To Bounce or Not to Bounce

KITP

January 2007

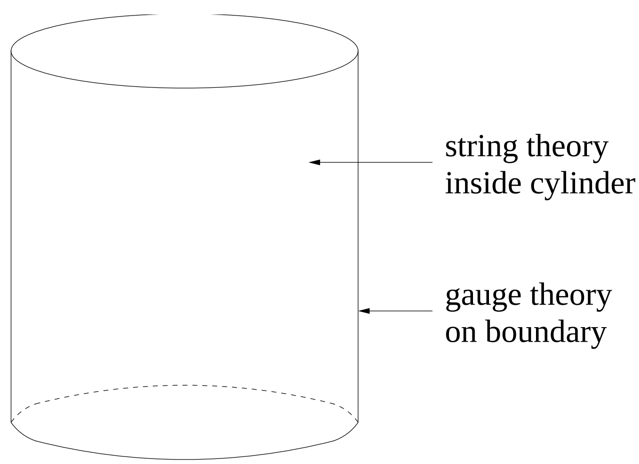
Thomas Hertog

w/ Gary Horowitz, Ben Craps and Neil Turok

The AdS/CFT correspondence

String theory with anti-de Sitter boundary conditions is equivalent to certain gauge theories living on the boundary of the AdS cylinder.

[Maldacena '97]



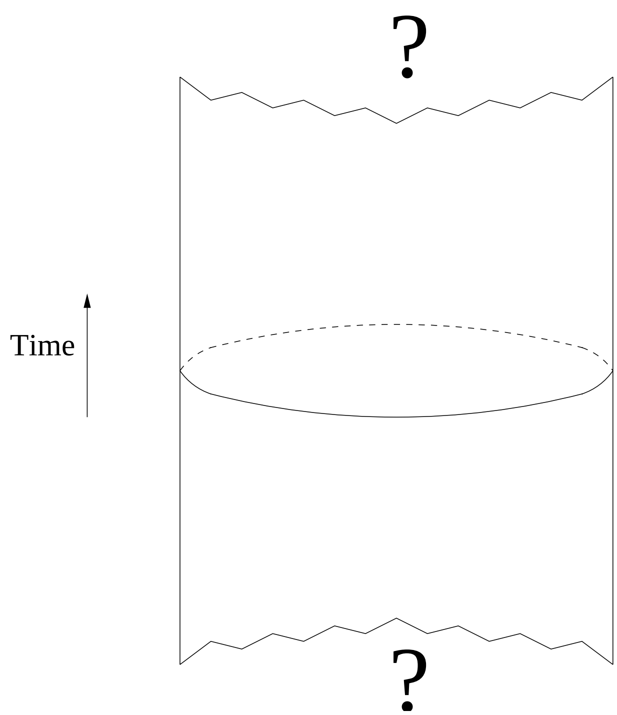
Strong/weak coupling duality,

$$l_{AdS}^4/l_s^4 \sim g_s N \sim \lambda$$

The finite N gauge theory is viewed as a *nonperturbative definition* of string theory with AdS boundary conditions.

Holographic (AdS) Cosmology

Generalization: SUGRA solutions where smooth asymptotically AdS initial data emerge from a big bang in the past and evolve to a big crunch in the future. [T.H & G. Horowitz '04]



Does the dual finite N gauge theory evolution give a fully quantum gravity description of the singularities?

Outline

- Cosmology with AdS_4 boundary conditions
- Dual CFT Evolution
- AdS_5 cosmology and its dual description

Setup

We consider the following action,

$$S = \int d^4x\sqrt{-g}\left[\tfrac{1}{2}R - \tfrac{1}{2}(\nabla\phi)^2 + 2 + \cosh(\sqrt{2}\phi)\right]$$

$\rightarrow$ consistent truncation of M-theory with $AdS_4 \times S^7$ boundary conditions.

Scalar, $\quad m^2 = -2 > m^2_{BF} = -9/4$

AdS in global coordinates,

$$ds^2 = -(1+r^2)dt^2 + \tfrac{dr^2}{1+r^2} + r^2 d\Omega_2$$

In all asymptotically AdS solutions, ϕ decays as

$$\phi(t,r,\Omega) = \tfrac{\alpha(t,\Omega)}{r} + \tfrac{\beta(t,\Omega)}{r^2}$$

Boundary Conditions

Standard (susy) boundary conditions on ϕ: $\beta = 0$

$$\phi = \frac{\alpha(t,\Omega)}{r} + \mathcal{O}(1/r^3)$$

$$g_{rr} = \frac{1}{r^2} - \frac{(1+\alpha^2/2)}{r^4} + O(1/r^5)$$

More generally: $\beta(\alpha) \neq 0$

$$\phi = \frac{\alpha(t,\Omega)}{r} + \frac{\beta(\alpha)}{r^2}$$

Conserved charges remain finite, but acquire explicit contribution from ϕ.

