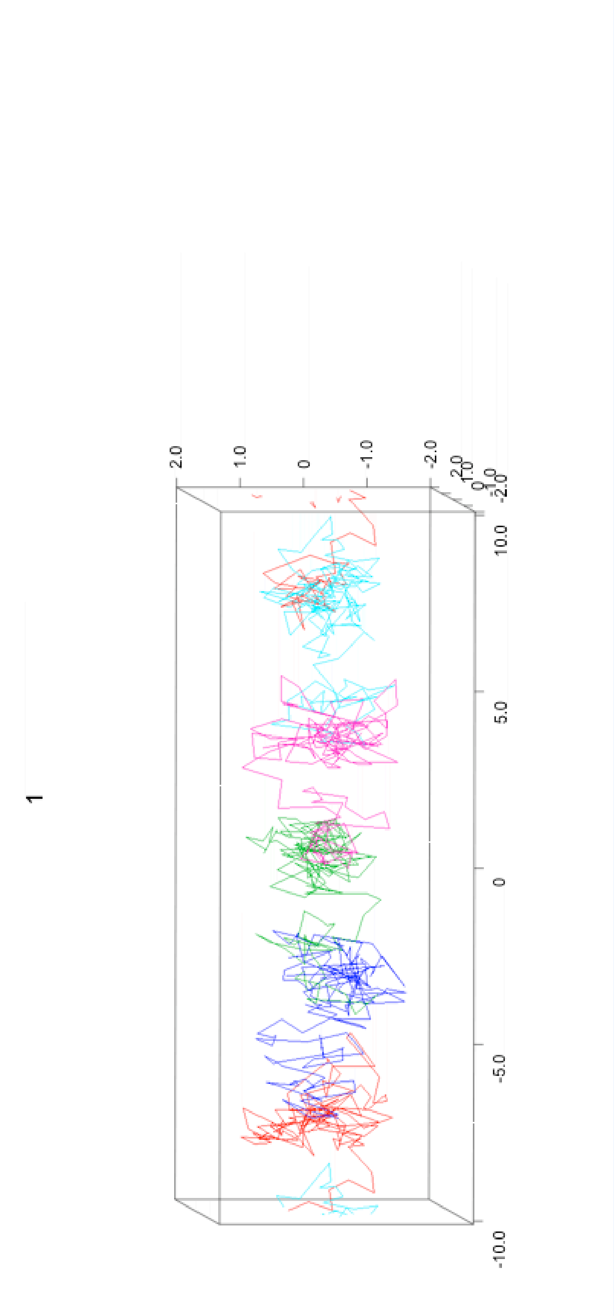
$$J_0 \simeq 7.2K$$





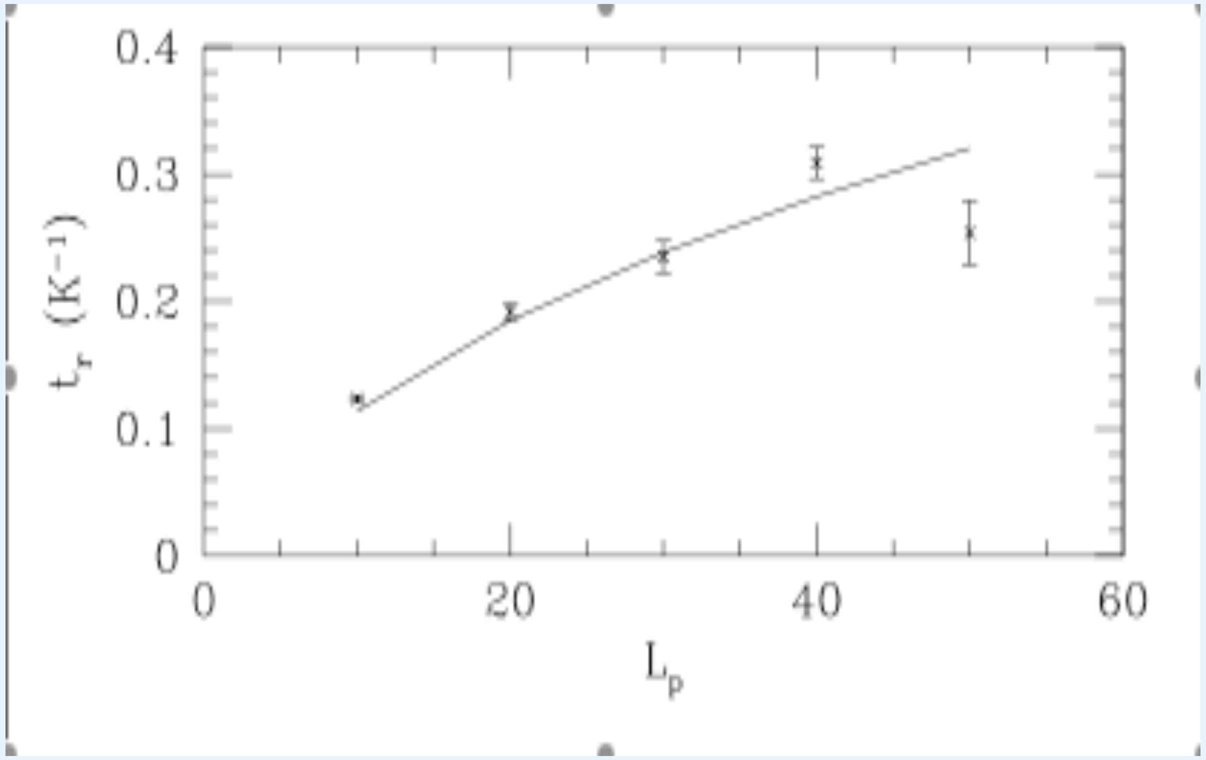
Exchange Mechanism

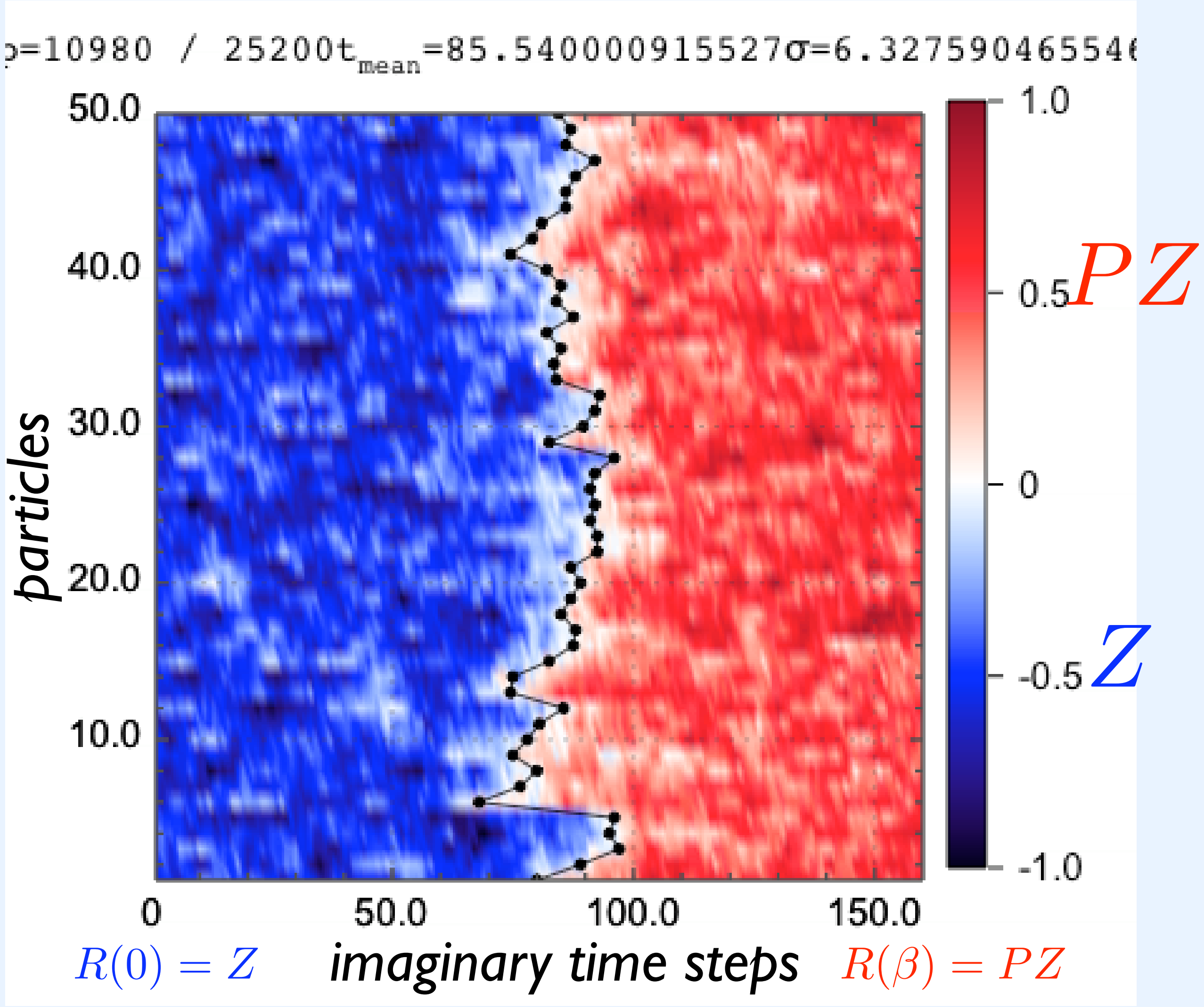
- “small” exchanges : all particles “must” exchange at the “same” imaginary time : instanton.



Exchange Mechanism

- "small" exchanges : all particles "must" exchange at the "same" imaginary time : instanton.
- What happens for longer ring exchanges





Conclusions

☑ Exchanges are localized in perfect crystal of ^4He

- more complicated type of exchanges
  - exchange of a few plans

✱ Questions

- In perfect crystals, can exchanges give a supersolid ?