Superdense Matter

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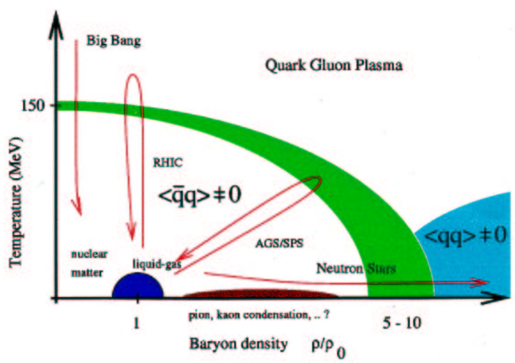


WHY HIGH BARYON DENSITY?

- IT'S PART OF THE PHASE DIAGRAM, STUPID!
- RELEVANT TO THE PHYSICS OF COMPACT OBJECTS.
- NON-PERTURBATIVE PHYSICS IN A PERTURBATIVE SETTING (MASS GAPS, χ SYM. BREAKING, HADR. BOUND STATES)

INTRODUCTION

- HIGH DENSITY QUARK MATTER IS A COLOR SUPERCONDUCTOR BECAUSE OF ATTRACTIVE INTERACTIONS IN THE COLOR $\bar{3}$ QQ CHANNEL

$$Q+Q \to Q+Q \quad \begin{cases} \bar{3} & \text{ATTR.} \\ 6 & \text{REP.} \end{cases}$$

$\rightsquigarrow \langle Q C \lambda_A \tau_A Q \rangle \neq 0$

- THE GROUND STATE OF $N_F=3$ QUARK MATTER IS PARTICULARLY SYMMETRIC. FOR $m_u = m_d = m_s$

$$\langle (Q_L)^a_i \, C \, (Q_L)^b_j \rangle = - \langle (Q_R)^a_i C (Q_R)^b_j \rangle$$
$$= \Phi \left(\delta^a_i \delta^b_j - \delta^a_j \delta^b_i \right)$$

"COLOR-FLAVOR-LOCKING"

SUPERFLUID PHASES

- ORDERPARAMETER

$$\Phi^{ab}_{ij} = \langle \psi^a_i C \gamma_5 \psi^b_j \rangle$$
$\leftarrow$ COLOR
$\leftarrow$ FLAVOR

[NOTE: MAY FIND ANISOTROPIC OR INHOMOGEN. PHASES]

- STUDY PHASESTRUCTURE IN PERT. THEORY

$\rightsquigarrow$ MINIMIZE $\Omega_{PERT}(\Phi^{ab}_{ij})$

- STRATEGY: CONSIDER Φ^{ab}_{ij} WHICH PRESERVE SUBGROUP OF $SU(N_f)_{L,R} \times SU(N_c)$

$\rightsquigarrow$ SYMMETRY CLASSES $\Phi^{ab}_{ij} = \Phi \left[M_{(1)} \right]^{ab}_{ij}, \ldots$

- IN MOST CASES: OPTIMUM Φ^{ab}_{ij} CHARACTERIZED BY

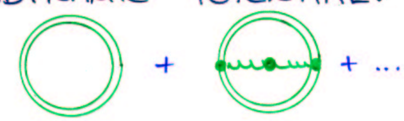
LARGEST RESIDUAL SYMMETRY GROUP

$V \subset SU(N_f)_L \times SU(N_f)_R$

CONSISTENT WITH MAXIMALLY GAPPED SPECTRUM

Grand Potential in Weak Coupling

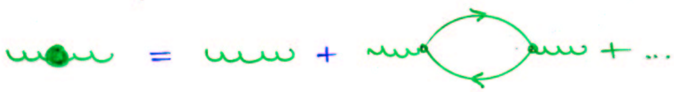
- Thermodynamic Potential: [Freedman, McLerran]

