Color superconducting quark matter

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Reviews:

M. Alford: **hep-ph/0102047** K. Rajagopal, F. Wilczek: **hep-ph/0011333**

I. Introduction to color superconductivity

Low temperatures and densities: confined/broken chiral symmetry phase of QCD.

High temperatures: quark-gluon plasma (QGP)

- chiral symmetry restored
- deconfinement
- signatures sought at heavy-ion colliders

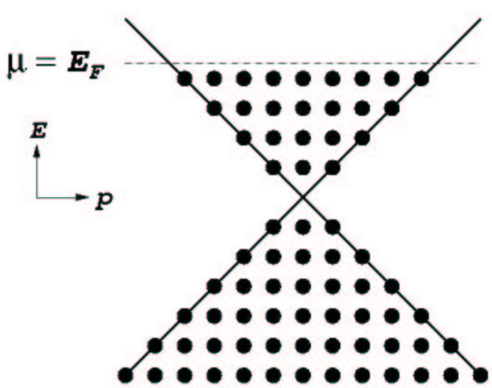
High densities: color superconductivity
quarks *pair* in color non-singlets. Various phases:

$N_f = 3$: CFL, chiral symmetry and baryon number broken.
$N_f = 2$: 2SC, chiral symmetry restored, baryon number unbroken.

Why is 2 flavors so different from 3 flavors?
What sort of quark matter do we expect in compact stars?

Quarks at very high density

At sufficiently high density and low temperature, there is a Fermi sea of almost free quarks.



$$F = E - \mu N$$

But quarks have attractive QCD interactions.

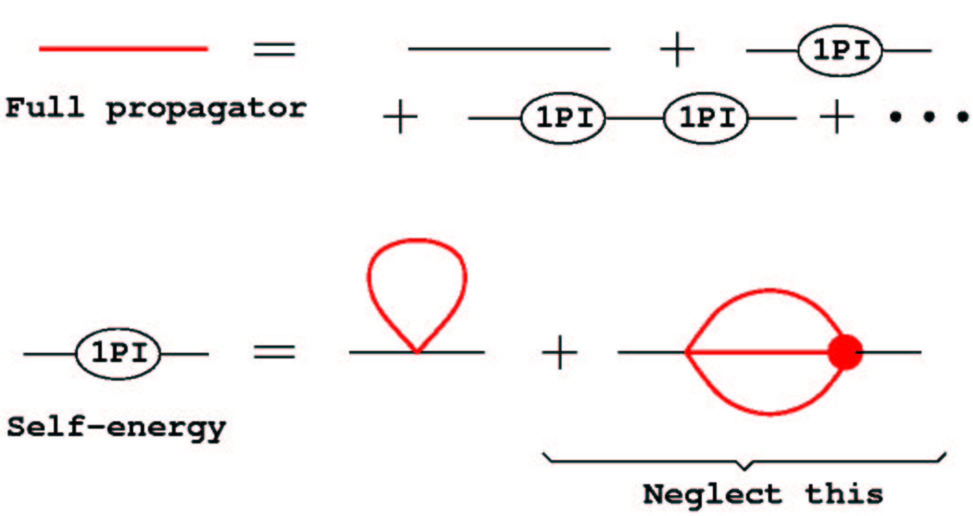
Any attractive quark-quark interaction causes pairing instability of the Fermi surface. This is the Bardeen-Cooper-Schrieffer (BCS) mechanism of superconductivity.

$$\langle qq \rangle \neq 0$$

Gap equation in field theory

The field-theoretic way to look for spontaneous symmetry breaking is to make an ansatz for the self-energy, and solve Schwinger-Dyson equations.

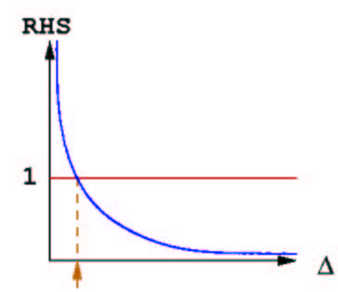
This is diagrammatic, but *non-perturbative*.



