ITP UCSB 05/28/02

HARD-THERMAL-LOOP QUASIPARTICLE APPROACH TO QCD THERMODYNAMICS AND QUARK SUSCEPTIBILITIES

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S	PRL 83 (1999) 2906
$S, \mathcal{N}$	PLB 470 (1999) 181
	PRD 63 (2001) 065003
χ	PLB 523 (2001) 143

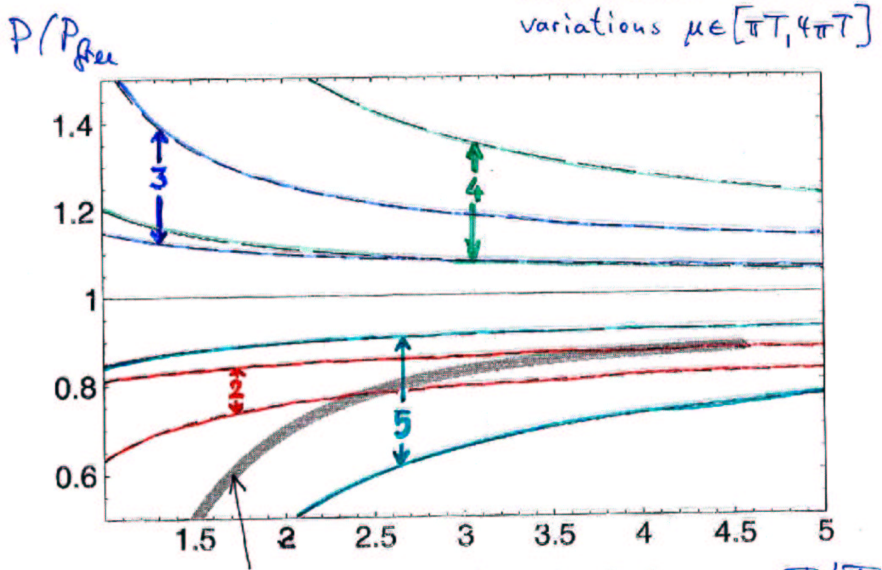
★ Approximately Self-Consistent Resummations of HTL Physics (as opposed to HTLpt of Andersen, Braaten, Strickland)

- comparison to lattice data including quark number suscept's
- new puzzle: off-diagonal quark suscept's

PROBLEM: No convergence of results from conventionally resummed PERTURBATION THEORY up to g^5 for $T \lesssim 10^5 T_c$

(Arnold + Zhai 94; Kastening + Zhai 95; Braaten + Nieto 96)

Renormalization scale variations $\mu \in [\pi T, 4\pi T]$



Lattice results from Boyd et al. (NPB 469 (1996))

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"Numerological" attempts to improve convergence:

Padé approximants, Borel transforms (Hatsuda, Kastening; Parwani; ...)

More promising:

SCREENED PERTURBATION THEORY

→ optimized/variational perturbation theory with screening mass as variational parameter ← PMS

(Karsch, Patkós, Petrecky '97; Chiku, Hatsuda '98; Andersen, Braaten, Strickland '01)

e.g. scalar φ^4:

$$\mathcal{L} = \underbrace{\mathcal{L}_0 - \tfrac{1}{2} m^2 \varphi^2}_{\mathcal{L}_0'} + \underbrace{\mathcal{L}_{int} + \tfrac{1}{2} m^2 \varphi^2}_{\mathcal{L}_{int}'}$$

WORKS GREAT

analogously for QCD: $-\frac{1}{2} m^2 \varphi^2 \longrightarrow \mathcal{L}_{HTL} \propto m_D^2 \to m^2_{variational}$

"HTLpt"

1-loop: ABS, PRL83 (1999), ...

