Phenomenology of Supersolids

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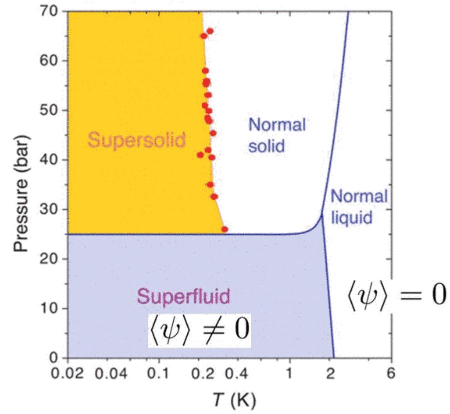
A. T. Dorsey, P. M. Goldbart, and J. Toner, Phys. Rev. Lett. **96**, 055301 (2006).

C.-D. Yoo and A. T. Dorsey, work in progress.

Outline

- Phenomenology-what can we learn without a microscopic model?
  - Landau theory of the normal solid to supersolid transition: coupling superfluidity to elasticity. Assumptions:
    - Normal to supersolid transition is continuous (2^{nd} order).
    - Supersolid order parameter is a complex scalar (just like the super*fluid* phase).
    - What is the effect of the elasticity on the transition?
  - Hydrodynamics of a supersolid:
    - Employ conservation laws and symmetries to deduce the long-lived hydrodynamic modes.
    - Mode counting: additional collective mode in the supersolid phase.
    - Use linearized hydrodynamics to calculate $S(\mathbf{q},\omega)$.

Landau theory for a super*fluid*



- Symmetry of order parameter

$\psi(\mathbf{x}) \to \psi(\mathbf{x}) e^{i\phi}$

- Broken U(1) symmetry for $T<T_c$.
- Coarse-grained free energy:

$\mathcal{F}_{\rm sf} = \int_{\mathbf{x}} \{ \frac{1}{2} c |\nabla\psi|^2 + \frac{1}{2} a(T) |\psi|^2 + \frac{w}{4!} |\psi|^4 \},$

$a(T) = a_0 (T - T_0)$

- Average over configurations:

$Z = \int D\psi D\psi^* e^{-\mathcal{F}_{\rm sf}/T}, \qquad F = -T \ln Z$

- Fluctuations shift T_0! T_C, produce singularities as a function of the reduced temperature $t=|(T-T_c)/T_c|$.
- Universal exponents and amplitude ratios.

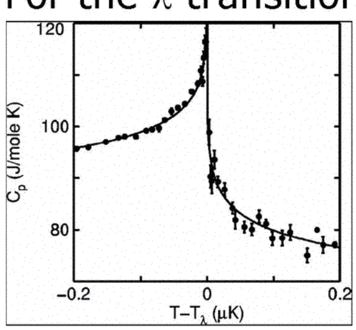
Specific heat near the λ transition

- The singular part of the specific heat is a correlation function:

$S = -\partial F/\partial T \propto -\partial F/\partial a(T) = \int_{\mathbf{x}} \langle |\psi(\mathbf{x})|^2 \rangle$

$C = T(\partial S/\partial T) \propto \int_{\mathbf{x}} \int_{\mathbf{x}'} \langle |\psi(\mathbf{x})|^2 |\psi(\mathbf{x}')|^2 \rangle \sim A_{\pm} |t|^{-\alpha}$

- For the λ transition, α= -0.0127.



Lipa et al., Phys. Rev. B (2003).

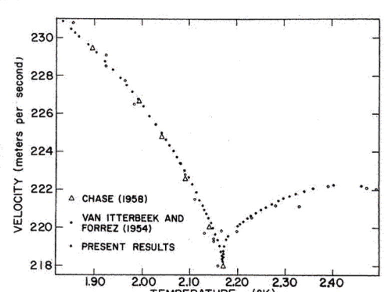


FIG. 7. Temperature dependence of the sound velocity in an extended region about the λ point.

