Color Collective Phenomena
@ RHIC & LHC

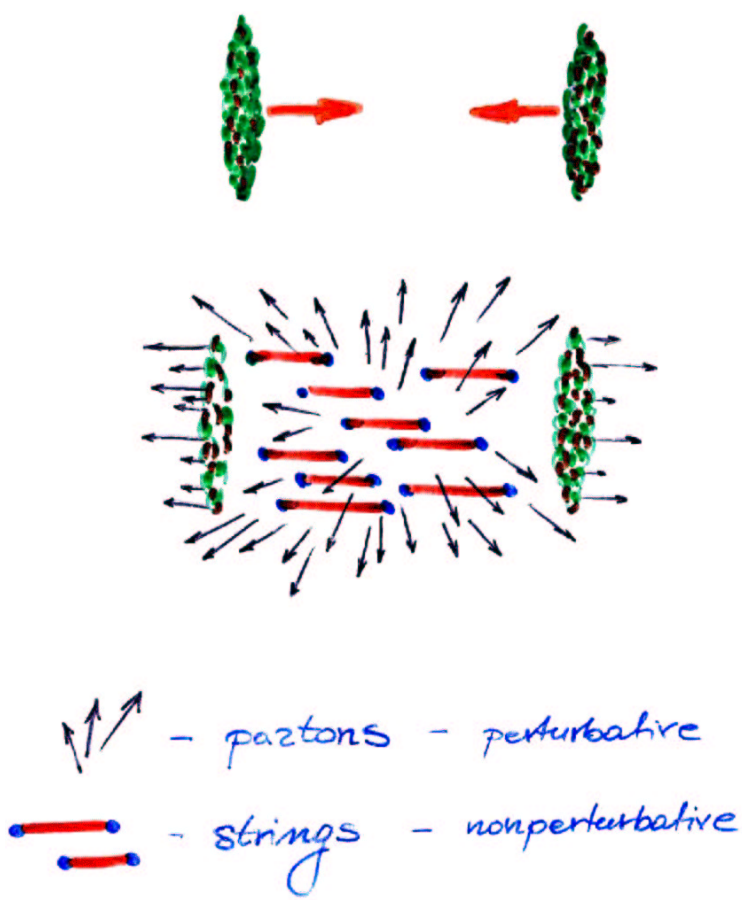
(in non-equilibrium parton system)

- Screening
- Plasmons Instabilities
- Quasiquarks

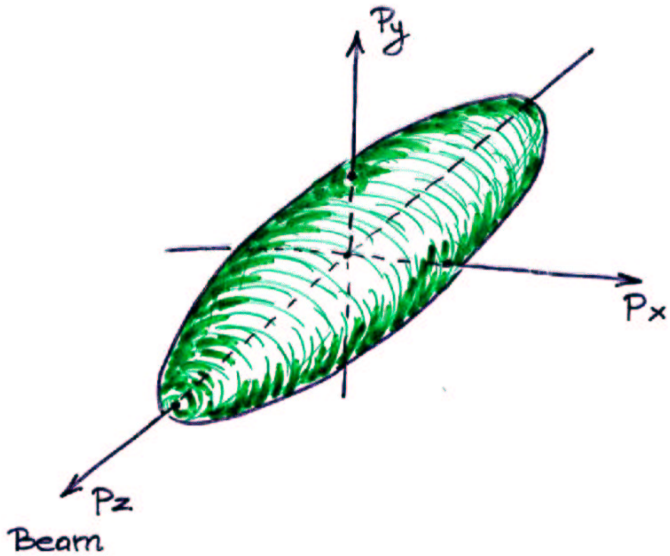
Theoretical tools

- Kinetic theory
- Diagrammatic approach

A-A collision at RHIC or LHC

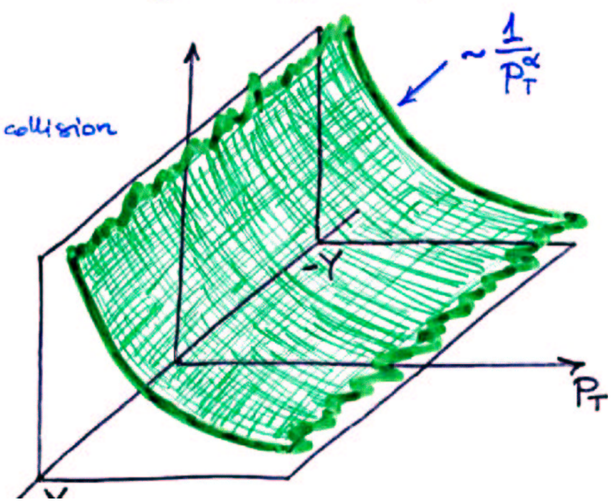


Parton Momentum Distribution



$$\frac{dN}{d^2p_T\, dy\, d\varphi} \sim \Theta(Y-y)\,\Theta(Y+y)\, h(p_T)$$

$\alpha = 6$ after 1-st collision



QGP transport theory

Distribution functions

Quarks: $Q(p,x)$
Antiquarks: $\bar{Q}(p,x)$ } 3×3 matrix in color space
Gluons: $G(p,x)$ 8×8

