The origin of the jamming critical exponents

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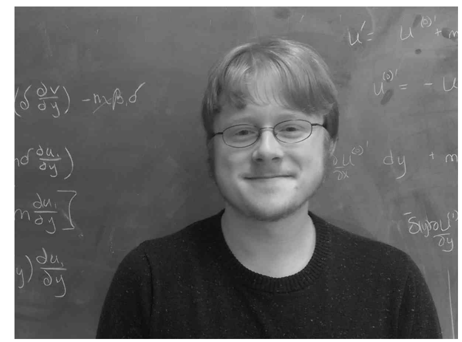

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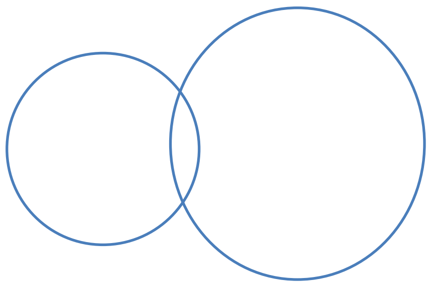
S. Chen, T. Bertrand, M. D. Shattuck, CSO, "Stress anisotropy in shear jammed packings of frictionless disks," *Phys. Rev. E* 98 (2018) 042906.

A. Boromand, A. Signoriello, F. Ye, C. S. O'Hern, and M. D. Shattuck, "Jamming of deformable polygons," *Phys. Rev. Lett*. 121 (2018) 248003.

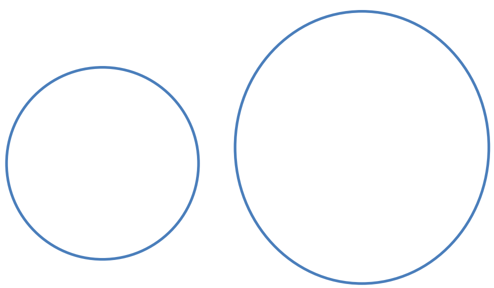
K. Vanderwerf, A. Boromand, M. D. Shattuck, and C. S. O'Hern, "The origin of the jamming critical exponents," in preparation (2019).

Jamming Transitions

Purely Repulsive, Frictionless Soft Sphere/Disk Model



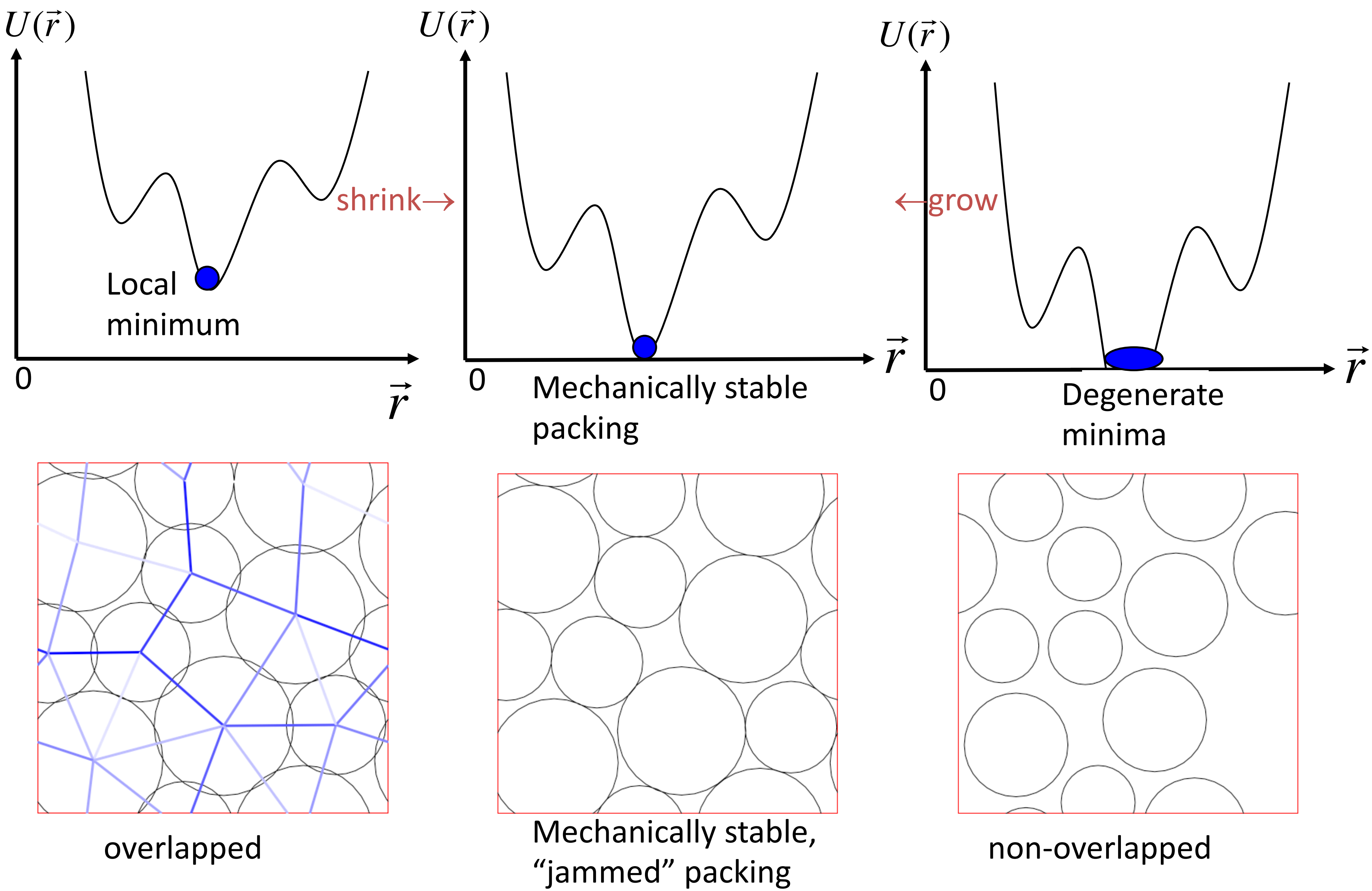
$$U\left(r_{ij}\right)=\frac{\varepsilon}{2}\left(1-\frac{r_{ij}}{\sigma_{ij}}\right)^{\alpha=2} \qquad r_{ij}<\sigma_{ij}$$



$$= 0 \qquad r_{ij}\geq\sigma_{ij}$$

$$\phi=\frac{\sum_{i=1}^{N}\pi R_i^2}{A_{box}}$$

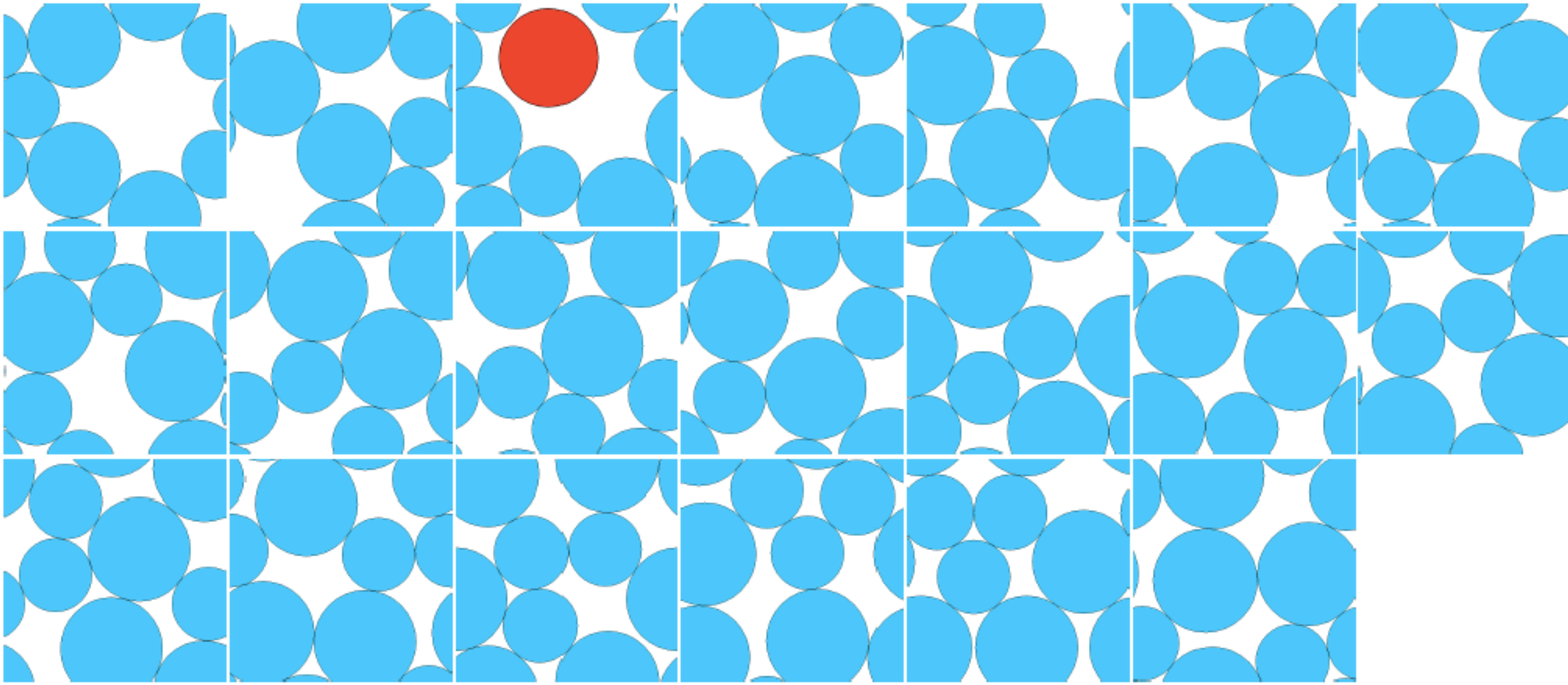
Potential Energy Landscape (PEL)



Isostatic packings, $N_c = N_c^{iso} = 2N - 1$

For N=6; N_c=11; N_p = 20 ~ exp[aN]

