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Spherically symmetric space-times in loop quantum gravity

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Plan

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- Spherically symmetric gravity
- The problem of dynamics and alternative proposals.
- Loop representation for spherical symmetry
- Dirac quantization of a simplified model.
- Expectations for more general models.

Introduction and apologies:

Spherically symmetric space-times include Schwarzschild and therefore the singularity.

They are the “next obvious thing” to try with loop quantum gravity after homogeneous space-times.

Work in progress, recent, we do not have results for the singularity…

Models always involve a tradeoff: using special features simplifies treatment but lessens the value as lessons for the full theory.

Things get unexpectedly complicated, we may need to use novel techniques not widely accepted.

It turns out that one can do a more traditional treatment using a special feature of spherical symmetry.

Spherically symmetric canonical quantum gravity:

Previous work on the spherical symmetry with the traditional variables for canonical gravity: Berger, Chitre, Moncrief, Nutku (1973), Lund (1973), Unruh (1976), and the definitive work (in vacuum): Kuchař (1994).

Kuchař does not simply use symmetry-reduced variables and proceed to a Dirac quantization, but makes a careful choice of canonical variables such that the quantization is immediate and the only dynamical variable is the mass. In this sense it can be seen as a “**microsuperspace** quantization”.

Such a quantization has so little in common with the full theory that we cannot learn anything about, for instance, the use of loop quantization or singularity elimination.

We would like to use less information about the model in question, i.e. just impose spherical symmetry and then proceed with the usual quantization program. We will see that this complicates things significantly.

Midi-superspaces: things complicate

Suppose one considers the usual spherical (spatial) metric,

$$ds^2 = A^2(r)dr^2 + B(r)^2(r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2)$$

And a appropriately spherical conjugate momenta. One will be left with one diffeomorphism constraint C^r and a Hamiltonian constraint H. They will satisfy the usual constraint algebra,

$$\{C, C\} \approx C$$

$$\{C, H\} \approx H$$

$$\{H, H\} \approx gC$$

Remarkably, even this simple model has “the problem of dynamics” of canonical quantum gravity. **This problem is absent in homogeneous cosmologies**.

The problem of dynamics, brief reminder

(Discussion oversimplified, see T. Thiemann "Insider's view" paper or Giesel and Thiemann's AQG papers for a more careful discussion)

In loop quantum gravity, in addition to the previous constraints there is a Gauss law. Viewed as a quantum constraint, the basis of solutions to Gauss' law is given by the spin network states. This space is endowed with a natural inner product and given certain assumptions is unique (LOST theorem).

In such space diffeomorphisms are represented by an operator, but the space is not weakly continuous, i.e. infinitesimal diffeomorphisms do not exist. Therefore one does not have a representation for the quantum diffeomorphism constraint, although one can construct diffeomorphism invariant states and therefore construct the kernel of the constraint.

This suggests one cannot find a representation of the quantum Hamiltonian constraint, since {H,H}~gC.

One can find a representation of the quantum Hamiltonian constraint acting on the space of diffeomorphism invariant states where {H,H}=0 (Thiemann's QSD (1996))

The latter construction appears to fail for technical reasons in spherical symmetry (Bojowald, Swiderski (2006)).

Alternative proposals

This problem has led several of us to seek alternatives to the Dirac quantization procedure to apply in the case of gravity. An example of this point of view is the "master constraint" program of Thomas Thiemann and collaborators (2004-present).

Our point of view is to attempt to define the continuum theory as a suitable limit of lattice theories that do not have the problem of the constraint algebra but that nevertheless provide a correspondence principle with the continuum theory. In a nutshell one can say that we wish to do for gravity what lattice QCD did for QCD. Unfortunately, when one discretizes complicated theories with constraints, usually the resulting discrete theories differ significantly from the continuum one (even in QCD, if not done carefully).

The constraints that are first class in the continuum become second class in the discrete theories.

They can be handled by the Dirac procedure by determining the Lagrange multipliers. The resulting theories therefore are considerably different from the continuum ones. We explored this approach for some time calling it "consistent discretizations".

Unfortunately it was never clear that one could control the continuum limit. The equations that determine the Lagrange multipliers are not guaranteed to have real solutions.

