# Real time confinement following a quench to a non-integrable model

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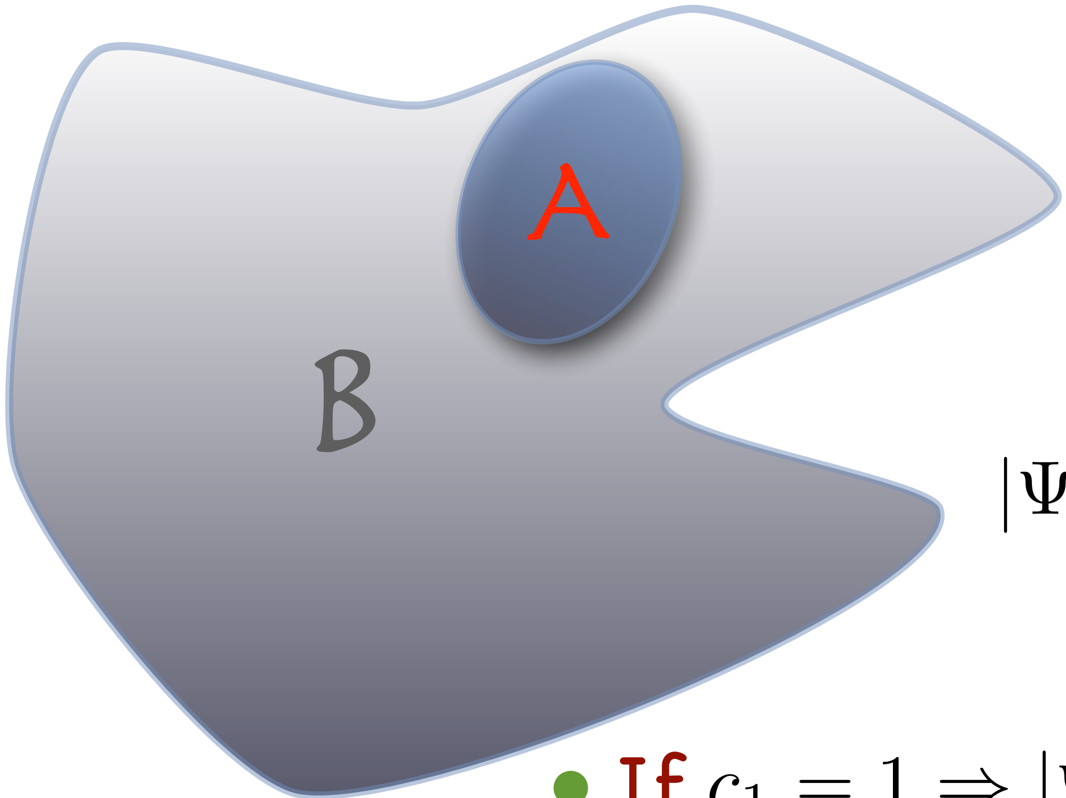
# Quantum quenches

- prepare the system in some initial state
  in many cases the ground state of a (local) Hamiltonian
- let it evolve **unitarily** with some other (local) Hamiltonian
  the system is **isolated**!
- questions about relaxation, thermalization
- role of integrability — GGE, prethermalization etc.

In this talk: small perturbations can have dramatic effects

# Entanglement entropy

Consider a system in a quantum state $|\Psi\rangle$




$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

Schmidt decomposition

$$|\Psi\rangle = \sum_n c_n |\psi_n\rangle_A |\psi_n\rangle_B \qquad c_n \geq 0, \ \sum_n c_n^2 = 1$$

- If $c_1 = 1 \Rightarrow |\Psi\rangle$ is unentagled
- If all $c_i$ are equal $\Rightarrow |\Psi\rangle$ is maximally entangled

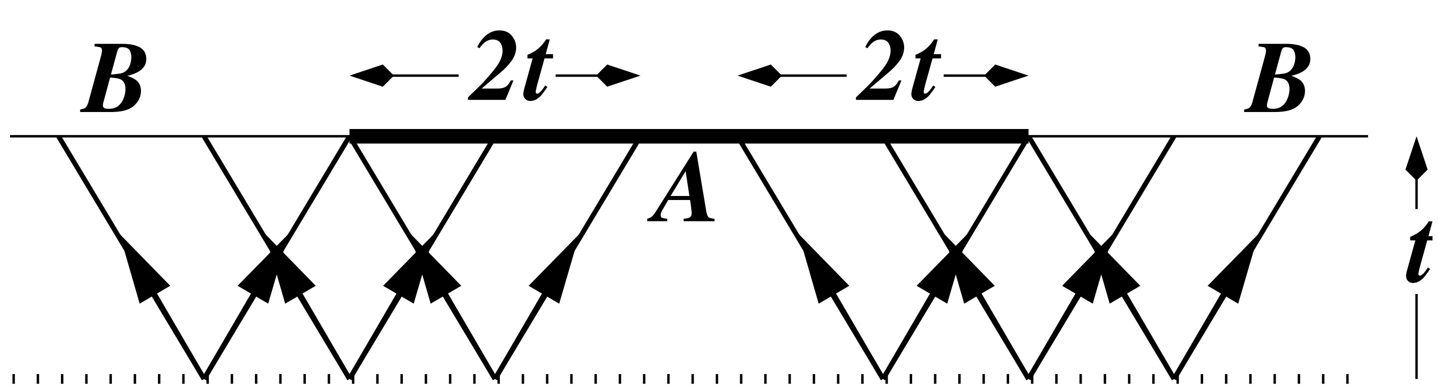
A natural measure is the entanglement entropy $(\rho_A = \mathrm{Tr}_B \rho)$

$$S_A \equiv -\mathrm{Tr}\rho_A \ln \rho_A = S_B$$
$$= -\sum_n c_n^2 \ln c_n^2$$

# Light cone spreading of entanglement

P. Calabrese, J. Cardy 2005

- After a global quench, the initial state $|\psi_0\rangle$ has an extensive excess of energy
- It acts as a source of quasi-particles at $t=0$. A particle of momentum p has energy $E_p$ and velocity $v_p = \mathrm{d}E_p/\mathrm{d}p$
- For $t>0$ the particles move semiclassically with velocity $v_p$
- Particles emitted from regions of size of the initial correlation length are entangled, particles from points far away are incoherent
- The point $x\in A$ is entangled with a point $x'\in B$ if a left (right) moving particle arriving at $x$ is entangled with a right (left) moving particle arriving at $x'$. This can happen only if $x \pm v_p t \sim x' \mp v_p t$



# Light cone spreading of entanglement

P. Calabrese, J. Cardy 2005

- The entanglement entropy of an interval $A$ of length $\ell$ is proportional to the total number of pairs of particles emitted from arbitrary points such that at time $t, x \in A$ and $x' \in B$

- Denoting with $f(p)$ the rate of production of pairs of momenta $\pm p$ and their contribution to the entanglement entropy, this implies

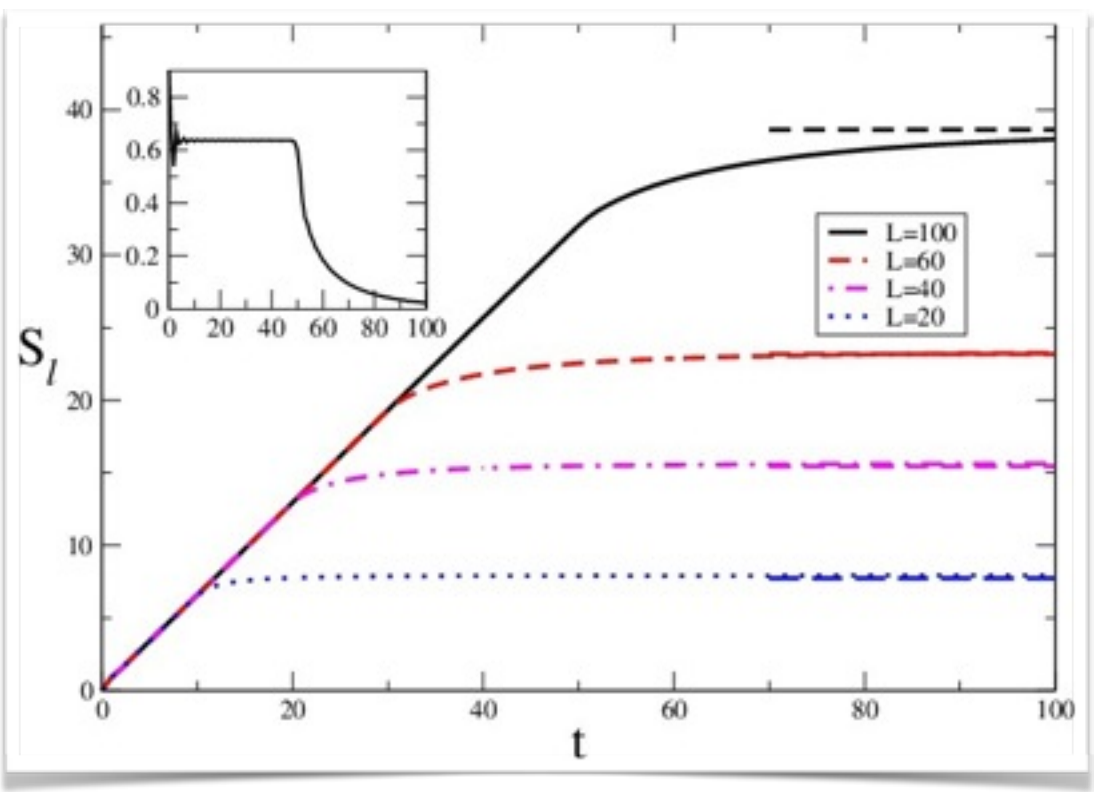
$$S_A(t) \approx \int_{x' \in A} dx' \int_{x'' \in B} dx'' \int_{-\infty}^{\infty} dx \int f(p) dp \delta\big(x' - x - v_p t\big) \delta\big(x'' - x + v_p t\big)$$

