Laughlin quasiparticle interferometer: Observation of anyonic statistics

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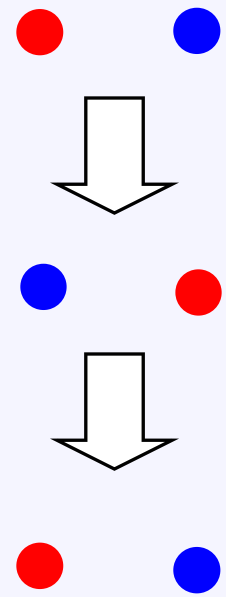
Quantum exchange statistics

- QM textbooks: two-particle wave function

$\Psi(r_1, r_2)$ acquires phase $\gamma = \pi\Theta$ upon exchange:

$$\Psi(r_1, r_2) = e^{i\pi\Theta}\,\Psi(r_2, r_1)$$



do again:

$$\Psi(r_1, r_2) = e^{i2\pi\Theta}\,\Psi(r_1, r_2)$$

single-valuedness of Ψ requires $e^{i2\pi\Theta} = +1$

⇒ statistics $\Theta = j$ (an integer)

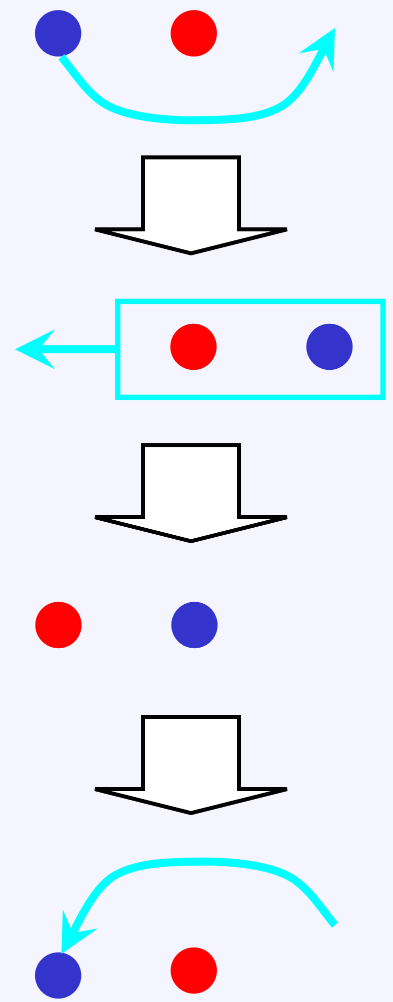
e.g., fermions: $\Theta_F = 1$, bosons: $\Theta_B = 0$

⇒ many-particle wave function symmetry, occupation numbers, Bose-Einstein and Fermi-Dirac statistical distributions, etc. follow

Exchange statistics in 2D

this derivation is not valid in 2D:

- exchange = half loop + translation
  (exchange)^2 = complete loop
- in 3D: loop with particle inside is **NOT** distinct from loop with no particle inside $\Rightarrow$ $\Theta = j$
- in 2D: loop with particle inside **IS** topologically distinct from loop with no particle inside



Leinaas, Myrheim 1977

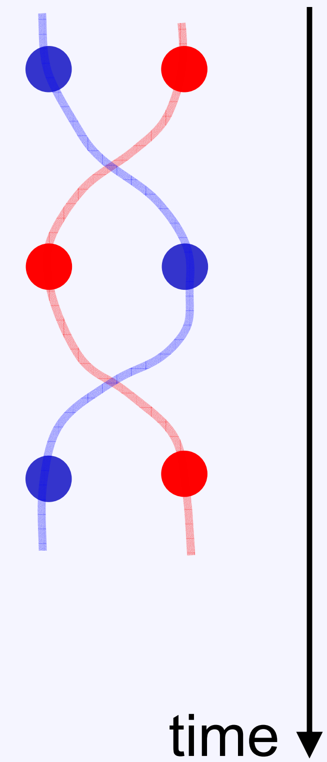
Exchange statistics in 2D

⇒ exchange ⇔ braiding

⇒ NO requirement for Θ to be an integer
e.g., Θ can be any real number

"anyons"

F. Wilczek 1982 -



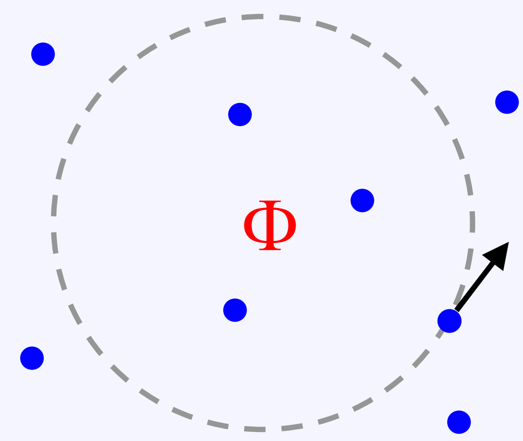
Q: are there such particles in Nature?

A: collective excitations of
a many-electron 2D system

e.g., elementary charged excitations
of a FQH fluid – Laughlin quasiparticles (LQPs)

Laughlin 1983, 1987; Haldane 1983; Halperin 1984;
Arovas, Schrieffer, Wilczek 1984; W.P. Su 1986

Adiabatic transport in magnetic field



- **electrons** $q=e,\ \Theta_e=1,\ N_e$ inside

encircling electron at z_0

Ψ acquires Berry phase $\exp(i\gamma)$

$$\gamma(T)=\frac{e}{\hbar}\Phi+2\pi\Theta_e N_e=2\pi\left(\frac{\Phi}{h/e}+\Theta_e N_e\right)$$

two contributions: Aharonov-Bohm + statistics

statistical contribution is **NOT** observable: $\exp(i2\pi\Theta_e N_e)=e^{i2\pi j}=+1$

$$\Phi=\oint_C A\cdot dl$$

$$\Theta N=\frac{i}{2\pi}\oint_C d\,z_0\left\langle\Psi(z_0;z_j)\middle|\frac{\partial}{\partial z_0}\Psi(z_0;z_j)\right\rangle$$

$z=x+iy$

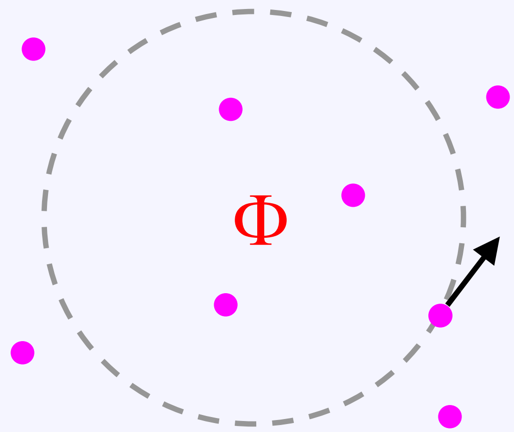
Aharonov, Bohm 1959

M.V. Berry 1984

Fractional statistics in 2D

- Laughlin quasihole in $f = 1/3$ FQHE

$$q = e/3, \quad \Theta_{1/3} = 2/3, \quad N_{QH}$$



Ψ_m of encircling quasihole acquires phase

$$\gamma_m = \frac{q}{\hbar}\Phi + 2\pi\,\Theta_{1/3} N_{QH} = 2\pi m$$

transition $m \to m+1$ $\Rightarrow$ period

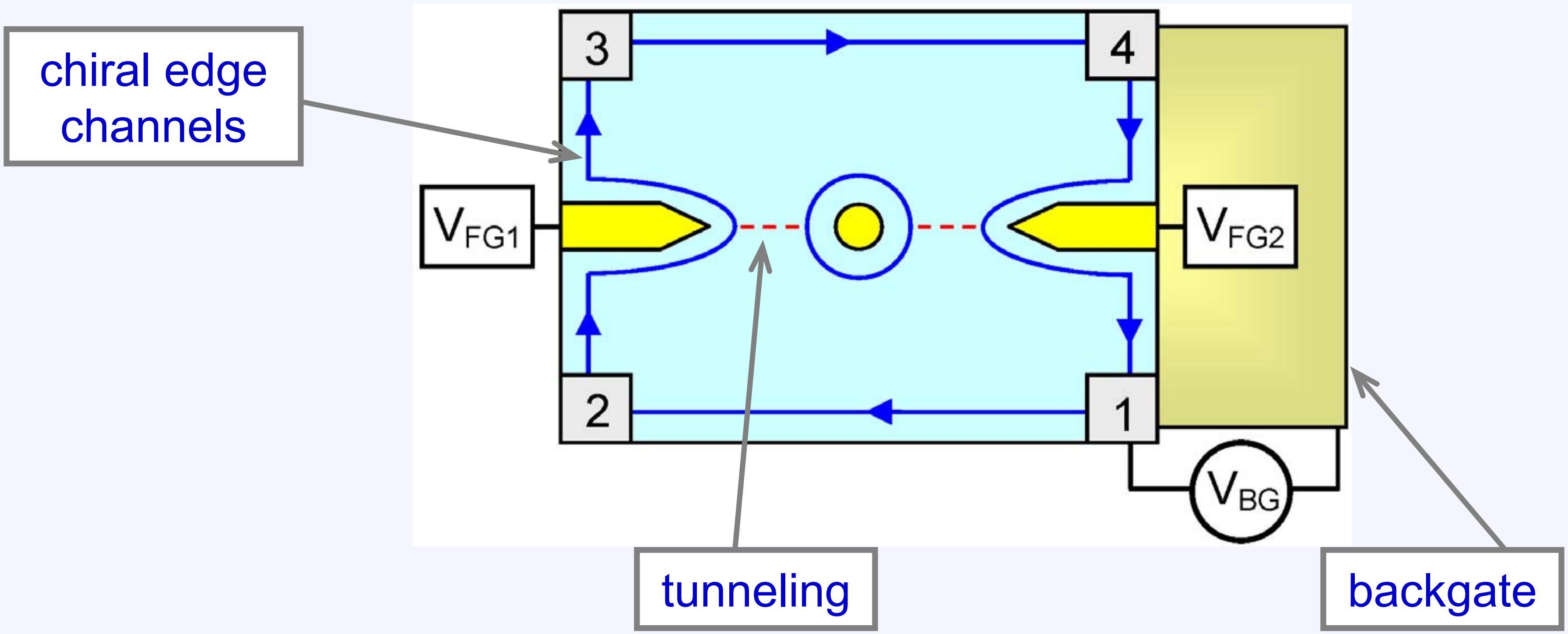
$$\Delta\gamma = \frac{q}{\hbar}\Delta\Phi + 2\pi\,\Theta_{1/3}\Delta N_{QH} = 2\pi$$

- when flux changes by $\Delta\Phi = h/e$, Ψ_m acquires A-B phase $\frac{he}{3\hbar e} = \frac{2\pi}{3}$

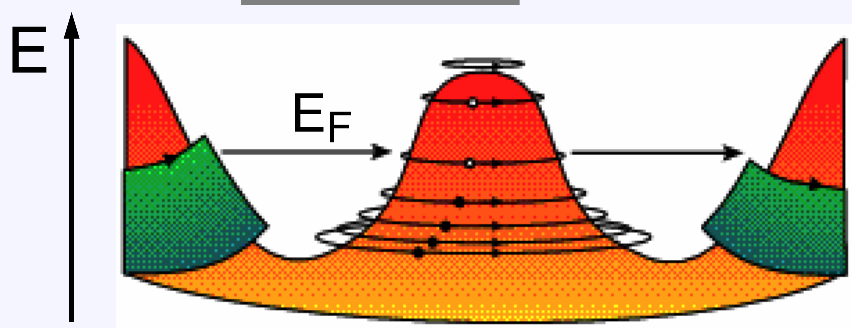
$\Rightarrow$ need $\Theta_{1/3} = 2/3$ for single-valued Ψ_m (period is h/e, NOT $3h/e$!)

$$\Delta\gamma = \frac{e}{3\hbar} \times \frac{h}{e} + 2\pi\frac{2}{3} \times 1 = 2\pi$$

