Transport Equations for the Quark-Gluon Plasma

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– transport theory

its use is justified when the concept of quasiparticle is well-defined

(~ it is enough long lived)

– transport equations can be derived from first principles ; they may provide a link between the microscopic physics (QCD) and the macroscopic one

in connection to RHIC :

* understanding the hydrodynamical behavior after the collision

  computation of transport coefficients such as conductivities, viscosities, etc transport theory can be used to study the behavior of the QGP at

$T \gg T_c$ $\qquad\qquad$ $g(T) \ll 1$

not clear whether the same methodology can be applied for $T \gtrsim T_c$ (relevant for RHIC)

for $T \gg T_c$, and close to equilibrium, transport theory has been shown to be a very efficient to describe the long distance physics of the QGP (to leading order! in g)

opening a door to numerical computations of dynamical quantities

QFT vs. transport theory

in QFT the transport coefficients are given in terms of Kubo relations, e.g.

shear viscosity

$$\eta = \frac{1}{20} \lim_{\omega \to 0} \frac{1}{\omega} \int d^4x \, e^{i\omega t} \langle [\underbrace{\pi_{kl}}_{\text{traceless part of the energy-momentum tensor}}(\vec{x},t), \pi_{kl}(0)] \rangle$$

bulk viscosity

$$\zeta = \frac{1}{2} \lim_{\omega \to 0} \frac{1}{\omega} \int d^4x \, e^{i\omega t} \langle [\tilde{\mathcal{P}}(\vec{x},t), \tilde{\mathcal{P}}(0)] \rangle$$

$$\tilde{\mathcal{P}} \equiv \mathcal{P} - \underset{\text{v. of sound}}{v_s^2} \, \varepsilon$$

their computation requires a complicated resummation of Feynman diagrams (ladder), a program that has only been completed for scalar theories!

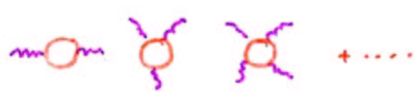
$\sim$ it is fully equivalent to a (linearized) Boltzmann equation at leading order in the coupling

Jeon, 96
Jeon and Yaffe, 96

gauge theories

Furthermore, one has to do the resummation of the hard thermal loops, which account for Debye screening and Landau damping

Pisarski
Braaten and Pisarski



- Transport coefficients in the QGP to leading order in g

$\eta \propto \frac{T^3}{g^4 \ln \frac{1}{g}}$ shear viscosity

$\sigma_{el} \propto \frac{T}{e^2 \ln \frac{1}{e}}$ electric conductivity

$\Rightarrow$ not known beyond leading log

$\sigma_{col} \propto \frac{m_D^2}{g^2 T \ln \frac{1}{g} + C}$ color conductivity

bulk viscosity ??

weak coupling regime. $T \gg T_c$

- up to now all the leading order results for the computed transport coefficients, as well as the hard thermal effective theory, can be computed from classical transport theory

  ~ quasiparticles propagating in the background of classical chromoelectromagnetic fields

  hard modes $\sim T$ quasiparticles

  soft modes $\sim gT$ classical fields

- these classical transport equations can be obtained from QCD, in the corresponding limit

  Heinz; Elze, Gyulassy and Vassak
  Mrowczynski
  Blaizot and Iancu,
  Bödeker, ...

  or one can alternatively start from pure classical considerations

  Heinz
  Kelly, Liu, Lucchesi, C.M.
  Litim and C.M.
  Laine and C.M.
  Jalilian-Marian, Jeon, Venugopalan and Wirstam

what can be said beyond leading order?

Classical Equations of Motion for Colored Particles

Q^a $\quad a = 1, \dots, N^2-1 \quad SU(N)$

$$\left.\begin{aligned} m \frac{dx^\mu}{d\tau} &= p^\mu \\ m \frac{dp^\mu}{d\tau} &= g\, Q^a F_a^{\mu\nu} p_\nu \\ m \frac{dQ^a}{d\tau} &= -g\, f^{abc} p^\mu A_\mu^b Q^c \end{aligned}\right\} \text{WONG EQUATIONS (Wong 70)}$$