To tackle this we proposed a new type of discretizations called "uniform discretizations" and they are defined by the following canonical transformation between instants n and n+1,

$$A_{n+1} = e^{\{\bullet, H\}}(A_n) \equiv A_n + \{A_n, H\} + \frac{1}{2}\{\{A_n, H\}, H\} + \cdots$$

Where A is any dynamical variable and H is a "Hamiltonian". It is constructed as a function of the constraints of the continuum theory. An example could be,

$$H(q,p) \;=\; 1/2 \sum_{i=1}^{N} \phi_i(q,p)^2$$

(More generally, any positive definite function of the constraints that vanishes when the constraints vanish and has non-vanishing second derivatives at the origin would do) Notice also that parallels arise with the "master constraint program".

These discretizations have desirable properties. For instance H is automatically a constant of the motion. So if we choose initial data such that H<ε, such statement would be preserved upon evolution.

So if we choose initial data such that H<ε then the constraints remain bounded throughout the evolution and will tend to zero in the limit ε->0.

We can also show that in such limit the equations of motion derived from H reproduce those of the total Hamiltonian of the continuum theory. For this we take $H_0=\delta^2/2$ and define $\lambda_i = \phi_i / \delta$ and therefore $\sum_{i=1}^{N} \lambda_i^2 = 1$

The evolution of a dynamical variable is given by

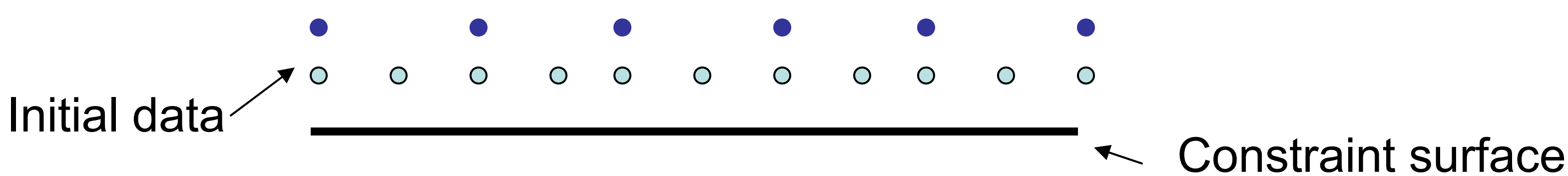
$$q_{n+1} = q_n + \sum_{i=1}^{N} \{q_n, \phi_i\} \lambda_i \delta + O(\delta^2)$$

and if we define $\dot{q} \equiv \lim_{\delta \to 0} (q_{n+1} - q_n)/\delta$,

One obtains in the limit,

$$\dot{q} = \sum_{i=1}^{N} \{q, \phi_i\} \lambda_i$$

Graphically,



The constants of motion of the discrete theory become in the continuum limit the observables ("perennials") of the continuum theory. Conversely, every perennial of the continuum theory has as a counterpart a set of constants of the motion of the discrete theory that coincide with it as a function of phase space in the continuum limit.

We therefore see that in the continuum limit we recover entirely the classical theory: its equations of motion, its constraints and its observables (perennials).

Notice that in the proof of the previous page we assumed the constraints are first class. If they are second class the same proof goes through but one has to use Dirac brackets. This is important for the case of field theories where discretization of space may turn first class constraints into second class ones

In this case one has several options: either one works with Dirac brackets, which may be challenging, or one works with ordinary Poisson brackets but takes the spatial continuum limit first. It may occur in that case that the constraints become first class. Then the method is applicable and leads to a quantization in which one has to take the spatial continuum limit first in order to define the physical space of states.
But probably the most promising avenue is to work with Poisson Brackets and stay with a sufficiently good discrete approximation of the continuum theory.

Quantization:

The discrete theories have no constraints, therefore many of the conceptual problems are bypassed.

To quantize the discrete theory one starts by writing the classical evolution equations $q_{n+1} = q_{n+1}(q_n, p_n), \quad p_{n+1} = p_{n+1}(q_n, p_n)$

One then defines a kinematical space of states H_k as the space of functions of N real variables ψ(q) that are square integrable. We define operators

$\hat{Q}, \ \hat{P}$ as usual and a unitary operator $\hat{U}$ such that,

$$\hat{Q}_n \equiv \hat{U}^{-1}\hat{Q}_{n-1}\hat{U} = \hat{U}^{-n}\hat{Q}_0\,\hat{U}^n\,, \quad \hat{P}_n \equiv \hat{U}^{-1}\hat{P}_{n-1}\hat{U} = \hat{U}^{-n}\hat{P}_0\,\hat{U}^n\,.$$

$$\hat{U} = e^{-i\hat{H}/\hbar}$$

This guarantees we will recover the classical evolution up to factor orderings, providing a desirable “correspondence principle” with the classical discrete theory.