2-loop: ABS + Petitgirard, hep-ph/0205085

↳ variational ✓ $\frac{\partial \mathcal{F}}{\delta m^2} = 0$ w/ solutions, $\not\propto m^2_{pe}, m^2_D, \ldots$

in contrast to old/standard HTL pert.th.:

- resummation of HTL's not only at soft momenta $\sim gT$, but throughout
- HTL's no longer L.O. for $\omega, k \sim T$ in general

→ additional UV subtractions and additional RS dependencies at any finite order

↑ suppressed by powers of g^2 only @ ≥ 3 loop order

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recap:

in contrast to LO thermal mass of φ^4-scalar, in gauge th.:

HTL quasiparticles have

momentum-dependent masses given by

poles of gluon/quark (photon/electron) propagators:

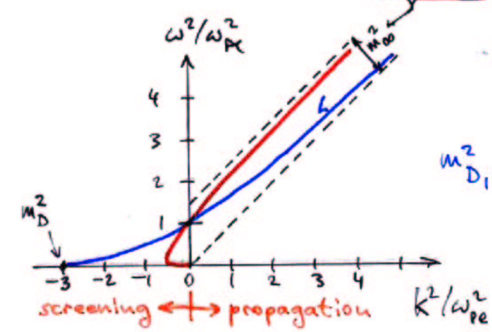
e.g. gauge bosons:

2 branches:

Landau damping acts

$$(L) \quad \omega^2 - k^2 = m^2_{D,LO} \left(1 - \frac{\omega}{2k} \log \frac{\omega + k}{\omega - k}\right) \equiv \hat{\Pi}_L$$

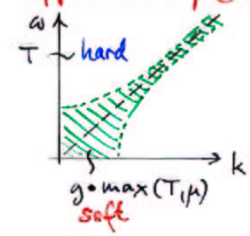
$$(T) \quad \omega^2 - k^2 = \hat{\Pi}_T = \underbrace{\tfrac{1}{2} m^2_{D,LO}}_{m^2_\infty} + \frac{\omega^2 - k^2}{2k^2} \hat{\Pi}_L$$



$$\omega^2_{pe,LO} = \frac{1}{3} m^2_{D,LO}$$

$$m^2_{D,LO} = \left(N + \frac{N_f}{2}\right) \frac{g^2 T^2}{3} + \sum_f \frac{g^2 \mu_f^2}{2\pi^2}$$

NLO corrections in HTL pert.th. ← applicability (Gelis 00)



$\delta\omega^2_{pe}/\omega^2_{pe} \approx -0.18 \sqrt{N}\, g$ (Schulz '83)

$\delta m^2_D / m^2_D = +(\sqrt{3N}/2\pi)\, g \log \frac{c}{g}$ (AR '93)

$\delta m^2_\infty / m^2_\infty < 0$, momentum-dependent!

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screened/optimized HTLpt

adds and subtracts $\mathcal{L}_{HTL}$ $\forall\, \omega, k$ (soft+hard)

LO relation $m_D^2 = 3\omega_{pe}^2 = 2m_\infty^2$ kept fixed → replaced by variational mass

Our approach: (B.I.R.)

approximately self-consistent propagator resummation through skeleton expansions (Luttinger, Ward '60; DeDominicis, Martin '64; Cornwall, Jackiw, Tomboulis)

free energy

$$\mathcal{F}[D] = \tfrac{1}{2}\mathrm{Tr}\ln D^{-1} - \tfrac{1}{2}\mathrm{Tr}\,\Pi D + \Phi[D]$$

$$\Phi[D] = \tfrac{-1}{12}\ominus - \tfrac{1}{8}\infty - \tfrac{1}{48}\ominus \;\ldots \qquad (2PI!)$$

$$\frac{\delta\mathcal{F}[D]}{\delta D} = 0 \quad\Longleftrightarrow\quad \tfrac{1}{2}\Pi[D] = \frac{\delta\Phi}{\delta D}$$

self consistent "Φ-derivable" approx. (Baym '62)

by truncation of Φ

take: 2-loop $\Phi \to \Phi_{2\text{-loop}} = -\tfrac{1}{12}\ominus - \tfrac{1}{8}\infty$

entropy $S = -\dfrac{d\mathcal{F}}{dT} = -\dfrac{\partial\mathcal{F}}{\partial T}\Big|_D$

densities $\mathcal{N}_i = -\dfrac{d\mathcal{F}}{d\mu_i} = -\dfrac{\partial\mathcal{F}}{\partial\mu_i}\Big|_D$

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Entropy and (quark) density in Φ-derivable approximations:

bosons:

$$S = -\int\frac{d^4k}{(2\pi)^4}\frac{\partial n}{\partial T}\,\mathrm{Im}\log D^{-1} + \int\frac{d^4k}{(2\pi)^4}\frac{\partial n}{\partial T}\,\mathrm{Im}\,\Pi\cdot\mathrm{Re}\,D + S'$$