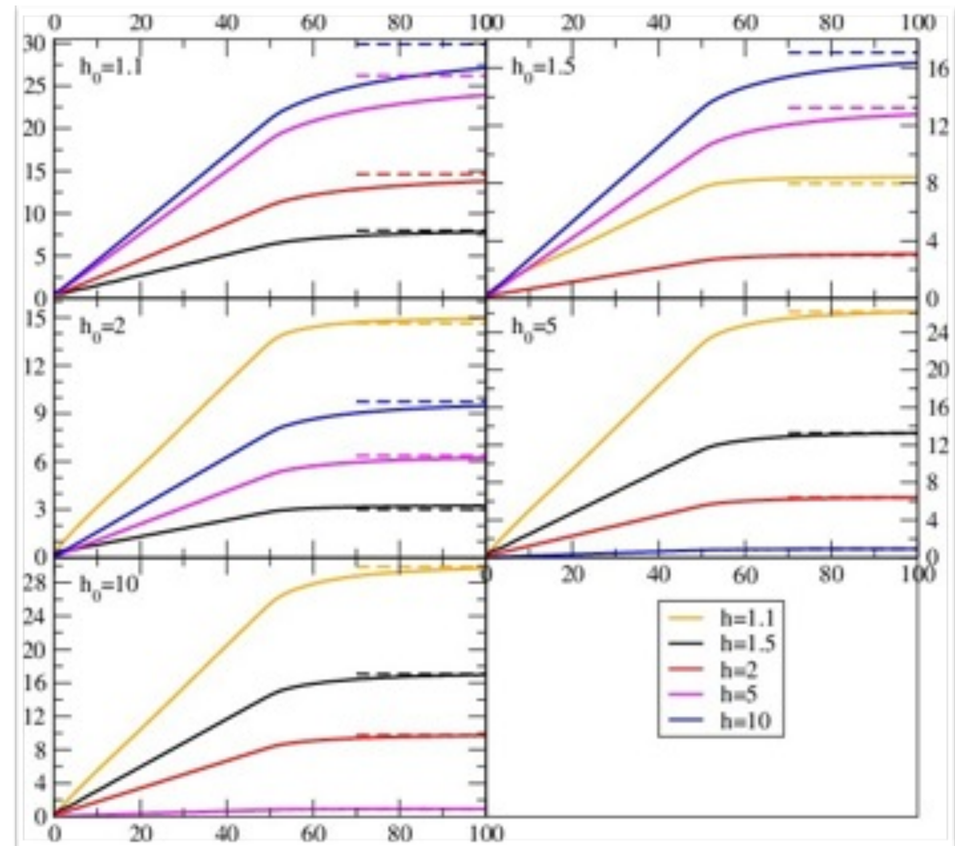
$$\propto t \int_0^{\infty} dp f(p) 2 v_p \theta(\ell - 2 v_p t) + \ell \int_0^{\infty} dp f(p) \theta(2 v_p t - \ell)$$

- When $v_p$ is bounded (e.g. Lieb-Robinson bounds) $|v_p| < v_{\max}$, the second term is vanishing for $2 v_{\max} < \ell$ and the entanglement entropy grows linearly with time up to a value linear in $\ell$

# Example: Transverse Field Ising chain

P. Calabrese, J. Cardy 2005




Analytically for $t, l \gg 1$ with $t/l$ constant

M. Fagotti, P. Calabrese, 2008

$$S(t) = t \int_{2|\epsilon'|t<\ell} \frac{d\varphi}{2\pi} 2|\epsilon'| H(\cos\Delta_\varphi) + \ell \int_{2|\epsilon'|t>\ell} \frac{d\varphi}{2\pi} H(\cos\Delta_\varphi)$$

$$\cos\Delta_\varphi = \frac{1 - \cos\varphi(h + h_0) + hh_0}{\epsilon_\varphi \epsilon^0_\varphi}$$

$$H(x) = -\frac{1+x}{2}\log\frac{1+x}{2} - \frac{1-x}{2}\log\frac{1-x}{2}$$

# Light cone spreading of correlations

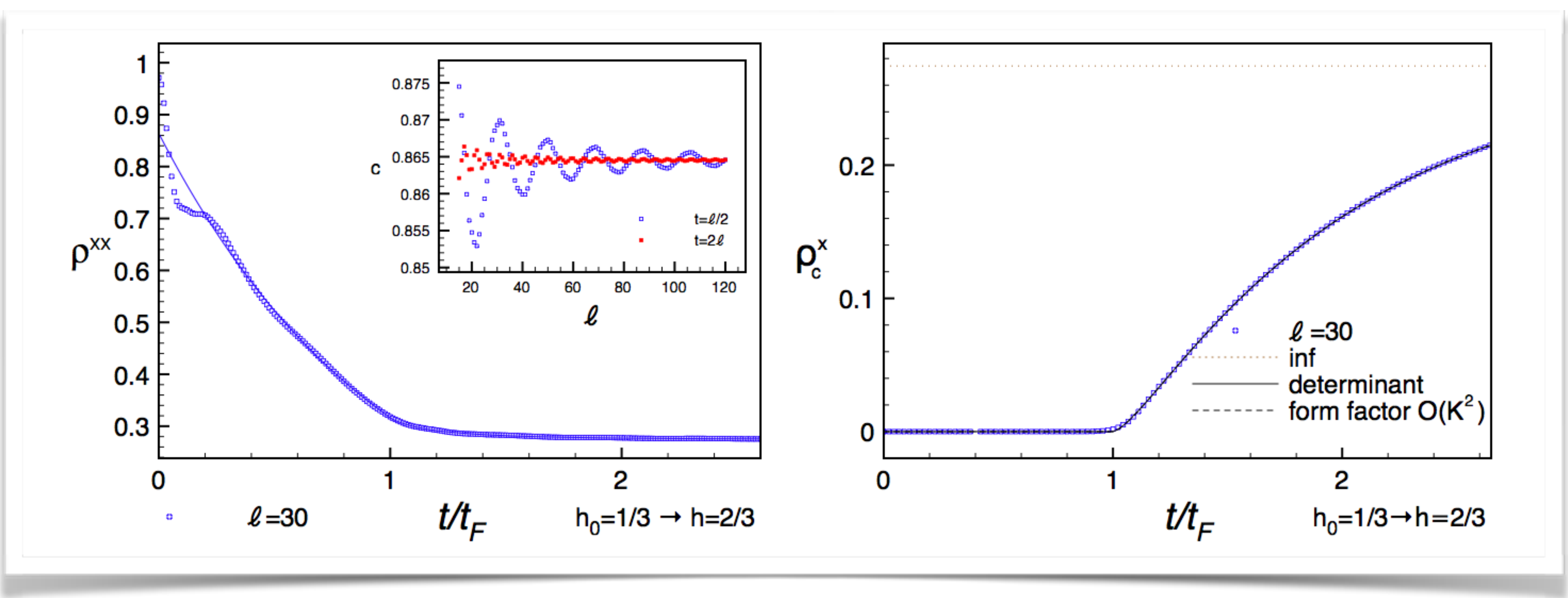
The same scenario is valid for correlations:

- **Horizon**: points at separation $r$ become correlated when left- and right-moving particles originating from the same point first reach them

- If $|v_p| < v_{\max}$, connected correlations are then frozen for $t < r/2v_{\max}$

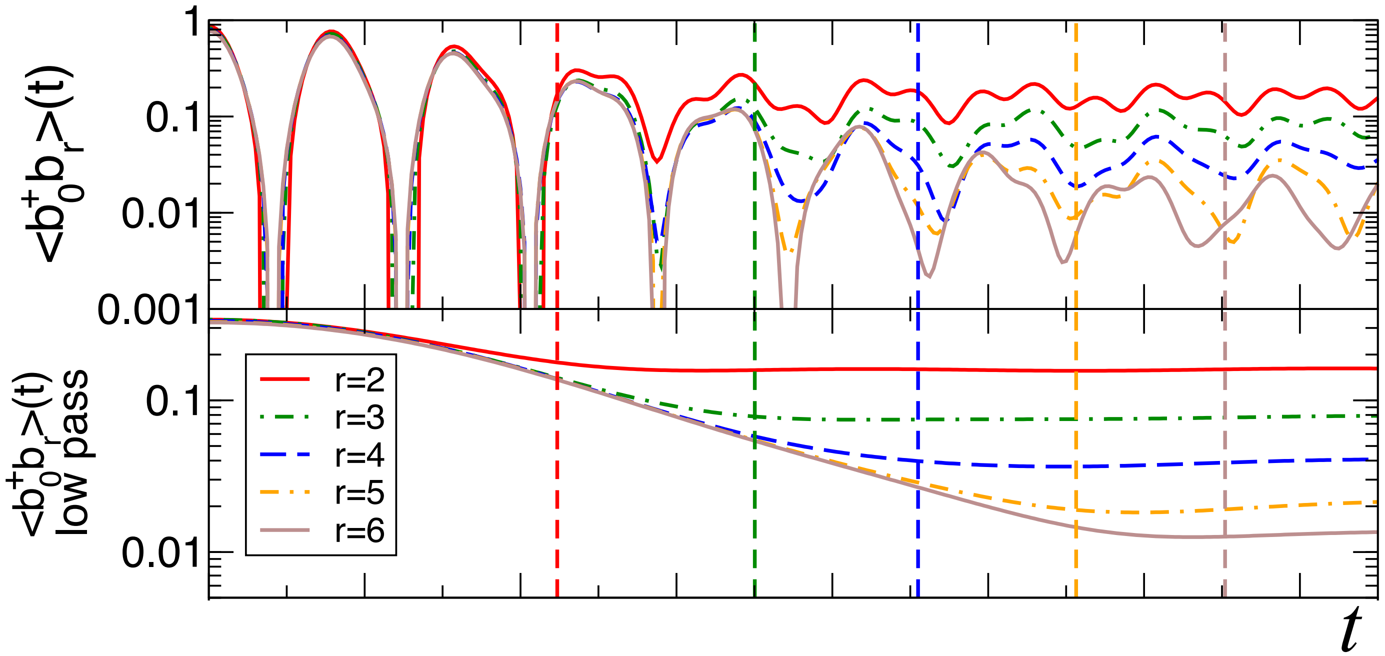
**Example**: Ising model within ferromagnetic phase