$\Omega =$ 

- Where $S =$ $= \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}$ "Normal" "Anomalous"

$$S^{-1} = S_0^{-1} + \Sigma \qquad \Sigma = \begin{pmatrix} 0 & \Delta \\ \Delta & 0 \end{pmatrix}$$ Gap

- Gluon Propagator

$D =$ 

$$D_E = \frac{1}{\vec{q}^{\,2} + 2m^2} \qquad m^2 = \frac{N_f}{4\pi^2} g^2 \mu^2$$ Debye Screening

$$D_M = \frac{1}{\vec{q}^{\,2} + i\frac{\pi}{2} m^2 \frac{\omega}{q}} \qquad \omega < |\vec{q}|$$ Landau Damping

- Variation of Ω ↳ Gap Equation

$\frac{\partial \Omega}{\partial \Phi} = 0$ ↳ 

- Get Grand Potential $\Omega = -P$

Degenaracy

$$\Omega = -\sum_i f_i \left(\frac{\mu^2}{4\pi^2}\right) \Delta_i^2 \log\left(\frac{\Delta_i}{\mu}\right)$$

Quasi-Particles $\qquad \sum_i f_i = 2 N_f N_c$

$$\Delta_i = c_i\, b_i\, \underbrace{(512\pi^4)\left(\frac{2}{N_f}\right)^{5/2} \mu g^{-5} \exp\left(-\frac{3\pi^2}{\sqrt{2}\, g}\right)}_{N_f=2 \text{ Result}}$$

Group Theory Factor

- Numbers: ($N_F = 2,3$)

$\Delta \simeq 100$ MeV

$T_c \simeq 50$ MeV

$\varepsilon \simeq -30$ MeV/fm^3

BEAUTIFUL CASE: $N_F=3$, $m_u=\ldots=m_s$

- FIND

$\phi^{ab}_{ij} = \Delta\left(\delta^a_i\delta^b_j - \delta^a_j\delta^b_i\right)$ COLOR-FLAVOR LOCKING [ARW '99]

$\langle ud\rangle = \langle us\rangle = \langle ds\rangle$
$\langle rb\rangle = \langle rg\rangle = \langle bg\rangle$ + COLOR & FLAVOR CORRELATED

[C.F. B-PHASE OF ^3He]

- SYMMETRIES

$SU(3)_L\times SU(3)_R\times [SU(3)]_C\times U(1)\longrightarrow SU(3)_{C+F}$

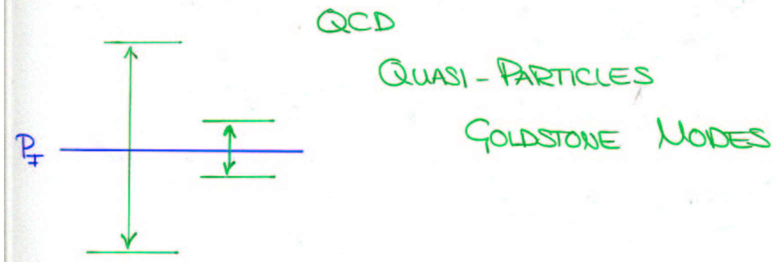
- NOVEL MECHANISM FOR χSB

$\langle\psi_L\psi_L\rangle$ BREAKS CHIRAL SYMMETRY BECAUSE

$$\underbrace{SU(3)_L \quad SU(3)_C}_{\text{LOCK}}\quad\underbrace{\quad SU(3)_R}_{\text{LOCK}}$$

- ALL QUARKS & GLUONS HAVE A GAP

EFFECTIVE THEORIES



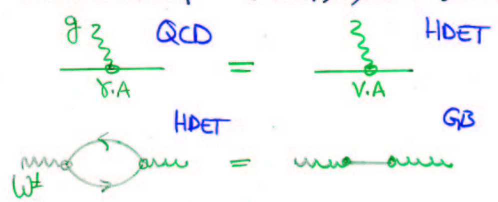
$$\mathcal{L}_{QCD} = \bar\psi(i\not{D}-\not{\mu})\psi - \frac{1}{4}(G^a_{\mu\nu})^2$$