Barmatz & Rudnick, Phys. Rev. (1968)

Sound speed

- What if we allow for local density fluctuations $\delta\rho$ in the fluid, with a bare bulk modulus B_0? The coarse-grained free energy is now

$$\mathcal{F}_{\rm sf} = \int_{\mathbf{x}} \{ \frac{1}{2} c |\nabla\psi|^2 + \frac{1}{2} a(T) |\psi|^2 + \frac{w}{4!} |\psi|^4 + \frac{1}{2} g_0 \delta\rho(\mathbf{x}) |\psi|^2 + \frac{1}{2} B_0 \delta\rho(\mathbf{x})^2 \}$$

- The "renormalized" bulk modulus B is then

$$B = B_0 - \frac{g_0^2}{4T} \int_{\mathbf{x}} \int_{\mathbf{x}'} \langle |\psi(\mathbf{x})|^2 |\psi(\mathbf{x}')|^2 \rangle$$

- The sound speed acquires the specific heat singularity (Pippard-Buckingham-Fairbank):

$$\delta c / c_0 = \delta B / 2B_0 \propto -C(t) \propto -|t|^{-\alpha}$$

Coupling superfluidity & elasticity

- Structured (rigid) superfluid: need anisotropic gradient terms: $c|\nabla\psi|^2 \to c_{\alpha\beta} \partial_\alpha \psi \partial_\beta \psi^*$
- Elastic energy: Hooke's law. 5 independent elastic constants for an hcp lattice: $u_{\alpha\beta} = \frac{1}{2}[\partial_\alpha u_\beta + \partial_\beta u_\alpha]$

$$\mathcal{F}_{\rm elastic}[u_{\alpha\beta}] = \mathcal{F}_{\rm elastic}[0] + \int_{\mathbf{x}} \frac{1}{2} \lambda_{\alpha\beta\gamma\delta}\, u_{\alpha\beta}\, u_{\gamma\delta}$$

- Compressible lattice: couple strain to the order parameter, obtain a strain dependent T_c.

$a(T, u_{\alpha\beta}) = a^{(0)}(T) + a^{(1)}_{\alpha\beta} u_{\alpha\beta} + \ldots$

- Minimal model for the normal to supersolid transition:

$$\mathcal{F}_{\rm ss} = \int_{\mathbf{x}} \left\{ \frac{1}{2} c_{\alpha\beta}\, \partial_\alpha \psi\, \partial_\beta \psi^* + \frac{1}{2} a^{(0)} |\psi|^2 + \frac{w}{4!} |\psi|^4 + \frac{1}{2} \lambda_{\alpha\beta\gamma\delta}\, u_{\alpha\beta}\, u_{\gamma\delta} + \frac{1}{2} a^{(1)}_{\alpha\beta}\, u_{\alpha\beta} |\psi|^2 \right\}.$$

Related systems

- Analog: XY ferromagnet on a compressible lattice. Exchange coupling will depend upon the local dilation of the lattice.

  $\mathcal{H} = -\sum_{\langle ij \rangle} J(\mathbf{R}_i - \mathbf{R}_j)\, \mathbf{S}_i \cdot \mathbf{S}_j$

  - Studied extensively: Fisher (1968), Larkin & Pikin (1969), De Moura, Lubensky, Imry & Aharony (1976), Bergman & Halperin (1976), …
  - Under some conditions the elastic coupling can produce a first order transition.
- Other systems:
  - Charge density waves: Aronowitz, Goldbart, & Mozurkewich (1990).
  - Spin density waves: M. Walker (1990s).
  - A15 superconductors: L.R. Testardi (1970s).

Universality of the transition

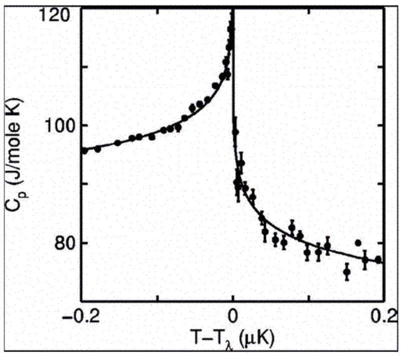
- De Moura, Lubensky, Imry & Aharony (1976): elastic coupling doesn't effect the universality class of the transition *if* the specific heat exponent of the rigid system is negative, which it is for the 3D XY model. *The critical behavior for the supersolid transition is in the 3D XY universality class*.
- But coupling does matter for the elastic constants:

$$\lambda_{\alpha\beta\gamma\delta} = -T\frac{\partial^2 F}{\partial u_{\alpha\beta}\partial u_{\gamma\delta}} = \lambda^{(0)}_{\alpha\beta\gamma\delta} - \frac{1}{4T} a^{(1)}_{\alpha\beta} a^{(1)}_{\gamma\delta} \int_{\mathbf{x}}\int_{\mathbf{x}'} \langle |\psi(\mathbf{x})|^2 |\psi(\mathbf{x}')|^2 \rangle_0$$

- Could be detected in a sound speed experiment as a dip in the sound speed.
- Anomaly appears in the "longitudinal" sound in a single crystal. Should appear in both longitudinal and transverse sound in polycrystalline samples.

Specific heat

High resolution specific heat measurements of the lambda transition in zero gravity.



J.A. Lipa et al.,
Phys. Rev. B **68**, 174518 (2003).

Specific heat near the putative supersolid transition in solid ^{4}He.

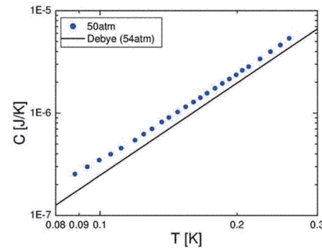
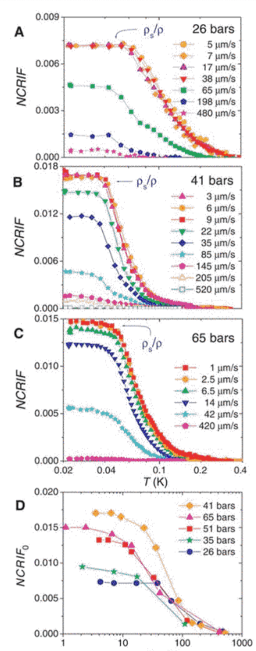


Fig. 2. Heat capacity data for pure solid ^{4}He under a pressure of 50bar in a volume of 0.08cc. The addenda heat capacity is negligible (see Fig. 3).

A.C. Clark & M.H.W. Chan,
J. Low Temp. Phys. **138**, 853 (2005).

Inhomogeneous strains

- Inhomogeneous strains result in a local T_c. The local variations in T_c will broaden the transition.
- Could "smear away" any anomalies in the specific heat.
- Strains could be due to geometry, dislocations, grain boundaries, etc.
- Need to prepare strain-free samples (?).
- N.B.: similar effects observed in superconductors (A15 compounds).



Hydrodynamics I: simple fluid

- Conservation laws and broken symmetries lead to long-lived "hydrodynamic" modes (lifetime diverges at long wavelengths).
- Simple fluid:
  - Conserved quantities are ρ, g_i, e.

$$\partial_t \rho + \partial_i g_i = 0 \qquad \text{(conservation of mass)},$$

$$\partial_t g_i + \partial_j \sigma_{ij} = 0 \qquad \text{(conservation of momentum)},$$

$$\partial_t e + \partial_i J_i^Q = 0 \qquad \text{(conservation of energy)}.$$

  - No broken symmetries.
  - 5 conserved densities) 5 hydrodynamic modes.
    - 2 transverse momentum diffusion modes $\omega = -iD_t q^2$.
    - 1 longitudinal thermal diffusion mode $\omega = -iD_T q^2$
    - 2 longitudinal sound modes . $\omega = \pm c_1 q$

Light scattering from a simple fluid

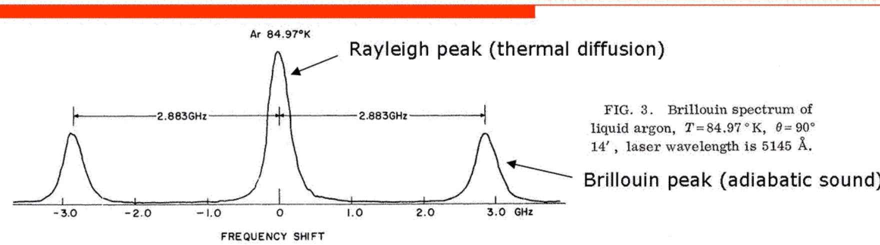


FIG. 3. Brillouin spectrum of liquid argon, $T = 84.97\,°\text{K}$, $\theta = 90°\,14'$, laser wavelength is 5145 Å.

