Adiabatic continuity and symmetry-twisting

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References:
1710.08923 w/ T. Misumi, N. Sakai
1711.10487 w/ Y. Kikuchi, T. Misumi, N. Sakai
1803.02430 w/ G. Dunne, M. Ünsal
1805.11423 w/ T. Sulejmanpasic
1812.02259 w/ M. Hongo, T. Misumi
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Path integral for Quantum Field Theory (QFT)

In many situations, computation of QFTs takes the form

$$Z = \int \mathcal{D}\phi \exp\left(-\frac{1}{g^2} S[\phi]\right).$$

Here, g^2 is the coupling constant, and

$$\phi : (\text{space time}) \to (\text{target space})$$

If we can compute this, we obtain the free energy

$$F = -\frac{1}{\text{volume}} \ln(Z),$$

so we can identify the phases of matter defined by QFTs.

Energy scale-dependent coupling

If $g^2 \ll 1$, we can expect the good approximation as $Z \sim \sum a_n g^{2n}$.

BUT, the statement $g^2 \ll 1$ itself is not always meaningful in QFT.

As we are interested in the low-energy states, we decompose

$$\{\phi\} = \{\phi_{<E}\} \oplus \{\phi_{>E}\},$$

and perform the path integral iteratively:

$$Z = \int \mathcal{D}\phi_{<E} \exp\left(-\frac{1}{g^2(E)} S^{(E)}[\phi_{<E}]\right),$$

$$\exp\left(-\frac{1}{g^2(E)} S^{(E)}[\phi_{<E}]\right) = \int \mathcal{D}\phi_{>E} \exp\left(-\frac{1}{g^2} S[\phi_{<E} + \phi_{>E}]\right).$$

Asymptotic freedom

The coupling $g^2(E)$ depends on the energy E, which we study.

If $g^2(E)$ decreases for smaller E, perturbation can be good.

But, we also encounter the opposite situation, for example,

$$g^2(E) \sim \frac{1}{\ln(E/\Lambda)} \qquad (E \gg \Lambda).$$

(Yang-Mills theory, 2d sigma model, ...) (Gross, Wilczek; Politzer, '73).

Big issue when we are interested in the ground states, as $E \sim 0$.

High temperatures

If we consider the finite temperature T, we can specify the value of running coupling:

$$g^2(T).$$

If $g^2(T) \ll 1$ (i.e. $T \gg \Lambda$), perturbation works well.

Can we use this to know about the ground states?

Thermodynamics tells

$$F = (\text{internal energy}) - T \times (\text{entropy}),$$

and we should minimize this.

Ground state $(T = 0) \Rightarrow$ Small energy (Hamiltonian) states.
Entropy can be small.

High-T $\Rightarrow$ Large entropy states.
As long as you gain F, you can cost the internal energy.

Because of this, ground state often prefers spontaneous breaking, while high-T washes out the order.
$\Rightarrow$ Very different states separated by phase transitions.

Adiabatic continuity and non-thermal b.c.

Introducing T = Putting a system on a cylinder $L = 1/T$.

The boundary condition is periodic/anti-periodic for boson/fermion, but we often encounter the phase transition as we change L.

Adiabatic continuity

We want to connect small and large L without phase transitions or sharp crossovers.

Can we establish it taking non-thermal b.c.?

(Ünsal, '07-, Dunne, Ünsal, '12-, Sulejmanpasic, '16, YT, Misumi, Sakai, '17)

2d $\mathbb{C}P^{N-1}$ sigma model

As a prototype example, we consider

$$S = \frac{1}{g^2}\int \sum_i |(\mathrm{d}+\mathrm{i}a)z_i|^2 + \mathrm{i}\frac{\theta}{2\pi}\int \mathrm{d}a.$$

z_i are complex fields with the constraint

$$\sum_i |z_i|^2 = 1,$$

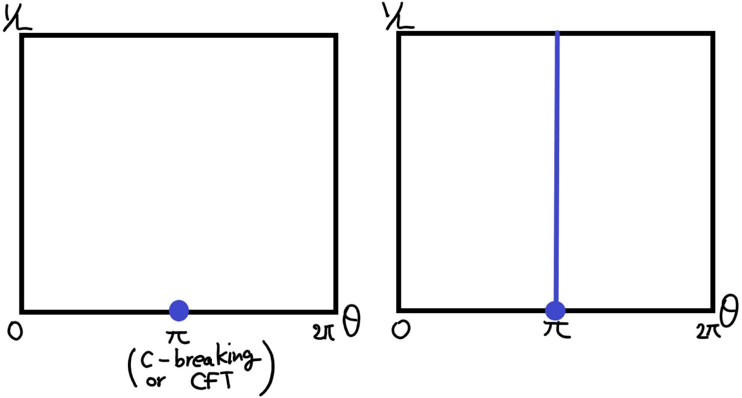
and a is the $U(1)$ gauge field.

