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DYNAMICS NEAR THE CRITICAL POINT: THE HOT RENORMALIZATION GROUP

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AT CRITICALITY : $T = T_c$

EXCITATIONS ARE GAPLESS (MASSLESS)

$$\omega_p \sim p^z \sim \Gamma_p = \text{WIDTH} \quad V_{group} = \frac{d\omega_p}{dp} \sim p^{z-1}$$

$$\lim_{p\to 0} \omega_p = \lim_{p\to 0} \Gamma_p = 0$$

$$t_{relax} \sim \frac{1}{\Gamma_p} \to \infty \quad \text{for } p \to 0$$

LONG WAVELENGTH EXCITATION RELAX VERY SLOWLY AT $T = T_c$. $\boxed{\Gamma_p \ll \omega_p}$

$\boxed{\text{CRITICAL SLOWING DOWN}}$

SUPPOSE TEMPERATURE DECREASES WITH TIME : $T(t)$

$$\text{COOLING TIME} \sim \frac{T(t)}{|\dot{T}(t)|}$$

IF $t_{cool} \gg t_{relax}$: the phase transition happens at equilibrium

IF $t_{cool} \ll t_{relax}$ $\begin{cases} \text{FLUCTUATIONS FREEZE OUT} \\ \text{P.T. OCCURS VERY FAST AND} \end{cases}$

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WE ARE INTERESTED ON THE REAL TIME EVOLUTION IN 3 + 1 DIMENSIONS OF THE $\lambda\,\phi^4$ MODEL. (and $\lambda\,(\vec{\phi}^{\,\prime})^2$) FOR HIGH TEMPERATURES AT AND NEAR T_c.

[T_c : 2nd order phase transition]

$|\vec{p}|, E \ll T \quad , \quad |\vec{x}|, t \gg \frac{1}{T}$

STATIC CRITICAL PHENOMENA: EFFICIENTLY TREATED BY RG IN 4-ϵ DIMENSIONS WILSON FIXED POINT $O(\epsilon)$. ϵ-expansion SET $\epsilon = 1$ AT THE END. D=3 space dimensions

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DYNAMICAL EVOLUTION:

$T \to \infty$ DIMENSIONAL REDUCTION MAY HAPPEN (EXPECTED)

1) WE WORK IN 5-ϵ EUCLIDEAN SPACE DIMENSIONS

2) WE DO OUR RG ANALYSIS (FOR $T\to\infty$) WE FIND A FIX POINT $O(\epsilon)$. ϵ-expansion etc.

3) WE SET $\epsilon = 1$ AND WORK ...

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SCALAR THEORY IN 5-ϵ SPACE DIM.

MATSUBARA FREQ. + EUCLIDEAN 4-ϵ MOMENTA

τ = imaginary time

$$\Phi(\vec{x},\tau) = \frac{1}{\sqrt{T}} \sum_{n\in\mathbb{Z}} \int \frac{d^{4-\epsilon}p}{(2\pi)^3}\, \phi(\vec{p},\omega_n)\, e^{i(\vec{p}\cdot\vec{x}-\omega_n\tau)}$$

$0 < \tau < 1/T \quad , \quad \vec{x} \in \mathbb{R}^{4-\epsilon} \qquad \omega_n = 2\pi n T$

["LORENTZ" $O(5-\epsilon)$] INVARIANCE IS BROKEN.

$O(4-\epsilon)$ ROTATIONAL INVARIANCE SURVIVES.

$$\mathcal{L}_E = \frac{Z}{2v_0^2}(\partial_\tau \Phi)^2 + \frac{Z}{2}(\vec{\nabla}\Phi)^2 + \frac{1}{2}\left[M(T)^2 + \delta m^2\right]\Phi^2 + \frac{\lambda_R + \delta\lambda}{4!}\Phi^4 .$$

v_0 = velocity of light , Z = wave function renormalization

[we may set $v_0=1$ but it gets renormalized]

$\Gamma^{(2)}(\vec{p}=0, \omega_n=0) = M^2(T) \Rightarrow$ CRITICAL POINT AT $M^2(T)=0$

AT CRITICALITY:

$$\Delta = \frac{1}{\vec{q}^{\,2} + S_n^2/v^2} . \qquad S_n = 2\pi n T ,\ n \in \mathbb{Z} \qquad \vec{q} \in \mathbb{R}^{4-\epsilon}$$

free propagator

... $T \int d^{4-\epsilon}q$ ...