e.g. mass of spherical symmetric solutions,

$$M = 4\pi(M_0 + \alpha\beta + \int_0^\alpha \beta(\tilde{\alpha})d\tilde{\alpha})$$

AdS-invariant boundary conditions

One-parameter class of functions $\beta_k(\alpha)$ that define AdS-invariant boundary conditions,

$$\beta_k = k\alpha^2$$

$$M = 4\pi(M_0 + \tfrac{4}{3}k\alpha^3)$$

Claim: For all $k \neq 0$, there exist smooth asymptotically AdS initial data that evolve to a singularity which extends to the boundary of AdS in finite global time.

Examples:

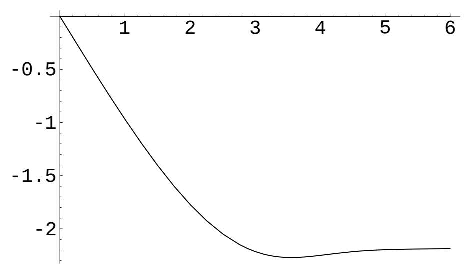
1. Evolution of rescaled soliton initial data

2. FRW cosmologies from analytic continuation of Euclidean instantons.

AdS Cosmology

$O(4)$ symmetric Euclidean instanton,

$$ds^2 = \frac{d\rho^2}{b^2(\rho)} + \rho^2 d\Omega_3, \qquad \phi(\rho) \sim \frac{\alpha}{\rho} + \frac{\beta}{\rho^2}$$



Lorentzian cosmology from analytic continuation:

- Inside lightcone from $\phi(0)$: FRW evolution to big crunch that hits boundary as $t \to \pi/2$.

- Asymptotically (at large r) one has

$$\phi = \frac{\alpha(t)}{r} + \frac{k\alpha^2(t)}{r^2} + O(r^{-3}), \qquad \alpha(t) = \frac{\alpha(0)}{\cos t}$$

Dual Field Theory

M Theory with $AdS_4 \times S^7$ boundary conditions is dual to the 2+1 CFT on a stack of M2 branes.

- With $\beta = 0$, $\phi \sim \alpha/r$ is dual to $\Delta = 1$ operator $\mathcal{O}$,

$$\mathcal{O} = \frac{1}{N} Tr T_{ij} \varphi^i \varphi^j$$

  and

$$\alpha \leftrightarrow \langle \mathcal{O} \rangle$$

- Taking $\beta(\alpha) \neq 0$ corresponds to adding a multitrace interaction $\int W(\mathcal{O})$ to the CFT, such that [Witten '02, Berkooz et al. '02]

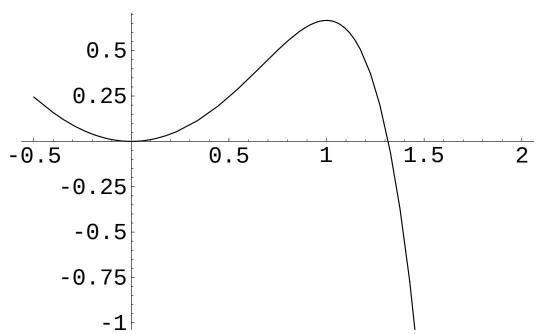
$$\beta = \frac{\delta W}{\delta \alpha}$$

Dual Field Theory

With $\beta_k = k\alpha^2$,

$$S = S_0 + \frac{k}{3}\int \mathcal{O}^3$$

The dual description of AdS cosmologies involves field theories that at first sight always contain at least one operator $\mathcal{O}$ with a potential that is unbounded from below.



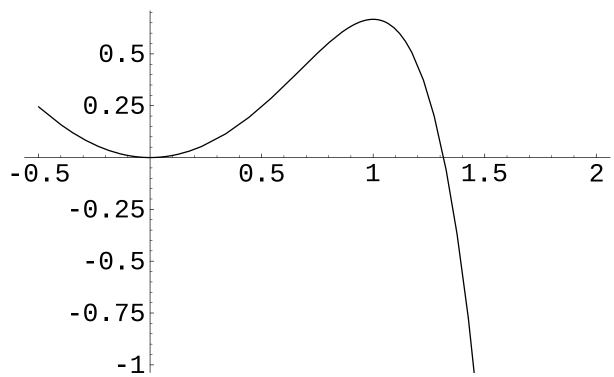
What is the CFT evolution dual to AdS cosmologies?