Gauge transformations:

$$Q(p,x) \longrightarrow U(x)\, Q(p,x)\, U^{+}(x)$$

$$G(p,x) \longrightarrow M(x)\, G(p,x)\, M^{+}(x)$$

$$M_{ab}(x) = \mathrm{Tr}\left[\tau_a\, U(x)\, \tau_b\, U^{+}(x)\right]$$

Probabilistic interpretation of

$\mathrm{Tr}\,Q$, $\mathrm{Tr}\,\bar{Q}$, $\mathrm{Tr}\,G$ <= gauge invariant

Baryon current:

gauge independent

- $$b^\mu(x) = \frac{1}{3}\int \frac{d^3p}{(2\pi)^3}\frac{p^\mu}{E}\left[\mathrm{Tr}\,Q - \mathrm{Tr}\,\bar{Q}\right]$$

Energy-momentum tensor:

- $$t^{\mu\nu}(x) = \int \frac{d^3p}{(2\pi)^3}\frac{p^\mu p^\nu}{E}\left[\mathrm{Tr}\,Q + \mathrm{Tr}\,\bar{Q} + \mathrm{Tr}\,G\right]$$

Color current: gauge dependent

- $$j^\mu(x) = -\frac{1}{2}g\int \frac{d^3p}{(2\pi)^3}\frac{p^\mu}{E}\left[Q - \bar{Q} - \frac{1}{3}\mathrm{Tr}(Q-\bar{Q}) + 2i\tau_a f_{abc}G_{bc}\right]$$

Transport equations

$$p_\mu \partial^\mu f(p,x) + e F_{\mu\nu}(x) p^\mu \frac{\partial}{\partial p_\nu} f(p,x) = C[f]$$

q: $$p_\mu \mathcal{D}^\mu Q + \frac{1}{2} g p^\mu \frac{\partial}{\partial p_\nu}\{F_{\mu\nu}, Q\} = C$$

$\bar{q}$: $$p_\mu \mathcal{D}^\mu \bar{Q} - \frac{1}{2} g p^\mu \frac{\partial}{\partial p_\nu}\{F_{\mu\nu}, \bar{Q}\} = \bar{C}$$

g: $$p_\mu \mathcal{D}^\mu G + \frac{1}{2} g p^\mu \frac{\partial}{\partial p_\nu}\{\mathcal{F}_{\mu\nu}, G\} = C_g$$

$$\mathcal{D}_\mu = \partial_\mu + ig[A_\mu, \ldots]$$

$$A^\mu = A^\mu_a \tau^a, \quad \mathcal{A}^\mu_{ab} = -if_{abc}A^\mu_c$$

A^μ: $$\mathcal{D}^\mu F_{\mu\nu} = j_\nu$$

$$j^\mu(x) = -\frac{1}{2}g\int \frac{d^3p}{(2\pi)^3}\frac{p^\mu}{E}\left[Q - \bar{Q} - \frac{1}{3}\mathrm{Tr}(Q-\bar{Q}) + 2i\tau_a f_{abc}G_{bc}\right]$$

Linear response analysis

colorless

$$Q_{ij}(p,x) = n(p)\,\delta_{ij} + \delta Q_{ij}(p,x)$$

$$\bar{Q}_{ij}(p,x) = \bar{n}(p)\,\delta_{ij} + \delta \bar{Q}_{ij}(p,x)$$

$$G_{ab}(p,x) = n_g(p)\,\delta_{ij} + \delta G_{ab}(p,x)$$

$i,j = 1,2,3$ $\quad\quad a,b = 1,2 \dots 8$

$$|n(p)| \gg |\delta Q(p,x)|$$

$$|\partial_p^\mu n(p)| \gg |\partial_p^\mu \delta Q(p,x)|$$

$$j^\mu = -\frac{1}{2} g \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu}{E} \Big[\delta Q - \delta \bar{Q} - \frac{1}{3} \mathrm{Tr}(\delta Q - \delta \bar{Q}) + 2i \tau_a f_{abc}\, \delta G_{bc} \Big]$$

Linearized transport equations

$$p^\mu D_\mu \delta Q = -g p^\mu F_{\mu\nu} \frac{\partial n}{\partial p_\nu}$$

$$p^\mu D_\mu \delta \bar{Q} = g p^\mu F_{\mu\nu} \frac{\partial \bar{n}}{\partial p_\nu}$$

$$p^\mu D_\mu \delta G = -g p^\mu \mathcal{F}_{\mu\nu} \frac{\partial n_g}{\partial p_\nu}$$

Solutions

$$\delta Q(\bar{p},x) = -g \int d^4y\, \Delta_p(x-y) \times U(x,y)\, p^\mu F_{\mu\nu}(y)\, U(y,x) \frac{\partial n}{\partial p_\nu}$$

$$\delta \bar{Q}(\bar{p},x) = \dots$$

$$\delta G(\bar{p},x) = \dots$$

$$p_\mu \partial^\mu \Delta_p(x) = \delta^{(4)}(x)$$

$$\Delta_p(x) = \frac{1}{E}\, \Theta(t)\, \delta^{(3)}(\bar{x} - \bar{v}t)$$

$$U(x,y) = P\exp\Big[-ig \int^{y} dz\, A^\mu(z)\Big]$$

Polarization tensor

$$j^\mu(k) = g^2 \int \frac{d^3p}{(2\pi)^3 2E}\, p^\mu \frac{\partial f}{\partial p_\lambda} \times \left[g^{\lambda\nu} - \frac{k^\lambda p^\nu}{p^\sigma k_\sigma + i0^+} \right] A_\nu(k)$$