At a classical level, since H is the sum of squares of the constraints, one has that the constraints are satisfied iff H=0. Quantum mechanically we can therefore impose the necessary condition $U\psi=\psi$ in order to define the physical space of state H_{phys}. More precisely, states ψ in H_{phys} are functions in the dual of a subspace of sufficiently regular functions (φ) of H_{kin} such that

$$\int \psi^* \hat{U}^\dagger \varphi dq = \int \psi^* \varphi dq,$$

This condition defines the physical space of states without having to implement the constraints of the continuum theory as quantum operators. We see similarities with the "master constraint" of Thiemann and collaborators.

The operators U allow to define the "projectors" onto the physical space of states of the continuum theory by,

$$\hat{P} \equiv \lim_{M\to\infty} C_M \hat{U}^M.$$

If such a limit exists for some C_M such that $\lim_{M\to\infty}\left(C_{M+1}/C_M\right)=1$ then $\hat{U}\hat{P}=\hat{P}$ and $\hat{U}\hat{P}\Psi=\hat{P}\Psi\ \forall\,\psi\in H_k$

We have analyzed several examples up to now:

M. Campiglia, C. Di Bartolo, R. Gambini, JP Phys. Rev. D **74**, 124012 (2006)

Mechanical systems with N Abelian constraints and in particular the formulation of 2+1 gravity of Noui and Perez. Here the method reproduces the usual Dirac quantization and the group averaging approach.

We also studied the case of a finite number of non Abelian constraints (for instance the case of imposing the generators of SU(2) as constraints). In this case we proved that the method reproduces the results of the standard Dirac quantization and the group averaging approach.

In the case of a non-compact group of constraints, like SO(2,1), the discrete theories exist and contains very good approximations of the classical behavior but the continuum quantum limit does not seem to exist. This parallels technical problems associated with the spectrum of H not containing zero that appear in the master constraint and other approaches as well.

(Gomberoff, Marolf IJMPD 8, 519 (1999); Dittrich, Thiemann CQG 23, 1067 (2006))

The last example suggests a point of view: the continuum limit is an achievable goal in the classical limit (for some states), but one could work with the discrete quantum theories close to the continuum limit as the fundamental framework.

Klauder NPB 547, 397 (1999); Dittrich, Thiemann CQG 23, 1089, (2006)

Spherical symmetry (at last!)

Previous work with the new variables, Bengtsson (1988) and Bojowald and Swiderski (2005, 2006). Choose connections and triads adapted to spherical symmetry,

$$A = A_x(x)\Lambda_3 dr + (A_1(x)\Lambda_1 + A_2(x)\Lambda_2)\, d\theta + ((A_1(x)\Lambda_2 - A_2(x)\Lambda_1)\sin\theta + \Lambda_3\cos\theta)\, d\varphi,$$

$$E = E^x(x)\Lambda_3 \sin\theta \frac{\partial}{\partial x} + \left(E^1(x)\Lambda_1 + E^2(x)\Lambda_2\right)\sin\theta \frac{\partial}{\partial\theta} + \left(E^1(x)\Lambda_2 - E^2(x)\Lambda_1\right)\frac{\partial}{\partial\varphi},$$

Λ's are generators of su(2).

It simplifies the constraints if one introduces a "polar" canonical transformation in the variables A_φ, P^φ, β, P^β

$$\begin{aligned} A_1 &= A_\varphi \cos\beta, \\ A_2 &= -A_\varphi \sin\beta, \\ E^\varphi &= \sqrt{(E^1)^2 + (E^2)^2}. \end{aligned}$$

$$\begin{aligned} P^\varphi &= 2E^1\cos\beta - 2E^2\sin\beta, \\ P^\beta &= -2E^1 A_\varphi \sin\beta + 2E^2 A_\varphi \cos\beta, \end{aligned}$$

To fix asymptotic problems (Bojowald, Swiderski), one does a further canonical change,

$$\begin{aligned} A_\varphi \to \bar{A}_\varphi &= 2\cos\alpha A_\varphi, \\ \beta \to \eta &= \alpha + \beta, \end{aligned}$$

$$P^\beta = P^\eta, \quad P^\varphi = 2E^\varphi\cos\alpha,$$

Leading to the canonical pairs $A_x, E^x, \overline{A}_\varphi, E^\varphi, \eta, P^\eta$.