ϕ=0.633

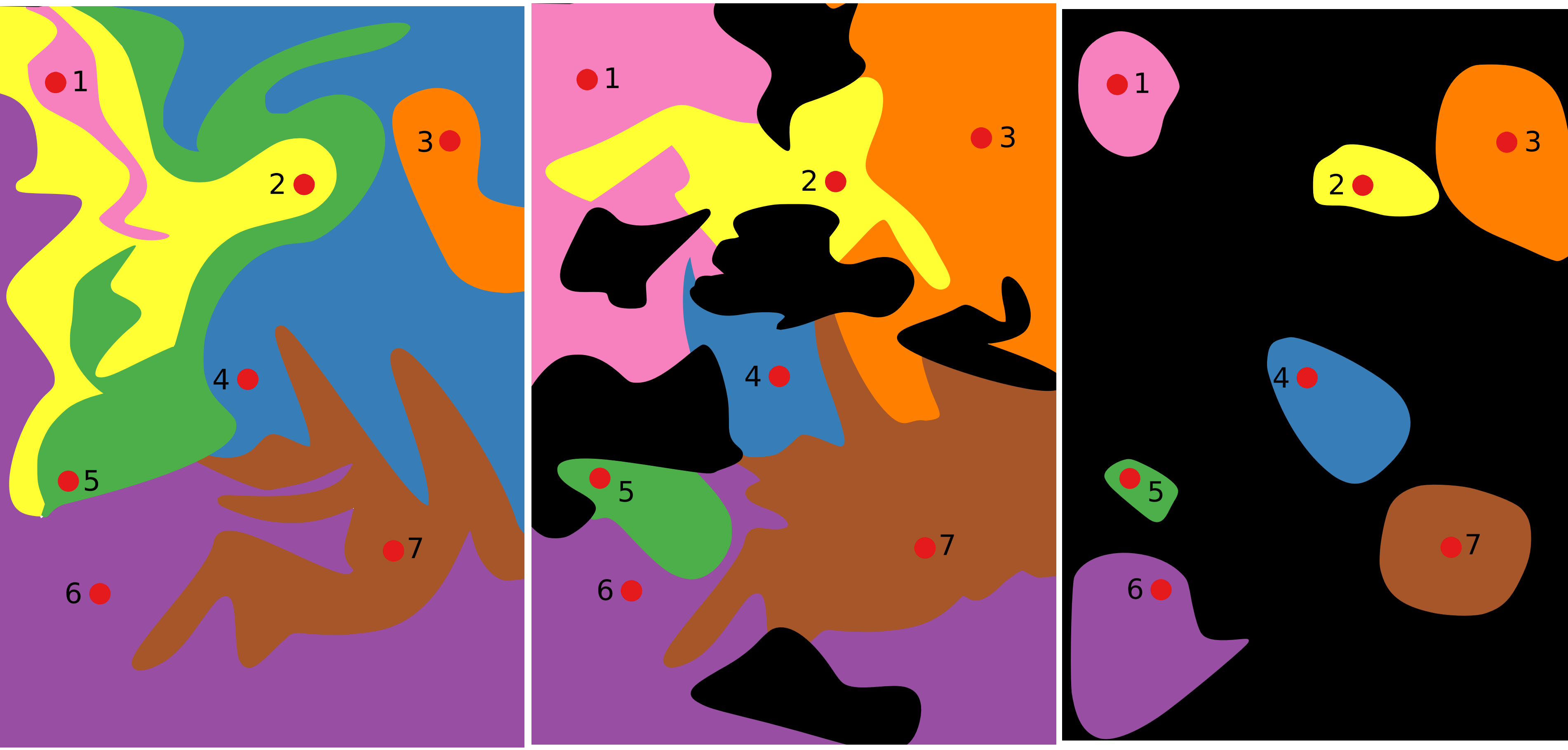


ϕ=0.772

ϕ=0.778

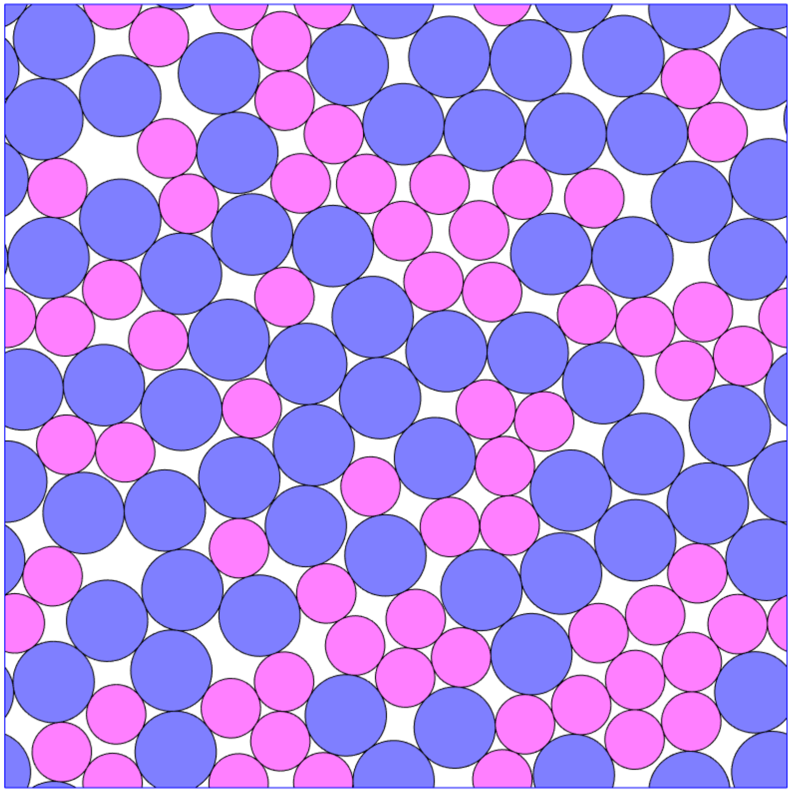
Basin volumes for hard spheres

disallowed

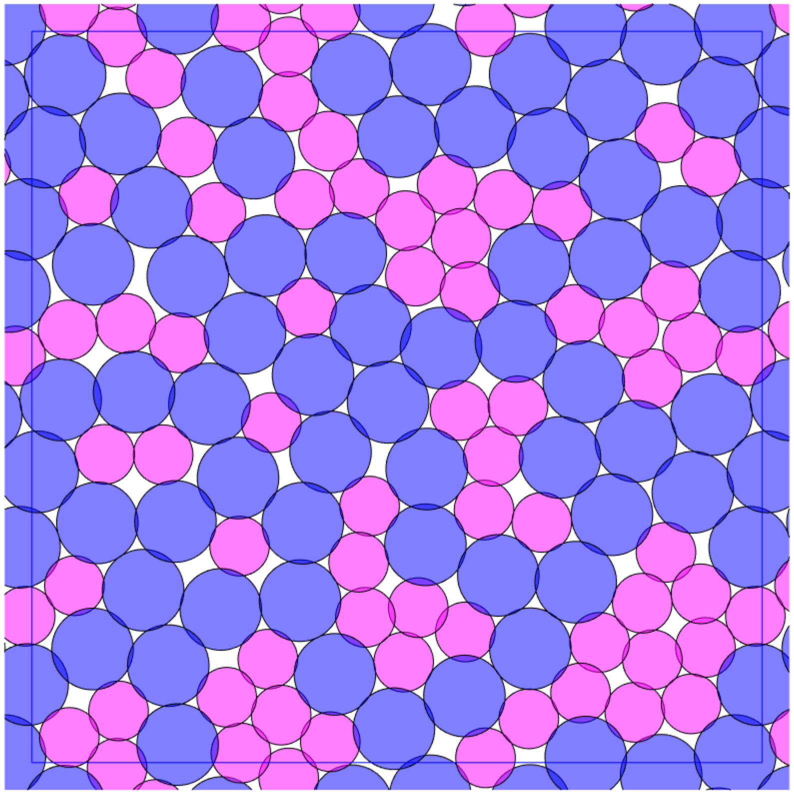


$\phi=0$ | $0 < \phi < \phi_J$ | $\phi \approx \phi_J$

Characterize the structural and mechanical properties of compressed packings at and above onset of jamming

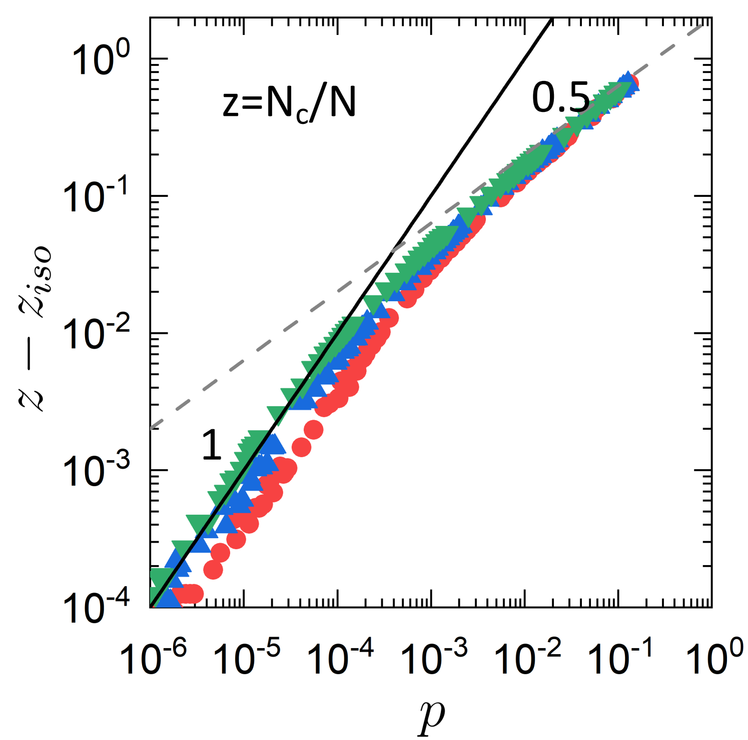


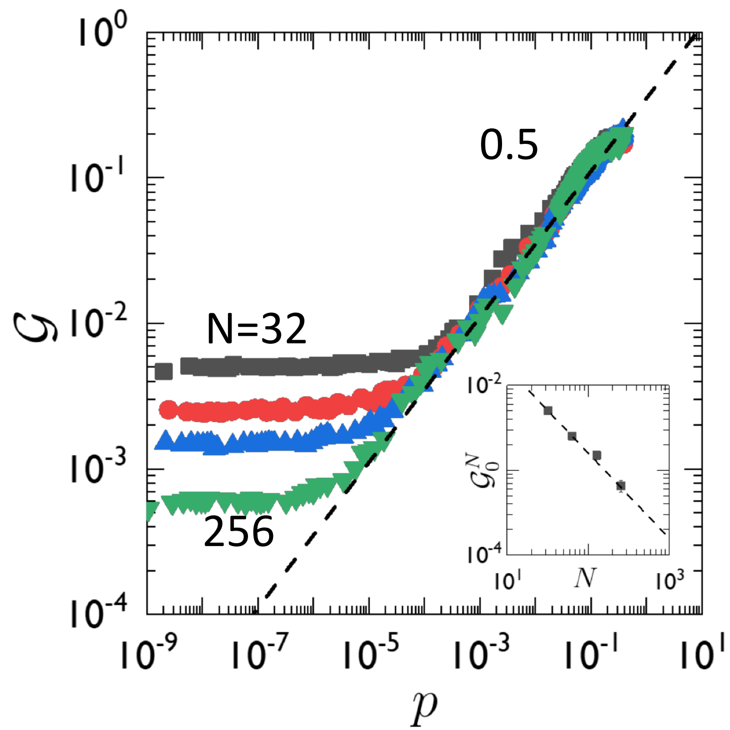
$p=0$; $\phi=\phi_J$



$p=10^{-2}$; $\phi>\phi_J$

Ensemble-averaged structural and mechanical properties of compressed packings above onset of jamming