Resonant tunneling via a Quantum Antidot



r =300 nm
potential hill



each RT peak ⇒ one more (or less) particle bound on antidot

Quantum Antidot experiments: fractional charge

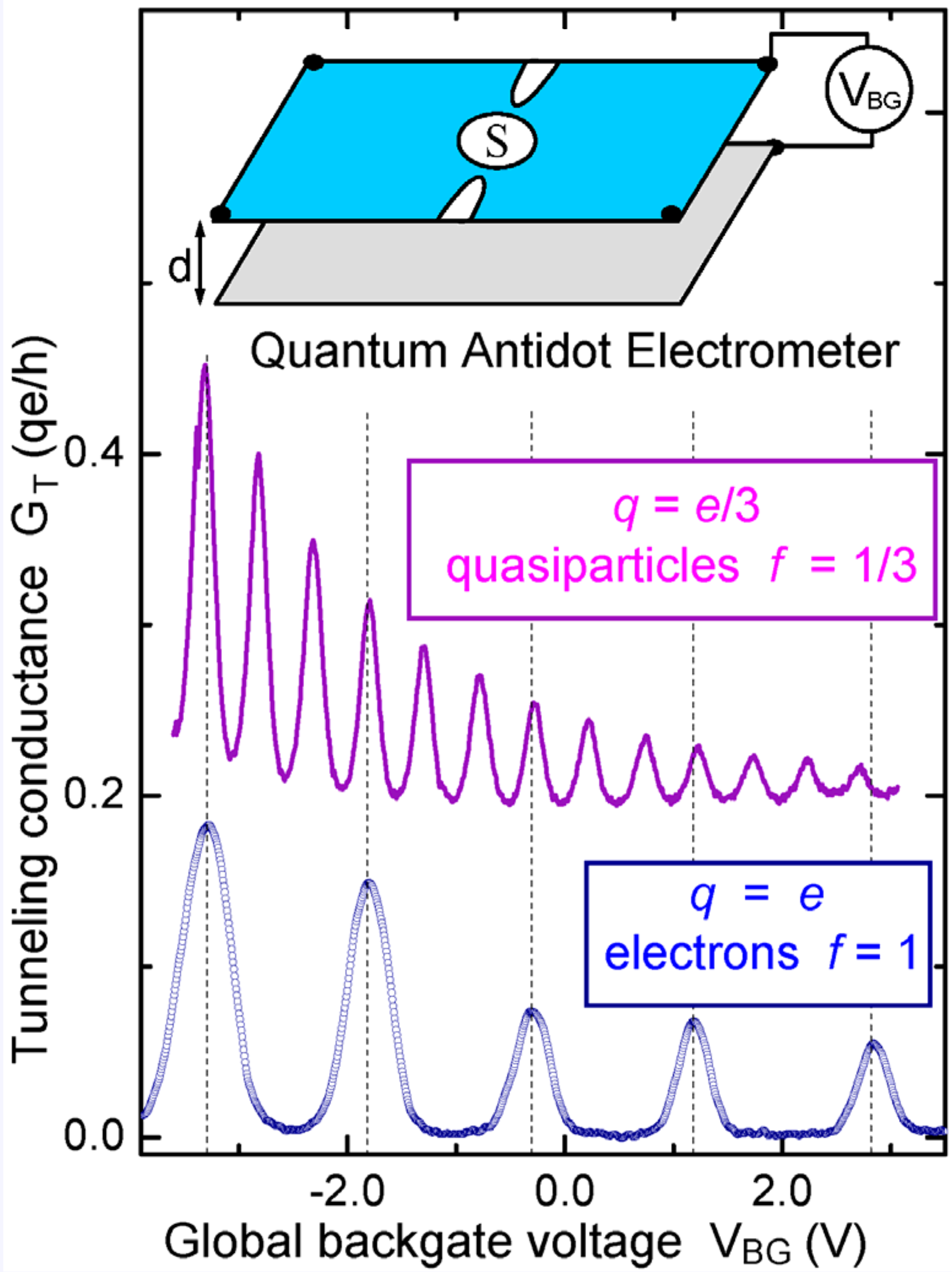
Goldman et al. 1995 -1997, 2001, 2005

$$q = \frac{\varepsilon\varepsilon_0 S}{d}\Delta V_{BG} \propto \Delta V_{BG}$$

$$3q_{1/3} = 1e\ (\pm 3\%)$$

$$\Rightarrow \quad q_{1/3} = e/3$$

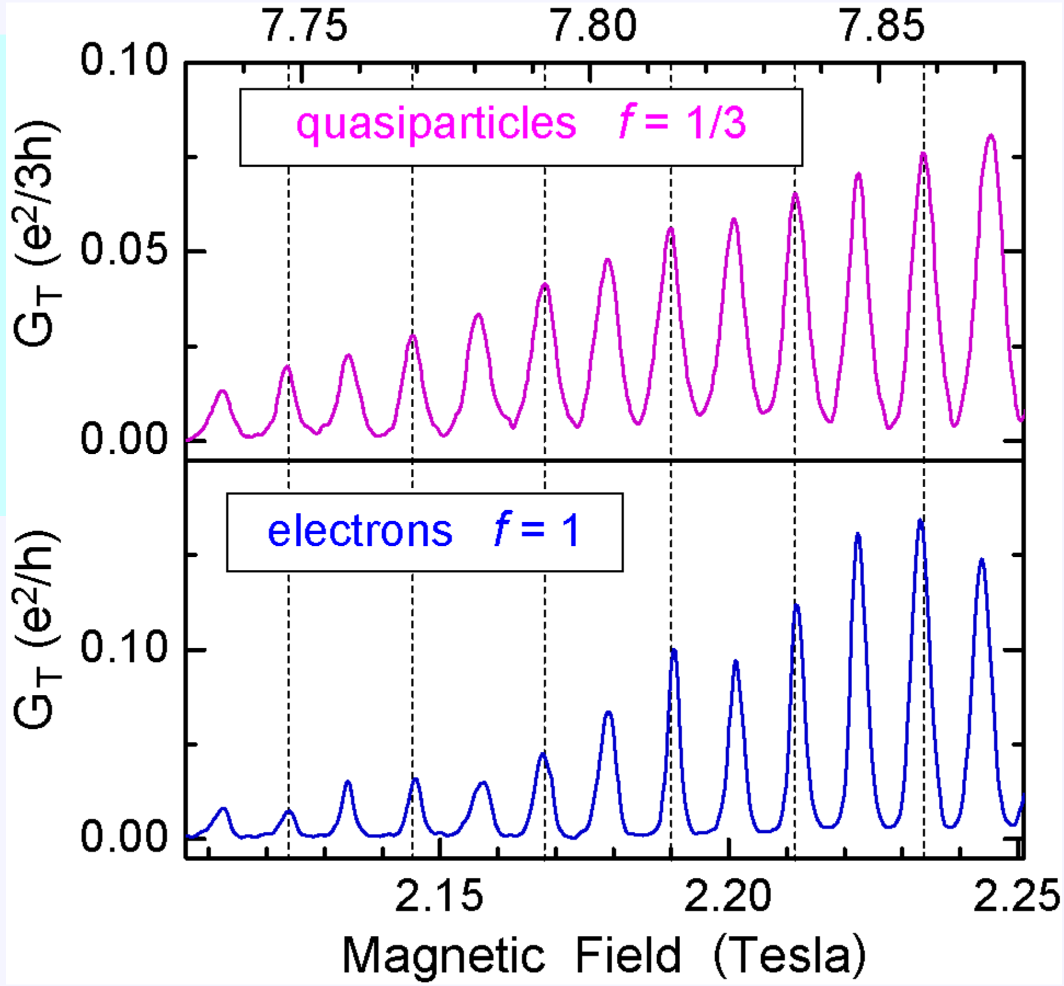
$$q_{2/5} = e/5$$



Quantum Antidot experiments:
anyon statistics

Goldman et al. 1995 -1997, 2001, 2005

$$S\Delta B = \Delta\Phi = h/e$$
$$\Delta N = 1$$
$$q = e/3$$

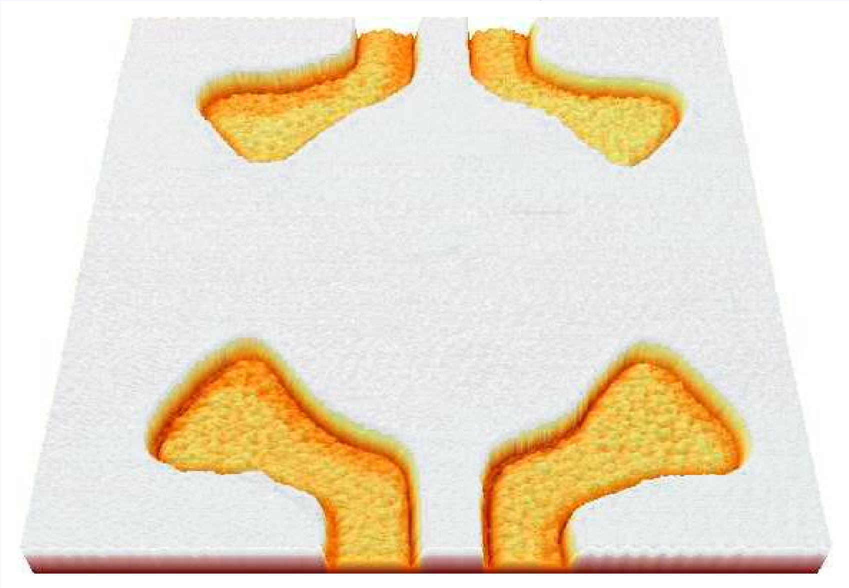


$\Downarrow$

$$\Delta\gamma = \frac{e/3}{\hbar}\Delta\Phi + 2\pi\Theta_{1/3}\Delta N = 2\pi\left(\frac{1}{3}+\frac{2}{3}\right) = 2\pi$$

consistent with $\Theta_{1/3} = 2/3$ …, but ensured by gauge invariance

Laughlin quasiparticle interferometer: Samples

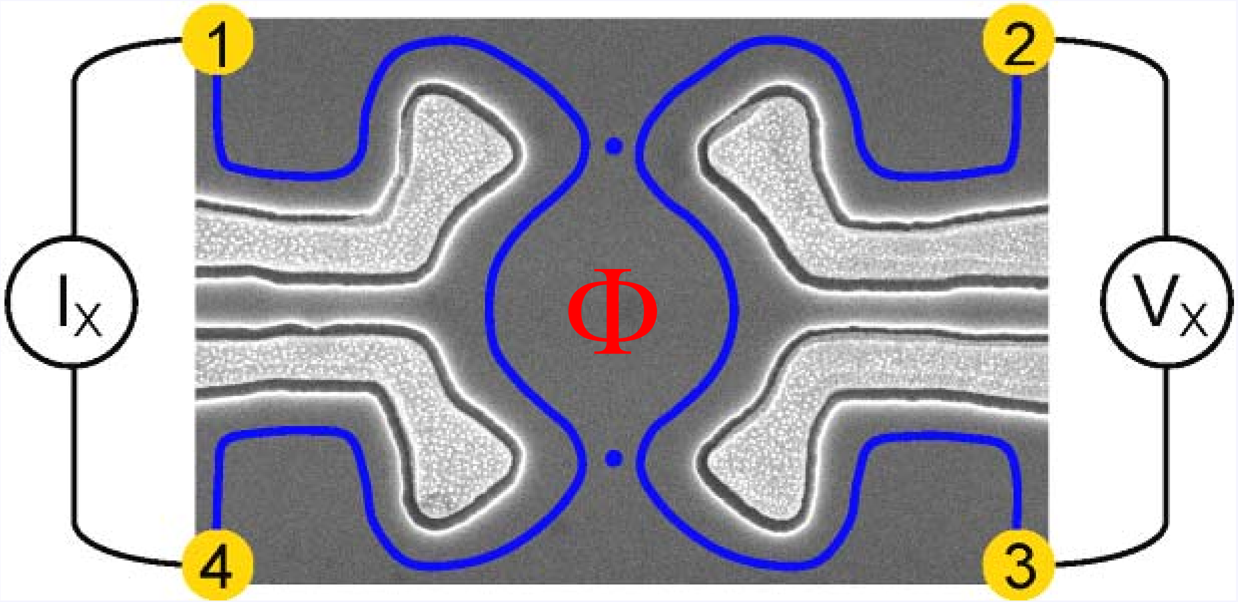


- 2D electrons ≈300 nm below surface in these low *n*, high μ GaAs/AlGaAs heterojunctions suitable for FQHE
- large island: 2,000 electrons lithographic island R≈1,050 nm
- etched 150 nm
- Au/Ti FrontGates in trenches