P. Calabrese, F. Essler, M. Fagotti 2011/12

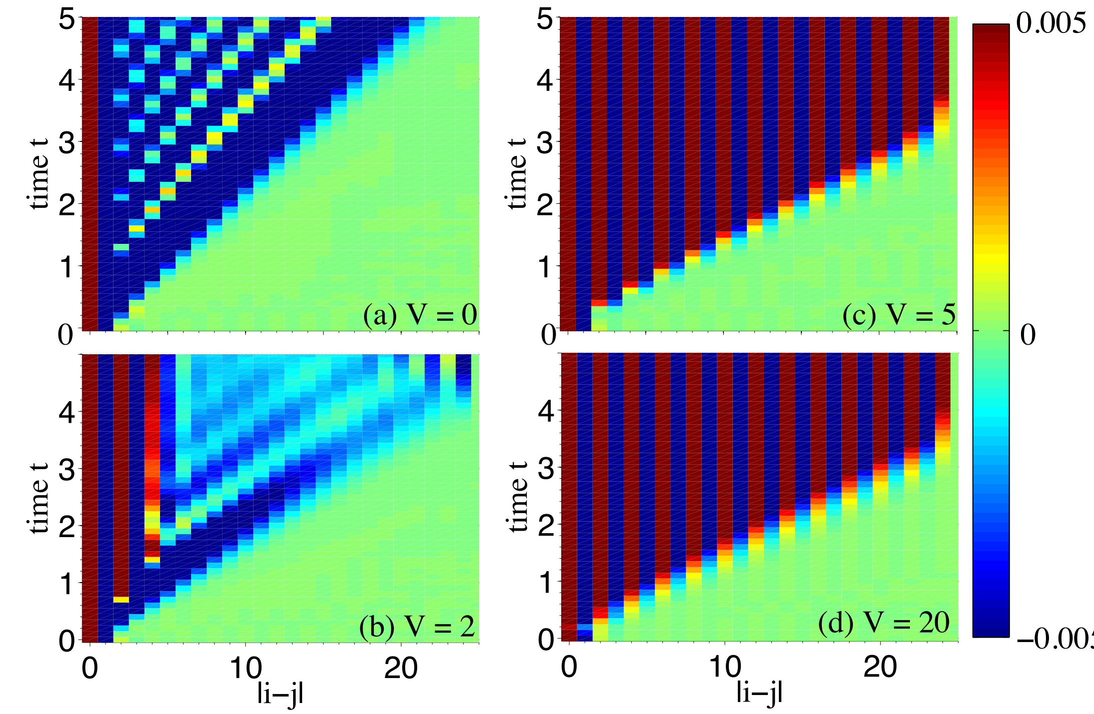


# Light cone in interacting models

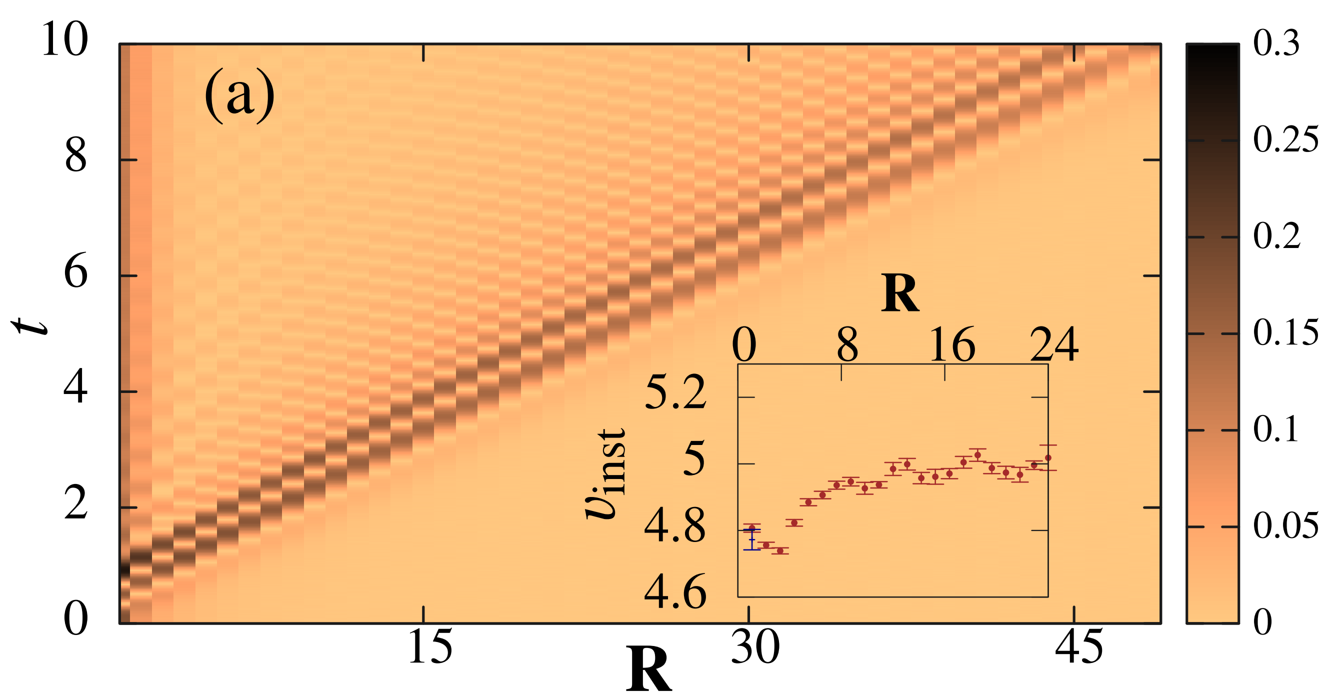
Kollath-Lauechli '08: Bose-Hubbard



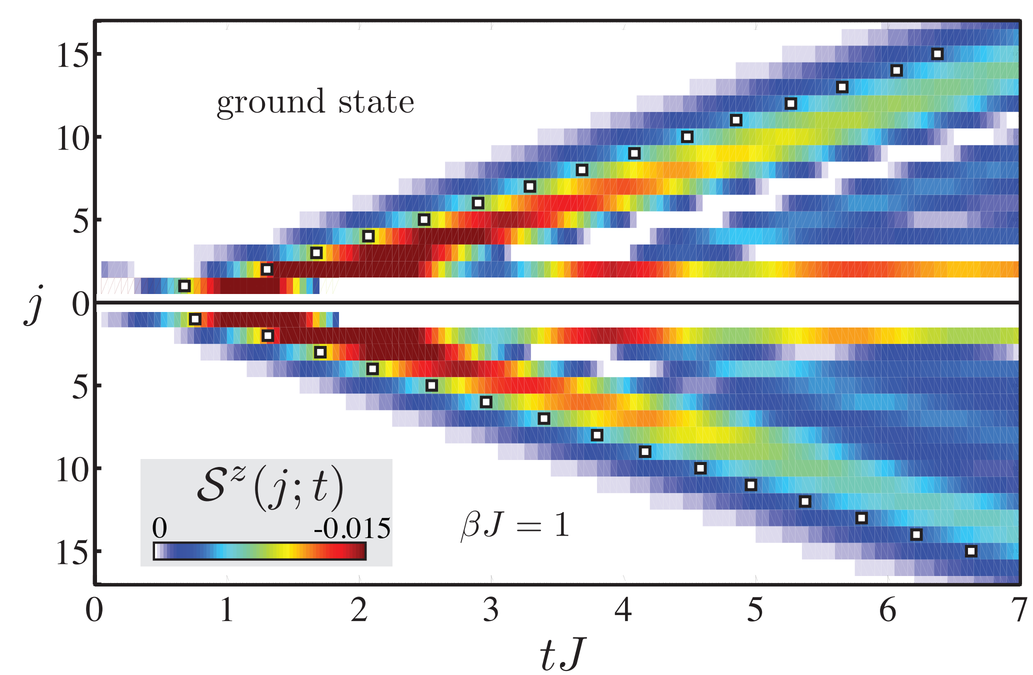

Manmana et al '08: interacting fermions




Carleo et al., '14: Bose-Hubbard




Bonnes, Essler, Lauchli '14: XXZ spin chain



# Light cone in experiments

M. Cheneau et al., Nature **481**, 484 (2012)

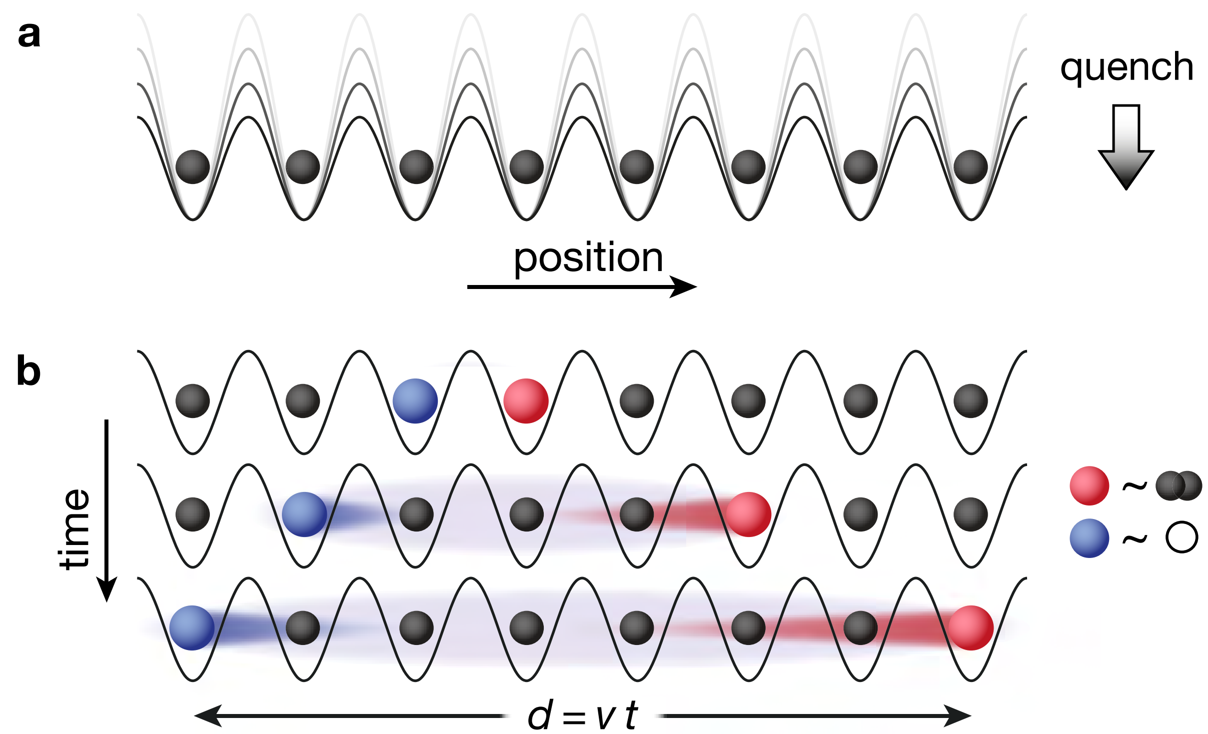
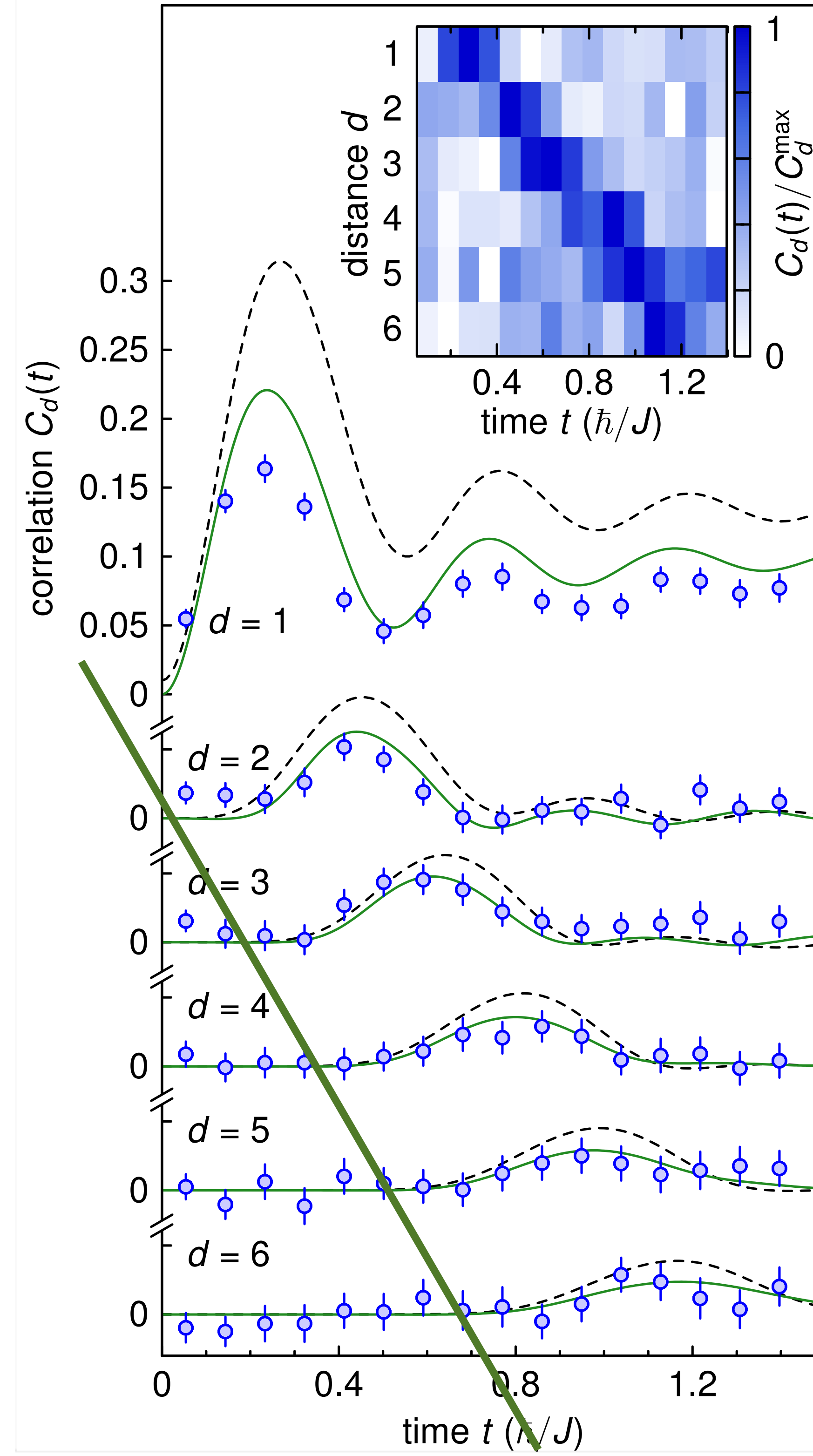


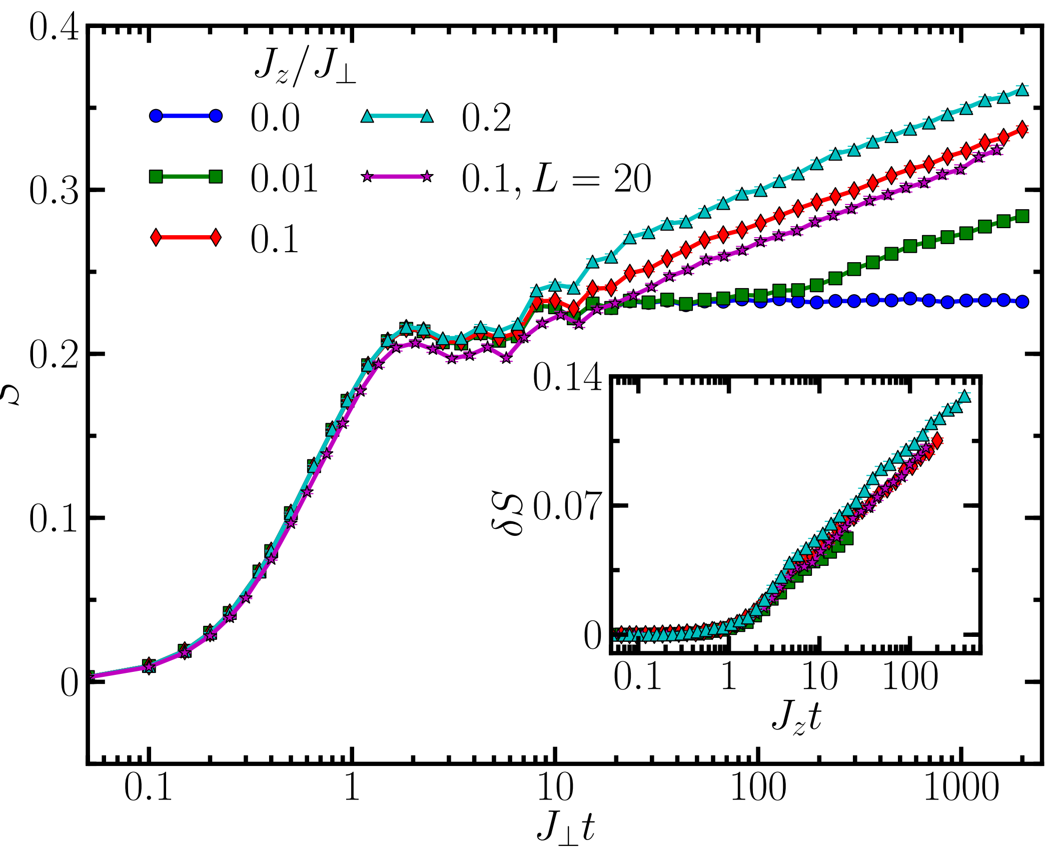

FIG. 1. **Spreading of correlations in a quenched atomic Mott insulator.** **a**, A 1D ultracold gas of bosonic atoms (black balls) in an optical lattice is initially prepared deep in the Mott-insulating phase with unity filling. The lattice depth is