Pions far from equilibrium

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J. B. , Nucl. Phys. **A699** (2002) 833

G. Aarts, J. B. , Phys. Rev. Lett. **88** (2002) 041603

G. Aarts, D. Ahrensmeier, R. Baier, J. B. , J. Serreau, hep-ph/0201308

Quantum fields far from equilibrium

Not close to thermal equilibrium in the weak coupling limit (effective descriptions based on separation of scales, gradient expansion, (non)linear response, ...)

1/N can provide a small nonperturbative expansion parameter

- even in extreme nonequilibrium situations!
- also capable of describing critical phenomena!

Leading order (LO): Long history (LO or "Hartree")

⇒ No late-time thermalization; valid at early times?

Cooper et al; Boyanovsky et al; ...

Beyond LO:

- $1/N$ expansion of the $1PI$ effective action

⇒ No late-time thermalization ("secular" in time)

Cooper, Dawson, Mihaila '97, '00; Bettencourt, Wetterich '98; Athan, Cooper, Dawson, Habib '00; Ryzhov, Yaffe '00

- $1/N$ expansion of the $2PI$ effective action (= two-particle irreducible generating functional for Green's functions)

⇒ Late-time thermalization of quantum fields from first principles (time-reversal invariant dynamics)

⇒ Quantitative description of extreme nonequilibrium quantum field dynamics J.B. '01 (NLO)

Test of $2PI - 1/N$ expansion in classical field theory

$2PI - 1/N$ expansion

Quantum field theory with classical action $S[\varphi]$ for real, scalar fields $\varphi_a(x)$, $a = 1, \ldots, N$ with $\lambda(\varphi_a\varphi_a)^2/(4!N)$ interaction.

Chiral $O(4) \sim SU_L(N_f = 2) \times SU_R(N_f = 2)$

$\Rightarrow$ "Linear sigma model" for light scalar and pseudo-scalar pion degrees of freedom $(\sigma, \vec{\pi})$

$2PI$ generating functional: Luttinger, Ward '60; Baym '62; Cornwall, Jackiw, Tomboulis '74

$$\Gamma[\phi, G] = S[\phi] + \frac{i}{2}\mathrm{Tr}\ln G^{-1} + \frac{i}{2}\mathrm{Tr}\, G_0^{-1}(\phi)\, G + \Gamma_2[\phi, G]$$

- parametrized by macroscopic field: $\phi_a(x) = \langle\varphi_a(x)\rangle$
- & composite field (exact connected propagator): $G_{ab}(x,y) = \langle\varphi_a(x)\varphi_b(y)\rangle - \langle\varphi_a(x)\rangle\langle\varphi_b(y)\rangle$

$\Gamma_2[\phi, G]$: contains $2PI$ diagrams; classical propagator: $G_0(\phi)$

$2PI - 1/N$ expansion: NLO: J. B. '01; Aarts, Ahrensmeier, Baier, J. B. , Serreau '02

$$\Gamma[\phi, G] \;=\; \underset{\sim N}{\Gamma^{\mathrm{LO}}[\phi, G]} \;+\; \underset{\sim 1}{\Gamma^{\mathrm{NLO}}[\phi, G]} \;+\; \underset{\sim 1/N}{\Gamma^{\mathrm{NNLO}}[\phi, G]} \;+\; \ldots$$

Equations of motion: (no further approximations at this level)

Variational $\Rightarrow$ time-reflection invariant, energy conserving

$$(1) \quad \frac{\delta\Gamma[\phi, G]}{\delta\phi_a(x)} = 0 \quad , \quad (2) \quad \frac{\delta\Gamma[\phi, G]}{\delta G_{ab}(x,y)} = 0\,.$$

Statistical and spectral components:

$$G(x,y) \;=\; G_>(x,y)\Theta_{\mathcal{C}}(x^0 - y^0) + G_>^*(x,y)\Theta_{\mathcal{C}}(y^0 - x^0)\,,$$

- symmetric propagator (anticommutator)

$$F(x,y) \;=\; \frac{1}{2}\Big(G_>(x,y) + G_>^*(x,y)\Big) \equiv \mathrm{Re}[G_>(x,y)]$$
$$\;=\; \frac{1}{2}\langle[\varphi(x), \varphi(y)]_+\rangle$$

- spectral function (commutator)

$$\rho(x,y) \;=\; i\Big(G_>(x,y) - G_>^*(x,y)\Big) \equiv -2\mathrm{Im}[G_>(x,y)]$$
$$\;=\; i\langle[\varphi(x), \varphi(y)]_-\rangle$$

Real-valued functions with symmetry properties

$$F(x,y) = F(y,x) \quad , \qquad \rho(x,y) = -\rho(y,x)\,.$$

F and ρ are lin. independent functions out of equilibrium!