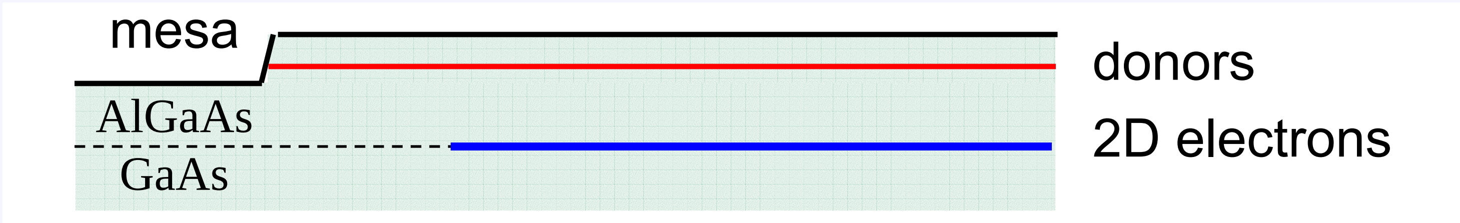
$$R_{XX} = V_X / I_X$$

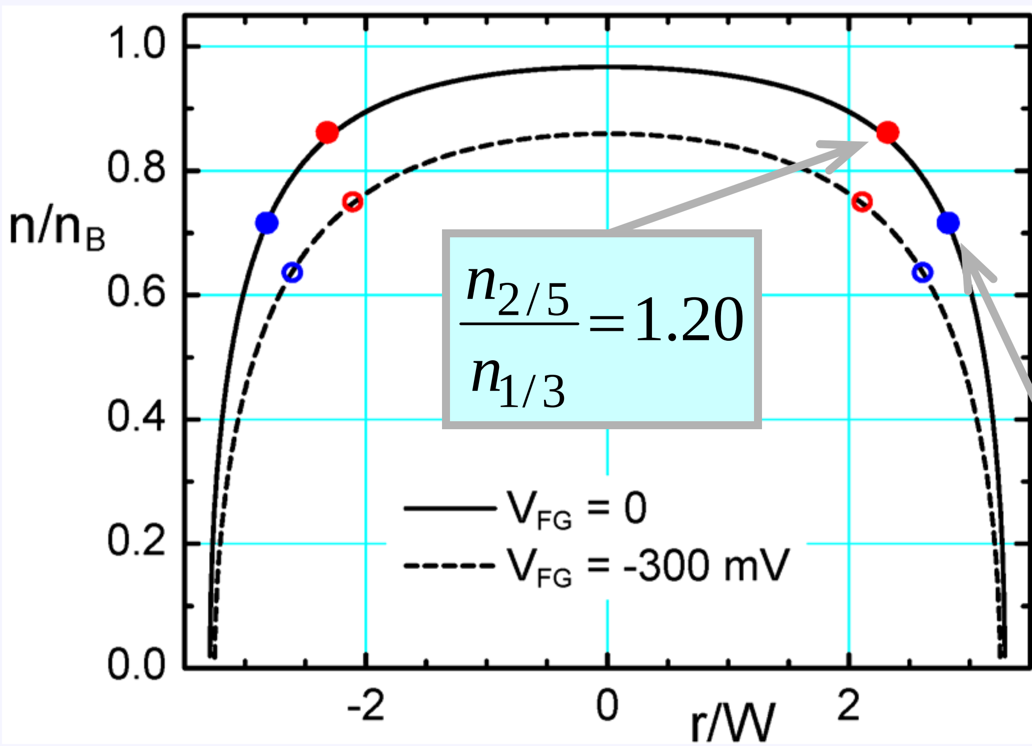
$$G_T \approx R_{XX} / R_{XY}^2$$

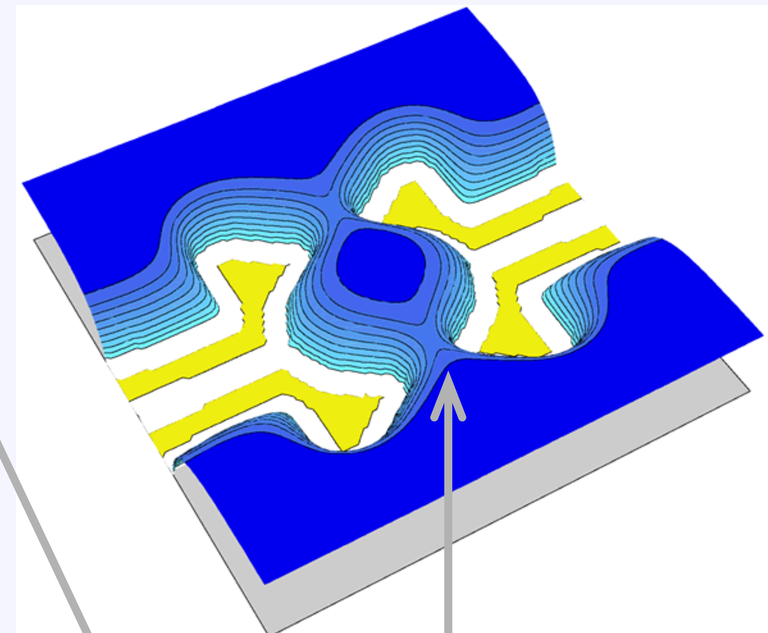
A-B flux Φ



Electron density profile of the island







circling edge channel defined by density at saddle points in constrictions

Calibration with electrons ⇒ A-B ring radius

$$2\Delta B_2 = 2.85\ \text{mT}$$

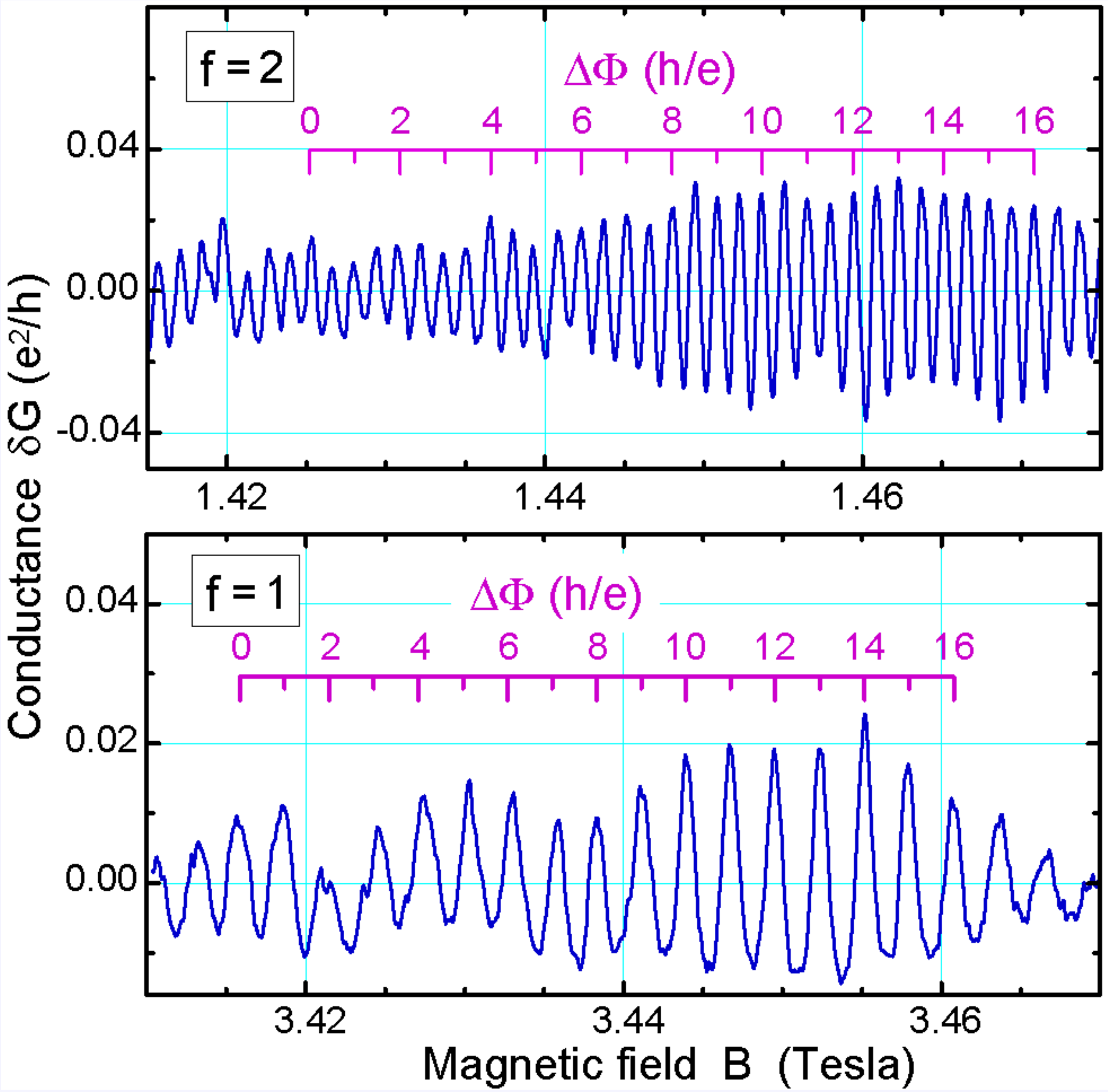
$$\Delta B_1 = 2.81\ \text{mT}$$

⇓

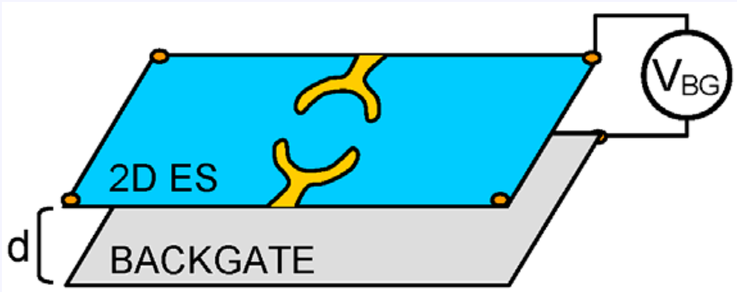
$$\pi r^2 \Delta B_1 = h/e$$

$$r = \sqrt{2\hbar / e\Delta B_1}$$

$$\approx 685\ \text{nm}$$



Calibration with electrons ⇒ backgate action



small perturbation:

$$\delta n / n \approx 0.0017$$

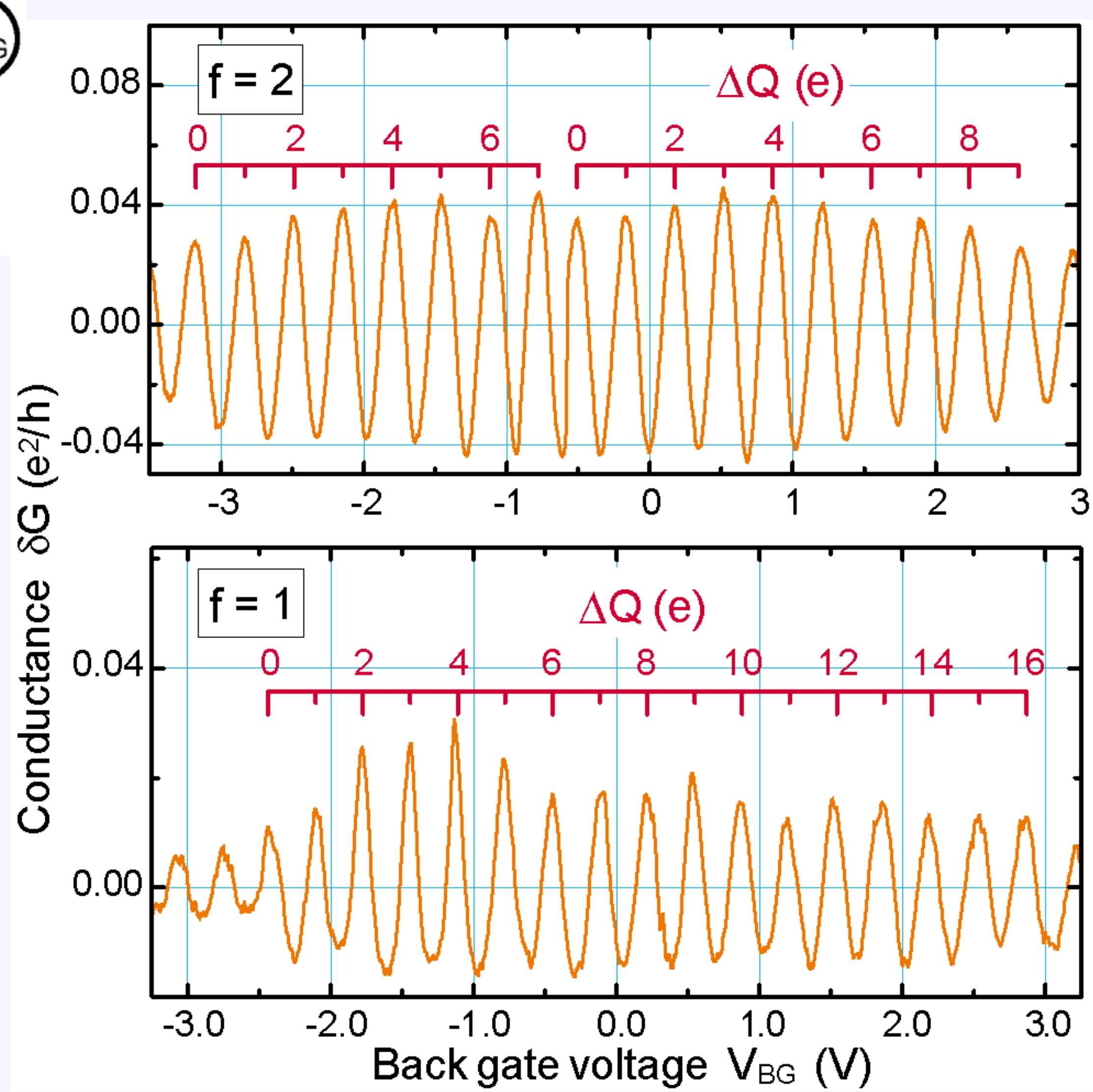
upon 1 Volt

$$\Delta Q = e, \ \Delta V_{BG}$$

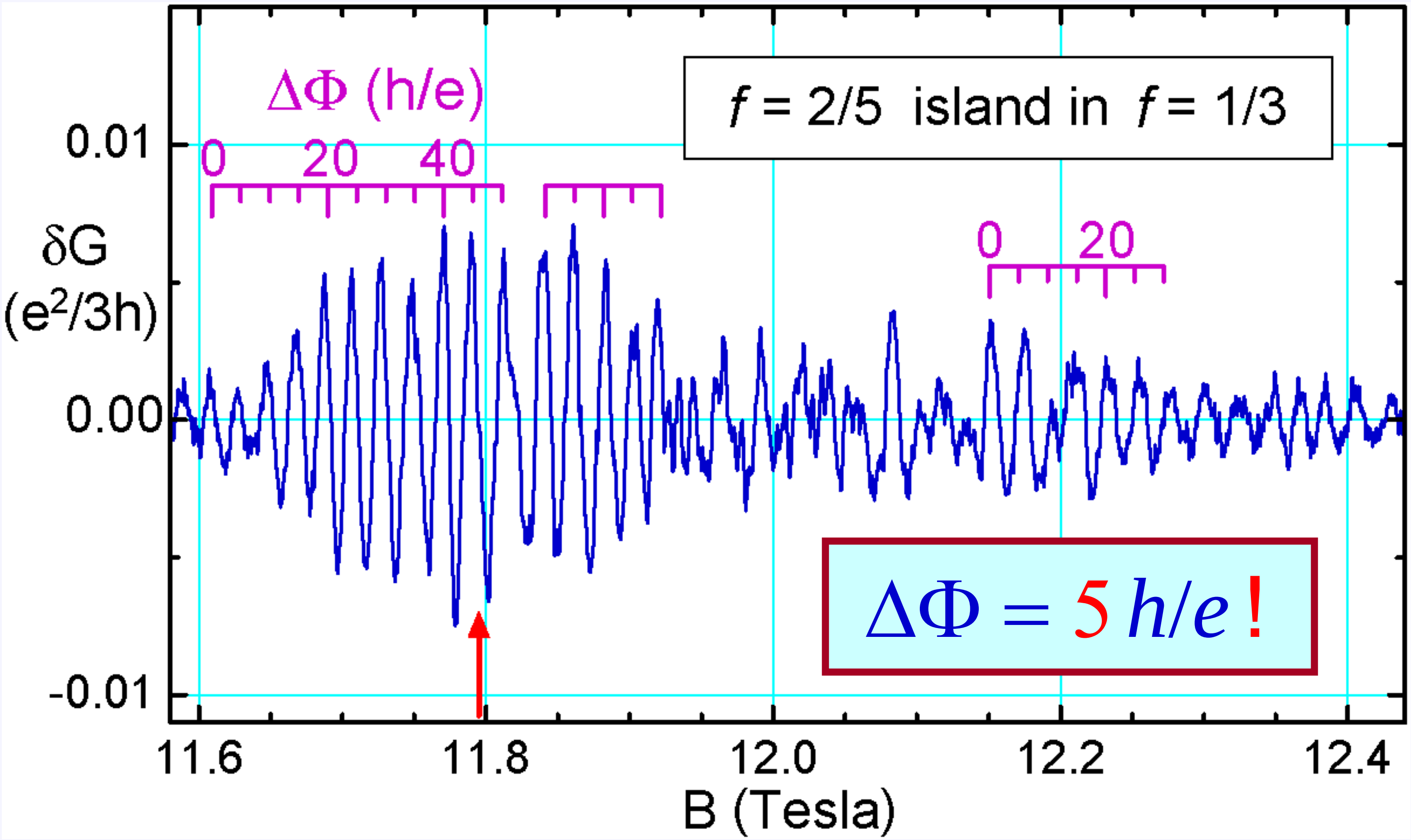
⇓

calibrate

$$\frac{\delta Q}{\delta V_{BG}}$$



Observation of an Aharonov-Bohm superperiod



Aharonov-Bohm interference of $e/3$ Laughlin quasiparticles
circling the island of the $f = 2/5$ FQH fluid

Observation of Aharonov-Bohm superperiod

Aharonov-Bohm superperiod of $\Delta_\Phi > h/e$
has never been reported before

discussion:

Derivation of Byers-Yang theorem uses a singular gauge transformation at the center of the A-B ring, where electrons are excluded

Present interferometer geometry has no electron vacuum within the A-B path $\Rightarrow$ BY theorem is not applicable (no "violation" of BY theorem)

N. Byers and C.N. Yang, PRL 1961; C.N. Yang, RMP 1962

LQP interferometer flux and charge periods

flux period

$\Delta_\Phi = 5h/e$

⇓

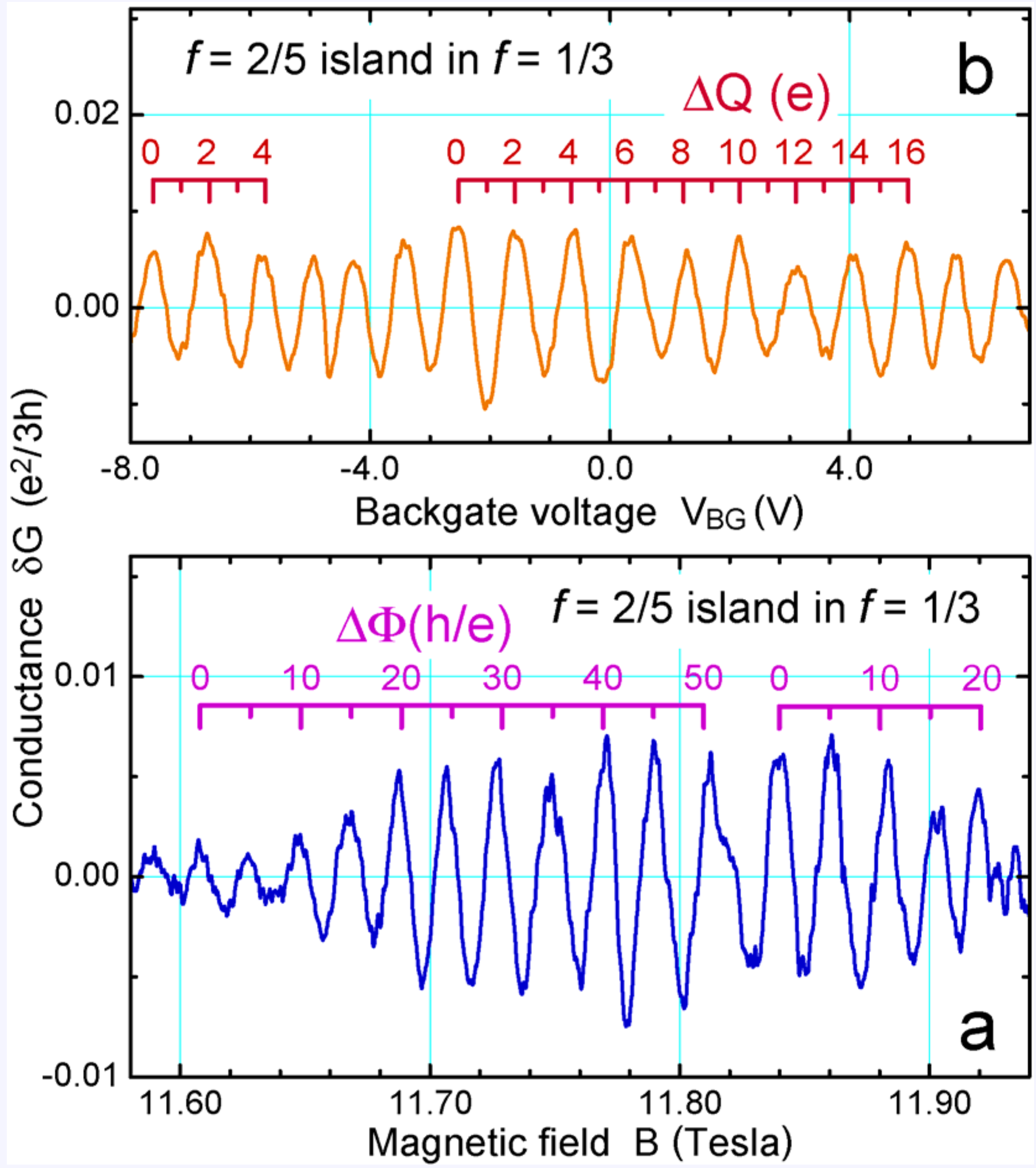
creation of ten

$e/5$ LQPs

in the island

backgate voltage period

$\Delta_Q = 2e = 10(e/5)$



(recall: $\delta\Phi = h/e$ creates two $e/5$ LQP in 2/5 fluid)

Q: How do we know the island filling?

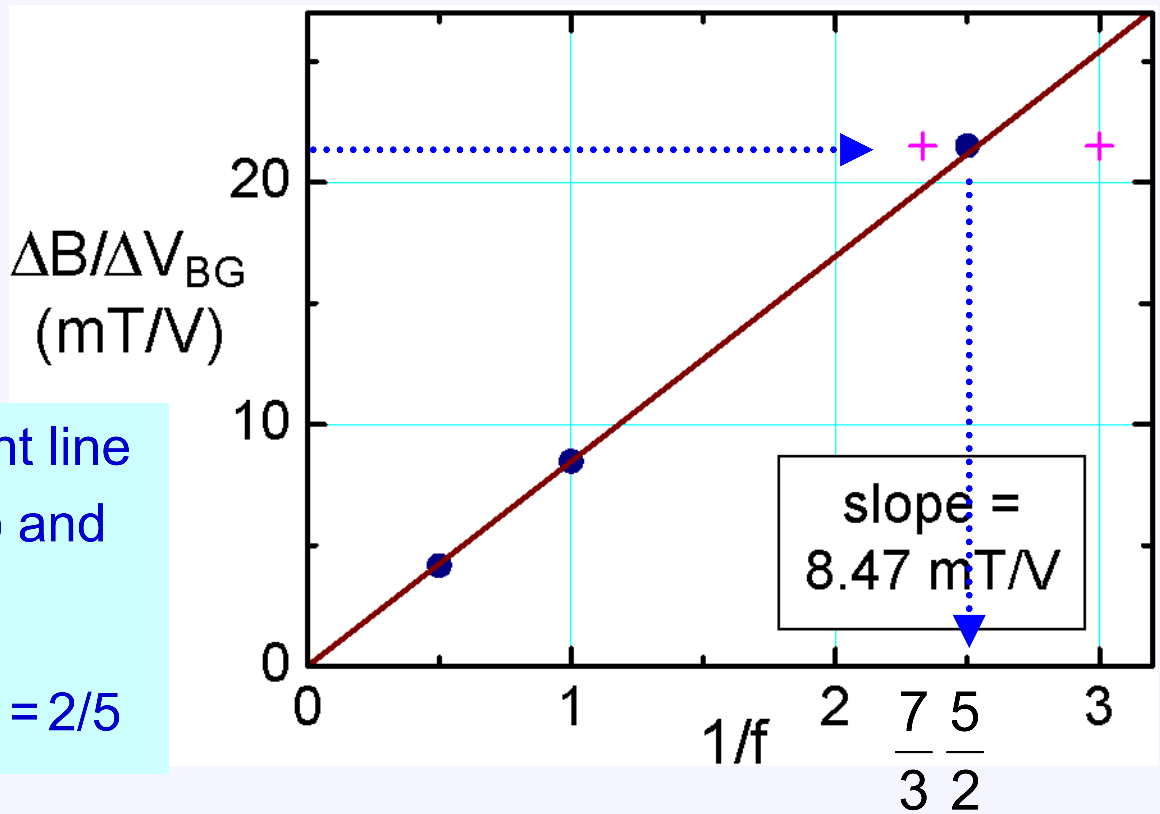
A: The ratio of oscillations periods is independent of island area S

$$\frac{S\Delta B}{S\Delta V_{BG}} \propto \frac{N_\Phi}{N_e} \equiv \frac{1}{f}$$

$$CdV = dQ$$

$$C \propto \frac{\varepsilon\varepsilon_0 S}{d}$$

$$SdV \propto dQ$$



Ratios fall on straight line forced through (0,0) and the $f = 1$ data point

⇒ island filling is $f = 2/5$

⇒ no edge depletion model is used to establish island filling

Q: How do we know 1/3 FQH fluid surrounds the island?

