The quark-gluon plasma and its quasiparticles.

ITP - 04/25/02

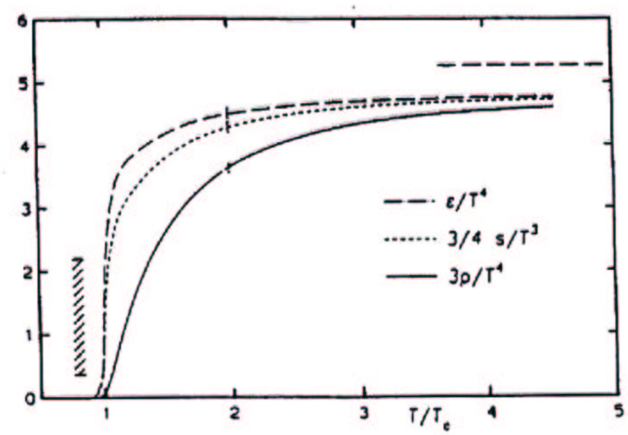
- quasiparticles : weakly coupled degrees of freedom (rel. for dynamics and static properties)
- QGP thermodynamics dominated by hard degrees of freedom ($k \sim T$)
- thermodynamics : entropy is easier
- "first principles" calculations of entropy (using weak coupling techniques) give excellent account of lattice data for $T \gtrsim 2.5\, T_c$

Work done in collaboration with
E. Iancu and A. Rebhan

J.P. Blaizot
SPhT - Saclay

SU(3) EQUATION OF STATE

From E. Laermann, Nucl.Phys. A610(1996)1c

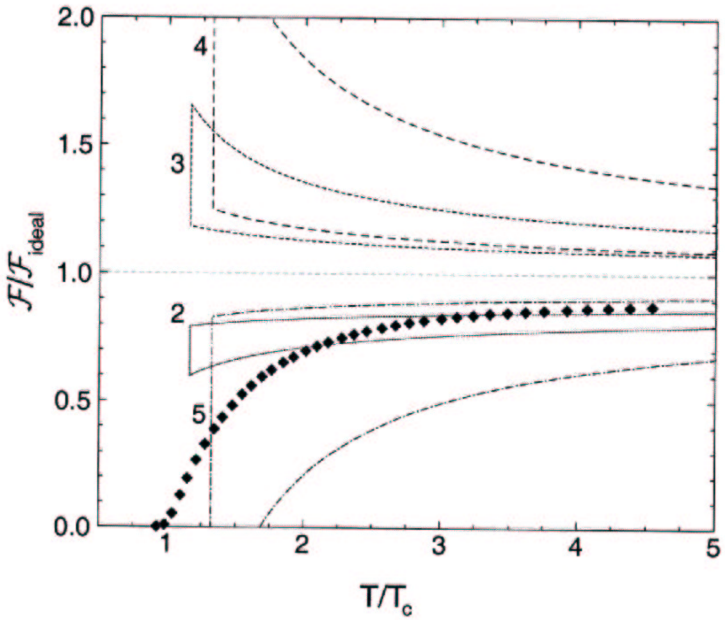


- Thermodynamical functions approach (slowly) the free gas limit at high T.
- **Question:** can we understand the deviation from the ideal gas behaviour with weak coupling calculations?

Perturbation theory up to order g^5

$$\frac{P}{P_0} = 1 + c_2\, g^2 + c_3\, g^3 + [c_4(\bar{\mu}/T) + c_4' \ln g]\, g^4 + c_5(\bar{\mu}/T)\, g^5 + \cdots$$

[Shuryak '78
Kapusta '79
Tomeila '83
Arnold & Zai '94
Zai & Kastening '95]



- Strong deviations from one order to another.
- Huge dependence upon $\pi T < \bar{\mu} < 4\pi T$
- Odd powers of g.