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PERTURBATIVE CALCULATION (Two Loops)

3+1 DIMENSIONS HIGH TEMPERATURE

AT CRITICALITY

euclidean

$$\Gamma^{(2)}(p,s) = p^2+s^2 - \Sigma(p,s) = p^2+s^2+\frac{1}{12}\left(\frac{\lambda T}{4\pi}\right)^2\left[\ln\frac{p^2+s^2}{\mu^2} - \frac{cs}{p}\log\frac{cs-p}{cs+p}\right],$$

Wick rotation $s = -i\omega + 0$, $\omega \in R$

($M(T)^2 = 0$, thermal mass cancelled by c.t.)

$$\mathrm{Re}\,\Gamma^{(2)}(p,\omega) = p^2-\omega^2+\frac{1}{12}\left(\frac{\lambda T}{4\pi}\right)^2\left[\left(1-\frac{\omega}{p}\right)\ln\left|\frac{\omega-p}{\mu}\right| + \left(1+\frac{\omega}{p}\right)\ln\left|\frac{\omega+p}{\mu}\right|\right]$$

quasiparticles: $\mathrm{Re}\,\Gamma^{(2)}(p,\omega_p)=0 \Rightarrow \boxed{\omega_p = p}$

width $\Gamma_p = -\frac{\mathrm{Im}\,\Gamma^{(2)}(p,\omega_p)}{2p} = \frac{1}{12\pi p}\left(\frac{\lambda T}{4}\right)^2 = \Gamma_p$

Γ_p grows unbounded for $p \to 0$

perturbative result not valid in the IR limit.

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Renormalized euclidean Lagrangian

$$\mathcal{L} = \frac{1}{2}\left[\frac{1}{v_B^2}\left(\frac{\partial\Phi_R}{\partial\tau}\right)^2 + (\vec\nabla\Phi_R)^2 + m_R^2\,\Phi_R^2\right] + \frac{g}{4!}\frac{\mu^\epsilon}{T}\Phi_R^4 ,$$

$\mu =$ normalization mass scale $\ll T$

$$\boxed{g \equiv \lambda\,\frac{T\mu^{-\epsilon}}{(4\pi)^{2-\epsilon/2}}}$$ effective coupling

$\Phi_R =$ renormalized field

$\Phi_R = \sqrt{Z_\phi}\,\Phi_B$

Renormalization conditions:

$\Gamma_R^{(2)}(p=0, s=0) = 0$ CRITICALITY CONDITION

$$\left.\frac{\partial\Gamma_R^{(2)}}{\partial p^2}\right|_{p=\mu,\ s=\mu v_R} = 1 , \quad \left.\frac{\partial\Gamma^{(2)}}{\partial s^2}\right|_{p=\mu,\ s=\mu v_R} = \frac{1}{v_R^2}$$

(no Lorentz invariance)

$\Gamma^{(4)}(p_i = S.P.(\mu), s_i = 0) = -\lambda_R$,

$v_R =$ renormalized speed of light

EXPLICIT CALCULATION:

$$\Gamma^{(4)}(\vec p_1, s_1, \vec p_2 s_2, \vec p_3 s_3, \vec p_4 s_4) = -\lambda Z_1 \mu^{\epsilon-1} +$$

$$+\lambda^2\left[H(\vec p_1+\vec p_2, s_1+s_2) + H(\vec p_1+\vec p_3, s_1+s_3) + \ldots\right.$$