then abruptly lowered, bringing the system out of equilibrium. **b**, Following the quench, entangled quasiparticle pairs emerge at all sites. Each of these pairs consists of a doublon (red ball) and a holon (blue ball) on top of the unity-filling background, which propagate ballistically in opposite directions. It follows that a correlation in the parity of the site occupancy builds up at time $t$ between any pair of sites separated by a distance $d = vt$, where $v$ is the relative velocity of the doublons and holons.



# Some no light cone spreadings

MBL, logarithmic growth of entanglement:

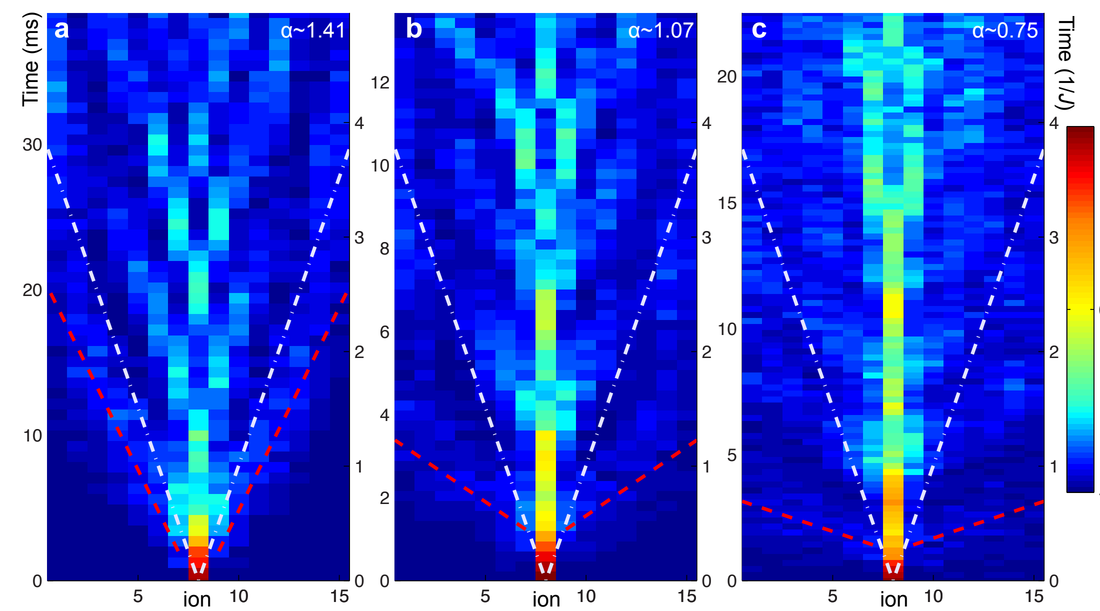
Bardarson, Pollmann, Moore '14




see also: De Chiara et al. '05
Burrell & Osborne '07
Vosk & Altman '13

Long-range interaction:

Jurcevic et al., Nature **511**, 202 (2014)