$$f(\mathbf{p}) \equiv n(\mathbf{p}) + \bar{n}(\mathbf{p}) + 6\, n_g(\mathbf{p})$$

$$j^\mu(k) = \Pi^{\mu\nu}(k) A_\nu(k)$$

$$\Pi^{\mu\nu}(k) = g^2 \int \frac{d^3p}{(2\pi)^3 2E}\, p^\mu \frac{\partial f}{\partial p_\lambda} \times \left[g^{\lambda\nu} - \frac{k^\lambda p^\nu}{p^\sigma k_\sigma + i0^+} \right]$$

$$k_\mu \Pi^{\mu\nu}(k) = 0$$

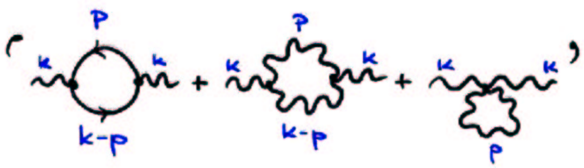
$$\Pi^{\mu\nu}(k) = \Pi^{\nu\mu}(k)$$

$$f(\mathbf{p} = \infty) = 0$$

Diagrammatic Approach*)

$m = 0$

$\Pi^{\mu\nu}(k) =$



Hard Loop approximation:

$$\omega,\ k_i \ll p_i \qquad i = 1,2,3$$

$$k_\mu \Pi^{\mu\nu}(k) = 0$$

$$\Pi^{ij}(k) = -\frac{g^2}{2} \int \frac{d^3p}{(2\pi)^3} \frac{f(\mathbf{p})}{E} \left[\delta^{ij} + \frac{k^i p^j + k^j p^i}{\omega E - \mathbf{k}\cdot\mathbf{p} + i0^+} - \frac{p^i p^j (\omega^2 - \mathbf{k}^2)}{(\omega E - \mathbf{k}\cdot\mathbf{p} + i0^+)^2} \right]$$

$$f(\mathbf{p}) = f(-\mathbf{p})$$

*) St. Mrów...., M.H. Thoma, Phys. Rev. D62 (2000) 036011

Screening

$$D^i(\omega=0,\bar{k}) = \varepsilon^{ij}(\omega=0,\bar{k})\, E^j(\omega=0,\bar{k})$$

$$\varepsilon^{ij}(k) = \delta^{ij} + \frac{1}{\omega^2}\Pi^{ij}(k)$$

Isotropic medium

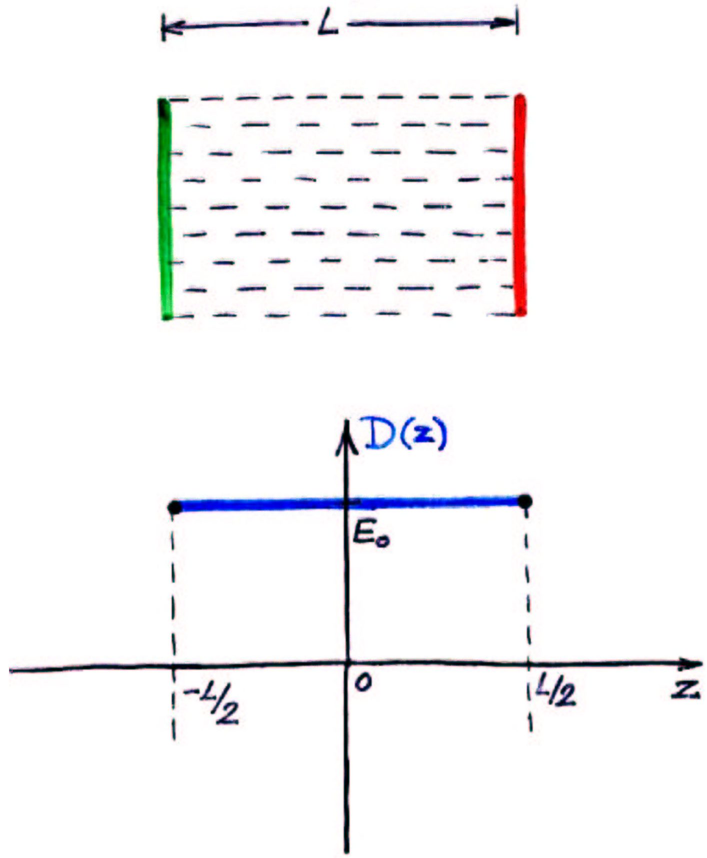
$$\varepsilon^{ij}(k) = \varepsilon_T(k)\left(\delta^{ij} - \frac{k^i k^j}{\bar{k}^2}\right) + \varepsilon_L(k)\frac{k^i k^j}{\bar{k}^2}$$