A: quantum Hall plateau

$R_{XY} = 3h/e^2$ at 12.3 T

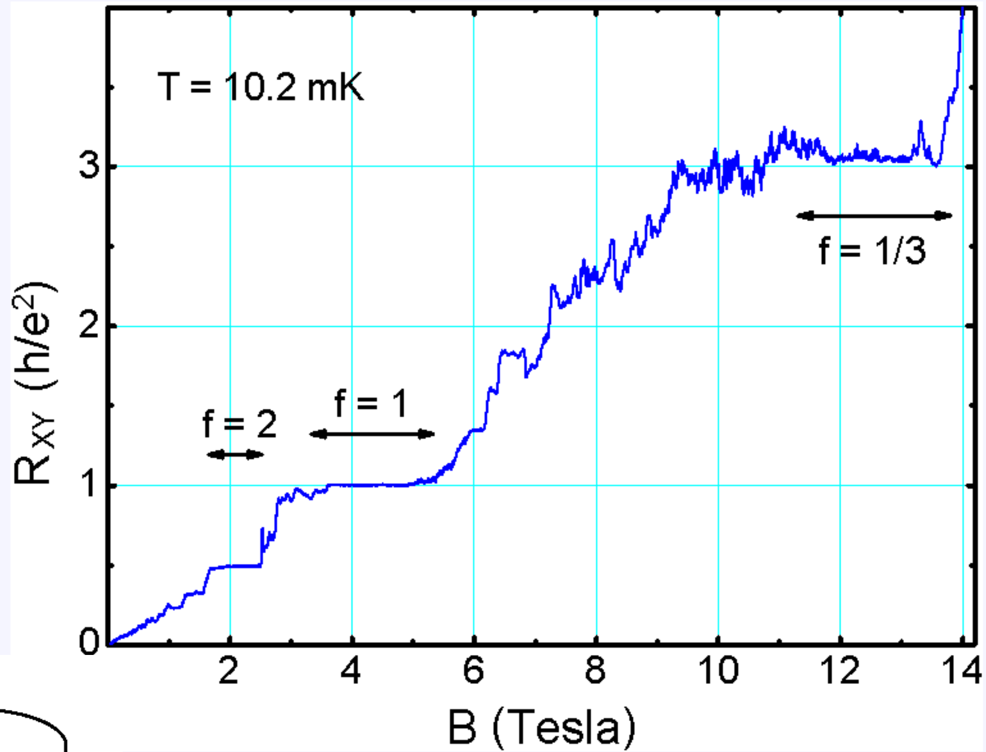
(island f = 2/5) confirms

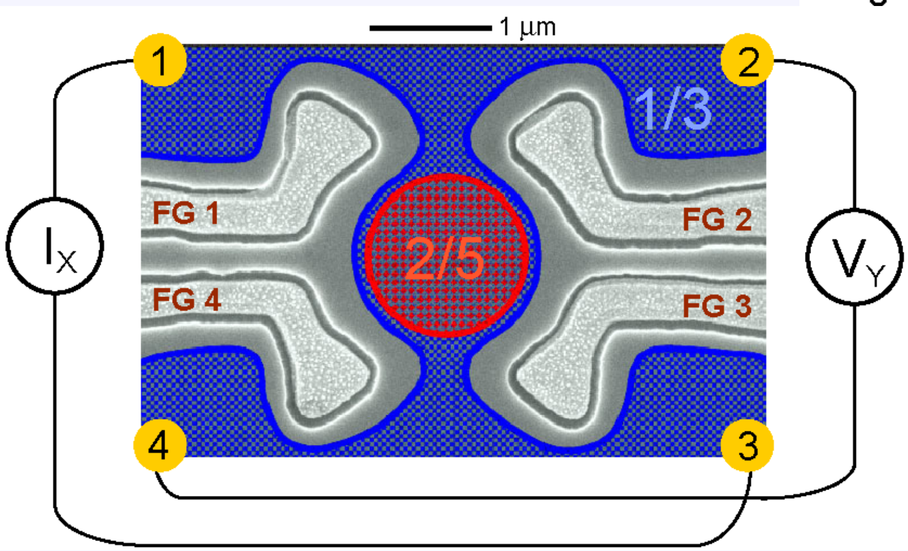
conduction through

uninterrupted f_C = 1/3

C = constriction

I = island

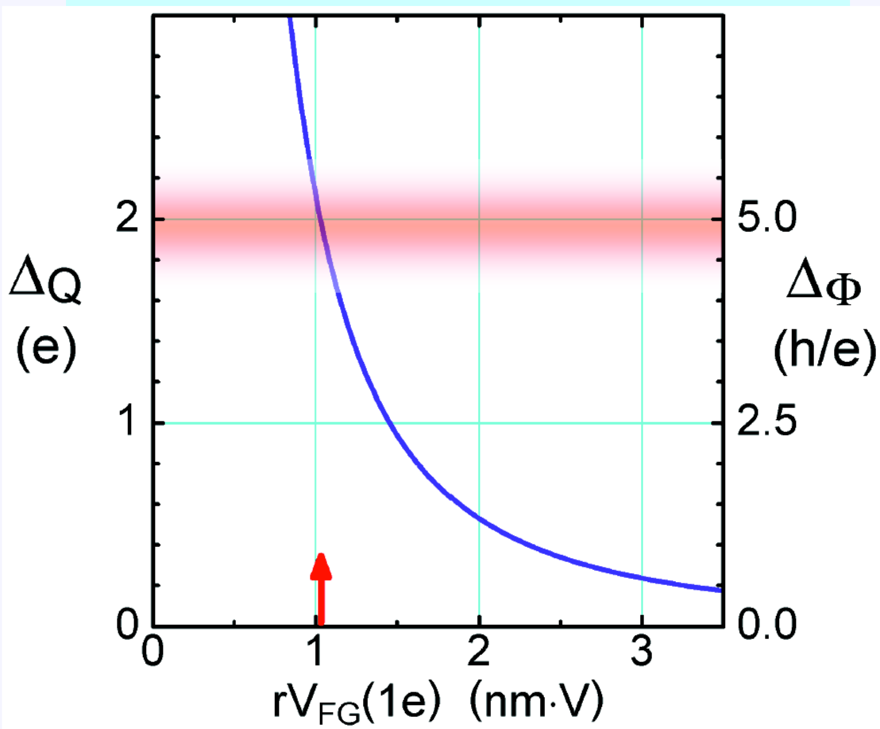


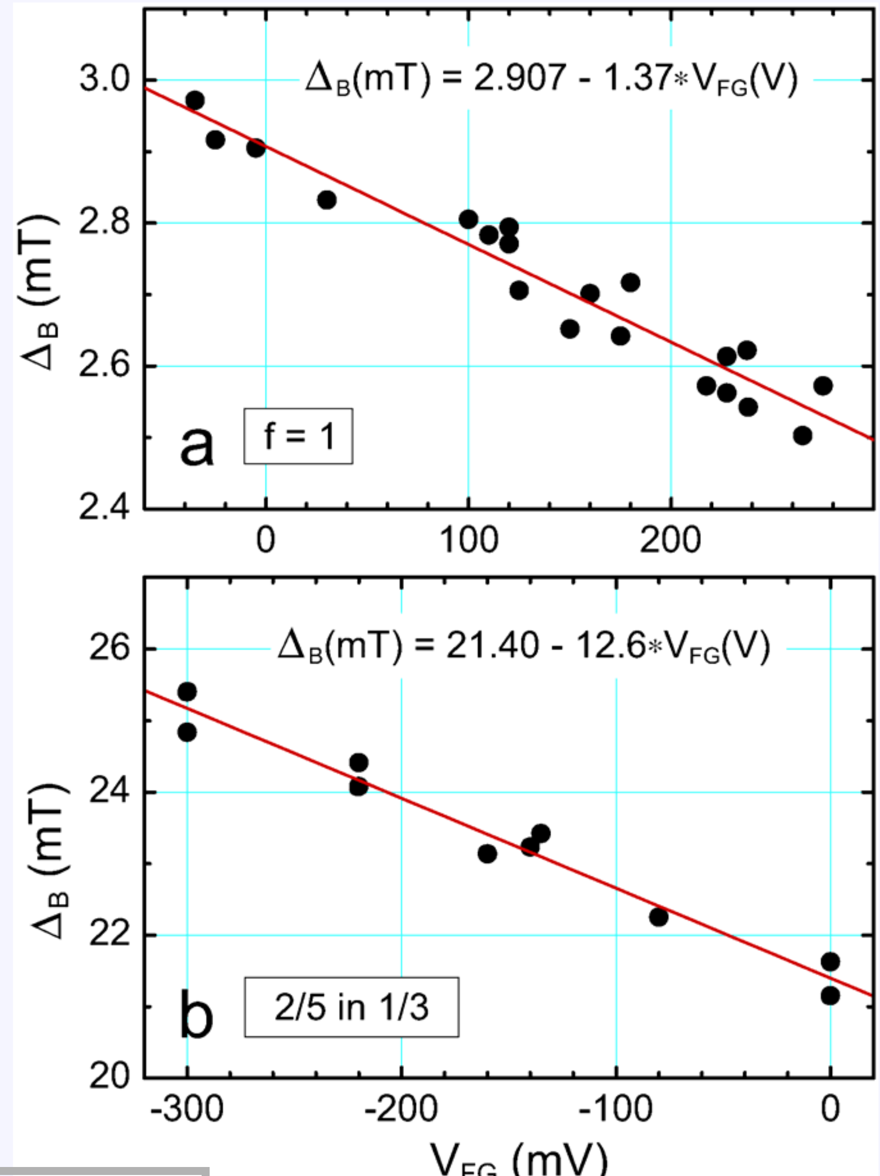


Q: How do we know the flux period is $5h/e$?

A: apply **front** gate voltage, measure *B*-field period Δ_B

⇒ scaling between integer and fractional regimes gives flux period Δ_Φ





scaling: $rV_{FG}(1e)$ equal for IQHE and FQHE

Statistics of Laughlin quasiparticles

- $f = 1/3$ LQPs: $q = e/3$ $f = 2/5$ LQPs: $q = e/5$
- Berry phase period $\Delta\gamma = 2\pi$ upon $\Delta\Phi = 5h/e$

⇒ an -*e*/3 encircling one more -*e*/3 and $\Delta N = 10$ of $f = 2/5$ LQPs:

$$\Delta\gamma = \frac{-e/3}{\hbar}\Delta\Phi + 2\pi\left(1\cdot\Theta_{1/3}^{1/3} + \Delta N\cdot\Theta_{2/5}^{-1/3}\right) = 2\pi$$

$$-\frac{5}{3} + \Theta_{1/3}^{1/3} + 10\cdot\Theta_{2/5}^{-1/3} = 1$$

e/3 statistics †

$$\frac{1}{3} + \Theta_{1/3}^{1/3} = 1 \quad\Rightarrow\quad \Theta_{1/3} = \frac{2}{3}$$

e/3 - e/5 relative

$$10\,\Theta_{2/5}^{-1/3} = 2 \quad\Rightarrow\quad \Theta_{2/5}^{-1/3} = \frac{1}{5}$$

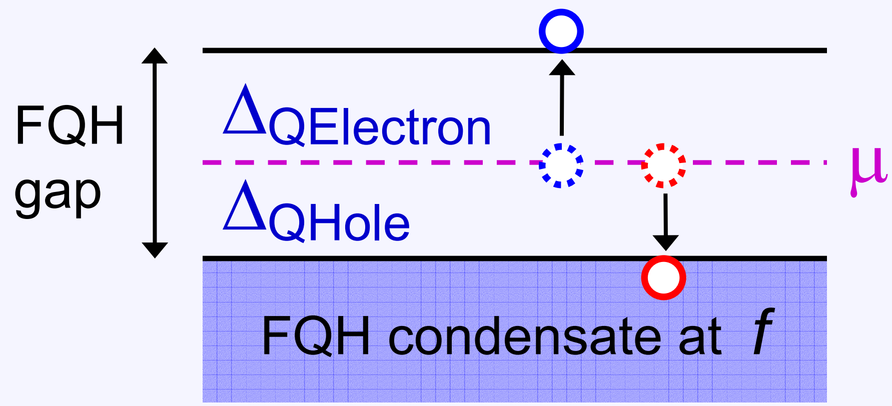
† same as in antidots, but now *no electron vacuum*

‡ $\Theta_{1/3} = 2/3$ (mod 1) no matter what

* inputs: q's (from prior antidot experiments), but NOT Θ's

How can one make FQH quasiparticles?