Finally, one is left with the following form for the constraints,

$$
\begin{aligned}
G &= P^\eta + (E^x)' \\
D &= P^\eta \eta' + E^\varphi \bar{A}'_\varphi - (E^x)' A_x .
\end{aligned}
$$

$$
\begin{aligned}
H = &-\frac{E^\varphi}{2\sqrt{|E^x|}} - \frac{A_x \bar{A}_\varphi \sqrt{|E^x|}}{2\gamma^2} - \frac{\bar{A}_\varphi^2 E^\varphi}{8\sqrt{|E^x|}\gamma^2} + \frac{((E^x)')^2}{8\sqrt{|E^x|}E^\varphi} \\
&- \frac{\sqrt{|E^x|}(E^x)'(E^\varphi)'}{2(E^\varphi)^2} - \frac{\bar{A}_\varphi \sqrt{|E^x|}\eta'}{2\gamma^2} + \frac{\sqrt{|E^x|}(E^x)''}{2E^\varphi} .
\end{aligned}
$$

To simplify matters further, we will fix the spatial coordinate gauge. This eliminates the diffeomorphism constraint, but still leaves a Gauss law and a Hamiltonian constraint with a first class algebra of constraints with structure functions, therefore still a challenging problem.

The choice is $E^x=a$ at the horizon, which in turn puts the latter at $x=0$. The variable a is a dynamical variable that is related to the mass of the space-time $a=M/2$. One solves $D=0$ for A_x and substituting in H one has an equation for the pair A_φ, E^φ.

The Hamiltonian constraint becomes,

$$H = -\frac{E^\varphi}{(x+a)\gamma^2}\left(\frac{\bar{A}_\varphi^2(x+a)}{8}\right)' - \frac{E^\varphi}{2(x+a)} + \frac{3(x+a)}{2E^\varphi} + (x+a)^2\left(\frac{1}{E^\varphi}\right)' = 0.$$

And the constraint algebra is,

$$\begin{aligned}
\{H(x), H(y)\} &= \left(\frac{\bar{A}_\varphi(y)}{2\gamma}H(y)\right)'\delta^3(x-y) - \frac{\bar{A}_\varphi(y)}{\gamma}H(y)\delta^3_{,x}(x-y), \\
\{G(x), H(y)\} &= 0, \\
\{G(x), G(y)\} &= 0.
\end{aligned}$$

So the model, although simplified, is still quite challenging (has structure functions in the constraint algebra). So in principle we cannot treat it with traditional techniques, we could use the “uniform discretizations”.

But it turns out that for this example one can introduce a trick that allows for the traditional treatment. Dividing the constraint by E^φ turns the Hamiltonian constraint **Abelian!** We will then see that one can discretize it in such a way that it remains first class upon discretization. Then one can quantize the discrete theory in the traditional way. But first let us construct a suitable loop representation for these models.

Loop representation for the spherically symmetric case:

Manifold is a line. “Graph” is a set of edges $g = \bigcup_i e_i$. The only variable that behaves as a connnection on the line is A_x. The variables η and A_φ are scalars, so in the loop representation one uses “point holonomies” to represent them.

To avoid presenting too many equations, I will write the states for the “gauge fixed” case we introduced. There the only variables in the bulk are E^φ and $2\gamma K_\varphi = A_\varphi$

$$\mathcal{H} = L^2\left(\otimes_N R_{\rm Bohr}, \otimes_N d\mu_0\right)$$

$$T_{g,\vec{\mu}}[K_\varphi] = \prod_{v \in V(g)} \exp\left(2i\mu_v \gamma K_\varphi(v)\right)$$

$$\hat{E}^\varphi_m = -i\ell^2_{\rm Planck} \frac{\partial}{\partial K_{\varphi,m}},$$

$$\hat{E}^\varphi_m T_{g,\vec{\mu}} = \sum_{v \in V(g)} \mu_m \gamma \ell^2_{\rm Planck} \delta_{m,n(v)} T_{g,\vec{\mu}},$$

Volume of an interval I

$$V(I) = 4\pi \sum_{m \in I} |E^\varphi_m|(x_m + a),$$

$$\hat{V}(I) T_{g,\vec{\mu}} = \sum_{v \in I} 4\pi |\mu_v| (x_v + \hat{a}) \gamma \ell^2_{\rm Planck} T_{g,\vec{\mu}}.$$

We can introduce a basis of loop states $|g, \vec{\mu}>$,

$$\langle K_m^\varphi | g, \vec{\mu}\rangle = T_{g,\vec{\mu}}[K],$$

and the Bohr measure guarantees that,

$$\langle g, \vec{\mu} | g', \vec{\mu}'\rangle = \delta_{g,g'}\delta_{\vec{\mu},\vec{\mu}'} .$$

