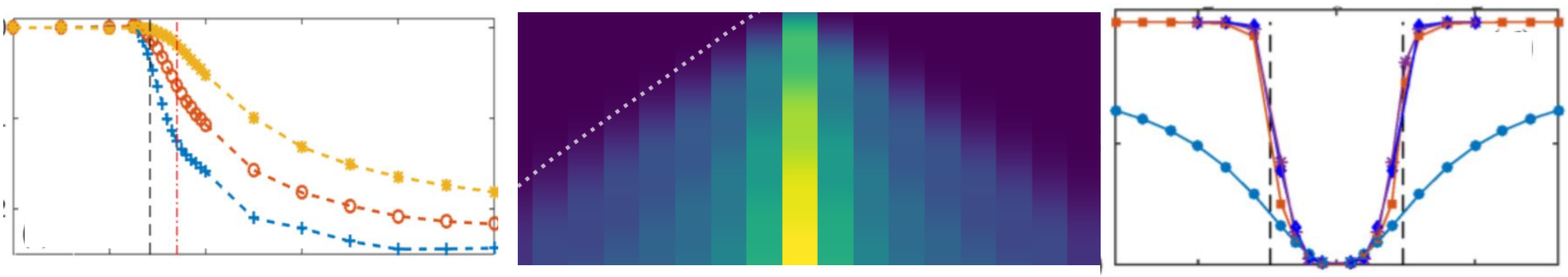
Many-body open quantum systems: about baths and transport

KITP

Thermodynamics of Quantum Systems



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Ministry of Education SINGAPORE

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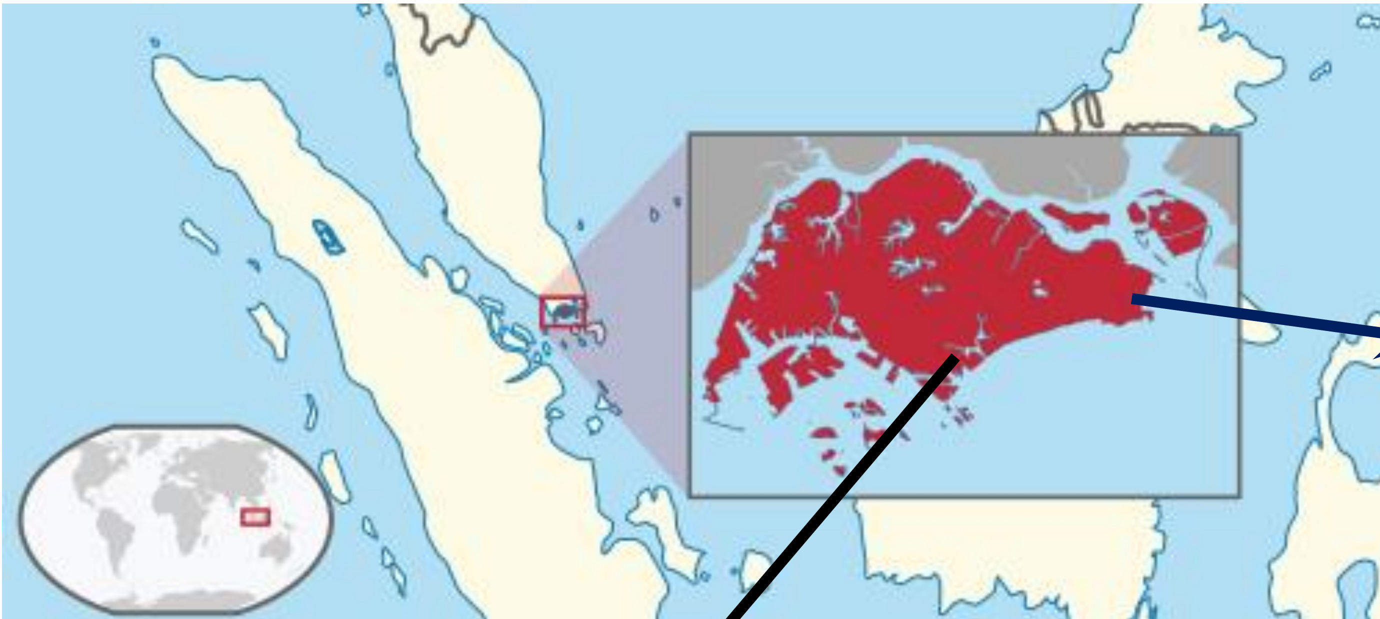
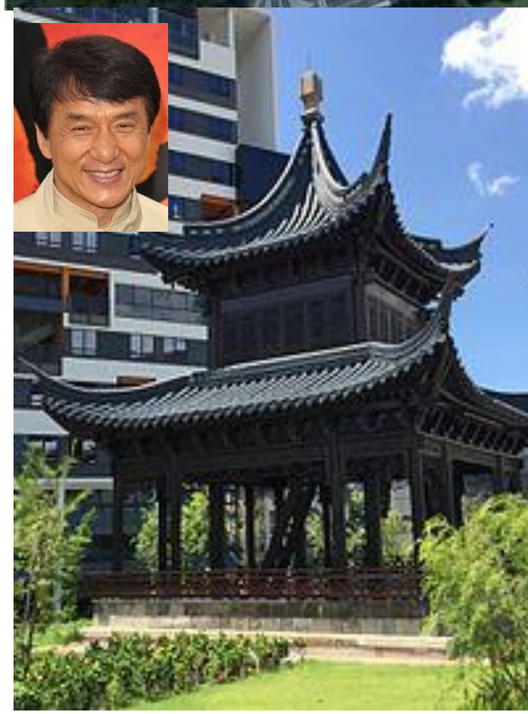


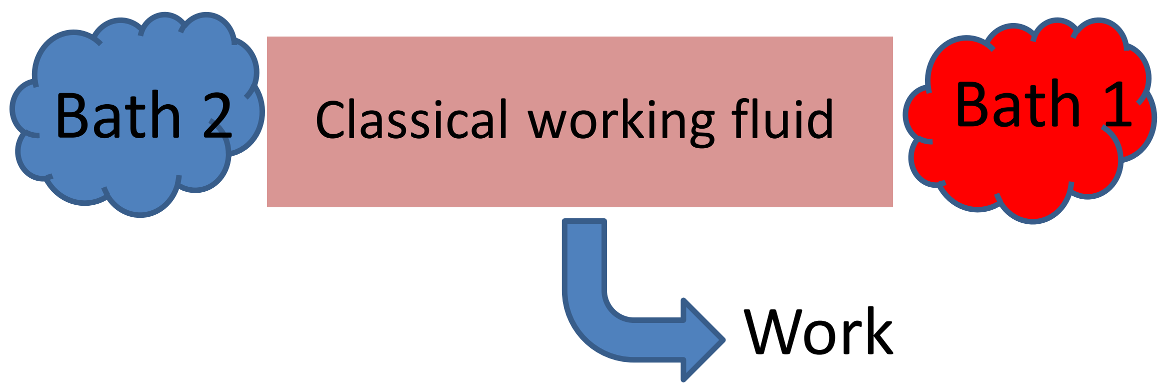
Singapore
170 km north of Equator
5.6 M people (USA 325M)
720 km^2 (USA 9.8M km^2)

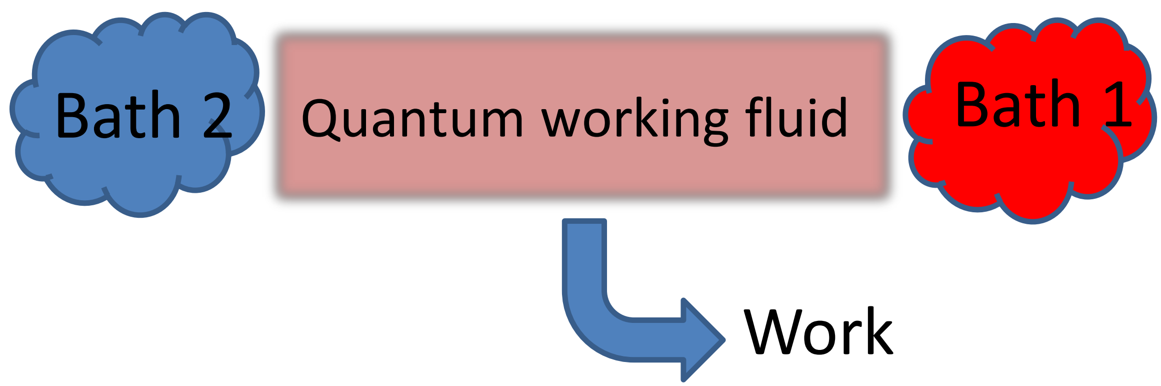
74% Chinese
13% Malay
9% Indian

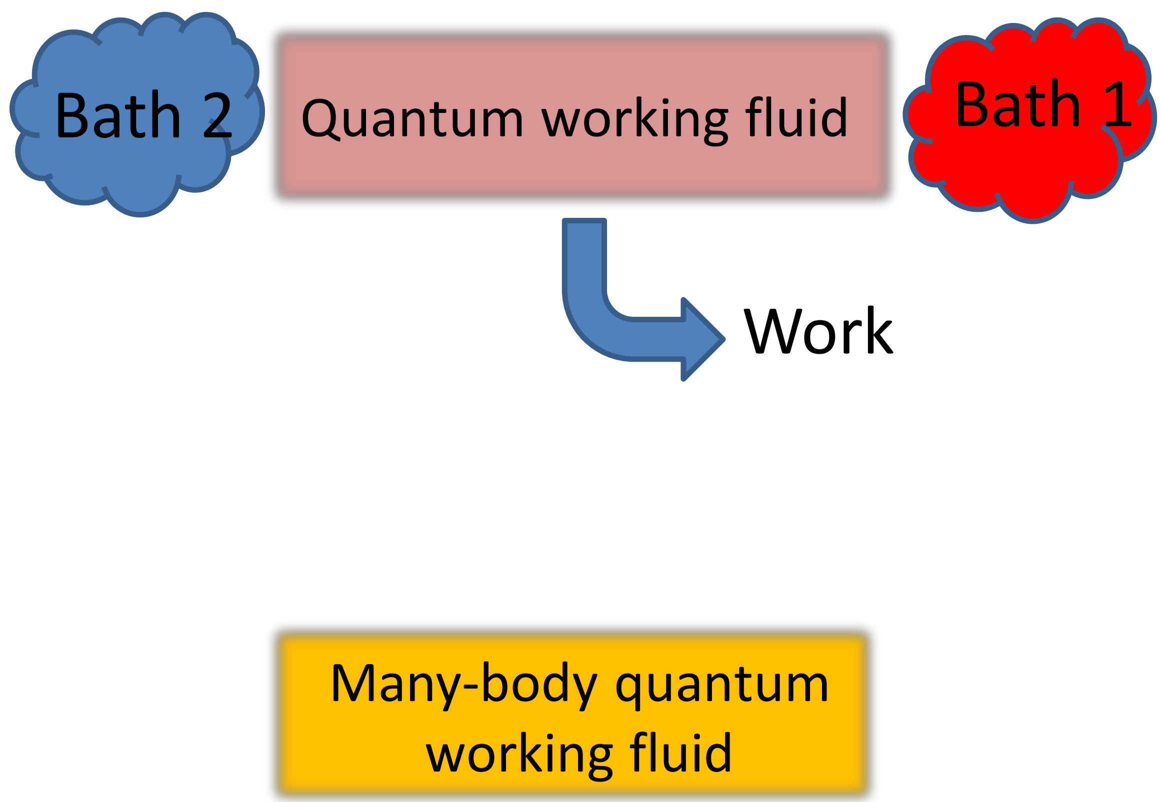
3 Universities with Physics
SUTD, NUS, NTU

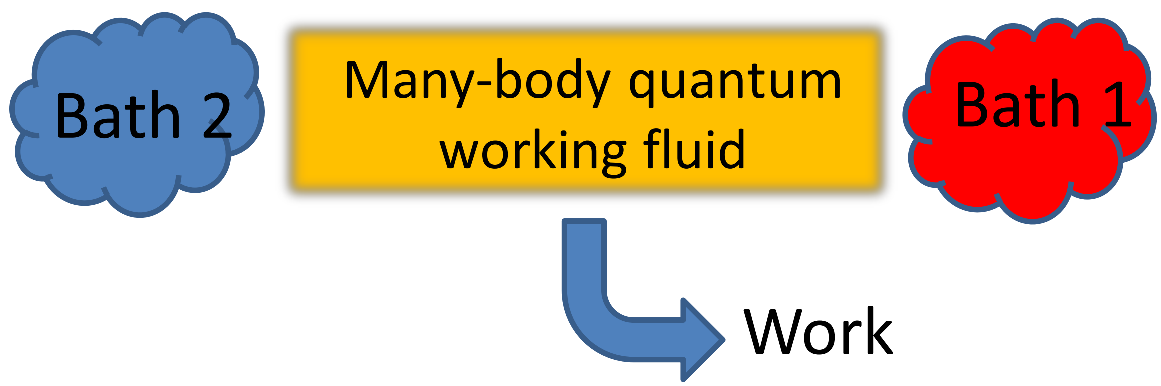
SINGAPORE UNIVERSITY OF TECHNOLOGY AND DESIGN



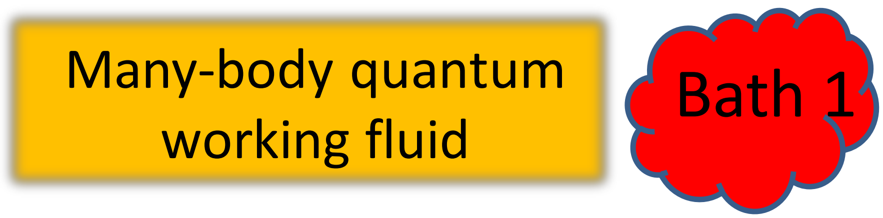

Bath 2
Classical working fluid
Bath 1
Work

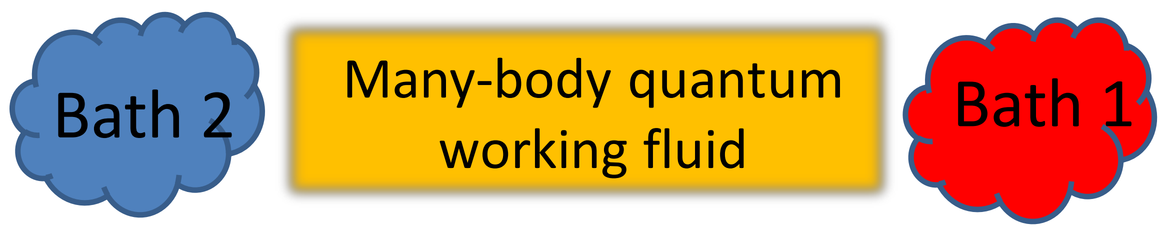

Bath 2
Quantum working fluid
Bath 1
Work

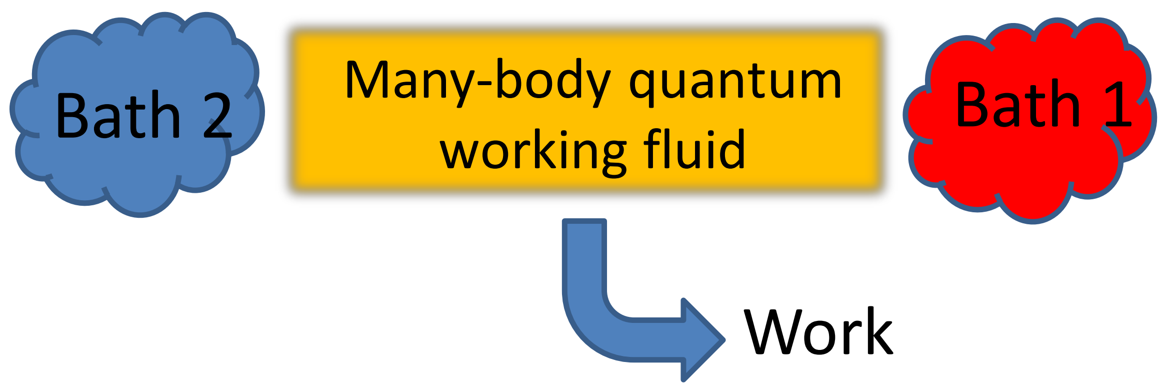

Bath 2
Quantum working fluid
Bath 1
Work
Many-body quantum working fluid


Bath 2
Many-body quantum working fluid
Bath 1
Work

Many-body quantum working fluid


Many-body quantum working fluid
Bath 1

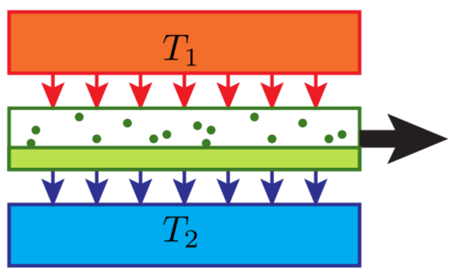

Bath 2
Many-body quantum
working fluid
Bath 1


Bath 2
Many-body quantum working fluid
Bath 1
Work

Many-body quantum working fluid

Many-body quantum working fluid

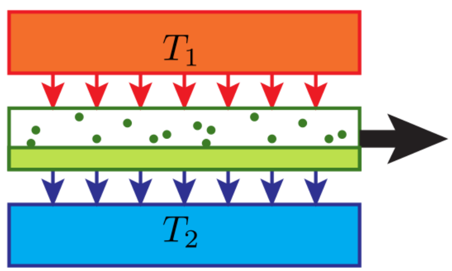
Phase transitions and latent heat in working fluid



Campisi, Fazio, Nat. Comm. (2016)

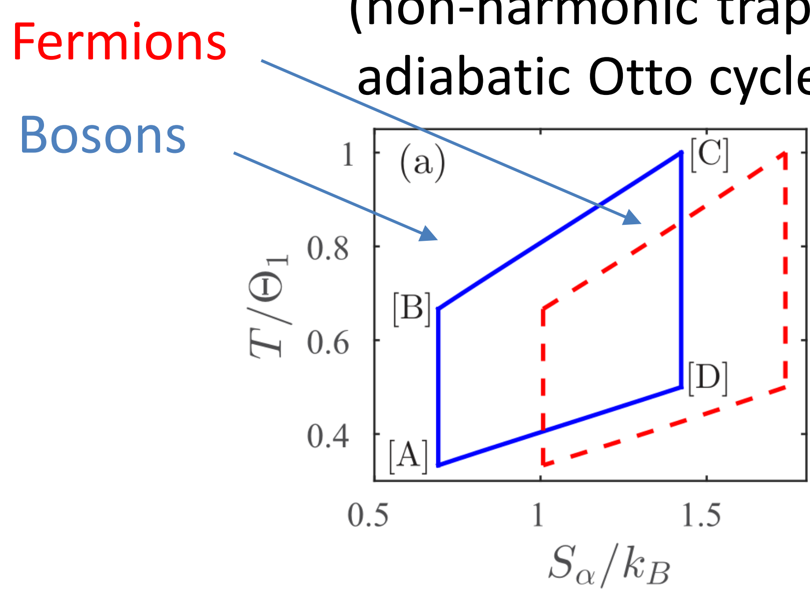
Many-body quantum working fluid

Phase transitions and latent heat in working fluid



Campisi, Fazio, Nat. Comm. (2016)

Bosonic vs Fermionic working fluid (non-harmonic traps, adiabatic Otto cycle)

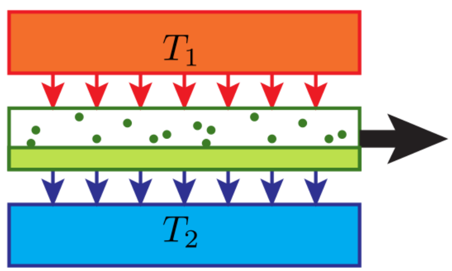


N=2 atoms

Zheng, Poletti, PRE (2015)