Similar decomposition for nonlocal part of Σ: Σ_F, Σ_ρ.

Note: In **thermal equilibrium** (not employed here)

$$F^{(\mathrm{eq})}(\omega, \mathbf{p}) \;=\; -i\Big(n_{\mathrm{B}}(\omega) + \frac{1}{2}\Big)\,\rho^{(\mathrm{eq})}(\omega, \mathbf{p})$$

with $n_{\mathrm{B}}(\omega) = (e^{\beta\omega} - 1)^{-1}$, fluctuation-dissipation th.

Evolution equations ($\phi = 0$):

$$(\Box_x + M^2(x;F))F(x,y) = -\int dz \left\{ \int_0^{x^0} dz^0\, \Sigma_\rho(x,z;F,\rho)\, F(z,y) - \int_0^{y^0} dz^0\, \Sigma_F(x,z;F,\rho)\, \rho(z,y) \right\}$$

$$(\Box_x + M^2(x;F))\rho(x,y) = -\int dz \int_{y^0}^{x^0} dz^0\, \Sigma_\rho(x,z;F,\rho)\, \rho(z,y)$$

⇒ Causal, time-reversal invariant equations with "**memory**" integrals. (Exact equations for known self-energies.)

NLO: J. B. , hep-ph/0105311 (NPA)

$$M^2(x;F) = m^2 + \lambda \frac{N+2}{6N} F(x,x)$$

$$\Sigma_F(x,y) = -\frac{\lambda}{3N}\Big(F(x,y)I_F(x,y) - \frac{1}{4}\rho(x,y)I_\rho(x,y)\Big)$$

$$\Sigma_\rho(x,y) = -\frac{\lambda}{3N}\Big(F(x,y)I_\rho(x,y) + \rho(x,y)I_F(x,y)\Big)$$

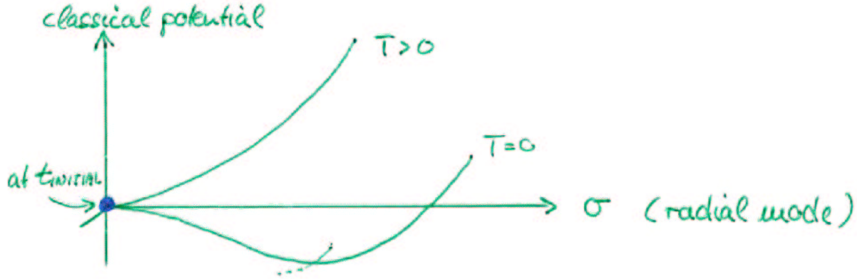
$\mathcal{O}(I_F) = \mathcal{O}(I_\rho) = \mathcal{O}(1)$: RHS $\equiv 0$ at LO ($N \to \infty$)

⇒ **No memory terms at LO** (additional (!) conserved quantity in this limit and F decouples from ρ).

$\phi \neq 0$: Aarts, Ahrensmeier, Baier, J. B. , Serreau, hep-ph/0201308

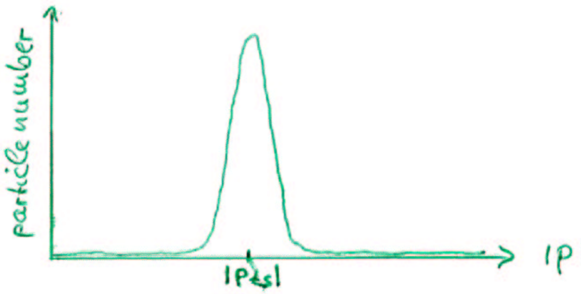
Initial condition scenarios

1. "Quench": Initially at high temperature, the system is rapidly "cooled" on time scales $\tau_{\rm cool} \ll \tau_{\rm relax}$ (sudden drop in effective mass)



Note: no phase transition in $1+1$ dimensions.

2. "Tsunami": Densely populated modes in a narrow momentum range around $p_{\rm ts}$ and $-p_{\rm ts}$ (reminiscent of two colliding wave packets moving with opposite and equal momentum).



Far from equilibrium initial conditions!