When the range of interaction is long enough there is no light cone

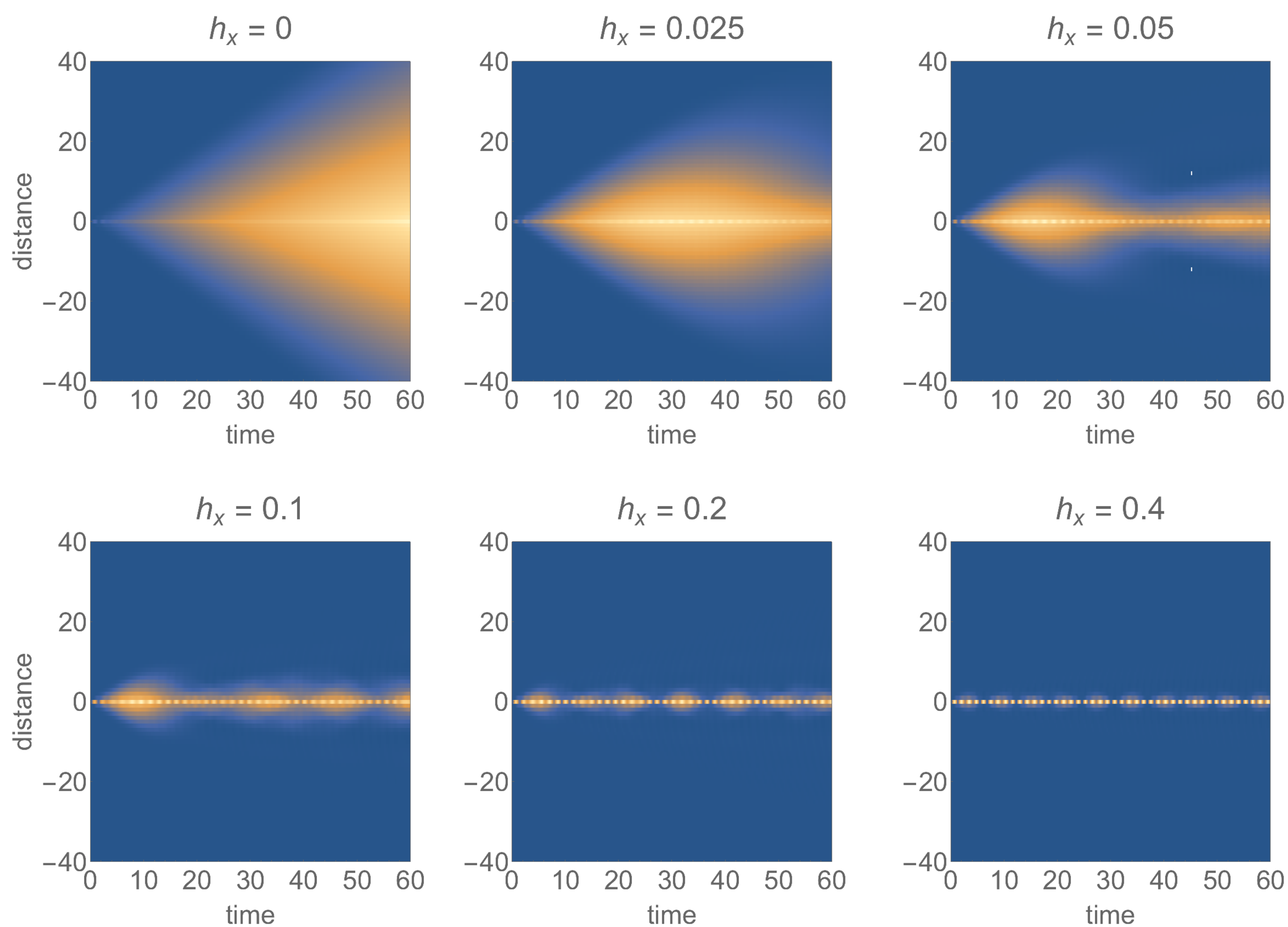
see also: Hauke & Tagliacozzo, '13
Schachenmayer et al. '13
Richerme et al. '14

# Suppression of the light cone

Starting from the ferromagnetic state (all spins up) and evolving with

$$H = -J \sum_{j=1}^{L} \left[ \sigma_j^x \sigma_{j+1}^x + h_z \sigma_j^z + h_x \sigma_j^x \right]$$

with $h_z = 0.25$. Connected longitudinal correlation $\langle \sigma_1^x \sigma_{m+1}^x \rangle_c$



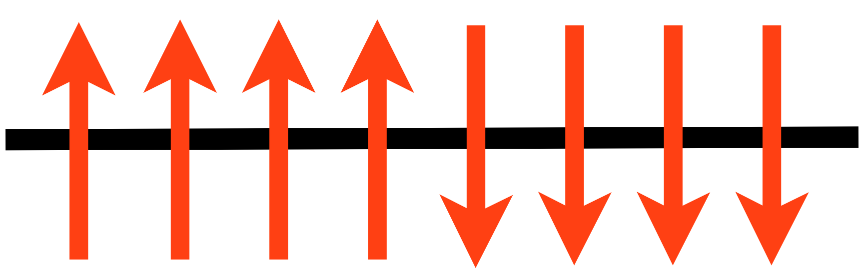

Why??

# Confinement in the Ising model

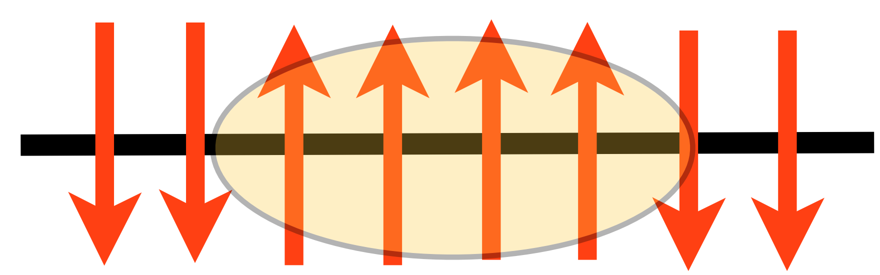
$$H = -J \sum_{j=1}^{L} \left[ \sigma_j^x \sigma_{j+1}^x + h_z \sigma_j^z + h_x \sigma_j^x \right]$$

McCoy & Wu '78

- For $h_x = 0$ free fermions with dispersion $\varepsilon(k) = 2J\sqrt{1 - 2h^z \cos k + {h^z}^2}$
- $h_z = 1$ separates two massive phases
- For $h_z < 1$ (ferro phase), the massive fermions can be seen as domain walls separating domains of magnetization $\sigma = (1 - h_z)^{1/8}$
- $h_x$ induces an attractive interaction between DW that for small enough $h_x$ can be approximated as a linear potential $V(x) = 2Jh_x\sigma\,|x|$
- DW do not propagate freely but get confined into **mesons**



Free DW



Bound state = meson

# Approximation for the meson spectrum

Consider two fermions in 1D with Hamiltonian                Rutkevich '08

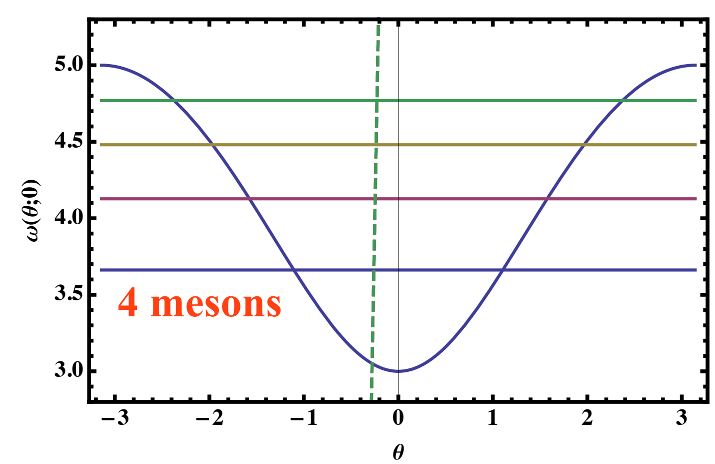
$$\mathcal{H} = \varepsilon(\theta_1) + \varepsilon(\theta_2) + \chi|x_2 - x_1| = \omega(\theta;\Theta) + \chi|x|$$

$$\omega(\theta;\Theta) = \varepsilon(\theta + \Theta/2) + \varepsilon(\theta - \Theta/2)$$

This can be quantized semiclassically a la Bohr-Sommerfeld

- The number and the energies of mesons depend on $h_x$, $h_z$, $\Theta$
- When $\omega$ has a single minimum one obtains

$h_x = 0.1,\ h_z = 0.25,\ \Theta = 0$

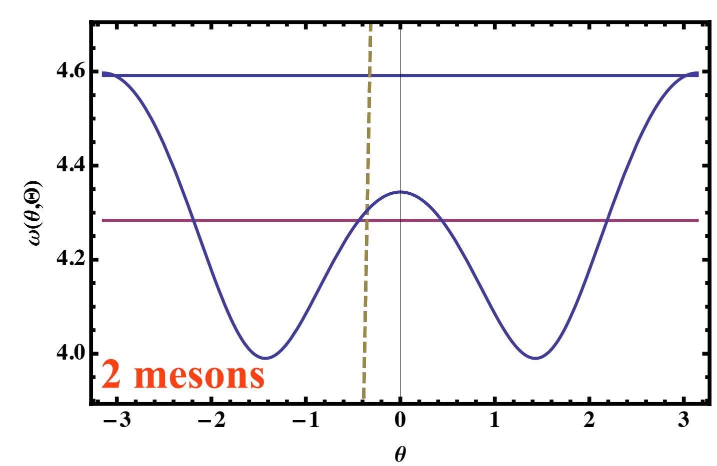



$$2E_n(\Theta)\theta_a - \int_{-\theta_a}^{\theta_a} \mathrm{d}\theta\, \omega(\theta;\Theta) = 2\pi\chi(n - 1/4)\,, \qquad n = 1, 2, \ldots$$

where $\theta_a$ is the solution of $\omega(\theta_a(n;\Theta);\Theta) = E_n(\Theta)$

- For two minima

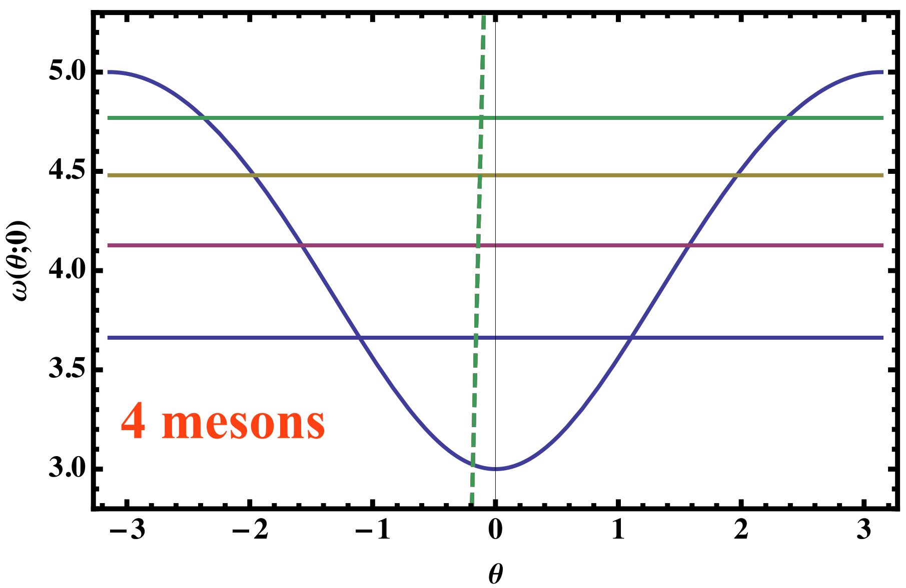
$h_x = 0.1,\ h_z = 0.5,\ \Theta = 3$




$$E_n(\Theta)(\theta_a - \theta_b) - \int_{-\theta_b}^{\theta_a} \mathrm{d}\theta\, \omega(\theta;\Theta) = \pi\chi(n - 1/2)\,, \qquad n = 1, 2, \ldots$$