P. A. Fleury and J. P. Boon, Phys. Rev. **186**, 244 (1969)

- Intensity of scattered light:

$$I(\mathbf{q},\omega) \propto S(\mathbf{q},\omega) \qquad S(\mathbf{q},\omega) = \langle \delta\rho(\mathbf{q},\omega)\delta\rho(-\mathbf{q},-\omega)\rangle$$

- Longitudinal modes couple to density fluctuations.
  - Sound produces the Brillouin peaks.
  - Thermal diffusion produces the Rayleigh peak (coupling of thermal fluctuations to the density through thermal expansion).

Hydrodynamics II: superfluid

- Conserved densities ρ, g_i, e .
- Broken U(1) gauge symmetry

$$\psi(\mathbf{r}) = |\psi(\mathbf{r})| e^{i\theta(\mathbf{r})}, \qquad \mathbf{v}_s = \frac{\hbar}{m}\nabla\theta.$$

- Another equation of motion:

$$\partial_t \theta = \mu/\hbar \qquad \text{Josephson relation.}$$

- 6 hydrodynamic modes:
  - 2 transverse momentum diffusion modes.
  - 2 longitudinal (first) sound modes.
  - 2 longitudinal second sound modes.
- Central Rayleigh peak splits into two new Brillouin peaks.

Light scattering in a superfluid

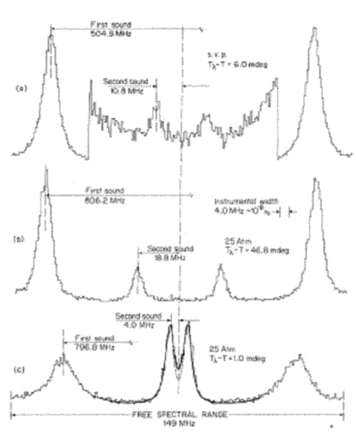


FIG. 1. Brillouin spectra showing first and second sound in superfluid He⁴. In the central region of the upper trace (a) the gain is increased by a factor of 10. The counting rate in the second sound in (a) is about 3 Hz.

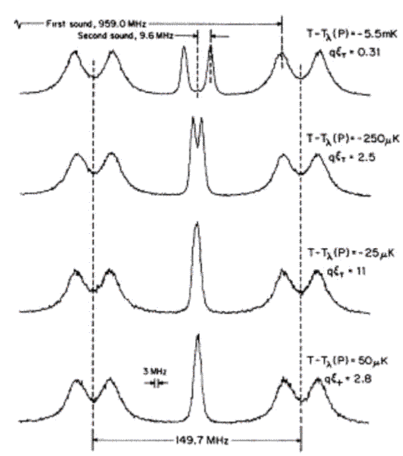


FIG. 6. Four Brillouin spectra taken at $P_\lambda = 23.1$ bars near the λ transition of liquid ^{4}He. In the top spectrum the frequency shifts of first and second sound are marked. For second sound, zero-frequency shift is at the center of the marked free spectral range (149.7 MHz). The instrumental linewidth (3 MHz) is also shown.

Winterling, Holmes & Greytak PRL 1973

Tarvin, Vidal & Greytak 1977

Solid "hydrodynamics"

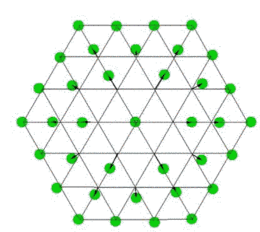
- Conserved quantities: ρ, g_i, e .
- Broken translation symmetry: u_i, i=1,2,3
- Mode counting: 5 conserved densities and 3 broken symmetry variables) 8 hydrodynamic modes. For an isotropic solid (two Lame constants λ and μ):
  - 2 pairs of transverse sound modes (4),
  - 1 pair of longitudinal sound modes (2),
  - 1 thermal diffusion mode (1).
- What's missing? Martin, Parodi, and Pershan (1972): diffusion of vacancies and interstitials.