$$z - z_{iso} \sim p^{\lambda}$$ λ=0.5,1

$$G \sim p^{\beta}$$ β=0,0.5

C. P. Goodrich, A. J. Liu, and S. R. Nagel. Phys. Rev. Lett. 109 (2012) 095704

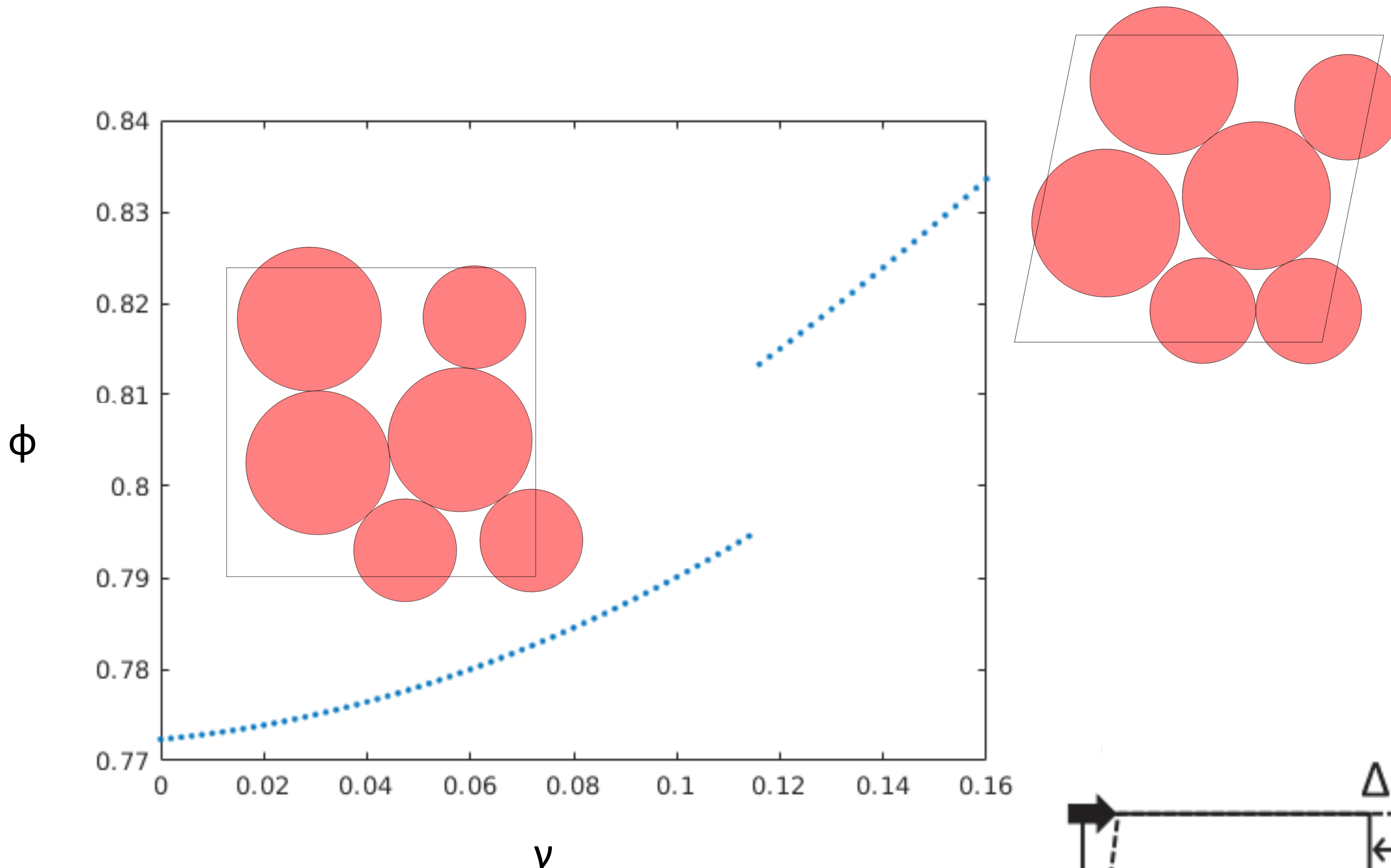
Open Questions

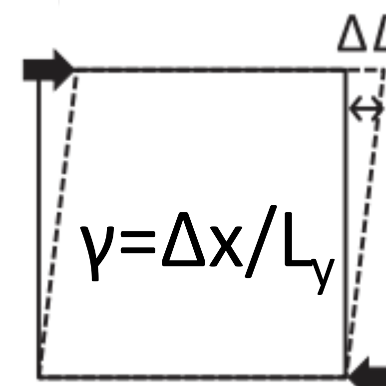
1. What determines the values of the power-law scaling Exponents λ and β for compressed disk packings? Why don't they depend on the interparticle potential?

2. How general are the exponents? For example, can they be used to describe compressed foams and emulsions?

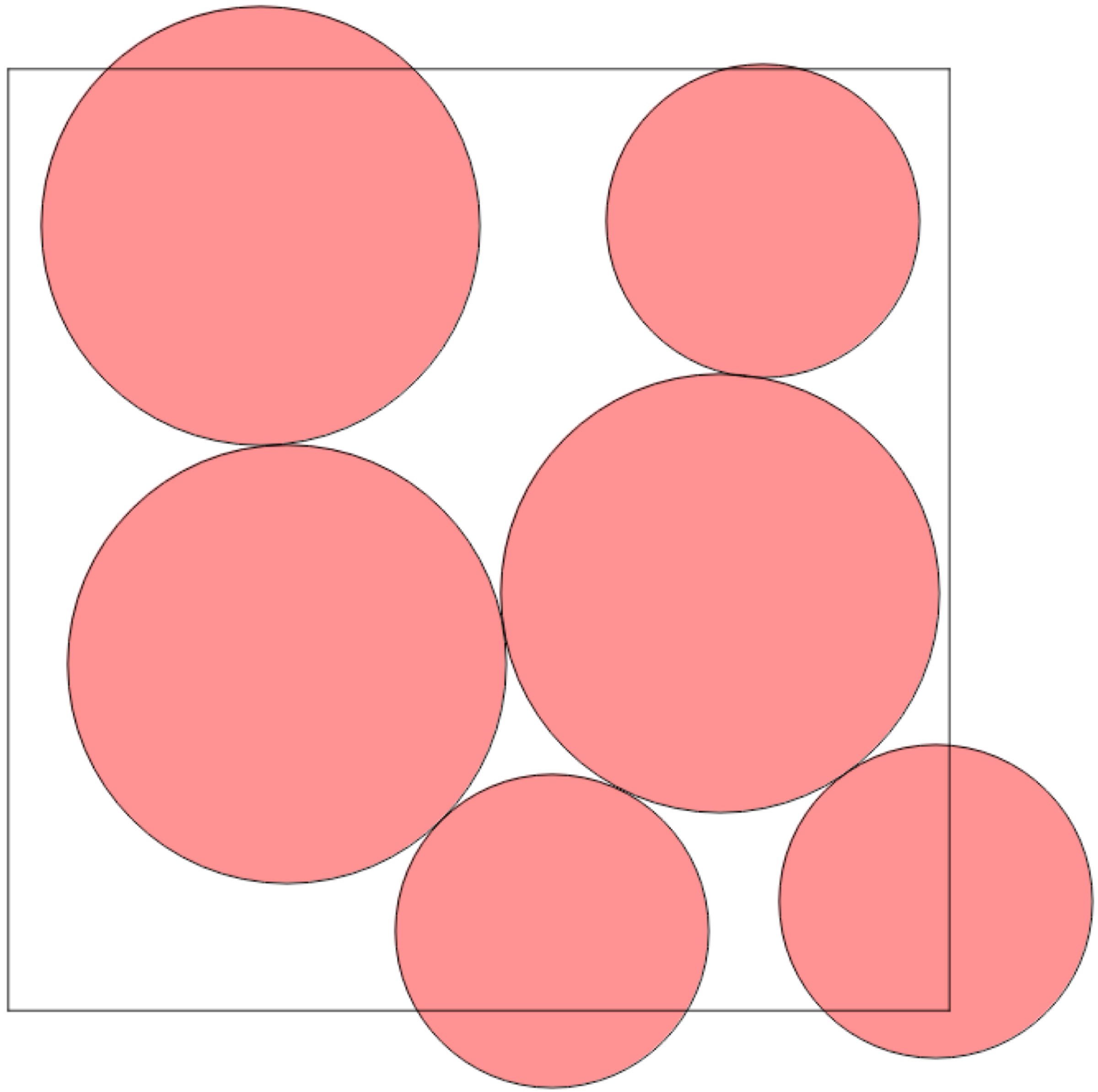
3. What are the power-law scaling exponents for packings of compressed, area-conserving deformable particles?

Simple Shear Families at Fixed Pressure, p=0

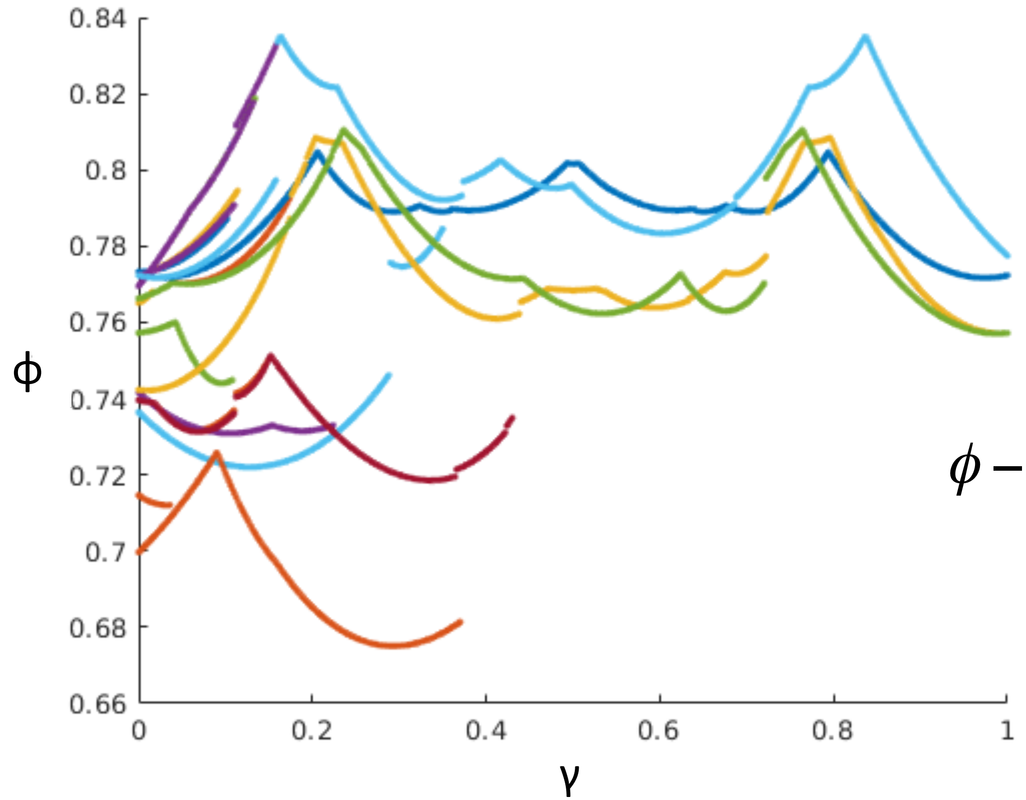




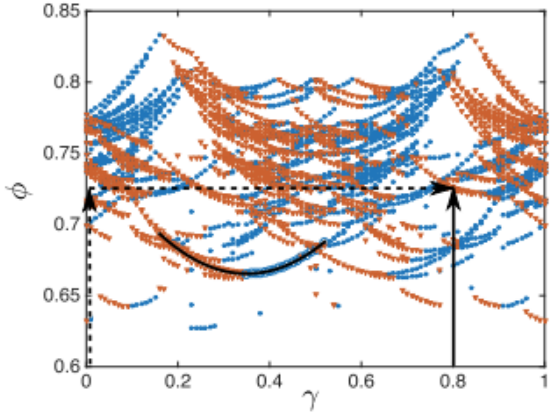
Simple shear
from strain
γ=0 to 0.16



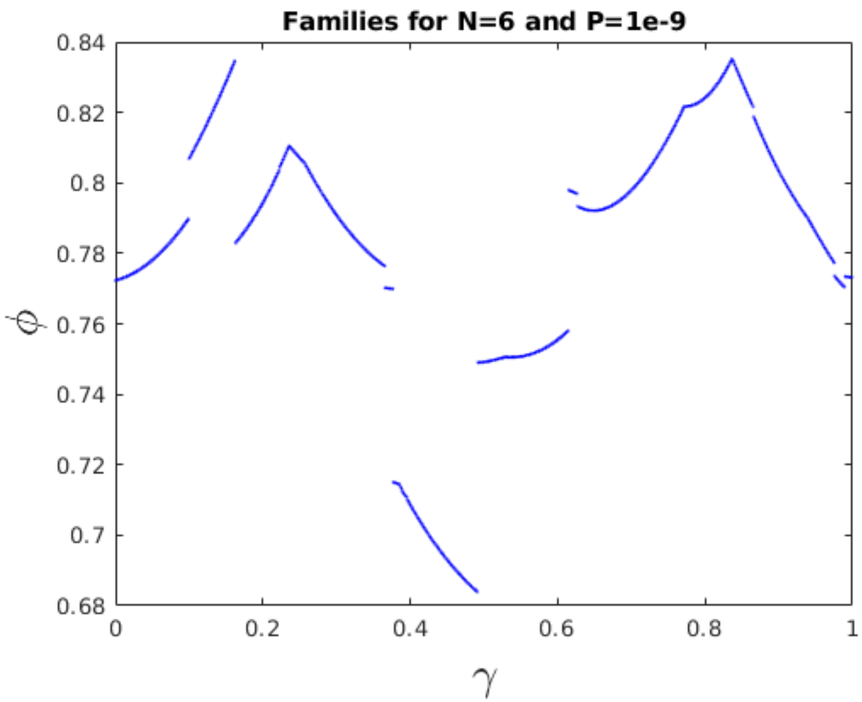
Shear families at p=0 are segments of parabolas