Symmetry

- $SU(N)/\mathbb{Z}_N$: $\vec{z}\to U\cdot\vec{z}$.
- C: $z\to z^*$, $a\to -a$. (Only at $\theta=0$ or π)

Thermal & symmetry-twist phase diagrams

$$z_n(x,\tau+L)=z_n(x,\tau) \quad \text{vs.} \quad z_n(x,\tau+L)=\mathrm{e}^{2\pi \mathrm{i} n/N} z_n(x,\tau).$$



With symmetry twist, the C-breaking at $\theta=\pi$ is persistent in L.

Semiclassical explanation for persistence

With the periodic b.c. with small L,

$$F(\theta) \sim -\mathrm{e}^{-N(\cdots)} \cos(\theta),$$

which comes out of (quantum) instanton (Affleck, '80).
Smooth θ dependence.

With the symmetry-twisted b.c., instanton fractionalizes into N constituents:

$$F_{\mathrm{twist}}(\theta) \sim \min_k \left[-N \cos\left(\frac{\theta + 2\pi k}{N} \right) \right].$$

Singularity at $\theta = \pi$ exists even at small L. (Dunne, Ünsal, '12)

Anomaly viewpoint of C-breaking

Spontaneous C-breaking of 2d $\mathbb{C}P^{N-1}$ model is a kinematical consequence due to anomaly:

(Lieb, Schultz, Mattis '61, Affleck, Lieb '86, Komargodski, Sharon, Thorngren, Zhou, '17)

Under the presence of $SU(N)/\mathbb{Z}_N$ background field (A, B_2),

$$C : Z_{\theta=\pi}[A, B_2] \to \exp\left(\mathrm{i}\int_{\mathbb{R}^2} B\right) Z_{\theta=\pi}[A, B_2].$$

For consistency, the ground state at $\theta = \pi$ breaks C spontaneously.

or, supports gapless excitations.

Anomaly of S^1-compactified theory

The anomaly of $\mathbb{C}P^{N-1}$ model disappear after naive compactification.
$\Rightarrow$ This is consistent with the preference of disordering at high-T.

With the symmetry-twisted b.c., there is a "center" symmetry, $\mathbb{Z}_N \subset SU(N)/\mathbb{Z}_N$:

$$z_n \mapsto z_{n+1},\ P = \mathrm{e}^{\mathrm{i}\int_{S^1} a} \mapsto \exp\left(\frac{2\pi \mathrm{i}}{N}\right) P.$$

$\mathbb{Z}_N$ symmetry has the same anomaly of 2d theory: $B_2 = B_1 \wedge \mathrm{d}x^2/L$,

$$C : Z^{(\mathrm{twist})}_{\theta=\pi}[B_1] \mapsto \exp\left(\mathrm{i}\int_{\mathbb{R}} B_1\right) Z^{(\mathrm{twist})}_{\theta=\pi}[B_1].$$

(YT, Misumi, Sakai, 1710.08923)

How does it evade the energy-entropy argument?

	Hilbert space picture (Meson spectrum)	Path integral picture (KK modes)	Anomaly picture
$\mathcal{Z}(\beta)$	Sing. 1 Adj. $1\quad 1\quad 1\quad \cdots$ $\Rightarrow \mathrm{tr}_{\mathrm{adj.}}(1) = N^2-1$	$\frac{2\pi}{\beta}$	$\mathbb{R}\times S^1_\beta$ $\beta=\infty$ anomaly in 2d No constraint by 't Hooft anomaly $\beta = 0$
$\mathcal{Z}_\Omega(L)$	Sing. q^0 Adj. $q^0\quad q^1\quad q^2\quad \cdots$ $\Rightarrow \mathrm{tr}_{\mathrm{adj.}}(\Omega) = -1$	$\frac{2\pi}{NL}$	$\mathbb{R}\times S^1_{L,\Omega}$ $L=\infty$ anomaly in 2d constraint by 't Hooft anomaly $L = 0$

$(q = \mathrm{e}^{2\pi\mathrm{i}/N})$

Huge cancellation exists due to the phase factor by symmetry-twist.

(Dunne, YT, Ünsal, 1803.02430, Sulejmanpasic ’16)

Strength of anomaly viewpoint

Anomaly makes the generalization to other QFTs much easier.

We can also study, for example,

- 2d matrix-type sigma models. ($\Leftarrow$ the rest of this talk)
- 4d QCD with $\gcd(N_c, N_f) > 1$ (YT, Kikuchi, Misumi, Sakai, 1711.10487, Shimizu, Yonekura, '17, Furusawa, YT, Itou, '20).

Because of the presence of anomaly under S^1 compactification, these systems enjoy the persistent order with symmetry-twisting.

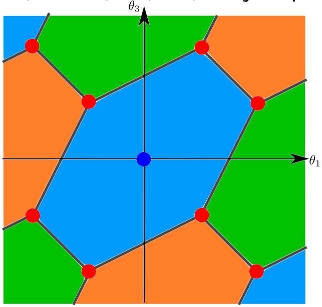
Flag manifold, $SU(N)/U(1)^{N-1}$, sigma model

This is the sigma model with $S[\Phi, a_i]$ with

- $\Phi = [\boldsymbol{\phi}_1, \ldots, \boldsymbol{\phi}_N]$: $SU(N)$-valued field
- a_i: $U(1)$ gauge fields ($i = 1, 2, \ldots, N-1$).