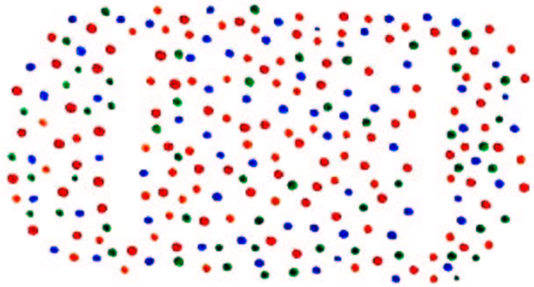
$$\varepsilon_L(\omega=0,\bar{k}) = 1 + \frac{m_D^2}{\bar{k}^2}$$

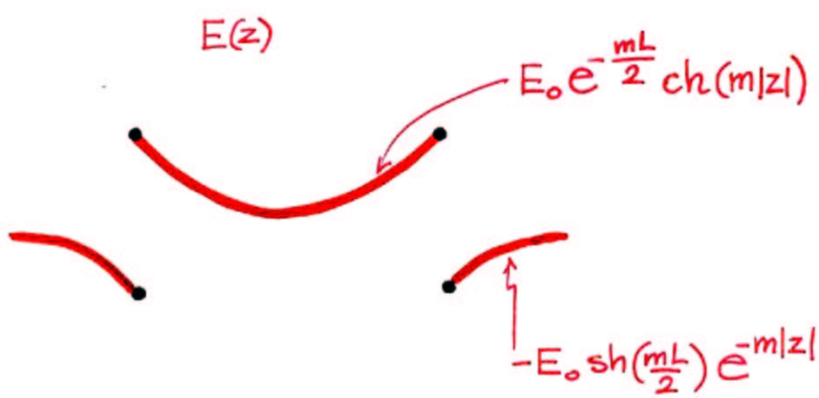
$$m_D^2 = -\frac{g^2}{2}\int\frac{d^3p}{(2\pi)^3}\frac{\bar{k}}{\bar{k}\cdot\bar{v}}\cdot\frac{\partial f(\bar{p})}{\partial\bar{p}}$$

$$A^0(\bar{k}) \sim \frac{1}{\bar{k}^2} \longrightarrow \frac{1}{\bar{k}^2+m_D^2}$$

Screening







- $D_z(k) = \int dz\, e^{ikz} D_z(z) = E_0 \frac{2}{k} \sin\frac{kL}{2}$
- $D_z(k) = \varepsilon^{zz}(\omega=0,k)\, E_z(k)$
- $\varepsilon^{zz}(\omega=0,k) = 1 + \frac{m^2}{k^2}$ m - screening mass
- $E_z(k) = E_0 \frac{2k}{k^2+m^2} \sin\frac{kL}{2}$

Chromodielectric tensor

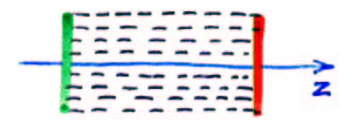
linear response

$$\varepsilon^{ij}(\omega,\bar{k}) = \delta^{ij} + \frac{2\pi\alpha_s}{\omega}\int\frac{d^3p}{(2\pi)^3}\frac{v^i}{\omega-\bar{k}\bar{v}+i0^+}\frac{\partial f(\bar{p})}{\partial p^l}\times$$
$$\times\left[\left(1-\frac{\bar{k}\bar{v}}{\omega}\right)\delta^{lj}+\frac{k^l v^j}{\omega}\right]$$

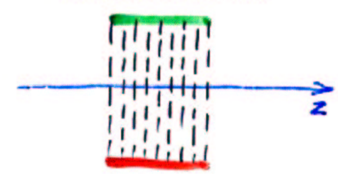
$f(\bar{p})$ – parton distribution function

$\bar{v} \equiv \frac{\bar{p}}{E}$

$$\varepsilon^{zz}(\omega=0,\bar{k}) = 1 + \frac{m_\parallel^2}{\bar{k}^2}$$



$$\varepsilon^{xx}(\omega=0,\bar{k}) = 1 + \frac{m_\perp^2}{\bar{k}^2}$$



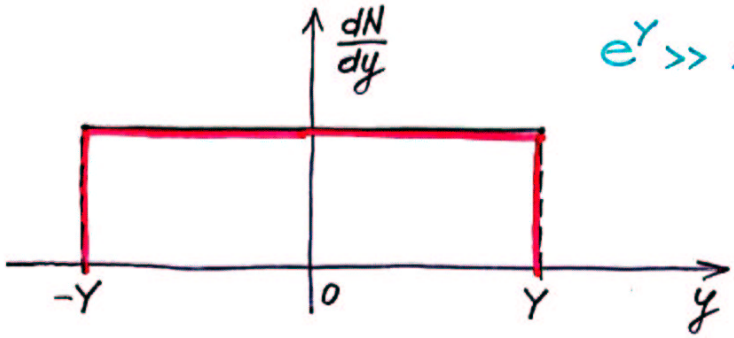
$$m_\parallel^2 = -2\pi\alpha_s\int\frac{d^3p}{(2\pi)^3}\frac{E}{p_z}\frac{\partial f}{\partial p_z}$$

$$m_\perp^2 = -2\pi\alpha_s\int\frac{d^3p}{(2\pi)^3}\frac{E}{p_x}\frac{\partial f}{\partial p_x}$$