Many-body quantum working fluid

Phase transitions and latent heat in working fluid



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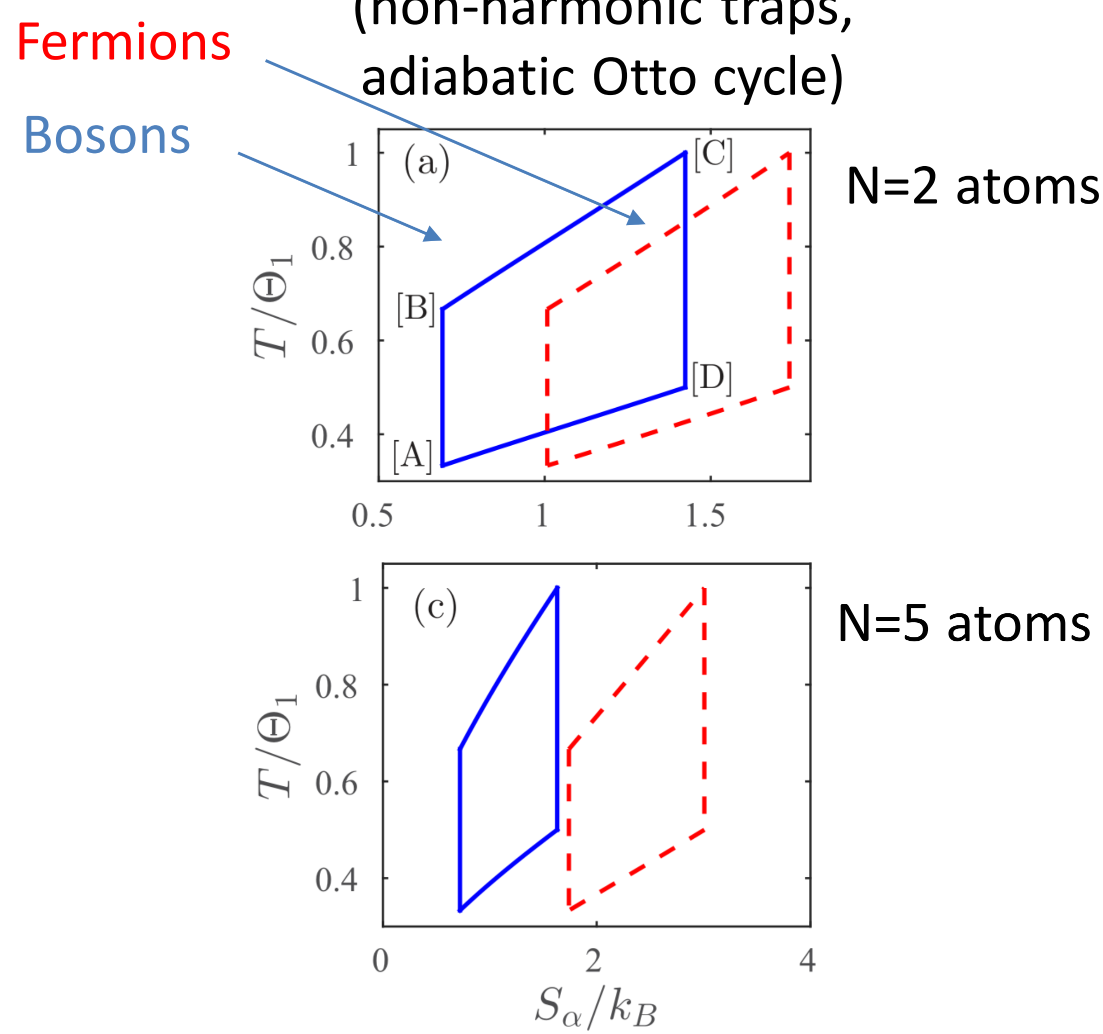

Zheng, Poletti, PRE (2015)

Many-body quantum working fluid

XXZ model

$$\hat{H} = \sum_{l=1}^{L-1} \left[J \left(\hat{\sigma}_l^x \hat{\sigma}_{l+1}^x + \hat{\sigma}_l^y \hat{\sigma}_{l+1}^y \right) + \Delta \hat{\sigma}_l^z \hat{\sigma}_{l+1}^z \right]$$

Many-body quantum working fluid

XXZ model

$$\hat{H} = \sum_{l=1}^{L-1} \Big[\underbrace{J\left(\hat{\sigma}_l^x \hat{\sigma}_{l+1}^x + \hat{\sigma}_l^y \hat{\sigma}_{l+1}^y\right)}_{\text{Kinetic energy}} + \underbrace{\Delta \hat{\sigma}_l^z \hat{\sigma}_{l+1}^z}_{\text{Interaction/Anisotropy}} \Big]$$

Kinetic energy

Interaction/Anisotropy

Many-body quantum working fluid

XXZ model

Kinetic energy

Interaction/Anisotropy

$$\hat{H} = \sum_{l=1}^{L-1} \left[J \left(\hat{\sigma}_l^x \hat{\sigma}_{l+1}^x + \hat{\sigma}_l^y \hat{\sigma}_{l+1}^y \right) + \Delta \hat{\sigma}_l^z \hat{\sigma}_{l+1}^z \right]$$

Many-body quantum working fluid

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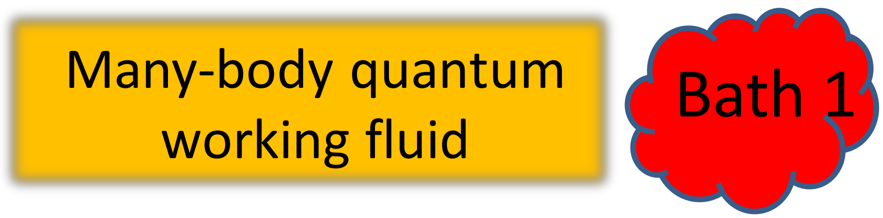
Kinetic energy: $J \left(\hat{\sigma}_l^x \hat{\sigma}_{l+1}^x + \hat{\sigma}_l^y \hat{\sigma}_{l+1}^y \right)$

Interaction/Anisotropy: $\Delta \hat{\sigma}_l^z \hat{\sigma}_{l+1}^z$

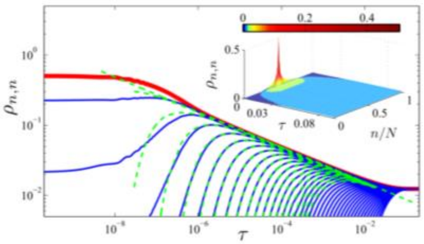
1D systems -> much more easily strongly correlated, tools


Many-body quantum working fluid
Bath 1

Many-body quantum working fluid

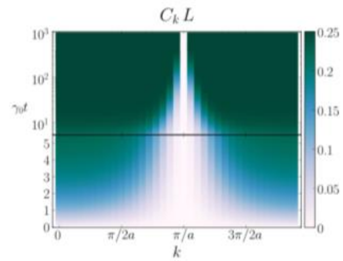
Bath 1

Probing Systems



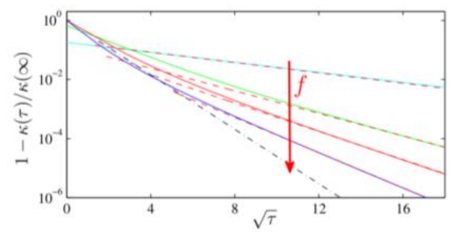
D.P. et al PRL (2012)

Generating Correlations



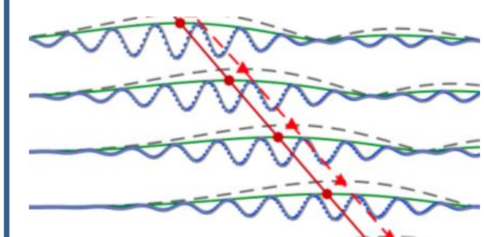
J.-S. Bernier et al. PRA (2013)

Relaxation Regimes



B.Sciolla. et al PRL (2015)

Spreading correlations

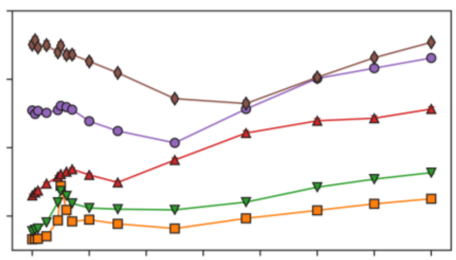


J.-S. Bernier. et al PRL (2018)

Time crystals

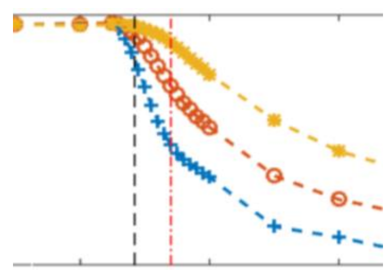
R.R.W. Wang et al PRE (2018)

Localization



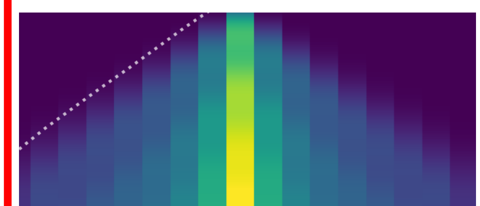
X. Xu et al. PRB (2018)

Stability



C. Guo et al. PRA (2018)

Methods



X. Xu et al. in preparation

Interplay between **interactions** and **dissipation**

- Steady states
- Out-of-equilibrium phase transitions
- Transient correlations
- Relaxation regimes
- Propagation correlations
- Methods
- …

Many-body quantum working fluid

Bath 1

Experiments

Ultracold gases experiments prided themselves to be clean and close.

Now they can be open and also disordered … but **in a controlled way**!

Many-body quantum working fluid

Bath 1

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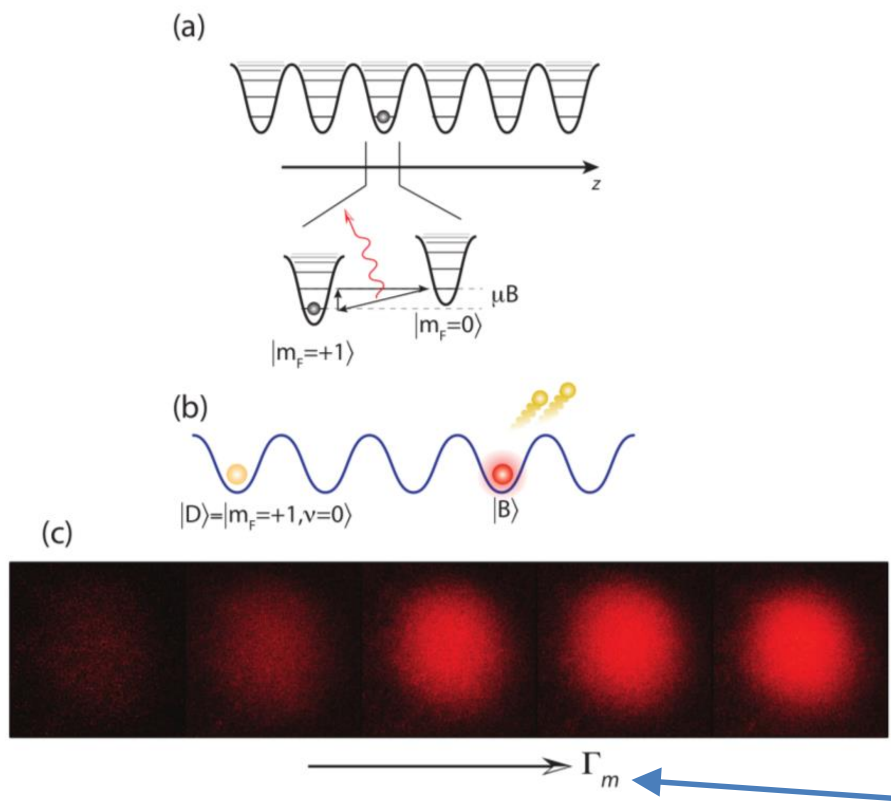


Take bosons in a lattice and shine light at low intensity on it such that there can be fluorescence.

Studying the light emitted you can learn about the system properties.

Laser intensity

Patil et al. Phys. Rev. Lett. (2015)

Many-body quantum working fluid

Bath 1

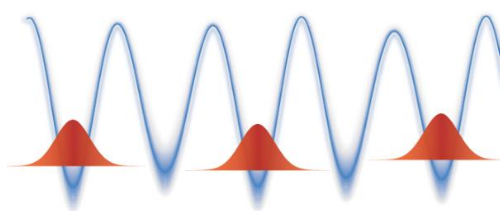
Experiments

Ultracold gases experiments prided themselves to be clean and close.

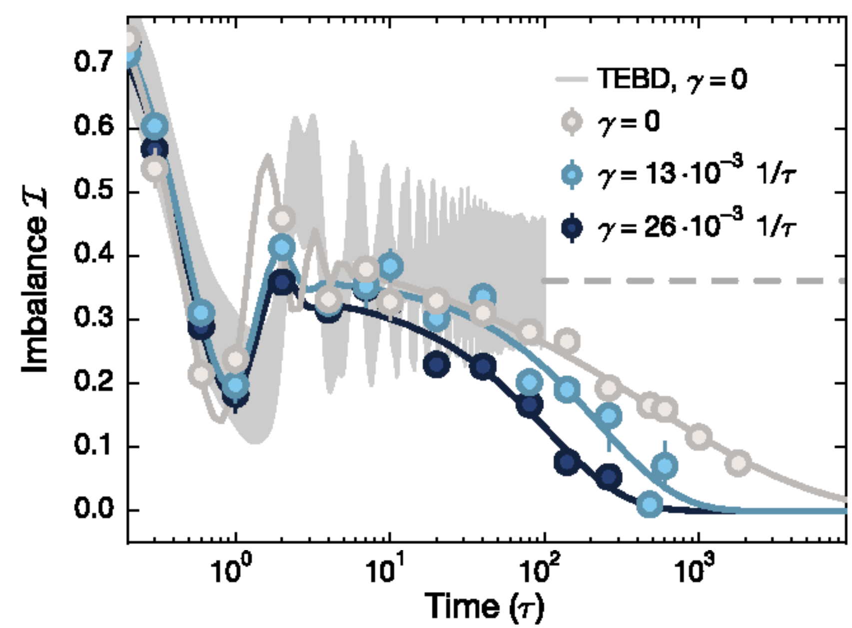
Now they can be open and also disordered ... but **in a controlled way**!

Take a strongly interacting disordered Hubbard model. It is many-body localized -> the population imbalance I between nearest sites stays large.

But once dephasing is applied the Imbalance will decay.



γ decay rate



Luschen et al. Phys. Rev. X (2017)

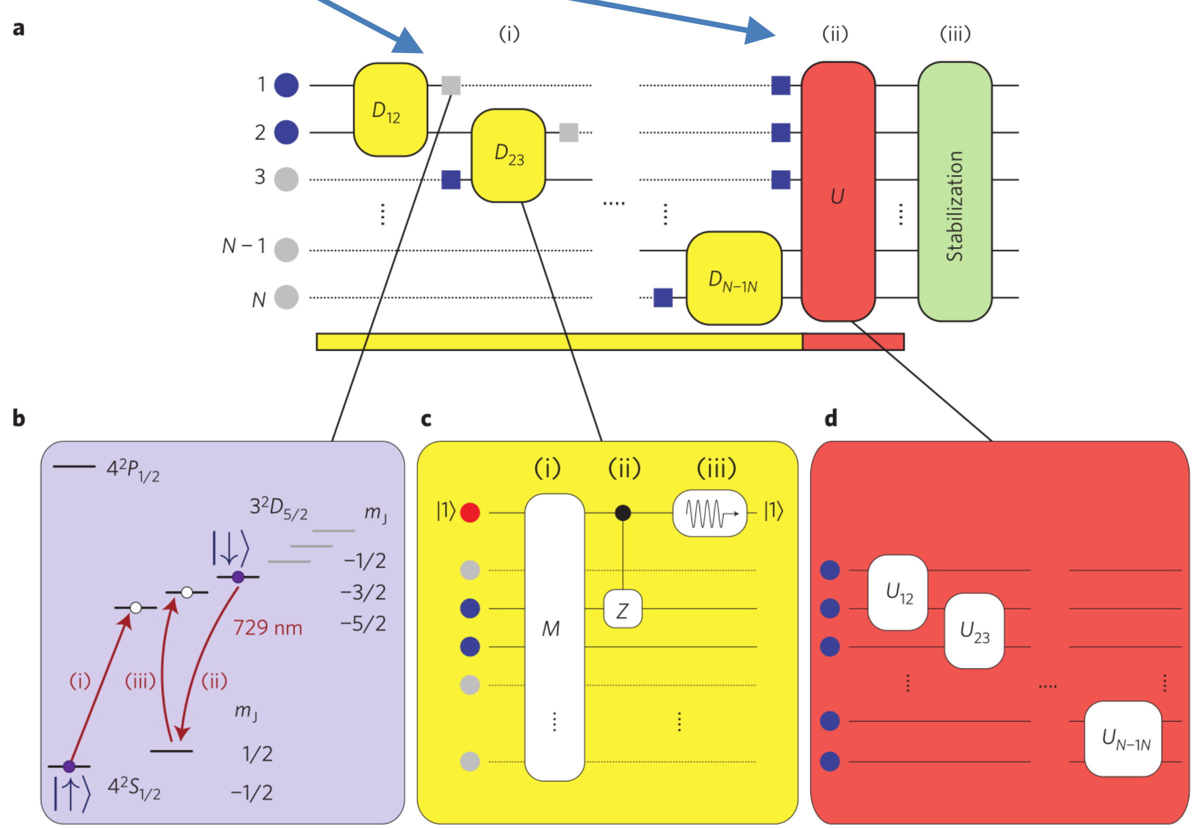
Many-body quantum working fluid

Bath 1

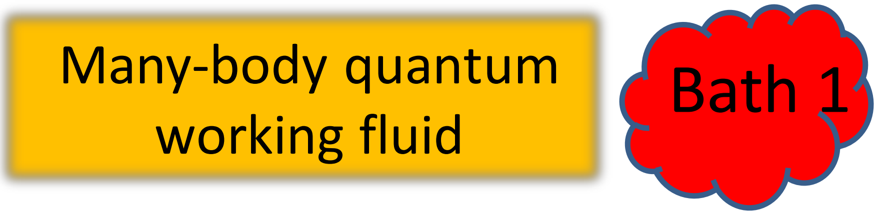
Experiments

Another way to study experimentally these systems is by using a quantum simulator.
Here trapped ions by Blatt's group.

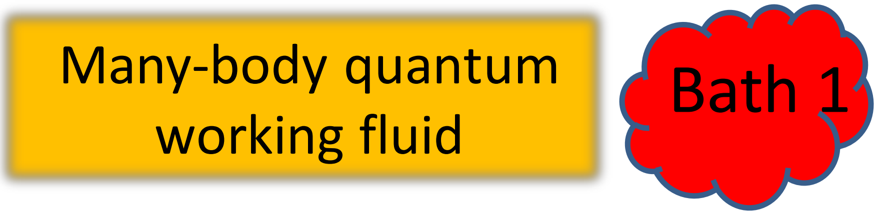
$$U = e^{L\,dt} = e^{(H+D)dt} \approx e^{Ddt}\, e^{Hdt}$$



Schindler et al. Nat Phys (2013)



Strategies to study manybody open quantum systems
(which cannot be diagonalized)



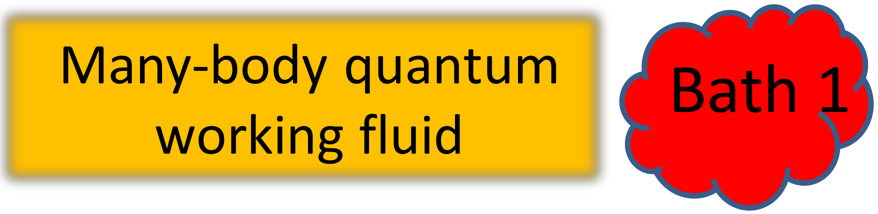
Strategies to study manybody open quantum systems
(which cannot be diagonalized)

Large (small) baths

- Couple the many-body system to another many-body system
- Represent the bath's harmonic oscillators as a chain

Local Lindblad master equations

- Local jump operators (with or without unraveling)



Strategies to study manybody open quantum systems
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Many-body quantum working fluid

Bath 1

Strategies to study manybody open quantum systems (which cannot be diagonalized)

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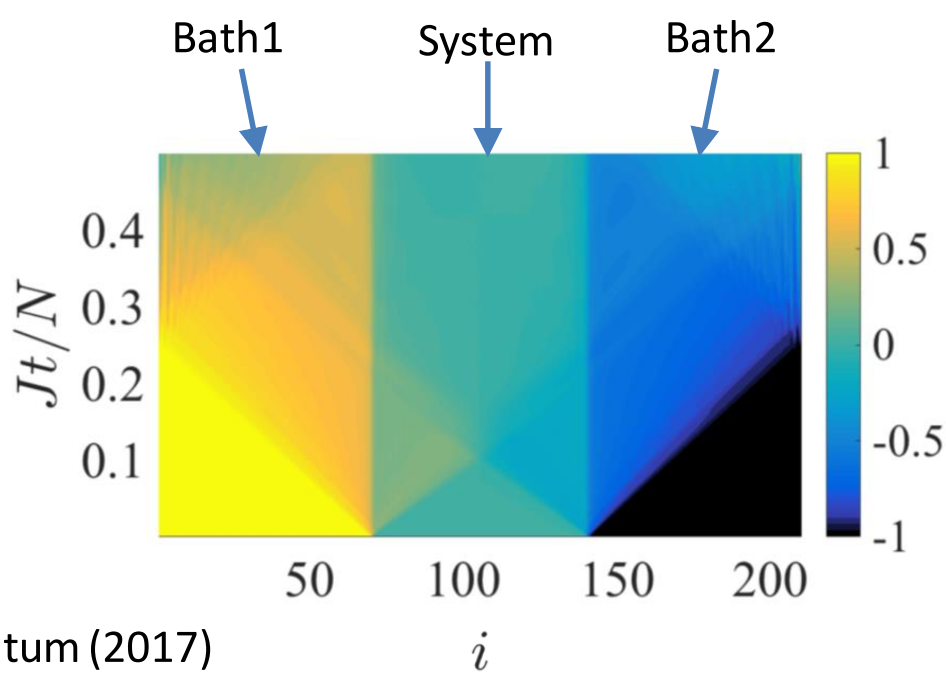

