Role of 3He Disorder in supersolids

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Discussions with Cristian Batista

1. Outlines of the GL theory on effects of disorder on supersolid

- Two order parameters needed for supersolids:

density and supercomponent

- Two types of disorder: heavier and lighter substitution atoms
- Lighter atoms (3He) might enhance Tc and
- will suppress stiffness

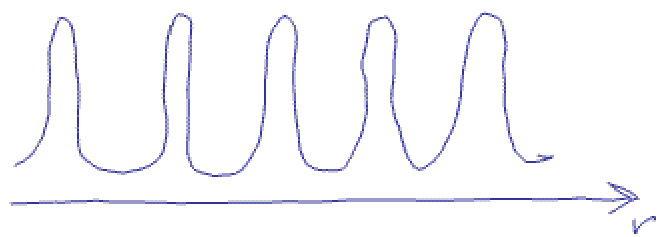
2. Comparison with experimental data most likely indicates that either as measured this is not Tc (see also M. Chan's remarks) or it is a highly nontrivial sstate.

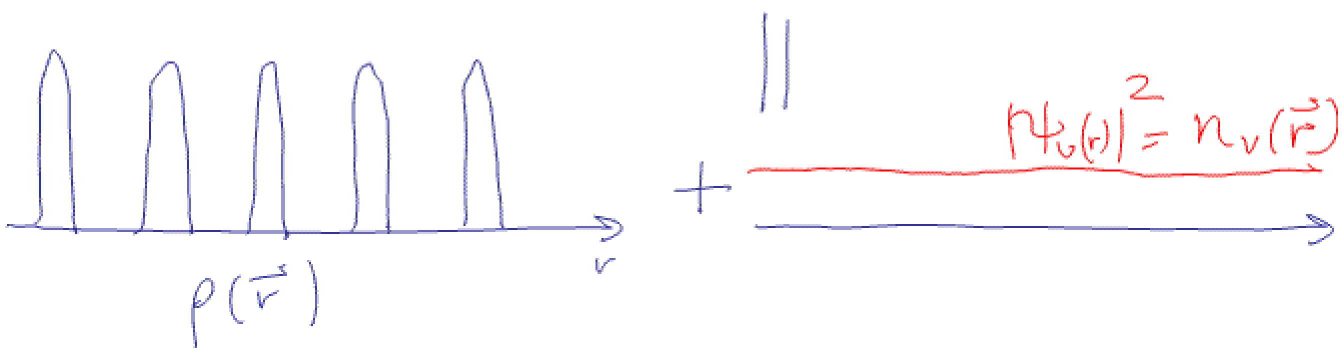
Two order parameters in supersolid state

Density $\rho(r) = \sum_{Q \in BZ} \rho_Q \, e^{iQr} + h.c.$

SS order parameter $\psi_v(r)$

4He wave function





Coarse graining to get to long length scale

Coupling between density and vacancy fields

Density-density coupling $\pm \rho(\vec{r}) |\Psi_v(\vec{r})|^2$

Grad-grad coupling $\pm \vec{\nabla}\rho \, \Psi_v^+ \nabla \Psi_v$ . . . higher order

+ higher order terms

GL description

$F = F_0 + F_{int}$

$F_0 = \alpha(1 - T/T_{c0}) |\Psi_v(\vec{r})|^2 + \beta |\Psi_v|^4$

$F_{int} = \lambda \delta\rho(r) |\Psi_v(\vec{r})|^2 + \gamma \vec{\nabla}\rho (\Psi_v^+ \vec{\nabla} \Psi + h.c)$

$\delta\rho(r)$ Density modulation due to external potential, disorder, 3He impurities

Simple GL arguments can not fix the sign of density-density coupling

Wave function description

Sites $i = 1 \ldots N$

Doubly occupied, “forbidden”

Each site has three possible states:

Singly occ and empty allowed

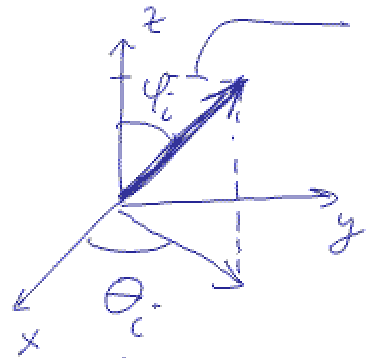
If all sites are occupied

$$\Psi = \prod_i b_i^{\dagger} |0\rangle \rightarrow |FM\rangle = |\uparrow\uparrow\uparrow..\rangle$$

