On scattering of higher spins in flat space

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Motivation:

• understand properties of theories with infinite number of states:
e.g. consistent massless higher spin theory in AdS (vector dual)
or tensionless limit of string theory in AdS (adjoint dual)

• HS theory in AdS is complicated:
action? locality? consider some simpler limit

• HS theory in flat-space ... no-go theorems ...
such theory may exist if relax locality condition?
what about its symmetries? trivial S-matrix?

There are more things in heaven and earth, Horatio,
Than are dreamt of in your philosophy.
- Hamlet (1.5.167-8)

Summary:

- construction of quartic HS interaction vertices
for single tower of massless even spins $s = 0, 2, 4, ...$
using the Lorentz-covariant S-matrix-based approach
- $000s$: 4-vertices making amplitudes on-shell gauge invariant
local for $s = 2, 4$ only
- locality can be restored by extending set of fields:
add extra tower of even spins $s > 0$ with specific couplings
- indications that extended local action has trivial S-matrix
- would be in agreement with soft limit constraints on S-matrix
from gauge invariance under assumption of locality
- underlying global symmetry of flat-space HS theory?
analogy with conformal extension of Einstein theory:
invariance under conformal HS algebra?
then possible reason for trivial S-matrix

Plan:

- scattering via massless HS exchanges:
0000 and 000s amplitudes

- constraints from gauge invariance of S-matrix in soft limit

- S-matrix approach to construction of gauge-invariant action:
  non-local 000s 4-vertices

- resolving non-locality by introducing extra tower of states

- conformal off-shell extension:
  Einstein theory and possible HS generalization

Massless higher spins in flat 4d space

- free theory: symmetric double-traceless rank s tensors

$$S = \int d^4x \, \partial^n \phi^{m_1 \dots m_s} \partial_n \phi_{m_1 \dots m_s} + \dots$$

$$\delta \phi_{m_1 \dots m_s} = \partial_{(m_1} \epsilon_{m_2 \dots m_s)}, \quad s = 0, 1, 2, \dots$$

- cubic interactions with linearized gauge invariance known
- quartic interactions? consistent interacting theory?
- various $s > 2$ "no-go theorems"

e.g. no minimal interactions – no long-range forces

[Weinberg; Cachazo, Benincasa ,...

review: Bekaert, Boulanger, Sundell 1007.0435]

- assumptions? locality of quartic and higher interactions
- demand gauge invariance: which type of non-locality required?
- resolve non-locality introducing new fields?
- resulting S-matrix is trivial? underlying symmetries?

Why of interest?

• tensionless limit of string theory in flat space?

degenerate, not well defined ...

but is well-defined in AdS:

"leading Regge trajectory" – massless tower of higher spins

• massless HS theory in AdS

  • consistent non-linear equations known [Vasiliev]

but complicated, many auxiliary fields, so far no action

  • action for physical Fronsdal fields

can be reconstructed in principle using AdS/CFT:

match correlators of boundary CFT

[Bekaert,Erdmenger,Ponomarev,Sleight 15; Taronna, Sleight 16]

  • cubic vertices known; quartic are complicated

issue of locality is subtle / unclear – kernels $f(a\,\partial)$, $\Lambda = 1/a^2$

• flat-space limit of AdS HS theory?

non-local theory for HS tower $s = 0, 1, 2, ...$?

infinite global symmetry? S-matrix ?

• consistent theory requires

– infinite tower of spins $s = 0, 1, 2, 3, ..., \infty$

– higher derivative (non-minimal) cubic interactions $(s_1 \leqslant s_2 \leqslant s_3)$

$\partial^n \phi_{s_1} \phi_{s_2} \phi_{s_3}, \quad s_2 + s_3 - s_1 \leqslant n \leqslant s_2 + s_3 + s_1$

e.g. l.c. 2-2-2 vertex – $\partial^2, \partial^4, \partial^6$ and 2-3-3 vertex – $\partial^4, ..., \partial^6$

[light-cone: Bengtsson, Bengtsson, Brink; Metsaev;

covariant: Fotopoulos, Tsulaia; Boulanger, Leclerc, Sundell;

Manvelyan, Mkrtchyan, Ruhl; Sagnotti, Taronna, ...]

• Noether procedure: deform $\delta\phi_s = \partial\epsilon_{s-1} + ...$, add 4-vertex,...

• 3-point coupling constants [Metsaev]

$$c_{s_1 s_2 s_3} = g \, \frac{\ell^{s_1+s_2+s_3-1}}{(s_1+s_2+s_3-1)!}$$

• two constants (cf. string th.): g and ℓ= length

structure of action:

$$\frac{1}{g^2}\int d^4x \Big[\sum_s \partial\phi_s \partial\phi_s + \sum \ell^{n-1} \partial^n \phi_{s_1}\phi_{s_2}\phi_{s_3} + \sum \ell^{k-2}\partial^k \phi^4 + \ldots \Big]$$

effectively "non-local": no. of ∂ grows with s and no. of ϕ

• ℓ should be hidden scale in background-independent generalization of HS theory in AdS (cf. Einstein theory)

Aim: find quartic interactions required by gauge invariance

Free higher spin action

- symmetric higher spin tensors

$$\phi_s(x,u) = \phi^{a_1\dots a_s}(x)\, u_{a_1}\dots u_{a_s}$$

- Fronsdal action: gauge-inv $\int \phi_s \,\Box\, \phi_s$, 2 d.o.f.

$$S^{(2)}[\phi_s] = \int d^4x\, \phi_s(x,\partial_u)\, \widehat{T}\left[\Box_x - (u\cdot\partial_x)\,\widehat{D}\right] \phi_s(x,u)\Big|_{u=0}$$

$$\widehat{T} = 1 - \tfrac{1}{4}u^2\partial_u^2, \qquad \widehat{D} \equiv (\partial_x\cdot\partial_u) - \tfrac{1}{2}(u\cdot\partial_x)\partial_u^2$$

- ϕ_s double-traceless

$$(\partial_u^2)^2\phi_s(x,u) = 0$$

- free equations

$$\left[\Box_x - (u\cdot\partial_x)\,\widehat{D}\right]\,\phi(x,u) = 0$$

• linearized gauge transformations

$$\delta_s^{(0)}\phi_s(x,u) = (u\cdot\partial_x)\varepsilon_{s-1}(x,u)$$

with traceless parameter $\quad \partial_u^2\,\varepsilon_{s-1}(x,u) = 0$

• de Donder gauge:

$$\widehat{D}\,\phi_s(x,u) = 0 \qquad \to \qquad \partial^{a_1}\phi_{a_1\ldots a_s} + \ldots = 0$$

$$S^{(2)}[\phi_s] = s!\int d^4x\;\phi_s(x,\partial_u)\,\widehat{T}\,\Box_x\,\phi_s(x,u)\,\Big|_{u=0}$$

• equations of motion

$$\Box_x\,\phi_s(x,u) = 0$$

Cubic interaction vertices:

- requiring gauge invariance of combined action

$\delta^{(0)}S^{(3)} + \delta^{(1)}S^{(2)} = 0$ [Manvelyan et al; Sagnotti,Taronna; Joung et al 11]

- traceless-transverse part of cubic vertex $\quad (\partial_{x_{ij}} \equiv \partial_{x_i} - \partial_{x_j})$

$$S^{(3)}[\phi_0, \phi_{s_2}, \phi_{s_3}] = c_{0 s_2 s_3} \int d^d x \Big[(\partial_{u_2} \cdot \partial_{x_{31}})^{s_2} (\partial_{u_3} \cdot \partial_{x_{12}})^{s_3} \times \phi_0(x_1) \phi_{s_2}(x_2, u_2) \phi_{s_3}(x_3, u_3) \Big]_{\substack{u_i = 0 \\ x_i = x}}$$

- $c_{s_1 s_2 s_3}$ fixed in l.c. approach [Metsaev 91]

$$c_{s_1 s_2 s_3} = g \frac{\ell^{s_1+s_2+s_3-1}}{(s_1+s_2+s_3-1)!}$$

- same $c_{s_1 s_2 s_3}$ for massless HS in AdS$_4$ constructed from AdS/CFT

  [Skvortsov 15; Sleight,Taronna 16]

Free HS propagator in de Donder gauge

$$\mathcal{D}_s(u,u';p) = -\frac{i}{p^2}\,\mathcal{P}_s(u,u')$$

$$\mathcal{P}_s(u,u') = \tfrac{2}{(s!)^2}\left(\tfrac{1}{2}\sqrt{u^2u'^2}\right)^s T_s\left(\tfrac{u\cdot u'}{\sqrt{u^2u'^2}}\right)$$

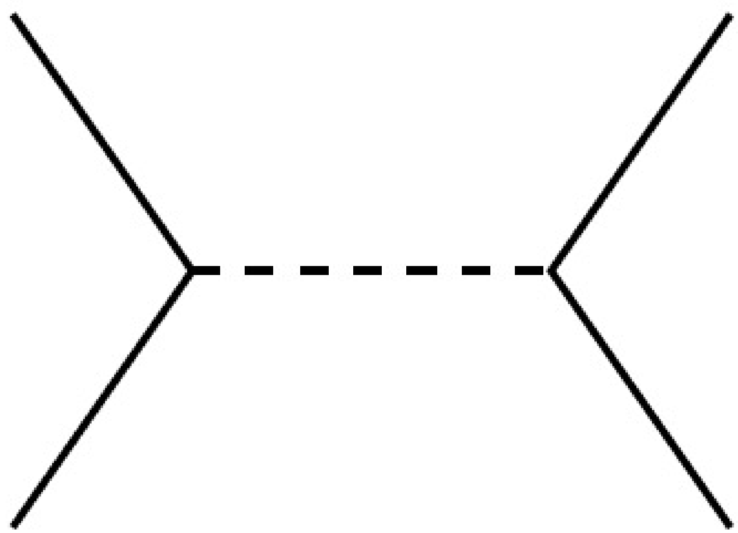
$$\begin{aligned} T_s(z) &\equiv \tfrac{s}{2}\sum_{k=0}^{[s/2]} \tfrac{(-1)^k(s-k-1)!}{k!(s-2k)!}(2z)^{s-2k} \\ &= \tfrac{1}{2}\left[\left(z+\sqrt{z^2-1}\right)^s + \left(z-\sqrt{z^2-1}\right)^s\right] \end{aligned}$$

T_s – Chebyshev polynomial of 1st kind

Cubic $0s_2s_3$ vertex : $\quad (p_{ij} \equiv p_i - p_j)$

$$\mathcal{V}(\partial_{u_2},\partial_{u_3};p_1,p_2,p_3) = 2ic_{0s_2s_3}(-ip_{31}\cdot\partial_{u_2})^{s_2}(-ip_{12}\cdot\partial_{u_3})^{s_3}$$

Consider scattering of spin 0 via all spin s exchanges



4-scalar scattering amplitude: exchange part

exchange of tower of higher spin fields

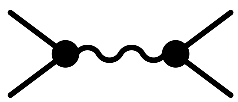
[Bekaert, Joung, Mourad 09; Ponomarev, AT 16]

• scalar: $s = 0$ member of HS tower

interactions with even spins only

contribution of contact 4-vertex is yet to be fixed

• s-channel exchange of spin j field

 $\equiv \quad \mathcal{A}^j_{exch}(\mathrm{s},\mathrm{t},\mathrm{u})$

Mandelstam variables $\quad (p_i^2 = p_i'^2 = 0, \quad \mathrm{s}+\mathrm{t}+\mathrm{u} = 0)$

$$\mathrm{s} \equiv -(p_1+p_2)^2, \quad \mathrm{t} \equiv -(p_1+p_1')^2, \quad \mathrm{u} \equiv -(p_1+p_2')^2$$

$$\mathcal{A}^j_{exch}(\mathrm{s},\mathrm{t},\mathrm{u}) = -\frac{ic^2_{00j}}{\mathrm{s}} 2^{-j+1}\,(\mathrm{t}+\mathrm{u})^j \;\; T_s\big(\tfrac{\mathrm{t}-\mathrm{u}}{\mathrm{t}+\mathrm{u}}\big)$$

$$\mathcal{A}_{exch}(\mathrm{s},\mathrm{t},\mathrm{u}) = \sum_{j=0,2,4,\ldots}^{\infty} \mathcal{A}^{j}_{exch}(\mathrm{s},\mathrm{t},\mathrm{u})$$

$$= -\frac{i}{\mathrm{s}}\Big[F\big(\sqrt{\mathrm{s}+\mathrm{t}}+\sqrt{\mathrm{t}}\,\big) + F\big(\sqrt{\mathrm{s}+\mathrm{t}}-\sqrt{\mathrm{t}}\,\big)\Big]$$

$$F(z) \equiv \sum_{j=0,2,4,\ldots}^{\infty} c^2_{00\,j}\,\big(\tfrac{z^2}{4}\big)^j = g^2\sum_{k=0}^{\infty}\tfrac{1}{[(2k-1)!]^2}\,\big(\tfrac{\ell^2 z^2}{4}\big)^{2k}$$

$$= \tfrac{1}{8}g^2\,(\ell z)^2\Big[I_0(\ell z) - J_0(\ell z)\Big]$$

- sum over spins is convergent
- non-trivial dependence on Mandelstam variables and ℓ
- full exchange amplitude

$$\widehat{\mathcal{A}}_{exch}(\mathrm{s},\mathrm{t},\mathrm{u}) = \mathcal{A}_{exch}(\mathrm{s},\mathrm{t},\mathrm{u}) + \mathcal{A}_{exch}(\mathrm{t},\mathrm{s},\mathrm{u}) + \mathcal{A}_{exch}(\mathrm{u},\mathrm{t},\mathrm{s})$$