Gap equation

$$1 = \frac{8K}{\pi^2}\int_0^{\Lambda} p^2 dp \left\{ \frac{1}{\sqrt{\Delta^2 + (p-\mu)^2}} \right\}$$

$$\Delta = \langle qq \rangle_{1PI}$$



Note BCS divergence as $\Delta \to 0$: there is *always* a solution, for any interaction strength K and chemical potential μ.

Roughly,

$$1 \sim K\mu^2 \ln(\Lambda/\Delta)$$

$$\Rightarrow \Delta \sim \Lambda \exp\left(-\frac{1}{K\mu^2}\right)$$

Superconducting gap is non-perturbative.

II. Different pairing patterns

Three massless flavors: Color-flavor locking (CFL)

Equal number of colors and flavors gives a special pattern of symmetry breaking:

$$\langle q_i^\alpha q_j^\beta \rangle \sim \delta_i^\alpha \delta_j^\beta + \kappa\, \delta_j^\alpha \delta_i^\beta$$

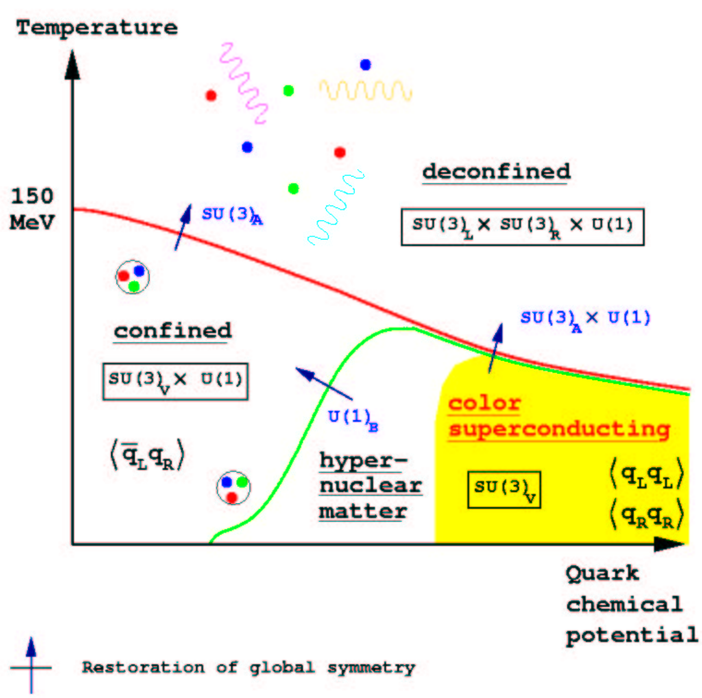
color α, β — This is invariant under equal and opposite
flavor i, j — rotations of color and (vector) flavor

$$SU(3)_{\text{color}} \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_Q} \times U(1)_B \to \underbrace{SU(3)_{C+L+R}}_{\supset U(1)_{\tilde{Q}}} \times \mathbb{Z}_2$$

- All quarks pair, so automatically color neutral.
- Breaks chiral symmetry, but *not* by a $\langle \bar{q} q \rangle$ condensate.
- There need be no phase transition between the low and high density phases: ("quark-hadron continuity")
- Unbroken "rotated" electromagnetism, $\tilde{Q}$, photon-gluon mixture.

Color-flavor locking phase diagram

(three massless flavors)



Quark-hadron continuity

Quark	$SU(2)_{C+V}$	$\tilde{Q}$	Hadron	$SU(2)_V$	Q
u	**2**	$+1$	p	**2**	$+1$
d		0	n		0
s	**2**	0	Ξ^0	**2**	0
s		-1	Ξ^-		-1
$u-d$	**3**	0	Σ^0	**3**	0
u		$+1$	Σ^+		$+1$
d		-1	Σ^-		-1
$u+d+\xi_- s$	**1**	0	Λ	**1**	0
$u+d-\xi_+ s$	**1**	0	—		