Mascarenhas et al.
Znidaric et al.
Biella et al.
Balachandran et al.
Ponomarev et al.
…

Mascarenhas et al., Quantum (2017)

Many-body quantum working fluid

Bath 1

Strategies to study manybody open quantum systems (which cannot be diagonalized)

Large (small) baths

- Couple the many-body system to another many-body system
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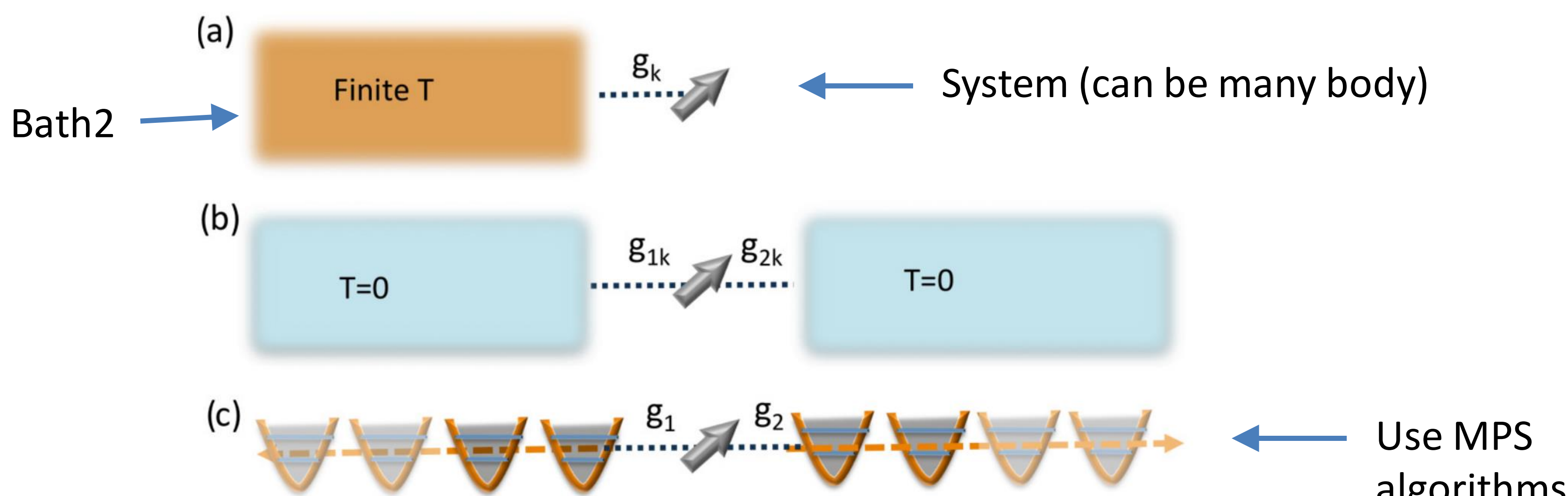
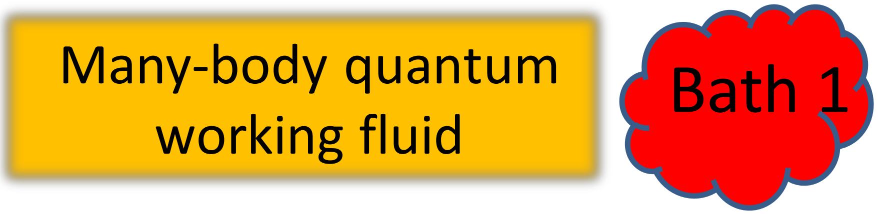
Thermofield transformation + star to chain mapping



de Vega and Banuls, PRA (2015)



Strategies to study manybody open quantum systems
(which cannot be diagonalized)

Local Lindblad master equations

- Local jump operators (with or without unraveling)

Many-body quantum working fluid

Bath 1

Strategies to study manybody open quantum systems
(which cannot be diagonalized)

Local Lindblad master equations

- Local jump operators (with or without unraveling)

$$\frac{d\hat{\rho}}{dt} = \mathcal{L}[\hat{\rho}] = -\frac{i}{\hbar}[\hat{H},\hat{\rho}] + \mathcal{D}[\hat{\rho}]$$

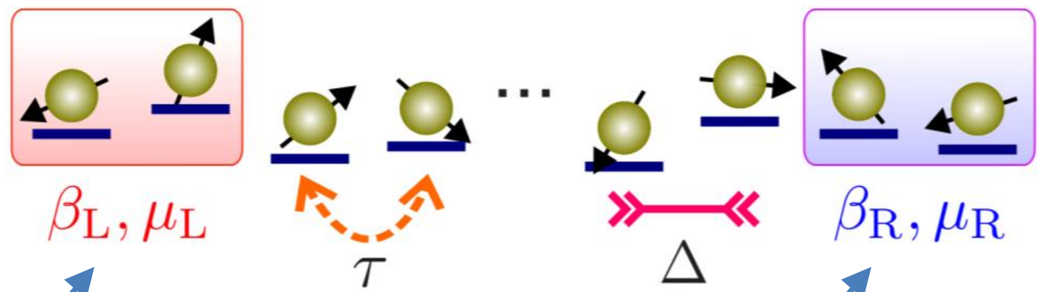
$$\mathcal{D}(\varrho) = \sum_{l,k=1}^{M^2-1} \gamma_{kl}(t) \left[V_k \varrho V_l^\dagger - \frac{1}{2}\{V_l^\dagger V_k, \varrho\} \right]$$

where V_k is local (most of the time single site).

Many-body quantum working fluid

Bath 1

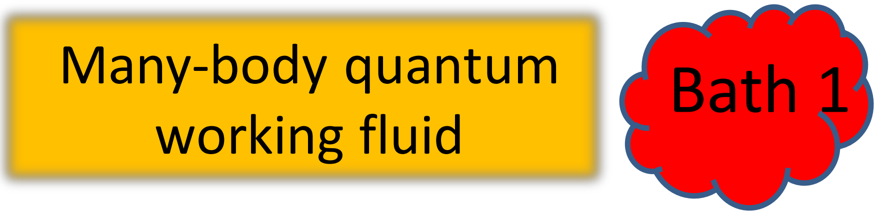
Local Lindblad master equations can be done in different ways. One of the most promising is



Portion of system in contact with bath L

Portion of system in contact with bath R

Prosen, Znidaric JStatMech (2009)
MendozaArenas et al (2018)



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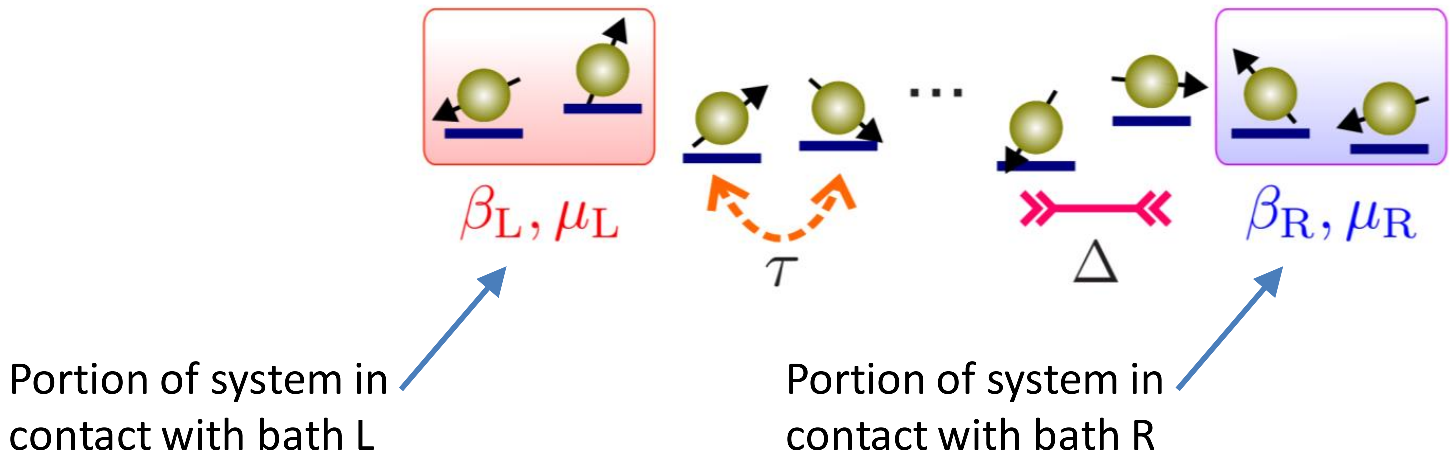
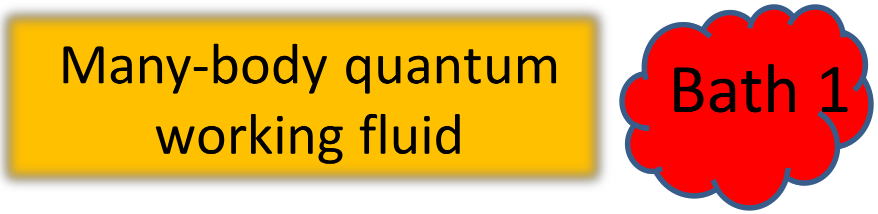


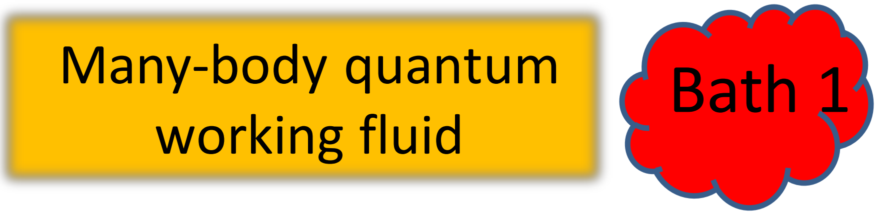
Prosen, Znidaric JStatMech (2009)
MendozaArenas et al (2018)

Another possibility is to use the method of surrogate Hamiltonian

Gelman et al., J. Chem. Phys. (2004)
Torrontegui, Kosloff NJP (2016)



How to effectively model baths for many-body quantum systems is something which deserves to be studied further.



How to effectively model baths for many-body quantum systems is something which deserves to be studied further.

And remember that one needs to be careful, because single site local Lindblad master equations have been shown to result in apparent violations of the second law.

Levy, Kosloff, EPL (2014)

Many-body quantum working fluid

Bath 1

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And remember that one needs to be careful, because single site local Lindblad master equations have been shown to result in apparent violations of the second law.

Levy, Kosloff, EPL (2014)

We show here one of the two approaches that we have been exploring.

1) Redfield master equation — Redfield, J Res Dev (1957)
2) Thermofield transformation — de Vega and Banuls, PRA (2015); Guo et al. PRA (2018)

before this … a little discussion on Matrix Product States and Operators

Many-body quantum working fluid

Bath 1

MPS

BIG PROBLEM!!!

- Size of $|\psi >$ scales as d^L where d is the size of the local Hilbert space while L is the size of the system

$$|\psi\rangle = \sum_{\sigma_1,\sigma_2,\cdots,\sigma_L} C^{\sigma_1,\sigma_2,\cdots,\sigma_L} |\sigma_1, \sigma_2, \cdots, \sigma_L\rangle$$

Many-body quantum working fluid

Bath 1

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$$|\psi\rangle = \sum_{\sigma_1,\sigma_2,\cdots,\sigma_L} C^{\sigma_1,\sigma_2,\cdots,\sigma_L} |\sigma_1, \sigma_2, \cdots, \sigma_L\rangle$$

- Rewrite $|\psi>$ as the product of a series of 3-dimensional tensors

$$c_{\sigma_1,\ldots,\sigma_L} = \sum_{a_1,a_2,\ldots,a_{L+1}} M^{\sigma_1}_{a_1,a_2} M^{\sigma_2}_{a_2,a_3} \ldots M^{\sigma_L}_{a_L,a_{L+1}}$$

- Now it scales polynomially with system size dD^2L

Standard tool for 1-dimensional quantum many body systems!

U. Schollwock, Ann. Phys. 326, 96 (2011)

Many-body quantum working fluid

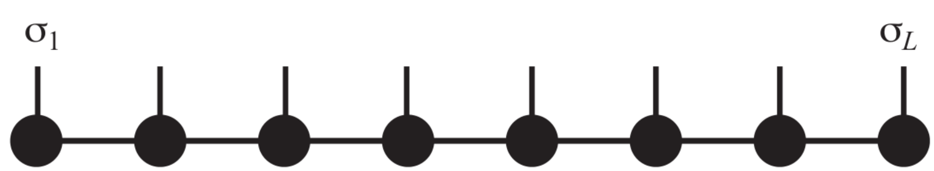
Bath 1

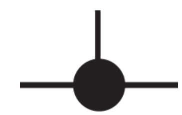
MPS

MPS

$$M^{\sigma_1}_{a_1,a_2} M^{\sigma_2}_{a_2,a_3} \ldots M^{\sigma_L}_{a_L,a_{L+1}}$$

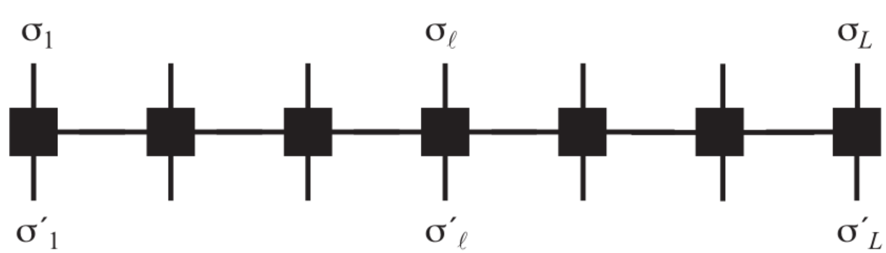
$$M^{\sigma_2}_{a_2,a_3}$$

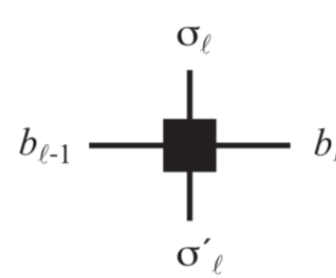




Matrix product operator MPO

$$W^{[\ell]\sigma_\ell\sigma'_\ell}_{b_{\ell-1},b_\ell}$$





U. Schollwock, Ann. Phys. 326, 96 (2011)

Many-body quantum working fluid

Bath 1

MPS

For the simulation we map the density operator to a state $|\hat{\rho}\rangle\rangle$ by the mapping

$$|n_1, n_2 \dots n_L\rangle\langle n'_1, n'_2 \dots n'_L|$$

$$\rightarrow |n_1, n_2 \dots n_L, n'_1, n'_2 \dots n'_L\rangle\rangle$$

Many-body quantum working fluid

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For the simulation we map the density operator to a state $|\hat{\rho}\rangle\rangle$ by the mapping

$$|n_1, n_2 \ldots n_L\rangle\langle n'_1, n'_2 \ldots n'_L| \quad \rightarrow \quad |n_1, n_2 \ldots n_L, n'_1, n'_2 \ldots n'_L\rangle\rangle$$

The evolution is then given by

$$\frac{d|\hat{\rho}\rangle\rangle}{dt} = -\frac{\mathrm{i}}{\hbar}\left(\hat{H}\otimes\mathbf{1} - \mathbf{1}\otimes\hat{H}^{\mathrm{t}}\right)|\hat{\rho}\rangle\rangle + \gamma\sum_j\left(\hat{n}_j\otimes\hat{n}_j - \frac{1}{2}(\hat{n}_j^2\otimes\mathbf{1} + \mathbf{1}\otimes\hat{n}_j^2)\right)|\hat{\rho}\rangle\rangle = \mathcal{L}|\hat{\rho}\rangle\rangle$$

example with dephasing

Many-body quantum working fluid

Bath 1

MPS

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$$|n_1, n_2 \dots n_L\rangle\langle n'_1, n'_2 \dots n'_L| \rightarrow |n_1, n_2 \dots n_L, n'_1, n'_2 \dots n'_L\rangle\rangle$$

The evolution is then given by

$$\frac{d|\hat{\rho}\rangle\rangle}{dt} = -\frac{\mathrm{i}}{\hbar}\left(\hat{H}\otimes\mathbf{1} - \mathbf{1}\otimes\hat{H}^{\mathrm{t}}\right)|\hat{\rho}\rangle\rangle + \gamma\sum_j\left(\hat{n}_j\otimes\hat{n}_j - \frac{1}{2}(\hat{n}_j^2\otimes\mathbf{1} + \mathbf{1}\otimes\hat{n}_j^2)\right)|\hat{\rho}\rangle\rangle = \mathcal{L}|\hat{\rho}\rangle\rangle$$

example with dephasing

And we use matrix product states (or exact diagonalization) with number conservation in bra and ket

$$|\hat{\rho}\rangle\rangle = \sum_{\{n_j, n'_k\}} M_1 \dots M_i \dots M_L |n_1, n_2 \dots n_L, n'_1, n'_2 \dots n'_L\rangle\rangle$$

Verstraete et al., PRL.(2004)

Many-body quantum working fluid

Bath 1

MPS

Using stochastic trajectories is of course also possible.

Daley, Adv. Phys. (2014)

However it is dynamics dependent whether the needed bond dimension D for an accurate evolution is less for stochastic evolution of for purification (evolution of the density matrix).

Bonnes, Lauchli, arXiv:1411.4831

Both methods are used.

Many-body quantum working fluid

Bath 1

Redfield with MPS

X. Xu
SUTD

C. Guo
SUTD/ZISTI

J. Thingna
Luxembourg

+ D.P.

Many-body quantum working fluid

Bath 1

Redfield

We consider a system and a bath.

System S bath B coupling

$$H_{\text{tot}} = H_S + H_B + \gamma S \otimes B$$

Many-body quantum working fluid

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Consider weak coupling, γ small, Born-Markov approximation $\rho_{\mathrm{tot}} \approx \rho(t) \otimes \rho_B$

It is possible to write a second-order master equation in this form

$$\frac{\partial \rho(t)}{\partial t} = -\,\mathrm{i}\left[H_{\mathrm{S}}, \rho(t)\right] - \mathcal{R}^{t}\left[\rho(t)\right]$$

Many-body quantum working fluid

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Where $\mathcal{R}^t[\,\cdot\,] = [S, \mathbb{S}(t)\,\cdot\,] + [\,\cdot\,\mathbb{S}^\dagger(t), S]$,

and $\mathbb{S}(t) = \int_0^t \tilde{S}(-\tau)\, C(\tau)\, d\tau$ with $\tilde{S}(\tau) = \mathrm{e}^{\mathrm{i}H_{\mathrm{S}}\tau} S \mathrm{e}^{-\mathrm{i}H_{\mathrm{S}}\tau}$

Evolution of operator

and $C(\tau) = \mathrm{tr}\left(e^{\mathrm{i}H_B\tau} B e^{-\mathrm{i}H_B\tau}\, B\, \rho_B\right)$

Bath 2-time correlations

Many-body quantum working fluid

Bath 1

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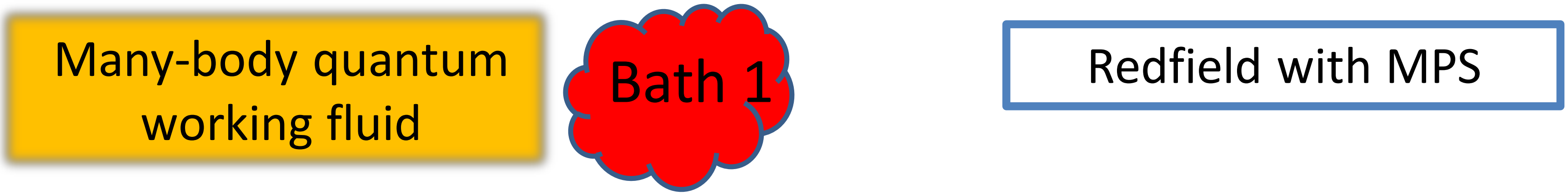
Where $\mathcal{R}^t[\,\cdot\,] = [S, \mathbb{S}(t)\,\cdot\,] + [\,\cdot\,\mathbb{S}^\dagger(t), S]$,

Solved usually by exact diagonalization of system Hamiltonian!