• Regge limit of exchange part: $\quad \mathrm{t} \to \infty, \ \mathrm{s}$=fixed

$$\widehat{\mathcal{A}}_{exch}(\mathrm{s},\mathrm{t},\mathrm{u}) \sim -\frac{ig^2}{\mathrm{s}}\ell^2\mathrm{t}\, I_0(\ell\sqrt{8\mathrm{t}}) \sim -\frac{ig^2}{\mathrm{s}}\frac{(\ell^2\mathrm{t})^{3/4}}{2^{5/4}\pi^{1/2}}\, e^{\ell\sqrt{8\mathrm{t}}}$$

• fixed angle limit:

$\mathrm{s},\mathrm{t},\mathrm{u} \to \infty, \ \ \frac{\mathrm{t}}{\mathrm{s}} = -\sin^2\frac{\theta}{2}, \ \ \frac{\mathrm{u}}{\mathrm{s}} = -\cos^2\frac{\theta}{2}, \ \ \theta =$fixed

$$\widehat{\mathcal{A}}_{exch}(\mathrm{s},\mathrm{t},\mathrm{u}) \sim ig^2|\mathrm{s}|^{3/4}\, e^{\ell\sqrt{|\mathrm{s}|}\ f(\theta)} \to \infty\,, \qquad\qquad f(\theta) > 0$$

• exponential growth: indication of UV divergences in loops

but this is not full amplitude: still to add 4-vertex contribution

[cf. string theory: Shapiro-Virasoro amplitude is UV-soft]

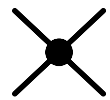
$$A_4 = g^2\frac{\Gamma(-1-\frac{1}{4}\alpha'\mathrm{s})\Gamma(-1-\frac{1}{4}\alpha'\mathrm{s})\Gamma(-1-\frac{1}{4}\alpha'\mathrm{s})}{\Gamma(2+\frac{1}{4}\alpha'\mathrm{s})\Gamma(2+\frac{1}{4}\alpha'\mathrm{s})\Gamma(2+\frac{1}{4}\alpha'\mathrm{s})}$$

$$\to \ \ g^2|\mathrm{s}|^{-6}(\sin\theta)^{-6}e^{-\alpha'|\mathrm{s}|\, h(\theta)} \to 0$$

$$h(\theta) = -\tfrac{1}{4}\Big(\sin^2\tfrac{\theta}{2}\log\sin^2\tfrac{\theta}{2} - \cos^2\tfrac{\theta}{2}\log\cos^2\tfrac{\theta}{2}\Big) > 0$$

4-vertex contribution

add contribution of 0-0-0-0 vertex



• expected to be effectively non-local: infinite series in ∂^n

• may cancel or "soften" large p behaviour of exchange

• try to guess 4-scalar vertex in flat-space HS action from its form in AdS action reconstructed from AdS/CFT

[Bekaert, Erdmenger, Ponomarev, Sleight 2015]: $\nabla \to \partial$

$$S^{(4)}[\phi_0] = g^2 \int d^4x \Big[\sum_{j=0}^{\infty} f_{2j}(\Delta_{x_{34}}) \left(\partial_{x_{12}} \cdot \partial_{x_{34}}\right)^{2j} \times \phi_0(x_1)\phi_0(x_2)\phi_0(x_3)\phi_0(x_4)\Big]_{x_i=x}$$

$$\Delta_{x_{34}} \equiv (\partial_{x_3} + \partial_{x_4})^2, \qquad \partial_{x_{12}} \equiv \partial_{x_1} - \partial_{x_2}$$

$f_{2j}(z)$ = infinite series in z, regular at $z=0$: no poles

• choose large z asymptotics same as in AdS 4-scalar vertex:

$$z\to\infty: \qquad f_{2j}(z)\to c_{2j}\frac{\ell^{4j-2}}{z}\,, \qquad c_{2j}=\tfrac{1}{[(2j-1)!]^2}$$

• then asymptotic contribution to 4-scalar amplitude is

$$\textstyle\sum_{j=0}^{\infty} f_{2j}(\mathrm{s})\,(\mathrm{t}-\mathrm{u})^{2j}=\frac{2\mathrm{t}+\mathrm{s}}{2\mathrm{s}}\left[I_0\big(2\ell\sqrt{2\mathrm{t}+\mathrm{s}}\big)-J_0\big(2\ell\sqrt{2\mathrm{t}+\mathrm{s}}\big)\right]$$

• contact term may cancel against the exchange contribution

– full amplitude may be trivial?

Remarks:

• soft UV behaviour is expected in higher-spin theory in AdS:
4-scalar amplitude = free CFT 4-point correlator ($\phi_0 \to \Phi^*\Phi$)

• Witten diagrams in AdS in Mellin representation
look similar to scatt amplitudes in flat space [Penedones 2010]
[AdS exch – Mellin ampl with poles related to dim's of ops;
contact interactions with $\partial^{2n} \to$ Mellin amps
$\sim n$-order polynomials in $\mathrm{s}, \mathrm{t}, \mathrm{u}$ =Mellin variables]

• total AdS scatt amp similar to Mellin transform (in u, v)
of 4-point correlator of spin 0 operator in free $O(N)$ CFT:
distribution $\delta(\mathrm{s} - \frac{1}{2})\delta(\mathrm{t} - \frac{3}{2}) + ...$ [Taronna; Bekaert et al]

• suggests (?) exch + 4-vertex tree-level 4-scalar amplitude
in flat space may also have trivial large p asymptotics

Comment on BCFW constructibility

• requires that amplitudes vanish under infinite complex shifts of momenta [Benincasa, Cachazo 07]
assumption of analyticity (vanishing at $\infty \to$ ampl can be reconstructed from poles and residues)

• leads to recurrence relations which allow to express any tree-level amplitude in terms of on-shell 3-point ones

• applied to 4-scalar amplitude would allow to determine quartic scalar self-coupling in terms of cubic vertices

• but BCFW construction can be applied only if cubic vertices satisfy non-trivial consistency condition - "four-particle" test;
cubic HS vertices fail to satisfy this test
[Fotopoulos, Tsulaia 08; Dempster, Tsulaia 12; Bengtsson 16]

• not clear if condition of BCFW constructibility should apply to an effectively non-local HS theory containing infinite number of fields with higher derivatives of any order in vertices
(e.g. assumption of analyticity may fail if sums over spins do not converge fast or give amplitudes $\sim$ distributions)

S-matrix approach to gauge-invariant interactions

• direct construction of gauge-inv action via Noether procedure: quadratic action, deform by higher-order terms while also deforming linearized gauge transformations to make full action invariant: ties construction of action $S = S_2 + S_3 + S_4 + ...$ to that of gauge transformations $\delta = \delta^{(0)} + \delta^{(1)} + \delta^{(2)} + ...$

$$\delta^{(0)} S_3 + \delta^{(1)} S_2 = 0 \ , \qquad \delta^{(0)} S_4 + \delta^{(1)} S_3 + \delta^{(2)} S_2 = 0, ...$$