Two-flavor color superconductor (2SC)

$$\langle q_i^\alpha q_j^\beta \rangle \sim \varepsilon_{ij}\varepsilon^{\alpha\beta 3}$$

color α, β, flavor i, j

This is a flavor singlet, color $\bar{\mathbf{3}}$.

$$SU(3)_{\text{color}} \times U(1)_Q \times SU(2)_L \times SU(2)_R$$
$$\to\ SU(2)_{\text{color}} \times U(1)_{\tilde{Q}} \times SU(2)_L \times SU(2)_R$$

- Two colors are special: color neutrality must be enforced.
- No global symmetries are broken, so no order parameter.
- Chiral symmetry is restored at high density.
- Unbroken "rotated" electromagnetism, $\tilde{Q}$, photon-gluon mixture.
- Unbroken "rotated" baryon number $\tilde{B} = \tilde{Q} + I_3$.

2+1 flavor phases

Three flavors ($m_s = 0$), CFL:

$\langle q_i^\alpha q_j^\beta \rangle \sim \delta_i^\alpha \delta_j^\beta - c\,\delta_j^\alpha \delta_i^\beta$ chiral symmetry broken

$$SU(3)_{\text{color}} \times SU(3)_L \times SU(3)_R \times U(1)_B \to SU(3)_{C+L+R}$$

2+1 flavors (low m_s), CFL:

u-s and d-s pairs differ from u-d, chiral symmetry broken

$$SU(3)_{\text{color}} \times SU(2)_L \times SU(2)_R \times U(1)_Y \times U(1)_B \to SU(2)_{C+L+R} \times U(1)_Y \xrightarrow{\langle\kappa\rangle} 1$$

2 flavors (high m_s), 2SC:

Strange quarks decouple, u and d pair,

$\langle q_i^\alpha q_j^\beta \rangle \sim \varepsilon_{ij}\varepsilon^{\alpha\beta 3}$ chiral symmetry unbroken

$$SU(3)_{\text{color}} \times U(1)_Q \times SU(2)_L \times SU(2)_R \to SU(2)_{\text{color}} \times U(1)_{\tilde{Q}} \times SU(2)_L \times SU(2)_R$$

III. Compact stars

Where in the universe is color-superconducting quark matter most likely to exist? In compact stars.

A quick history of a compact star.

A star of mass $M \gtrsim 10 M_\odot$ burns Hydrogen by fusion, ending up with an Iron core. Core grows to Chandrasekhar mass, collapses $\Rightarrow$ supernova. Remnant is a compact star:

mass	radius	density	initial temp
$\sim 1.4 M_\odot$	$\mathcal{O}(10\text{ km})$	$\gtrsim \rho_{\text{nuclear}}$	~ 30 MeV

The star cools by neutrino emission for the first million years.

Signatures of color superconductivity in compact stars

Transport properties, mean free paths, conductivities, viscosities, etc.

1. Glitches and crystalline ("LOFF") pairing
2. Cooling by neutrino emission, neutrino pulse at birth
3. r-mode instability

Equation of state

Pressure of quark matter relative to hadronic vacuum

$$p \sim \mu^4 + \Delta^2 \mu^2 - B$$

If bag constant is large enough to bring quark matter close to stability, a superconducting gap Δ may have large effects.

Quark matter in compact stars

- Weak equilibrium

$$\begin{aligned} u &\rightarrow d\, e^+\, \bar{\nu} & \mu_u &= \bar{\mu} - \tfrac{2}{3}\mu_e \\ u &\rightarrow s\, e^+\, \bar{\nu} & \mu_d = \mu_s &= \bar{\mu} + \tfrac{1}{3}\mu_e \end{aligned}$$

- Electromagnetic neutrality

$$\tfrac{2}{3}N_u - \tfrac{1}{3}N_d - \tfrac{1}{3}N_s - N_e = 0$$

But there may be a globally neutral mixture of positive nuclear matter with negative quark matter