MASSIVE QUASIPARTICLE MODELS

A. Peshier, B. Kämpfer, O. Pavlenko and G. Soff, Phys. Rev. D54 (1996) 2399

- **Simple parametrization:**

$$E_k^2 = k^2 + m^2(T) \qquad n_k = \frac{1}{e^{E_k/T} - 1}$$

$$P = -T \int \frac{d^3k}{(2\pi)^3} \log\left(1 - e^{-E_k/T}\right) - B(T)$$

$$\varepsilon = \int \frac{d^3k}{(2\pi)^3} n_k E_k + B(T)$$

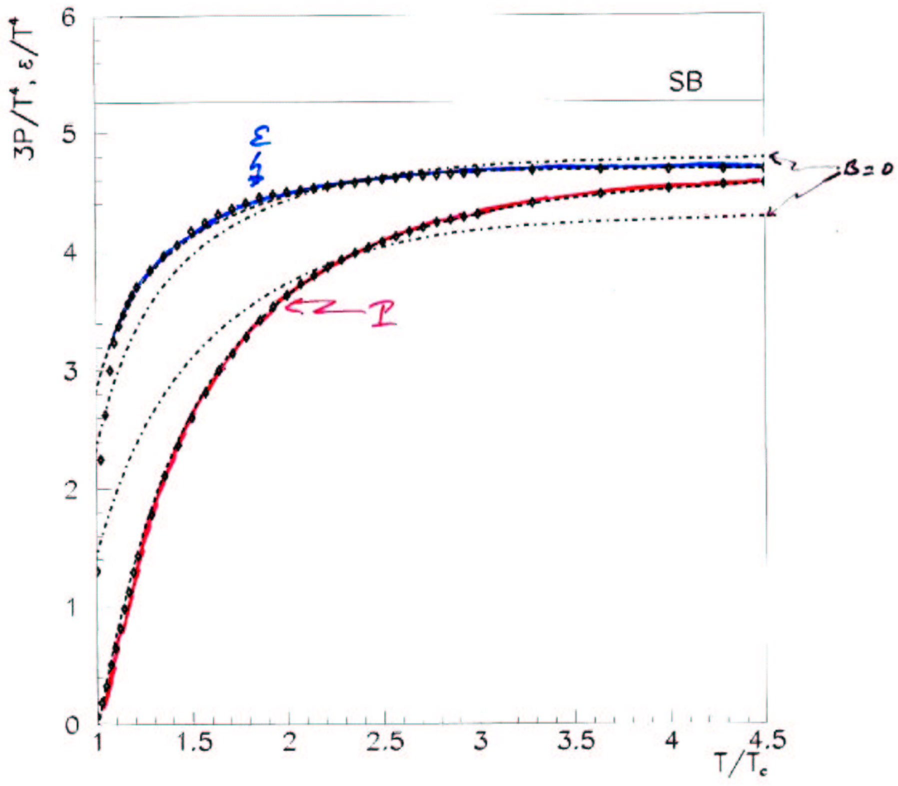
$$s = \int \frac{d^3k}{(2\pi)^3} \left[(1 + n_k)\log(1 + n_k) - n_k \log n_k\right]$$

$$\varepsilon + P = Ts \qquad s = \frac{dP}{dT} \longrightarrow B(T)$$

- **Fit nicely lattice data**

from P. Levai and U. Heinz hep-ph/9710463

Fig.4. SU(3), N_f=0 --- EOS + Lattice QCD data



SCALES, DEGREES OF FREEDOM and FLUCTUATIONS

$$\langle A^2\rangle \approx \int \frac{d^3k}{(2\pi)^3}\frac{N_k}{\varepsilon_k} \qquad N_k = \frac{1}{e^{\varepsilon_k/T}-1} \qquad D_j = \partial_j - igA_j$$