Vacancies and interstitials

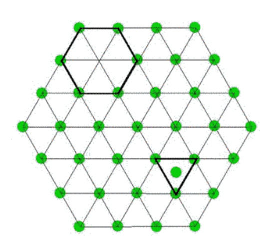
- Local density changes arise from either lattice fluctuations or vacancies and interstitials.

$$\delta\rho = \delta\rho_\Delta - \rho_0 \nabla \cdot \mathbf{u}$$

- Neglect of vacancies will leads to a missing mode.
- In classical solids vacancies diffuse slowly. Density of vacancies is small at low temperatures.
- Does ^{4}He have zero point vacancies?



$$\delta\rho = -\rho_0 \nabla \cdot \mathbf{u}$$



$$\delta\rho_\Delta = (\delta\rho_{\text{interstitial}} - \delta\rho_{\text{vacancy}})$$

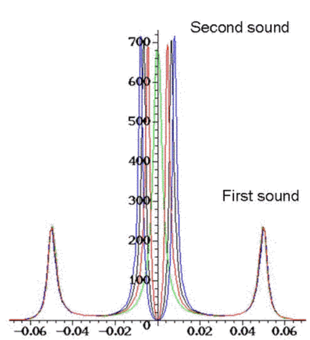
Supersolid hydrodynamics

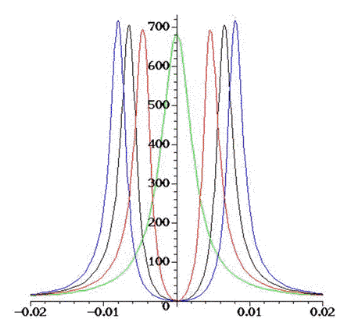
- Conserved quantities: ρ, g_i, e
- Broken symmetries: u_i, gauge symmetry.
- Mode counting: 5 conserved densities and 4 broken symmetry variables) 9 hydrodynamic modes.
  - 2 pairs of transverse sound modes (4).
  - 1 pair of longitudinal sound modes (2).
  - 1 pair of longitudinal "fourth sound" modes (2).
  - 1 longitudinal thermal diffusion mode.
- Use Andreev & Lifshitz hydrodynamics to derive the structure function (isothermal, isotropic solid). New Brillouin peaks below T_c.

Structure function for supersolid

$$\frac{c_L^4}{D_\Delta \rho_0}\frac{\beta S(\tilde{\mathbf{q}},\tilde{\omega})}{2} \simeq \frac{[\delta+\Omega(2-\delta)]\,\tilde{q}^4}{[\tilde{\omega}^2-(1+\Omega)\,\tilde{q}^2]^2+[(\delta+2\Omega)\,\tilde{q}^2\tilde{\omega}]^2}+\frac{\delta^2\Omega\tilde{q}^4}{(\tilde{\omega}^2-\delta\Omega\tilde{q}^2)^2+[(1-2\Omega)\,\tilde{q}^2\tilde{\omega}]^2}$$
$$+\frac{\tilde{q}^2\,[\delta+2\Omega\,(2-\delta)]\,(\tilde{\omega}^2-\delta\Omega\tilde{q}^2)}{(\tilde{\omega}^2-\delta\Omega\tilde{q}^2)^2+[(1-2\Omega)\,\tilde{q}^2\tilde{\omega}]^2},$$

$$\tilde{\omega}=D_\Delta\omega/c_L^2,\ \tilde{q}=D_\Delta q/c_L,\ \Omega=\rho_{s0}/\rho_0,\ \delta=1/(\rho_0\chi c_L^2).$$





Summary

- Constructed a Landau theory of the normal solid to supersolid transition. Coupling to the elastic degrees of freedom doesn't change the critical behavior.
  - Predicted anomalies in the elastic constants that should be observable in sound speed measurements.
  - Noted the importance of inhomogeneous strains in rounding the transition.
- Used linearized hydrodynamics to derive the structure function of a model supersolid. A new collective mode emerges in the supersolid phase, which might be observable in light scattering.