To leading order in $1/N$, $\quad < \mathcal{O} > \to \infty$

Toy Model Field Theory

Neglecting the nonabelian structure ($\mathcal{O} \leftrightarrow \varphi^2$), the potential becomes

$$V = \frac{1}{8}\varphi^2 - \frac{k}{3}\varphi^6$$



This admits an exact homogeneous classical (zero energy) solution,

$$\varphi(t) \sim \frac{1}{k^{1/4}\cos^{1/2} t}$$

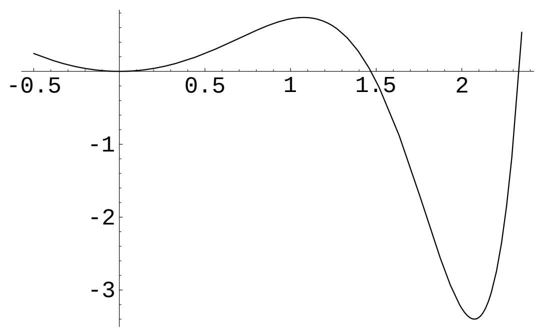
which reproduces time evolution of SUGRA solutions.

$\longrightarrow$ semiclassically and at large N, the CFT evolution ends in finite time.

Regularization

Regularize by adding quartic interaction $\epsilon\mathcal{O}^4$,

$$V = \frac{1}{8}\varphi^2 - \frac{k}{3}\varphi^6 + \frac{\epsilon}{4}\varphi^8$$



This changes bulk boundary conditions to

$$\beta_{k,\epsilon} = -k\alpha^2 + \epsilon\alpha^3,$$

- small change instanton initial data, $M_i \sim \epsilon$
- potentially significant change bulk evolution in regime $\alpha^2 > k/\epsilon$, i.e. near the singularities

Black Holes with Scalar Hair

Static spherical solutions

$$ds_4^2 = -h(r)e^{-2\delta(r)}dt^2 + h^{-1}(r)dr^2 + r^2 d\Omega_2^2$$

of Einstein eqs,

$$h\phi_{,rr} + \left(\frac{2h}{r} + \frac{r}{2}\phi_{,r}^2 h + h_{,r}\right)\phi_{,r} = V_{,\phi}$$

$$1 - h - rh_{,r} - \frac{r^2}{2}\phi_{,r}^2 h = r^2 V(\phi)$$

$$\delta_{,r} = -\frac{1}{2}\phi_{,r}^2$$

Asymptotically:

$$\phi(r) = \frac{\alpha}{r} + \frac{\beta}{r^2}$$

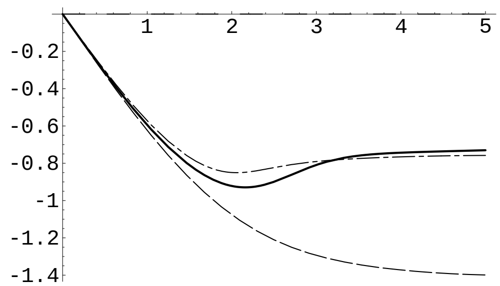
Regularity at horizon R_e determines $\phi_{,r}(R_e)$.

Integrating field equations outward yields a point in (α, β) plane for each pair (R_e, ϕ_e).

Repeating for all ϕ_e gives curve $\beta_{R_e}(\alpha)$.

Black Holes with Scalar Hair

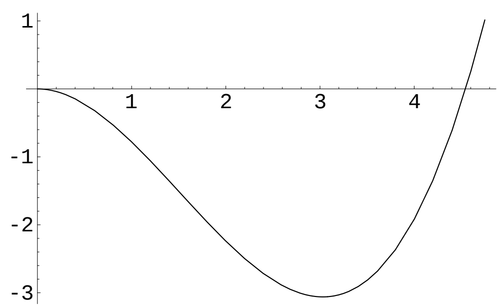
Some curves $\beta_{R_e}(\alpha)$:



For given boundary conditions $\beta(\alpha)$, the hairy black hole solutions are given by the intersection points,

$$\beta_{R_e}(a) = \beta(\alpha)$$

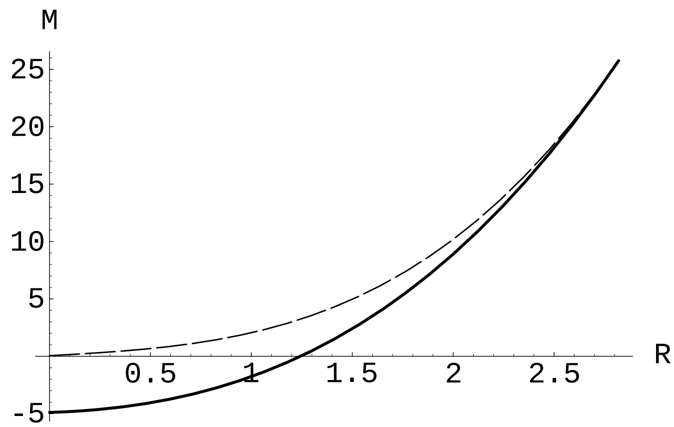
In particular, with $\beta_{k,\epsilon} = -k\alpha^2 + \epsilon\alpha^3$,



$\longrightarrow$ two branches of hairy black holes, corresponding to typical excitations about the $\alpha \neq 0$ vacua of $V(\phi)$.