$\Rightarrow$ the color charges are also phase-space variables

the color current

$$j_a^\mu(x) = g \int d\tau\, Q_a(\tau)\, \dot{x}^\mu(\tau)\, \delta^{(4)}(x - x_u(\tau))$$

is covariantly conserved

$$(D_\mu j^\mu)_a(x) = \partial_\mu j_a^\mu(x) + g\, f_{abc} A_\mu^b(x)\, j_c^\mu(x) = 0$$

Abelian case

$$Q_a \longrightarrow q \qquad (D_\mu F^{\mu\nu})_a = j_a^\nu$$
$$F_{\mu\nu}^a \longrightarrow F_{\mu\nu}$$

Classical Transport Theory for Colored Particles

(U. Heinz, 84)

$f(x^\mu, p^\mu, Q_a) \equiv$ classical probability distribution of finding a particle in the state (x^μ, p^μ, Q_a)

Physical constraints:

- on mass-shell evolution and positivity of the energy

$$dP = \frac{d^4p}{(2\pi)^3}\, 2\,\Theta(p_0)\, \delta(p^2 - m^2)$$

- conservation of group invariants

e.g.: SU(3)

$$dQ = d^8Q\, \delta(Q_a Q_a - c_2)\, \delta(d_{abc} Q^a Q^b Q^c - c_3)$$

Q^a: is a constrained variable

but one can also work with "unconstrained" variables (Darboux variables)

$f(x,p,Q_a)$ evolves in time via a TRANSPORT EQUATION

$$\frac{df}{d\tau} = C[f]$$

in a collisionless case $C = 0$

$$p^\mu \left(\frac{\partial}{\partial x^\mu} - g\, Q_a F^a_{\mu\nu} \frac{\partial}{\partial p^\nu} - g f_{abc} A^b_\mu Q^c \frac{\partial}{\partial Q^a} \right) f(x,p,Q_a) = 0$$

$$(D_\mu F^{\mu\nu})_a (x) = J^\nu_a(x)$$

$$J^\nu_a(x) = \sum_{\text{species}} \sum_{\text{helicities}} j^\nu_a(x)$$

$$j^\nu_a(x) = g \int dP\, dQ\, p^\nu Q_a f(x,p,Q_a)$$

self-consistent set of
NON-ABELIAN VLASOV-BOLTZMANN equations

$$(D_\mu j^\mu)_a (x) = 0$$

the equation can be solved perturbatively $g \ll 1$
as an expansion in $g Q^a$ (collisionless)
(M. Laine, CM, 01)

$$f = f^{(0)} + f^{(1)} + f^{(2)} + \dots$$

$$\overbrace{p^\mu \left(\frac{\partial}{\partial x^\mu} - g f^{abc} A^b_\mu \frac{\partial}{\partial Q^a} \right)}^{\hat{D}} f^{(n+1)} = \overbrace{g\, Q_a F^a_{\mu\nu} p^\mu \frac{\partial}{\partial p_\nu}}^{\hat{L}} f^{(n)}$$

$$f^{(n)} = \left(\frac{1}{p \cdot \hat{D}} \hat{L} \right)^{(n+1)} f^{(0)}$$

if $f^{(0)} = f^{\text{equilibr.}} = \dfrac{1}{e^{p_0/T} \pm 1}$

and one solves the above equation for $n = 0$
one reproduces the hard thermal loop effective theory (Kelly, Liu, Lucchesi, C.M., 94)

$$j^\mu_a(x) = g \int dP\, dQ\, p^\mu Q_a f^{(1)}(x,p,Q)$$

$$j^\mu_a(x) = \frac{\delta \Gamma_{HTL}}{\delta A^a_\mu(x)}$$

but EXACT solutions can also be found

$$\frac{d}{d\tau} f = 0$$

Look for constants of motion under the Wong dynamics

i) homogeneous limit $\quad \partial_i A_0^a = 0 \quad i = 1,2,3$

$$f_\pm(x,p,Q) = \frac{1}{e^{(|\vec{p} \pm g Q^a \vec{A}^a| \pm \mu)/T} + 1}$$

for quarks / antiquarks

(similar solutions for gluons)

ii) statit limit $\quad \partial_0 A_\mu^a = 0$

$$f_\pm(x,p,Q) = \frac{1}{e^{[(p_0 \pm g Q^a A_0^a) \pm \mu]/T} + 1}$$

Exact classical effective action in the static case from fermionic degrees of freedom