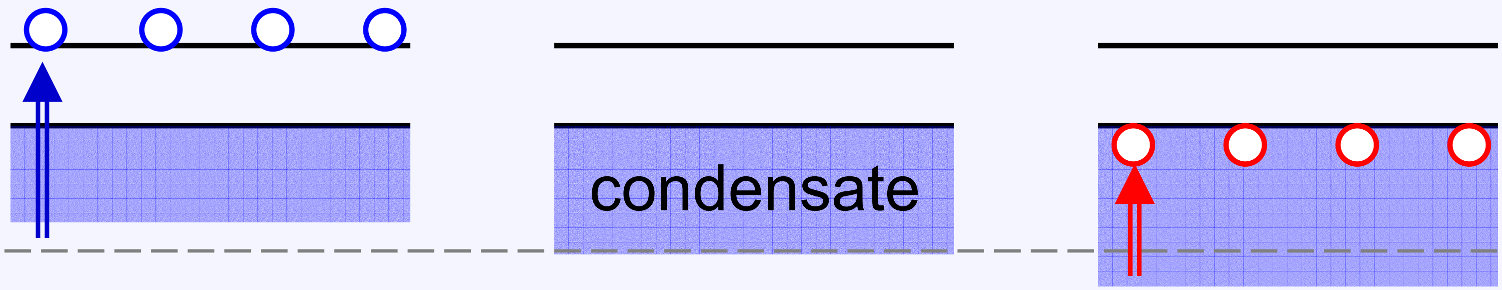
- large 2D electron system
  (include donors = neutral)



A: give energy (e.g., thermally, or shine light),
excite QElectron-QHole pairs from FQH condensate (vacuum)
$\Rightarrow$ remains neutral, unchanged $\nu = f$

How can one make FQH quasiparticles?

A: change *B*, electron density *n* is fixed $\Rightarrow$ ν changes; remains neutral †



$B < B_f$

$\nu = \frac{B_f}{B} f > f$

$B = B_f$

"exact filling"

$\nu = f$

$B > B_f$

$\nu = \frac{B_f}{B} f < f$

quantum number: $f = \frac{\sigma_{XY}}{e^2 / h}$

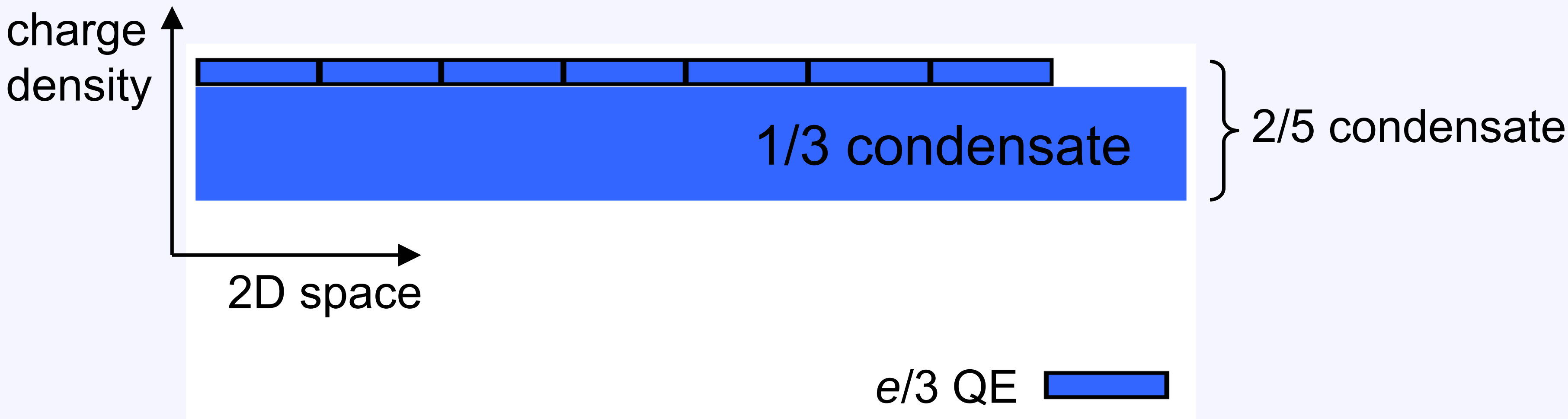
variable: $\nu = \frac{hn}{eB}$

† addition of flux does not "push charge": each *h*/*e* excites +*fe* in quasiholes out of condensate (within the area of applied flux), AND condensate charges by –*fe*

- same effects achieved by changing *n* at a fixed *B*, relevant variable is ν

Microscopic structure of the 2/5 condensate

- Haldane-Halperin hierarchy: ($f = 2/5$) is ($f = 1/3$ + MDD of $e/3$ LQE)



Maximum Density Droplet of $e/3$ LQE

anyonic statistics fixes occupation:

one $e/3$ quasielectron per area $5S_0$

resulting density: $e/15S_0$, or $\nu = 1/15$

$\Rightarrow$ total density: $1/3 + 1/15 = 2/5$

$$S_0 \equiv 2\pi \ell_0^2 = h/eB$$

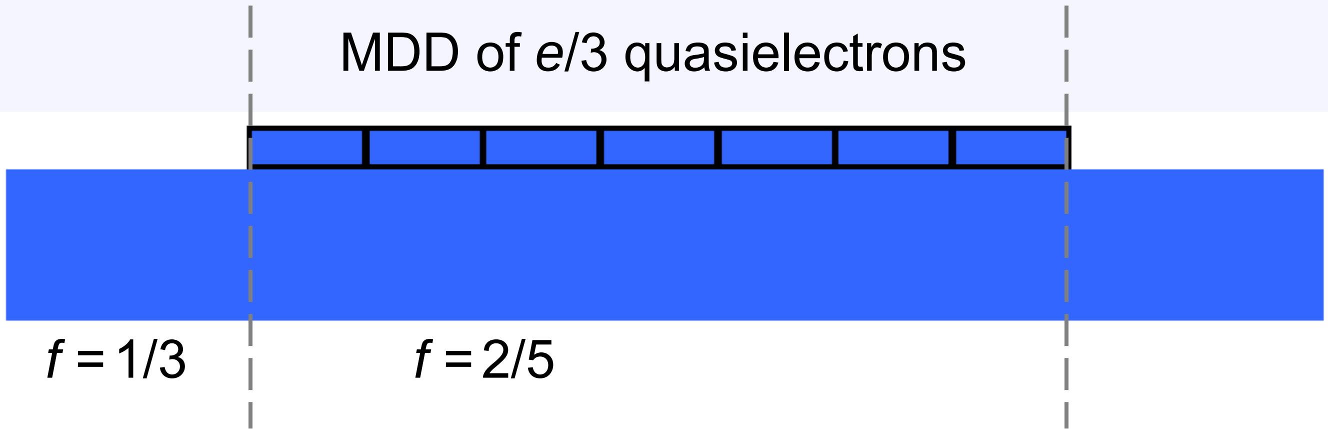
$$\nu = nS_0$$

$$\rho = e\nu$$

condensing $e/5$ QEs obtain $f = 3/7$, etc. Haldane 1983; Halperin 1984

2/5 island enclosed by 1/3

- Haldane-Halperin hierarchy theory, exact filling



quantized quasiparticles

$\delta\rho = -e/15S_0$ over $5S_0$ $\Rightarrow$ $-e/3$ QE

$\delta\rho = +e/15S_0$ over $3S_0$ $\Rightarrow$ $+e/5$ QH

$\delta\rho = -e/35S_0$ over $7S_0$ $\Rightarrow$ $-e/5$ QE

Q: What happens when filling is varied?

A: e.g., increase B ⇒ decrease $S_0 = h/eB$

1. number of S_0 fitting into island area increases by $\frac{1}{2}S_0$ per excited $+e/5$ quasihole, $10(\frac{1}{2}S_0) = 5S_0$

2. increase B ⇒ increase condensate density $n_{2/5} = 2eB/5h$
   the 1/3 condensate charges by $-5(e/3)$
   the MDD layer charges by $-5(e/15) = -e/3$ } total of $-2e$

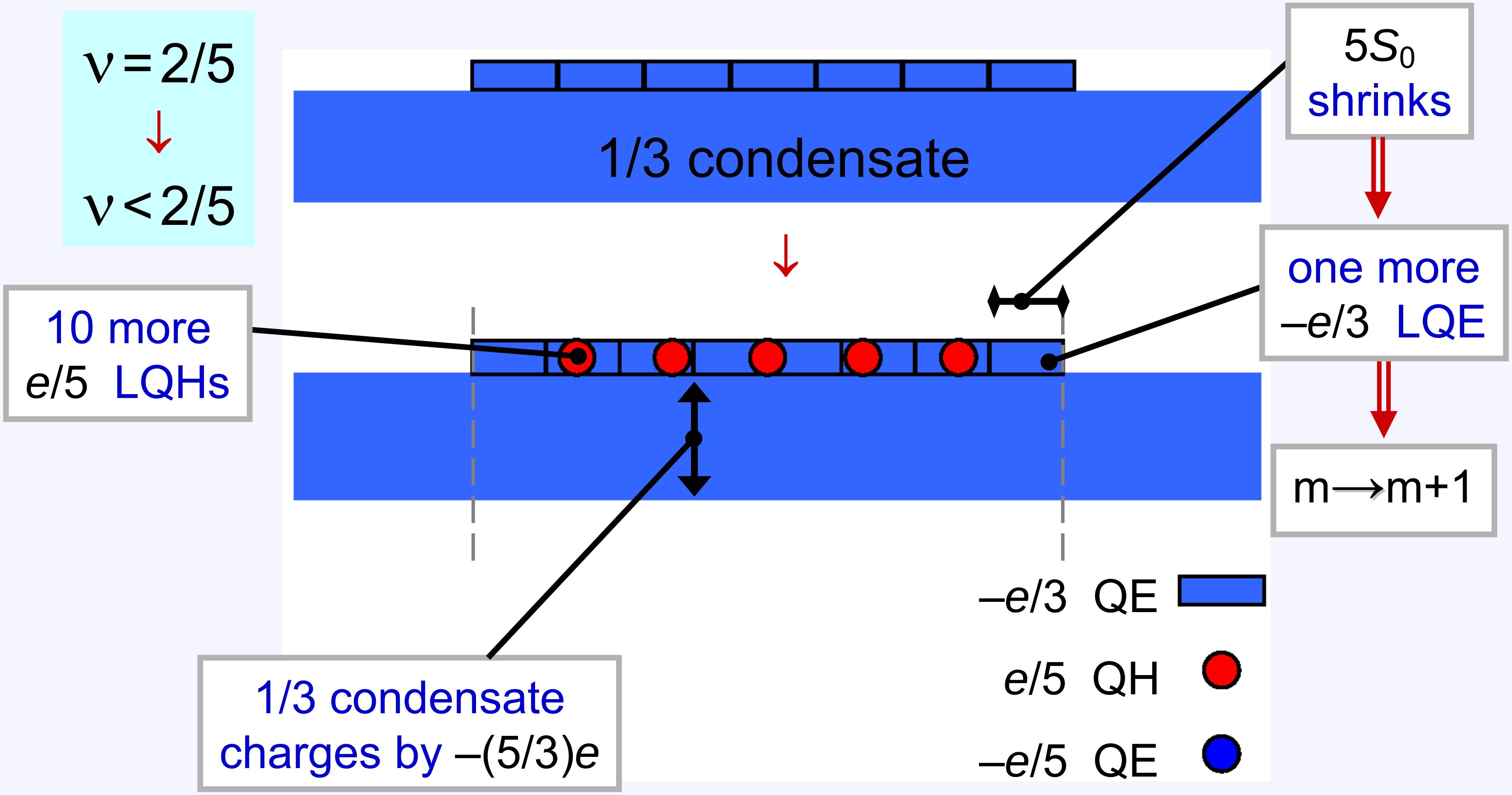
3. excite ten $e/5$ quasiholes in the island — total of $+2e$

⇒ $m \rightarrow m+1$: period is $5S_0$ (one more $-e/3$ in MDD)

⇒ island remains neutral; one more $-e/3$ quasielectron and 10 $+e/5$ quasiholes excited per addition of $5h/e$ to island

What happens when filling is varied?