• more efficient approach: start with S-matrix and demand its on-shell gauge invariance: advantage - only linearized transformations $\delta^{(0)}$ act on physical amplitudes

non-linear $\phi\epsilon$ terms in $\delta\phi \sim \partial\epsilon + \phi\,\epsilon + \ldots$,

relate Green's functions to Green's functions but projected out by leg amputation to get S-matrix element

• linearized gauge transformations

$$\delta^{(0)}\phi_s = \partial\epsilon_{s-1} \quad \rightarrow \quad \delta\phi_{\mu_1\cdots\mu_s}(p) = p_{(\mu_1}\epsilon_{\mu_2\cdots\mu_s)}(p)$$

• non-trivial case: if S_3 is invariant under linearized g.t.
only up to eqs of motion need to add S_4 vertex:
3-point vertex in higher point amplitude – variation leads to
$p^2 \times \frac{1}{p^2}$ – higher point violation of invariance
– add higher vertex to cancel

Example: scalar electrodynamics

$L = \partial^m \phi^* \partial_m \phi + i A^m (\phi^* \partial_m \phi - \phi \partial_m \phi^*) + A^m A_m \phi^* \phi$

$\delta A_m = \partial_m \epsilon, \quad \delta \phi = i \phi \epsilon$

$A(1)\phi(2)\phi(3)A(4)$ scattering amplitude:

$A_m \to \zeta_m(p) e^{ip\cdot x}, \quad p \cdot \zeta = 0$

$$\mathcal{A}_{\rm exch} = \frac{1}{p_{12}^2} \zeta_1 \cdot p_2 \, \zeta_4 \cdot p_3 + \frac{1}{p_{13}^2} \zeta_1 \cdot p_3 \, \zeta_4 \cdot p_2$$

• gauge transformation in leg 1: $\delta \zeta_1 = p_1 \epsilon_1, \quad \delta \phi = 0$

$\delta \mathcal{A}_{\rm exch} = (\zeta_4 \cdot p_3 + \zeta_4 \cdot p_2) \epsilon_1 = -\zeta_4 \cdot p_1 \, \epsilon_1$

• can be cancelled by adding contact $A^m A_m \phi^* \phi$ vertex

$\mathcal{A}_{\rm cont} = \zeta_1 \cdot \zeta_4 \to \ \delta \mathcal{A}_{\rm cont} = p_1 \cdot \zeta_4 \, \epsilon_1$

• thus 4-point vertex can be found from condition
of linearized gauge invariance of on-shell amplitude

• To get information about structure of possible 4-vertices consider 0-0-0-s tree-level amplitude:

(i) find exchange contribution

(ii) add general 4-vertex contribution

(iii) impose on-shell gauge invariance w.r.t. spin s leg

(iv) determine "minimal" 4-vertex required by gauge invariance

• Parametrization of $000s$ 4-vertex in momentum space:

$$\mathcal{L}_{000s} = \sum_{k=0}^{s-2} V_{sk}(p_1, p_2, p_3) \times \phi_0(p_1)\, (2ip_2 \cdot \partial_u)^k \phi_0(p_2)\, (2ip_3 \cdot \partial_u)^{s-k} \phi_0(p_3)\, \phi_j(p_4, u)$$

• Aim: constrain coefficient functions V_{sk}

by demanding that S-matrix element $000s$ is gauge invariant

Gauge-invariance constraints on S-matrix

• soft momentum expansion of massless higher spin amplitudes and gauge invariance constraints: [Low 58; Weinberg 64; Bern et al 14]

• soft limit of massless HS theory with generic 3-couplings

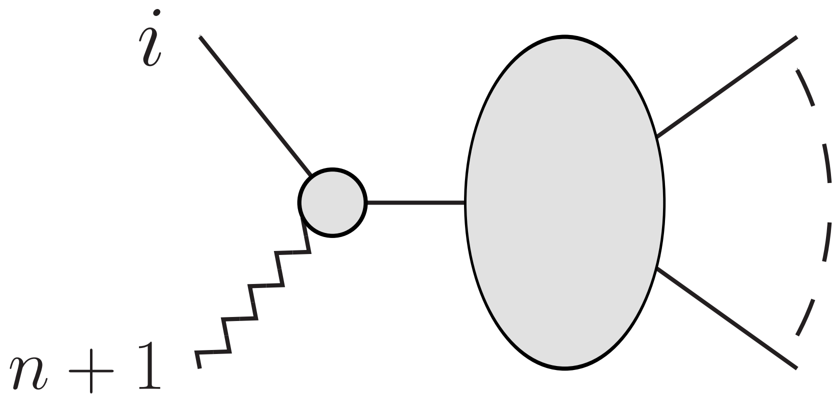
• assume locality: all poles in momentum in amplitudes may only come from on-shell propagators of particles present in the original action

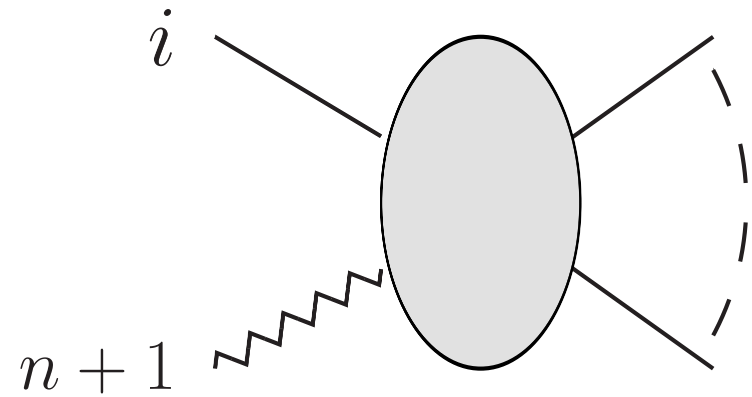
• restrict to leading order of soft momentum expansion: extends [Weinberg 64] to arbitrary couplings of HS [Taronna 11]

Soft momentum expansion of $0...0s$ amplitude

n spin-0 and one spin-s with $p_{n+1} \equiv q \to 0$

two contributions: with pole at $q \to 0$ and without





$$\mathcal{A}^{\mu_1\cdots\mu_s}(p_1,\ldots,p_n,q) = \mathcal{P}^{\mu_1\cdots\mu_s}(p_1,\ldots,p_n,q) + \mathcal{R}^{\mu_1\cdots\mu_s}(p_1,\ldots,p_n,q)$$

$$\mathcal{P}^{\mu_1\cdots\mu_s} \to \sum_i \sum_{s'_i} \frac{p_i^{\mu_1}\ldots p_i^{\mu_s}}{q\cdot p_i} P_{s'_i}(u,u')\left[(p_i-q)\cdot\partial_u\right]^{s'_i} W_{s'_i}(p_i+q,\partial_{u'})$$

$(p_i+q)^2 = 2q\cdot p_i + q^2 \;\to\; 2q\cdot p_i$

$P_s(u,u')$ – projector in spin-s propagator

$W_{s'_i}$ – Green's function with all but i-th leg (p_i+q) on shell

- for $q=0$: W is n-point amplitude

W is then gauge-invariant: $(W_{s_i'})_{q\to 0} = W_{s_i'}(p_i, \partial_{u'})$

$$W_{s_i'}(p_i, \partial_{u'}) P_{s_i'}(p_i, u') = 0 \,, \qquad s_i' \neq 0$$

$$W_{s_i'}(p_i, \partial_{u'})(p_i \cdot u')^k P_{s_i'-k}(p_i, u') = 0 \,, \qquad k = 1, \ldots s_i'$$