$$f(\bar{p}) = f_q(\bar{p}) + \bar{f}_q(\bar{p}) + 2N_c f_g(\bar{p})$$

1) **Flat y-distribution**

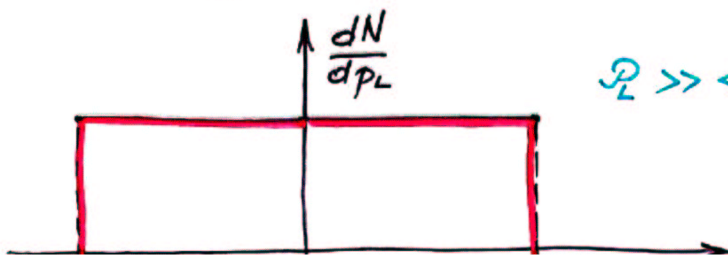
$$f(\vec{p}) = \frac{1}{2Y}\,\Theta(Y-y)\,\Theta(Y+y)\,h(p_\perp)\,\frac{1}{p_\perp \operatorname{ch} y}$$



$e^{Y} \gg 1$

2) **Flat p_L-distribution**

$$f(\vec{p}) = \frac{1}{2\mathcal{P}_L}\,\Theta(\mathcal{P}_L-p_L)\,\Theta(\mathcal{P}_L+p_L)\,h(p_\perp)$$



$\mathcal{P}_L \gg \langle p_T \rangle$

(central collisions)

$$\begin{cases} m_{\parallel}^2 = \dfrac{\pi}{3}\,\dfrac{\alpha_s}{r_0^2 A^{2/3}}\,\dfrac{1}{Y}\left(N_q + N_{\bar q} + \dfrac{9}{4}N_g\right) \\ m_\perp^2 \cong m_\parallel^2 \quad \text{- stable system } (e^{Y} \gg 1) \\ m_\perp^2 \cong \infty\,! \quad \text{- unstable system} \end{cases}$$

$\alpha_s = 0.1,\quad r_0 = 1.1\,\text{fm},\quad A = 197,\quad N_q = N_{\bar q} = 0$

$$\lambda_\parallel = m_\parallel^{-1} = \begin{cases} 0.9\ \text{fm}/c & \text{RHIC} \\ 0.3\ \text{fm}/c & \text{LHC} \end{cases}$$

RHIC: $N_g = 570,\ Y = 2.5$ LHC: $N_g = 8100,\ Y = 5.0$

Biró, Müller, Wang, Phys. Lett. B283 ('92) 171

$L \gtrsim 1\,\text{fm} \qquad L > \lambda$

- **Strings dissolve**

Quasiparticles

- Quasigluons (Plasmons)
- Quasiquarks

Dispersion relation Plasmons

$$\left[k^2 g^{\mu\nu} - k^\mu k^\nu - \Pi^{\mu\nu}(k)\right] A_\nu(k) = 0$$

$$\det\left[k^2 g^{\mu\nu} - k^\mu k^\nu - \Pi^{\mu\nu}(k)\right] = 0$$

$$k_\mu \Pi^{\mu\nu}(k) = 0$$

$$\varepsilon^{ij}(k)\, E^i(k)\, E^j(k) = \Pi^{\mu\nu}(k)\, A_\mu(k) A_\nu(k)$$

$$\varepsilon^{ij}(k) = \delta^{ij} + \frac{1}{\omega^2}\Pi^{ij}(k)$$ chromodielectric tensor

$$\left[\quad \det\left[\mathbf{k}^2 \delta^{ij} - k^i k^j - \omega^2 \varepsilon^{ij}(k)\right] = 0\right.$$

$$\left[\quad \varepsilon^{ij}(k) = \delta^{ij} + \frac{g^2}{2\omega}\int\frac{d^3p}{(2\pi)^3}\frac{v^i}{\omega - \mathbf{k}\cdot\mathbf{v} + i0^+} \times \frac{\partial f(\mathbf{p})}{\partial p_n}\left[\left(1 - \frac{\mathbf{k}\cdot\mathbf{v}}{\omega}\right)\delta^{nj} + \frac{k^n v^j}{\omega}\right]\right.$$