Quickly difficult for many-body systems


Many-body quantum working fluid
Bath 1
Redfield with MPS

Many-body quantum working fluid

Bath 1

Redfield with MPS

We now show how to study large(r) system with Redfield and MPS

We consider the system coupled to the bath only in the centre of the chain

$$S = \mathbf{1} \bigotimes \cdots \bigotimes \mathbf{1} \bigotimes \sigma_0^x \bigotimes \mathbf{1} \bigotimes \cdots \bigotimes \mathbf{1}$$

This can be seen as an MPO which we then evolves in time

Many-body quantum working fluid

Bath 1

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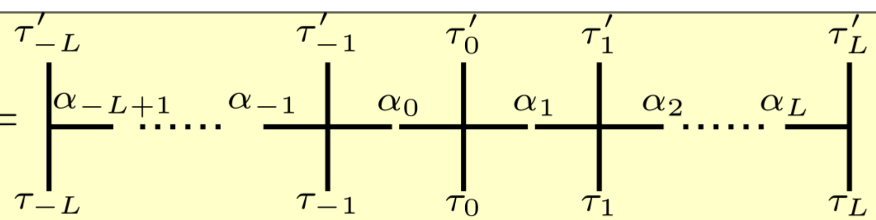
$\mathbb{S}(t) =$



Trotterized evolution and convolution with bath correlations

$$\tilde{S}(\tau) = \mathrm{e}^{\mathrm{i}H_{\mathrm{S}}\tau} S \mathrm{e}^{-\mathrm{i}H_{\mathrm{S}}\tau} \qquad \mathbb{S}(t) = \int_0^t \tilde{S}(-\tau)\, C(\tau)\, d\tau$$

Many-body quantum working fluid

Bath 1

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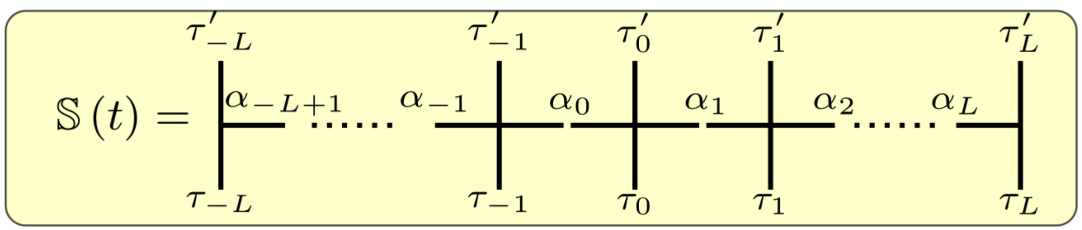

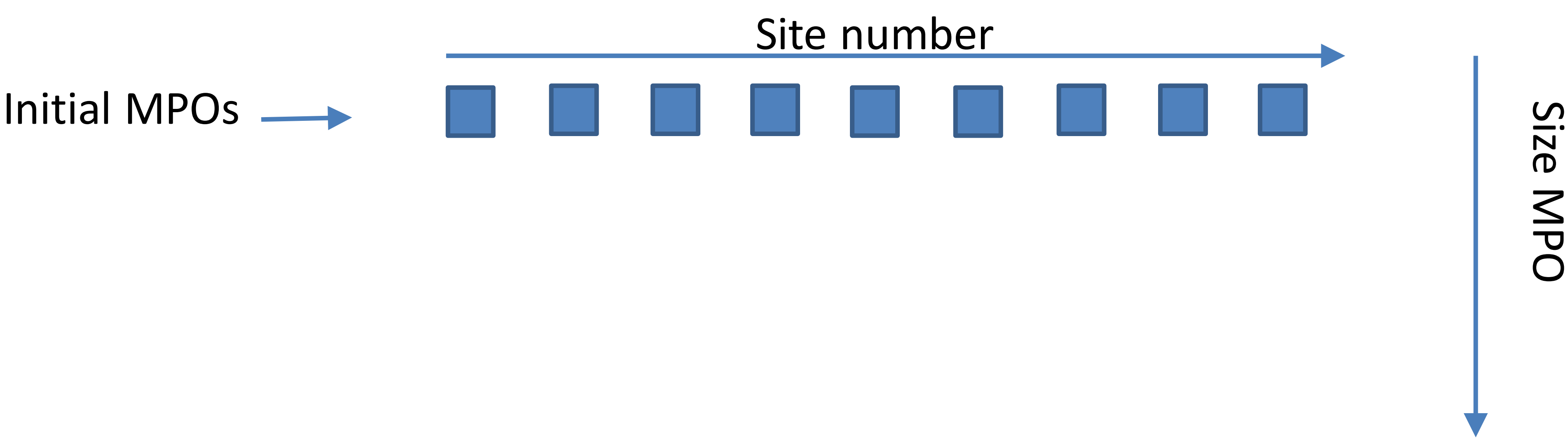

Many-body quantum working fluid

Bath 1

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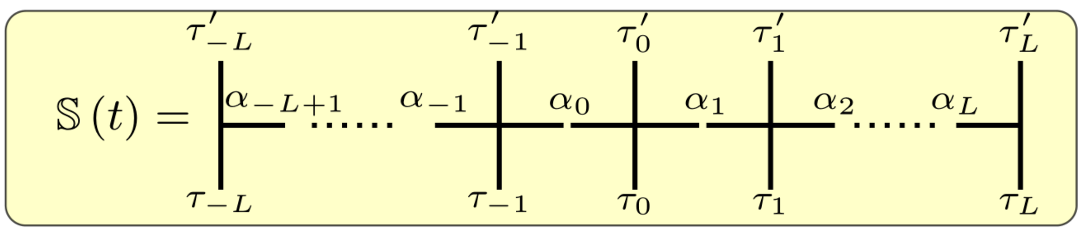

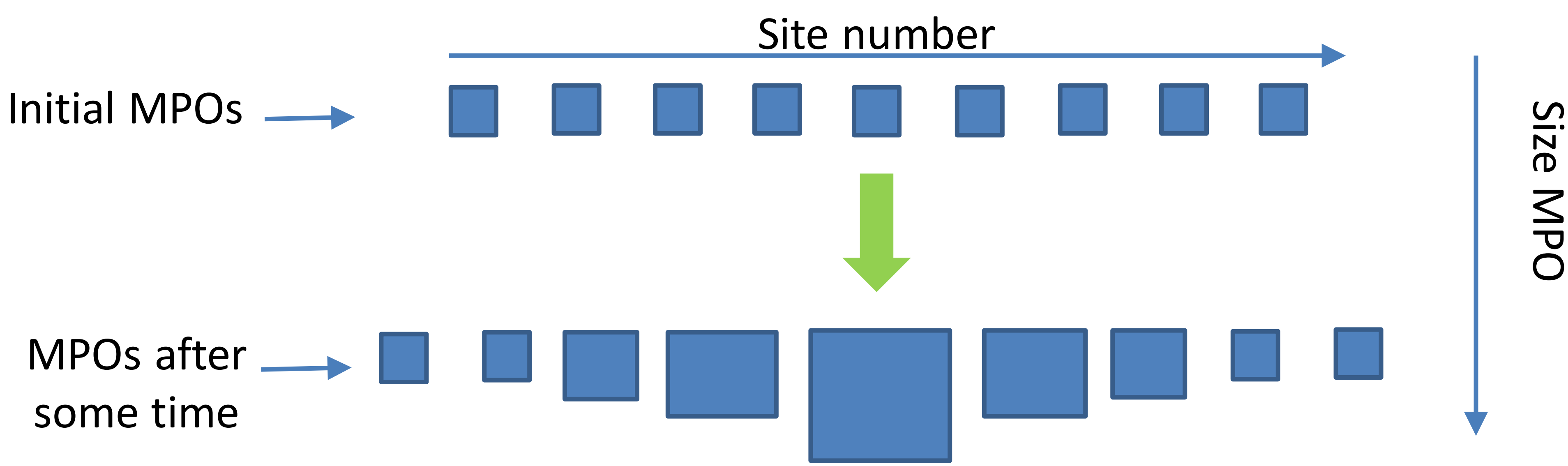

Many-body quantum working fluid

Bath 1

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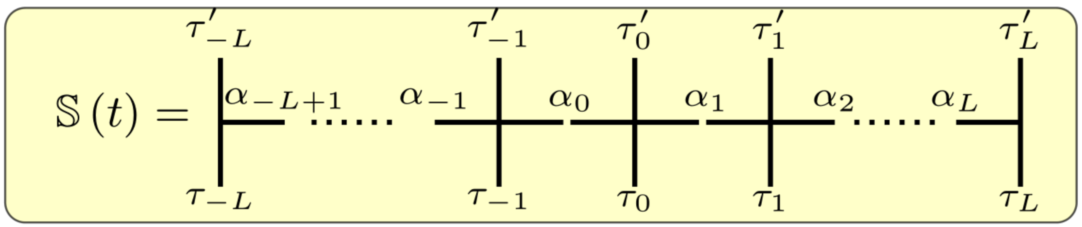
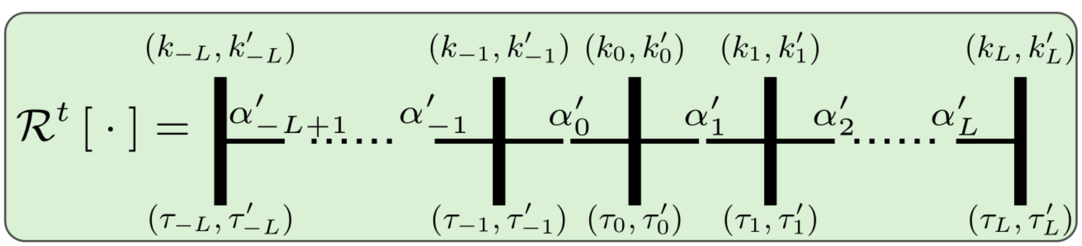


Superoperator as MPO



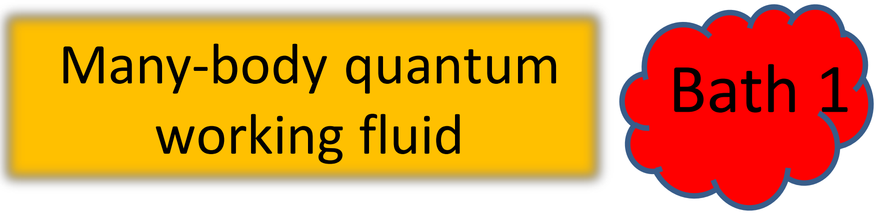
Redfield with MPS

Many-body quantum working fluid

Bath 1

Then we use the MPO to evolve the MPS following (we use Runge-Kutta)

$$\frac{\partial \rho(t)}{\partial t} = -\mathrm{i}\left[H_{\mathrm{S}}, \rho(t)\right] - \mathcal{R}^{t}\left[\rho(t)\right]$$



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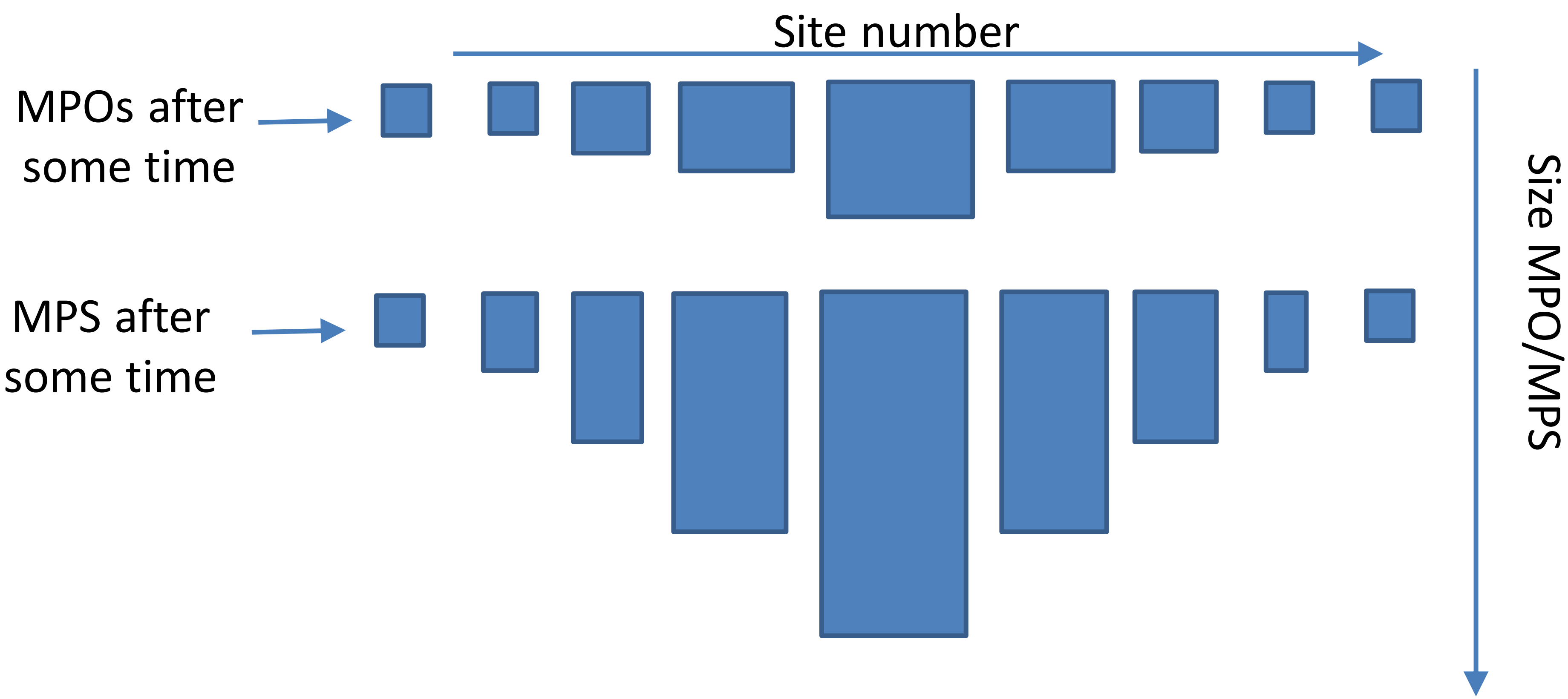


Many-body quantum working fluid

Bath 1

What we got ....

$$H_{\text{tot}} = H_S + H_B + \gamma S \otimes B$$

We consider and XXZ chain

$$H_{\text{S}} = \sum_{l=-L}^{L-1} \left[J \left(\sigma_l^x \sigma_{l+1}^x + \sigma_l^y \sigma_{l+1}^y \right) + \Delta \sigma_l^z \sigma_{l+1}^z \right] + h \sum_{l=-L}^{L} \sigma_l^z ,$$

bath of harmonic oscillators $H_B = \sum_{n=1}^{\infty} \left[\frac{p_n^2}{2m_n} + \frac{m_n \omega_n^2 x_n^2}{2} \right]$

and coupling in the middle $S = \sigma_0^x$ $B = -\sum_{n=1}^{\infty} c_n x_n$

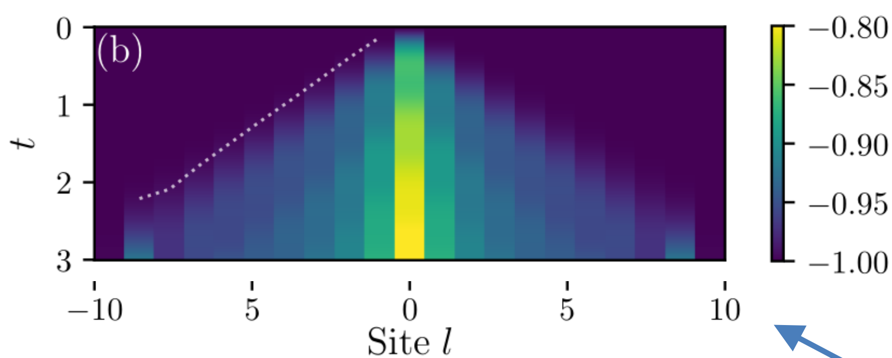
Many-body quantum working fluid

Bath 1

What we got ….

Let us look at the local magnetization versus time for different positions

$$\langle \sigma_l^z \rangle$$



Ballistic propagation for the tails

Non-ballistic in the centre

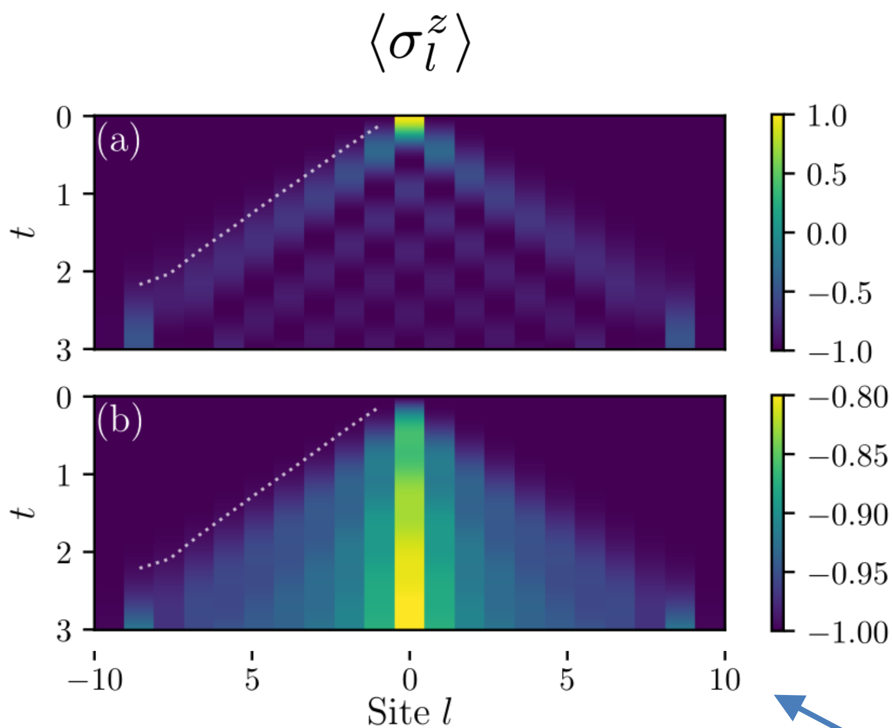
$\Delta = 5, h = 0.5, \omega_c = 20, T = 2, \gamma = 0.02$

Many-body quantum working fluid

Bath 1

What we got ....

Let us look at the local magnetization versus time for different positions



Unitary evolution of single spin flipped down

It explains the tails

Ballistic propagation for the tails

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$\Delta = 5, h = 0.5, \omega_c = 20, T = 2, \gamma = 0.02$

Many-body quantum working fluid

Bath 1

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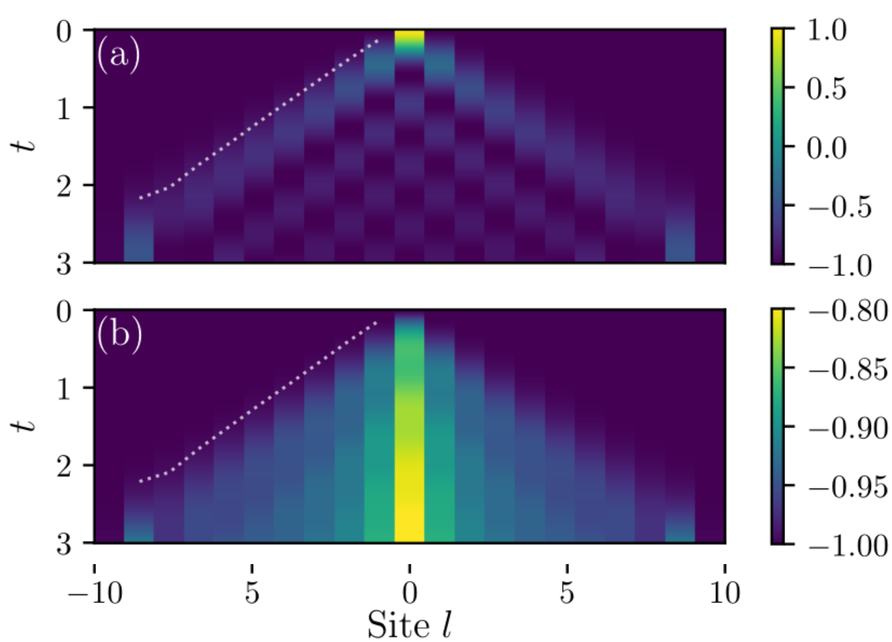
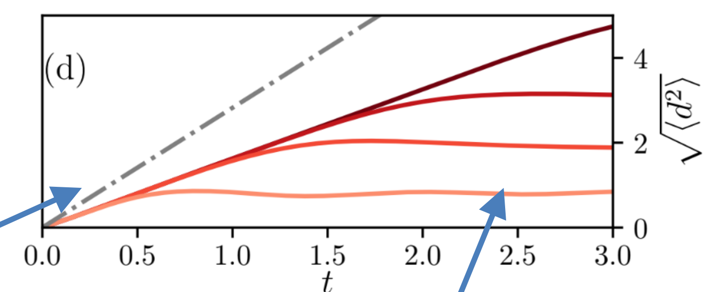


To study the evolution we look at how the perturbation due to the bath propagates in the system.

$$\langle d^2 \rangle = \sum_l \langle \sigma_l^u \rangle \, l^2 \; / \; \sum_l \langle \sigma_l^u \rangle$$



Unitary evolution

Dissipative evolution for 5, 9, 13 and 21 spins

$\Delta = 5, h = 0.5, \omega_c = 20, T = 2, \gamma = 0.02$

Many-body quantum working fluid

Bath 1

What we got ....