# Approximation for the meson spectrum

$h_x = 0.1,\ h_z = 0.25,\ \Theta = 0$




The four masses are

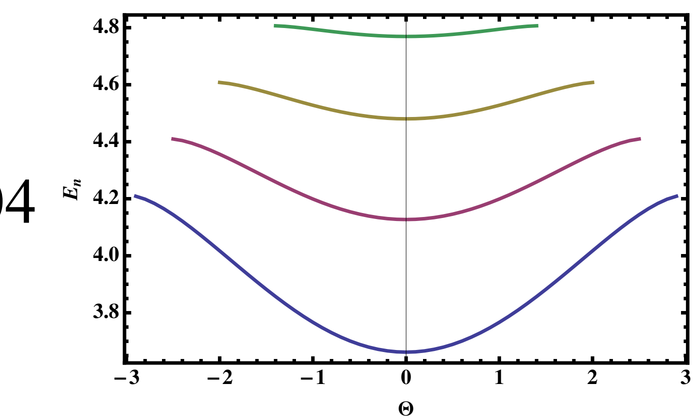
$m_1 = 3.662 \quad m_2 = 4.127 \quad m_3 = 4.48 \quad m_4 = 4.77$

$E_n(\Theta)$ is the dispersion relation of the mesons

$$v_n(\Theta) = \frac{dE_n(\Theta)}{d\Theta}$$

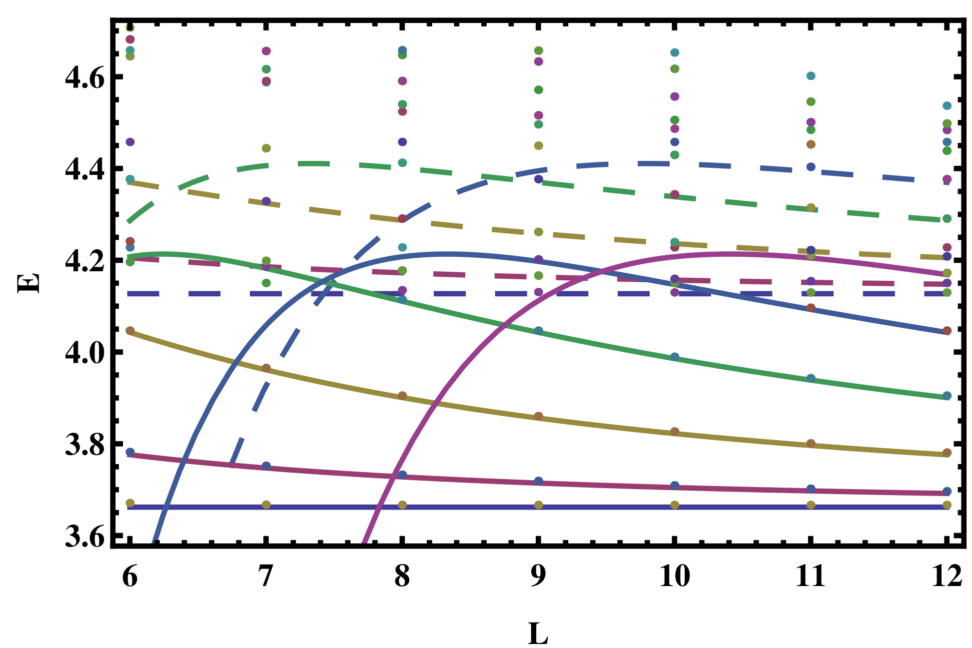
$v_{max} = 0.274, 0.166, 0.094, 0.004$

$v_{max}$ of DW = 0.5




Comparison with exact diagonalization:

$h_x = 0.1,\ h_z = 0.5$



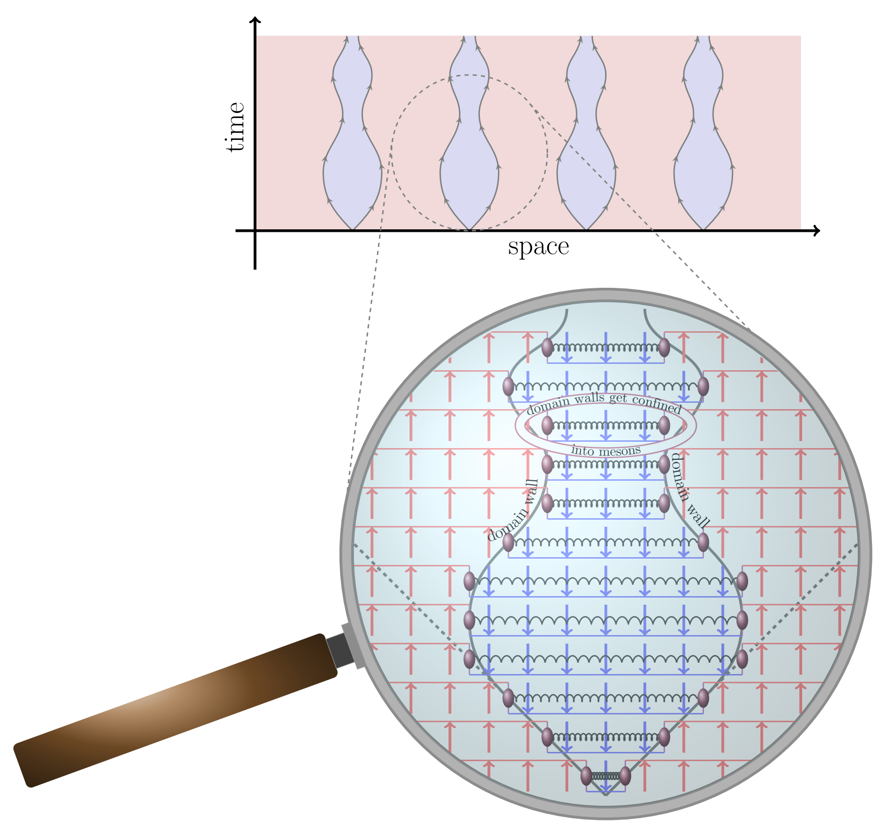

2nd meson 1pt states

1st meson 1pt states

# Back to quenches

What happens if there are mesons in the spectrum of the post-quench Hamiltonian in the quasi-particle picture?

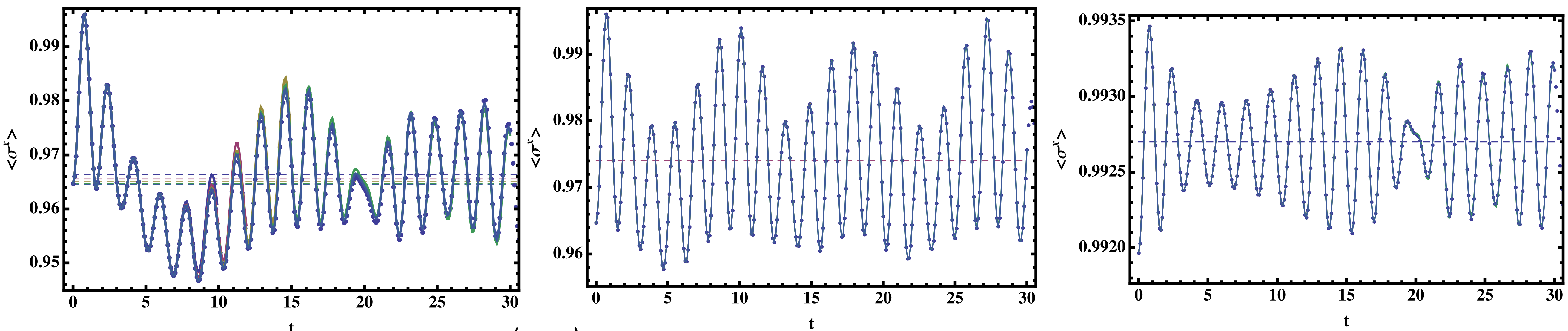
- $|\psi_0\rangle$ acts as a source of quasi-particles at $t = 0$
- pairs of particles move in opposite directions with velocity $v_p$
- moving away the quasi-particles feel the attractive interaction
- The interaction will eventually turn the particles back



# 1-point function $\langle \sigma_x \rangle$

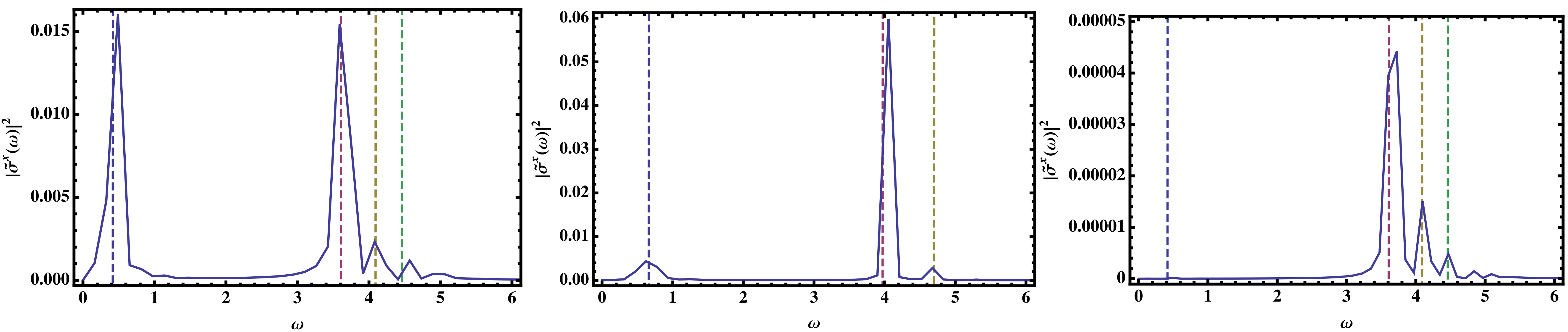
## Quenches from ferro to ferro

$h_z^0$=0.5, $h_x^0$=0, $h_z$=0.25, $h_x$=0.1 | $h_z^0$=0.5, $h_x^0$=0, $h_z$=0.25, $h_x$=0.2 | $h_z^0$=0.25, $h_x^0$=0, $h_z$=0.25, $h_x$=0.1