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$$s = 2\pi m T$$

$$H(p,s) = \frac{T}{2} \sum_{m \in \mathbb{Z}} \int \frac{d^{4-\epsilon}q}{(2\pi)^{4-\epsilon}} \frac{\mu^{2\epsilon-2}}{\left[q^2 + \left(\frac{2\pi T m}{v^2}\right)^2\right]\left[(\vec{q}+\vec{p})^2 + \frac{(2\pi T)^2 (n+m)^2}{v^2}\right]}$$

For high temperatures: $T \gg p, s$

$$H(p,s) = H_{asi}(p,s) + \text{corrections}$$

MAtsubara zero mode

$$H_{asi}(p,s) = \frac{T}{2} \int \frac{d^{4-\epsilon}q}{(2\pi)^{4-\epsilon}} \frac{\mu^{2\epsilon-2}}{q^2\left[(\vec{q}+\vec{p})^2 + \frac{s^2}{v^2}\right]} +$$

$$+ \frac{\mu^{2\epsilon-2}}{8\epsilon\, \pi^{2+\epsilon/2}} \Gamma\left(1+\frac{\epsilon}{2}\right) \zeta(\epsilon)\, T^{1-\epsilon}.$$

HIGH TEMPERATURE LIMIT OF THE SUM OF NON-ZERO MATSUBARA MODES

$$\text{corrections} = -\frac{\mu^{2\epsilon-2}\, \Gamma\left(1+\frac{\epsilon}{2}\right) \zeta(3+\epsilon)}{192\, \pi^{4+\epsilon/2}\, T^{1+\epsilon}} \left[p^2 + \frac{s^2}{v^2}(1-\epsilon)\right]$$

$$\left[1 + O\left(\frac{p^2, s^2}{T^2}\right)\right]$$

$$H_{asi}(p,s) =$$

$$-\frac{T}{2}\mu^{-\epsilon} \frac{\Gamma\left(\frac{\epsilon}{2}-1\right)}{(4\pi)^{2-\epsilon/2}} \left(\frac{s^2/v^2 + p^2}{\mu^2}\right)^{-\frac{\epsilon}{2}} F\left(\frac{\epsilon}{2}, 1-\frac{\epsilon}{2}; 2-\frac{\epsilon}{2}; \frac{p^2}{p^2+s^2/v^2}\right) +$$

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$H(p,s)$ is ANALYTIC AT $\epsilon = 0$. (POLES $\frac{1}{\epsilon}$ CANCEL)

$\epsilon = 0$ MEANS $d = 5$: one-loop diagrams are finite for odd dimensions.

FOR $T \to \infty$ FIRST TERM DOMINATES; 'DIMENSIONAL REDUCTION' PROVIDED $\epsilon > 0$

For $T \gg p, s$ $\quad 0 < \epsilon \ll 1$

WE FIND

$$\lambda H(p,s) = \frac{1}{2} g(\mu) \left[\frac{2}{\epsilon} - \left(1 + \frac{s^2}{v^2 p^2}\right) \log \frac{s^2 + p^2 v^2}{\mu^2 v^2}\right.$$

$$\left. + \frac{s^2}{v^2 p^2} \log \frac{s^2}{v^2 \mu^2} + \ln 4\pi + 2 - \gamma + O(\epsilon)\right]$$

TWO POINTS FUNCTION

$$\Gamma^{(2)}(p,s) = p^2 + \frac{s^2}{v^2} - \lambda^2 \Sigma(p,s)$$

MATSUBARA ZERO MODE

$$\Sigma(p,s) \underset{T \gg p,s}{=} \frac{T^2}{6} \mu^{2\epsilon-2} \int \frac{d^{4-\epsilon}q}{(2\pi)^{4-\epsilon}} \frac{d^{4-\epsilon}k}{(2\pi)^{4-\epsilon}} \frac{1}{q^2 k^2 \left[(\vec{p}+\vec{k}+\vec{q})^2 + \frac{s^2}{v^2}\right]}$$

$$+ T^{2-2\epsilon} \mu^{2\epsilon-2} \Gamma(\epsilon) \left[s^2 \ldots \right. \qquad A(\epsilon) = \frac{2}{\epsilon} + (\ldots$$