We compare **Redfield** to master equations in Lindblad limit

1) Singular coupling limit

$$C\left(\tau\right) \approx 2\gamma T \delta\left(\tau\right)$$

2) Local Hamiltonian

neglect couplings between sites

Wichterich et al. PRE (2007)

Many-body quantum working fluid

Bath 1

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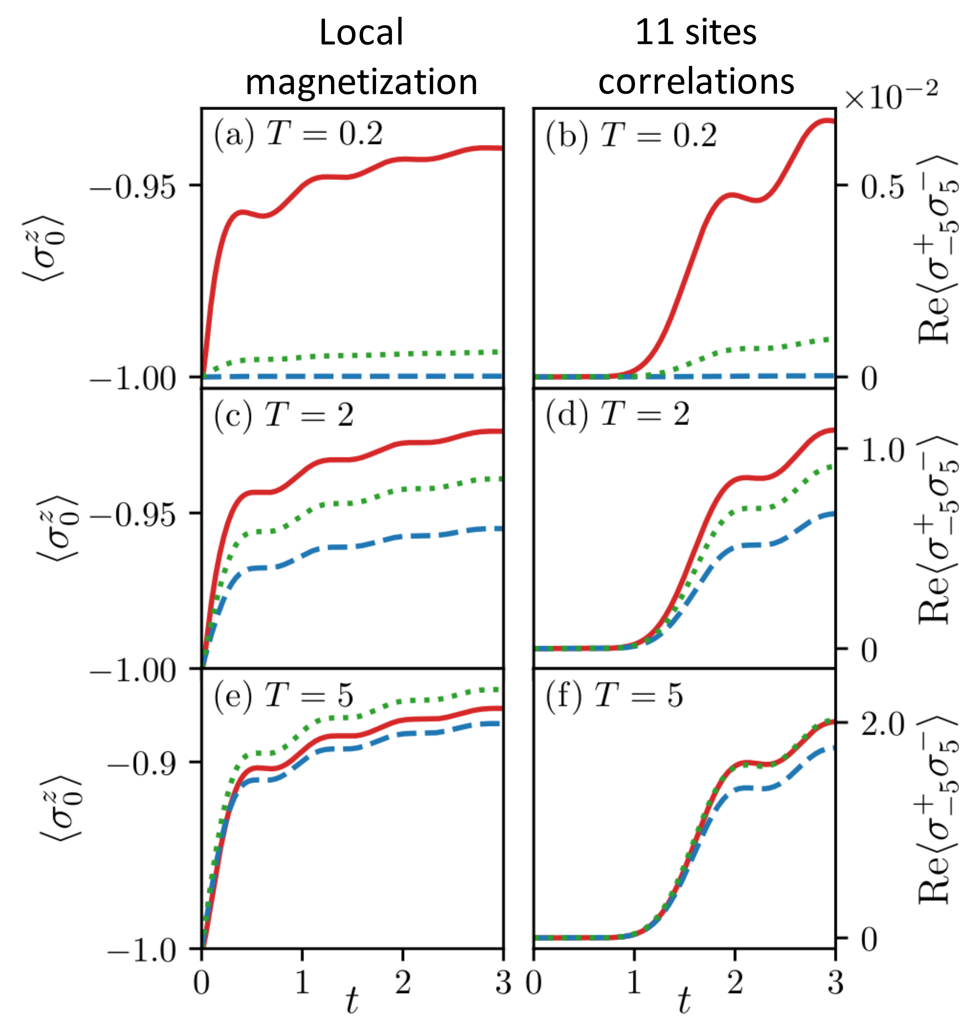
Both approximations work better for larger temperatures.



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Many-body quantum working fluid

Bath 1

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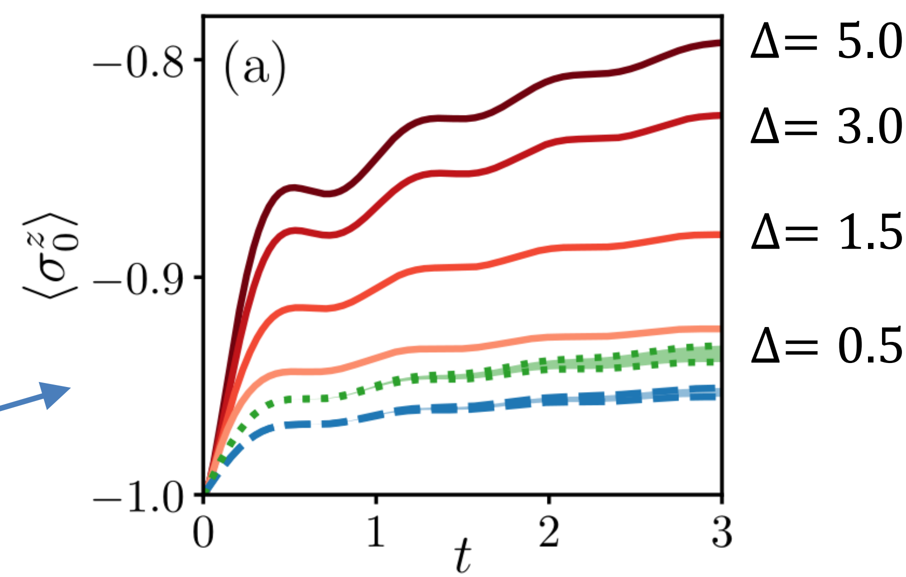
$$C(\tau) \approx 2\gamma T \delta(\tau)$$

2) Local Hamiltonian
neglect couplings between sites
Wichterich et al. PRE (2007)

Both approaches which give Lindblad evolutions struggle to capture the dependence on the interaction

Different **interactions**



$h = 0.5, \omega_c = 20, T = 2, \gamma = 0.02$

Many-body quantum working fluid

Bath 1

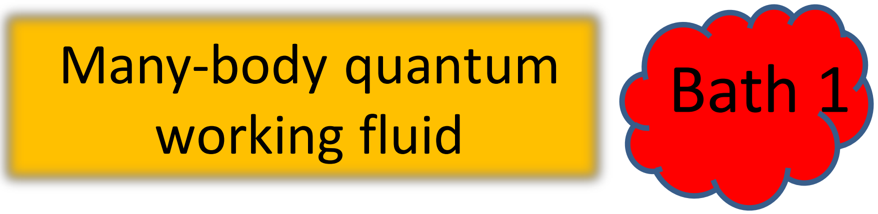
Conclusions

Many-body quantum systems can be a useful working fluid

Important to model properly baths for them

We showed how to use the Redfield master equation for large systems

We showed that thanks to this master equation we can explore a much broader set of regimes.



Conclusions

Many-body quantum systems can be a useful working fluid

Important to model properly baths for them

We showed how to use the Redfield master equation for large systems

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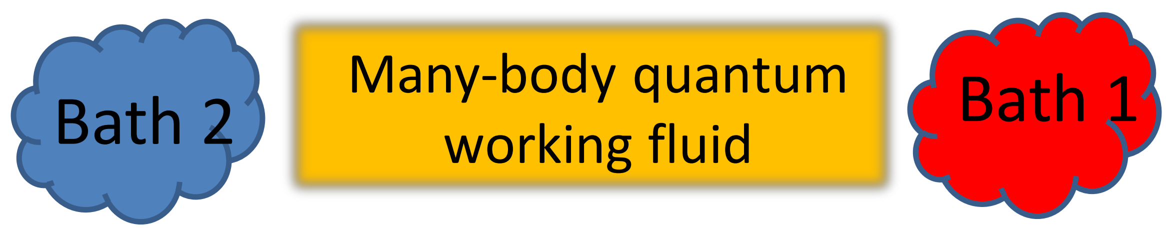
Outlook

Steady states with Redfield and MPS?
Time-dependence?
...


Bath 2
Many-body quantum
working fluid
Bath 1

Bath 2 | Many-body quantum working fluid | Bath 1

Rectification

V. Balachandran

E. Pereira

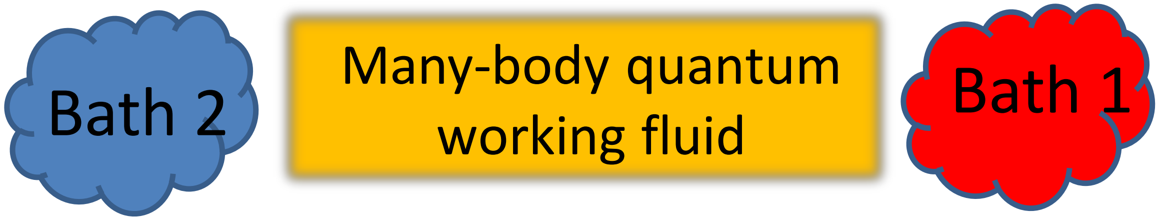
G. Benenti

G. Casati

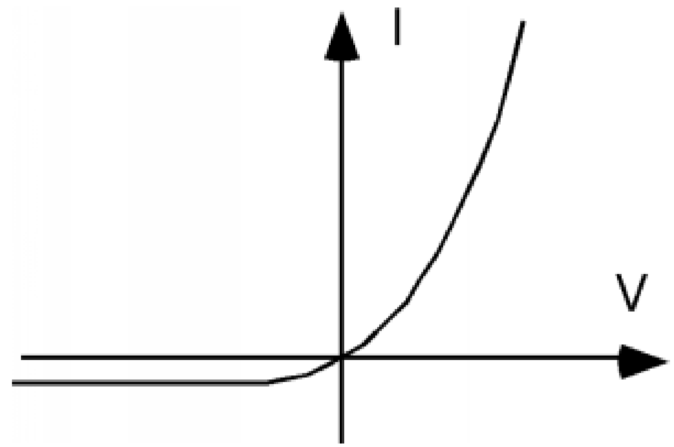
+ D.P.

Physical Review Letters 120, 200603 (2018)

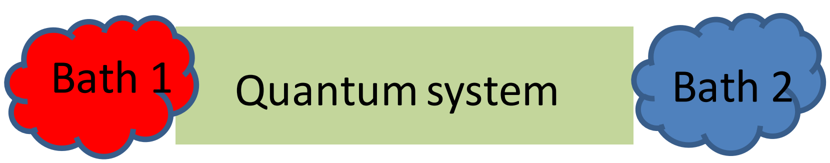
Rectification



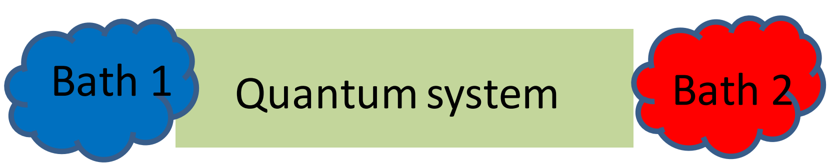
Typical diode characteristic curve with current vs voltage



$\mathcal{J}_f$ Current in forward bias



$\mathcal{J}_r$ Current in reverse bias



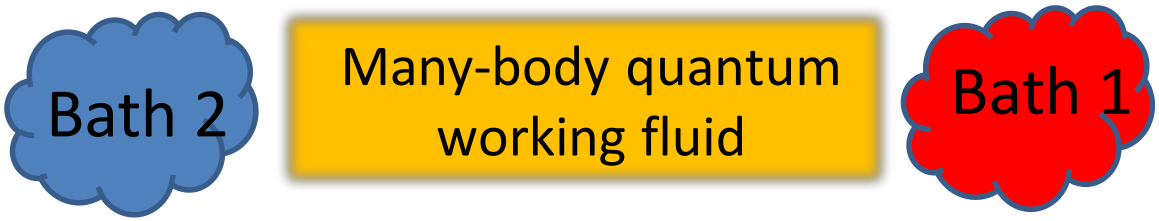
Rectification is characterized by

$$\mathcal{R} = -\frac{\mathcal{J}_f}{\mathcal{J}_r}$$

The Contrast is also useful

$$\mathcal{C} = \left|\frac{\mathcal{J}_f + \mathcal{J}_r}{\mathcal{J}_f - \mathcal{J}_r}\right|$$

Rectification



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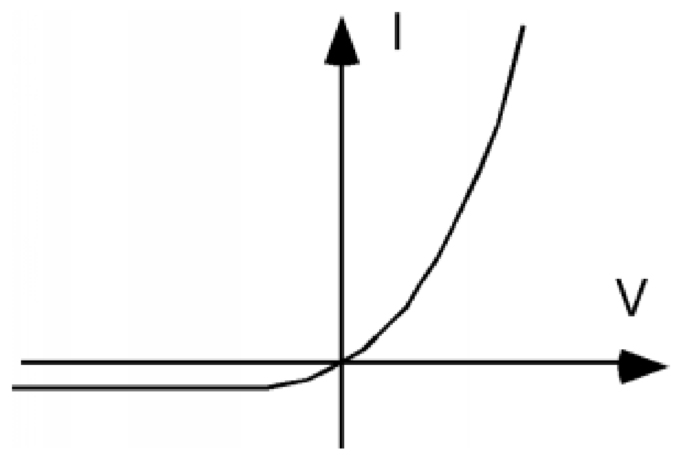

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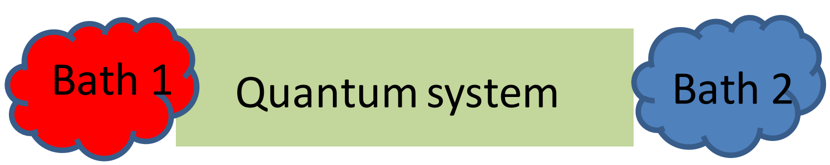

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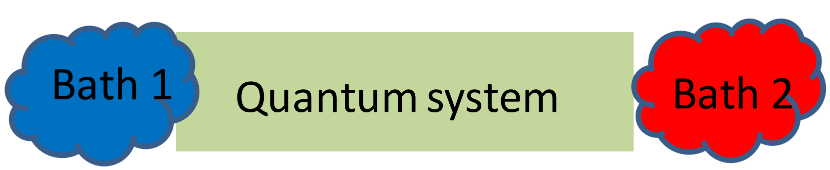


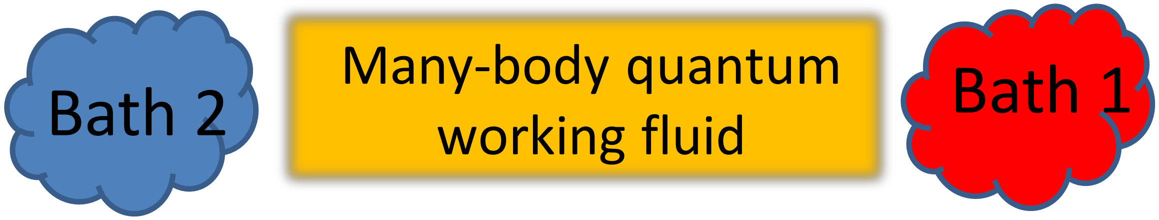
Rectification is characterized by

$$\mathcal{R} = -\frac{\mathcal{J}_f}{\mathcal{J}_r} \quad \to \infty$$

The Contrast is also useful

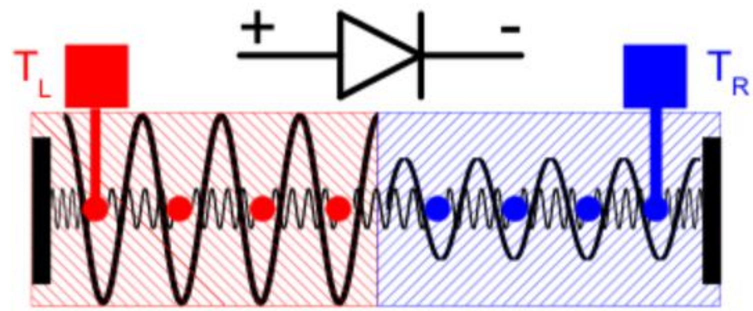
$$\mathcal{C} = \left|\frac{\mathcal{J}_f + \mathcal{J}_r}{\mathcal{J}_f - \mathcal{J}_r}\right| \quad \to 1$$

for perfect diode

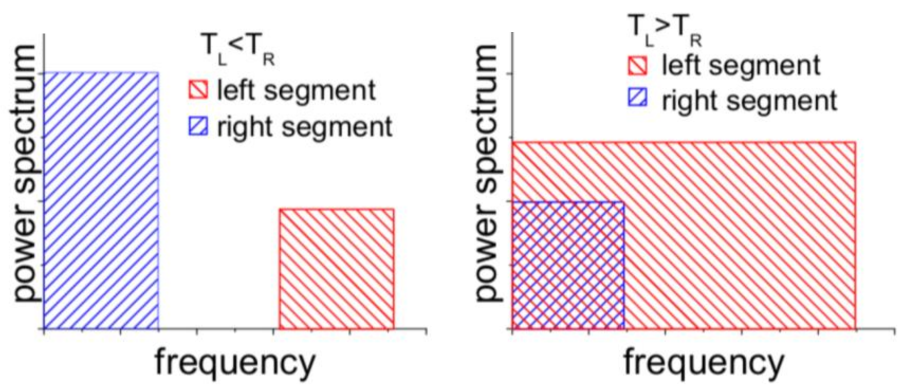


Rectification

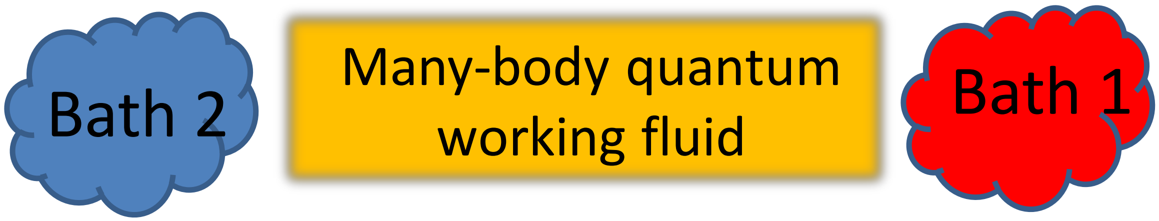
Consider a classical chain of particles with **nonlinear** couplings and reflection symmetry broken.



These ingredients allow a diode to function.