## Power spectrum of $\langle \sigma_x \rangle$

iTEBD vs. ED with $L$=8,10,12



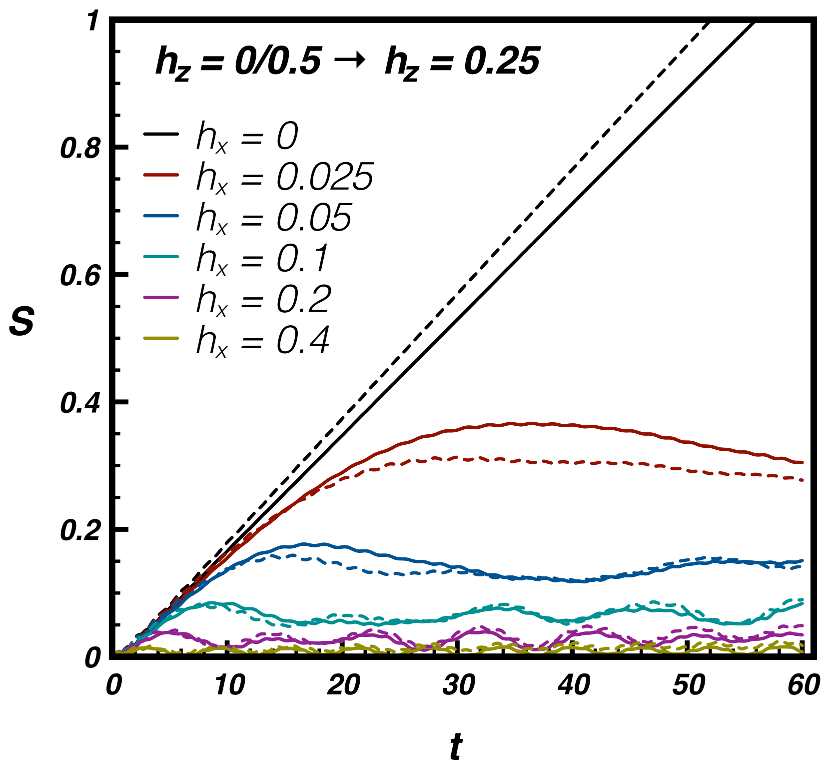

$m_2$-$m_1$ =0.46, $m_1$ =3.7, $m_2$ =4.1, $m_3$ =4.5

$m_2$-$m_1$ =0.68, $m_1$ =4.0, $m_2$ =4.7

$m_2$-$m_1$ =0.46, $m_1$ =3.7, $m_2$ =4.1, $m_3$ =4.5

# Half-chain entanglement entropy

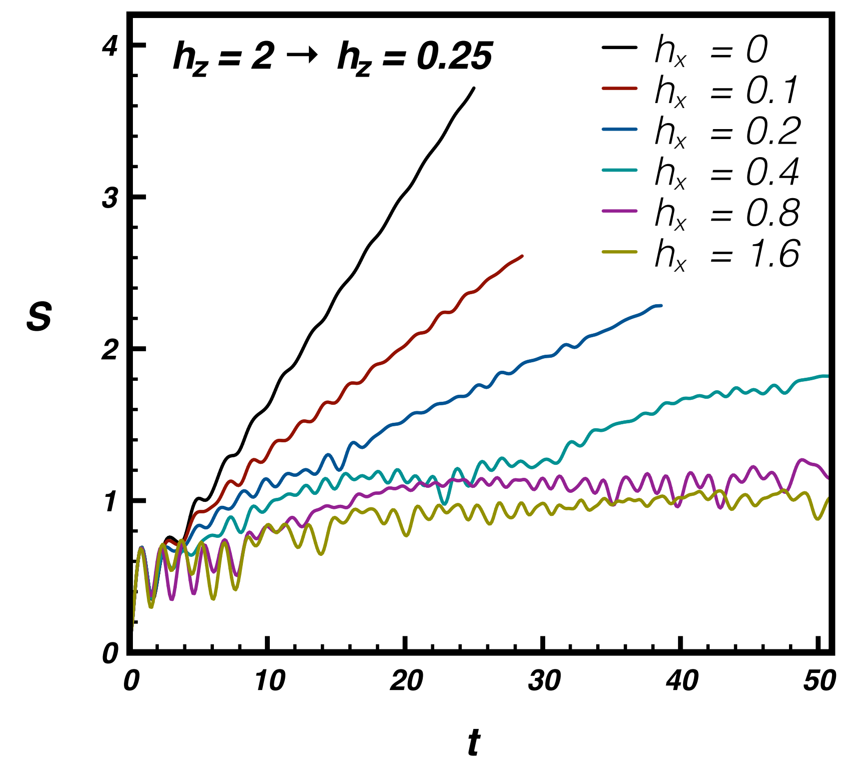
## Ferro to Ferro




The entanglement entropy does not grow indefinitely but seems to oscillate around a finite value

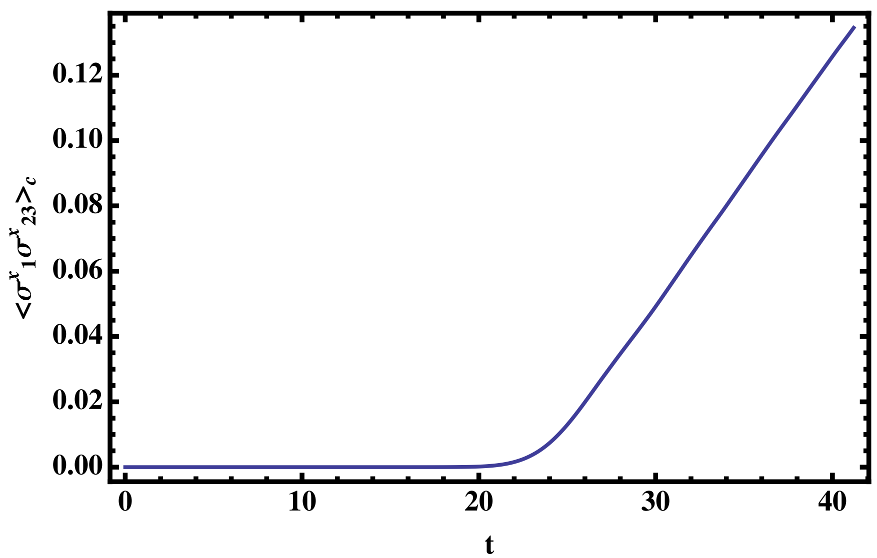
(almost) NO Light-cone!