$$\mathbf{v} = \mathbf{p}$$

stable fluctuation

equilibrium

unstable fluctuation

nonequilibrium

$\bar{k} = (k, 0, 0)$

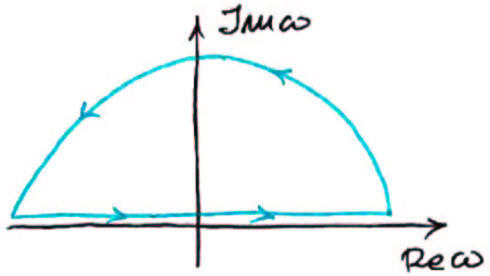
$\bar{E} = (0, 0, E)$

unstable configuration

$\bar{k} = (k,0,0) \qquad \bar{E} = (0,0,E)$

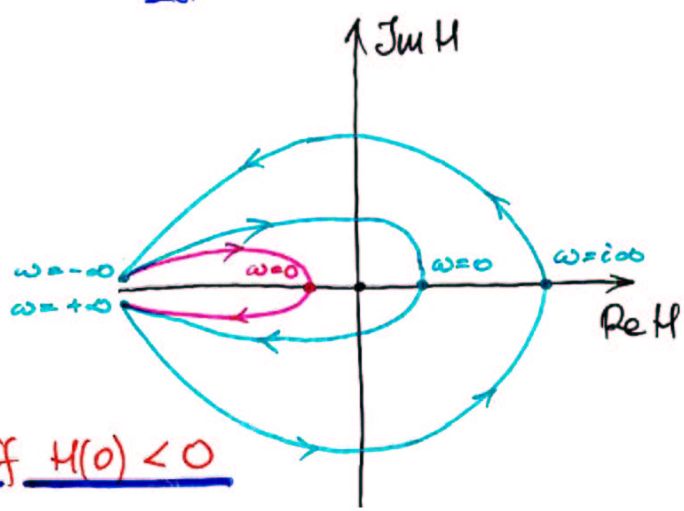
$$\boxed{H(\omega) \equiv k^2 - \omega^2 \varepsilon^{zz}(\omega,k) = 0}$$

Nyquist method



$$\int_C \frac{d\omega}{2\pi i}\frac{1}{H}\frac{dH}{d\omega} = \text{"number of zeros of } H \text{ in } C\text{"}$$

$$= \frac{1}{2\pi i}\Delta \ln H \quad \text{– phase gain along } C$$



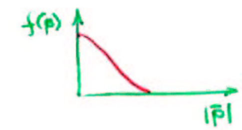
Instability if $H(0) < 0$

$\bar{k} = (k,0,0)$
$\bar{E} = (0,0,E)$

$$\boxed{H(\omega) \equiv k^2 - \omega^2 \varepsilon^{zz}(\omega,k) = 0}$$

$$\varepsilon^{zz}(\omega,k) = 1 + \frac{2\pi\alpha_s}{\omega}\int\frac{d^3p}{(2\pi)^3}\frac{v_z}{\omega - k v_x + i0}\frac{\partial f(\bar{p})}{\partial p_z}$$

Penrose criterion



There are unstable solutions if $H(0) < 0$

$$H(0) = k^2 - \chi^2$$

1) $\chi^2 = \frac{\alpha_s}{4\pi}\frac{e^Y}{Y} p_T^{min} h(p_T^{min}) \geq 0$

flat y-distribution, $e^Y \gg 1$

2) $\chi^2 = \frac{\alpha_s}{4\pi} p_L^{max} h(p_T^{min}) > 0$

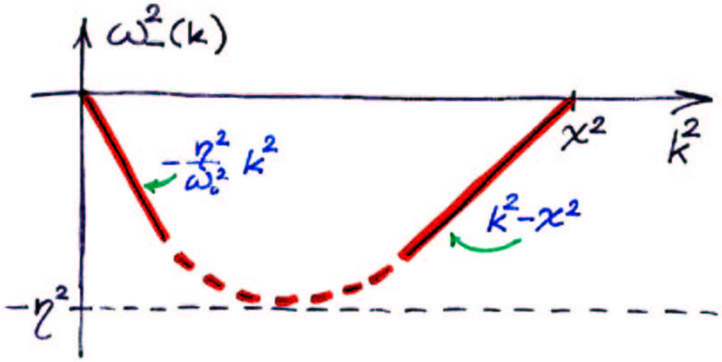
flat p_L-distribution, $\langle p_L \rangle \gg \langle p_T \rangle$

$$\boxed{k^2 - \omega^2 \varepsilon^{zz}(\omega, k) = 0}$$

$\left|\frac{\omega}{k}\right| \gg 1$ $\qquad\qquad$ $\left|\frac{k}{\omega}\right| \gg 1$

$$\boxed{k^2 - \omega^2 + \omega_0^2 + \eta^2 \frac{k^2}{\omega^2} = 0} \qquad \boxed{k^2 - \omega^2 - \varkappa^2 = 0}$$

$\omega_0^2, \eta^2, \varkappa^2$ - constants



$$\tau = \frac{1}{\mathrm{Im}\,\omega}, \quad \tau_{min} = \frac{1}{\eta}$$

$$\eta^2 = \frac{25\pi}{48} \frac{\alpha_s}{r_0^2 A^{2/3}} \frac{1}{Y} \left(N_q + N_{\bar{q}} + \frac{9}{4} N_g\right)$$

for central collision

$\alpha_s = 0.3$ for RHIC, $r_0 = 1.1$ fm, $A = 197$, $N_q = N_{\bar{q}} = 0$

$\alpha_s = 0.1$ for LHC,

$\tau_{min} = 0.4$ fm/c, RHIC: $N_g = 570$, $Y = 2.5$

$\tau_{min} = 0.3$ fm/c, LHC: $N_g = 8100$, $Y = 5.0$

Biró, Müller, Wang, Phys. Lett. B283 (192) 171

There is enough time for the instability to develope.