- gauge invariance of full amplitude requires for any q

$$q_{\mu_s} \mathcal{A}^{\mu_1 \ldots \mu_s}(p_1, \ldots, p_n, q) = 0$$

leading term in $q \to 0$:

$$\sum_i \sum_{s_i'} p_i^{\mu_1} \ldots p_i^{\mu_{s-1}} \; s_i'! \, W_{s_i'}(p_i, \partial_{u'}) P_{s_i'}(p_i, u') = 0$$

- assumed locality: dropped $\mathcal{R}$-term that has no poles in q
- gauge inv of $W_{s_i'}(q=0)$: only terms with $s_i' = 0$ non-zero
- left with W_0 = $\mathcal{A}^{0\ldots 0}(p_1, \ldots, p_n)$

$$\mathcal{A}^{0\ldots 0}(p_1, \ldots, p_n) \sum_i p_i^{\mu_1} \ldots p_i^{\mu_{s-1}} = 0$$

• as $\sum_i p_i^{\mu_1} \dots p_i^{\mu_{s-1}}$ does not, in general, vanish if $s > 2$:

$$\mathcal{A}^{0\dots 0}(p_1, \dots, p_n) = 0$$

local action $\to$ scattering amplitude =0 [Weinberg]

Soft momentum expansion of $s_1 \dots s_n s$ amplitude

again $\mathcal{A} = \mathcal{P} + \mathcal{R}$, for $q \to 0$

$$\mathcal{P}^{\mu_1 \dots \mu_s}(p_1, \dots, p_n, q) \to \sum_{i, s'_i} V^{\mu_1 \dots \mu_s}_{s, s_i, s'_i}(q, p_i, \partial_u) \frac{P_{s'_i}(u, u')}{q \cdot p_i} W_{s'_i}(p_i + q, \partial_{u'})$$

$W_{s'_i}$: all but the i-th leg $(q + p_i)$ on shell

for $q = 0$ subject to gauge-invariance constraints

3-vertices $V^{\mu_1 \dots \mu_s}_{s, s_i, s'_i}(q, p_i, \partial_u)$ gauge inv on shell [Manvelyan et al]

non-trivial contribution to spin s gauge inv constraint:

$$
\begin{aligned}
0 =& q \cdot \partial_{u_q} \Big[u_q^{\mu_1} \dots u_q^{\mu_s} \mathcal{A}^{\mu_1 \cdots \mu_s}(p_1, \dots, p_n, q) \Big] \\
=& \sum_{i, s_i'} F_{j, j_i, s_i'}(q, p_i; u_q; \partial_u) \, W_{s_i'}(p_i + q, \partial_{u'}) \, P_{s_i'}(u, u') \\
&+ \; q \cdot \partial_{u_q} \Big[u_q^{\mu_1} \dots u_q^{\mu_s} \mathcal{R}^{\mu_1 \cdots \mu_s}(p_1, \dots, p_n, q) \Big]
\end{aligned}
$$

leading order at $q \to 0$: using explicit form of vertices

$$
\begin{aligned}
0 =& \sum_i c_{s s_i s_i} \tfrac{1}{s_i!} (u_q \cdot p_i)^{s-1} \phi_{s_i}(p_i, \partial_u^{s_i}) \, W_{s_i'}(p_i, \partial_{u'}) \, P_{s_i'}(u, u') \\
=& \; \mathcal{A}^{s_1, \dots, s_n}(p_1, \dots, p_n) \sum_i c_{s s_i s_i} (u_q \cdot p_i)^{s-1}
\end{aligned}
$$

- $s = 2$: $c_{2 s_i s_i}$ must be same for all s_i (can use $\sum_k p_k = 0$)

– spin 2 coupling must be universal

- $s > 2$: sum cannot vanish for generic on-shell momenta

• thus gauge invariance requires that either $\mathcal{A}^{s_1\dots s_n}=0$
or constraint on coupling consts: $c_{ss_is_i}=0\ , \quad s_i<s$
– no cubic diagonal coupling of spin-s with $s_i<s$ fields

• 0...0s amplitude as special case:
if $c_{s00}\neq 0$ and assume locality then $\mathcal{A}^{0\dots 0}_n=0$
$n=3$: trivially absent
$n=4$: vanishing comes from constraint on 5-point $0000s$
(assuming locality of 5-vertex)

• assumed locality:
may still get gauge-inv S-matrix in a non-local theory
but if recover locality (by adding extra fields, relaxing unitarity)
then total gauge-inv amplitude should still vanish

0-0-0-s exchange amplitude: [Roiban, AT 17]

- no constraint from soft limit: $\mathcal{A}_3^{000} \equiv 0$ – go beyond soft limit
- use 0-0-s' and 0-s'-s: $\phi_s \to \zeta_s(p)\, e^{ip\cdot x}$

$\zeta_s(p, q^s) \equiv \zeta_{m_1 \dots m_s}(p)\, q^{m_1} \dots q^{m_s}, \quad p_{ij} = p_i \cdot p_j, \quad p_i^2 = 0$

- s-channel:

$$\mathcal{A}_{\rm exch} = -\frac{ig^2}{p_{12}^2} \sum_{s'} \frac{\ell^{2s'+s-2}}{(s'-1)!(s+s'-1)!} (p_{12}^2)^{s'} T_{s'}\big(\tfrac{p_{13}^2 - p_{23}^2}{p_{12}^2}\big)\, \zeta_s(p_4, p_3^s)$$

$$T_s(z) = \tfrac{1}{2}\Big[(z + \sqrt{z^2-1})^s + (z - \sqrt{z^2-1})^s\Big]$$

$$\mathcal{A}_{\rm exch} = -\frac{2ig^2}{p_{12}^2}\Big[F_s(z_+) + F_s(z_-)\Big]\, \zeta_s(p_4, p_3^s)$$

$$F_s(z) = z^{2-s}\Big[I_s(z) - J_s(z)\Big]\,, \qquad z_\pm = \ell\big(\sqrt{p_{13}^2} \pm \sqrt{p_{12}^2 + p_{13}^2}\big)$$