- **Hard** degrees of freedom: the plasma **particles**

$$k \sim T \qquad \langle A^2\rangle_T \sim T^2 \qquad g\bar{A} \sim gT \ll k$$

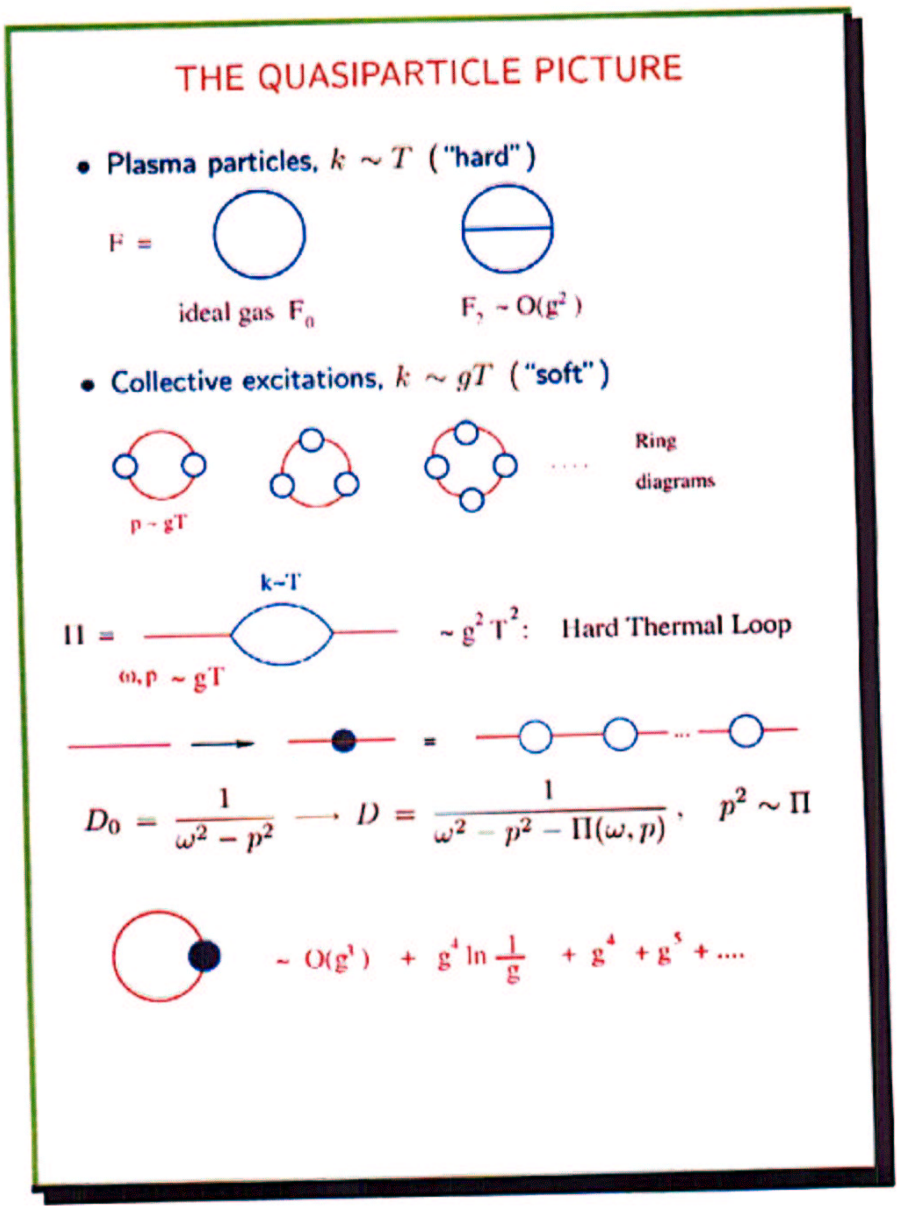
- **Soft** degrees of freedom: **collective modes**