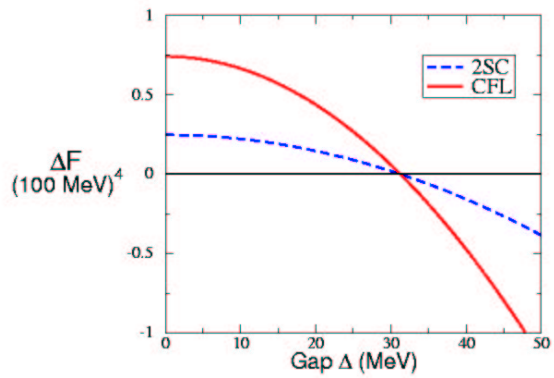
- Color neutrality. In unitary gauge, number of red, green, blue quarks must be the same. The cost of projecting to a color singlet is then negligible.

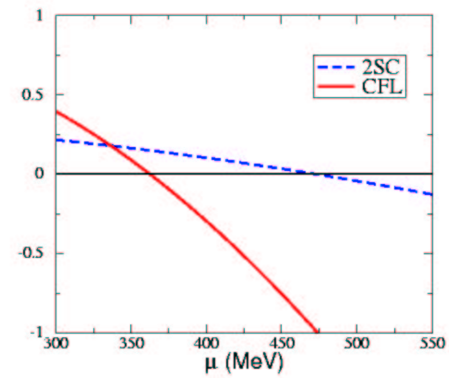
$$\langle Q_3 \rangle = \langle Q_8 \rangle = 0, \quad T_3 = \mathrm{diag}(1, -1, 0), \quad T_8 = \mathrm{diag}(1, 1, -2)$$

Does the 2SC phase occur in compact stars?

The CFL phase is innately neutral, but the 2SC phase needs an electrostatic potential to render it neutral. This imposes a free energy cost. Expanding free energy to $(M_s/\mu)^4$ and $(\Delta/\mu)^2$, we find that 2SC never occurs.

Plot free energy $\Delta F = F - F_{\text{unpaired}}$:

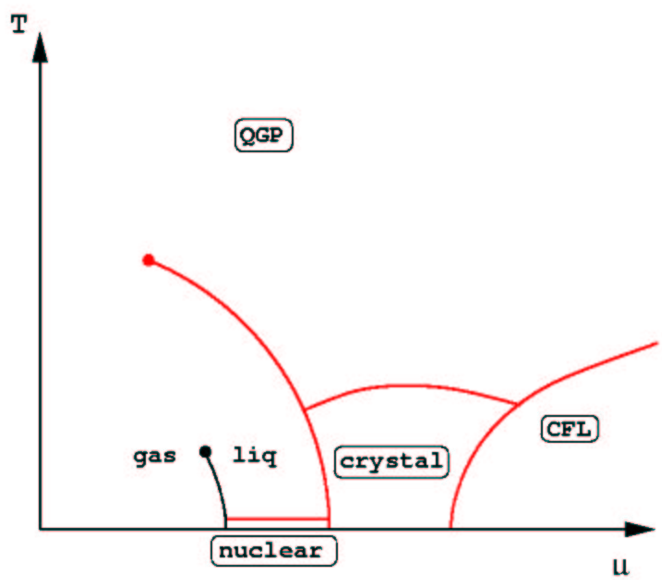




Fixed $M_s = 250$ MeV Δ and M_s smaller in CFL

"Unpaired" region will have crystalline pairing or $J \neq 0$ self-pairing.

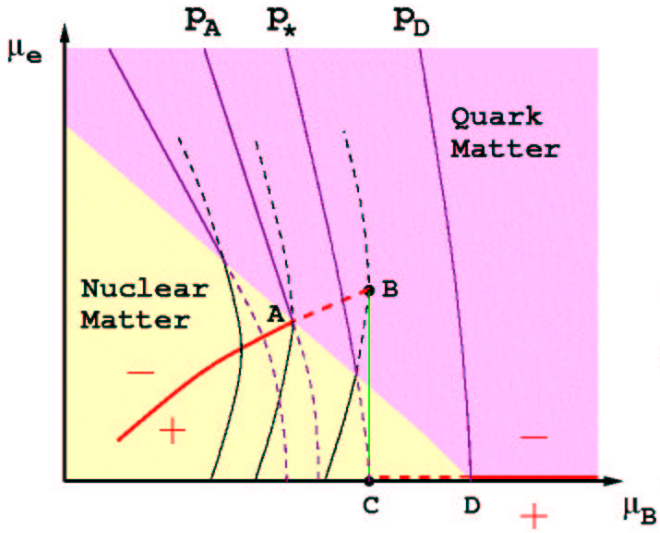
Possible bulk nuclear/quark matter phase diagram