H-H hierarchy illustration of increasing B by $5h/e$ through island



total island charge: $e/3 + 5e/3 - 10(e/5) = 0$

Topological order of FQH condensates

$\Rightarrow$ period is $5S_0 = 5h/eB$

change B: $\Delta\Phi = B(5S_0) = 5h/e$

change n: $\Delta Q = en\,(5S_0) = 2e$

discussion:

- period is determined by anyonic statistics $\Theta_{1/3}$ of $f = 1/3$ LQPs, and $\Theta^{1/3}_{2/5}$ of $e/3$ circling $f = 2/5$ LQPs, both fitting the same period (the two are related by the H-H hierarchy construction)
- exchange of charge in units of $1e = 3(e/3) = 5(e/5)$ is not allowed by the topological order of the 1/3 and 2/5 FQH condensates (topological order determines anyonic statistics of LQPs)

Counting composite fermions right (not just ±1 CF)

H-H hierarchy

CF hierarchy

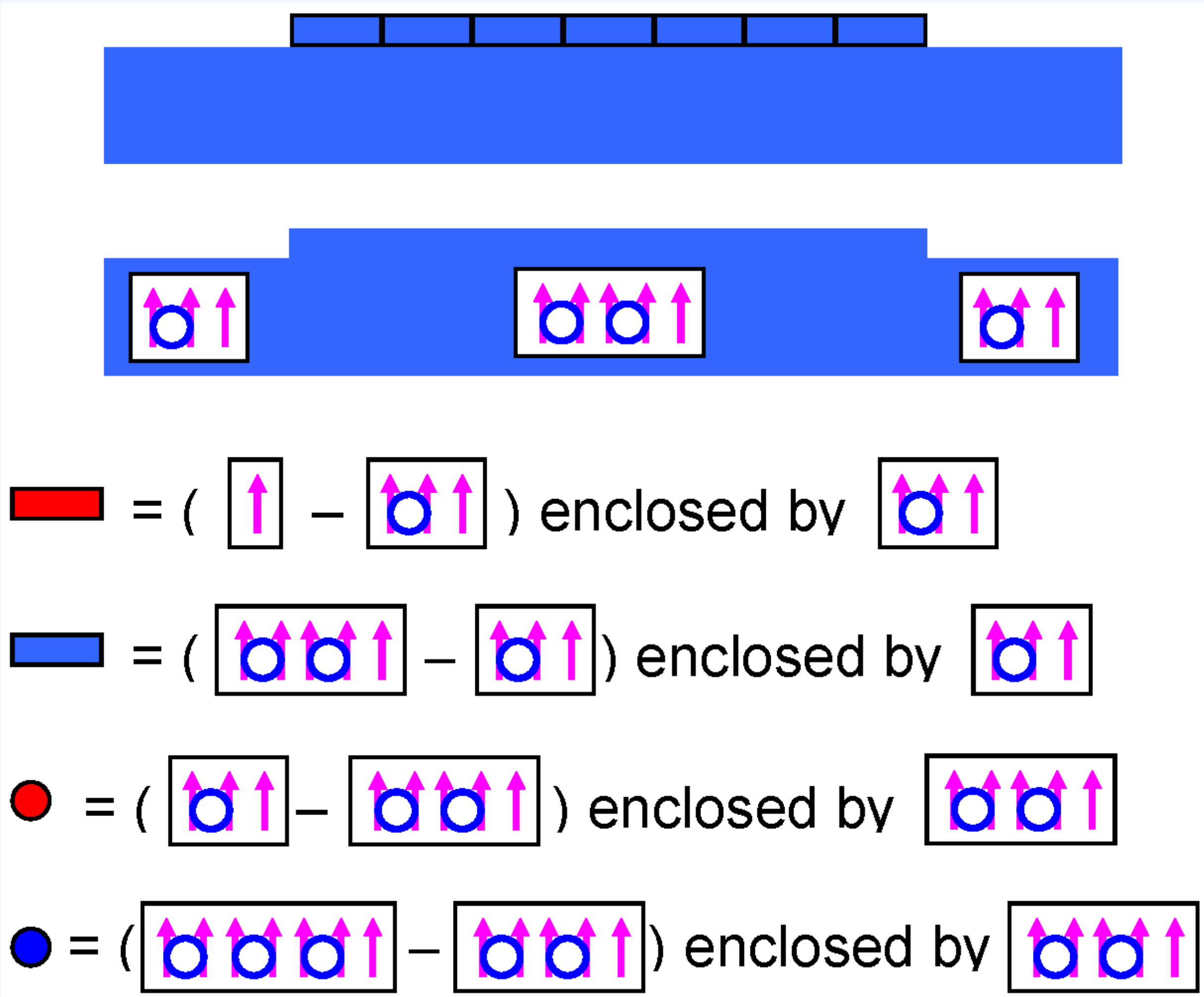
CF build of LQPs

$e/3$, $1S_0$

$-e/3$, $5S_0$

$e/5$, $3S_0$

$-e/5$, $7S_0$

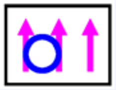


⇒ entirely equivalent to H-H theory at microscopic level

Counting composite fermions

increase B by $5h/e$ through S ⇒ decrease $S_0 = h/eB$

$B \rightarrow B'$ $\quad S' = S \quad (S/S_0)' = (S/S_0) + 5$

create ten $e/5$ = replace ten  by  = gain $20S_0$

$20S_0 + 5S_0 = 5{\times}5S_0$ goes to make five new condensate 

⇒ ten two-vortex CFs from LQHs and external $5h/e$ goes to create five new 2/5 CF condensate blocks

⇒ island remains neutral, excited ten $e/5$ quasiholes

⇒ divide by 5, get period h/e (assumes flux quantization)

need LQP statistics, just counting CFs gives wrong period

A-B oscillations vs. T

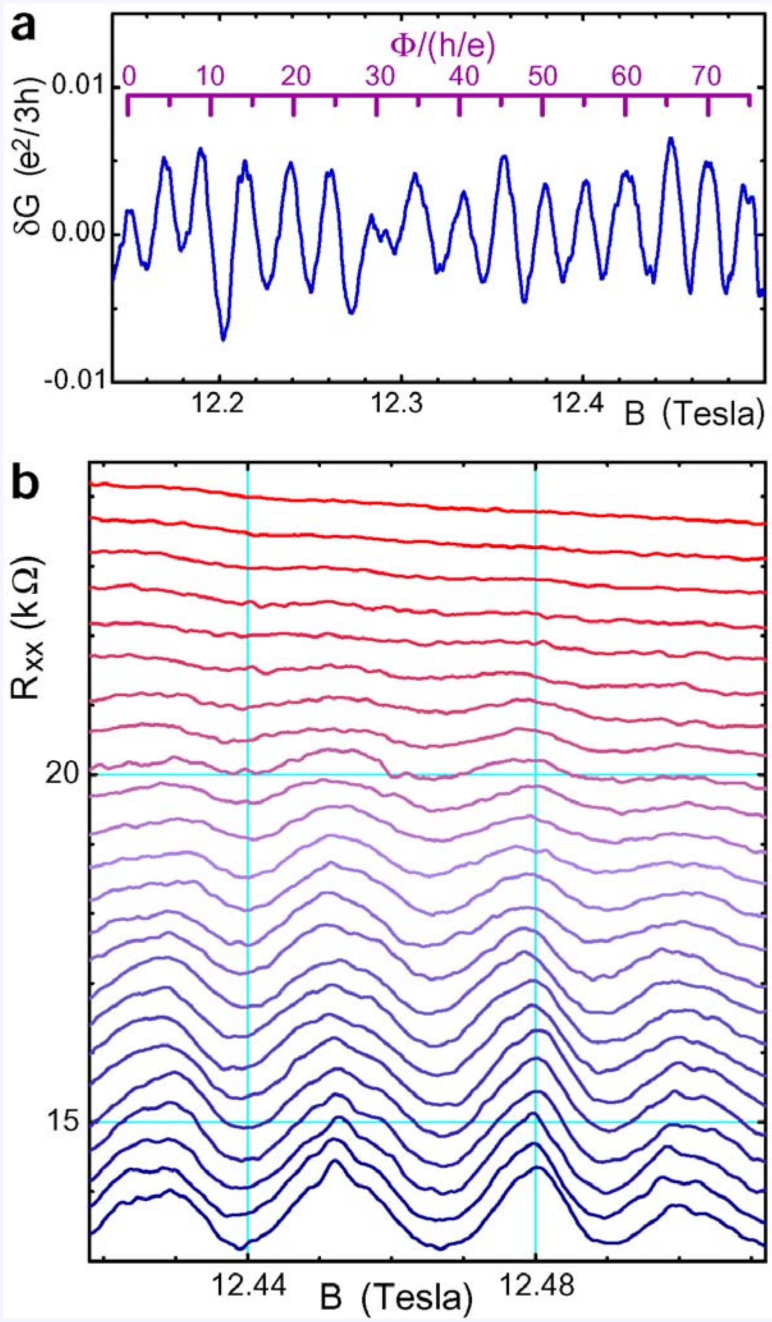
interference of *e*/3 quasiparticles circling the *f* = 2/5 island

10.2 ≤ Temperature ≤ 141 mK

Δ_Φ = 5*h*/*e* persists to highest T

⇒ experimental demonstration of robustness of topological statistical interaction

number of 2/5 LQPs in the island is well-defined so long as T << $^{2/5}\Delta$ gap ≈2 Kelvin at 12 Tesla



each next trace is shifted by 0.4 kΩ

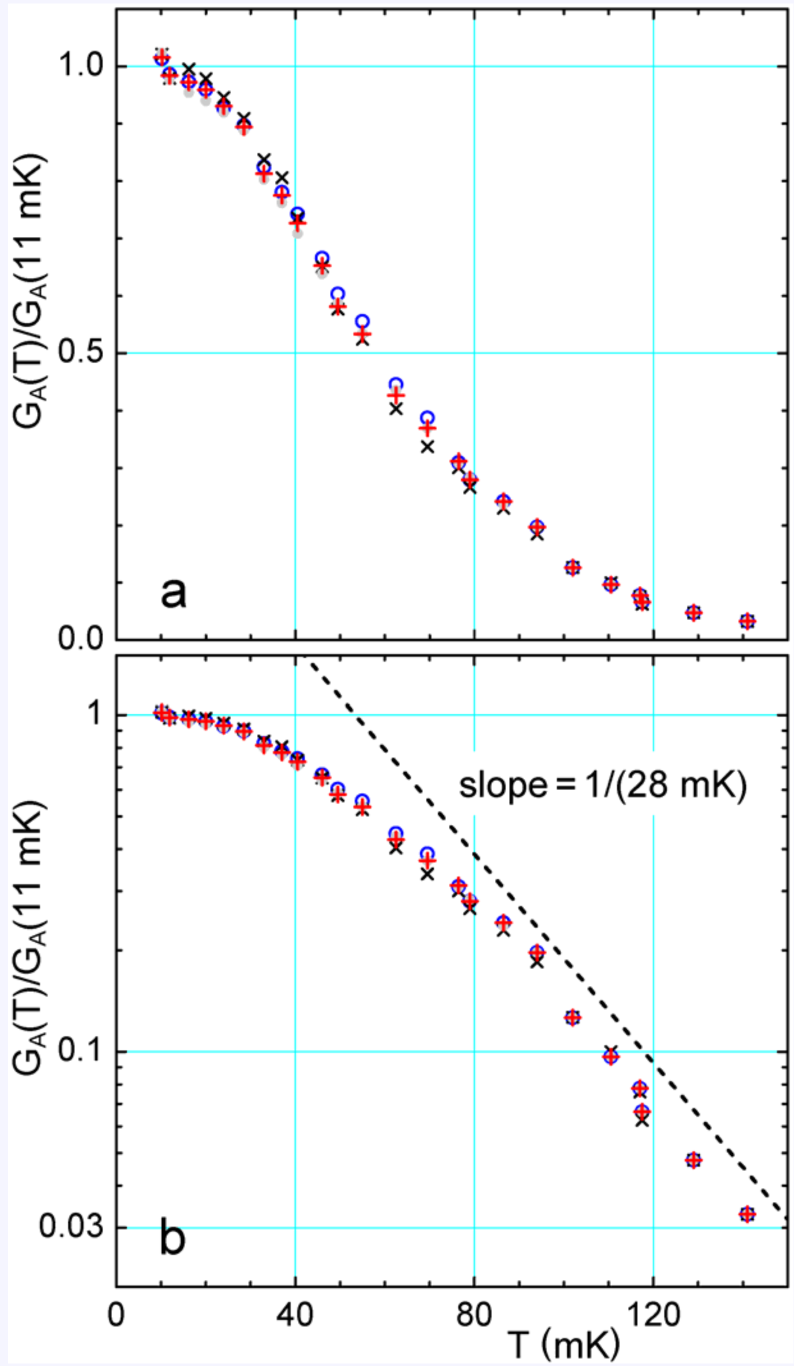
Thermal dephasing of conductance amplitude

theory:

chiral Luttinger liquid (χLL) A-B interferometer

C. Chamon *et al.* PRB 1997

- "oscillation frequency" $$\omega_0 = \frac{4\pi u}{C}$$
- finite ac bias: Hall voltage V_H=7.2 μV, in the $V_H \rightarrow 0$ limit
- high-T: $$G \propto \exp(-T/T_0)$$