(n: n_{BE})

$$S' = -\frac{\partial(T\Phi)^{2\text{-loop}}}{\partial T}\Big|_D + \int\frac{d^4k}{(2\pi)^4}\frac{\partial n}{\partial T}\,\mathrm{Re}\,\Pi\cdot\mathrm{Im}\,D = 0 \qquad \text{per d.o.f.} \quad (O(3\text{-loop}))$$

fermions: (Dirac)

$$S_f = -2\int\frac{d^4k}{(2\pi)^4}\frac{\partial f}{\partial T}\,\mathrm{tr}\left\{\mathrm{Im}\log\gamma_0 S^{-1} - \mathrm{Im}(\gamma_0\Sigma)\,\mathrm{Re}(S\gamma_0)\right\} + S'_f$$

(f: n_{FD})

$S'_{total} = O(3-\text{loop})$ still holds — Vanderheyden+Baym 98; B.I.R. 99

non-zero chemical potential:

$$\mathcal{N} = -2\int\frac{d^4k}{(2\pi)^4}\frac{\partial f}{\partial\mu}\,\mathrm{tr}\left\{\;\text{———''———}\;\right\} + \mathcal{N}'$$

$\mathcal{N}' = O(3\text{-loop})$

Up to 1 integration constant (↔ bag constant) this allows one to (re)construct

$$P = -\mathcal{F} = -\Omega(\mu, T)/V$$

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Properties of S and $\mathcal{N}$ (2-loop Φ-derivable)

- S, $\mathcal{N}$ are UV-finite functionals

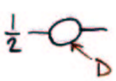
  all integrals involve

  $$\frac{\partial n}{\partial T},\ \frac{\partial f}{\partial T} \text{ or } \frac{\partial f}{\partial \mu} \xrightarrow{\text{exp'ly}} 0 \quad \text{for } \omega \to \pm\infty$$

- though:

  1-loop gap equation is not

  $$\Pi = 2\,\frac{\delta \Phi[D]^{2\text{-loop}}}{\delta D}$$

  i.e. $D^{-1} - D_0^{-1} = \frac{1}{2}$  $+ \frac{1}{2}$ 

  but from HTL perturbative counting of g's:

  $$\Pi^{\text{div}} \sim g^4 \sim S', \mathcal{N}'$$

- similar problem with gauge invariance:

  Φ-derivable approximations a priori don't respect gauge invariance

  but again: gauge dependences in $\Pi \sim S', \mathcal{N}'$

  "when inserted in S, $\mathcal{N}$ contribute to perturbative order of"

→ shall use

finite, gauge invariant + gauge independent approximations to Π

to wit: HTL and some NLO corr. thereof

within standard HTL pert. th.

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LO "interaction" term in S, $\mathcal{N}$:

(gluonic contribution only:)

$$S_{SB} + S_2 = -2N_g \int \frac{d^4k}{(2\pi)^4} \frac{\partial n}{\partial T} \Big\{ \underbrace{\mathrm{Im} \log(-\omega^2+k^2)}_{\to \frac{4\pi^2}{45} N_g T^3} - \mathrm{Im}\frac{\Pi_T}{\omega^2-k^2} + \mathrm{Im}\,\Pi_T\, \mathrm{Re}\frac{1}{\omega^2-k^2} \Big\}$$

dominated by hard momenta ⇒ only D_T

$$S_2 = -2N_g \int \frac{d^4k}{(2\pi)^4} \frac{\partial n}{\partial T} \underbrace{\mathrm{Re}\,\Pi_T(\omega,k)}_{m_\infty^2} \underbrace{\mathrm{Im}\frac{1}{\omega^2-k^2}}_{-\pi\,\varepsilon(\omega)\,\delta(\omega^2-k^2)}$$

entirely given by spectral data of hard modes where HTL remains applicable because of $\delta(\omega^2-k^2)$

general formula:

$$S_2 = -T\Big\{ \sum_B \frac{m_{\infty B}^2}{12} + \sum_F \frac{M_{\infty F}^2}{24} \Big\}$$

$$\mathcal{N}_2 = -\frac{1}{8\pi^2} \sum_F \mu_F M_{\infty F}^2$$

Plasmon term in $S, \mathcal{N}$ $\propto g^3$

(where strict perturbation theory becomes inadequate)

