Dynamics for spherical spin glasses: Disorder dependent initial conditions

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Joint work with Eliran Subag

Overview

- **Limiting Langevin dynamics for spherical spin glasses**
- **Disorder dependent initial conditions**
- **Low temperature Gibbs measure: near pure case**

Langevin dynamics for spherical spin glasses

Langevin particles $\mathbf{x}_t = (x_t^i)_{1 \le i \le N} \in \mathbb{R}^N$,

$$d\mathbf{x}_t = -f_L'(\|\mathbf{x}_t\|^2/N)\mathbf{x}_t dt - \beta \nabla H_{\mathbf{J}}(\mathbf{x}_t) dt + d\mathbf{B}_t$$

$\mathbf{B}_t$ is N-dimensional Brownian motion
$\|\mathbf{x}\|^2 = \langle \mathbf{x}, \mathbf{x} \rangle$, Euclidean norm

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Langevin dynamics is invariant for (random) Gibbs measure

$$G_{2\beta,\mathbf{J}}^{N,L}(A) = Z_{2\beta,\mathbf{J}}^{-1} \int_A e^{-2\beta H_{\mathbf{J}}(\mathbf{x}) - N f_L(N^{-1}\|\mathbf{x}\|^2)} d\mathbf{x}, \qquad A \subset \mathbb{R}^N .$$

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$f_L(r) = L(r-1)^2 + r^{2m}$, as $L \to \infty$ get $\mathbf{x}_t$ near $\mathbb{S}^N = S^{N-1}(\sqrt{N})$.

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$$G^{N,L}_{2\beta,\mathbf{J}}(A) = Z^{-1}_{2\beta,\mathbf{J}} \int_A e^{-2\beta H_{\mathbf{J}}(\mathbf{x}) - N f_L(N^{-1}\|\mathbf{x}\|^2)} d\mathbf{x}, \qquad A \subset \mathbb{R}^N .$$

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$H_{\mathbf{J}} : \mathbb{R}^N \longrightarrow \mathbb{R}$ centered Gaussian of covariance

$$\mathrm{Cov}\big(H_{\mathbf{J}}(\mathbf{x}), H_{\mathbf{J}}(\mathbf{y})\big) = N\nu\big(N^{-1}\langle \mathbf{x}, \mathbf{y}\rangle\big), \quad \nu(r) := \sum_{p=2}^{m} b_p^2 r^p .$$

Band initial condition, conditional disorder

$$dx_t = -f_L'(\|x_t\|^2/N)x_t dt - \beta \nabla H_{\mathbf{J}}(x_t)dt + d\mathbf{B}_t$$

For $q_\star \in (0,1]$, $\|\boldsymbol{\sigma}\| = \sqrt{N} q_\star$, $|q| \le q_\star$ let $x_0 \sim \mu^q_{\boldsymbol{\sigma}}$ (band IC).

$\mu^q_{\boldsymbol{\sigma}}$ is uniform measure on sub-sphere $\{x \in \mathbb{S}^N : \frac{1}{N}\langle x, \boldsymbol{\sigma}\rangle = q\}$.

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For $\mathbf{J}$ conditional to $\{H_{\mathbf{J}}(\boldsymbol{\sigma}) = -NE_\star\, ,\ \nabla H_{\mathbf{J}}(\boldsymbol{\sigma}) = -G_\star \boldsymbol{\sigma}\}$

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For J conditional to $\{H_J(\boldsymbol{\sigma}) = -NE_\star,\ \nabla H_J(\boldsymbol{\sigma}) = -G_\star \boldsymbol{\sigma}\}$

study empirical covariance, integrated response & spatial-overlap:

$$\widehat{C}_N(s,t) = \frac{1}{N}\langle x_s, x_t\rangle, \quad \widehat{\chi}_N(s,t) = \frac{1}{N}\langle x_s, B_t\rangle, \quad \widehat{q}_N^{\boldsymbol{\sigma}}(s) = \frac{1}{N}\langle x_s, \boldsymbol{\sigma}\rangle.$$

Thermodynamical convergence of dynamics

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$\exists (C_L, \chi_L, q_L)$ non-random (depends on $E_\star, G_\star, q_\star, q$), so

$$\Lambda_N(\boldsymbol{\sigma}) := \Big\{ \|\widehat{C}_N - C_L\|_T + \|\widehat{\chi}_N - \chi_L\|_T + \|\widehat{q}_N^{\boldsymbol{\sigma}} - q_L\|_T \Big\} \overset{N\to\infty}{\longrightarrow} 0$$

for any $T < \infty$ (in L_p WRT $\mathbf{B}$, $\mathbf{x}_0$ and the conditional $\mathbf{J}$).

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$\chi_L(s,t) = \int_0^t R_L(s,u)du$, $R_L(s,s) = 1$, $C_L(0,0) = 1$, $q_L(0) = q$,

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solve for $s > t$ explicit integro-differential equations.

Limiting spherical dynamics ($L \to \infty$)

$(R_L, C_L, q_L) \to (R, C, q)$, with $C(s,s) = 1$ and $\forall s \geq t$,

$$\partial_s R(s,t) = -\mu(s) R(s,t) + \beta^2 \int_t^s R(u,t) R(s,u) \nu''(C(s,u)) du,$$

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(to get $\mu(s)$ set $1 + 2\partial_s C(s,t)|_{t=s} = 0$; recall $C(s,t) = C(t,s)$).

$v_\star(\cdot)$ (explicit in $\nu'(\cdot)$ and $q_\star$) is linear in $(E_\star, G_\star)$.