FM state

Rotate spin on each site to allow
finite amplitude of empty site

$$\Psi \to \prod_i e^{i S_i^- n_i^+ \eta_i}|FM\rangle = |\nearrow \nearrow \nearrow \nearrow\rangle$$



tilt angle: determines how strong admixture into superconducting gapped mode

xy phase angle

Goldstone phase gapless mode

$$\Psi = \prod_i \left(b_i^+ \cos\varphi_i + e^{i\theta_i} \sin\varphi_i \right)|0\rangle$$

Boson creation amplitude

Empty site amplitude

Prob of empty $= n_v = \sin^2\varphi_i$

$n_v + n_b = 1$

Prob of single occupied $= n_b = \cos^2\varphi_i$

Effects of 3He impurities on SS

- 3He requires more “shoulder” space in 4He matrix for zero point motion
- It is an attractive site for vacancies
- Increases Tc in GL?!
- Illustrated in WF approach

(a) $H = H_0 + H_{int}$ $\qquad \psi = \prod_i \left(b_i^+ \cos\varphi_i + e^{i\theta_i} \sin\varphi_i\right)|0\rangle$

$$H_{int} = \sum_i U_i b_i^+ b_i$$

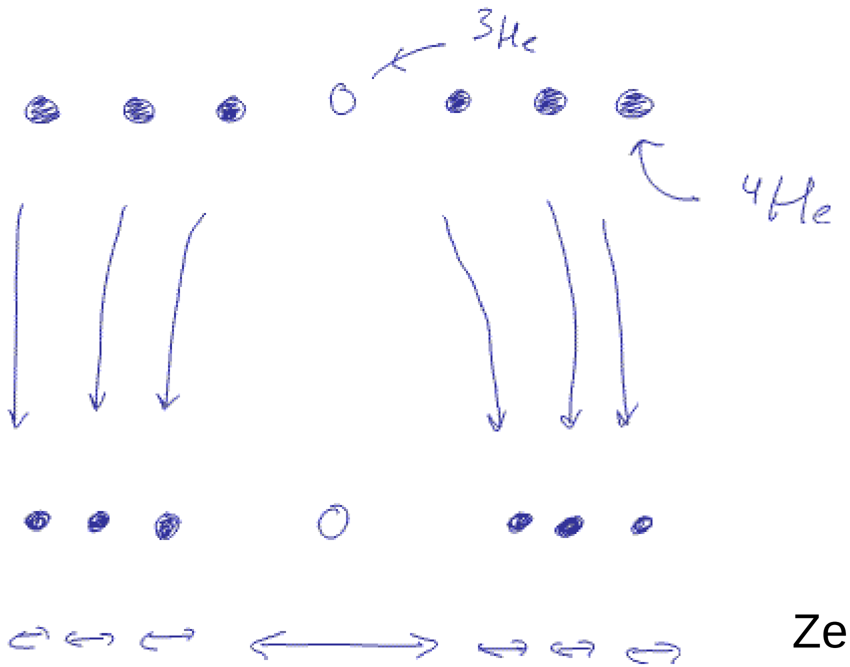
$$\langle \psi | b_i^+ b_i | \psi \rangle = \cos^2\varphi_i = 1 - \sin^2\varphi_i$$

$$\langle \psi | H_{int} | \psi \rangle = \sum_i U_i \cos^2\varphi_i \sim -\sum_i U_i \sin^2\varphi_i$$
$$= -\int d\vec{r}\, U(r) |\psi_v(r)|^2$$

Potential that is repulsive for bosons is attractive for vacancies

$$n_v + n_b = 1$$

3He has larger zero point motion amplitude



Zero point motion amplitude

Pushes 4He aside

Less of n_b = more of n_v

Take $$U(\vec{r}) = \lambda \rho_3(\vec{r})$$

$\rho_3(\vec{r})$ Is local 3He density
is an attractive region
for vacancy

Not a random mass term

$$\alpha\left(1 - T/T_c^0\right)\psi_v^2 - \lambda\langle\rho_3\rangle\psi_v^2$$

$$\downarrow$$

$$0.75 \lesssim \frac{\delta T_c}{T_c} \sim \frac{\lambda}{\alpha}\langle\rho_3\rangle \longrightarrow \frac{\lambda}{\alpha}\cdot 10^{-5}$$

$$\frac{\lambda}{\alpha} \sim 10^5 - 10^4 \qquad \text{HUGE!}$$

Anti Anderson theorem

Contrast to SC case and Anderson Theorem (no Tc enhancement)