recall:

in pressure and conv'ly resummed pert. th. given by Debye mass in electrostatic propagator:

$$P_3 = -N_g T\int\frac{d^3k}{(2\pi)^3}\left[\log\left(1+\frac{\hat m_D^2}{k^2}\right)-\frac{\hat m_D^2}{k^2}\right] = N_g\frac{\hat m_D^3\, T}{12\pi}$$

now soft as well as hard contributions:

$$S^{(3)}_{\mathrm{soft}} = -N_g\,\mathrm{tr}\int\frac{d^4k}{(2\pi)^4}\underbrace{\frac{\partial n}{\partial T}}_{\sim\frac{1}{\omega}}\Big\{\mathrm{Im}\left[\log(1+D_0\hat\Pi)-\hat\Pi D_0\right] - \mathrm{Im}\,\hat\Pi\;\mathrm{Re}\,(\hat D - D_0)\Big\}$$

$$= \underbrace{\frac{\partial P_3}{\partial T}\Big|_{\hat m_0}}_{\frac{1}{4} S_3} + \underbrace{N_g\int\frac{d^4k}{(2\pi)^4}\frac{1}{\omega}\Big\{2\,\mathrm{Im}\,\hat\Pi_T\;\mathrm{Re}\,(\hat D_T - D_T^0) - \mathrm{Im}\,\hat\Pi_L\;\mathrm{Re}\,(\hat D_L - D_L^{(0)})\Big\}}_{0 \text{ (numerical result)}}$$

? larger contribution from hard momenta:

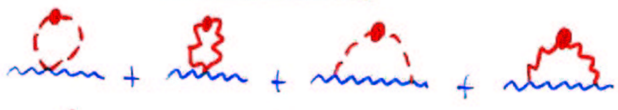
$$S^{(3)}_{\mathrm{hard}} = -N_g\int\frac{k\,dk}{2\pi^2}\frac{\partial n(k)}{\partial T}\underbrace{\mathrm{Re}\,\delta\Pi_T(\omega=k)}_{\delta m_\infty^2(k)}$$

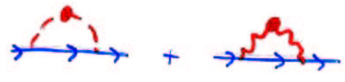
"massive" reorganization of resummed pert. th.!

similarly for S_q

for density: $\mathcal{N}^3 = \mathcal{N}^3_{\mathrm{hard}}$ (all plasmon effects from δM_∞^2)

Completion of plasmon term requires δm_∞^2 and δM_∞^2

$\delta m_\infty^2 =$  $\Big|_{\omega=k}$

δM_∞^2 :  $\Big|_{\omega=k}$

although $\hat m_\infty^2$ and $\hat M_\infty^2$ are simple constant masses,

$\delta m_\infty^2 = \delta m_\infty^2(k)$, $\quad \delta M_\infty^2 = \delta M_\infty^2(k)$

simplification:

define average NLO asymptotic masses through

$$\bar\delta m_\infty^2 = \frac{\int k\,dk\,n'(k)\,\mathrm{Re}\,\delta\Pi_T(\omega=k)}{\int k\,dk\,n'(k)} \qquad \text{etc.}$$

$$\longrightarrow \bar\delta m_\infty^2 = -\frac{1}{2\pi}g^2 N T \hat m_D$$

$$\bar\delta M_\infty^2 = -\frac{1}{2\pi}g^2 C_F T \hat m_D$$

AIM: "Next-to-leading order approximation" defined as

$$S_{NLA} = S_{HTL}\big|_{\mathrm{soft}} + S_{\bar m_\infty^2}\big|_{\mathrm{hard}}$$

separation scale $\Lambda = \sqrt{2\pi T\,\hat m_D \cdot c_\Lambda}$

$c_\Lambda = \frac{1}{2}\ldots 2$ keeps Λ well between hard $(2\pi T)$ and soft $(\hat m_D)$ for all g of interest

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"Strictly perturbative" inclusion of δm_∞^2 leads to tachyonic masses for $g \sim 1$

e.g. $N=3$: $\bar{m}_\infty^2 = \frac{1}{2} g^2 T^2 \left(1 - \frac{3}{\pi} g + O(g^2)\right)$

BUT: no specific problem of QCD!