Terraneo et al. PRL (2002)
Li et al PRL (2004)
…
Li et al. Rev Mod Phys (2012)
…



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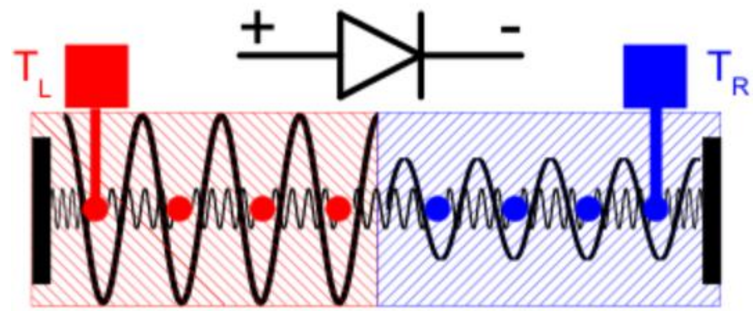

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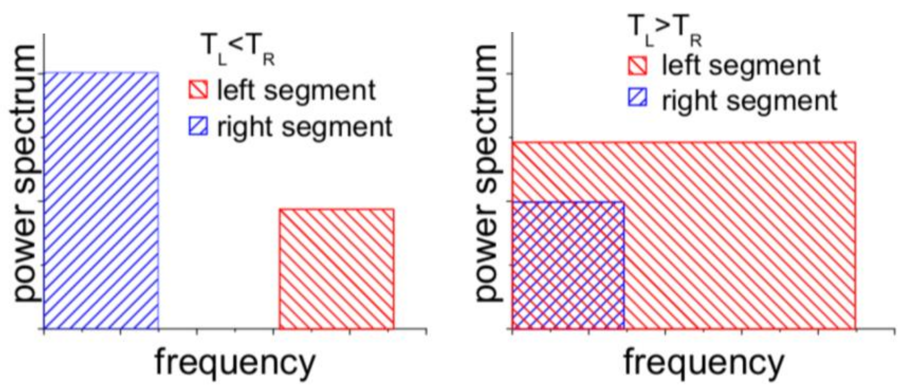
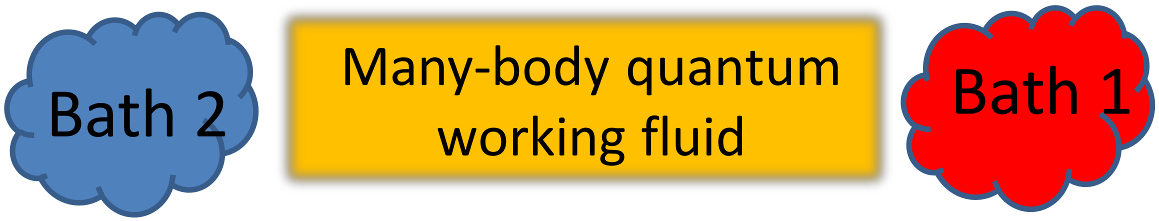


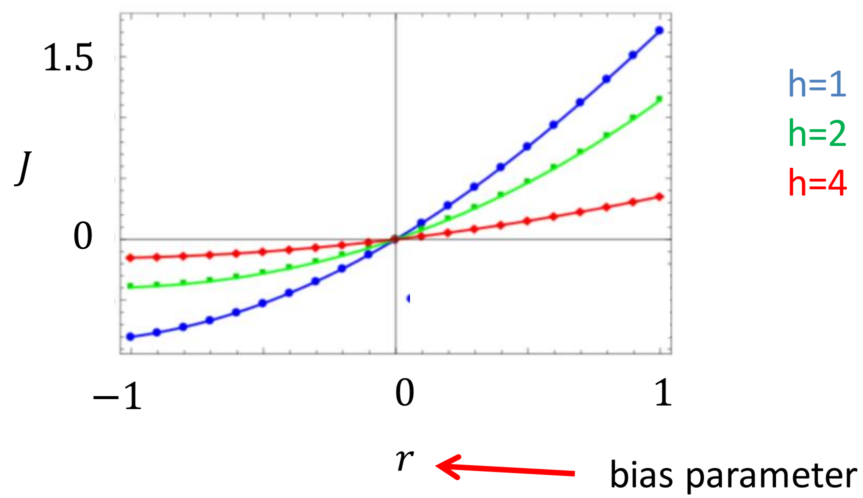
Terraneo et al. PRL (2002)
Li et al PRL (2004)
...
Li et al. Rev Mod Phys (2012)
...

It is natural to try to understand what happens in the quantum regime
nonlinear → strongly interacting



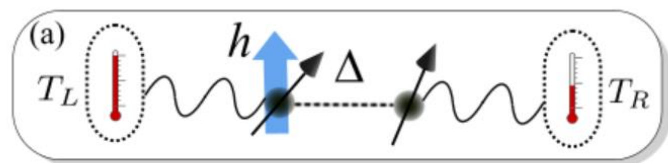
Rectification

Rectification in XXZ chain (3 spins) with transverse field
Landi et al PRE (2014)



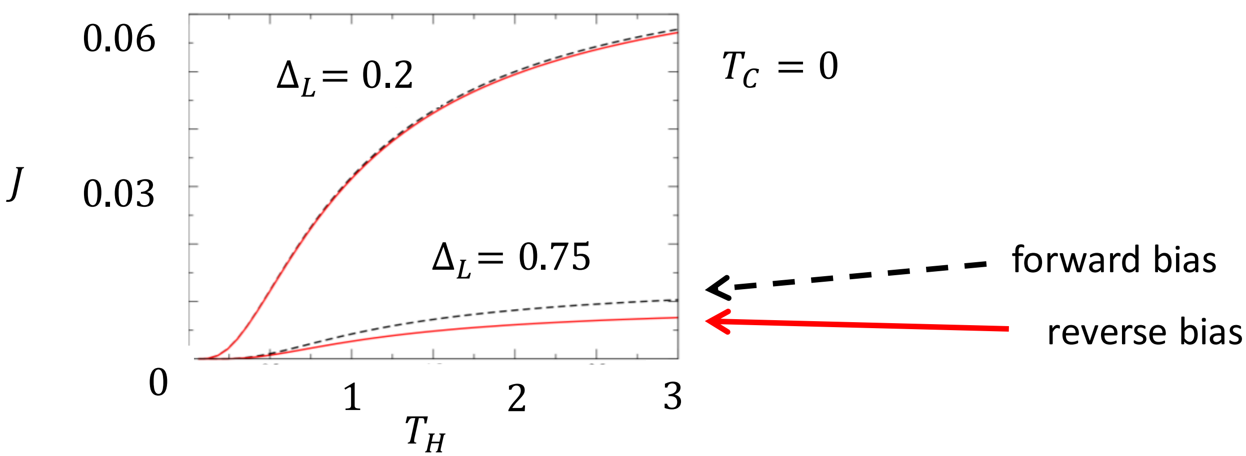
Optimal rectification
Werlang et al PRL (2014)

$$H_S = \frac{h}{2}\,\sigma_z^L + \frac{\Delta}{2}\,\sigma_z^L\sigma_z^R$$

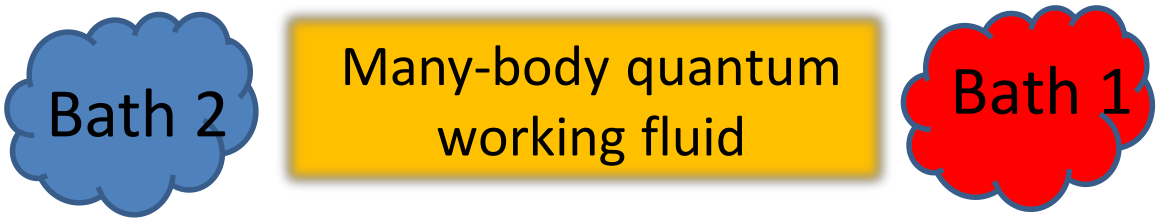


also works from D. Segal, A. Dhar ...

Thermal transport in one-dimensional spin heterostructures
Arrachea et al PRB (2009)



XX chain coupled to XY chain and magnetic field in baths



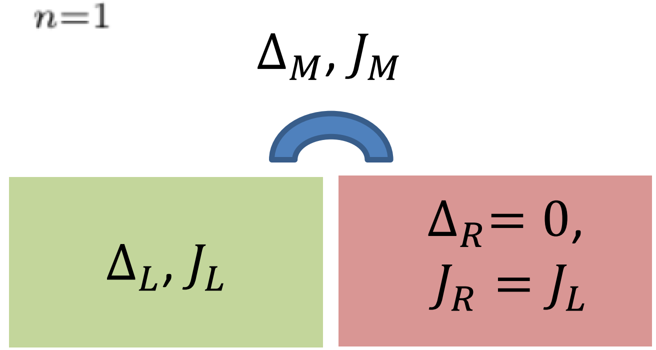
Rectification

We focus on larger quantum spin chains and we focus on the role of the anisotropy.
No external magnetic fields.

Anisotropy
Interaction

Segmented XXZ chain

$$\hat{H} = \sum_{n=1}^{N-1} \left[J_n (\hat{\sigma}_n^x \hat{\sigma}_{n+1}^x + \hat{\sigma}_n^y \hat{\sigma}_{n+1}^y) + \Delta_n \hat{\sigma}_n^z \hat{\sigma}_{n+1}^z \right]$$



$\Delta_n = \Delta_L$	$J_n = J_L$	**Interacting** left-half of the chain
$\Delta_n = \Delta_R = 0$	$J_n = J_R$	**Non-interacting** Right-half
$\Delta_n = \Delta_M = 0$	$J_n = J_M$	Junction of two half-chains

Bath 2 | Many-body quantum working fluid | Bath 1

Rectification

We focus on larger quantum spin chains and we focus on the role of the anisotropy.
No external magnetic fields.
We describe the evolution of the system by

$$\frac{d\hat{\rho}}{dt} = \mathcal{L}(\hat{\rho}) = -\frac{\mathrm{i}}{\hbar}[\hat{H},\hat{\rho}] + \sum_{n=1,N} \mathcal{D}_n(\hat{\rho})$$

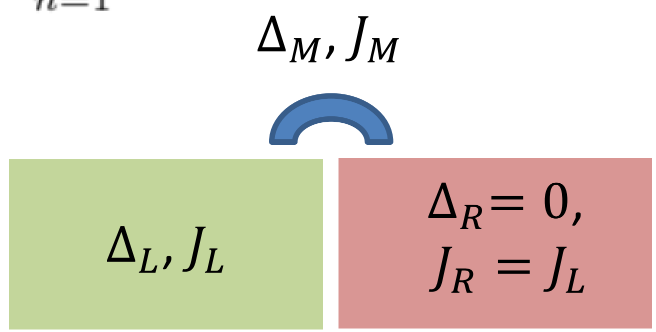
$$\mathcal{D}_n(\hat{\rho}) = \gamma \left[\lambda_n \left(\hat{\sigma}_n^+ \hat{\rho} \hat{\sigma}_n^- - 1/2\left\{\hat{\sigma}_n^- \hat{\sigma}_n^+, \hat{\rho}\right\}\right) + (1-\lambda_n)\left(\hat{\sigma}_n^- \hat{\rho} \hat{\sigma}_n^+ - 1/2\left\{\hat{\sigma}_n^+ \hat{\sigma}_n^-, \hat{\rho}\right\}\right)\right]$$

(non-equilibrium) spin baths
They set the local magnetization

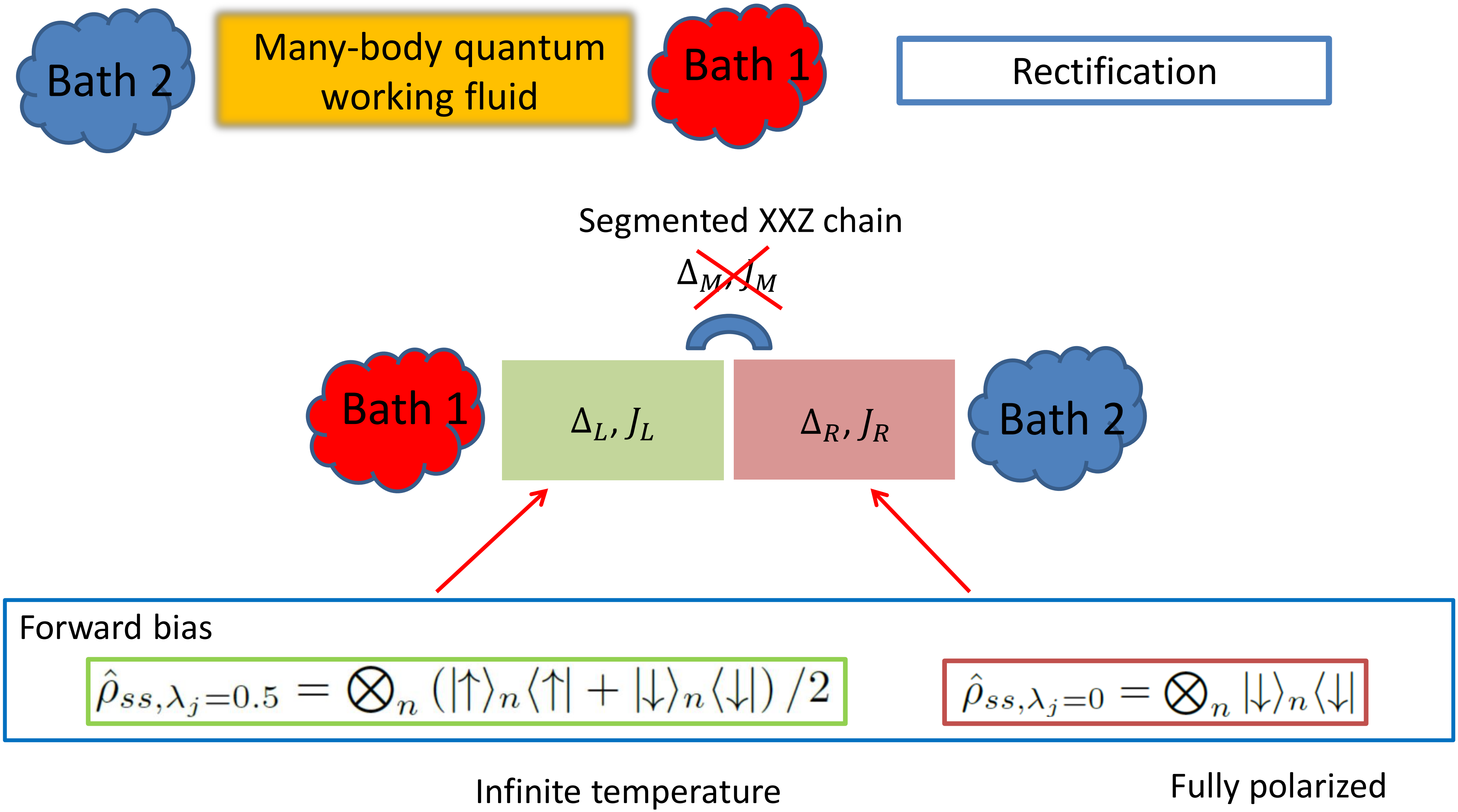
Anisotropy
Interaction

Segmented XXZ chain

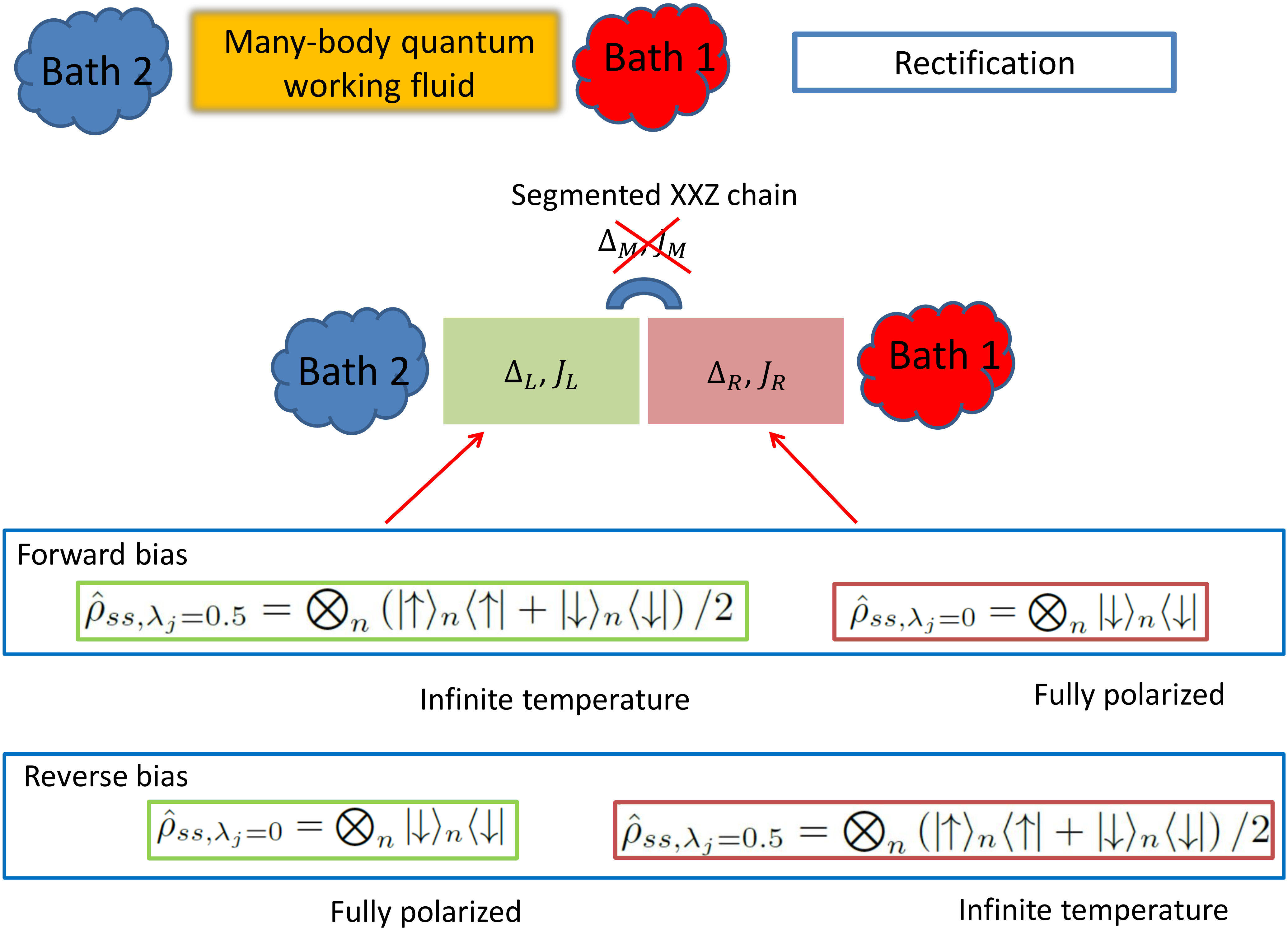
$$\hat{H} = \sum_{n=1}^{N-1}\left[J_n(\hat{\sigma}_n^x \hat{\sigma}_{n+1}^x + \hat{\sigma}_n^y \hat{\sigma}_{n+1}^y) + \Delta_n \hat{\sigma}_n^z \hat{\sigma}_{n+1}^z\right]$$



$\Delta_n = \Delta_L$	$J_n = J_L$	Interacting left-half of the chain
$\Delta_n = \Delta_R = 0$	$J_n = J_R$	Non-interacting Right-half
$\Delta_n = \Delta_M = 0$	$J_n = J_M$	Junction of two half-chains



Bath 2
Many-body quantum working fluid
Bath 1
Rectification
Segmented XXZ chain
Δ_M, J_M
Bath 1
Δ_L, J_L
Δ_R, J_R
Bath 2
Forward bias
$\hat{\rho}_{ss,\lambda_j=0.5} = \bigotimes_n \left(|\uparrow\rangle_n\langle\uparrow| + |\downarrow\rangle_n\langle\downarrow|\right)/2$
$\hat{\rho}_{ss,\lambda_j=0} = \bigotimes_n |\downarrow\rangle_n\langle\downarrow|$
Infinite temperature
Fully polarized


Bath 2
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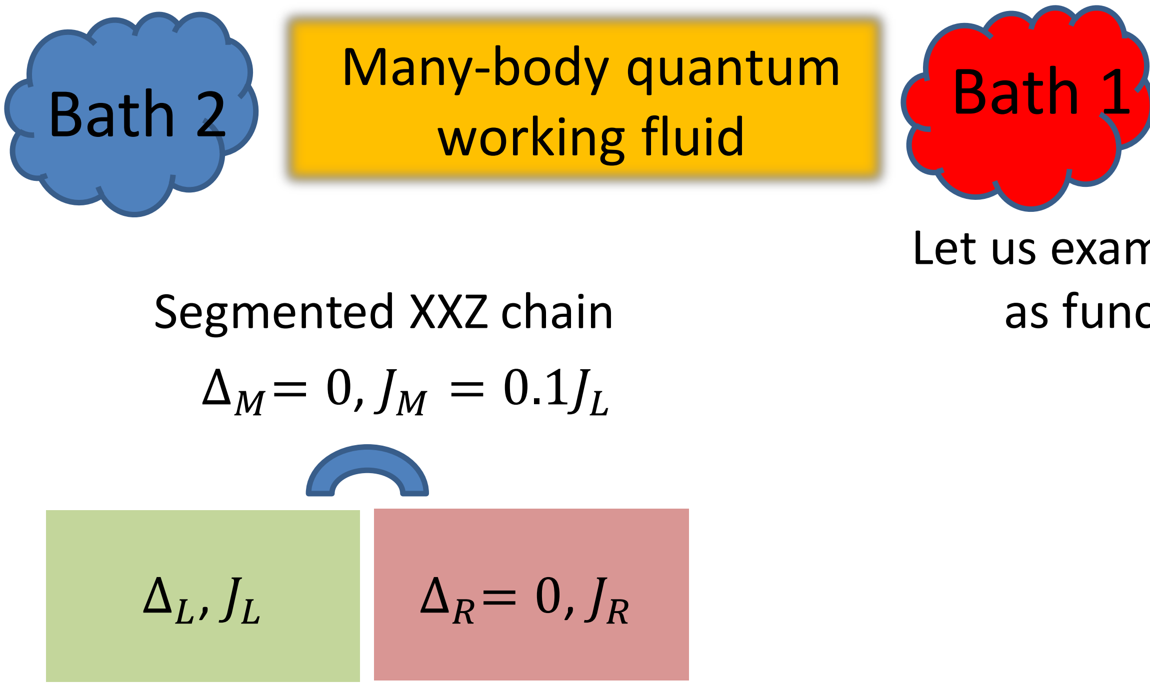
$\hat{\rho}_{ss,\lambda_j=0} = \bigotimes_n |\downarrow\rangle_n\langle\downarrow|$
Infinite temperature
Fully polarized
Reverse bias
$\hat{\rho}_{ss,\lambda_j=0} = \bigotimes_n |\downarrow\rangle_n\langle\downarrow|$
$\hat{\rho}_{ss,\lambda_j=0.5} = \bigotimes_n \left(|\uparrow\rangle_n\langle\uparrow| + |\downarrow\rangle_n\langle\downarrow|\right)/2$
Fully polarized
Infinite temperature