Hairy Black Hole Mass

Mass of hairy black holes:



$\longrightarrow$ the large $M \sim \epsilon$ hairy black hole is the natural endstate of evolution if the instanton initial data are evolved with $\beta_{k,e} = -k\alpha^2 + \epsilon\alpha^3$ boundary conditions.

- As $\epsilon \longrightarrow 0$, one has for the $M \sim \epsilon$ black hole

$$R_e \longrightarrow \infty, \qquad \phi_e \longrightarrow \phi_i(0)$$

- Bulk evolution independent of ϵ for a while.

Back to Cosmology

If the field theory is regular, then AdS/CFT suggests evolution to a big crunch can be viewed as evolving to an equilibrium state in the dual theory.

What corrections are sufficient?

Bulk: black hole forms when $\beta(\alpha) \to -C$ for large α.

Hence it is sufficient that at large ϕ,

$$V(\phi) \geq -C\phi^2$$

$\to$ V can be unbounded from below, as long as a wave packet does not reach infinity in finite time.
(i.e. H is automatically self-adjoint)

Equilibration can happen because inhomogeneities first grow when V" decreases and then become dynamically important when V" increases again to $-C$.

AdS_5 cosmology

Consider now the following action,

$$S = \int d^5x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}(\nabla\phi)^2 + 2e^{2\phi/\sqrt{3}} + 4e^{-\phi/\sqrt{3}}\right]$$

$\longrightarrow$ consistent truncation of string theory with $AdS_5 \times S^5$ boundary conditions.

Scalar, $\quad m^2 = -4 = m^2_{BF}$

In all asymptotically AdS solutions, ϕ decays as

$$\phi(t, r, \Omega) = \frac{\beta(t,\Omega)\ln r}{r^2} + \frac{\alpha(t,\Omega)}{r^2}$$

For boundary conditions

$$\beta_\lambda = -\lambda\alpha$$

there are instanton initial data that 'probably' produce a big crunch.

Dual Field Theory

String theory with $AdS_5 \times S^5$ boundary conditions is dual to $\mathcal{N}$=4 super Yang-Mills theory in $D = 4$.

- For $\beta = 0$, $\phi \sim \alpha/r^2$ is dual to $\Delta = 2$ operator $\mathcal{O}$,

$$\mathcal{O} = Tr(\varphi_1^2 - \varphi_2^2)$$

  and $\alpha \leftrightarrow \langle \mathcal{O} \rangle$

- Taking $\beta(\alpha) = -\lambda\alpha$ corresponds to adding a potential term

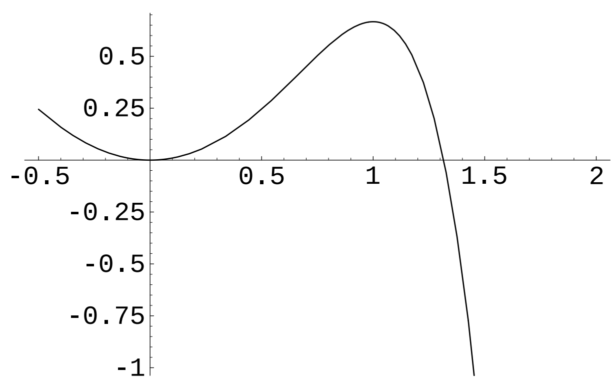
$$S = S_{YM} - \frac{\lambda}{2} \int \mathcal{O}^2$$

But this is essentially $\lambda\varphi^4$ in $D = 4$, which is renormalizable with only logarithmic corrections to the classical (unbounded) potential. [Witten '02]

Unstable Field Theories

The dual description of AdS cosmologies involves field theories that are genuinely unstable.

$$V = \frac{1}{8}\varphi^2 - \frac{\lambda}{2}\varphi^4$$



What are the principles?

Possible simplification since V" continues to decrease.

$\longrightarrow$ consider homogeneous mode $\varphi(t) = x(t)$.

"Quantum mechanics with unbounded potentials."

Quantum Mechanics

A right-moving wave packet in $V(x)$ reaches infinity in finite time.

To ensure probability is not lost at infinity one can construct a self-adjoint extension of the Hamiltonian, by carefully specifying its domain. [Carreau et al. '90]

$\rightarrow$ unitary evolution for all time.

What happens?

The center of a wave packet follows essentially the classical trajectory. When it reaches infinity, however, it bounces back.

$\rightarrow$ The AdS/CFT correspondence indicates that evolution continues, with an immediate transition from a big crunch to a big bang.