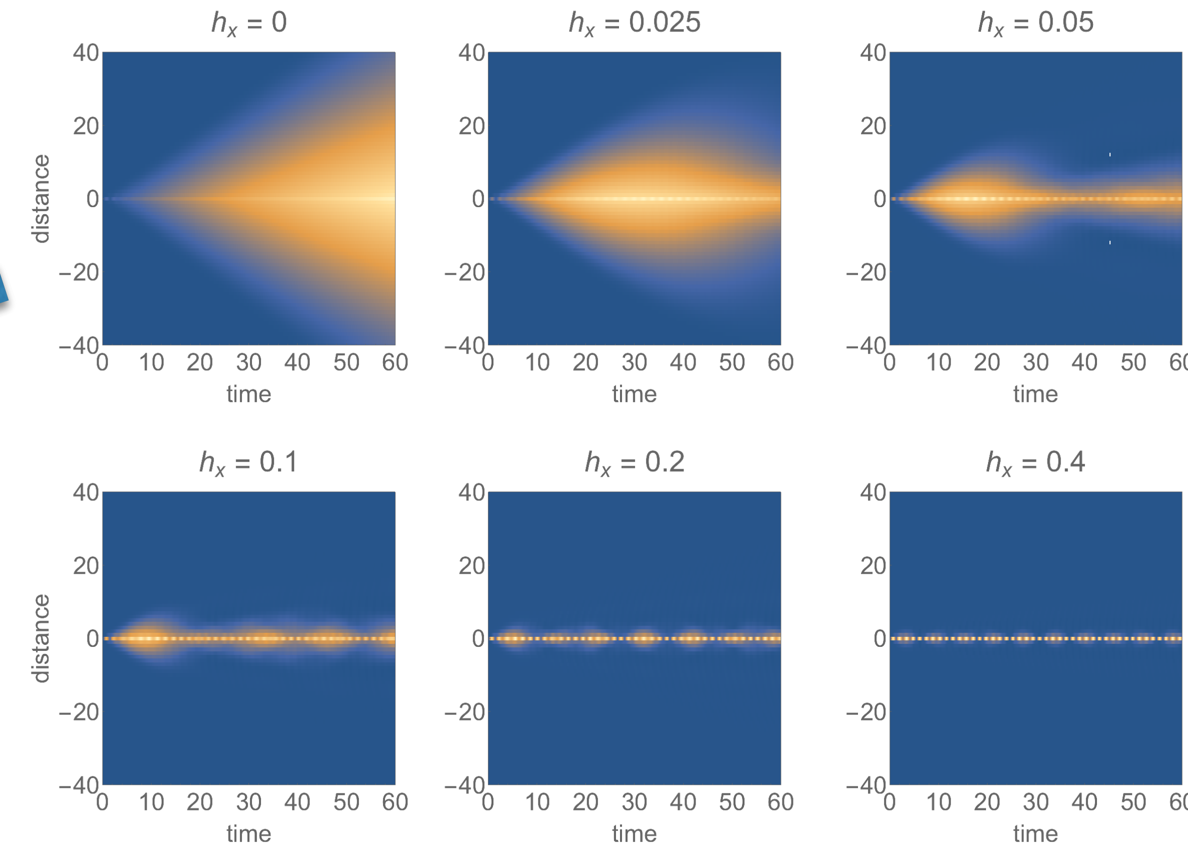
## Para to Ferro

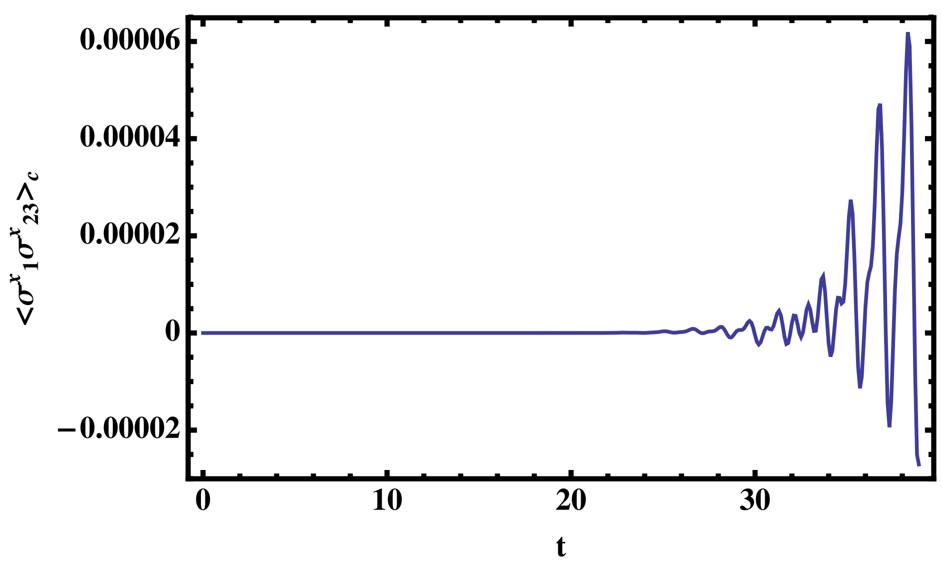


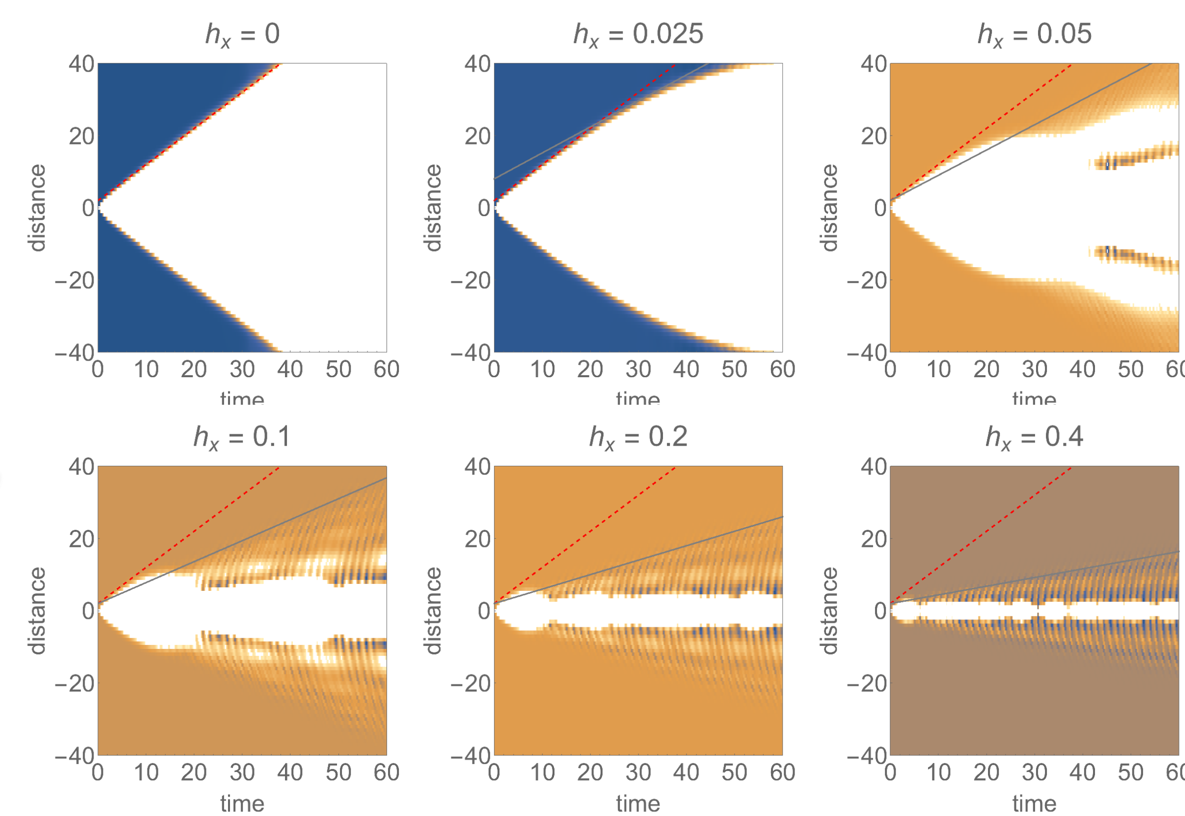

The entanglement entropy grows but slower compared to the integrable case

# Zooming in: escaping correlations



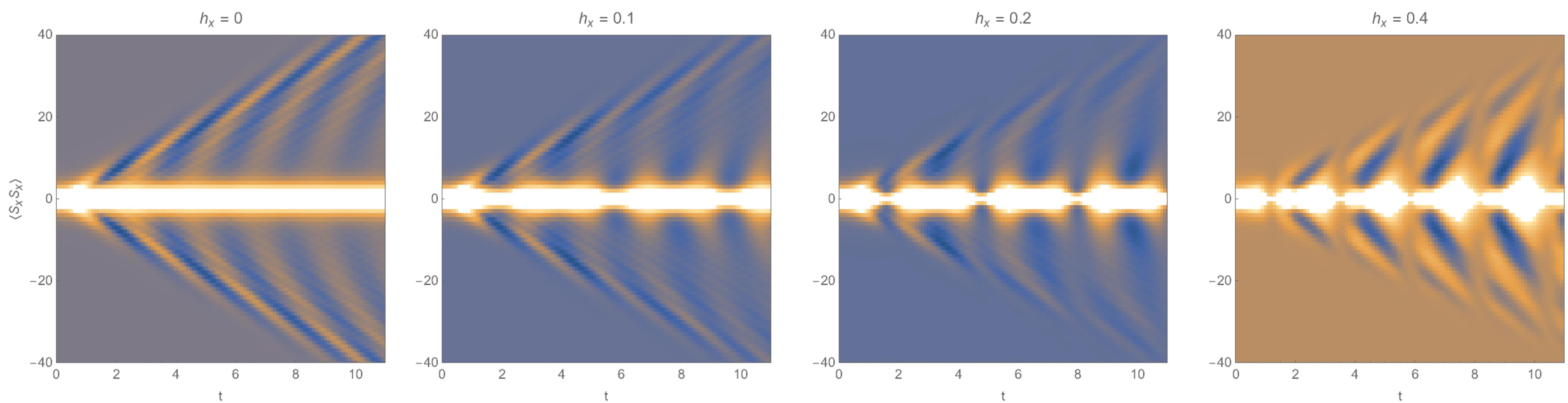



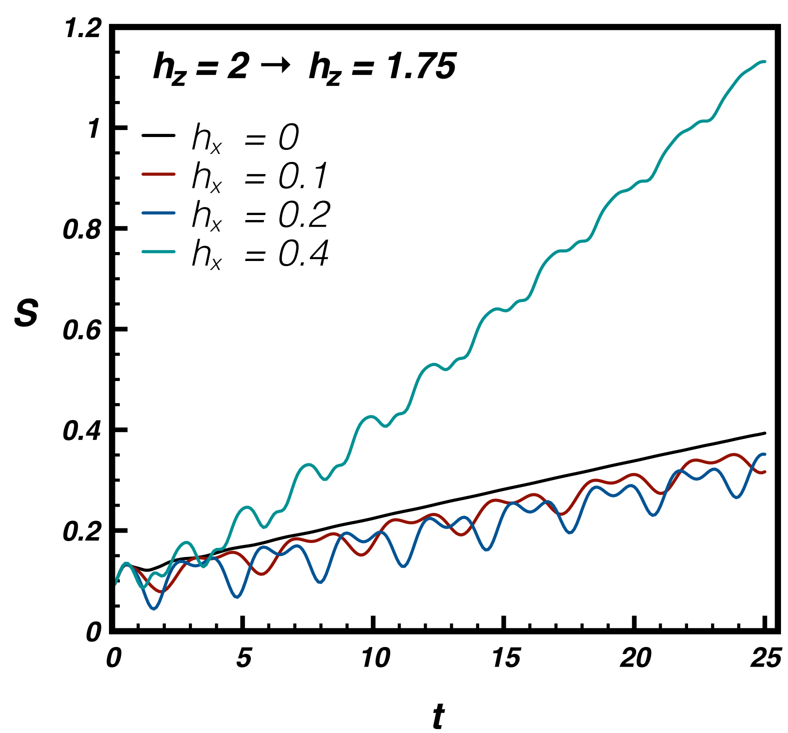

3 orders of magnitude!

# Quench in the paramagnetic phase

$h_z^0=2,\ h_x^0=0,\ h_z=1.75$

$h_x=0$ $\quad$ $h_x=0.1$ $\quad$ $h_x=0.2$ $\quad$ $h_x=0.4$

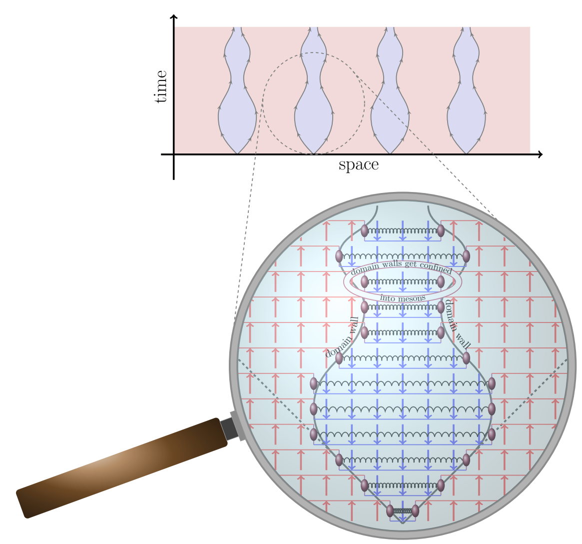



Change for small hx is perturbative.
For large hx new fast excitations appear. No confinement.

# Conclusions

In the Ising chain, confinement changes the light cone spreading of correlations and entanglement




## Questions:

- Is it a general property of other cond-mat models featuring confinement? Presumably yes, possible to check numerically
- Is it true in higher dimensions? e.g. in QCD?

  maybe holography can offer some hints