T-dependence is different from RT and CB

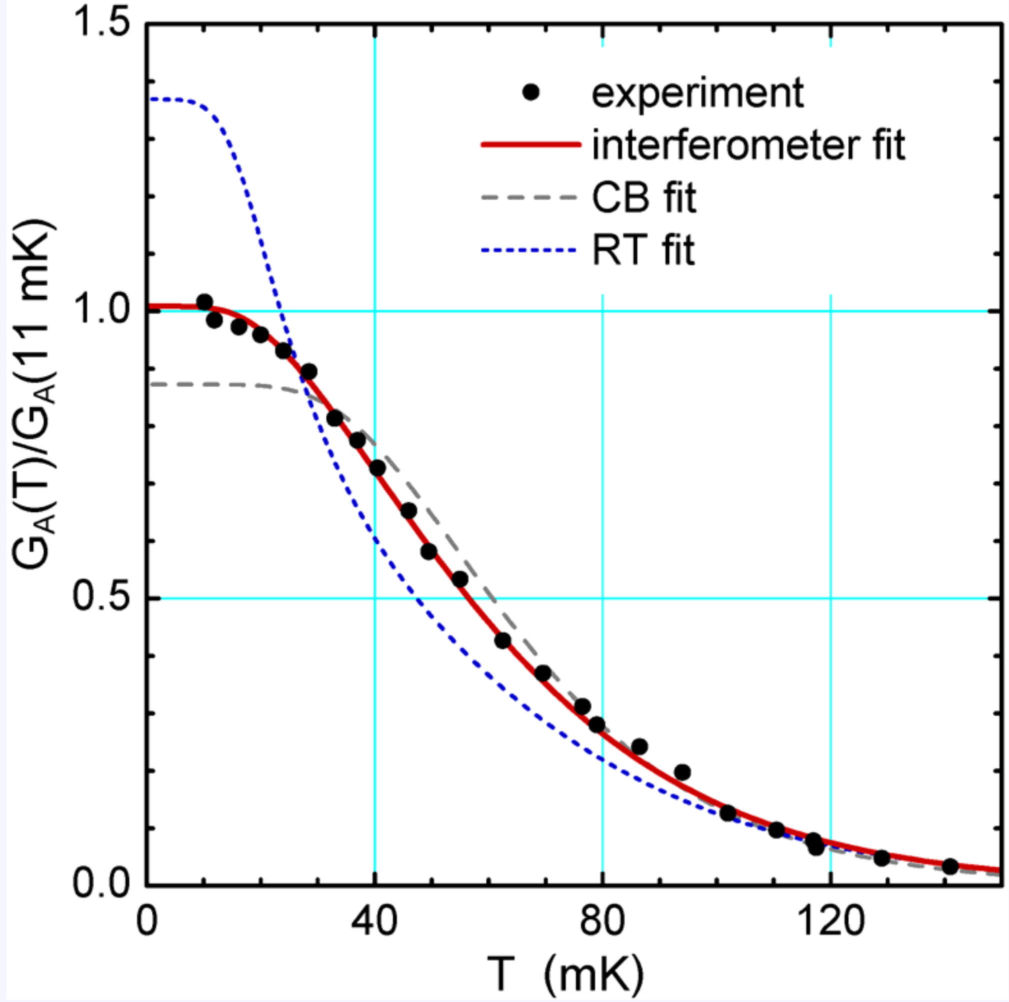
theoretical fits:

interferometer –
Chamon et al. 1997

RT – single particle
resonant tunneling

CB – “orthodox”
Coulomb blockade †

† the undershoot of CB fit at low *T* is not curable !



“electron heating temperature”
$T_H = 18$ mK for quantum antidot

Maasilta & Goldman, PRB 1997

Direct observation of anyonic statistics

- no fit to a detailed model is necessary: single-valuedness of wave function of the encircling *e*/3 LQP requires quantum statistics to be fractional

$$\Theta_{1/3} = \frac{2}{3}$$

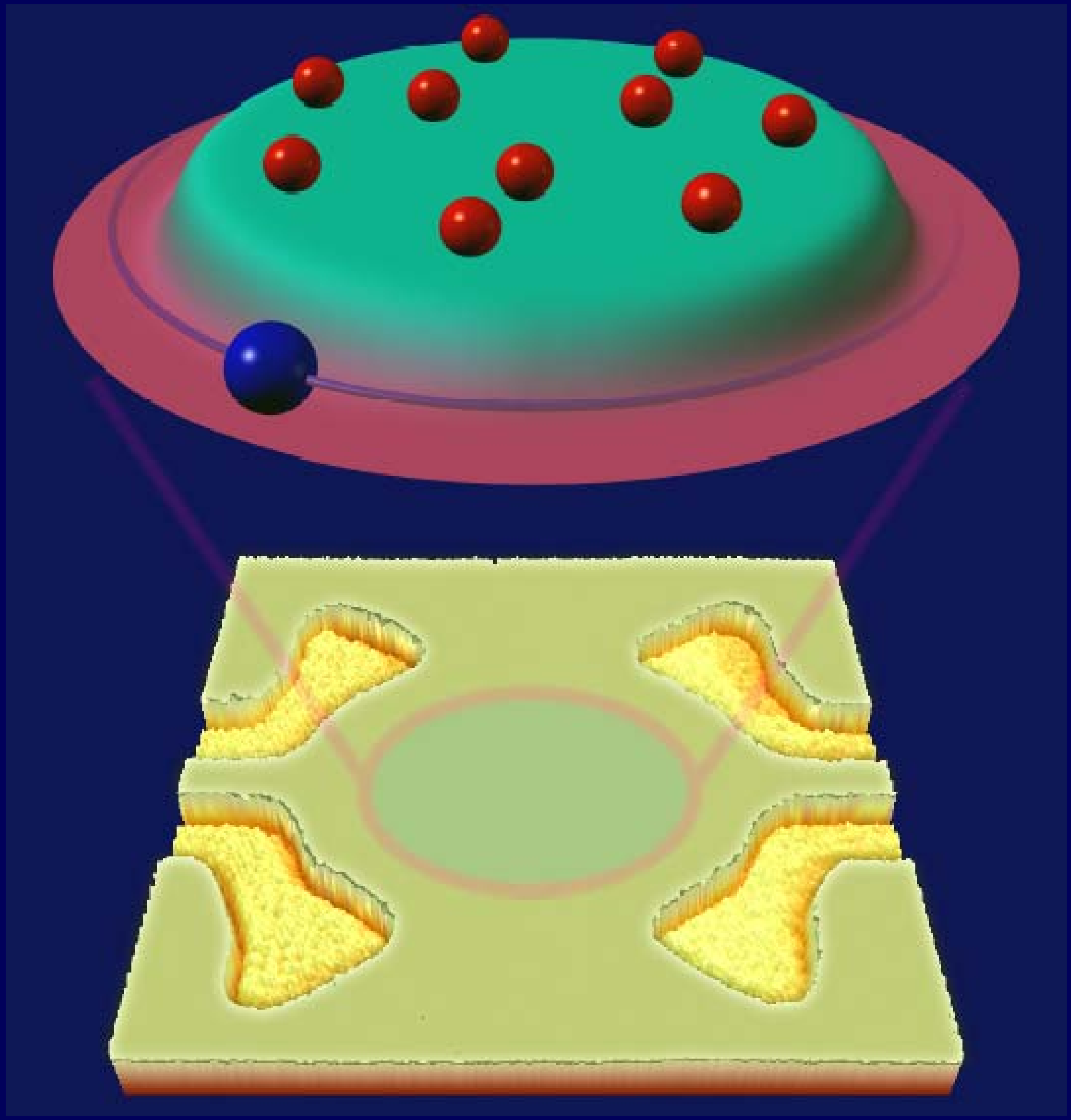
$$\Theta_{2/5}^{1/3} = \frac{1}{5}$$

- direct: experiment closely models definition of anyonic quantum statistics in 2D

- the only input: LQP charges *e*/3, *e*/5 have been measured directly in quantum antidots

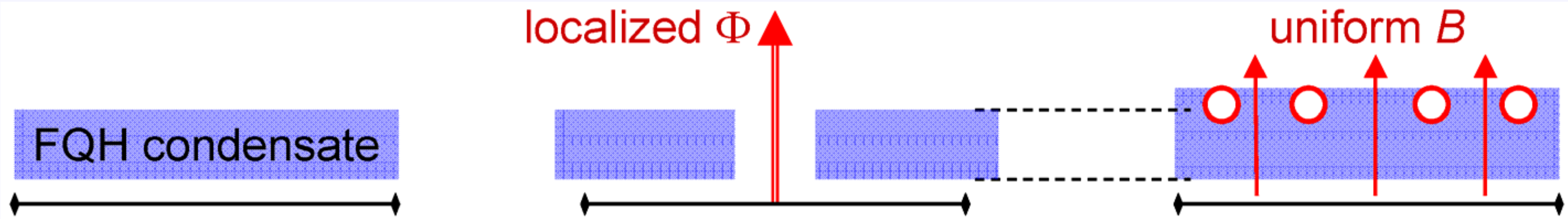
- thermal dephasing fits well A-B interferometer theory; demonstrates robustness of statistical interaction

Thanks for attention!



“nothing but charge transfer” counterarguments

• here’s NOT Laughlin gedanken experiment geometry: flux is real, 2D electrons in uniform field *B*, which can’t be gauge-transformed to zero



$\nu = f$ exact filling $\quad$ $\nu \neq f$ ⇒ charging U $\quad$ $\nu \neq f$ true ground state

• addition of flux does not “push charge”: each *h*/*e* excites +*fe* in quasiholes out of condensate (within the area of applied flux), AND condensate charges by –*fe*

⇒ total FQH fluid is neutral (net charging = huge Coulomb energy)

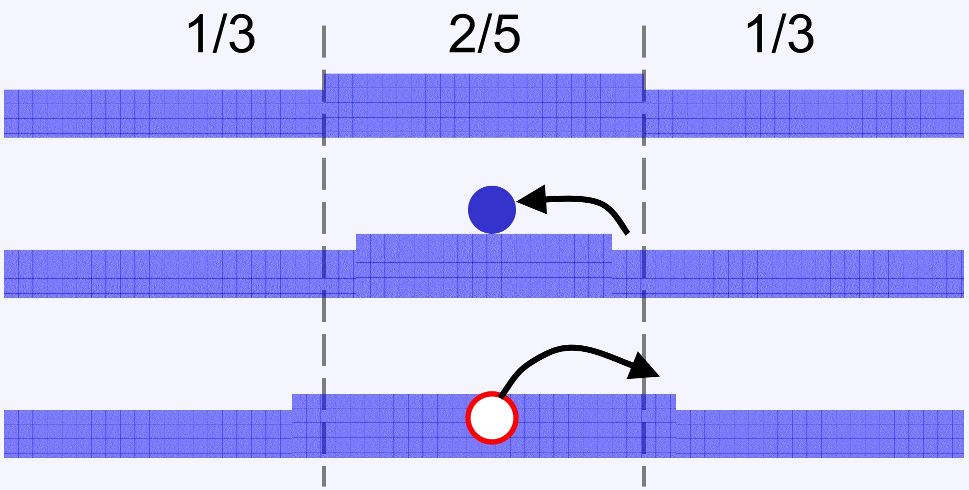
⇒ predicted periods are wrong: $\Delta_\Phi = (5/2)h/e$, $\Delta_Q = 1e$

"quasiparticles allowed, but ... ?" model Jain et al. 2006

- considers "transitions" of CFs to/from island to 2/5-1/3 boundary and to surrounding 1/3, requiring total CF number be fixed, ignores statistics
- overlooks simple excitation of LQPs from condensate, resulting in $\nu \neq f$, as occurs in experiment

$\Rightarrow$ charge transfer is allowed in units of $e/5$ and/or $e/3$, contrary to experiment

"By definition, when CF is added in the interior of the island, it shifts the island edge by an amount that encloses two additional flux quanta, giving an excess boundary charge of [2e/15]" – but, LQPs are excited from FQH condensate, no "boundary charge" results: *redistribution* of electronic charge between LPQs and condensate



(PHY102: $\Phi = BS$, adding flux does not imply increasing *S*, can increase *B*)