- add t and u channels: full $\mathcal{A}_{\rm exch}$

• impose linearized gauge invariance condition

$\delta\zeta_{m_1\ldots m_s}(p) = p_{(m_1}\epsilon_{m_2\ldots m_s)}$

on full amplitude: $\mathcal{A}_4 = \mathcal{A}_{\rm exch} + \mathcal{A}_{cont}$

$$\delta\mathcal{A}_{\rm exch} = -2sg^2\Big[F_s(z_+) + F_s(z_-)\Big]\,\epsilon_{s-1}(p_4, p_3^{s-1}) + \ldots$$

cancel this against variation of contribution of 0-0-0-s vertex

$\sum_{k=0}^{s/2} V_{sk}(p_1, p_2, p_3)\,\phi_0(p_1)(p_2{\cdot}\partial_u)^k\phi_0(p_2)(p_3{\cdot}\partial_u)^{s-k}\phi_0(p_3)\,\zeta_s(p_4, u)$

$$\delta\mathcal{A}_{\rm cont} = sV_{s0}(p_1, p_2, p_3)\,p_{24}^2\,\zeta_{s-1}(p_4, p_2^{s-1}) + \ldots$$

• find required 4-point vertex V_{000s}

get “minimal” solution consistent (?) with locality

• gauge-invariance: gives relation of V_{sk} to Bessels in $\mathcal{A}_{\rm exch}$

• local solution for 4-vertex exists only for $s = 2$ and $s = 4$

• $s=2$:

local 4-vertex exists:

$$V_{20} = \frac{g^2}{p_{12}^2}\Big(F_2(z_+) + F_2(z_-) \\ -\frac{1}{2}\left[p_{13}^2 R_2(p_{13}^2) + p_{23}^2 R_2(p_{23}^2) + p_{12}^2 R_2(p_{12}^2)\right]\Big)$$

$R_s(x) \equiv \frac{1}{2x}\left[I_s(\sqrt{-x}) - J_s(\sqrt{-x})\right]$ $\quad x \to 0$ residue of $F_2(x)$

• particular form of gauge-invariant 0-0-0-2 amplitude:

for special choice of local 4-vertex

$$\mathcal{A} = g^2\left[p_{13}^2 R_2(p_{13}^2) + p_{23}^2 R_2(p_{23}^2) + p_{12}^2 R_2(p_{12}^2)\right] \\ \times\left(\frac{\zeta_2(p_4,p_3^2)}{p_{12}^2} + \frac{\zeta_2(p_4,p_2^2)}{p_{13}^2} + \frac{\zeta_2(p_4,p_1^2)}{p_{23}^2}\right)$$

• still need to fix possible extra terms in 4-vertex:

requires study of other amplitudes

• $s = 4$:

local 4-vertex $\sim R_4 \sim$ Bessels

particular form of gauge-invariant

exchange + contact 0004 amplitude:

$$\mathcal{A} = U(p_1, p_2, p_3)\, \zeta_4\big(p_4, (p_{12}^2 p_2 - p_{13}^2 p_3)^4\big) - \frac{i p_{12}^2}{15 p_{13}^2}\, \zeta_4(p_4, p_2^4) + \ldots$$

$$U = \left(\frac{1}{p_{13}^2} + \frac{1}{p_{23}^2}\right) R_4(p_{12}^2) + \text{ cycle}$$

• $s > 4$: no local 4-vertex exist [Roiban, AT; Taronna]

• cf. constraint of soft theorem:

if assume locality then gauge invariance of $000ss'$

implies vanishing of $000s$

• similar conclusions assuming BCFW constructibility

[Benincasa, Cachazo; Benincasa, Conde; Dempster, Tsulaia]

Required non-local terms for $s \geqslant 6$

- minimal required non-local 4-vertex

to make $000s$ amplitude gauge invariant

coefficient functions $V_{s0}(p_1, p_2, p_3)$ have poles ($\ell = 1$)

s	$V_{s0}(p_1, p_2, p_3)$
6	$\frac{1}{2^2\times 3!\times 7!!}\frac{p_{13}^2+p_{23}^2}{p_{12}} + \text{local}$
8	$\frac{1}{2^3\times 4!\times 9!!}\frac{p_{13}^2+p_{23}^2}{p_{12}} + \frac{1}{2^2\times 3!\times 5!\times 11!!}\frac{p_{13}^4+p_{23}^4}{p_{12}} + \text{local}$
10	$\frac{1}{2^4\times 5!\times 11!!}\frac{p_{13}^2+p_{23}^2}{p_{12}} + \frac{1}{2^3\times 3!\times 6!\times 13!!}\frac{p_{13}^4+p_{23}^4}{p_{12}} + \frac{1}{2^2\times 5!\times 7!\times 15!!}\frac{p_{13}^6+p_{23}^6}{p_{12}} + \text{local}$

$$V_{s0}^{\text{nonloc}}(p_1, p_2, p_3) = -\frac{1}{p_{12}} \sum_{l=0}^{s/2-3} \kappa_{sl}\left(p_{13}^{2l+2} + p_{23}^{2l+2}\right)$$

$$\kappa_{sl} = \frac{1}{2^{s/2-1} l!(2l+1)!!(l+\frac{s}{2})!(2l+s+1)!!}$$

• 4-vertex in position space $\quad (\phi(u) \equiv \phi(x,u))$

$$\mathcal{L}_{000s}^{\rm nonloc} = g^2 \sum_{l=0}^{s/2-3} \phi_0 \left(\partial^{\mu_1} \ldots \partial^{\mu_{2l+2}} \phi_0\right) \frac{1}{\Box} \left[\partial_{\mu_1} \ldots \partial_{\mu_{2l+2}} (\partial_u \cdot \partial)^s \phi_0\right] \phi_s(u)$$

• observe factorization in sum over s: $\quad \mathrm{C}_{sl} \equiv \frac{\sqrt{8}g}{2^{s/2-l}(l+\frac{s}{2})!(2l+s+1)!!}$

$$\sum_{s=6,8,\ldots} \mathcal{L}_{000s}^{\rm nonloc} = \sum_{l=0}^{\infty} \mathrm{C}_{0l}\, \phi_0 \left(\partial^{\mu_1} \ldots \partial^{\mu_{2l+2}} \phi_0\right)$$
$$\times \frac{1}{\Box} \sum_{s=6+2l}^{\infty} \mathrm{C}_{sl} \left[\partial_{\mu_1} \ldots \partial_{\mu_{2l+2}} (\partial_u \cdot \partial)^s \phi_0\right] \phi_s(u)$$

• suggests that may eliminate non-locality by introducing additional spin $s = 2, 4, 6, \ldots$ ghosts-like fields ψ_s

$$\mathcal{L}(\phi,\psi) = -\tfrac{1}{2}\sum_{l=0}^{\infty}\psi_{2l+2}\Box\psi_{2l+2} - \sum_{l=0}^{\infty}\Big[\mathrm{C}_{0l}\,\phi_0(\partial\cdot\partial_v)^{2l+2}\phi_0$$

$$+\sum_{s=2l+6}^{\infty}\mathrm{C}_{sl}\,\big((\partial_u\cdot\partial)^s(\partial_v\cdot\partial)^{2l+2}\phi_0\big)\phi_s(u)\Big]\psi_{2l+2}(v)$$