NLO correction of thermal mass in scalar $g^2\varphi^4$ exactly the same problem:

$$m^2 = g^2 T^2 \left(1 - \frac{3}{\pi} g + \cdots\right)$$

whereas solution of 1-loop gap equation

$$m^2 = \underline{\bigcirc^{\leftarrow m^2}}$$

monotonic function in g

better approximation:

$$m^2 = g^2T^2 - \frac{3}{\pi} g^2 T \underset{\text{neg. feedback}}{\underset{\downarrow}{m}}$$

$$\longrightarrow \frac{m}{T} = \left[g^2 + \frac{9}{4\pi^2} g^4\right]^{\frac{1}{2}} - \frac{3}{2\pi} g^2$$

monotonic in g ✓

rather similar results by simple Padé approximant $m^2 = \dfrac{g^2T^2}{1 + \frac{3}{\pi} g}$

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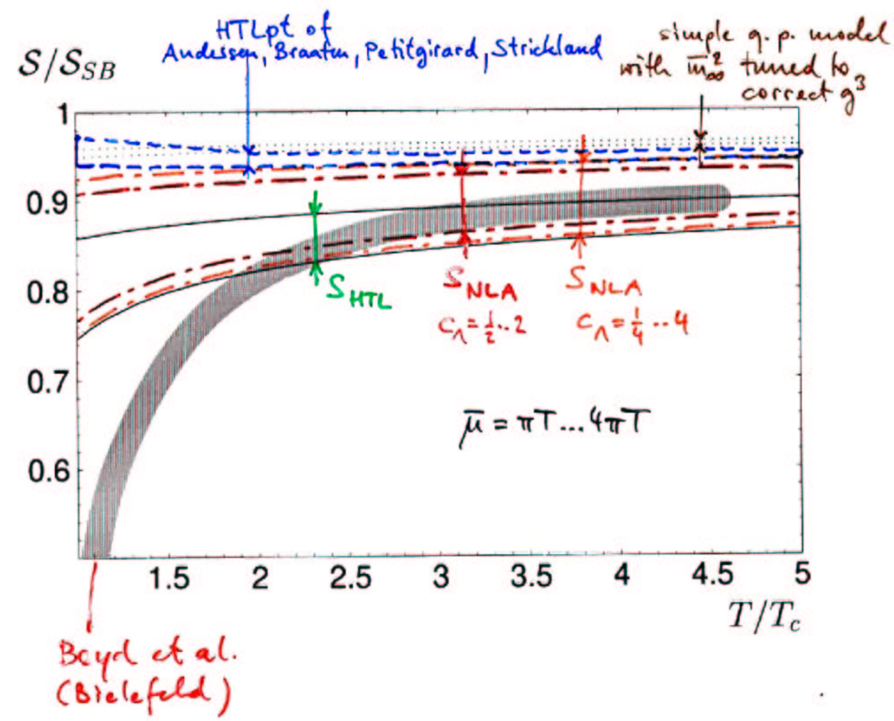
Comparison with lattice results (pure glue)

assuming $\alpha_s(\bar{\mu})$ from 2-loop RG eq.

$T_c = 1.14\, \Lambda_{\overline{MS}}$

$\bar{\mu} \in (\pi T, 4\pi T)$



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New application: (bec. of new lattice results for:)

QUARK NUMBER SUSCEPTIBILITIES

experim. interest: (event-by-event) fluctuations
(Asakawa, Heinz, Müller; Jeon, Koch, Bleicher, ...)

theoret. interest: similar problems w/ pert. th. (+ new one)

$$\chi_{ij} = \frac{\partial \mathcal{N}_i}{\partial \mu_j}\Big|_{\mu=0}$$ ← for lattice

HTL: all $m_i = 0$ ⟶ 2 different susc's χ ... diagonal
$\tilde{\chi}$ ... off-diagonal