Kinetic energy and stiffness

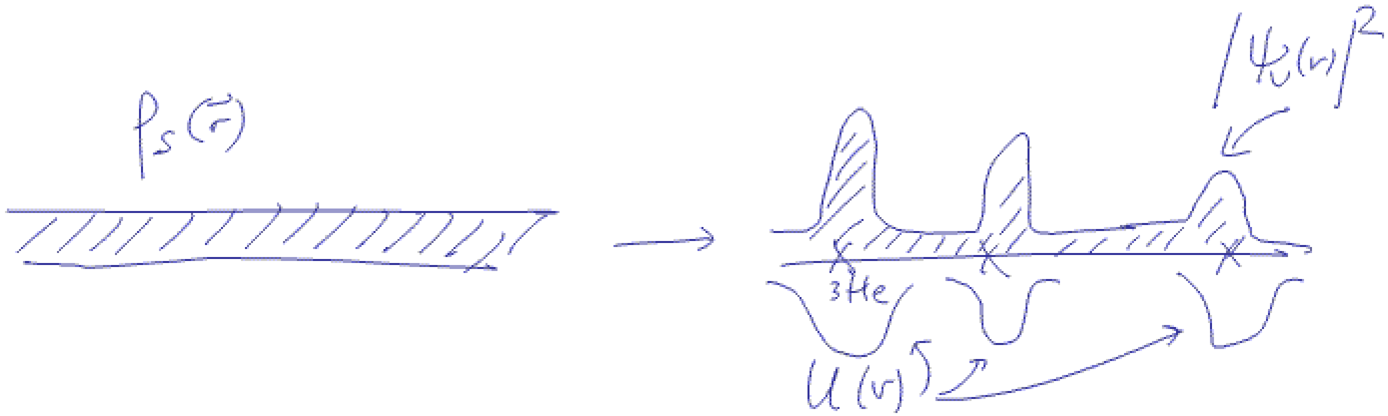
$$\langle \psi | \sum_{ij} t \; b_i^+ b_j | \psi \rangle = t \cos\varphi_i \sin\varphi_i \cos\varphi_j \sin\varphi_j \cos(\theta_i - \theta_j)$$

$$\simeq \sin^2\varphi(\vec{r}) \cos(\theta_i - \theta_j) \rightsquigarrow \rho_s(\vec{r}) (\nabla\theta)^2$$

assuming φ - small

$$\rho_s(r) \propto \sin^2\varphi(\vec{r}) = n_v(\vec{r})$$

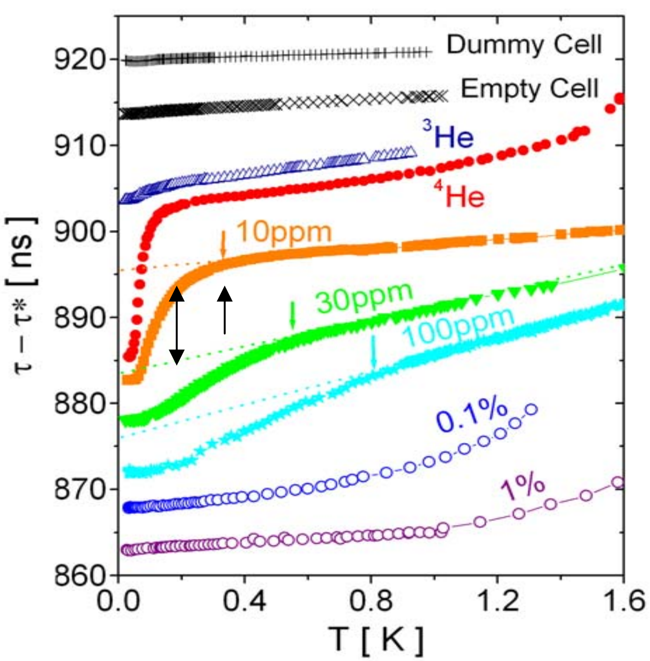
Locally stiffness increases. Global stiffness (q ->0) will decrease as is the case of granular SC
Possibility where this can be modified: 3He induces extra vacancies