Mixed NM-CFL phase in compact stars

Glendenning, Phys. Rev. D46, 1274 (1992)

$$Q = \frac{\partial p}{\partial \mu}\bigg|_{\mu_B}$$



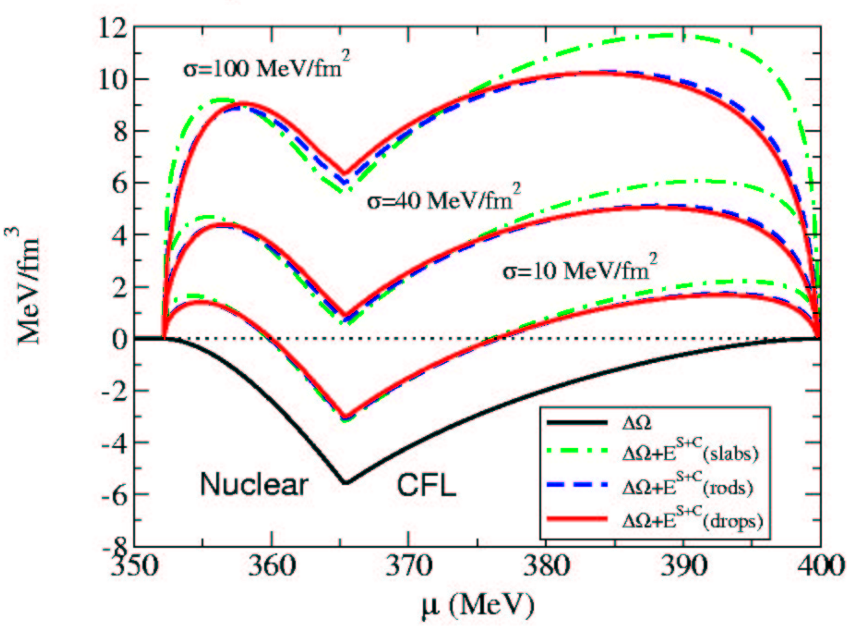
(1) Impose local neutrality: NM to CFL transition at one radius, $A \to B \to C \to D$.

(2) Only impose global neutrality: NM to CFL transition over a range of radii, following *mixed phase* co-existence line $A \to D$.

Mixed phase is favored if surface tension is small, so positive NM and negative CFL can comingle.

Sharp interface or mixed phase?

Free-energy advantage of sharp transition over mixed phase, $F_{\text{mixed}} - F_{\text{sharp}}$.



For $\sigma \gtrsim 40$ MeV, the sharp interface is favored.

Phenomenology of the boundary structure

Mixed phase: neutrinos have short mean free path due to coherent scattering off droplets. Could affect time-signature of supernova neutrino pulse.

Sharp interface: Density changes discontinuously by a factor of 2. Affects mass-radius relationship, and also gravitational waves emitted in collisions between compact stars.

IV. Looking forward

- Compact-star phenomenology:
  - Crystalline phase and glitches
  - Nuclear-quark interface: mixed phase
  - conductivity and emissivity (neutrino cooling)
  - shear and bulk viscosity (r-mode spin-down)
- Other phenomenology:
  - Diquark condensate model of *zero density* confining phase.
  - Role of quark pairing in heavy-ion collisions.
- Other questions:
  - “Kaon” condensation in CFL phase
  - Better weak-coupling calculations, include vertex corrections
  - Go beyond mean-field, include fluctuations.