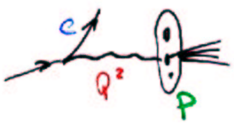
Perturbative Saturation: Systems Large & Small

Why is small x physics interesting?

DIS: photon "counts" partons



$x = \frac{Q^2}{Q^2 + W^2}$ ← W^2 energy of $\gamma^* p$ system

The cross section is proportional to the number of partons

$$\sigma_{DIS} \simeq \frac{\alpha_{em}}{Q^2} \sum_i e_i^2 N^i_{parton}$$

Lower x → boost the proton

Partonic density grows

How does the gluon cloud evolve to low x?

BFKL evolution for $\varphi(\vec{k}, b)$ - density of gluons with momentum $\vec{k}$ at impact parameter b.

$$\frac{d}{dt}\varphi(k,b) = \alpha_s N_c \int dk' db' \, K(k,k',b,b') \, \varphi(k',b')$$

$t = \ln \frac{1}{x}$

At large t:

$$\varphi(k,b) \sim \exp\left\{\omega t - \frac{\ln^2(b^2 k k_0)}{a^2 t}\right\}$$

$$\omega = 4\ln 2 \, \frac{\alpha_s}{\pi} N_c \; ; \quad a^2 = 14 \, \alpha_s N_c$$

a) Density at small b grows exponentially

$b < b_{diff} \simeq \exp\sqrt{\alpha_s t}$: $\varphi(b) \sim e^{\omega t}$

Violates unitarity : $\sigma = \int d^2 b \, \varphi(b) \sim e^{\omega t}$

$\Big[$ b) Spreads in impact parameter

$b = \exp\{\varepsilon \alpha_s t\}$ ⇒ $\varphi(b) \sim e^{(\omega - \varepsilon^2 \alpha_s)t}$

In BFKL this effect is subleading $\Big]$

BFKL : scattering probability $P > 1$.

The basic problem : double counting

$P \leq 1 \neq N$

scattering probability — number of partons

In a dense system the scattering probability is NOT proportional to the number of partons. It has to saturate at 1.

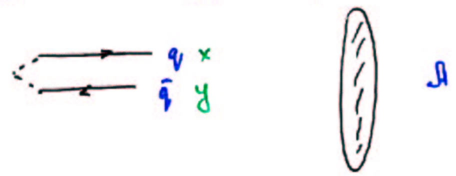
color glass condensate
or
cold gluon cloud

- Balitsky (94) + Kovchegov (99) $\Rightarrow$
  BK equation ("fan" diagramms)
- JKLW (97) ("fan" + "pomeron loops")
  BBK equation (beyond BK)

BK equation

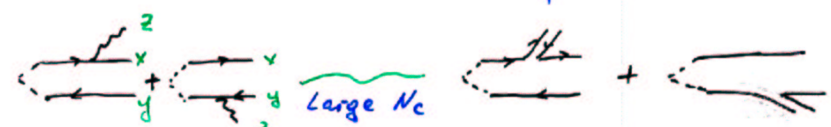
Target "rest" frame

Projectile - $\bar{q}q$ dipole



Scatters with probability $N(x,y)$

Lower x - boost the dipole



$\alpha_s t \frac{(x-z)_i}{(x-z)^2}$ $\qquad$ $\alpha_s t \frac{(y-z)_i}{(y-z)^2}$

$\Rightarrow$

$\alpha_s t \frac{(x-y)^2}{(x-z)^2 (y-z)^2}$

Scattering probability :

$$N(xz, yz) = N(xz) + N(yz) - N(xz)\,N(yz)$$

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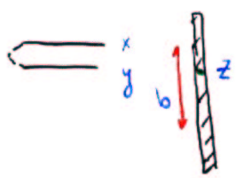
Finally:

$$\frac{dN(xy)}{dt} = \frac{\alpha_s N_c}{2\pi}\int d^2z\, \frac{(x-y)^2}{(x-z)^2(y-z)^2}\Big[N(xz)+N(yz)-N(xz)N(yz)-\underbrace{N(xy)}_{\text{virtual correction}}\Big]$$

$N\to 1$, RHS $\to 0$: $N(xy)\le 1$ - saturates

Numerical studies (translational inv. $x-y \ll \frac{x+y}{2}$, $x-z \ll \frac{x+z}{2}$)



Qualitatively: $N(xy) = 1-\exp\{-(x-y)^2 Q_s^2(t)\}$

$(x-y)^2 < Q_s^{-2}$ $\qquad N = (x-y)^2 Q_s^2(t)$ $\leftarrow \alpha_s N_c \frac{dG}{db^2}$

$(x-y)^2 > Q_s^{-2}$ $\qquad N\to 1$

$$Q_s^2 \sim \exp\left\{\frac{4\alpha_s N_c}{\pi} * t\right\}$$

At fixed $\vec b$ the scattering probability saturates

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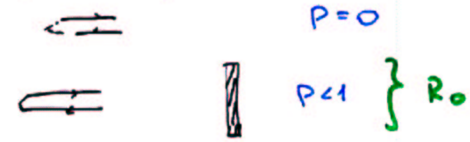
Does this answer all the questions?

No, it does not.