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WE FIND FOR $T \gg p, s$, $1 \gg \epsilon > 0$

$$\Gamma^{(2)}(p,s) = p^2 + \frac{s^2}{v^2} - \frac{g^2(\mu)}{3\epsilon}\,\frac{s^2}{v^2}\log\left(\frac{s^2}{v^2\mu^2}\right) +$$

$$+ O(g^2\epsilon^0, g^3)$$

(notice the lack of Lorentz invariance)

RENORMALIZATION GROUP EQUATIONS IN THE CRITICAL THEORY

(bare functions are μ-independent)

NEW PIECE

$$\left[\mu\frac{\partial}{\partial\mu} + \beta_g\frac{\partial}{\partial g} + \beta_v\frac{\partial}{\partial v} - \frac{N}{2}\gamma\right]\Gamma^{(N)}_{(p_i,\frac{s_i}{v},\, g,\mu,T)} = 0$$

$$\beta_g = \mu\frac{\partial g}{\partial\mu}\Big|_{\lambda_0, T, v_0} \quad , \quad \beta_v = \mu\frac{\partial v}{\partial\mu}\Big|_{\lambda_0, T, v_0}$$

$$\gamma = \mu\frac{\partial}{\partial\mu}\log Z_\phi\Big|_{\lambda_0, T, v_0} \qquad v_0^2 = \frac{Z_\phi}{Z_v}\,v^2$$

TO LOWEST ORDER:

$$\beta_g = -\epsilon g + 3g^2 + O(g^3, \epsilon g^2) \quad \text{(AS USUAL}_{\text{STATIC}}^{\text{IN } 4-\epsilon)}$$

$$\beta_v = v\left[\frac{g^2}{3\epsilon} - \frac{\gamma}{2}\right] + O(g^3, g^2\epsilon)$$

F.P. $g^* = \frac{\epsilon}{3}$, $v^* = 0$

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RGE SOLUTION:

$$\Gamma^{(N)}(\rho p_i, \rho\frac{s_i}{v}, T, g, \mu) = \rho^{4-N+(1-\epsilon)(1-\frac{N}{2})}$$

$$e^{-\frac{N}{2}\int_1^\rho \frac{dx}{x}\gamma(g(x), v(x), \frac{T}{\mu x})}\;\Gamma^{(N)}\left(p_i, \frac{s_i}{v(\rho)}, \frac{T}{\rho}, g(\rho), \mu\right)$$

$g(\rho)$ and $v(\rho)$ RUN as:

$$\rho\frac{dg}{d\rho} = \beta_g\left(g(\rho), \frac{T}{\mu\rho}, v(\rho)\right) \; ; \; \rho\frac{dv}{d\rho} = \beta_v\left(g(\rho), \frac{T}{\mu\rho}, v(\rho)\right)$$

$$g(1) = g \qquad\qquad v(1) = v$$

$$\lim_{\rho\to 0} g(\rho) = \frac{\epsilon}{3} + O(\epsilon^2) \quad \text{(wilson's fixed point)}$$

$$\lim_{\rho\to 0}\gamma = \frac{\epsilon^2}{54} + O(\epsilon^3) = \eta$$

$$\boxed{v(\rho) \underset{\rho\to 0}{=} \rho^{z-1}\, v \quad \text{where} \quad z = 1 + \frac{\epsilon}{27} + O(\epsilon^2)}$$

$v(0) = 0$ for $\epsilon > 0$.