⟶ follow relaxation process

LO (or 'Hartree') fixed points

LO equations: $M^2(x;F) = \left(m^2 + \lambda \frac{N(+2)}{6N}\, F(x,x)\right)$

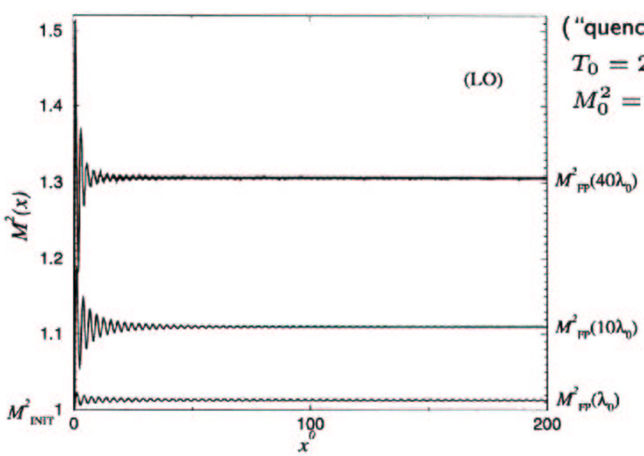
$$\left(\Box_x + M^2(x;F)\right) F(x,y) = 0$$
$$\left(\Box_x + M^2(x;F)\right) \rho(x,y) = 0$$

- "collisionless approximation"
- **additional conserved quantity:** particle number

$$n_0(p) + \frac{1}{2} = \left(F(t,t';p)\,\partial_t\partial_{t'}F(t,t';p) - (\partial_t F(t,t';p))^2\right)^{\frac{1}{2}}\Big|_{t=t'}$$

Cooper, Habib, Kluger, Mottola '97; Aarts, Bonini, Wetterich '00

⇒ Nonthermal, initial $n_0(p)$ determines late-time behavior:



("quench": $T_0 = 2M_{\rm INIT}$, $M_0^2 = 2M^2_{\rm INIT}$)

$M^2_{\rm FP} = \{1.01, 1.11, 1.31\} M^2_{\rm INIT} \simeq M^2_{n_0}$ for $\lambda/\lambda_0 = \{1, 10, 40\}$,

$\lambda_0 = 0.5 M^2_{\rm INIT}$

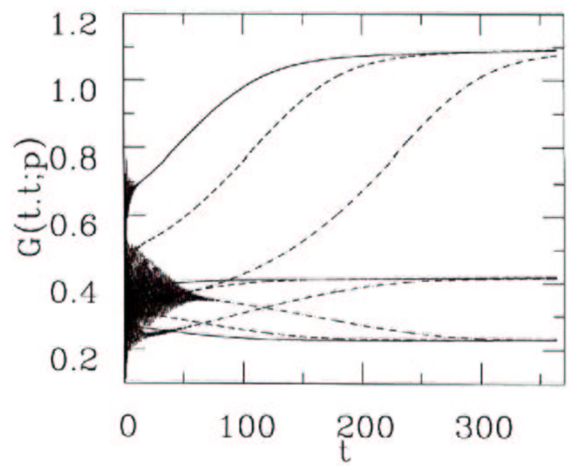
$$M^2_{n_0} = m^2 + \lambda\frac{N(+2)}{6N}\int\frac{dp}{2\pi}\left(n_0(p)+\frac{1}{2}\right)\frac{1}{\sqrt{p^2+M^2_{n_0}}}$$

$\{1.01, 1.10, 1.30\} M^2$

Beyond LO: Overview — thermal fixed point

Very different nonequilibrium initial conditions:

⇒ universal asymptotic late-time behavior determined by initial energy density

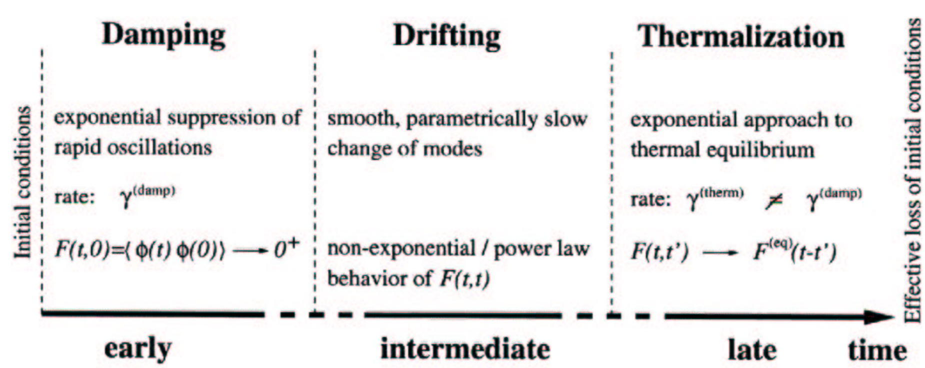


Propagator for different initial conditions with *same* $\langle E\rangle$

($p = 0, 3, 5$; all in initial mass units)