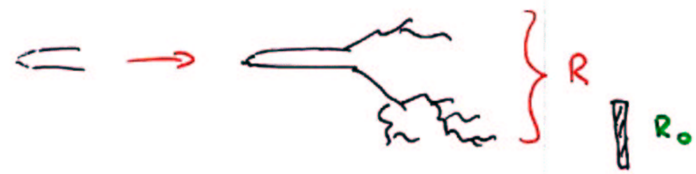
$$\sigma_{tot} = 2\int d^2b\, P(b)$$

Initially $(t=t_0)$



$\sigma < 2\pi R_0^2$

After evolution to t



P(R) does not vanish $\qquad \sigma_{tot}\sim 2\pi R^2(t)$

To satisfy Froissart bound $R(t)\sim t$

How fast does R grow?

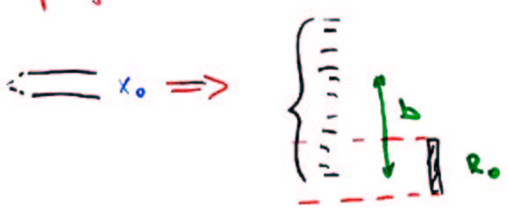
BK : evolution of the projectile wave function is linear

$n(\bar{x}, \bar{b} \mid \bar{x}_0)$ evolves according to BFKL

density of dipoles of size $\bar{x}$ at a distance $\bar{b}$ from the original dipole of size $\bar{x}_0$.

$$n(x, b, x_0, t) \propto \frac{1}{x^2} \exp\left\{ \omega t - \ln \frac{16 b^2}{x x_0} - \frac{\ln^2 \frac{16 b^2}{x x_0}}{a^2 t} \right\}$$

The projectile swells :



The target is characterized by $Q_s(t_0)$

Scattering probability is unity if there is at least one dipole of size $x = Q_s^{-1}(t_0)$ in the overlap area.

The maximal impact parameter

$$b^2_{max} = \frac{1}{16} x_0 Q_s^{-1}(t_0) \exp\left\{ \frac{\alpha_s N_c}{\pi} \varepsilon t \right\}$$

$$\varepsilon = 7 \zeta(3) \left[-1 + \sqrt{1 + \frac{8 \ln 2}{7 \zeta(3)}} \right]$$

Or $\frac{\alpha_s N_c}{\pi} \frac{\varepsilon}{\omega} \simeq .87$ (Just smaller than BFKL)

[numerical value of ε is not reliable, but parametrically OK]

So :

As long as $R(t) < R_0$ σ_{tot} is geometric

$$\sigma = \pi R_0^2 + \underbrace{2\pi R_0 x_0 \exp\left\{ \frac{\alpha_s N_c}{2\pi} \varepsilon (t - t_0) \right\}}_{\text{"surface" correction}}$$

As $R(t) \sim x_0 \exp\{\alpha_s \varepsilon t\} = R_0$ the growth is nonunitary

BK is limited to $t < \frac{1}{\ldots} \ln \frac{R_0}{\ldots}$

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Is this such a bad news?

Beyond PT - confinement cuts off the long range Coulomb fields

Even within PT: BK equation is not valid for calculating σ_{TOT}.

BK breaks down when the density in the projectile wave function is large

$$n \sim e^{\omega t} \Rightarrow P_{\text{interaction}} \simeq \alpha_s k^2 n = 1 \quad \text{at}$$

$$t \propto \frac{1}{\alpha_s} \ln \frac{1}{\alpha_s}$$

Wave function saturation effects ≡ Pomeron loops become important.

The equation that sums them has not been analysed yet.

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What do Pomeron loops do?

They must slow down the growth of the density and also of σ_{TOT}.

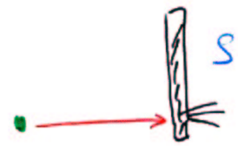
Can they unitarize σ_{TOT}? -

very unlikely.

PT ⇒ massless gluons ⇒ Coulomb fields

Heisenberg argument: exponential tails → $\sigma \sim t^2$

power tails ⇒ $\sigma \sim e^{\lambda t}$



$\rho(b)$ - density

$E(b) = s\rho(b) > m_\pi$

If $\rho(b) \sim e^{-mb} \Rightarrow b_{max} = \frac{1}{m} \ln \frac{s}{m^2}$

If $\rho(b) \propto b^{-\lambda} \Rightarrow b_{max} \propto s^{1/\lambda}$

with perturbative massless gluons σ_{TOT} is most likely nonunitary.

Is this the Soft Pomeron ?

Two distinct perturbative mechanisms for exponential growth of σ.

1. Growth of density in dilute "small" systems

$$\sigma \sim s^{.4} \qquad (\text{BFKL} \stackrel{?}{=} \text{hard pomeron})$$

2. Growth of transverse size - in saturated "black" systems

$$\sigma \sim s^{\lambda} \qquad \lambda = ? \quad \text{but} \quad \lambda < .4$$

Are these the "Two Pomerons" ?

Would be nice: growth due to PT (very natural with Coulomb fields) - nonperturbative physics needed only to ultimately unitarize in asymptotia.

Can the soft Pomeron be perturbative ?

Completely crazy ?

Constituent quarks $r \sim .3\,\text{fm}$ - small

What if they are both small and "black"?

pp :

$$\sigma_{pp} \sim 3 \times 3 \times 2\pi r^2 \simeq 50\ \text{mb} \qquad (\text{just about right})$$

$$\frac{\sigma_{\pi p}}{\sigma_{pp}} \simeq \frac{2}{3} \qquad \text{that's OK}$$

$$\frac{\sigma_{elastic}}{\sigma_{tot}} \simeq \frac{1}{5} \ll \underset{\text{black disk}}{\frac{1}{2}} \qquad \text{consistent}$$

So maybe...

If yes, the soft Pomeron intercept is calculable from the perturbative evolution equation BBK.