FOR $N = 2$ WE FIND FROM ABOVE

STATIC SCALING NEW

$$\Gamma^{(2)}(p, \frac{s}{v}, T, g, \mu) \underset{p\to 0}{=} \mu^2\left(\frac{p}{\mu}\right)^{2-\eta}\,\Phi\left(\frac{s}{p^z}\,\frac{\mu^{z-1}}{v}\right)$$

η

WE OBTAIN $\not{z}$ BY MATCHING WITH PERTURBATION THEORY TO TWO LOOPS (10)

$$\Gamma^{(2)}\left(p, \frac{s}{v}\right) \underset{\substack{p\to 0\\ s\to 0}}{=} p^{2-\eta}\mu^{\eta}\left[1 + \left(\frac{s\,\mu^{z-1}}{v\,p^{z}}\right)^{\frac{4-2\eta}{z}}\right]$$

$$z = 1 + \frac{\epsilon}{27} + O(\epsilon^2)$$

PERTURBATIVE CALCULATION

$$= p^{2-\eta}\mu^{\eta}\left\{1 + \frac{s^2}{v^2}\,\frac{\mu^{2z-2}}{p^{2z}}\left[1 - \frac{\epsilon}{27}\log\left(\frac{s^2}{v^2p^2}\right)\right] + O(\epsilon^2)\right\}$$

REAL TIME : WICK ROTATION

$$s = -i\omega + 0 \qquad \omega \in \mathbb{R}$$

$$\Gamma^{(2)}_{\substack{(p,\omega)\\ p\ll T\\ \omega\ll T}} = p^{2-\eta}\mu^{\eta}\left\{1 - \frac{\omega^2}{v^2}\,\frac{\mu^{2z-2}}{p^{2z}}\left[1 - \frac{\epsilon}{27}\ln\left(\frac{\omega^2}{v^2p^2}\right)\right] - \frac{i\pi\epsilon}{27}\,\frac{\omega^2}{v^2}\,\frac{\mu^{2z-2}}{p^{2z}}\,\mathrm{sg}(\omega)\right\}$$

quasiparticles: zeros of $\mathrm{Re}\,\Gamma^{(2)}(p,\omega) = 0$

$$\boxed{\omega(p) = v\,p^{z}\,\mu^{1-z}} \qquad z = 1 + \epsilon/27 + O(\epsilon^2)$$

gapless resonance ANOMALOUS DISPERSION

group velocity $v_g = \frac{\partial\omega}{\partial p} = z\,v\,\mu^{1-z}\,p^{z-1} \to 0$

PROPAGATOR FOR THE QUASIPARTICLES (11)

$$G(p,\omega) = -\frac{1}{\Gamma^{(2)}(p,\omega)} \underset{\omega\simeq\omega(p)}{\simeq} \frac{v\,\mu^{-\eta}\,p^{-2+\eta+z}}{2\mu^{z-1}\left[\omega - \omega(p) + i\Gamma(p)\right]}$$

$$\omega(p) = p^{z}\,v\,\mu^{1-z}\ , \qquad \Gamma_p = \frac{\pi\epsilon}{54}\,\omega_p$$

The width Γ_p vanishes for $p \to 0$

CRITICAL SLOWING DOWN

x) NOTICE THAT a) $v\,\mu^{1-z}$ IS RG INVARIANT

b) $\frac{\pi}{54} = 0.058...$ IS A SMALL NUMBER

$$\frac{\Gamma_p}{\omega_p} = \frac{\pi\epsilon}{54} \ll 1$$

GENERALIZATION TO THE $O(N)$ MODEL

$$\vec{\phi} = (\phi_1, \dots, \phi_N)\ , \qquad \frac{\lambda}{N}\left(\vec{\phi}^{\,2}\right)^2$$

$$\boxed{z = 1 + \epsilon\,\frac{N+2}{(N+8)^2} + O(\epsilon^2)} \qquad \Big|^{N=1}\ 1 + \frac{\epsilon}{27}$$

$$\eta = \epsilon^2\,\frac{N+2}{2(N+8)^2} + O(\epsilon^3)$$

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NON CRITICAL THEORY: $T > T_c$