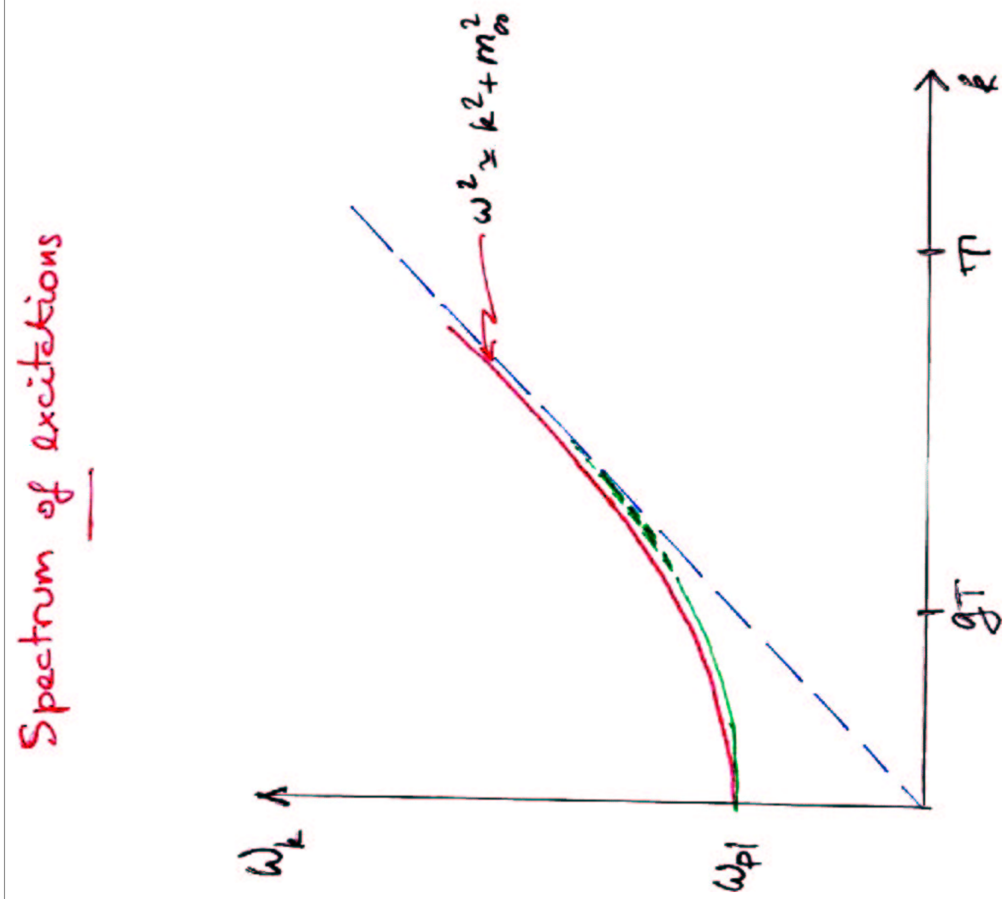
$$k \sim gT \qquad \langle A^2\rangle_{gT} \sim gT^2 \qquad g\bar{A} \sim g^{3/2}T \ll gT$$

- **Ultrasoft** degrees of freedom: **unscreened** magnetic fluctuations.

$$k \sim g^2T \qquad \langle A^2\rangle_{g^2T} \sim g^2T^2 \qquad g\bar{A} \sim g^2T$$

- Thermodynamical functions at high T are dominated by hard degrees of freedom


THE QUASIPARTICLE PICTURE
• Plasma particles, $k \sim T$ ("hard")
F =
ideal gas F_0
$F_2 \sim O(g^2)$
• Collective excitations, $k \sim gT$ ("soft")
Ring diagrams
$p \sim gT$
$k \sim T$
$\Pi =$
$\omega, p \sim gT$
$\sim g^2 T^2$: Hard Thermal Loop
$D_0 = \frac{1}{\omega^2 - p^2} \longrightarrow D = \frac{1}{\omega^2 - p^2 - \Pi(\omega, p)}, \quad p^2 \sim \Pi$
$\sim O(g^3) + g^4 \ln\frac{1}{g} + g^4 + g^5 + \ldots$