J. B., Cox '00; J. B. '01

Here: Three characteristic regimes:

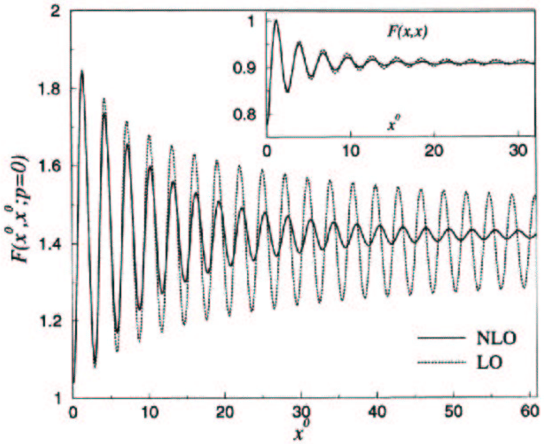


→ Time scales/Role of scattering/Nonequilibrium phenomena

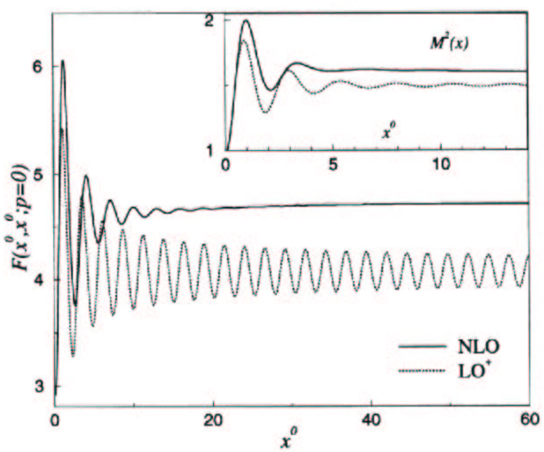
Early-time behavior

Equal-time correlations: (same "quench" as before)

Weak effective coupling ($\lambda/6N = 0.083\, M^2_{\rm INIT}$ for $N = 10$):

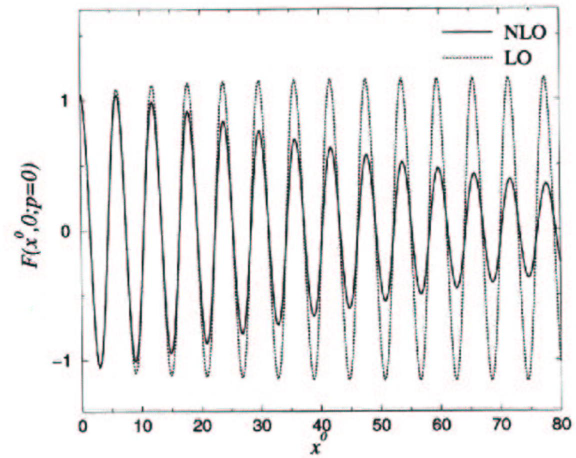


Stronger coupling ($\lambda/6N = 0.17\, M^2_{\rm INIT}$ for $N = 4$):



$\Rightarrow$ Better LO/NLO agreement for mode averages: $M(x)$, $F(x,x)$

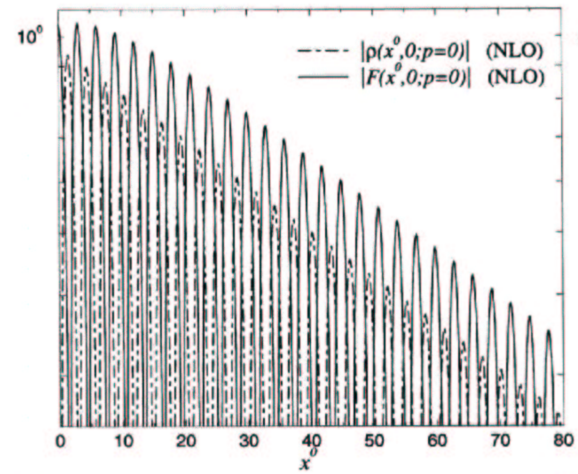
Unequal-time correlations: ($\lambda/6N = 0.083 M^2_{\rm INIT}, N = 10$)



