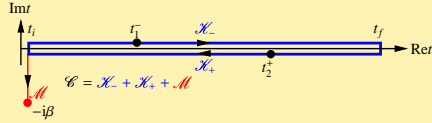


Fundamentals

A generating functional with **local** and **bilocal** sources is defined [1,2,3]:

$$Z[J, K] = N \exp D\phi \exp \left[iS[\phi] + i \int J_1 \phi_1 + \frac{1}{2} \int [K_{12} \phi_1 \phi_2]_{12} x \right],$$

where ϕ is a scalar field. $\{f_{12..}\}_{12..}$ denotes the integration over space-time arguments 1, 2, ... The time argument is running along the **Schwinger-Keldysh real-time contour**. Through the **KMS-condition** the Green's functions of the imaginary-time formalism, defined along the **vertical part of the contour**, can be obtained by analytic continuation of the corresponding real-time quantities.



The **two-particle irreducible (2PI) effective action functional** is then defined as

$$W[J, K] = -i \ln Z[J, K], \quad \varphi_1 = \frac{\delta W[J, K]}{\delta J_1}, \quad \frac{1}{2} (\varphi_1 \varphi_2 + iG_{12}) = \frac{\delta W[J, K]}{\delta K_{12}},$$

$$\Gamma[\varphi, G] = W[J, K] - \int J_1 \varphi_1 - \frac{1}{2} \int [K_{12} (\varphi_1 \varphi_2 + iG_{12})]_{12}$$

A formal saddle-point expansion of the path integral yields

$$\Gamma[\varphi, G] = S_0[\varphi] + \frac{1}{2} \text{Tr} \ln(-iG^{-1}) + \frac{i}{2} \text{Tr} [D_{12}^{-1} (G_{12} - D_{12})]_{12}$$

$$+ \Phi[\varphi, G] \equiv \text{all closed 2PI interaction diagrams with } D_{12} = (-\square - m^2)^{-1}$$

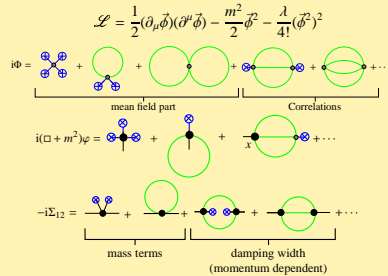
The vanishing of the auxiliary sources yields a **closed set of self-consistent equations of motion** for the **mean field** and the **Green's function**:

$$\frac{\delta \Gamma[\varphi, G]}{\delta \varphi} = \frac{\delta \Gamma[\varphi, G]}{\delta G} = 0,$$

which defines the self-energy in terms of the **self-consistent propagator** to be

$$\Sigma_{12} = 2i \frac{\delta \Phi[\varphi, G]}{\delta G_{12}}$$

Thus Φ is a generating functional for **skeleton self-energy diagrams**. As an example we take the $O(N)$ - ϕ^4 model



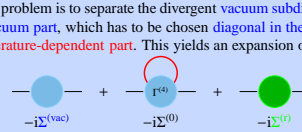
Truncating the Φ -functional at a certain diagram order yields self-consistent Dyson approximations which have the following properties

- expectation values of Noether currents are **exactly conserved**
- Γ is an approximation for the thermodynamic potential **consistent with the dynamics of the mean field and the Green's function**

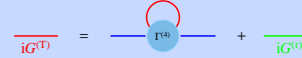
Renormalization

As in perturbation theory the self-consistent diagrams are **divergent**. The additional problem is that self-consistency yields additional divergences which are not seen directly by the structure of the diagrams, the so-called **hidden divergences**. Our aim is to prove that the self-consistent approximation schemes of the here considered kind can be renormalized with **temperature-independent counter terms**. For that purpose we expand the functional (and thus also the self-energy and the source-term) around the **solution for the same functional at $T = 0$** . The vacuum problem can be renormalized by using the **BPHZ-renormalization formalism as in perturbation theory** since the power counting of propagators and vertices remains the same as in perturbation theory.

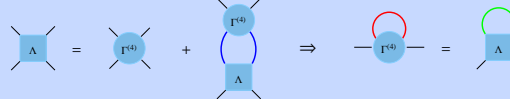
At **finite temperature** the problem is to separate the divergent **vacuum subdiagrams**. To this end we split the propagator in the **vacuum part**, which has to be chosen **diagonal in the real-time matrix formalism** and the **remaining temperature-dependent part**. This yields an expansion of the self-energy



Power counting shows that these parts are of momentum power 2, 0 and -2 respectively. Thus the loop from closing the vacuum four-point function $\Gamma^{(4)}$ with a temperature part of the Green's function in the second diagram is still **logarithmically divergent**. Thus we have to split the propagator further:



The first diagram contains the self-energy of momentum power 0 which is **linear in $G^{(T)}$** . Thus it fulfills a linear integral equation for a **pure vacuum four-point function subdiagram**