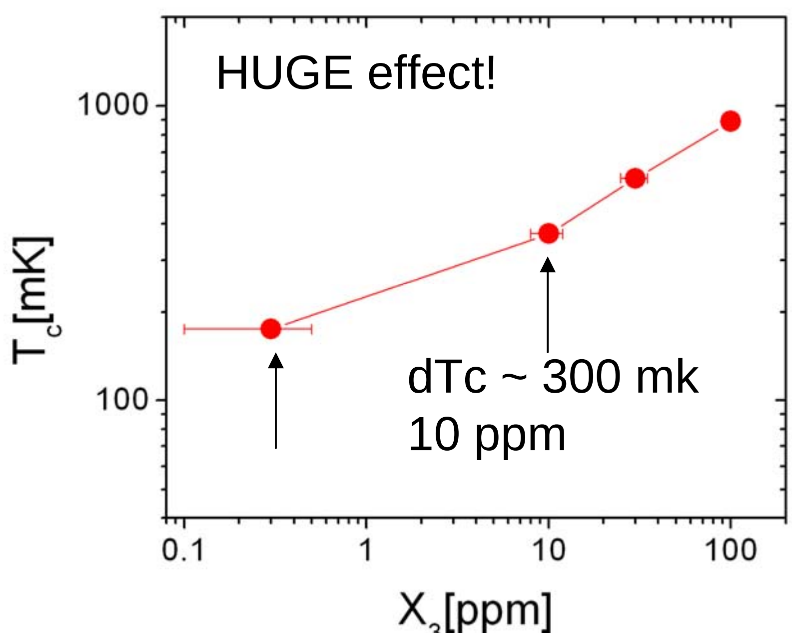


Comparison with experiments

- Tc will go up but not as much as

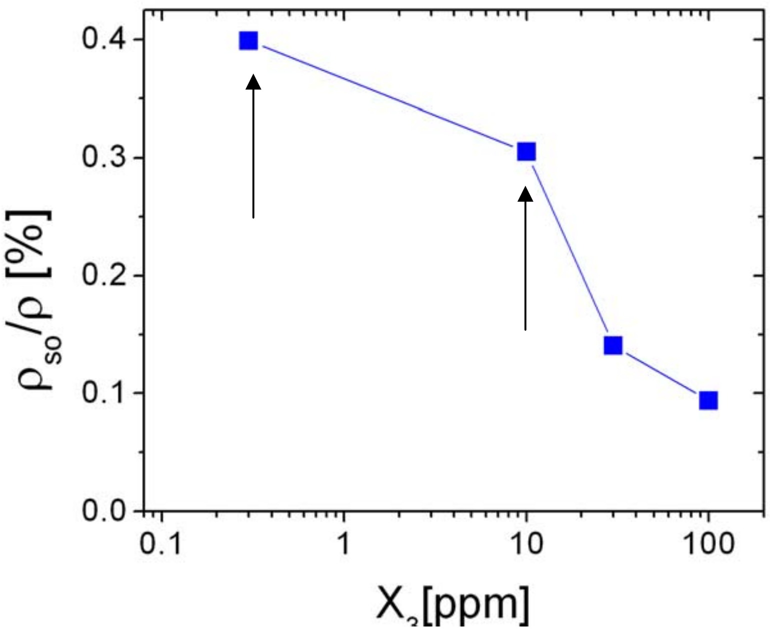
what is measured by Chan et al. It is most likely due to vortices/dislocations …





Numbers

- Stiffness goes down but by a more modest amount



$$\rho_s^{(0)} |\nabla \psi|^2 - d \langle \rho_3 \rangle |\nabla \psi|^2$$

$$\frac{\delta \rho_s}{\rho_s} \sim \frac{d \langle \rho_3 \rangle}{\rho_s} \sim \frac{d \cdot 10^{-5}}{10^{-3}} \sim d \cdot 10^{-2}$$

$$d \sim 25$$

Conclusion

- Supersolid state can benefit from lighter atoms if they attract vacancies.
- We have devloped a GL theory that does predict Tc increase but ot on the observed scale.

- Tc increase seen by PSU group is HUGE

for any known to date superstate: Tc nearly doubles by adding 10^-3 % impurities

- Implication is that either this is not Tc or this is a highly nontrivial superstate.