Spectrum of color fluctuations

$\langle j_a^\mu(x) \rangle = 0$ average color current

$$M_{ab}^{\mu\nu}(x) \stackrel{df}{=} \langle j_a^\mu(x_1)\, j_b^\nu(x_2) \rangle =$$

$$= \frac{1}{8} g^2 \delta^{ab} \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu p^\nu}{E^2} f(\bar{p})\, \delta^{(3)}(\bar{x} - \bar{v}t)$$

$x \equiv x_1 - x_2$, $x \equiv (t, \bar{x})$

$\bar{v} \equiv \bar{p}/E$

parton distribution function

$$M_{ab}^{\mu\nu}(k) = \frac{\pi}{4} g^2 \delta^{ab} \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu p^\nu}{E^2} f(\bar{p})\, \delta(\omega - \bar{k}\bar{v})$$

$k \equiv (\omega, \bar{k})$

$$\int d^4x\, e^{ikx} \delta^{(3)}(\bar{x} - \bar{v}t) = \int dt\, e^{i(\omega - \bar{k}\bar{v})t}$$

- $\bar{J} \parallel$ beam - z-axis
- $\bar{k} \perp$ beam
- $\omega \approx 0$

Flat y-distribution

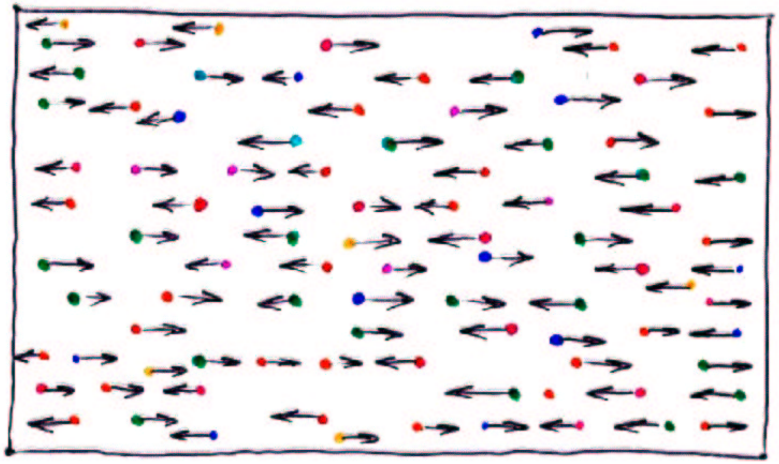
$$M^{zz}(\omega=0, k_x) \approx e^{Y} \cdot M_{eq}(\omega=0, k_x)$$

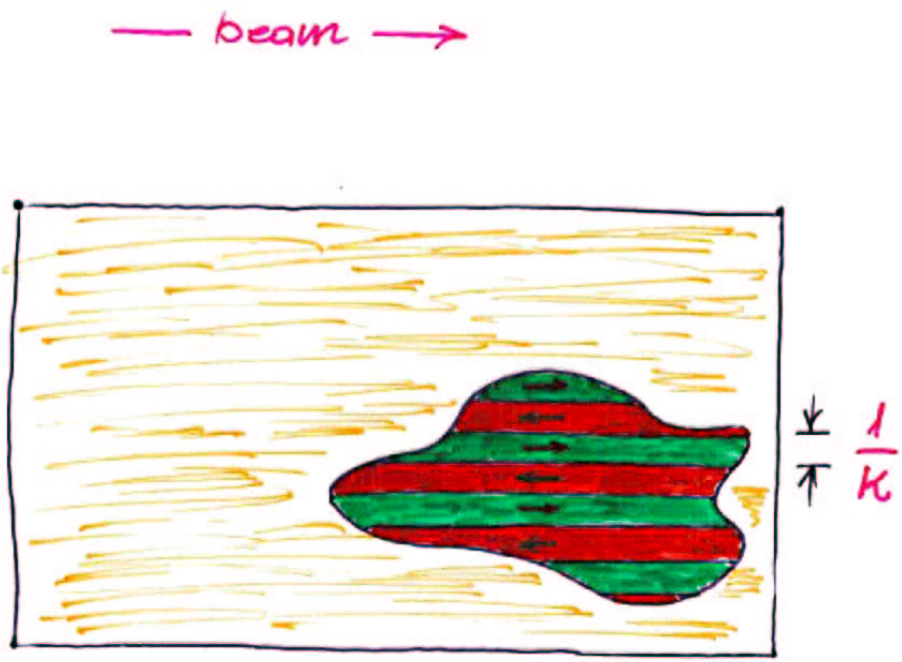
equilibrium fluctuations

Flat p_L-distribution

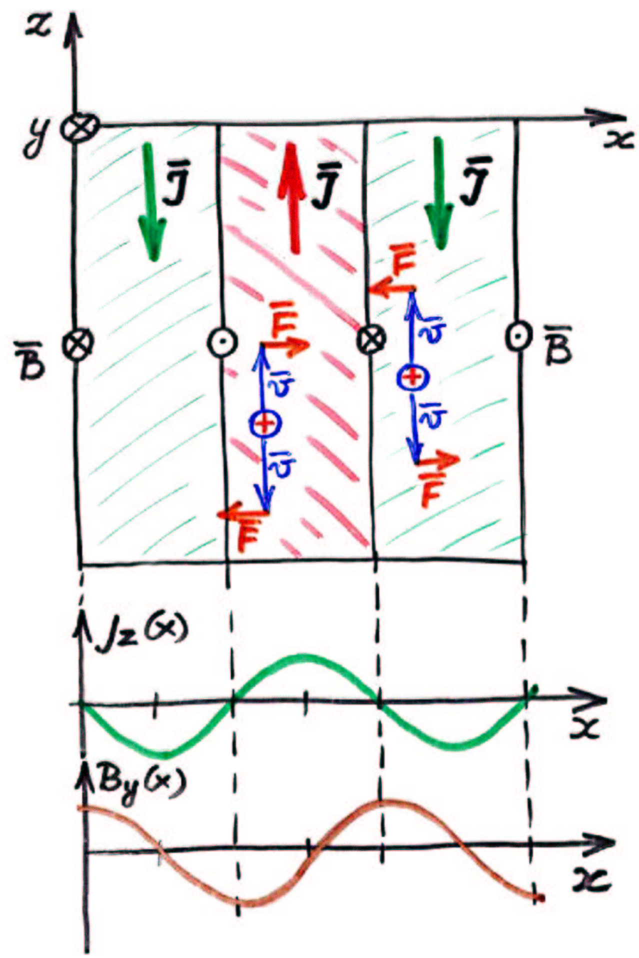
$$M^{zz}(\omega=0, k_x) \approx \frac{\mathcal{P}_L}{\langle p_T \rangle} \cdot M_{eq}(\omega=0, k_x)$$