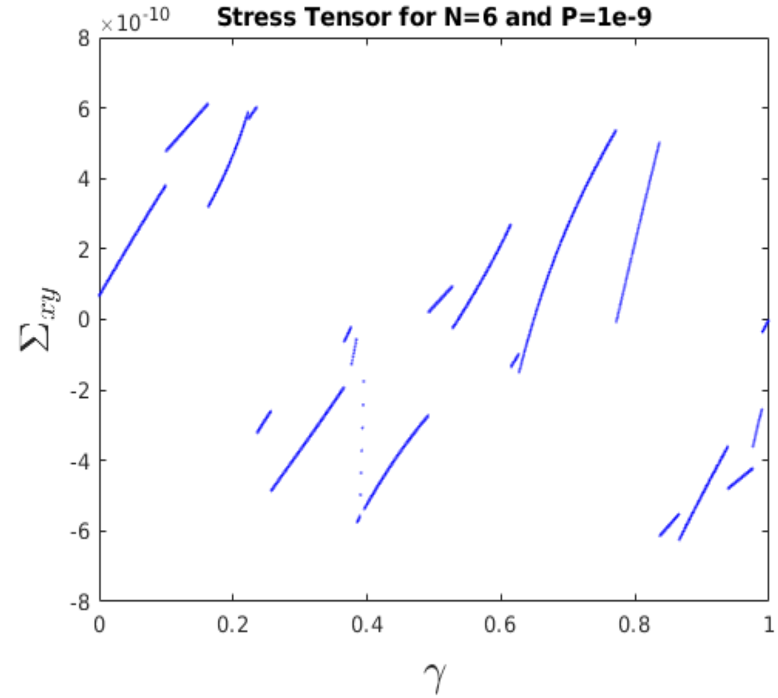


$$\phi - \phi_0 = A\left(\gamma - \gamma_0\right)^2$$



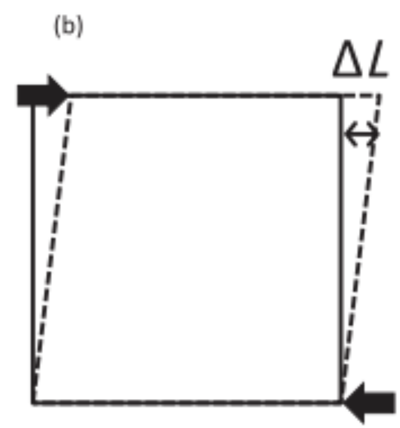
Stress Tensor of Shear Families





$$\Sigma_{xy} = \frac{1}{A}\sum_{i>j} f_{xij} r_{yij}$$

$\gamma = \Delta x / L_y$



Definition of Shear Modulus

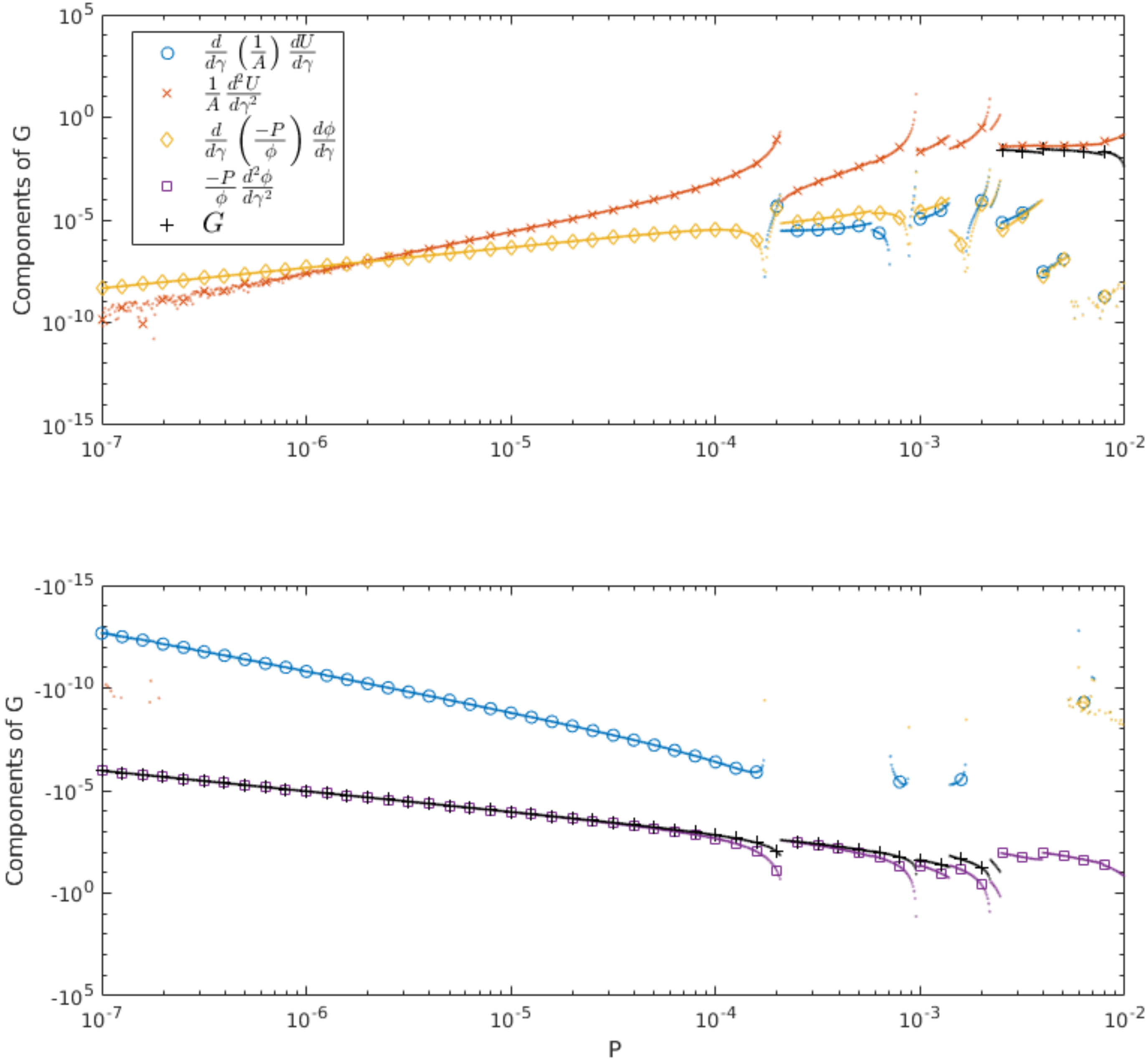
$$\Sigma_{xy} = -\frac{1}{A}\left(\frac{dH}{d\gamma}\right)_p = \frac{p}{\phi}\frac{d\phi}{d\gamma} - \frac{1}{A}\frac{dU}{d\gamma}$$

$$G = -\frac{d\Sigma_{xy}}{d\gamma} \approx \frac{1}{A}\frac{d^2U}{d\gamma^2} - \frac{p}{\phi}\frac{d^2\phi}{d\gamma^2}$$