$p\to p_F$

$$\mathcal{L}_{HDET} = \psi^\dagger(v.D)\psi + \Delta\psi\psi + \ldots$$

$$\mathcal{L}_{GB} = \frac{f_\pi^2}{4}\mathrm{Tr}(\nabla_0\Sigma\nabla_0\Sigma^\dagger) + \ldots$$

- MATCHING (HONG,...) (SON, STEPHANOV,...)



GOAL

- DETERMINE GROUNDSTATE IN THE PRESENCE OF QUARK MASSES AND EXTERNAL FIELDS

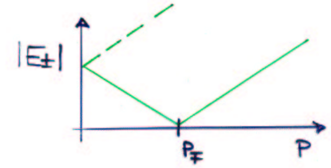
$$M = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix} \qquad \mu_e Q = \mu_e \begin{pmatrix} 2/3 & & \\ & -1/3 & \\ & & -1/3 \end{pmatrix}$$

- DETERMINE SPECTRUM OF EXCITATIONS, INTERACTIONS, COUPLING TO EXT. FIELDS, ...

HIGH DENSITY EFFECTIVE THEORY

- QUASI PARTICLES

$$E_\pm = -\mu \pm \sqrt{p^2+m^2} \simeq -\mu \pm |\vec{p}|$$



- E_- (ANTI-PARTICLE) MODES DECOUPLE $\hookrightarrow$ DEFINE

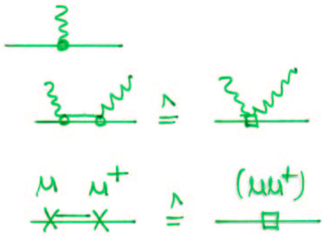
$$\psi(x) = e^{i\mu v\cdot x}\left[\psi_+ + \psi_-\right] \quad \hookrightarrow \psi_\pm = e^{-i\mu v\cdot x}\left(\frac{1 \pm \vec{\alpha}\cdot\vec{v}}{2}\right)\psi$$

- INTEGRATE OUT ψ_- (ORDER BY ORDER IN $\frac{1}{\mu}$ AND g)

$$\mathcal{L} = \psi_L^\dagger (iv\cdot D)\psi_L$$

$$+ \frac{1}{2p_F}\psi_L^\dagger\left[(\vec{\alpha}_\perp\cdot\vec{D})^2 + MM^\dagger\right]\psi_L$$

$$+ (L \leftrightarrow R) + \ldots$$

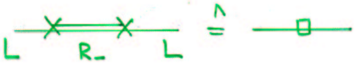


- BCS PHENOMENON LEADS TO IR-DIVERGENCE

$$\mathcal{L} \rightarrow \mathcal{L} + \underbrace{\left[\psi_L C\Delta\psi_L - (L\leftrightarrow R)\right]}_{\text{"FREE PART"}} - \underbrace{\left[\psi_L C\Delta\psi_L - (\ )\right]}_{\text{"INTERACTION"}}$$

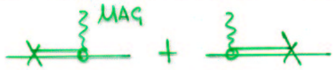
MASS CORRECTIONS

- FIRST TERM



$$\hat{=} \quad \frac{1}{2p_F}\psi_L^\dagger M M^\dagger \psi_L$$

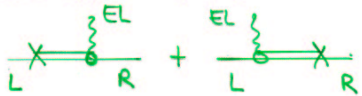
- MASS CORRECTIONS TO $\psi^\dagger v\cdot A\psi$?



$$= 0$$

$$\{\gamma_0, \vec\gamma\} = 0$$

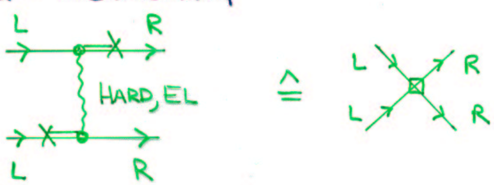
- ELECTRIC PART



$$\sim (1 - \vec v\cdot\vec v\,')$$

VANISHES IN FORWARD DIR.