Spectrum of excitations
$\omega^2 \simeq k^2 + m_\infty^2$
k
T
gT
ω_k
ω_{pl}

"SCREENED PERTURBATION THEORY"

- **Scalar field**

[Karsch, Patkos, Petreczky, PLB401(1997)]

$$\mathcal{L} = \mathcal{L}_0 - \frac{1}{2}m^2\phi^2 + \frac{1}{2}m^2\phi^2 + \mathcal{L}_{int}$$
$$= \mathcal{L}'_0 + \mathcal{L}'_{int}$$

- **QCD: HTL perturbation theory**

[Andersen, Braaten, Strickland]

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{HTL} - \mathcal{L}_{HTL} + \mathcal{L}_{int}$$
$$= \mathcal{L}'_0 + \mathcal{L}'_{int}$$

DIMENSIONAL REDUCTION

[Kajantie, Laine, Rummukainen, and Schroder (2000)]

- Effective 3-d theory for $\omega_n = 0$

$$\mathcal{L}_{eff} = \frac{1}{4}(F^a_{ij})^2 + \frac{1}{2}(D_i A^a_0)^2 + \frac{1}{2}m^2_D(A^a_0)^2 + \lambda(A^a_0)^4 + \delta\mathcal{L}$$

$$D_i = \partial_i - ig\sqrt{T}A_i$$

- NB. *Singles out one moment of the spectral density:*

$$D(\omega = 0, k) = \int_{-\infty}^{+\infty} \frac{\mathrm{d}k_0}{2\pi}\frac{\rho(k_0, k)}{k_0}$$

SKELETON EXPANSION

[Luttinger & Ward (60), De Dominicis & Martin (64)]

- Expression of the free energy $\mathcal{F}$ in terms of the full propagator D

$$\mathcal{F}[D] = \frac{1}{2}\mathrm{Tr}\ln D^{-1} - \frac{1}{2}\mathrm{Tr}\Pi D + \Phi[D],$$

$\Phi[D] =$ 

- Stationarity property

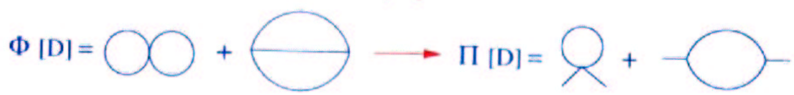
$$\Pi[D] = 2\frac{\delta\Phi}{\delta D} \Longrightarrow \frac{\delta\mathcal{F}[D]}{\delta D} = 0.$$

- Self-consistent approximations [Baym (62)]

Select some particular skeletons in Φ and solve:

$$D^{-1} = D_0^{-1} + \Pi[D]$$

- 2-loop approximation for $\Phi[D]$:



- The entropy $\mathcal{S}[D]$:

$$\mathcal{S} = -\frac{\mathrm{d}\mathcal{F}}{\mathrm{d}T} = -\left.\frac{\partial\mathcal{F}}{\partial T}\right|_D$$

TECHNICALITIES

$$\int_0^\beta \mathrm{d}\tau \int \mathrm{d}^3x \to \sum_{n,even} V \int \frac{\mathrm{d}^3k}{(2\pi)^3}$$

$V =$ volume, $k^\mu = (i\omega_n, \mathbf{k})$, $\omega_n = 2n\pi T$

$\sum_{n,even} \longrightarrow$ standard contour integration:

$$\Omega/V = \int \frac{d^4k}{(2\pi)^4} N(\omega)\mathrm{Im}\left(\log(-\omega^2+k^2+\Pi) - \Pi D\right) + T\Phi[D]/V$$

where $N(\omega) = 1/(e^{\beta\omega}-1)$.