- At NLO correlations with initial time are suppressed

$\Rightarrow$ Scattering essential for effective loss of details of initial conditions (prerequisite for thermalization)

- Exponential damping: **early-time scale** $1/\gamma^{(\rm damp)}$

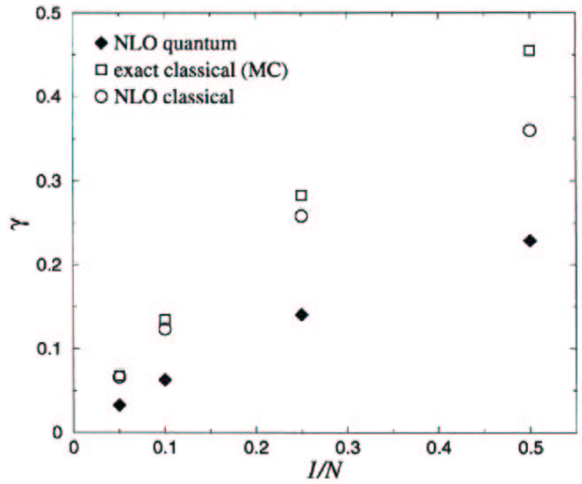


$\sim e^{-\gamma^{(\rm damp)} t}$

$\gamma^{(\rm damp)} \simeq 0.016 M_{\rm INIT}$

Time-reversal invariance: no 'damping out', $F(x^0, 0; p) \to 0^+$

Parametric behavior: (strong coupling $\lambda/M^2_{\rm INIT} = 30$)



- Damping time $\tau^{(\rm damp)} \sim 1/\gamma$ scales $\sim N$ for large N
  $\Rightarrow$ Cannot be captured by LO approximation
- Damping enhanced if quantum corrections are neglected (classical theory)

Classical statistical field theory: "exact result" test

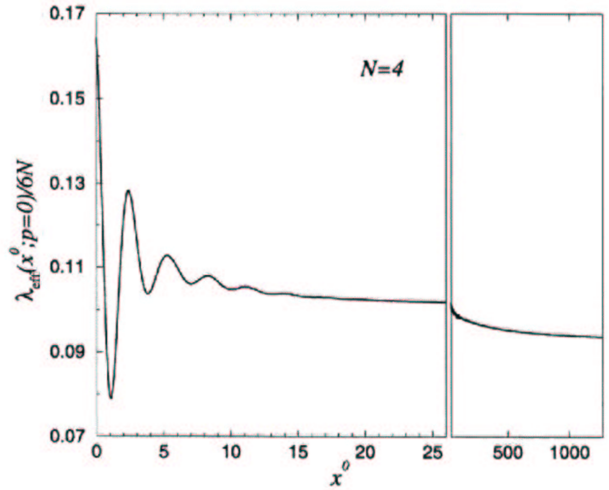
1. Sample initial conditions from initial probability distribution
2. Solve numerically the *known* classical field equation:
   $\left[\Box_x + m^2 + \lambda\phi^2(x)/6N\right]\phi_a(x) = 0, \quad \phi^2 \equiv \sum_{a=1}^N \phi_a\phi_a$
3. Construct ensemble averages (here: 5-8 $\times 10^4$ members)

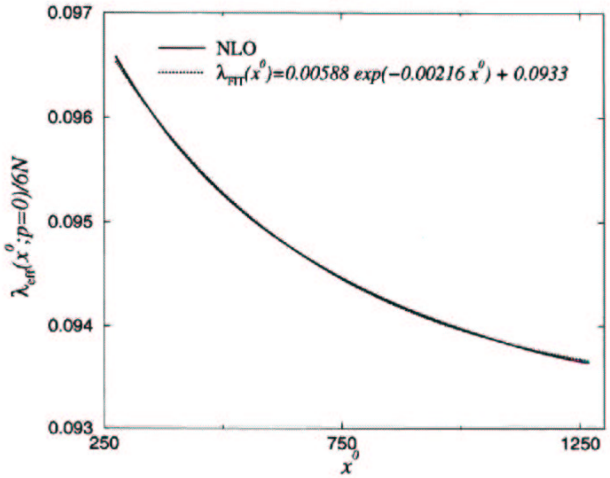
- Includes all orders in $1/N$ (!)
- Compare with NLO $2PI$ result for the *classical* field theory:

$\Rightarrow$ Convergence of classical NLO and "exact" (MC) results already for moderate values of N (!)

