Local observation in quantum gravity

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Based on:
hep-th/0512200, w/ Marolf and Hartle;
hep-th/0612191, w/ M. Gary
(hep-th/0604072)

Quantum nature of spacetime singularities
KITP

The problem:

How to describe local observation in a quantum-mechanical theory with gravity?

Motivation:

- Singularities: local phenomena; likewise their resolution ~ local issue
- Connect to QFT
- Limitations -- should take seriously? (QM guide: unobservable features not part of theory)
- Locality -- likely not fundamental?

The basic issue:

In QFT (all physics except gravity):

$\mathcal{O}(x)$ gauge invariant $\Rightarrow$ local observable

describe our measurements ...

Gravity: gauge symmetry $\supset$ diffeomorphisms

$$\delta_\xi \mathcal{O}(x) = \xi^\mu \partial_\mu \mathcal{O}(x) \neq 0$$

$\therefore \quad \mathcal{O}(x)$ is not observable

What observables in a theory w/ GR+QFT reduce to local observables of QFT?

Proposed resolution: via a relational approach

Einstein (1916)

DeWitt (1962, 1967)

Page and Wooters (1983)

Banks (1985)

Hartle (1986)

Rovelli (1990, 1991, 2002)

Tsamis and Woodard (1992)

Smolin (1993)

Ashtekar, Tate, Uggla (1993)

Marolf (1994)

Gambini, Porto, Pullin (2003-2006)

Dittrich (2004-2006)

Thiemann (2004-2006)

Pons and Salisbury (2005)

...

For a complete treatment, presumably need full description of fundamental degrees of freedom ...

But ... it should be possible to reconcile gauge symmetry and local observation in the low energy effective theory (small parameter: $p_i \cdot p_j / M_P^2$)

And: such considerations could constrain allowed degrees of freedom

Basic points of this talk:

- Following a relational approach, one can write down diff invariant relational observables
- In certain states, certain of these approximately reduce to local observables of QFT ("quasi-local")
- There appear to be fundamental limitations to such locality implied by basic quantum and gravitational principles

Example: "Z-model" (broad brush)

Begin w/ a field theory w/ local operator $\mathcal{O}(x)$

Introduce four fields Z^i, and a state $|\Psi\rangle$ such that

$$\langle\Psi|Z^i|\Psi\rangle = \lambda\delta^i_\mu x^\mu$$

Define operators

$$\mathcal{O}_{\xi^i} = \text{``}\int d^4x\sqrt{-g}\,\delta(Z^i(x)-\xi^i)\,\mathcal{O}(x)\text{''}$$

$$\langle \Psi | Z^i | \Psi \rangle = \lambda \delta^i_\mu x^\mu$$

$$\mathcal{O}_{\xi^i} = \text{“} \int d^4x \sqrt{-g}\, \delta(Z^i(x) - \xi^i)\, \mathcal{O}(x) \text{ ”}$$

Then:

$$\langle \Psi | \mathcal{O}_{\xi_1} \cdots \mathcal{O}_{\xi_N} | \Psi \rangle \approx \mathcal{O}(x_1^\mu) \cdots \mathcal{O}(x_N^\mu)$$

where

$$x_A^\mu = \frac{1}{\lambda} \delta_i^\mu \xi_A^i$$

Precise, explicit illustration in 2d gravity ...

2d Liouville gravity:

$X^0, \cdots, X^i$ c massless scalar fields

$$\mathcal{Z} = \int \mathcal{D}g \mathcal{D}X e^{iS[X,g]}$$

$$g_{ab} = e^{\phi} \hat{g}_{ab}$$

$$= \int \mathcal{D}\phi \mathcal{D}X e^{i(S_L[\phi,\hat{g}]+S[X,\hat{g}])}$$

w/

$$S_L = \frac{c-25}{48\pi} \int d^2x \sqrt{\hat{g}} \left(\frac{1}{2} \hat{g}^{ab} \partial_a \phi \partial_b \phi + \hat{R}\phi \right)$$

Simplest case -- c=25: $\phi \leftrightarrow X^{25}$

Analogue to Z-model (and fully diff invt):

$$Z^i \leftrightarrow X^0, X^1$$

$$\mathcal{O} \leftrightarrow \mathcal{O}[X^2, \cdots, X^{24}]$$

$$|\Psi\rangle \leftrightarrow \ ?$$

$$\mathcal{O}_\xi \leftrightarrow \ ?$$

$|\Psi\rangle$: $x^a = (t, \theta)$; cylinder

$$\langle\Psi|X^0|\Psi\rangle = p^0 t$$

$$\langle\Psi|X^1|\Psi\rangle = R\theta$$

w/ $X^1 \cong X^1 + 2\pi R \ , \ R \gg 1$

winding state of string!