SYMMETRIC PHASE $\langle \Phi \rangle = 0$

$$\Gamma^{(2)}_R(p=0, s=0) = M_R^2(T) \neq 0$$

TO TWO LOOPS AND LEADING ORDER IN ϵ $[g^* = O(\epsilon)]$

$$\Gamma_R^{(2)}(p,s,m_R(T)) = p^2 + \mu^2 m_R^2 \left[1 + \frac{1}{2} g \log(m_R^2 e^?) \right] +$$

$$+ \frac{s^2}{v^2}\left[1 - \frac{g^2}{3\epsilon} \log m_R^2\right] - \frac{g^2}{3\epsilon}\left[m_R^2 \mu^2 + \frac{s^2}{v^2}\right] \log\left[1 + \frac{s^2}{v^2 \mu^2 m_R^2}\right] + O(g^3, g^2\epsilon)$$

m_R^2 = dimensionless

$$m_R^2 = \frac{Z_\phi}{Z_m} m^2_{bare}(T) \quad , \quad Z_m = 1 + \frac{g}{\epsilon} + O(g^2, g\epsilon)$$

RGE:

NON-CRITICAL

$$\left[\mu \frac{\partial}{\partial \mu} + \beta_g \frac{\partial}{\partial g} + \beta_v \frac{\partial}{\partial v} + \beta_m \frac{\partial}{\partial m_R^2} - \frac{N}{2}\gamma\right]$$

$$\Gamma^{(N)}\left(p_i, \frac{s_i}{v}, g, m, \mu\right) = \Delta \Gamma^{(N)}$$

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new off-criticality:

$$\beta_m = \mu \left.\frac{\partial m_R^2}{\partial \mu}\right|_{m_0, T, \lambda_0} = -m_R^2 \left[2 - g + O(g^2, g^2\epsilon)\right]$$

$g^* = \epsilon/3$

m_R now runs

$$\boxed{m^{1/\nu} = |T - T_c|}$$

$$\rho \frac{d m_R^2}{d\rho} = \beta_m(g, m_R, v) \Rightarrow m^2(\rho) = \frac{m^2(1)}{\rho^{1/\nu}}$$

$$\frac{1}{\nu} = 2 - \frac{\epsilon}{3} + O(\epsilon^2), \quad \nu = \frac{1}{2} + \frac{\epsilon}{12} + O(\epsilon^2)$$

TWO POINTS FUNCTION:

NEW OFF CRITICALITY

$$\Gamma^{(2)}(p,s) \underset{p,s \to 0}{=} \mu^2 \left(\frac{p}{\mu}\right)^{2-\eta} \Phi\left(\frac{s}{p^2} \frac{\mu^{\eta - 1}}{v}, p\xi\right)$$

$$\xi = \text{correlation length} = \frac{[m^2(1)]^{-\nu}}{\mu} = \frac{1}{\mu}|T - T_c|^{-\nu}$$

ALSO,

$$\Gamma^{(2)}(p,s) = \mu^2 (\mu \xi)^{\eta - 2} \Psi\left(\frac{s\, \xi^z}{v \mu^{1-z}}, p\xi\right)$$

in particular, the susceptibility

$$\chi^{-1} = \Gamma^{(2)}(0,0) = C\, \xi^{\eta-2} = C' |T - T_c|^{\nu(2-\eta)}$$

$$\chi \sim |T - T_c|^{-\gamma} \qquad \gamma = \nu(2 - \eta)$$

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MATCHING WITH PERTURBATION THEORY TO TWO LOOPS

$$\Gamma^{(2)}(p,s) = \mu^2 (T-T_c)^{\gamma} \left\{ (p\xi)^2 + \left[1 + \left(\frac{s\,\xi^{z}}{\nu\,\mu^{1-z}} \right)^2 \right]^{2-\eta} \right\}$$

ASSUMING EXPONENTIATION.