•) diagonal

$$\chi_0 = \frac{NT^2}{3} \quad \text{(ideal gas limit)}$$

conv. pert. th.: ($N_c = 3$)

$$\frac{\chi}{\chi_0} = 1 - 2\frac{\alpha_s}{\pi} + 8\sqrt{1+\frac{N_f}{6}}\left(\frac{\alpha_s}{\pi}\right)^{\frac{3}{2}} + 12\left(\frac{\alpha_s}{\pi}\right)^2 \log\frac{\alpha_s}{\pi} + O(\alpha_s^2)$$

undetermined scale of log

g^3 (plasmon-) contribution spoils pert. th. for $T \lesssim 700\, T_c$

($\mathcal{F}$: $10^5\, T_c$)

pert. result through g^3: $\chi > \chi_0$ for $T \lesssim 40\, T_c$

lattice : $\chi < \chi_0$ ($T \lesssim 5 T_c$)

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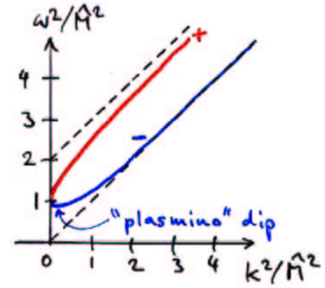
2-loop Φ-derivable quark density

$$\mathcal{N}_i = -4N_c \int \frac{d^4k}{(2\pi)^4} \frac{\partial f(\omega)}{\partial \mu_i} \Big[\mathrm{Im} \log \Delta_+^{-1} + \mathrm{Im} \log (-\Delta_-)^{-1} - \mathrm{Im}\,\Sigma_+ \,\mathrm{Re}\,\Delta_+ + \mathrm{Im}\,\Sigma_- \,\mathrm{Re}\,\Delta_- \Big] + O(\text{3-loop} \sim g^4)$$

$$\Delta_\pm^{-1} \equiv -[\omega \mp (k + \Sigma_\pm)] \qquad \Sigma_\pm^{HTL} \propto \hat{M}_i^2 = \frac{g^2 C_f}{8}\left(T^2 + \frac{\mu_i^2}{\pi^2}\right)$$



⇓

no mixing of quark flavor in HTL-approximation

$\tilde{\chi} \equiv 0$ (also for $\mu \neq 0$)

off-diagonal

NLO: all $O(g^3)$ contributions come from

$\delta M^2_{\infty i}$ = 

● $\propto m^2_{D,LO}$ .... involves all quark flavor

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Diagonal quark number susceptibility

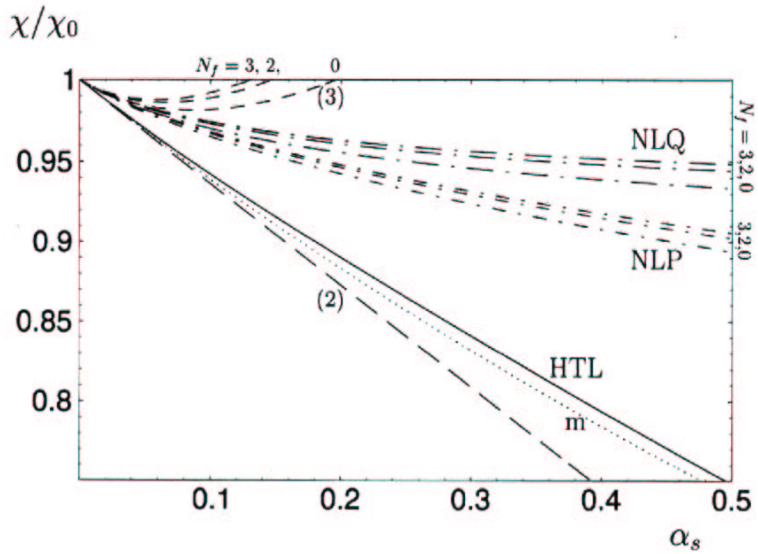
— — — : conv. perturbation theory

· · · · · · m : simple qu. particle model with const. mass $m = M_\infty$

——— : HTL → 2-loop Φ-derivable

—·—·— : NL approximations

NLQ : quadratic gap eq.
NLP : Padé resummed $\mathcal{O}(g)$ corr. to M_∞



HTL : no $\chi^{(3)}$; $\chi^{(4)} = N\left(0.0431 \log\frac{T}{\bar{\mu}} + 0.0028\right)\frac{\hat{M}_4}{T^2}$