- integrating out ψ_j gives also other non-local terms

$$\mathcal{L}^{\mathrm{nonloc}}_{0000} = \sum_{l=0}^{\infty}(\mathrm{C}_{0l})^2\ \phi_0\partial^{\mu_1}\ldots\partial^{\mu_{2l+2}}\phi_0\ \frac{1}{\Box}\ \phi_0\partial_{\mu_1}\ldots\partial_{\mu_{2l+2}}\phi_0$$

$$\mathcal{L}^{nonloc}_{00s_1s_2} = \sum_{l=0}^{\infty}\mathrm{C}_{s_1l}\ \mathrm{C}_{s_2l}\ \big[(\partial_{u_1}\cdot\partial)^{s_1}\partial_{\mu_1}\ldots\partial_{\mu_{2l+2}}\phi_0\big]\phi_{j_1}(u_1)$$

$$\times\frac{1}{\Box}\big[(\partial_{u_2}\cdot\partial)^{s_2}\partial_{\mu_1}\ldots\partial_{\mu_{2l+2}}\phi_0\big]\phi_{s_2}(u_2)$$

• assume that these non-local quartic terms are indeed present
then extra contact contribution to 0000 amplitude

$$(A^{ex}_{\rm s})_{0000}\Big|_{\rm pole} = -\,\frac{2ig^2}{p_{12}}\ {\rm s}_{13}\big[I_0(\sqrt{8p_{13}})-J_0(\sqrt{8p_{13}})\big]$$

$$(A^{ct}_{\rm s})_{0000}\Big|_{\rm pole} =4i\sum_{l=0}^{\infty}({\rm C}_{0l})^2(p_{13})^{2l+2} = 2ig^2\frac{p_{13}}{p_{12}}\big[I_0(\sqrt{8p_{13}})-J_0(\sqrt{8p_{13}})\big]$$

• sum vanishes – cancellation of s-channel pole in 0000
suggests full 0000 amplitude should vanish ?
• same may expect for $s>0$ – if add proper non-minimal terms
• if local and gauge-invariant but non-unitary extended action
exists – such theory may have a trivial S-matrix
consistent with expectations based on soft theorem
• same should then apply to non-local HS theory:
consequence of hidden infinite-dimensional symmetry?

Conformal off-shell extension

• candidate symmetry: higher spin conformal symmetry –
symmetry of higher derivative conformal higher spins $\int d^4x \, \phi \, \Box^s \, \phi$

• analogy: Weyl gravity and conformal extension of Einstein
$\int d^4x \sqrt{g} \, (R + 6\partial^m \varphi \partial_m \varphi)$ have same symmetries

• similar conformal extension of Fronsdal ∂^2 theory?
requires tower of auxiliary ghost fields

• non-local if one eliminates ghost fields:
non-local action with extra gauge symmetry but same S-matrix

• $S_E(h) = \int d^4x \sqrt{g} R$
depends on traceless t_{mn} and trace h parts of h_{mn}

$h_{mn} = t_{mn} + \frac{1}{4}\eta_{mn}\mathrm{h} \, , \qquad t_{mn} \equiv h_{mn} - \frac{1}{4}\eta_{mn}\mathrm{h} \, , \qquad \mathrm{h} \equiv h^m_m$

• h – unphysical – can be gauged away on shell:
does not appear as asymptotic state in S-matrix

• integrate out h – non-local effective action for t_{mn}

$$\bar{S}_E(t) = \int d^4x (t\partial^2 t + \partial\partial t \partial^{-2} \partial\partial t + \partial^2 ttt$$
$$+\partial^2 t \partial^{-2} \partial t \partial t + \partial^2 tttt + \partial t \partial t \partial^{-2} \partial t \partial t + \ldots)$$

produces same Einstein S-matrix for gravitons

cf. $S_W(t) = \int d^4x \sqrt{g}\, C^2 = \int d^4x (t\partial^4 t + \partial^4 ttt + \partial^4 tttt + \ldots)$

• closed form of such action: [Fradkin, Vilkovisky 75]

$$S' = \int d^4x \sqrt{g} \big(R - \tfrac{1}{6} R \Delta^{-1} R \big)$$

Weyl-invariant off shell extension of Einstein theory

• generalize to HS case: quadratic plus cubic action for Fronsdal HS fields $\phi_{m_1 \ldots m_s}$ subject to double-tracelessness

(i) split into "physical" traceless t_s + "ghost-like" trace h_{s-2}

(ii) integrate out h_{s-2}

• resulting non-local action for t_s should lead to same S-matrix: analog of conformal off-shell extension of Einstein theory invariant under infinite dim conformal HS symmetry?

Integrating out the trace from the Einstein action

$$L_E(h) = \sqrt{g}R = X_1 + X_2 + X_3 + X_4 + \ldots\,,$$
$$X_1 = \partial_m\partial_n h_{mn} - \partial^2\mathrm{h} = \partial_m\partial_n t_{mn} - \tfrac{3}{4}\partial^2\mathrm{h}$$
$$X_2 = \tfrac{3}{4}\partial_k t_{mn}\partial_k t_{mn} - \tfrac{1}{2}\partial_k t_{mn}\partial_n t_{mk} + t_{mn}\partial^2 t_{mn} - \partial_n t_{kn}\partial_m t_{km}$$
$$+ \tfrac{3}{32}(\partial_k\mathrm{h})^2 + \tfrac{1}{4}\partial_m t_{mn}\partial_n\mathrm{h} + \tfrac{1}{2}t_{mn}\partial_m\partial_n\mathrm{h}$$
$$X_3 = \tfrac{3}{4}t_{mn}\partial_m t_{sr}\partial_n t_{sr} + t_{ms}\partial_m t_{nr}\partial_n t_{sr} + \tfrac{1}{2}t_{ns}\partial_m t_{nr}\partial_r t_{sm} + \ldots$$

Solving for h:

$$\bar{L}_E(t) = \bar{L}_E^{(2)}(t) + \bar{L}_E^{(3)}(t) + L_E^{(4)}(t) + \ldots$$
$$\bar{L}_E^{(2)}(t) = -\tfrac{1}{4}\partial_k t_{mn}\partial_k t_{mn} + \tfrac{1}{2}\partial_k t_{mk}\partial_n t_{mn} + \tfrac{1}{6}\partial_m\partial_n t_{mn}\partial^{-2}\partial_k\partial_r t_{kr}$$
$$= \tfrac{1}{2}C_{mnkl}\partial^{-2}C_{mnkl} = \tfrac{1}{4}t_{ab}P^{ab}_{mn}\partial^2 t^{mn}$$
$$P^{ab}_{mn} = P^a_{(m}P^b_{\ n)} - \tfrac{1}{3}P^{ab}P_{mn}\,, \qquad P_{mn} = \eta_{mn} - \frac{\partial_m\partial_n}{\partial^2}$$

$$\bar{L}_E^{(3)}(t) = X_3(t) + \tfrac{1}{3}X_1(t)\partial^{-2}X_2(t)\,, \qquad X_n(t) \equiv X_n(t,\mathrm{h}=0)$$