Because of self-consistency condition important cancelations occur in $\mathcal{S} = -d\mathcal{F}/dT$:

$$\mathcal{S} = -\int \frac{d^4k}{(2\pi)^4}\frac{\partial N(\omega)}{\partial T}\mathrm{Im}\log D^{-1}(\omega,k)$$

$$+\int \frac{d^4k}{(2\pi)^4}\frac{\partial N(\omega)}{\partial T}\mathrm{Im}\Pi(\omega,k)\mathrm{Re}D(\omega,k) + \mathcal{S}'$$

with

$$\mathcal{S}' \equiv -\frac{\partial(T\Phi)}{\partial T}\Big|_D + \int \frac{d^4k}{(2\pi)^4}\frac{\partial N(\omega)}{\partial T}\mathrm{Re}\Pi\,\mathrm{Im}D.$$

For the two-loop skeletons (*), $\mathcal{S}' = 0$.

(*) B. Vanderheyden and G. Baym, J. Stat. Phys **93** (1998) 843

THE 2-LOOP ENTROPY

$$\mathcal{S} = -\int \frac{\mathrm{d}^4p}{(2\pi)^4}\frac{\partial N}{\partial T}\left\{\mathrm{Im}\ln D^{-1} - \mathrm{Im}\Pi\,\mathrm{Re}D\right\}$$

- Effectively one-loop expression
- Manifestly ultraviolet-finite
- Perturbatively correct up to order g^3

- **Why entropy ?**
  - $\mathcal{S}$ is most directly related to the quasiparticle spectrum
  - Residual interactions start contributing at order 3-loop
  - Up to order g^3, the self-energy of the hard particles is needed only on the light-cone: $\rightarrow$ "thermal masses" for the plasma particles

$$\omega(p) = \sqrt{p^2+m_\infty^2}, \quad m_\infty \sim gT$$

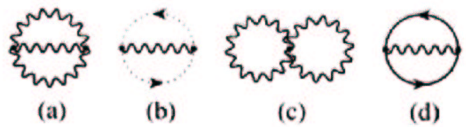
- Reconstructing the pressure $\mathcal{P} = -\mathcal{F}$:

$$\mathcal{P}(T) = \int_{T_0}^{T} \mathrm{d}T'\,\mathcal{S}(T') + \mathcal{P}(T_0)$$

with $\mathcal{P}(T_0)$ taken from the lattice data.

QCD

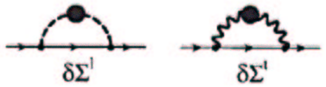
- 2-loop skeletons



- Approximate self-consistency

- In "leading order" $\Pi \sim \Pi_{HTL}$ and $\Sigma \sim \Sigma_{HTL}$

- Corrections to hard particles: "NLA"



Approximately Self-Consistent Entropy

Gauge invariance?

- Replace strict self-consistency by **gauge-invariant** approximations to Π, correct up to order g^3
- Compute the entropy **exactly** with these approximations for Π

$\omega, p \sim gT$: $\Pi_{soft} \approx \Pi_{HTL}$

$\omega, p \sim T$: $\Pi_{hard}(\omega^2 \sim p^2)$

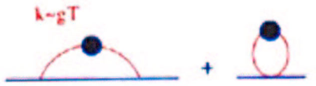
- **The HTL, or leading, approximation:**

$\Pi = \Pi_{HTL}$ at all momenta $\Longrightarrow$ $\boxed{\mathcal{S} = \mathcal{S}_{HTL}}$