Late-time behavior

Effective four-point function: ($\lambda/6N = 0.17M^2_{\rm INIT}$, $N = 4$)



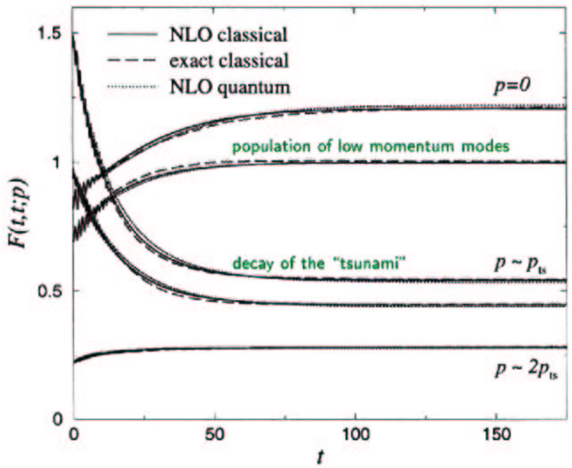


$\Longrightarrow$ **exponential** with $\gamma^{(\rm damp)}/\gamma^{(\rm therm)} \sim \mathcal{O}(10)$

Different time scales characterizing early- and late-time exponential behavior ("fast damping/slow thermalization").

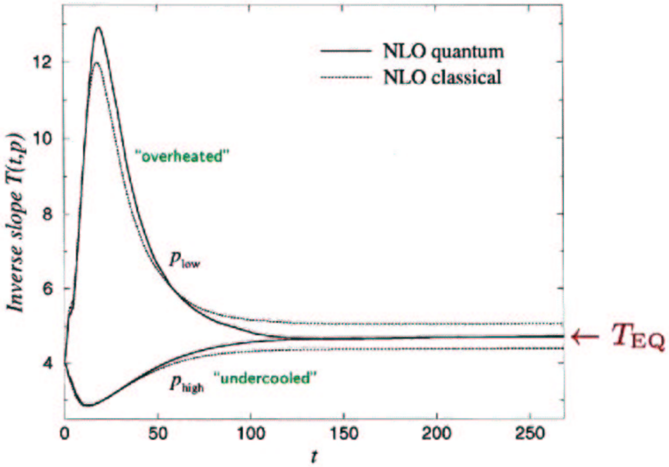
High particle number density $n(t;p)$ peaked initially around $p = |p_{\rm ts}| \simeq 4M_{\rm INIT}$, thermal "background" $(T_0 = 4M_{\rm INIT})$

"tsunami":



$n(t;p) > 0.35$ for $p \lesssim 2p_{\rm ts}$

Quantum vs classical thermalization:



Inverse slope: $T(t,p) \equiv -n(t,\epsilon_p)[n(t,\epsilon_p)+1](dn/d\epsilon_p)^{-1}$

$\Rightarrow$ constant for $n = 1/[e^{\epsilon/T_{\rm EQ}} - 1]$ (Bose-Einstein)

(Effective particle number: $n(t,\epsilon_p) + \frac{1}{2} \equiv [F(t,t';p)\,\partial_t\partial_{t'}F(t,t';p)]^{1/2}|_{t=t'}$

Conclusions

- $2PI$–$1/N$ expansion: powerful quantitative tool for nonequilibrium dynamics from first principles
  $\longrightarrow$ includes scattering, memory and off-shell effects
- Valid far from equilibrium as well as near critical points
- $O(N)$–/"linear σ–" model (spatially homogeneous, symmetric): three time regimes,
  (1) early-time exponential damping of oscillations,
  (2) intermediate-time drifting (nonexponential/ power law)
  (3) late-time exponential thermalization
- (1) – (3) **not** described by leading order ("collisionless") approximation (LO valid only for $t \ll \tau^{(\rm damp)}$, scattering crucial to remove spurious fixed points)
- "Quench" & "tsunami": $\tau^{(\rm damp)} \ll \tau^{(\rm therm)}$
  $\longrightarrow$ very efficient damping, "slow" thermalization

In progress: 3+1 dimensions

$\longrightarrow$ QFT study of formation of DCCs — with J. Serreau

$\longrightarrow$ Critical phenomena — with M. Alford

$\longrightarrow$ Early Universe (p)reheating — with D. Ahrensmeier, R. Baier, J. Serreau

Most pressing:

Extension to gauge theories (Ward identities!)