wrong sign

NL : $\chi^{(3)}$ complete; $\chi^{(4)}$ not (but within scope

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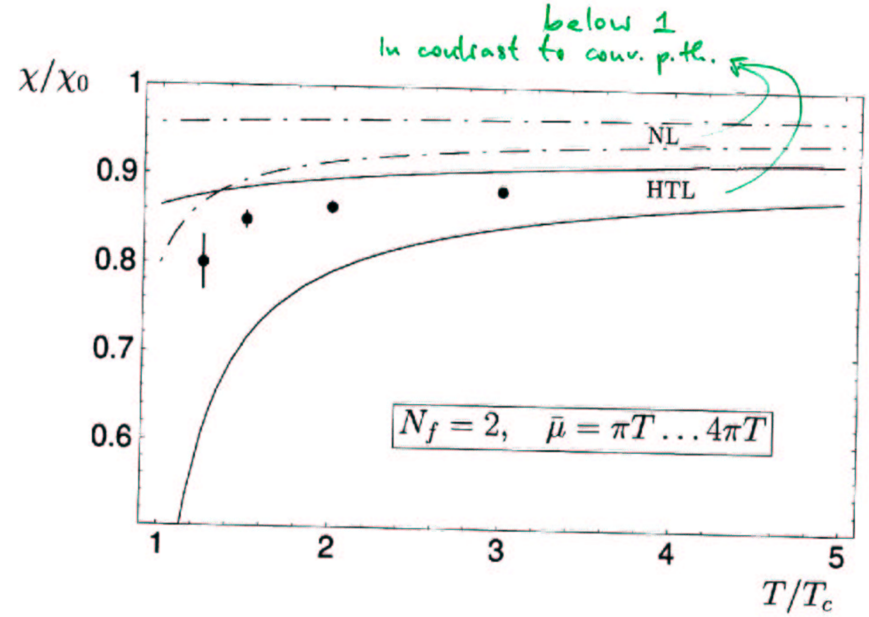
Comparison with lattice results
by Gavai + Gupta + Majumdar, PRD65(2002)

(assuming $T_c/\Lambda_c = 0.49$
$N_f = 2$ dynamical staggered quarks, $\frac{m}{T_c} = 0.1$)



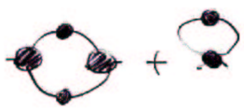
Comment on

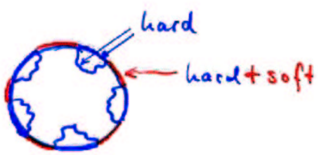
Chakraborty, Mustafa + Thoma EJP C (2002)

CMT use 1-loop HTLpt for

$$\chi(T) = \beta \int d^3x \left\langle \bar\Psi \gamma_0 \Psi(x) \cdot \bar\Psi \gamma_0 \Psi(0) \right\rangle$$

$\chi_0^{1\text{-loop}} =$  $= \frac{NT^2}{3}$

$\chi^{1\text{-loop}}_{HTLpt} =$ 

i.e. $\frac{\partial^2}{\partial\mu^2}$ 

claim good agreement with lattice data, but

- g^2 contribution incomplete

 $\sim g^2$

- g^3 completely missed out

no soft ~~non~~ resummed anywhere at 1-loop

$\Rightarrow$ only mindbogglingly complicated 2-loop HTLpt comparable!

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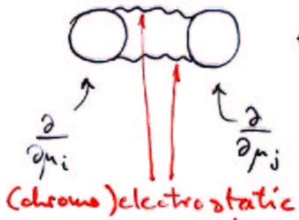
•) off-diagonal quark susceptibility

need 2 independent fermion loops

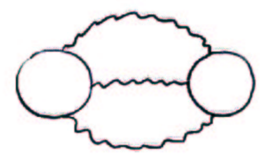
 $\in \chi^{NL}_{2\text{-loop}}$ ()

propagators $\longrightarrow$ lin. divergence screened by m_{Debye}

$$\longrightarrow \chi_{ij} = \frac{g^4 (N^2-1) T \mu_i \mu_j}{16\pi^2 \, m_D} \sim g^3 \cdot \mu_i \mu_j$$

$\Rightarrow$ $\tilde\chi$ @ $\mu=0$ vanishes

LO contribution to $\tilde\chi|_{\mu=0}$:



"bugblatter" diagram
[S. D. Adams, HHGG I]

log. divergence screened by m_{Debye}

$$\longrightarrow \tilde\chi = O\left(g^6 \log\tfrac{1}{g}\, T^2\right)$$

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coefficient of $g^6 \log\frac{1}{g}$ in easy to calculate:

need only $\lim_{k\to 0} \frac{\partial}{\partial\mu}$ $\Big|_{\omega=0,\ \mu=0}$

derivative $\frac{\partial}{\partial\mu}$ like A^0 vertex @ $\omega, k=0$;

symmetric A^0 vertices like $\left(\frac{\partial}{\partial\mu}\right)^n$; $\partial \bigcirc = \partial p^{free}(m_i,\mu_i,T)$

$\rightarrow$ effective vertex (C-odd)

$$= \frac{g^3}{3!}\,\mathrm{Tr}[A_0^3]\sum_i \frac{\partial^3}{\partial\mu_i^3}\,p^{free}(m_i;\mu_i,T)$$

$$\xrightarrow{m\to 0} \frac{g^3}{3!}\,\mathrm{Tr}[A_0^3]\sum_i \frac{2}{\pi^2}\mu_i$$

(Korthals-Altes, Pisarski, Sinkovics, '00
Hart, Laine, Philipsen, '00
Bödeker, Laine, '01)

el. static

$$\frac{\tilde\chi}{\chi_0} \simeq \simeq -\frac{1}{128N}\underbrace{d^{abc}d^{abc}}_{\frac{(N^2-1)(N^2-4)}{N}}\left(\frac{g}{\pi}\right)^6 \log\frac{T}{m_D}$$

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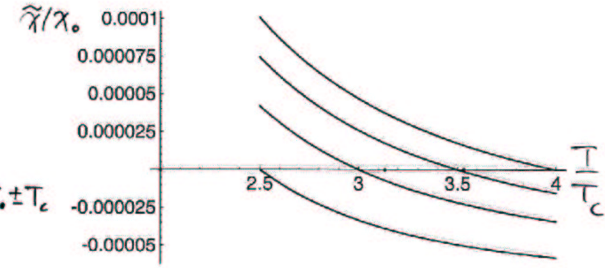
N=3:

off-diagonal susc. $\frac{\tilde\chi}{\chi_0} = -\frac{10}{9\pi^3}\,\alpha_s^3 \log\frac{c}{\alpha_s} + O(\alpha_s^{7/2})$

undetermined

$\log\frac{c}{\alpha_s} = O(1) \Rightarrow \tilde\chi/\chi_0 \gtrsim 10^{-4}$ for $T \lesssim 3T_c$

if c such that accidental zero at some T_0/T_c,
$\tilde\chi/\chi_0 \gtrsim 10^{-5}$ for $T = T_0 \pm T_c$



•) ∃ lattice results: $\tilde\chi/\chi_0$:

Gottlieb et al. PRL59('87): $-0.001(2)$
PRD55('97): $-0.0007(15)$ @ $1.5\,T_c$

consistent with both, zero and LO perturbative result

most recently:

Gavai, Gupta, Majumdar PRD65('02):

$\tilde\chi/\chi_0 \lesssim 10^{-6}$ $\quad +4(3)\cdot 10^{-6}$ @ $1.5\,T_c$
$+7(7)\cdot 10^{-7}$ @ $3\,T_c$
(dyn. staggered quarks)
$m/T_c = 0.1$

NEW PUZZLE ?

New kind of breakdown of pert. th.?
(Hitherto p.th. underestimated effects!)

Underestimated lattice errors?
(for χ/χ_0 errors are few$\times 10^{-3}$!)

CONCLUSIONS + OUTLOOK

- In contrast to Debye-resummed static pert. th., full resummation of HTL quasiparticle effects reproduces lattice data remarkably well down to $\sim 3T_c$
- Φ-derivable approximation for $S, \mathcal{N}$ resums most of interactions in spectral properties of weakly interacting quasiparticles
- NLO corrections to dispersion laws at hard momenta resums large part of soft physics which normally spoils conv'ly resummed pert. th.
- Further improvements possible in calculating m_∞, M_∞ as fct of $\vec{k}$ $(\sim T)$ in standard HTL pert. th.
  - resummation of NNLO $\sim g^4 \log\frac{1}{g}$
- New puzzle: smallness of off-diagonal $\tilde{\chi}$ on lattice vs. LO pert. th. $\sim g^6 \log\frac{1}{g} \cdot T^2$