$$A_\mu \equiv A_\mu^a Q^a$$

$$\Gamma = N_f \int dQ \left\{ \left(\frac{7\pi^2}{180} T^4 + \frac{\mu^2 T^2}{6} + \frac{\mu^4}{12\pi^2} \right) - g \frac{\mu}{3} \left(T^2 + \frac{\mu^2}{\pi^2} \right) A_0 + \frac{g^2}{2} \left(\frac{T^2}{3} + \frac{\mu^2}{\pi^2} \right) A_0^2 - \mu \frac{g^3}{3\pi^2} A_0^3 + \frac{g^4}{12\pi^2} A_0^4 \right\}$$

– Bödeker, Laine (dynamical case)

this would agree with the one-loop fermionic contribution to the static effective potential in QFT if

$$\int dQ\, Q_{a_1} Q_{a_2} \cdots Q_{a_n} \stackrel{?}{=} \mathrm{Tr}\left[T_{a_1} T_{a_2} \cdots T_{a_n} \right]_{sym}$$

it works for $n \leq 3$

it fails for $n = 4$ unless the particles are in high colour dimensional representations

$\sim$ the colour can be treated classically

how should one treat the low colour dimensional representations ?

- computation is ok for U(1), (QED)

Result for a general representation of SU(2)

$$\int dQ\, Q^a Q^b Q^c Q^d = L(R)\,(\delta^{ab}\delta^{cd} + \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc})$$

$$(1) \Rightarrow \int dQ\,(A_0^a Q^a)^4 = \underbrace{\frac{1}{5}\, d_R\, C_2^2(R)}\,(A_0^a A_0^a)^2$$

$$\int dQ = d_R$$

$$(2) \Rightarrow \mathrm{Tr}\,(A_0^4) = \underbrace{\frac{1}{5}\, d_R\, C_2(R)\left(C_2(R) - \frac{1}{3}\right)}\,(A_0^a A_0^a)^2$$

$d_R = 2j+1$ dimension of the representation

$C_2(R) = j(j+1)$ quadratic Casimir

For high j (1) is a good approximation of (2)

Alternatives ?

$$W(x,p) = \omega(x,p)\,\mathbb{1} + \frac{T^a}{2}\,\omega^a(x,p)$$

$$\left[p\cdot D, W(x,p)\right] + g\, p^\mu \left\{ F_{\mu\nu}, \frac{\partial}{\partial p^\nu} W(x,p)\right\} = 0$$

$\Rightarrow$ it does not even reproduce the correct operators !!

Going beyond classical transport theory

treat color quantum mechanically

$$f(x,p,Q) \Longrightarrow W(x,p)$$

matrix in color space

for matter in the fundamental representation

$$W(x,p) = \bar{\omega}(x,p)\,\mathbb{1} + \frac{T^a}{2}\,\omega^a(x,p)$$

$$\left[p\cdot D, W(x,p)\right] + g\, p^\mu \left\{ F_{\mu\nu}, \frac{\partial}{\partial p_\nu} W(x,p)\right\} = 0$$

$$j_a^\mu(x) = g \int \frac{d^3p}{(2\pi)^3}\,\frac{p^\mu}{E}\,\mathrm{Tr}\left(T_a W(x,p)\right)$$

e.g. for SU(2)

$$\begin{cases} p^\mu \partial_\mu \bar{\omega}(x,p) + \dfrac{g}{2}\, p^\mu F_{\mu\nu}^a(x)\, \dfrac{\partial}{\partial p_\nu}\,\omega^a(x,p) = 0 \\ p^\mu D_\mu^{ab}\,\omega^b(x,p) + g\, p^\mu F_{\mu\nu}^a(x)\,\dfrac{\partial}{\partial p_\nu}\,\bar{\omega}(x,p) = 0 \end{cases}$$

$\Rightarrow$ solving the equations to order g^3 in the static case : numerical mismatch with QFT and non-local pieces arise !!

* what are the transport equations beyond leading order ??

* computation of the bulk viscosity

* computation of different transport coefficients beyond leading $\log\left(\frac{1}{g}\right)$ (and beyond g)