- QQ - SCATTERING



$$g^2\left(\frac{M^2}{p_F^2}\right)\frac{1-\vec v\cdot\vec v\,'}{2p_F^2(1-\vec v\cdot\vec v\,')} \sim g^2\left(\frac{M^2}{p_F^4}\right)$$

CASALBUONI & GATTO '99

EFFECTIVE CHIRAL THEORY

- L/R CONDENSATES

$$X_i^a = \epsilon_{ijk}\epsilon^{abc}\langle(\psi_L)_j^b(\psi_L)_k^c\rangle^*$$

$$Y_i^a = \epsilon_{ijk}\epsilon^{abc}\langle(\psi_R)_j^b(\psi_R)_k^c\rangle^*$$

- COLLECTIVE EXCITATIONS

$$\Sigma = XY^\dagger = \mathrm{Exp}\left(i\frac{\phi^a\lambda^a}{f_\pi}\right) \qquad (\pi, K, \eta)$$

NOTE: $\Sigma \sim (LR^\dagger)$ VS. $\Sigma \sim (LL)^*(RR)$

- EFFECTIVE THEORY (CFLχTH)

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}\left(\nabla_0\Sigma\nabla_0\Sigma^\dagger - v_\pi^2\nabla_i\Sigma\nabla_i\Sigma^\dagger\right)$$

$$+ A\,\mathrm{Tr}(M\Sigma^\dagger)\,Ve^{i\theta} + \ldots \qquad \sim U_A(1) \text{ ANOMALY}$$

$$+ B_1\left[\mathrm{Tr}(M\Sigma^\dagger)\right]^2 + \ldots$$

$$\mathcal{L} \sim f_\pi^2\Delta^2\left(\frac{\vec\partial}{\Delta}\right)^{N_1}\left(\frac{\partial_0 + \frac{MM^\dagger}{p_F}}{\Delta}\right)^{N_2}\left(\frac{M^2}{p_F^2}\right)^{N_3}(\Sigma)^{N_4}(\Sigma^{-1})^{N_5}$$

NOTE: $\Lambda_{\chi SB} \simeq \Delta \ll 4\pi f_\pi$

Matching, Part I

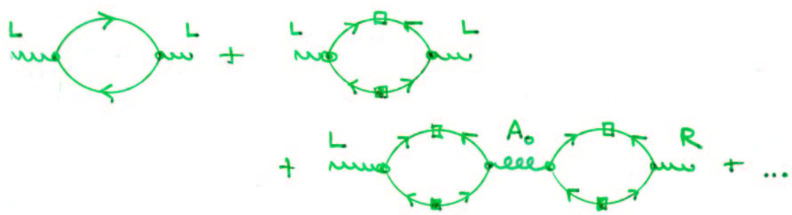
- Determine f_π : Gauge $SU(3)_L \times SU(3)_R$

$$\partial_\mu \Sigma \to \nabla_\mu \Sigma = \partial_\mu \Sigma - i W_\mu^L \Sigma + i \Sigma W_\mu^R$$

- Higgs Phenomenon

$$\mathcal{L} = \frac{f_\pi^2}{4} \mathrm{Tr}\left((W_0^L - W_0^R)^2\right) + \ldots \quad \leadsto \quad m_W^2 = f_\pi^2$$

- Microscopic Theory [Son, Stephanov]



$$\leadsto \quad f_\pi^2 = \frac{21 - 8\log 2}{18}\left(\frac{p_F^2}{2\pi^2}\right)$$