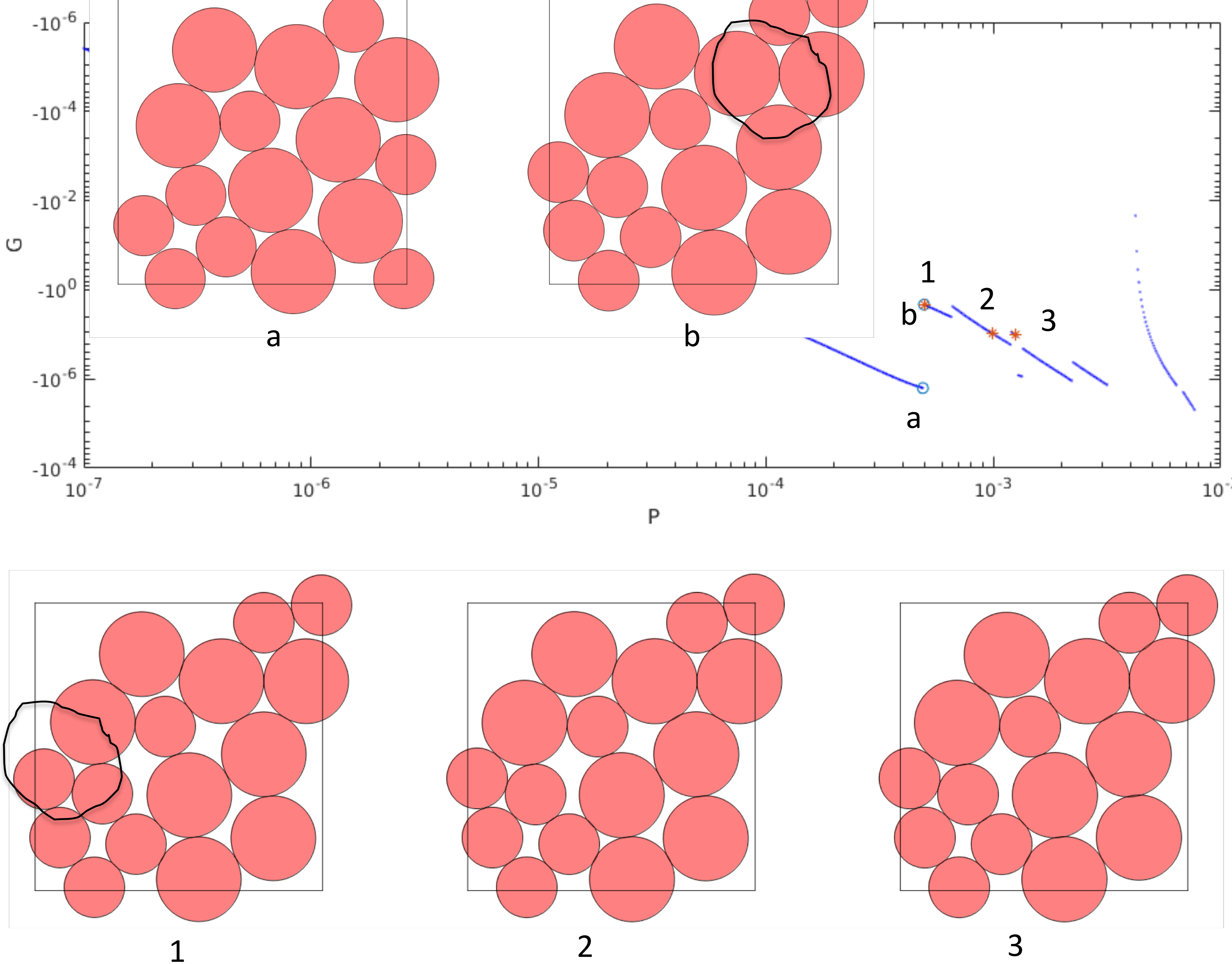
$$G \approx G_0 - \frac{A}{\phi}p$$

Not $p^{1/2}$ and decreases with p

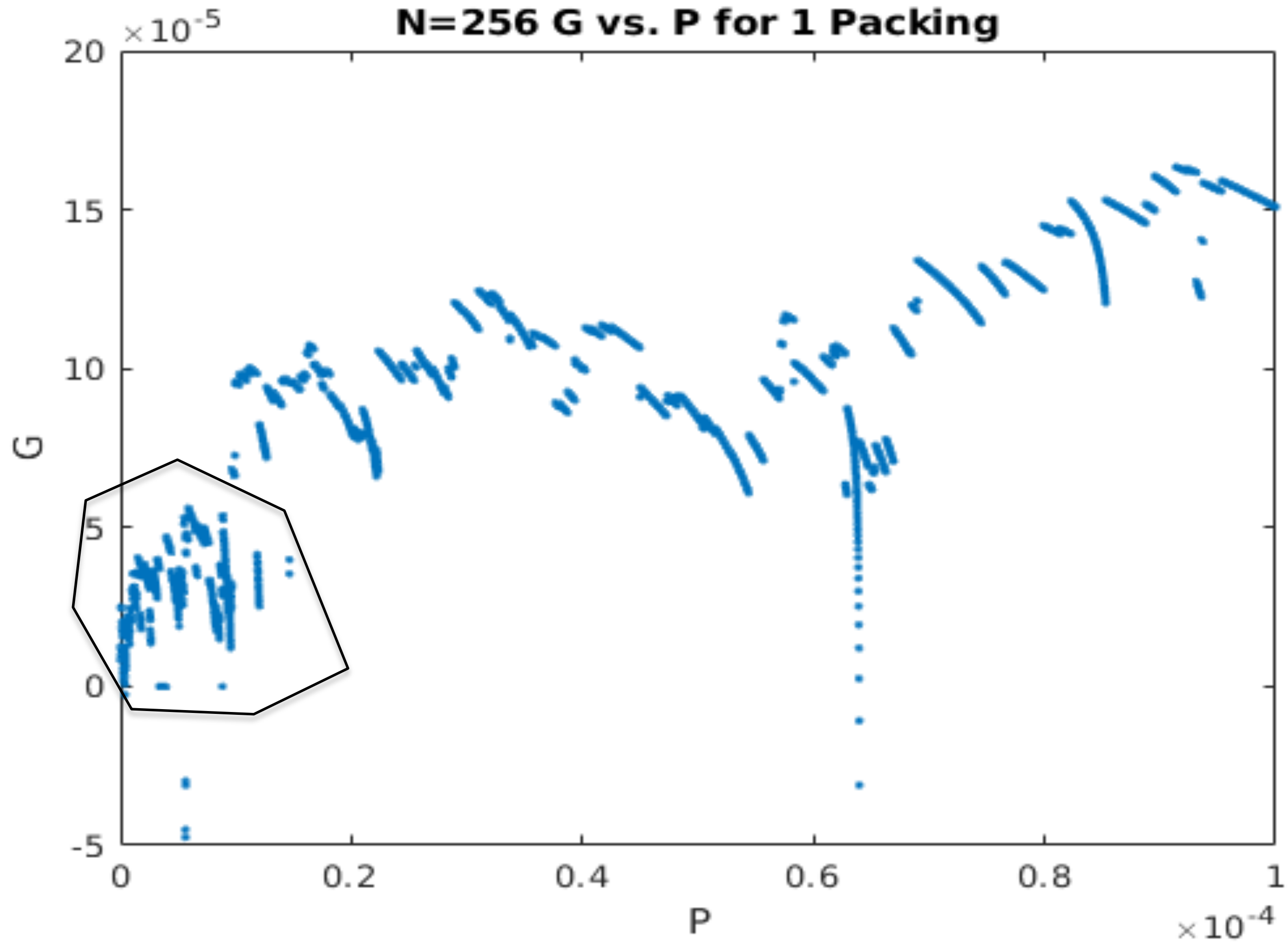
N=16



$\frac{d}{d\gamma}\left(\frac{1}{A}\right)\frac{dU}{d\gamma}$
$\frac{1}{A}\frac{d^2U}{d\gamma^2}$
$\frac{d}{d\gamma}\left(\frac{-P}{\phi}\right)\frac{d\phi}{d\gamma}$
$\frac{-P}{\phi}\frac{d^2\phi}{d\gamma^2}$
G
Components of G
Components of G
P