Rectification



Let us examine **currents**, **rectification** and **contrast** as function of anisotropy and chain length

$\Delta_R = 0$

$J_L = J_R$

N=4
N=6
N=8
N=10

$$\mathcal{R} = -\frac{\mathcal{J}_f}{\mathcal{J}_r}$$ **Rectification**

$$\mathcal{C} = \left|\frac{\mathcal{J}_f + \mathcal{J}_r}{\mathcal{J}_f - \mathcal{J}_r}\right|$$ **Contrast**

Bath 2

Many-body quantum working fluid

Bath 1

Rectification

Let us examine **currents**, **rectification** and **contrast** as function of anisotropy and chain length

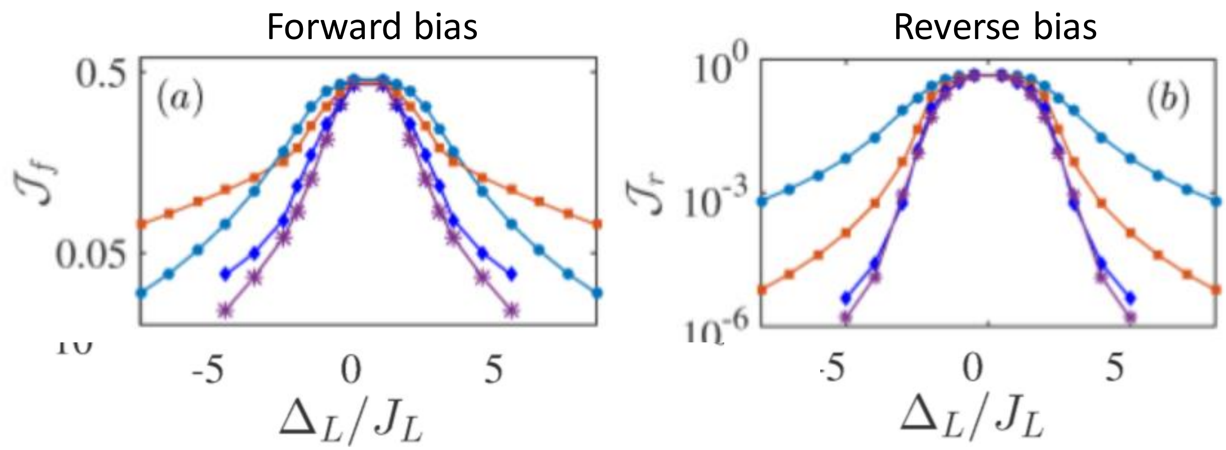
Segmented XXZ chain

$\Delta_M = 0, J_M = 0.1 J_L$

Δ_L, J_L | $\Delta_R = 0, J_R$

$\Delta_R = 0$

$J_L = J_R$

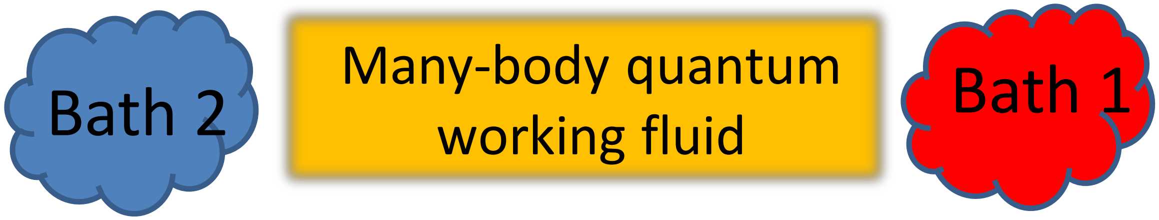


N=4
N=6
N=8
N=10

Forward and Reverse bias **currents** vs anisotropy of left half

$$\mathcal{R} = -\frac{\mathcal{J}_f}{\mathcal{J}_r}$$ **Rectification**

$$\mathcal{C} = \left|\frac{\mathcal{J}_f + \mathcal{J}_r}{\mathcal{J}_f - \mathcal{J}_r}\right|$$ **Contrast**

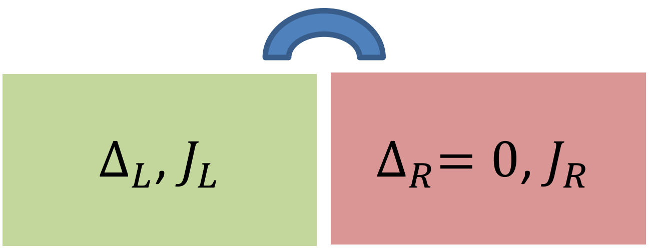


Rectification

Let us examine **currents**, **rectification** and **contrast** as function of anisotropy and chain length

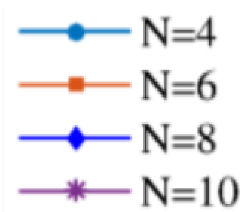
Segmented XXZ chain

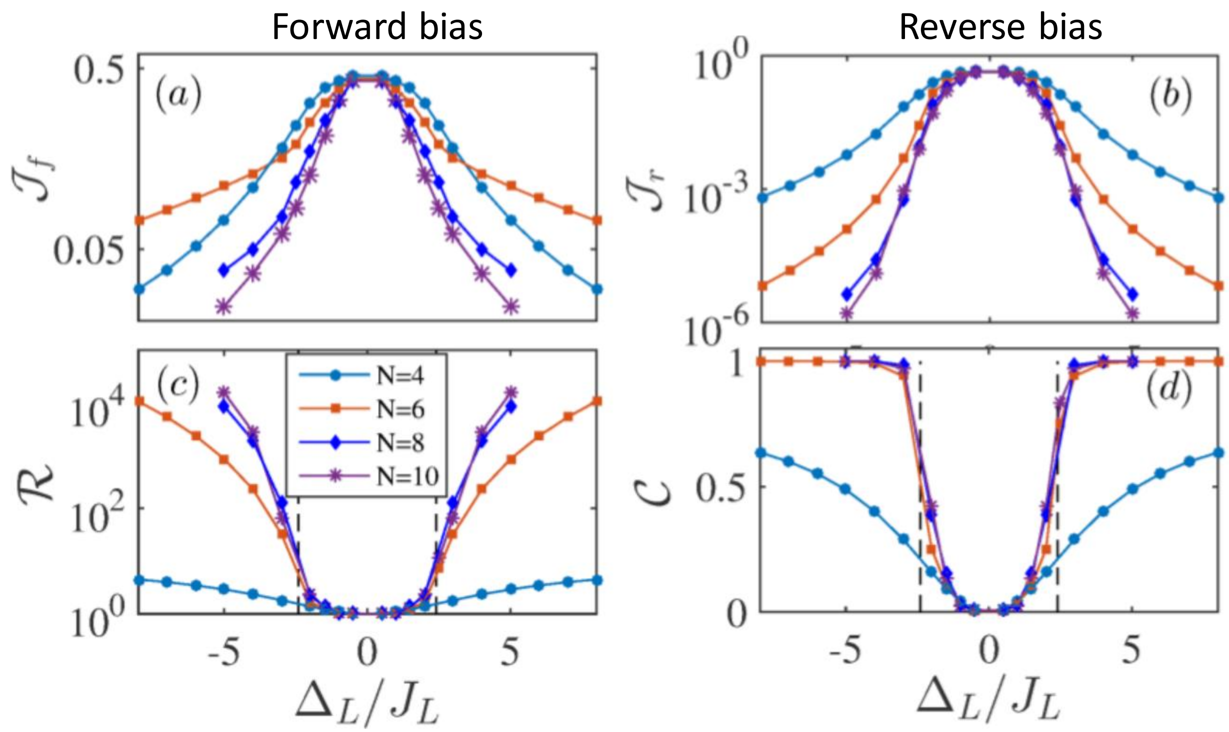
$\Delta_M = 0, J_M = 0.1 J_L$



$\Delta_R = 0$

$J_L = J_R$



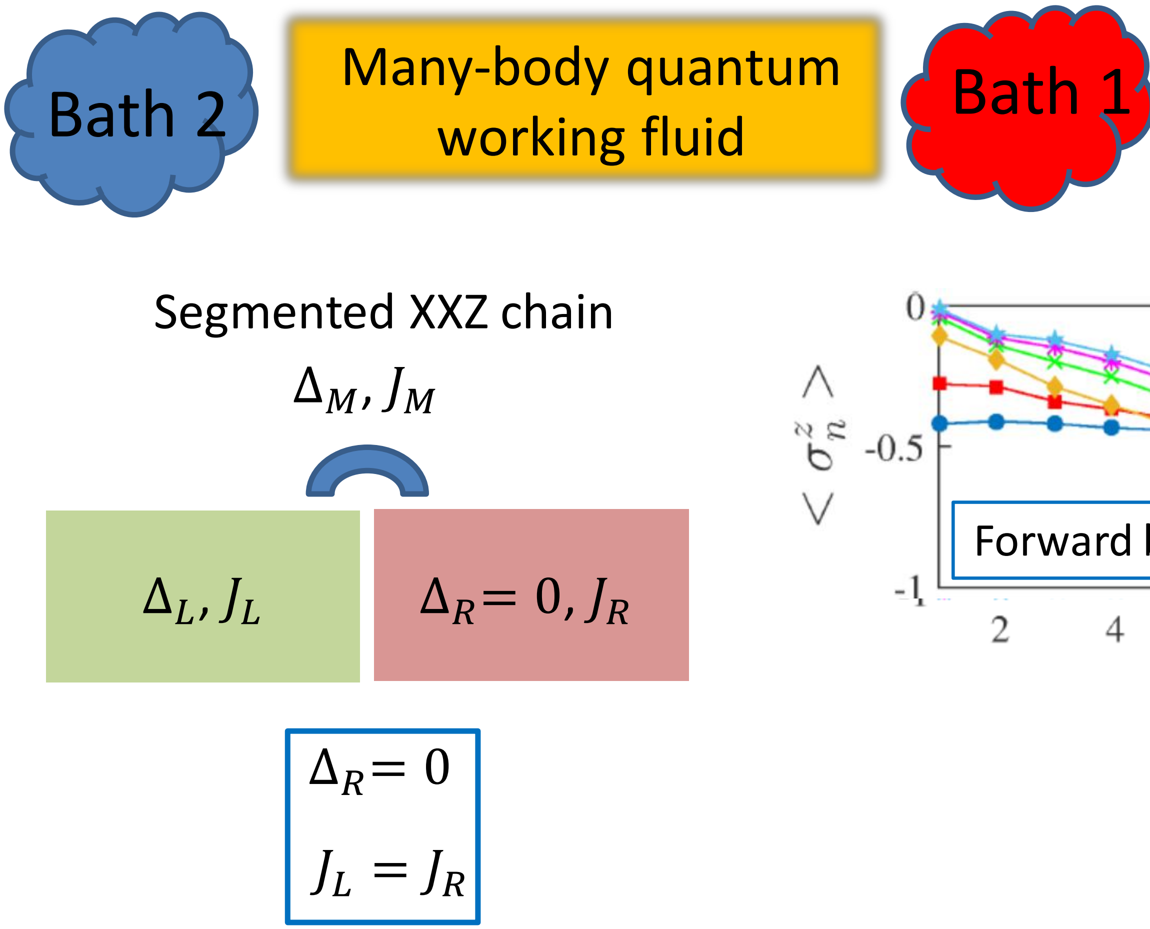


$$\mathcal{R} = -\frac{\mathcal{J}_f}{\mathcal{J}_r}$$ **Rectification**

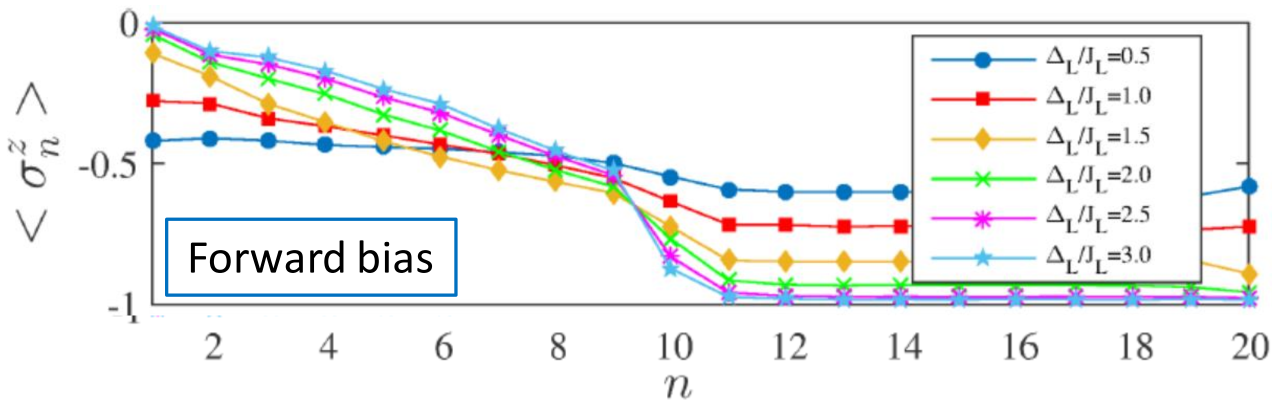
$$\mathcal{C} = \left| \frac{\mathcal{J}_f + \mathcal{J}_r}{\mathcal{J}_f - \mathcal{J}_r} \right|$$ **Contrast**

Rectification and **Contrast** vs anisotropy of left half

Rectification



Magnetization profiles



Bath 2

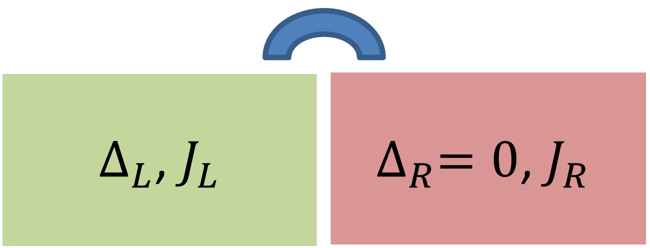
Many-body quantum working fluid

Bath 1

Rectification

Segmented XXZ chain

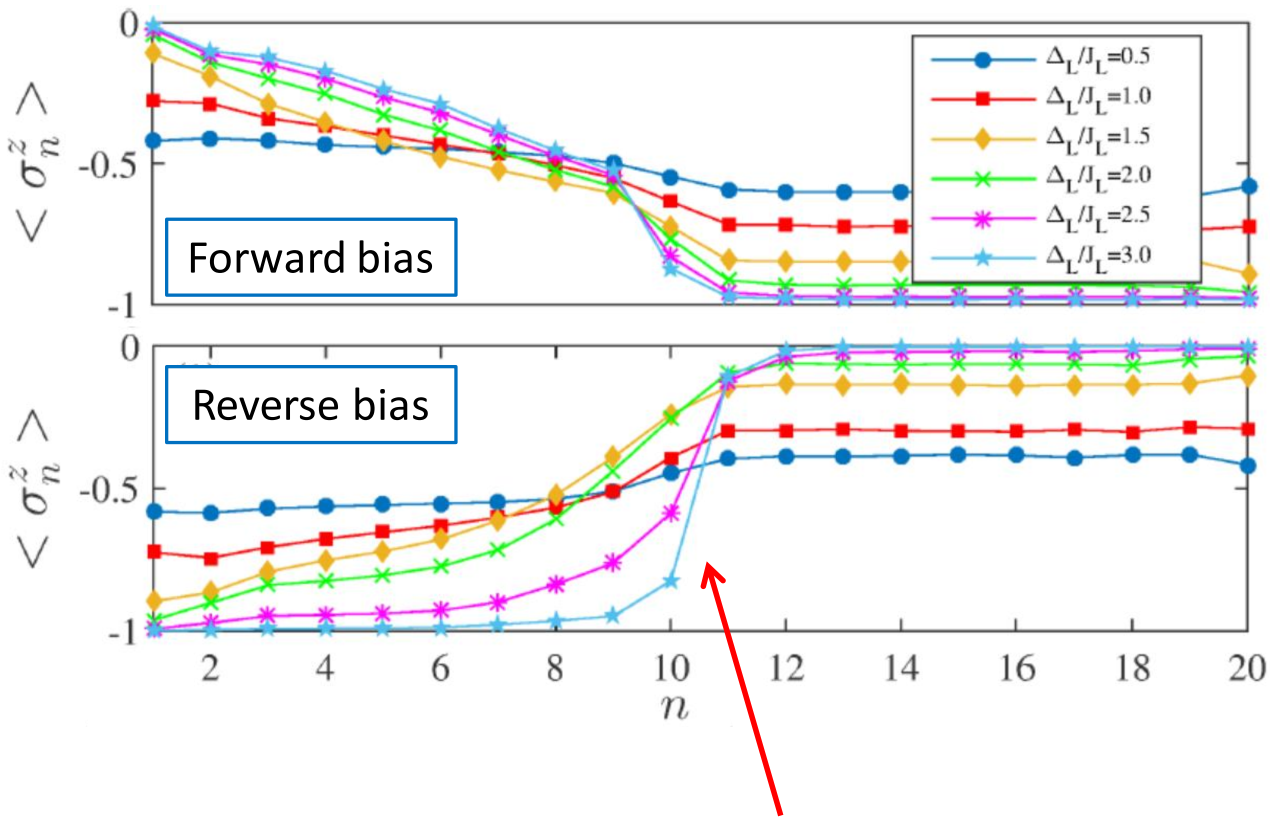
Δ_M, J_M



$\Delta_R = 0$

$J_L = J_R$

Magnetization profiles

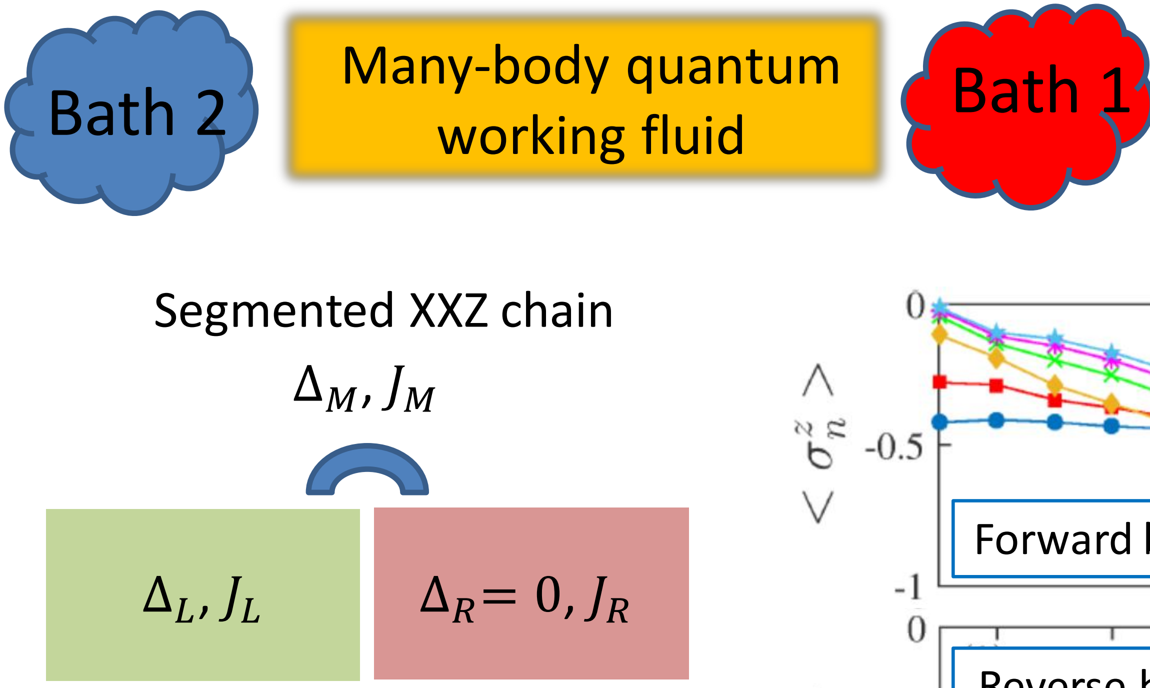


Insulating?

Which is the **critical value of anisotropy**?