REAL TIME: WICK ROTATION $s = -i\omega + 0$

$\Rightarrow$ QUASIPARTICLES

$\mathrm{Re}\,\Gamma^{(2)}(p,\omega)$ VANISHES AT

$$\omega_p = \nu\mu \sqrt{\left(\frac{p}{\mu}\right)^{2z} + (\mu\xi)^{-2z}}$$

$$\Gamma^{(2)} \underset{\omega \to \omega_p}{\simeq} [\omega - \omega_p + i\Gamma_p]/Z_p$$

$$\Gamma_p = \frac{\pi\epsilon}{54}\,\nu\mu^{1-z}\,\frac{p^z}{\sqrt{1+(p\xi)^{-2z}}} = \frac{\pi\epsilon}{54}\,\nu\mu^{1-z} \begin{cases} p^z, & p \text{ fixed}, \xi \to \infty \\ p^{2z}\xi^{z}, & \xi \text{ fixed}, p \to 0 \end{cases}$$

In BOTH LIMITS $\Gamma_p \to 0$ for $p \to 0$

CRITICAL SLOWING DOWN

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MOREOVER,

$$\frac{\Gamma_p}{\omega_p} = \frac{\pi\epsilon}{54}\,\frac{1}{1+(p\xi)^{-2z}} < \frac{\pi\epsilon}{54} \ll 1$$

WHAT WE HAVE DONE?

STATICAL CRITICAL PHENOMENA IN

$R^{4-\epsilon}_{\vec{x}} \otimes S^1_{\tau}$ S^1:

(INFRA-RED BEHAVIOUR), $0 < \tau < T$

FINITE SIZE SCALING IN τ (TRICKY!)

$0 < \epsilon \ll 1$

$\downarrow$ WICK ROTATION $\tau \to it$

DYNAMICAL CRITICAL PHENOMENA

$R^{4-\epsilon}_{\vec{x}} \otimes R$

t = REAL TIME

FINALLY WE SET $\epsilon = 1$

$R^3 \otimes R$: 3+1 MINKOWSKI SPACETIME

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CONCLUSIONS

DYNAMICS

AT OR NEAR CRITICALITY IN $\lambda \Phi^4$ THEORY

a) HIGH T BEHAVIOUR $(p, \omega \ll T)$
GOVERNED BY DIMENSIONAL REDUCTION
$\Rightarrow$ ZERO MATSUBARA MODES

b) ϵ-expansion in $\boxed{5-\epsilon}$ space-time dimensions

c) effective coupling $g = \frac{\lambda T}{\mu^{\epsilon}}$ RUNS TO AN
INFRARED STABLE POINT OF ORDER ϵ

d) THE SPEED OF LIGHT ALSO
RUNS AS $\boxed{v(p) = v\, p^{z-1}} \to 0$
$z = 1 + \frac{\epsilon}{24} + O(\epsilon^2)$ DYNAMICAL CRITICAL EXPONENT

e) QUASIPARTICLE DISPERSION LAWS:
ANOMALOUS SCALING

$$\omega_p^2 = (v p)^2 \left[\left(\frac{p}{\mu}\right)^{2z} + (p\xi)^{-2z} \right] \quad , \quad \Gamma_p = \frac{\pi \epsilon}{54} v p \frac{\left(\frac{p}{\mu}\right)^2}{\sqrt{1+(p\xi)^{-2z}}}$$

$\mu\xi = |T-T_c|^{-\nu} \to \infty$ $\qquad \frac{\Gamma_p}{\omega_p} \ll 1 \Rightarrow$ CRITICAL SLOWING DOWN

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z describes a NEW UNIVERSALITY CLASS OF DYNAMICAL CRITICAL PHENOMENA

APPLICATIONS: THE CHIRAL PHASE TRANSITION IN QCD HAS BEEN ARGUED TO BE IN THE SAME UNIVERSALITY CLASS AS THE O(4) LINEAR SIGMA MODEL

BROKEN PHASE: PION RELAXATION

O(N)
$$z = 1 + \epsilon \frac{N+2}{(N+8)^2} + O(\epsilon^2)$$

FUTURE PERSPECTIVES: HIGHER ORDERS IN ϵ
RESUMMATION AT $\epsilon = 1$, GAUGE THEORIES HTL

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