Here the kernel of this *s*-channel Bethe-Salpeter equation is defined by $\Gamma^{(4)}_{12,34} = \frac{\delta^4 \Phi}{\delta G_{12} \delta G_{34}}$. The **vacuum four-point function Λ** can be renormalized with the BPHZ-technique of renormalization theory. Here again the use of the **2PI-formalism is crucial**: The kernel $\Gamma^{(4)}$ is 2PI with respect to cuts separating the pairs (12) and (34) of its external points and thus no subdivergences overlap with parts inside $\Gamma^{(4)}$, which means in momentum space:

$$\Gamma^{(4)}(l, p) - \Gamma^{(4)}(l, 0) = O(l^{-\alpha}) \text{ with } \alpha > 0$$

Thus due to Weinberg's power counting theorem the renormalized version of the Beth-Salpeter equation reads

$$\Lambda(p, q) = \Lambda(0, 0) + \Gamma^{(4)}(p, q) - \Gamma^{(4)}(0, 0)$$

$$+ i \int \frac{d^4 l}{(2\pi)^4} [\Gamma^{(4)}(p, l) - \Gamma^{(4)}(0, l)] [G^{(\text{vac})}]^2(l) \Lambda(l, q)$$

$$+ i \int \frac{d^4 l}{(2\pi)^4} \Lambda(0, l) [G^{(\text{vac})}]^2(l) [\Gamma^{(4)}(l, q) - \Gamma^{(4)}(l, 0)]$$

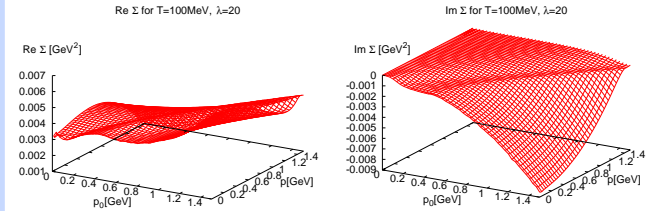
The constants $\Lambda(0, 0)$ and $\Gamma(0, 0)$ are fixed by the **renormalization condition**. Here we have chosen both constants to be $\lambda/2$.

The loop integral from closing Λ with G^* is finite since $G^*(l) = O(l^{-6})$. For details of the proof see [4]. The renormalization formalism can be implemented in numerical algorithms as we have shown in [5].

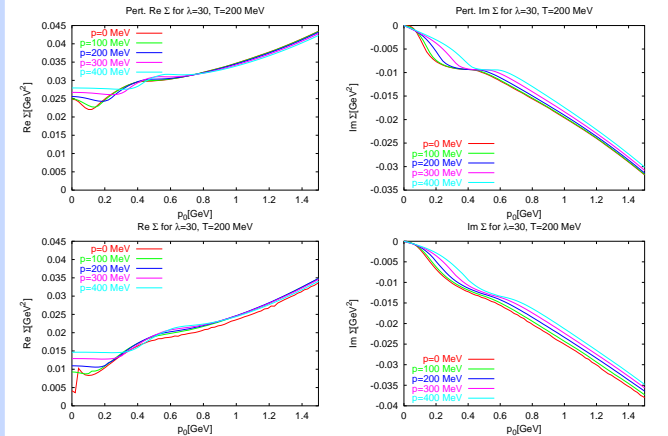
[1] T. D. Lee, C. N. Yang, Phys. Rev. **117** (1961) 22
 [2] G. Baym, Phys. Rev. **127** (1962) 1391
 [3] M. Cornwall, R. Jackiw, E. Tomboulis, Phys. Rev. **D10** (1974) 2428
 [4] H. van Hees, J. Knoll, Phys. Rev. **D65** (2002) 025010, hep-ph/0107200
 [5] H. van Hees, J. Knoll, Phys. Rev. D (2002), in press, hep-ph/0111193
 [6] H. van Hees, J. Knoll, Phys. Rev. D (2001), to be published, hep-ph/0203008

Numerical Results

We have evaluated the self-consistent self-energy for the simple ϕ^4 -model with full energy-momentum dependence at finite temperature. The vacuum self-energy and four-point functions were renormalized within an on-shell scheme such that the renormalized vacuum mass is $m = 140$ MeV and the running coupling constant is λ at zero energy and momentum. For the self-energy both the **tadpole** and the **sunset** diagrams were taken into account without further approximations.



The comparison between a perturbative calculation with the same diagrams and the self-consistent calculation looks as follows.



On one hand the finite spectral width in the self-consistent propagator leads to a **further broadening and a smoothing** of the self-energy. On the other hand the tadpole contribution tends to shift the mass upwards, so that there is less available phase space leading to less strength in the width. This effect is weakened through the fact that the tadpole diagram gives a smaller contribution if the effective mass becomes greater. At higher couplings and/or temperatures the contribution of the real part of the self-energy overrules the tadpole contribution leading to an effective downshift of the mass.

- self-consistent solutions resum to **any order of the expansion parameter for Φ**
- ⇒ violation of **crossing symmetry**
- ⇒ violation of **global and local gauge symmetry**
- investigation of symmetry properties and a scheme to **recover global symmetries**: [6]
- local gauge symmetries: Work in progress!