• in transverse gauge $\partial_m t_{mn} = 0$

$$\bar{X}_1 = 0\,, \qquad \bar{X}_2 = \tfrac{3}{4}\partial_k t_{mn}\partial_k t_{mn} - \tfrac{1}{2}\partial_k t_{mn}\partial_n t_{mk} + t_{mn}\partial^2 t_{mn}\,,$$

$$\bar{X}_3 = -\tfrac{1}{4}t_{ab}\partial_a t_{mn}\partial_b t_{mn} + t_{ab}\partial_a t_{mn}\partial_n t_{mb} - \tfrac{1}{2}t_{ab}\partial_n t_{ma}\partial_n t_{mb} + \ldots$$

$$\bar{X}_4 = -\tfrac{1}{16}t_{mn}t_{mn}(\partial_r t_{ab}\partial_r t_{ab} - 2\partial_r t_{ab}\partial_b t_{ar}) + \ldots$$

$$\bar{L}_E(t) = -\tfrac{1}{4}\partial_k t_{mn}\partial_k t_{mn} + \bar{X}_3(t) + \bar{X}_4(t) + Y_4\,, \qquad Y_4 = \tfrac{1}{6}\bar{X}_2\partial^{-2}\bar{X}_2$$

3-graviton amplitude is given by $\bar{X}_3(t)$

• non-local contribution Y_4 to 4-graviton amplitude

$$Y_4 = \tfrac{1}{6}\Big[\tfrac{3}{8}\partial^2(t_{mn}t_{mn}) - \tfrac{1}{2}\partial_k\partial_n(t_{mn}t_{mk})\Big]\tfrac{1}{\Box}\Big[\tfrac{3}{8}\partial^2(t_{ab}t_{ab}) - \tfrac{1}{2}\partial_r\partial_b(t_{ab}t_{ar})\Big]$$

• non-local h-exchange term contributes to S-matrix?

h "unphysical" – "wrong" sign of kin term, pure gauge on shell

same 3-graviton amplitude – should be no change to S-matrix

constructed by BCFW prescription: unitarity-based arguments

imply that scattering amplitudes should be cut-constructible

• BCFW representation: Einstein 4-point S-matrix in terms of
t^3 physical vertex (at complex momenta) – trace not involved
• complete 4-graviton amplitude =
t_{mn} exchange $\bar{X}_3 \partial^{-2} \bar{X}_3$ + local $\bar{X}_4(t)$ + non-local $Y_4(t)$
is physical and gauge-independent
but split between exchange and contact contributions depends
on (on-shell) gauge or particular choice of polarization tensors
• non-local term Y_4 (not gauge-invariant) does not contribute
to S-matrix in special gauge or for choice of polarization tensors
for which on-shell matrix element of Y_4 vanishes
• cf . similar choices in gauge theories
when 4-gluon vertex does not contribute:
it is under this special choice BCFW construction applies
• in general, unphysical trace exchange cancels
other unphysical (time-like, etc) parts of the t_{mn} exchange

Conformal off-shell extension of Einstein theory

• same $\bar{L}_E(t)$ obtained by integrating out h can be found from Weyl-invariant off-shell extension of Einstein theory

$$S(g,\phi) = S_E(\phi^2 g) = \int d^4x \sqrt{g}\big(R\,\phi^2 + 6\,\partial^m\phi\partial_m\phi\big)$$

invariant under $g'_{mn} = \lambda^2(x) g_{mn}, \ \ \phi' = \lambda^{-1}(x)\,\phi$

• perturbatively equivalent to the Einstein theory if assume ϕ has a non-zero constant vacuum value in flat space

i.e. expansion $g_{mn} = \eta_{mn} + h_{mn}, \ \ \phi = 1 + \varphi$

• if fix the Weyl gauge $\varphi = 0 \to$ Einstein theory

or if solve for φ in terms of the metric $\to$

non-local "conformal off-shell extension" of Einstein gravity

• gives equivalent S-matrix but has an additional Weyl symm

$$\phi(g) = 1 + \varphi(g) \,, \qquad -\nabla^2\varphi + \tfrac{1}{6}R(1+\varphi) = 0$$

$$\varphi = -\tfrac{1}{6}\Delta^{-1}R \,, \quad \Delta \equiv -\nabla^2 + \tfrac{1}{6}R$$

$$S_c(g) \equiv S(g,\phi(g)) = 6\int d^4x\sqrt{g}\,\phi(g)\Delta\phi(g)$$

$$= \int d^4x\sqrt{g}\Big(R - \tfrac{1}{6}R\Delta^{-1}R\Big)$$

• Weyl symmetry – can fix traceless gauge on h_{mn}:
S_c depends only on traceless graviton t_{mn} even off-shell
• resulting action is equivalent to $\bar{S}_E = \int d^4x\bar{L}_E(t)$
found by integrating out h from the Einstein action:
either gauge-fixing $\varphi = 0$ and solving for h
or first gauge-fixing $\mathrm{h} = 0$ and solving for φ
leads to same action for t_{mn}

Higher spin generalization?

• Weyl gravity $\rightarrow$ conformal higher spin theory
invariant under conformal higher spin symmetry
generalizing both reparametrizations and algebraic Weyl symm

• conformal extension of Einstein theory $\rightarrow$
2-derivative higher spin generalization?
should contain extra tower of ghost-like "compensator" fields
making it invariant under conformal higher spin symmetry

• solving for this extra tower of fields should give
non-local action with extra higher spin conformal symmetry
depending only on "physical" traceless parts t_s
of the original (double-traceless) Fronsdal fields ϕ_s

• equivalent action (leading to same S-matrix)
from integrating out traces h_{s-2} of the fields ϕ_s in
massless HS Lagrangian $L = \sum_s \phi_s \partial^2 \phi_s + V_3(\phi) + V_4(\phi) + \ldots$

• kin term in non-local action depends only on traceless t_s
represented in terms of linearized Weyl tensors $C_s \sim \partial^s t_s$
conf HS theory: $L_2 = C_s C_s = t_s \Box^s t_s + ...$
conf Fronsdal: $L_2 = C_s \Box^{1-s} C_s = t_s \Box t_s + ...$
• analogy with "extended" cubic+ quartic theory from
condition of on-shell gauge invariance:
also has extra "ghost-like" HS fields ψ_j needed for locality
• suggests interpretation of additional fields ψ_j as
conformal compensators of conformal off-shell extension
that should not appear as asymptotic states in S-matrix
• action should have extra hidden symmetry
– conformal higher symmetry –
• while speculative this proposal
may be explaining possible triviality of resulting S-matrix

Conclusions

• motivation to study flat space HS theory:
limit or simplified version of massless HS theory in AdS

• using S-matrix gauge invariance to constrain Lagrangian:
exist local quartic Lagrangians such that
0002 and 0004 amplitudes are gauge invariant
but $000s$ with $s > 4$ require non-local 4-vertices

• may be eliminated by tower of extra ghost-like HS fields

• requires additional 0000 vertex that cancels
exchange part of 0000 amplitude
same may apply for other amplitudes: full S-matrix trivial?

• tests required – e.g. gauge invariance of the $00s_1s_2$ amplitude

• analogy with conformal off-shell extension:
higher symmetry explaining triviality of S-matrix?