This theory appears in the study of quantum $SU(N)$ spin chain

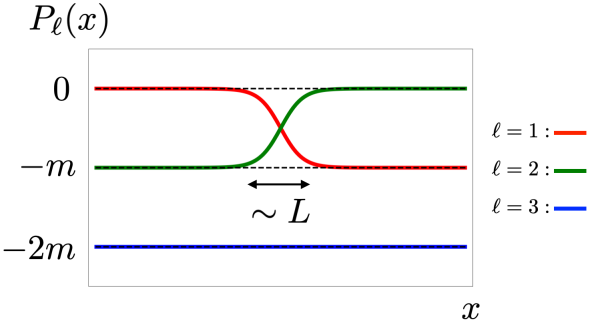
(Bykov, '12-, Lajkó, Wamer, Mila, Affleck, '17, YT, Sulejmanpasic, 1805.11423)



Red blobs = $SU(3)_1$ WZW model, or trimerized phase.

Fractional instanton in symmetry-twisted case

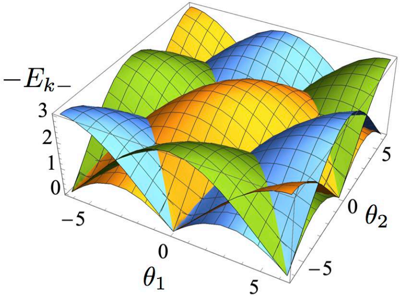
Under $\mathbb{Z}_N$ twisted b.c., the minimal BPS configuration has fractional topological charges: (Hongo, Misumi, YT, 1812.02259)

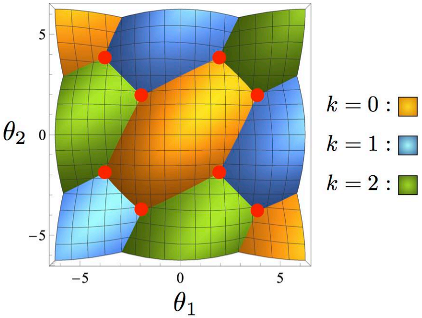


$P_\ell = \exp\left(\mathrm{i} \int_{S^1} a_\ell\right)$.

Ground-state energies with DIGA

On small L, DIGA with fractional instantons can be computed:





It reproduces the global nature of the 2d phase diagram expected by anomaly. (Hongo, Misumi, YT, 1812.02259)

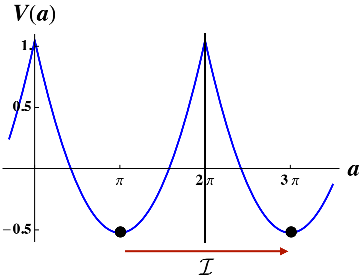
Open issue: Gapless vs. GS degeneracy

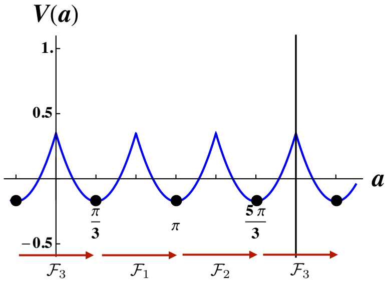
In the symmetry-twisted compactification, we can preserve the anomaly of 2d theory.

Since 1d quantum mechanics have the gap, this anomaly is always matched by the ground-state degeneracy.

Can we distinguish whether 2d theory is gapless or degenerate?

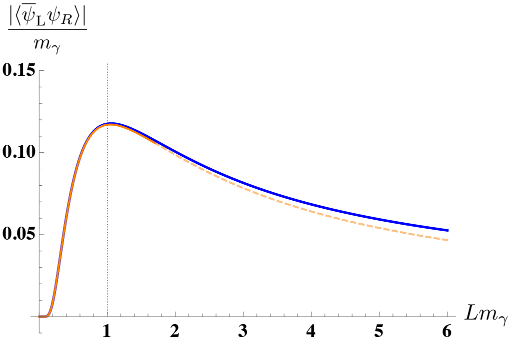
As a trial, we map the theory to N-flavor massless Schwinger model.
(Misumi, YT, Ünsal, 1905.05781)





Again, quantum instanton fractionalizes by symmetry twist.
(Smilga, '93, Shifman, Smilga, '94)

Chiral condensate with twisted b.c.:



Solid curve is the exact result: $\langle\overline{\psi}_L\psi_R\rangle \sim L^{-(1-1/N)}$ as $L\to\infty$.
Dashed curve is the semiclassical result for small L:
$\langle\overline{\psi}_L\psi_R\rangle \sim (NL)^{-1}\exp\left(-\frac{\pi}{NLm_\gamma}\right)$.

Summary

- Anomaly interpretation for the usefulness of symmetry twist
- Explicit realization of anomaly matching in semiclassical regime
- Idea works nicely for both vector-like and matrix-like QFTs
- Gapless vs. degeneracy in 2d limit?