Perturbative content: $\mathcal{O}(g^2) + \frac{1}{4}\mathcal{O}(g^3)$

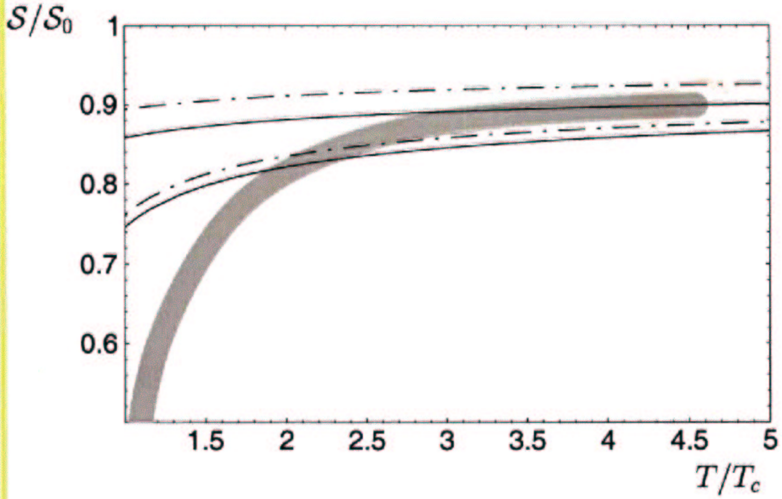
- **The next-to-leading approximation:**

$\Pi_{soft} = \Pi_{HTL}$, $\Pi_{hard} = \Pi_{HTL} + \delta\Pi \Longrightarrow \boxed{\mathcal{S} = \mathcal{S}_{NLA}}$

$\delta\Pi(\omega = p) \equiv$  $= O(g^3T^2)$ (1)

Perturbative content: $\mathcal{O}(g^2) + \mathcal{O}(g^3)$

The entropy for SU(3) Yang-Mills Theory



The entropy of pure SU(3) gauge theory normalized to the ideal gas entropy $\mathcal{S}_0$.

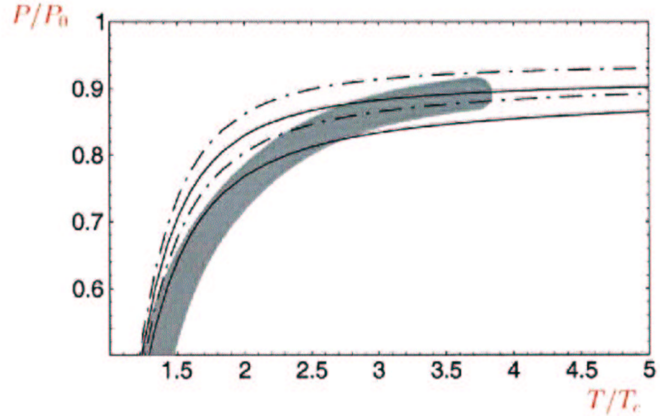
Full lines: $\mathcal{S}_{HTL}$.

Dashed-dotted lines: $\mathcal{S}_{NLA}$.

(The 2-loop β-function is used for the running coupling constant $\alpha_s(\bar{\mu})$. The $\overline{\text{MS}}$ renormalisation scale is $\pi T < \bar{\mu} < 4\pi T$.)

The dark grey band: lattice result by Boyd et al (1996).

RESULTS (2)
Quark-Gluon Plasma with two massless flavours



The pressure for a quark-gluon plasma with $N_f = 2$ versus the lattice results by Karsch et al. (2000).

Full lines: $\mathcal{P}_{HTL}$.

Dashed-dotted lines: $\mathcal{P}_{NLA}$.

CONCLUSIONS

What has been done?

- Identification of relevant degrees of freedom ("quasiparticles") whose mutual interactions are weak
- Check consistency of the approach in weak coupling regime where explicit calculations can be done (up to order g^3)
- When the coupling grows the quasiparticles are non perturbatively renormalized, but their interactions remain weak
- A test of the procedure is the comparison with "exact" results: good agreement with lattice data (when available) down to $T \sim 2.5T_c$

Extensions in progress

- Finite chemical potential
- From $N(\mu, T)$ and $S(\mu, T)$ one can reconstruct $P(\mu, T)$, using lattice data to fix the integration constant (e.g. $P(\mu = 0, T)$)