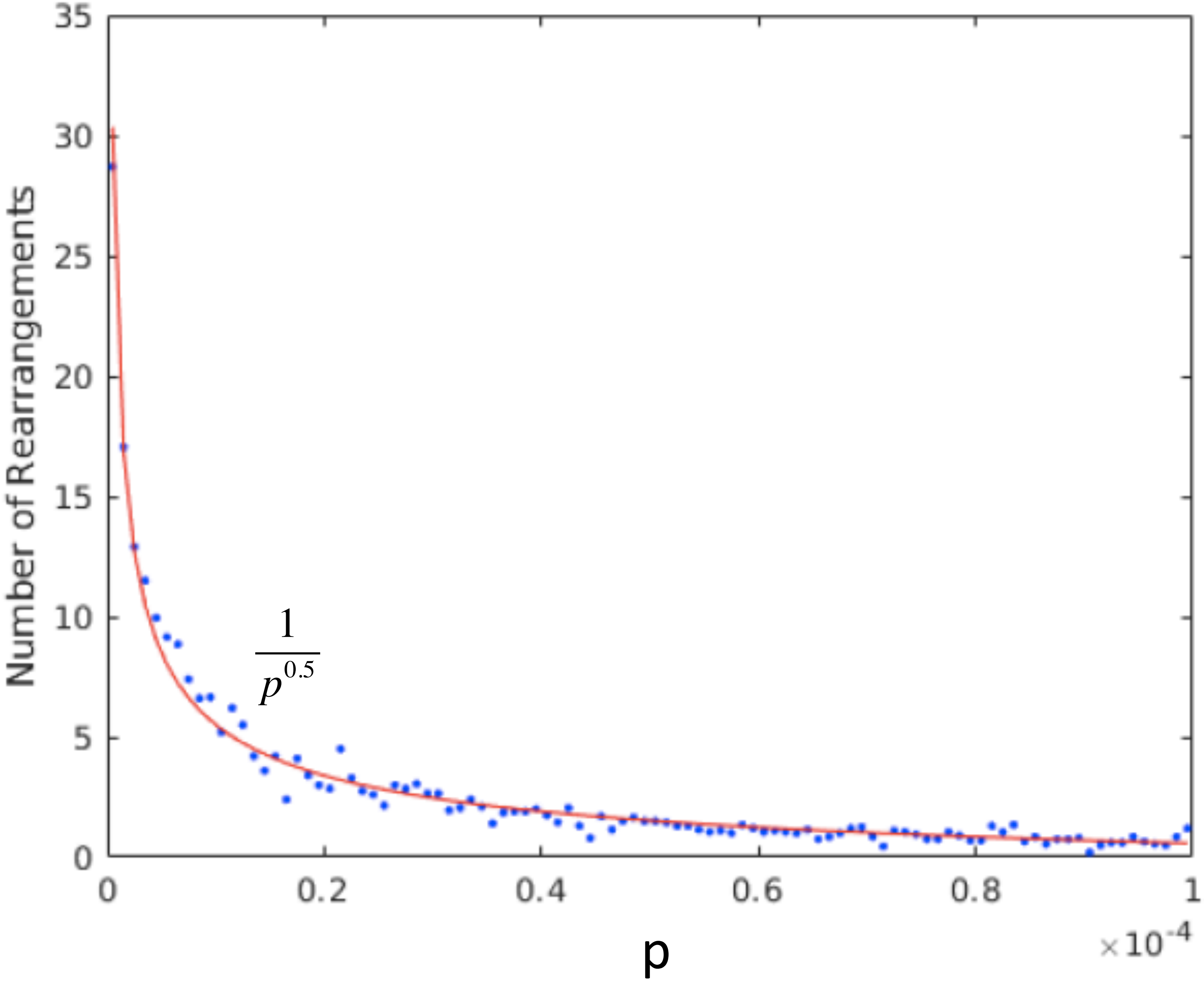


G
P
a
b
1
2
3

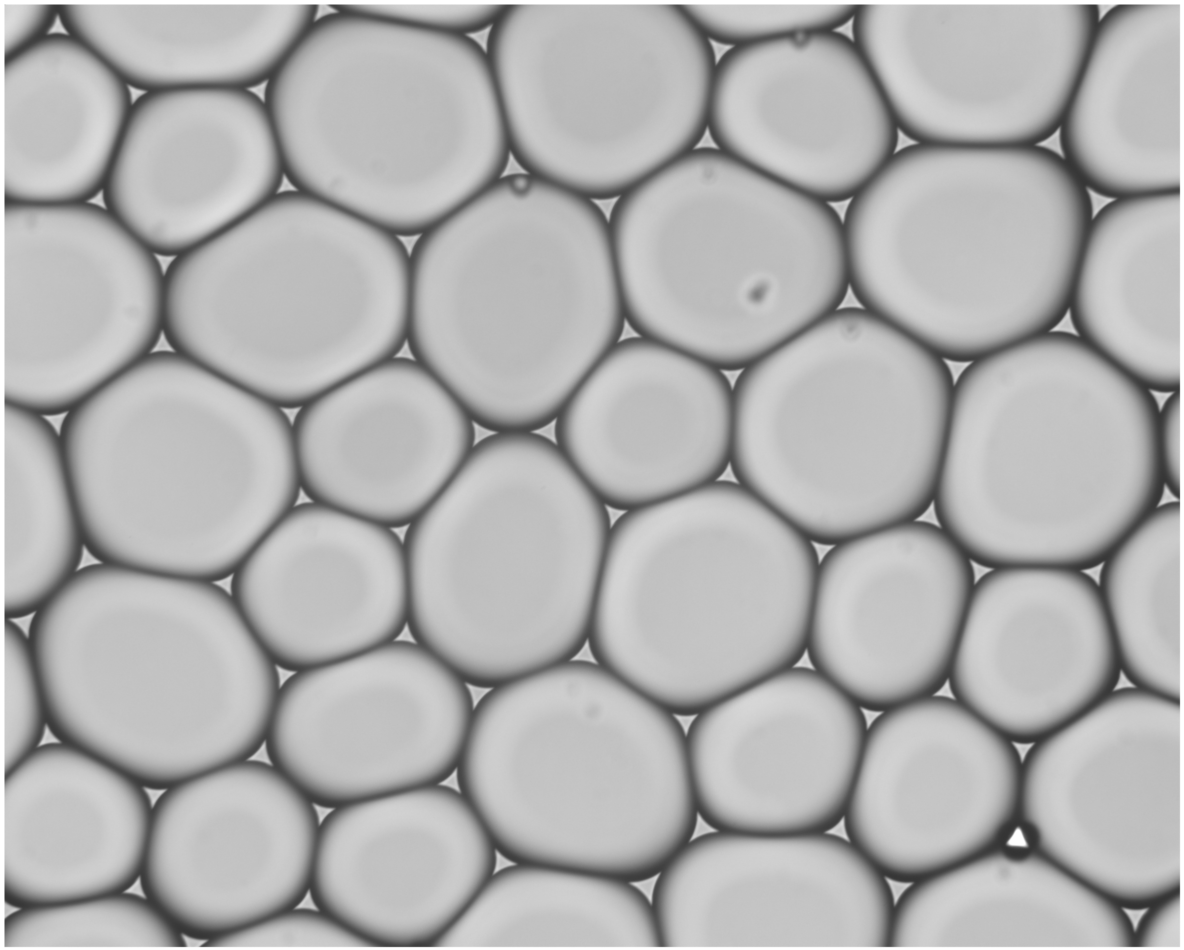


N=256 G vs. P for 1 Packing
×10-5
20
15
10
5
0
-5
G
0
0.2
0.4
0.6
0.8
1
P
×10-4

Rearrangements are important for determining power-law scaling exponents

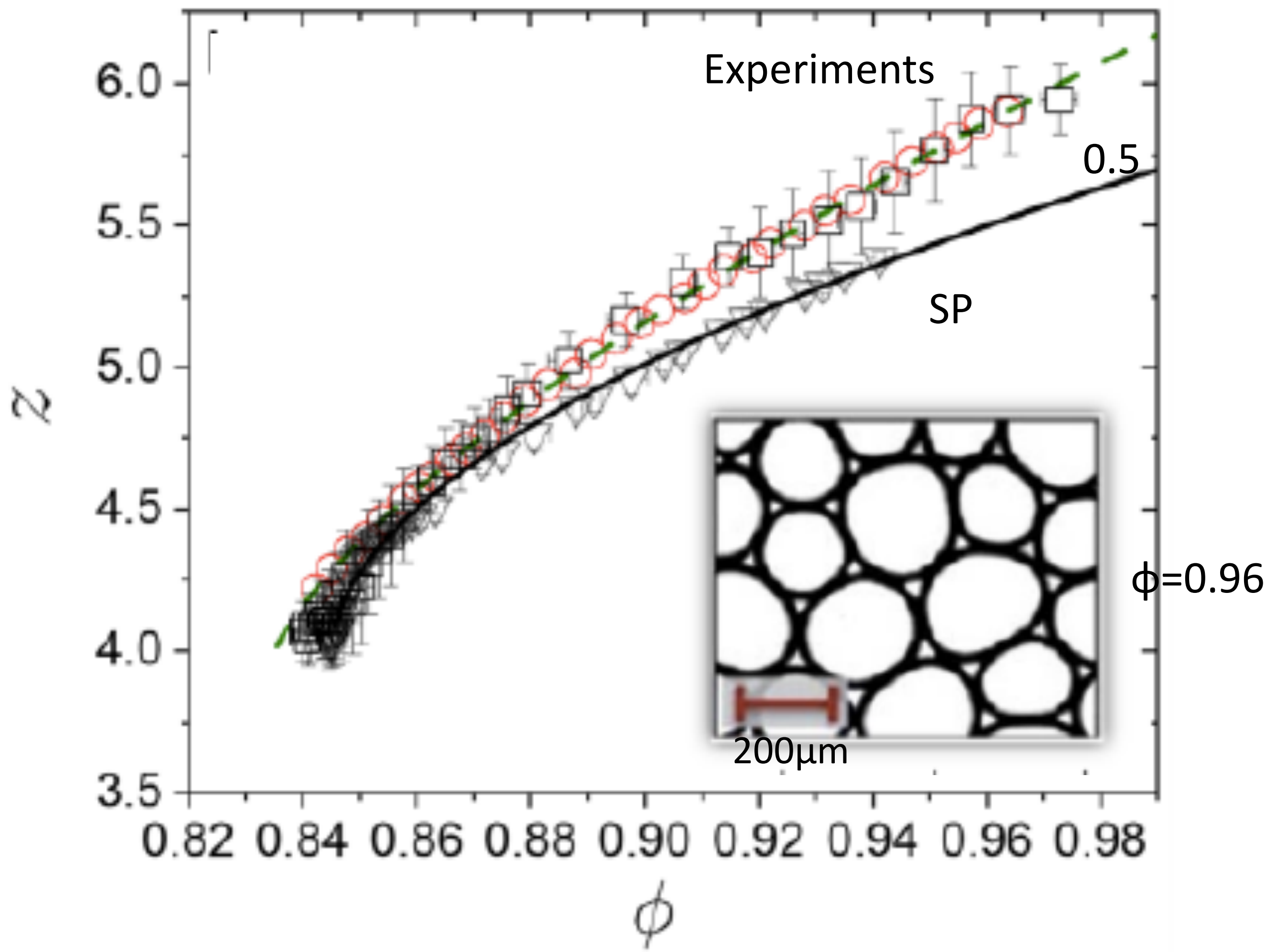


How general are the soft-sphere power-law exponents? For example, can they be used to describe compressed foams and emulsions?



ϕ=0.96;
Shift to “real” ϕ

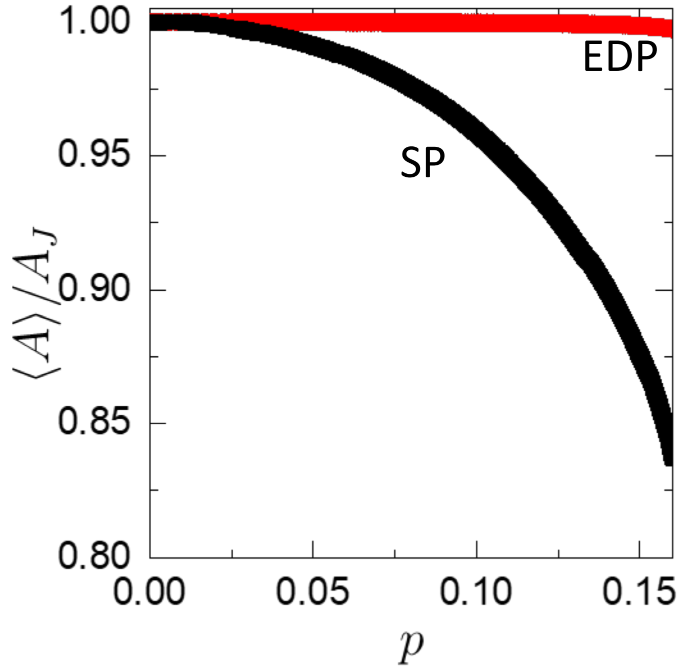
K. W. Desmond, P. J. Young, D. Chen, and E. R. Weeks, “Experimental study of forces between quasi-two-dimensional emumlsion droplets near jamming,” Soft Matter 9 (2013) 3424.