- Note: $f_\pi \sim p_F \gg \Delta$

Also: $p_F = 500$ MeV $\leadsto f_\pi \simeq 100$ MeV

Matching, Part II

- Vacuum Energy, from $\mathcal{L}_{eff}$

$$\Delta\mathcal{E} = -B_1 \left[\mathrm{Tr}(M)\right]^2 - B_2 \mathrm{Tr}(M^2) \qquad (\Sigma = 1)$$

- Microscopic Theory

$$\Delta\mathcal{E} \sim \left(\frac{g^2}{p_F^4}\right)\left[p_F^2 \Delta \log(\Delta)\right]^2 \left\{(\mathrm{Tr}(M))^2 - \mathrm{Tr}(M^2)\right\}$$

$$\sim \Delta^2 \left\{(\mathrm{Tr}(M))^2 - \mathrm{Tr}(M^2)\right\}$$

$$\leadsto \quad B_1 = -B_2 = \frac{3\Delta^2}{4\pi^2}$$

[Son, Stephanov, '00; T.S. '01]

- Meson Masses

$$m_{\pi^\pm}^2 = \frac{3\Delta^2}{4 f_\pi^2}(m_u + m_d) m_s$$

$$m_{K^+}^2 = \frac{3\Delta^2}{4 f_\pi^2}(m_u + m_s) m_d$$

- Note: $m_{GB} \sim 10$ MeV , $m_K < m_\pi$

P. Bedaque, T. Schaefer, '01

Matching, Part III

- Consider $1/p_F$ expansion

$$\mathcal{L} = \psi_L^\dagger\left(p_0 - \epsilon_p - \frac{MM^\dagger}{2p_F}\right)\psi_L + \frac{\Delta}{2}\psi_L C \psi_L$$
$$+ \left(L \leftrightarrow R,\ M \leftrightarrow M^\dagger\right) + O(1/p_F^2)$$

- $MM^\dagger$, $M^\dagger M$ enter as gauge fields

$$W_L = \frac{MM^\dagger}{2p_F} \qquad \psi_L \to L\psi_L \qquad \psi_R \to R\psi_R$$
$$W_R = \frac{M^\dagger M}{2p_F} \qquad W_L \to L W_L L^\dagger + i L \partial_0 L^\dagger$$

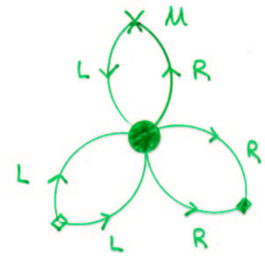
- Implement gauge sym. in eff. Lagrangian

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}\left(\nabla_0\Sigma \nabla_0\Sigma^\dagger\right) + \dots$$

$$\nabla_0\Sigma = \partial_0\Sigma + i\frac{MM^\dagger}{2p_F}\Sigma - i\Sigma\frac{M^\dagger M}{2p_F}$$

Matching, Anomalous Part

- $\mathrm{Tr}(M\Sigma)$ term related to instantons



Instanton size

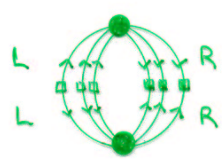
$$\rho \sim (p_F)^{-1} \ll \Lambda_{QCD}^{-1}$$

- Compute shift in vac. energy $\Delta\mathcal{E} = -A\,\mathrm{Tr}(M)$

$$A = C\cdot\phi^2\cdot\left(\frac{8\pi^2}{g^2}\right)^6\left(\frac{\Lambda_{QCD}}{p_F}\right)^{12}\cdot\Lambda_{QCD}^{-3}\,e^{i\theta}$$

$$\phi = \frac{3\sqrt{2}\pi}{g}\Delta\left(\frac{p_F^2}{2\pi^2}\right) \qquad \nabla_0$$