“Transverse point holonomies” and triads are well defined operators,

$$h_\varphi(v,\rho) \equiv \exp\left(i\rho A_\varphi(v)\right) = \exp\left(2i\rho\gamma K_\varphi(v)\right),$$

$$\hat{h}_\varphi(v_i,\rho)|g,\vec{\mu}>= |g, \mu_{v_1}, \ldots, \mu_{v_i}+\rho, \ldots>,$$

$$\hat{E}_m^\varphi|g,\vec{\mu}>= \sum_{v\in V(g)} \mu_v \gamma \ell_{\rm Planck}^2 \delta_{m,n(v)}|g,\vec{\mu}> .$$

And one can do the “Thiemann trick” (calculation omitted) for the non-polynomial portion of the Hamiltonian constraint (as in the full theory and LQC), and that the inverse of the triad is a bounded operator,

$$\widehat{\frac{{\rm sgn}\,(E_m^\varphi)}{\sqrt{E_m^\varphi}}}|g,\vec{\mu}>= \frac{2}{\sqrt{\gamma}\ell_{\rm Planck}\rho}\sum_{v\in V(g)}\delta_{m,n(v)}\left(|\mu_v+\frac{\rho}{2}|^{\frac{1}{2}} - |\mu_v-\frac{\rho}{2}|^{\frac{1}{2}}\right)|g,\vec{\mu}> .$$

With this one can represent the Abelian Hamiltonian constraint in the loop representation. We start from a classical discretization that is written in terms of quantities that are easy to promote to operators in the loop representation (i.e. replace connections by “small holonomies”, etc)

$$H = \left(\frac{(x+a)^3}{(E^\varphi)^2}\right)' - 1 - \frac{1}{4\gamma^2}\left((x+a)\bar{A}_\varphi^2\right)'.$$

$$H_m^\rho = \frac{1}{\epsilon}\left[\left(\frac{(x_m+a)^3\epsilon^2}{(E_m^\varphi)^2} - \frac{(x_{m-1}+a)^3\epsilon^2}{(E_{m-1}^\varphi)^2}\right) - \epsilon - \frac{1}{4\gamma^2\rho^2}\left((x_m+a)\sin^2\left(\rho\bar{A}_{\varphi,m}\right) - (x_{m-1}+a)\sin^2\left(\rho\bar{A}_{\varphi,m-1}\right)\right)\right]$$

It turns out that it is relatively easy to solve the constraint in the connection representation. One rewrites it as,

$$E_m^\varphi = \pm\frac{(x_m+a)\epsilon}{\sqrt{1-\frac{a}{x_m+a}+\frac{1}{4\gamma^2\rho^2}\sin^2\left(2\rho\gamma K_m^\varphi\right)}},$$

And imposing it as a quantum operator leads to states,

$$\Psi[K_m^\varphi,\tau,a] = C(\tau,a)\exp\left(\pm\frac{i}{\ell_{\rm Planck}^2}\sum_m f[K_m^\varphi]\right),$$

And f is an explicit function of elliptic integrals.

The bottomline is that one recovers the same quantization as Kuchař, one has a wavefunction that depends on the mass C(a,τ), and imposing the constraint on the boundary one gets,

$$C(\tau, a) = C_0(a) \exp\left(-\frac{ia\tau}{2\ell^2_{\text{Planck}}}\right)$$

So one is left with only a function of the mass C_0(a) as the wavefunction of the theory, with no dynamics.

How does the constraint look like in the loop representation? Start from classically rewriting the constraints

$$E^\varphi_m = \pm \frac{(x_m + a)\epsilon}{\sqrt{1 - \frac{a}{x_m+a} + \frac{1}{4\gamma^2\rho^2}\sin^2\left(2\rho K^\varphi_m\right)}},$$

as O_m =1. Then the quantum version of O is,

$$\hat{O}_m = \left(\sqrt{1 - \frac{a}{x_m + a} + \frac{1}{4\gamma^2\rho^2}\sin^2\left(2\rho \hat{K}^\varphi_m\right)}\,\frac{\hat{E^\varphi_m}}{(x_m + a)\epsilon}\right)^2$$