Satisfies 2d WdW eqn (= Virasoro constraints)

$$\leftrightarrow (p^0)^2 = \frac{R^2}{4} - 2$$

$\mathcal{O}_\xi$:

Let $\mathcal{O}[X^2, \cdots, X^{24}]$ have conformal dim Δ

Then

$$\hat{\mathcal{O}}(k_0, k_1) = \int d^2x \sqrt{\hat{g}} \mathcal{O} e^{ik_0 X^0 + ik_1 X^1 + ik_{25} X^{25}}$$

is diff invt

$$\Leftrightarrow k_{25} = \pm 2 \sqrt{\frac{1-\Delta}{2} - \frac{k_a k^a}{4}}$$

$\therefore$ Fourier transform w.r.t. $k_a = (k_0, k_1)$!

$$\hat{\mathcal{O}}(k_0, k_1) = \int d^2x \sqrt{\hat{g}}\mathcal{O}e^{ik_0X^0+ik_1X^1+ik_{25}X^{25}}$$

$$\rightarrow \hat{\mathcal{O}}(\hat{t}, \hat{\theta}) = \int d^2k \; e^{-\sigma^2 k_a^2} \; e^{2ik^0p^0\hat{t}-ik^1R\hat{\theta}} \; \hat{\mathcal{O}}(k_0, k_1)$$

(Note: $\sigma \gtrsim 1$, $\sim$ resolution)

...our quasi-local observables

Then:

$$\langle\Psi|\hat{\mathcal{O}}(\hat{t}_1,\hat{\theta}_1)\cdots\hat{\mathcal{O}}(\hat{t}_N,\hat{\theta}_N)|\Psi\rangle$$

$$\approx\int\prod_{i=1}^{N}d^2x_i e^{-(p^0/\sigma)^2(t_i-\hat{t}_i)^2-(R/2\sigma)^2(\theta_i-\hat{\theta}_i)^2}f(x_i)$$

$$\cdot\left\langle\prod_{i=1}^{N}\mathcal{O}_i(x_i)\right\rangle$$

... Localized!

Resolution: $\Delta t\sim\sigma/p^0$, $\Delta\theta\sim\sigma/R$

Resolution, and limitations

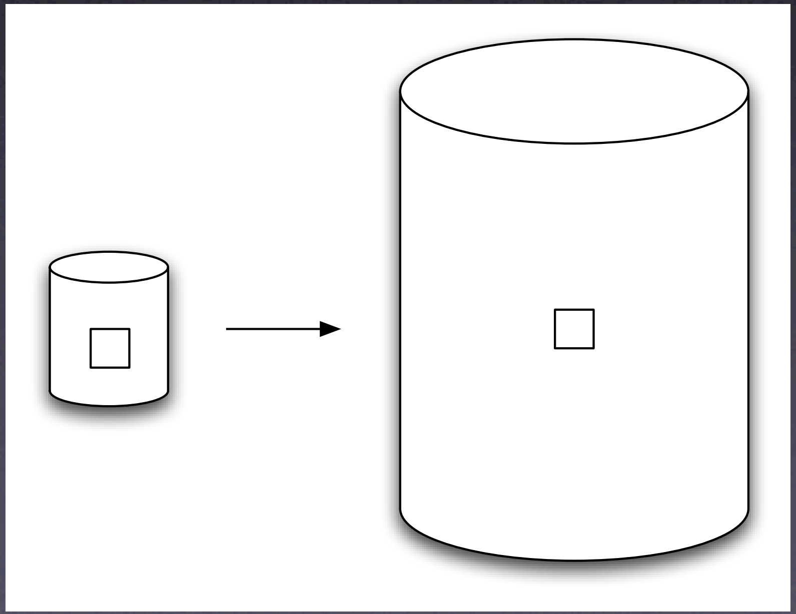
$$\Delta t \sim \sigma / p^0 \quad , \quad \Delta\theta \sim \sigma / R$$

$p^0 \ , \ R \ \sim$ X-field momentum

String intuition:

Target space resolution:

$$\Delta X \gtrsim 1$$



2d gravity:

Increase p^0 , R $\Rightarrow$ arbitrarily fine resolution

(gravity ~ trivial; conformal invariance)

4d gravity:

Increase field momentum $\Rightarrow$ large gravitational backreaction!

Suggests:

basic limitations

"locality bounds," instrumentation bounds

(see hep-th/0604072, hep-th0512200, +refs)

Other comments:

1) Generalizations

specific examples of more general framework

2) Measurement

For certain observables in certain states, can recover Copenhagen measurement theory...

3) Strings?

a) String field theory

Gauge symmetry constraints; one proposal:

$$\partial_g S[\Psi]$$

b) AdS/CFT

Assuming local data (<R) encoded, expect:

relational observables in the NxN matrix

Summary:

- Can recover local observables of QFT from a relational approach

  specific operators

  specific states

- Such reconciliation of local observation and diff invariance can be explicitly illustrated in 1+1 dim quantum systems

- There are fundamental limits to such locality

  quantum effects

  gravitational effects (>2d)

- These suggest fundamental limitations to local dynamics in quantum gravity

“Ultimate detector”

Suppose we wish to instrument a region of space of size R with a state capable of making measurements at resolution r

This requires exciting fields with momenta 1/r in each "cell" of size r. Total energy:

$$E \sim \frac{1}{r}\left(\frac{R}{r}\right)^3$$

Condition for small grav. backreaction:

$$R \gtrsim \frac{1}{M_P^2}\frac{1}{r}\left(\frac{R}{r}\right)^3$$

→ Strong ~holographic constraint:

$$N(R) \sim (M_P R)^{3/2}$$

(c.f. 't Hooft; Cohen, Kaplan, Nelson)

Possibly get $N(R) \sim (M_P R)^2$, accounting for grav DOF (or different eq. of state??)