- Note: η'-mass is a 2-instanton effect



COLLECT EVERYTHING

- COLLECT $O(m)$, $O(m^2)$ AND LEADING $O(\mu^4)$

$$m_{\pi^\pm} = \mp \frac{m_d^2 - m_u^2}{2p_F} + \left[\frac{2A}{f_\pi^2}(m_u+m_d) + \frac{4B}{f_\pi^2}(m_u+m_d)m_s \right]^{1/2}$$

$$m_{K^0,\bar{K}^0} = \mp \frac{m_s^2 - m_d^2}{2p_F} + \left[\frac{2A}{f_\pi^2}(m_d+m_s) + \frac{4B}{f_\pi^2}(m_d+m_s)m_u \right]^{1/2}$$

NEGATIVE FOR K^0 $(S=+1)$

POSITIVE FOR $\bar{K}^0$ $(S=-1)$

KAON CONDENSATION

- WITHOUT INSTANTONS, $m_{K^0}=0$ FOR

$$m_s > m_s\big|_{CRIT} \simeq 3\cdot m_u^{1/3}\Delta^{2/3}$$

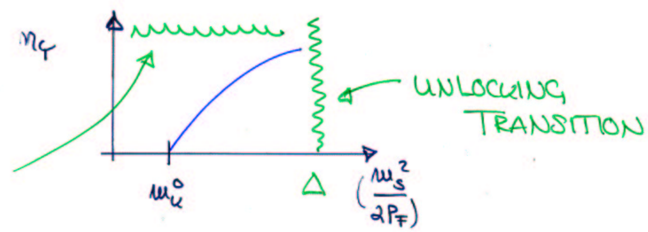
→ ONSET OF K^0-CONDENSATION

→ NEW VACUUM $\Sigma \simeq \exp(i\alpha\lambda_4)$

$$V(\alpha) = -f_\pi^2 \left\{ \frac{m_s^2}{4p_F}\sin^2(\alpha) + (m_{K}^0)^2(\cos(\alpha)-1) \right\}$$

- HYPERCHARGE DENSITY

$$n_Y = f_\pi^2 \mu_{EFF} \left\{ 1 - \frac{(m_K^0)^4}{\mu_{EFF}^4} \right\} \qquad \mu_{EFF} = \frac{m_s^2}{2p_F}$$



$$n_Y\big|_{MAX} \simeq f_\pi^2 \mu\big|_{CRIT} \simeq \left(\frac{p_F^2}{2\pi^2}\right)\cdot\Delta$$

Strange Goings on in High Density Quark Matter

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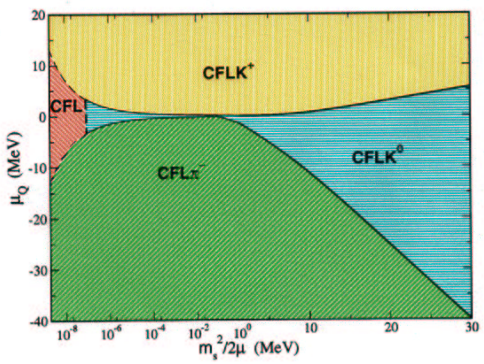

D. KAPLAN & S. REDDY, 2001

FINAL REMARKS

- EFFECTIVE THEORIES PROVIDE SYSTEMATIC TOOL FOR COMPUTATIONS IN CFL & OTHER HIGH DENSITY PHASES
- RESPONSE OF CFL TO LARGE M_S: "CFL − K CONDENSATION"
- "CLASSIC" [A.K.A KAPLAN-NELSON] VS. "CFL" K-CONDENSATION
  STRANGENESS DEFICIENCY
  VS. STRANGENESS OVERSATURATION
- NEUTRON STARS: ν-OPACITIES /EMISSIVITIES
  AMUSING FACT: $\pi^0 \to \nu\bar{\nu}$ IMPORTANT
- $N_c=2$ $(\mu\to\infty)$ INTERESTING TOY MODEL IN ORDER TO STUDY $\mu=0$ $(N_c\to\infty)$