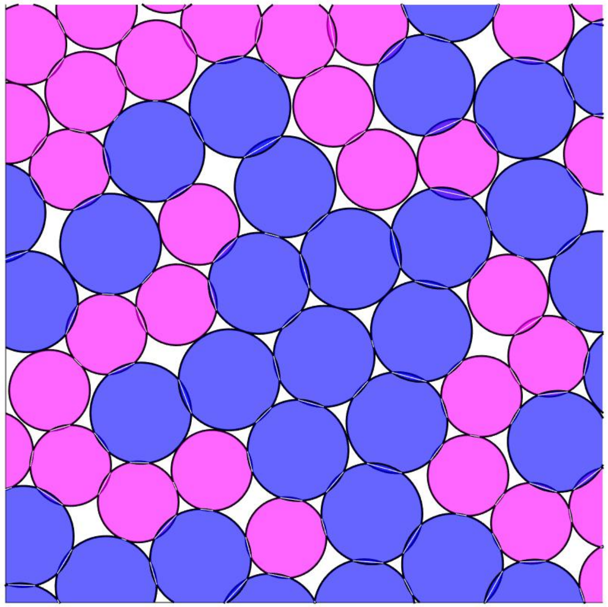
$$z = z_0 + Ap^{\lambda}$$

z_0=4; ϕ_J=0.84; $\lambda \approx$0.4-1.0

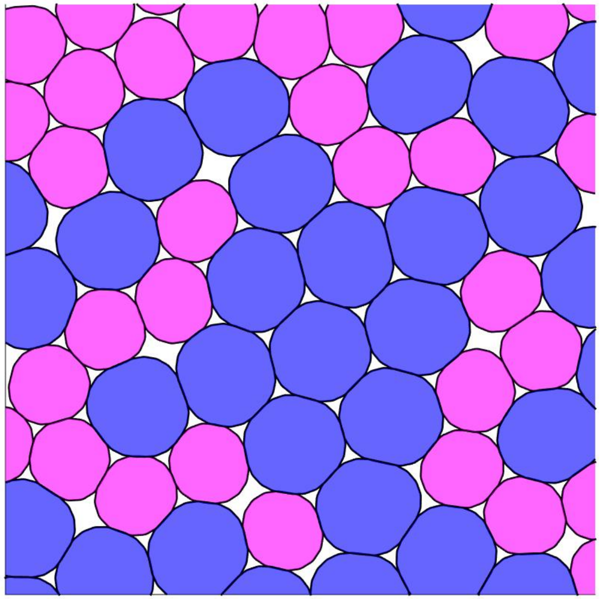
Conservation of particle area



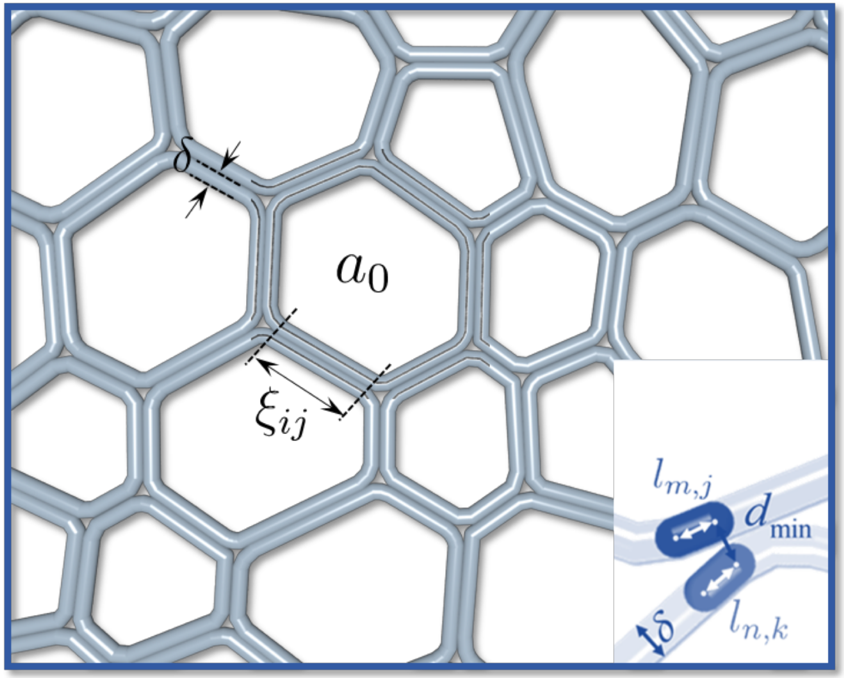
SP



EDP

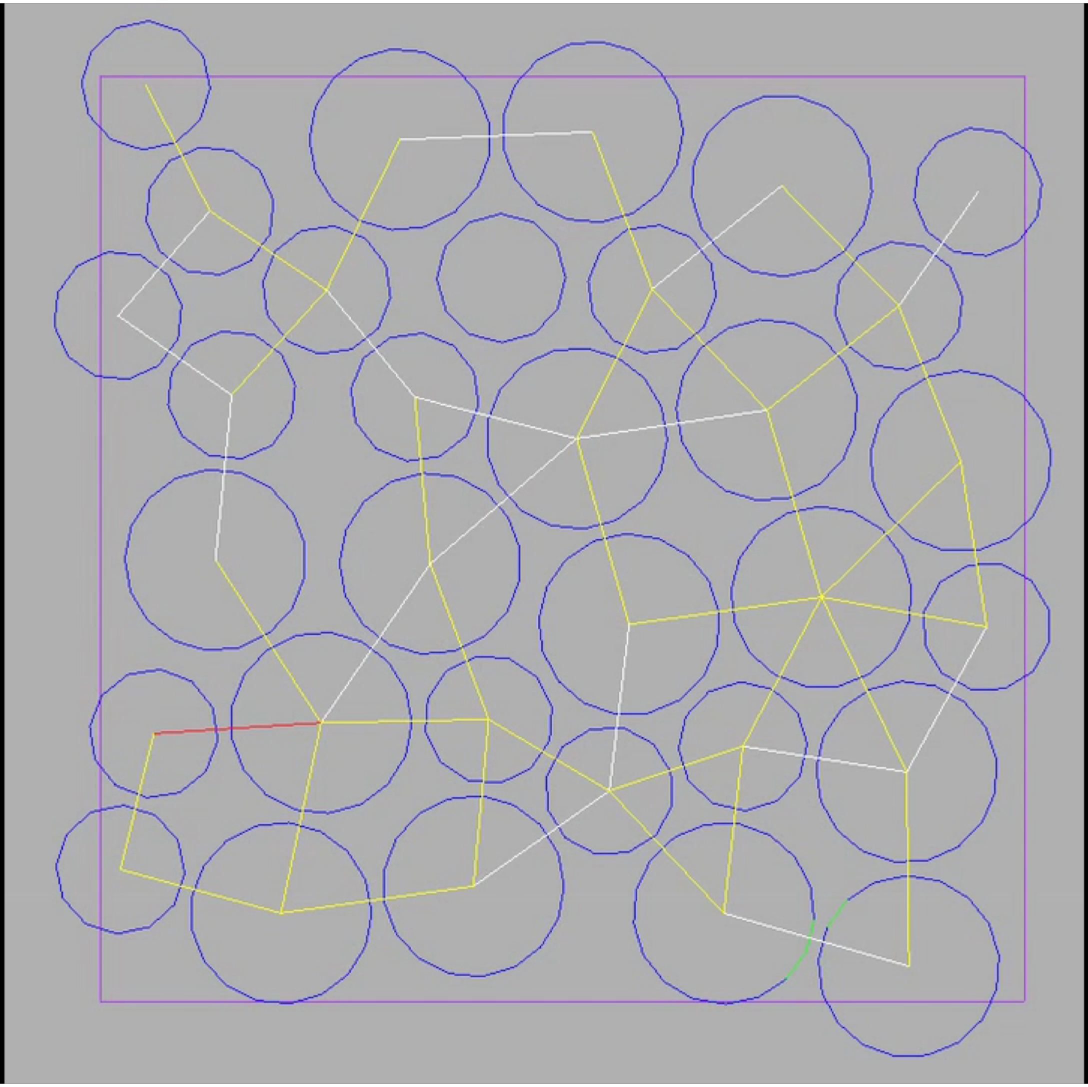


Elastically Deformable Particle Model

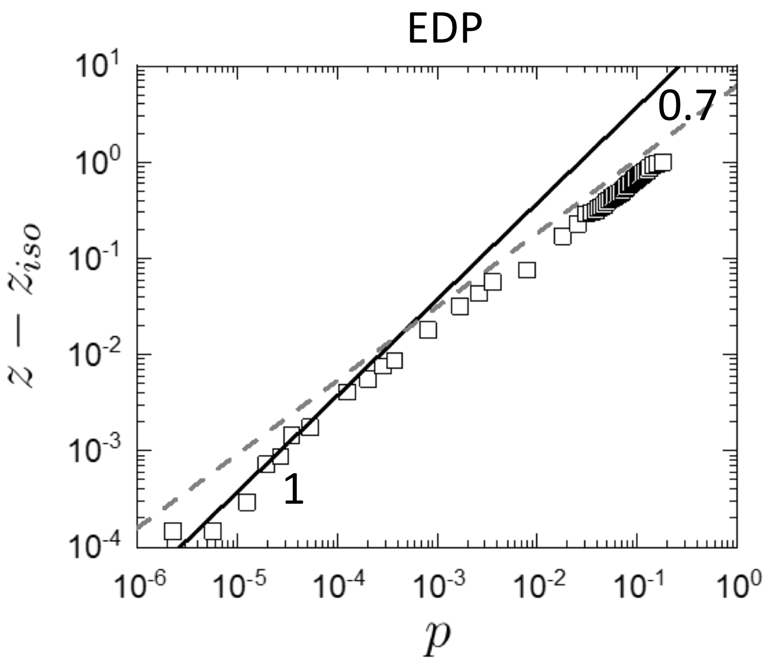


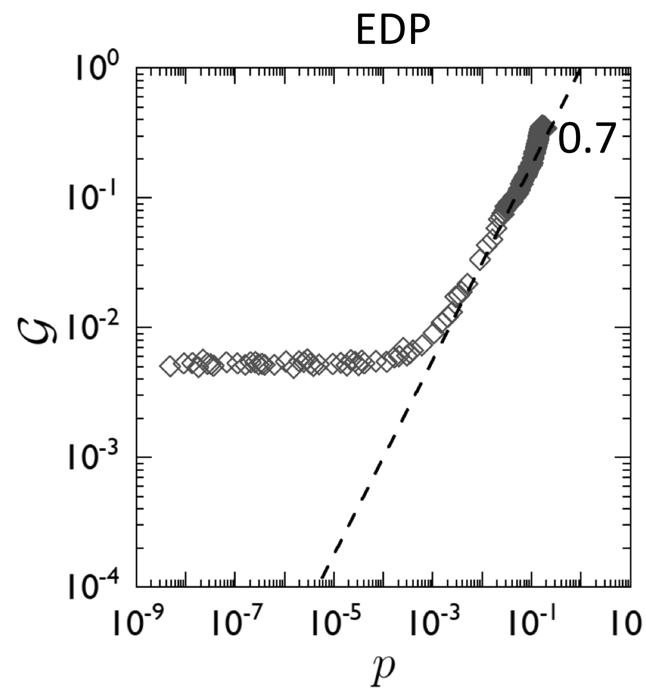
$$U = \gamma \sum_{m=1}^{N} \sum_{i=1}^{N_v} l_{m,i} + \frac{k_a}{2} \sum_{m=1}^{N} (a_m - a_0)^2 + U_{\text{int}}.$$

$$U_{\text{int}} = \sum_{m=1}^{N} \sum_{n>m}^{N} \sum_{j=1}^{N_v} \sum_{k=1}^{N_v} \frac{k_r}{2} (\delta - d_{\text{min}})^2 \times \Theta(\delta - d_{\text{min}}),$$



Ensemble-averaged structural and mechanical properties of compressed packings above onset of jamming