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$\mathbf{x}_0$ uniform on $\langle \mathbf{x}, \boldsymbol{\sigma} \rangle = Nq$, $\{H_{\mathbf{J}}(\boldsymbol{\sigma}) = -NE_\star, \nabla H_{\mathbf{J}}(\boldsymbol{\sigma}) = -G_\star \boldsymbol{\sigma}\}$.

Disorder dependent initial conditions

Consider critical points $\boldsymbol{\sigma}$ of $H_{\mathbf{J}}$ on $q_\star \mathbb{S}^N$
with value $H_{\mathbf{J}}(\boldsymbol{\sigma}) \approx -NE_\star$ and radial derivative $\partial_\perp H_{\mathbf{J}}(\boldsymbol{\sigma}) \approx -\sqrt{N} G_\star q_\star$

$$\mathscr{C}_\star^N(\delta) := \Big\{ \boldsymbol{\sigma} \in q_\star \mathbb{S}^N : \nabla_{\mathrm{sp}} H_{\mathbf{J}}(\boldsymbol{\sigma}) = 0, \\ \Big| \frac{H_{\mathbf{J}}(\boldsymbol{\sigma})}{N} + E_\star \Big| + \Big| \frac{\partial_\perp H_{\mathbf{J}}(\boldsymbol{\sigma})}{\|\boldsymbol{\sigma}\|} + G_\star \Big| < \delta \Big\}$$

($\nabla_{\mathrm{sp}} H_{\mathbf{J}}(\boldsymbol{\sigma})$ gradient WRT standard differential structure on $\mathbb{S}^N$).

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For $E_\star > 0,\ G_\star > 2\sqrt{\nu''(q_\star)}$, $\mathbf{x}_0 \sim \mu_{\boldsymbol{\sigma}}^q$ any $\epsilon > 0,\ T < \infty$,

$$\frac{1}{\mathbb{E}|\mathscr{C}_\star^N(\delta_N)|} \mathbb{E}\Big[\sum_{\boldsymbol{\sigma} \in \mathscr{C}_\star^N(\delta_N)} \mathbb{P}_{\boldsymbol{\sigma},\mathbf{J}}^{N,q}(\Lambda_N(\boldsymbol{\sigma}) > \epsilon) \Big] \stackrel{N\to\infty}{\longrightarrow} 0\,.$$

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Further, if

$$\lim_{b\to 0^+}\lim_{N\to\infty} \mathbb{P}\left\{|\mathscr{C}_\star^N(\delta_N)| > b\mathbb{E}|\mathscr{C}_\star^N(\delta_N)|\right\} = 1, \qquad (\dagger)$$

then

$$\frac{1}{|\mathscr{C}_\star^N(\delta_N)|}\sum_{\boldsymbol{\sigma}\in\mathscr{C}_\star^N(\delta_N)} \mathbb{P}_{\boldsymbol{\sigma},\mathbf{J}}^{N,q}\left\{\Lambda_N(\boldsymbol{\sigma}) > \epsilon\right\} \overset{N\to\infty}{\longrightarrow} 0\,, \quad \text{in prob.}$$

Low temperature Gibbs measure: pure p-spins

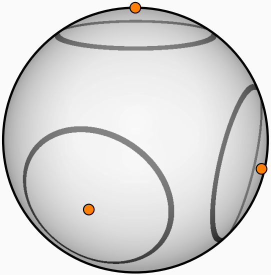
Theorem (Subag '17)

For the pure spherical models with $p \geq 3$ and large enough β,

$$\forall \epsilon > 0: \quad \lim_{k\to\infty} \lim_{N\to\infty} \mathbb{P}\left\{ G_{\beta,\mathbf{J}}^{N,\infty}\left(\cup_{i\leq k}\mathrm{Band}_i\right) > 1-\epsilon \right\} = 1.$$

$$G_{\beta,\mathbf{J}}^{N,\infty}(A) = Z_{\beta,\mathbf{J}}^{-1}\int_A e^{-\beta H_{\mathbf{J}}(\mathbf{x})}d\mathbf{x}, \qquad A \subset \mathbb{S}^N.$$

Band_i = spherical band, width $o(\sqrt{N})$ and radius $\sqrt{N}q$ around $\boldsymbol{\sigma}^{(i)} \in \mathbb{S}^N$, the i-th lowest local minimum of $H_{\mathbf{J}}(\mathbf{x})$.



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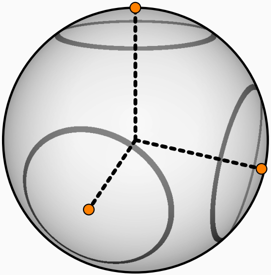
$\boldsymbol{\sigma}^{(i)}$ are roughly orthogonal.



Low temperature Gibbs measure: near pure case

Theorem (Ben Arous, Subag, Zeitouni '18) [informal version]

For models **'close' enough to pure,** and large β, we have (essentially) the same picture but with modified definition for the bands.

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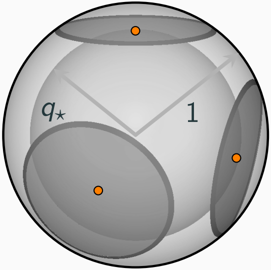
For models **'close' enough to pure,** and large β, we have (essentially) the same picture but with modified definition for the bands.

Now centers are local minima $\boldsymbol{\sigma}$ of $H_{\mathbf{J}}(\cdot)$ on $q_\star\,\mathbb{S}^N$ and their bands (on $\mathbb{S}^N$) are

$$\mathrm{Band}(\boldsymbol{\sigma}) = \left\{ \mathbf{x} \in \mathbb{S}^N : \left|\frac{1}{N}\langle \mathbf{x}, \boldsymbol{\sigma}\rangle - q_\star^2\right| \leq \delta_N \right\}.$$



Thank you!