This can be immediately represented in the loop representation as,

$$\left(1-\frac{a}{x_m+a}+\frac{1}{8\gamma\rho}\right)\frac{\mu_m^2\gamma^2\ell_{\rm Planck}^4}{(x_m+a)^2\epsilon^2}\Psi\left(\mu_m\right)-\left[\frac{\mu_m^2\gamma^2\ell_{\rm Planck}^2}{(x_m+a)^2\epsilon^2}+\frac{2\mu_m\gamma\ell_{\rm Planck}^4}{(x_m+a)^2\epsilon^2}\right]\frac{\Psi\left(\mu_m+4\rho\right)}{16\gamma\rho}$$
$$-\left[\frac{\mu_m^2\gamma^2\ell_{\rm Planck}^2}{(x_m+a)^2\epsilon^2}-\frac{2\mu_m\gamma\ell_{\rm Planck}^4}{(x_m+a)^2\epsilon^2}\right]\frac{\Psi\left(\mu_m-4\rho\right)}{16\gamma\rho}=\Psi\left(\mu_m\right).$$

Notice the parallels with the expression that Abhay Ashtekar wrote in his talk for the Loop Quantum Cosmology case.

This is suggestive, since it might imply a similar resolution for the Schwarzschild singularity as one had for the cosmological one. However, detailed calculations in horizon penetrating coordinates would be needed to confirm this.

The above recursion relation can be explicitly solved and one can show that the solution is the “loop transform” of the solution we found in the connection representation,

$$\Psi_r\left(\mu_m\right)=\int_0^{\pi/(\rho\gamma)}dK^{\varphi}\Psi\left(K\right)\exp\left(2\rho K^{\varphi}\gamma\mu_m(r)\right).$$

r identifies superselection sector.

Use of uniform discretizations:

What if we had not made use of the trick of Abelianizing the constraints? Then the only approach we know is to use the “uniform discretizations”. It turns out that one can explicitly construct a discretization of the constraint that exhibits some of the challenges that one expects in the full theory.

Namely, one constructs the evolution operator (exponential of the sum of the square of constraints), and one can estimate a bound for the minimum eigenvalue of the exponent, using the states of the Abelian model,

$$\langle\Psi|\hat{\mathbb{H}}|\Psi\rangle = C_1\frac{\epsilon^3}{a^3} + C_2\frac{\ell^2_{\rm Planck}}{a^3} + C_3\frac{\ell^4_{\rm Planck}}{a\epsilon^3} + \sum_{n=3}^{8} C_{n+1}\frac{\ell^{2n}_{\rm Planck}}{a\epsilon^{n+1}},$$

Significant ambiguity

The operator therefore does not have a quantum continuum limit, $\varepsilon \to 0$, $l_{\rm Planck}$ finite. On the other hand it does have a classical continuum limit, $\varepsilon \to 0$, $l_{\rm Planck} \to 0$.

Solving the model becomes a (hard) problem in quantum mechanics, akin to solid state physics. It will have to be tackled via variational, perturbative or numerical methods.

Since we cannot take the quantum continuum limit, we can ask, what is the minimum eigenvalue of H? We can again estimate an upper bound for this by evaluating <Ψ|H|Ψ> with the states we constructed in the Abelian model. If one does that one finds that $\epsilon = \sqrt[3]{\ell^2_{\rm Planck} a}.$

So the discrete theory has a minimum value for the length. For a Solar sized mass, The eigenvalue of H is 10^{-80}. So the discrete theory approximates very well the continuum one.

One last intriguing observation:

As in LQC, one expects the value of the parameter of the "transverse point holonomy", ρ, to take a finite minimum value, $h_\varphi(v,\rho) \equiv \exp\left(i\rho A_\varphi(v)\right) = \exp\left(2i\rho\gamma K_\varphi(v)\right)$

If one acts with this holonomy on a spin network state, one adds an element of volume, $\Delta V = 4\pi\rho(x_v + a) l^2_{\rm Planck}$

If one considers the volume of a shell of width Δx asymptotically one has N elements of volume per shell,

$N = x\dfrac{\Delta x}{l^2_{\rm Planck}}$, so for a finite shell $N \approx \dfrac{b^2 - a^2}{l^2_{\rm Planck}}$ Bekenstein bound ?

Summary:

- One can study spherically symmetric space-times using loop quantum gravity.
- One needs to use special features of spherical symmetry to apply the traditional Dirac quantization technique.
- The setup is ready, we need to extend it to horizon penetrating coordinates to make statements about the singularity.
- Without using special tricks the problem is hard, and it offers a promising arena to test new ideas for handling the problem of dynamics in canonical quantum gravity, like the “uniform discretization” approach.