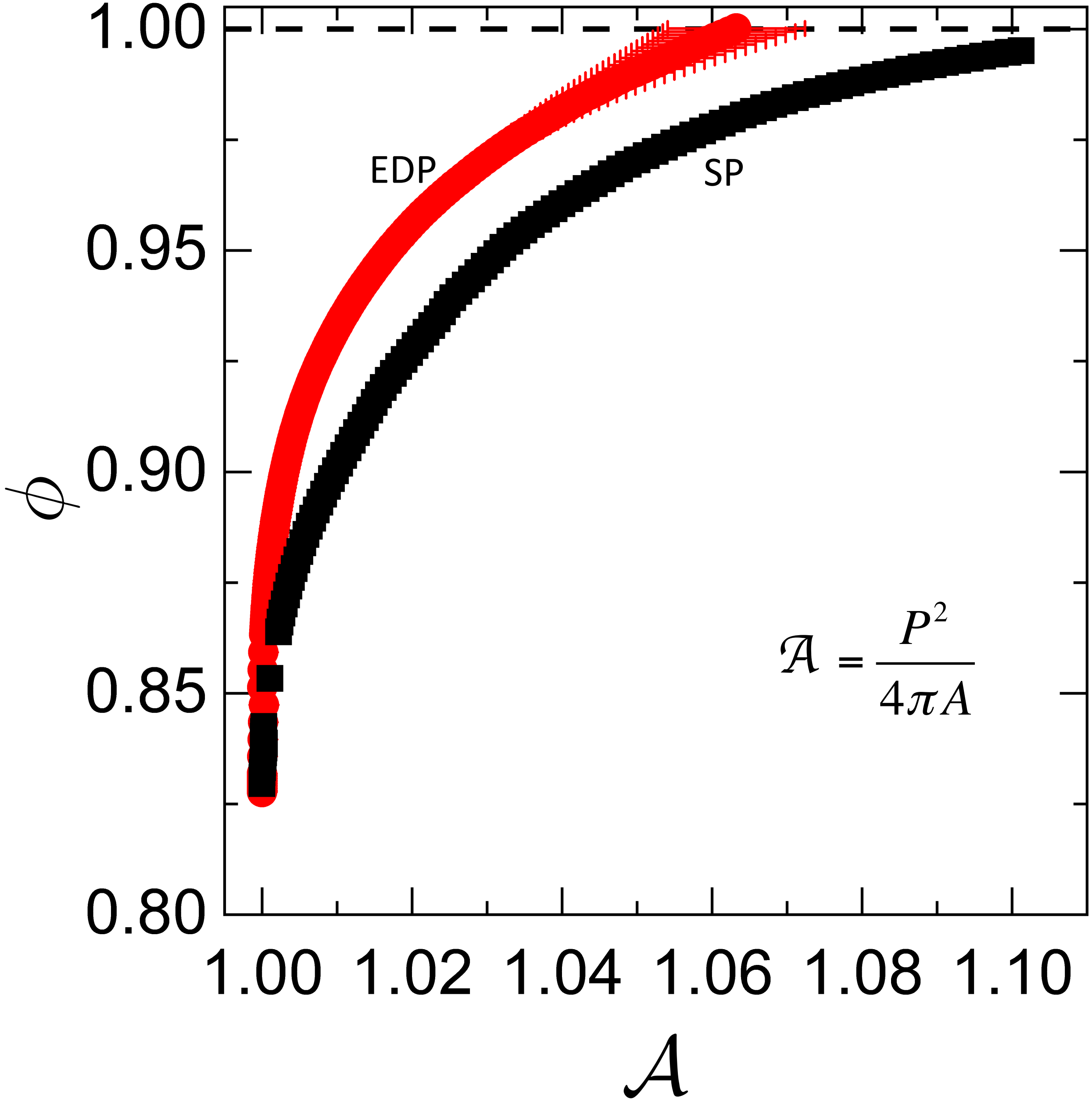




$$z - z_{iso} \sim p^{\lambda}$$ λ=0.7,1

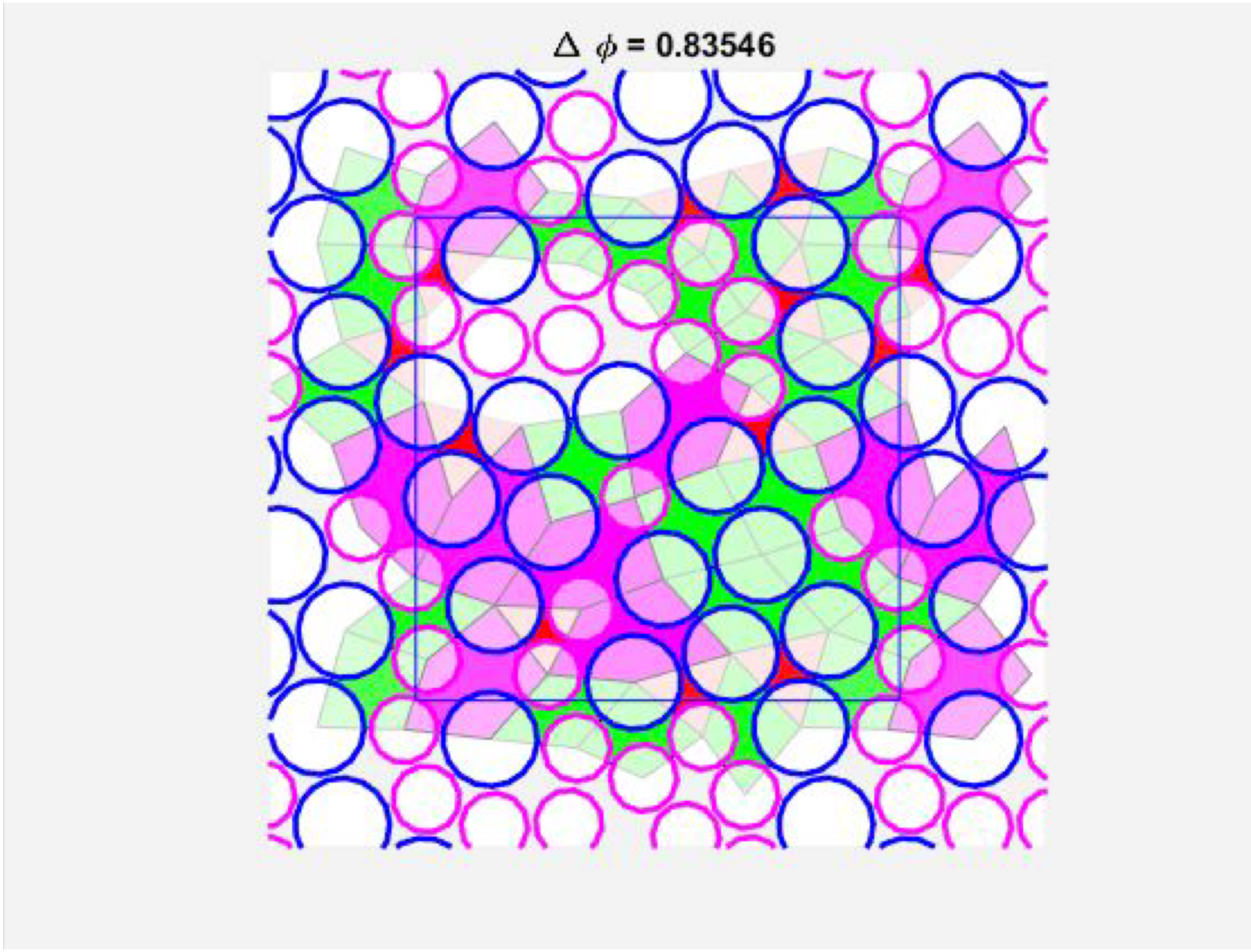
$$G \sim p^{\beta}$$ β=0,0.7

Deformable particles make more contacts.



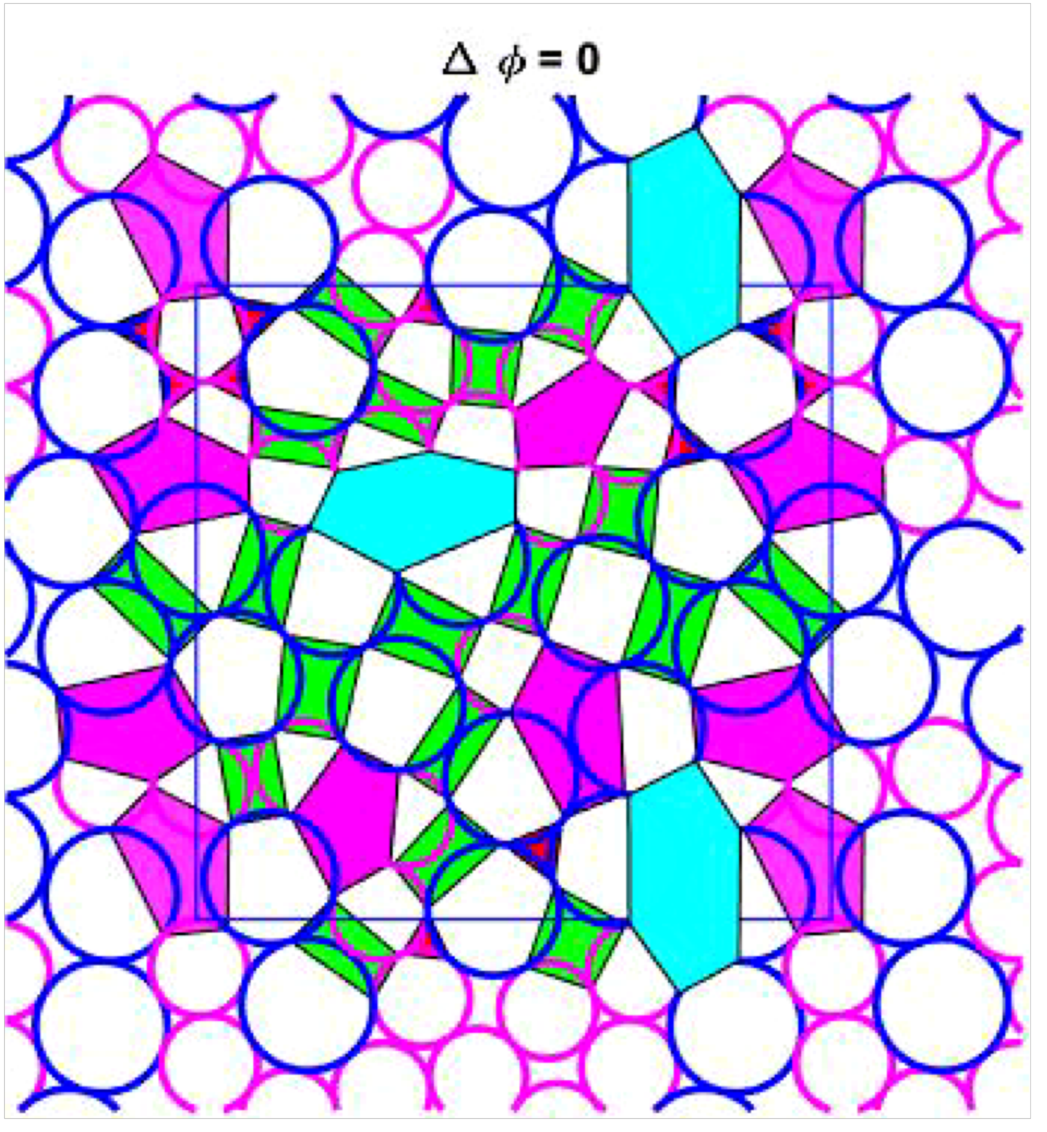
1.00
0.95
0.90
0.85
0.80
ϕ
EDP
SP
$\mathcal{A} = \frac{P^2}{4\pi A}$
1.00
1.02
1.04
1.06
1.08
1.10
$\mathcal{A}$

Void Space: EDP

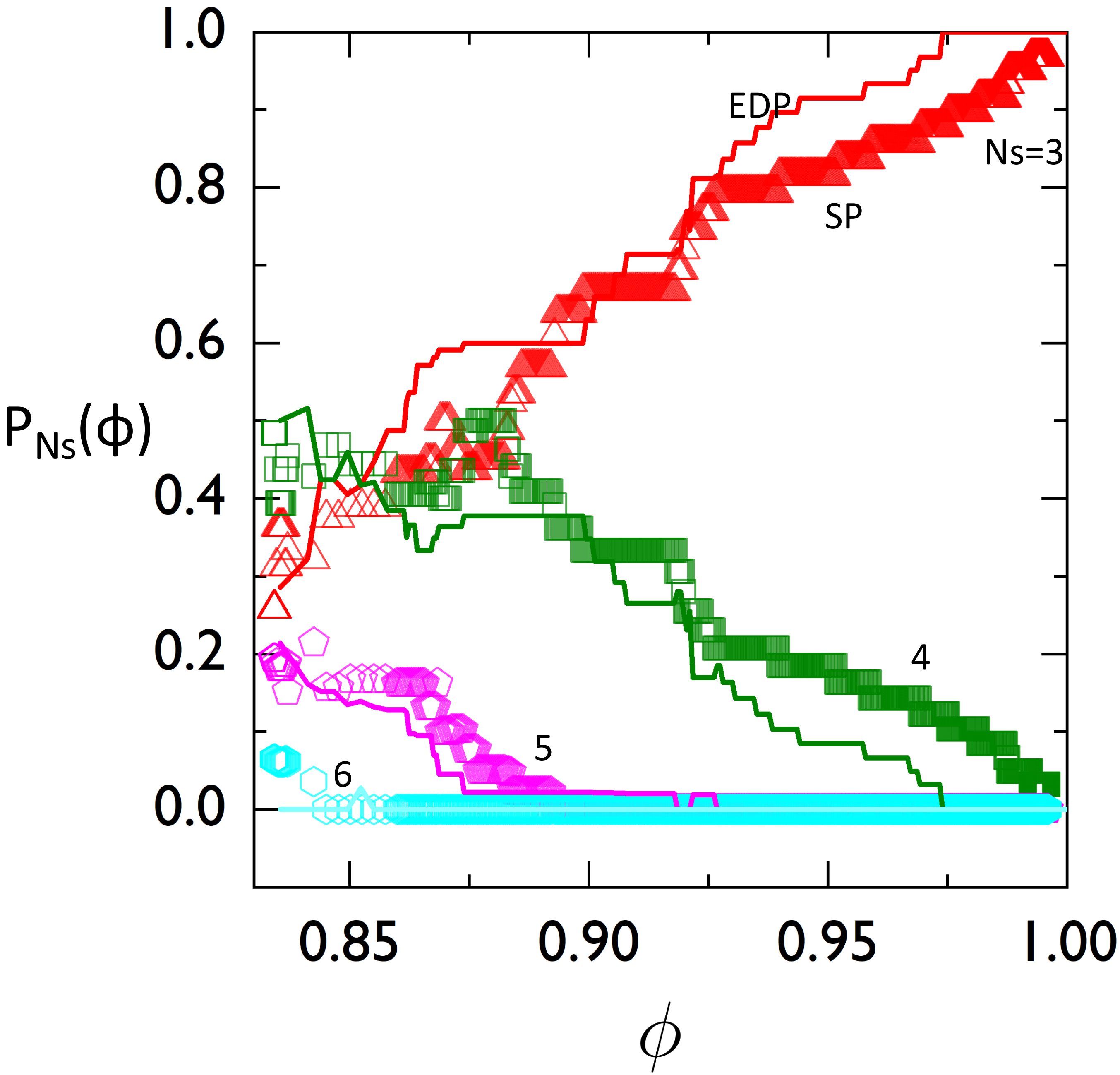


Pink: 5 sides; green: 4 sides; red: 3 sides

Void Space: SP



Pink: 5 sides; green: 4 sides; red: 3 sides



1.0
0.8
0.6
0.4
0.2
0.0
0.85
0.90
0.95
1.00
$P_{Ns}(\phi)$
ϕ
EDP
SP
Ns=3
4
5
6

Conclusions/Future Directions

1. The power-law scaling exponents for $z-z_{iso}$ and G are determined by the statistics of particle rearrangements.

2. Adding area-conserving particle deformability changes the power-law scaling exponents and void statistics.

3. Need to understand different types of particle rearrangements that occur at low versus high pressure (since they give rise to different power-law scaling exponents).

4. Do particle rearrangement statistics depend on particle shape, giving rise to different exponents for every shape?