Amplification of fluctuations!



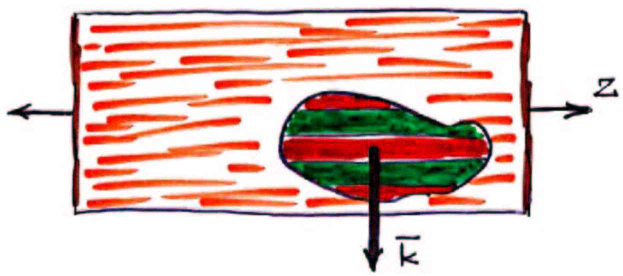


Mechanism of filamentation

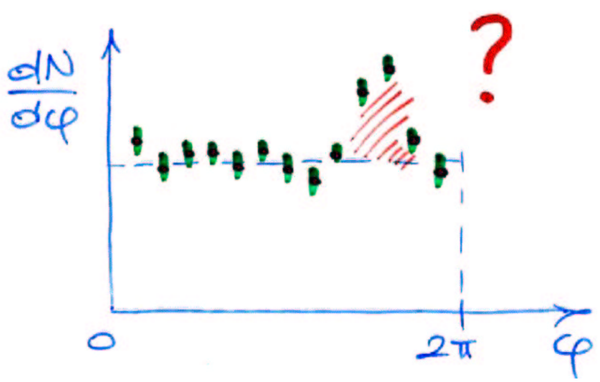


How to detect instability

Instability growth generates transverse collective flow!



Look for azimuthal event-by-event fluctuations.



$\bar{b} = 0$

small p_T

Quasiquark dispersion relation

$m = 0$

$$\det(K - \Sigma(k)) = 0$$

$\Sigma(k) =$ 

Hard Loop Approximation

$\omega, k_i \ll p_i \qquad i = 1,2,3$

$$\Sigma(k) = \frac{g^2}{6}\int \frac{d^3p}{(2\pi)^3}\frac{f(\bar{p})}{E}\cdot\frac{\gamma_0 - \bar{v}\bar{\gamma}}{\omega - \bar{k}\cdot\bar{v} + i0^+}$$

$$\Sigma(k) = \gamma^0\Sigma^0(k) - \bar{\gamma}\,\bar{\Sigma}(k)$$

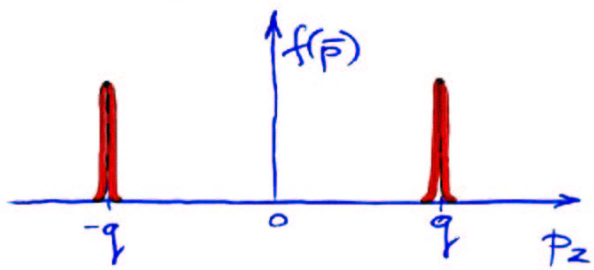
$$(\omega - \Sigma^0(k))^2 - (\bar{k} - \bar{\Sigma}(k))^2 = 0$$

$k \equiv (\omega, \bar{k})$

Extreme anisotropy

Two-stream system:

$$f(\bar{p}) = (2\pi)^3 \rho \left[\delta^{(3)}(\bar{p}-\bar{q}) + \delta^{(3)}(\bar{p}+\bar{q})\right]$$



$$\Sigma^0 = \mu^2 \frac{\omega}{\omega^2 - k_z^2}, \quad \Sigma^x = \Sigma^y = 0$$

$$\Sigma^z = \mu^2 \frac{k_z}{\omega^2 - k_z^2} \qquad \mu^2 = \frac{8}{3} g \frac{\rho}{|q|}$$

$$\omega_\pm^2(k_\perp, k_z) = \mu^2 + k_z^2 + \frac{1}{2}k_\perp^2 \pm \sqrt{\mu^2 k_\perp^2 + \frac{1}{4}k_\perp^4}$$

Two stable modes! $\qquad k_\perp^2 = k_x^2 + k_y^2$

$$\langle p_z^2 \rangle \gg \langle p_\perp^2 \rangle \Rightarrow v_z \cong \pm 1, \; v_T \cong 0$$

Conclusions

- Direction dependent screening
- Unstable plasmons
- Stable quasiquarks