What is the **mechanism** behind such good rectification?

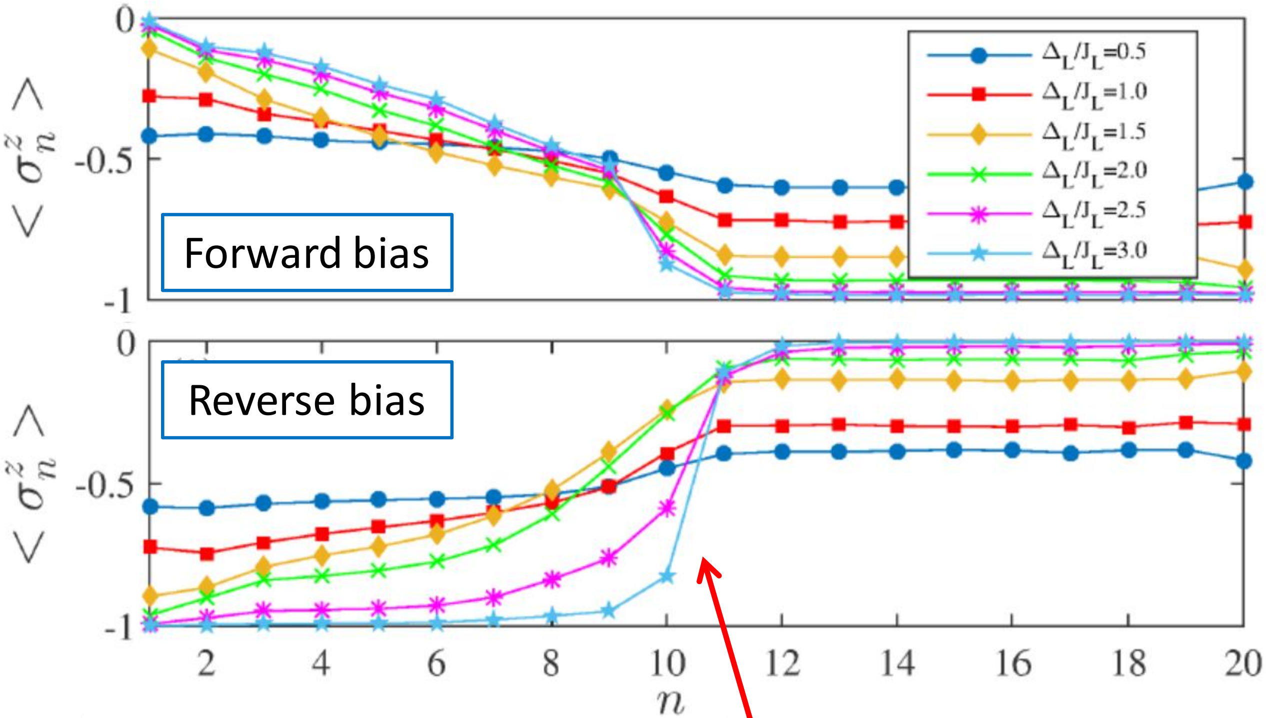
Rectification



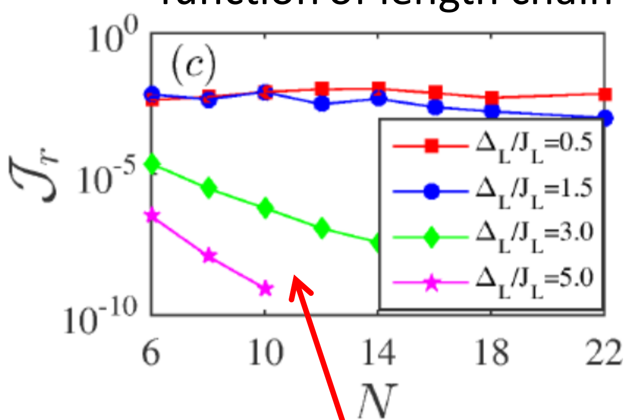
$\Delta_R = 0$

$J_L = J_R$

Magnetization profiles



Reverse current as function of length chain

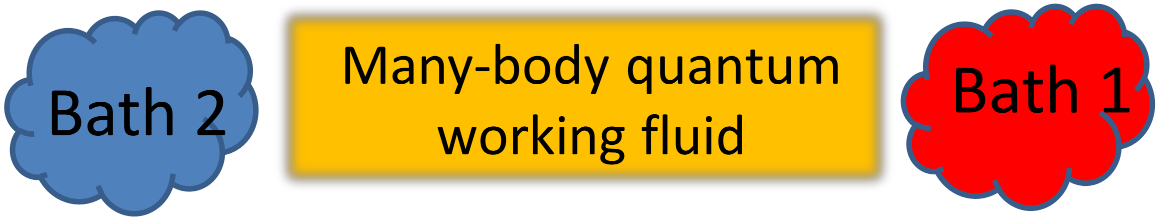


Smaller anisotropy

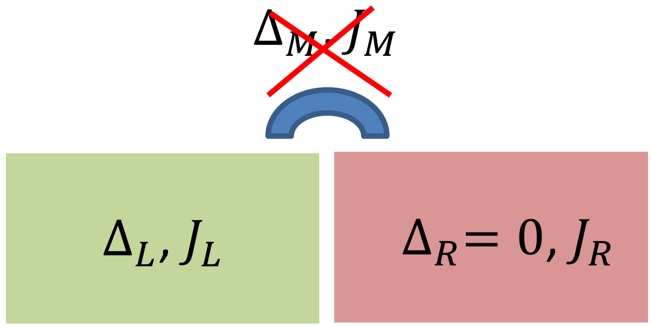
Larger anisotropy

Insulating?
Which is the **critical value of anisotropy**?
What is the **mechanism** behind such good rectification?

Rectification

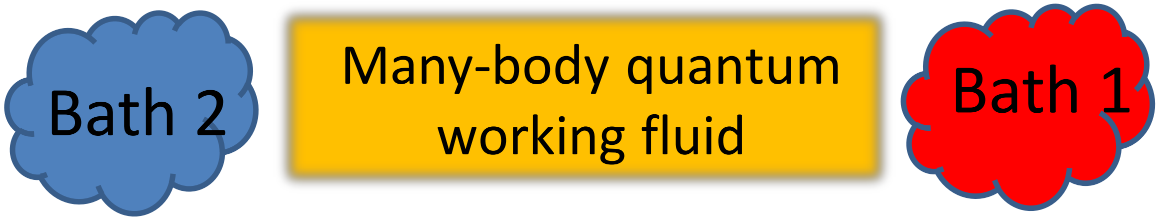


Let us consider the reverse bias case



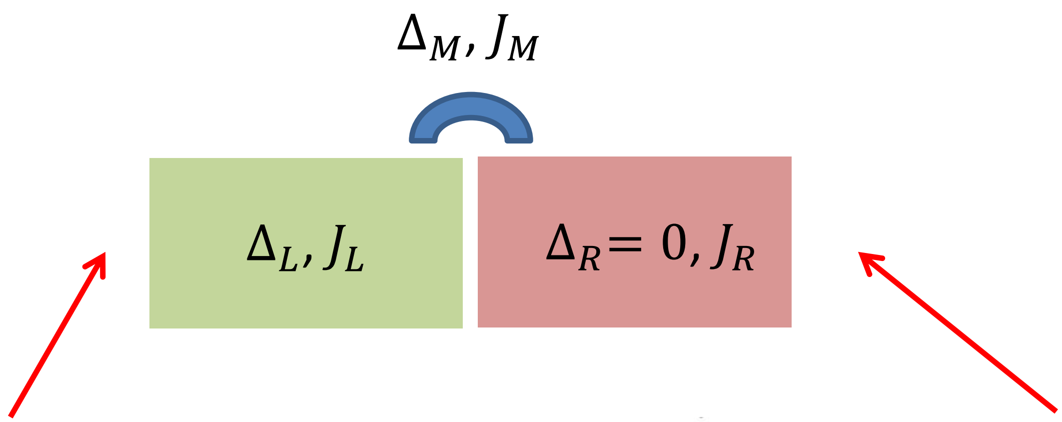
$$\hat{\rho}_{ss,\lambda_j=0} = \bigotimes_n |{\downarrow}\rangle_n\langle{\downarrow}|$$

$$\hat{\rho}_{ss,\lambda_j=0.5} = \bigotimes_n \left(|{\uparrow}\rangle_n\langle{\uparrow}| + |{\downarrow}\rangle_n\langle{\downarrow}|\right)/2$$



Rectification

Let us consider the reverse bias case

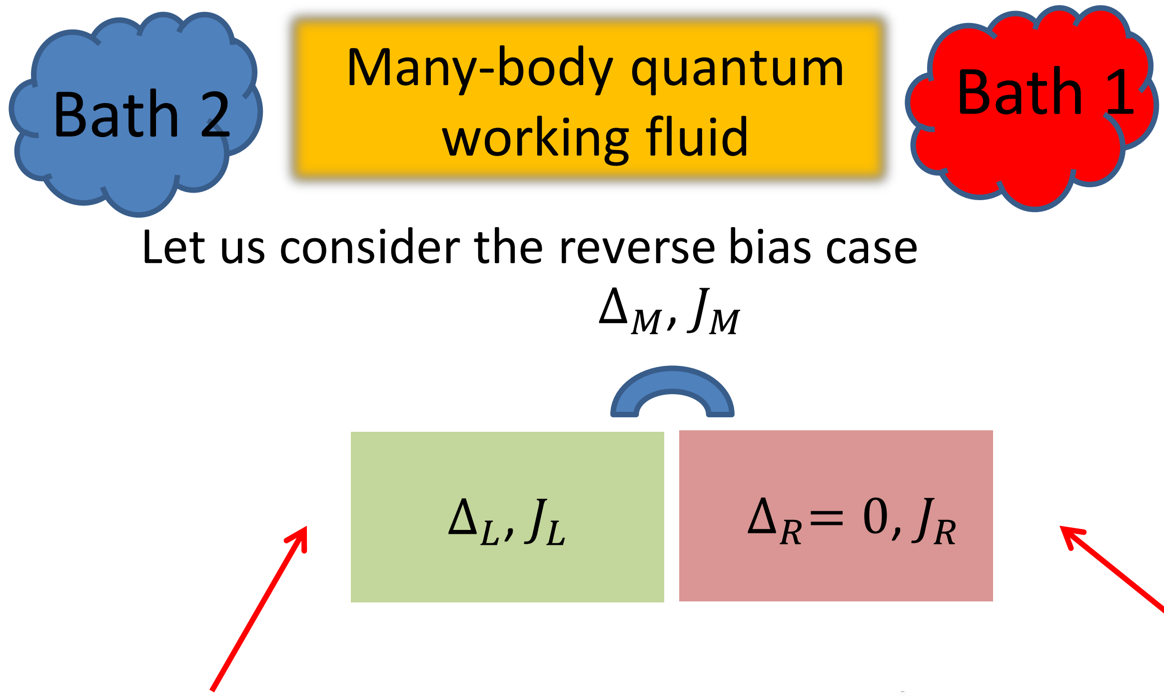


$$\hat{\rho}_{ss,\lambda_j=0} = \bigotimes_n |\downarrow\rangle_n\langle\downarrow|$$

$$\hat{\rho}_{ss,\lambda_j=0.5} = \bigotimes_n \left(|\uparrow\rangle_n\langle\uparrow| + |\downarrow\rangle_n\langle\downarrow|\right)/2$$

Can a spin flip $\sigma^+_{N/2}\,\sigma^-_{N/2+1}$ be generated at the boundary without energy cost?

Rectification



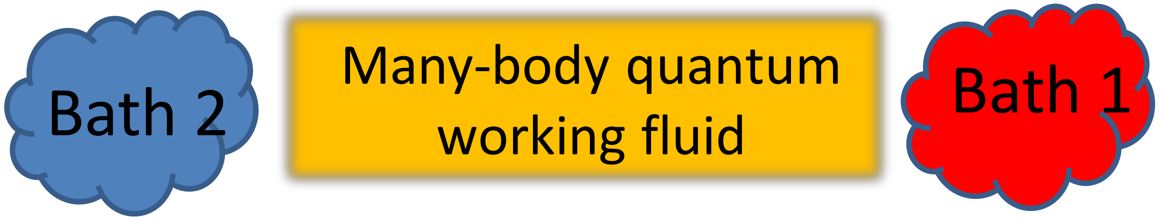
$$\hat{\rho}_{ss,\lambda_j=0} = \bigotimes_n |\downarrow\rangle_n\langle\downarrow| \qquad \hat{\rho}_{ss,\lambda_j=0.5} = \bigotimes_n \left(|\uparrow\rangle_n\langle\uparrow| + |\downarrow\rangle_n\langle\downarrow|\right)/2$$

Can a spin flip $\sigma^+_{N/2}\,\sigma^-_{N/2+1}$ be generated at the boundary without energy cost?

We analyse the excitation spectrum of each half of the chain.
The left side is given by the eigenvalues of

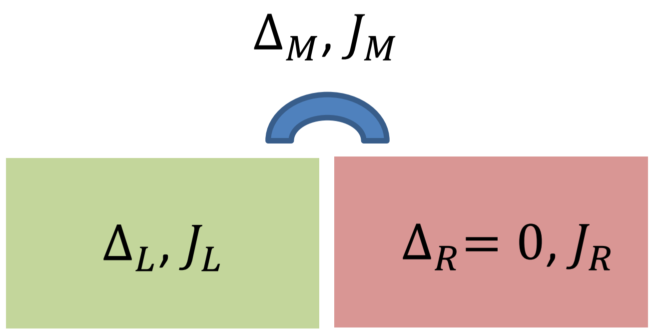
$$\mathbb{M}(\Delta) = \begin{bmatrix} -2\Delta_L & 2J_L & 0 & \dots & \dots & \dots \\ 2J_L & -4\Delta_L & 2J_L & 0 & \dots & \dots \\ 0 & 2J_L & -4\Delta_L & 2J_L & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & 0 & 2J_L & -4\Delta_L & 2J_L \\ \dots & \dots & 0 & 0 & 2J_L & -2\Delta_L \end{bmatrix}$$

If there is an **energy gap** in the **overall system** (left and right halves), it can become **insulating**!



Rectification

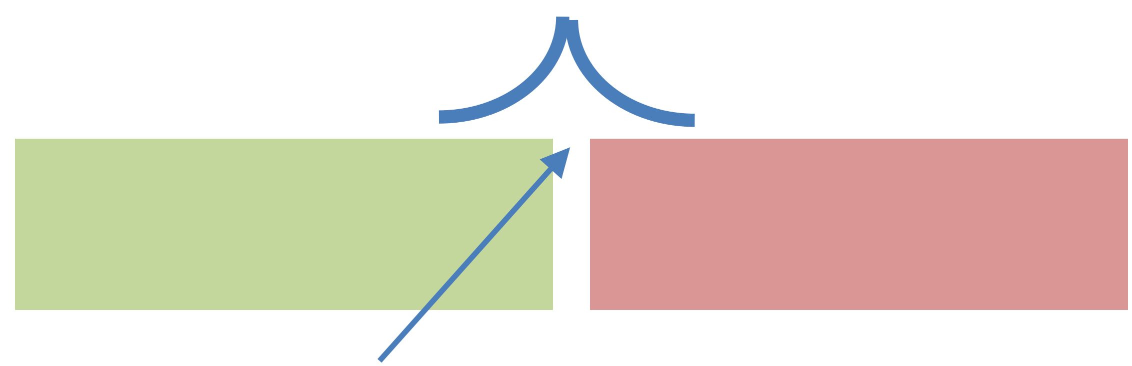
Let us consider the reverse bias case



$\hat{\rho}_{ss,\lambda_j=0} = \bigotimes_n |\downarrow\rangle_n\langle\downarrow|$

$\hat{\rho}_{ss,\lambda_j=0.5} = \bigotimes_n \left(|\uparrow\rangle_n\langle\uparrow| + |\downarrow\rangle_n\langle\downarrow|\right)/2$

Can a spin flip $\sigma^+_{N/2}\,\sigma^-_{N/2+1}$ be generated at the boundary without energy cost?



Excitations are gapped and localized at the boundary: **but only in reverse bias**

Bath 2

Many-body quantum working fluid

Bath 1

We test this in detail on a small system of 10 spins

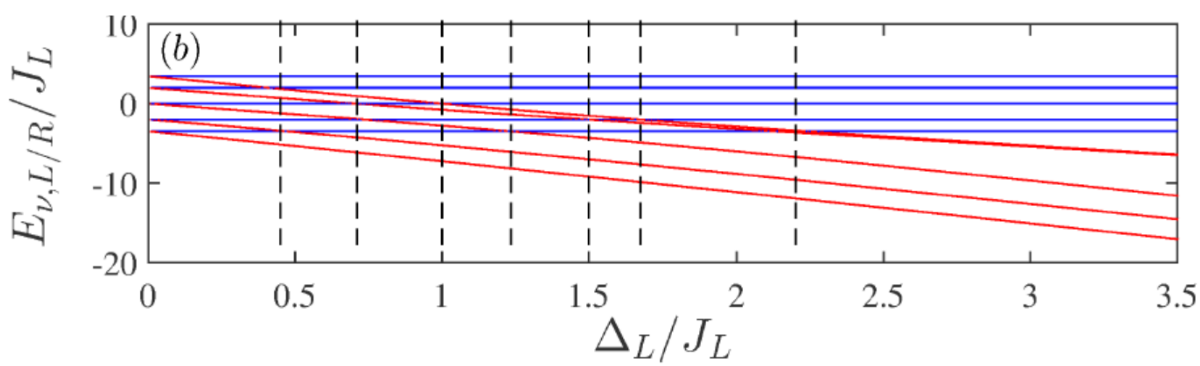
Rectification

very small

Δ_M, J_M

Δ_L, J_L

$\Delta_R = 0, J_R$



Spectrum of right half of the chain

Spectrum of left half of the chain

Bath 2

Many-body quantum working fluid

Bath 1

Rectification

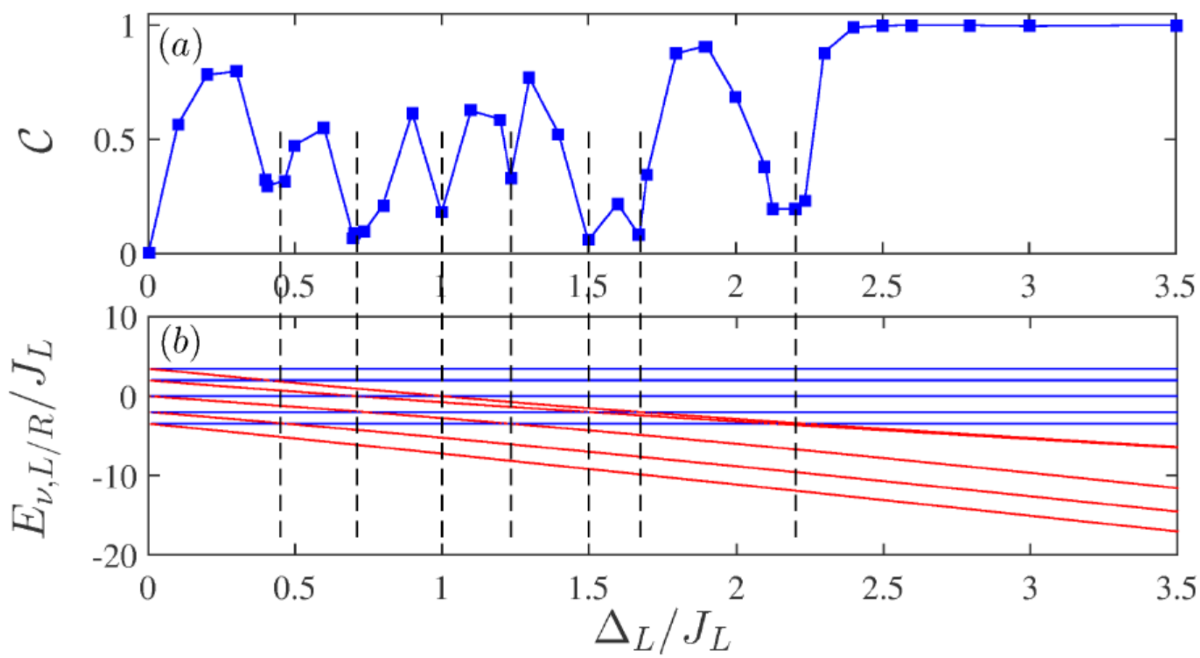
very small

Δ_M, J_M

Δ_L, J_L

$\Delta_R = 0, J_R$

We test this in detail on a small system of 10 spins



Spectrum of right half of the chain

Spectrum of left half of the chain

Bath 2 | Many-body quantum working fluid | Bath 1

Rectification

very small

Δ_M, J_M

Δ_L, J_L | $\Delta_R = 0, J_R$

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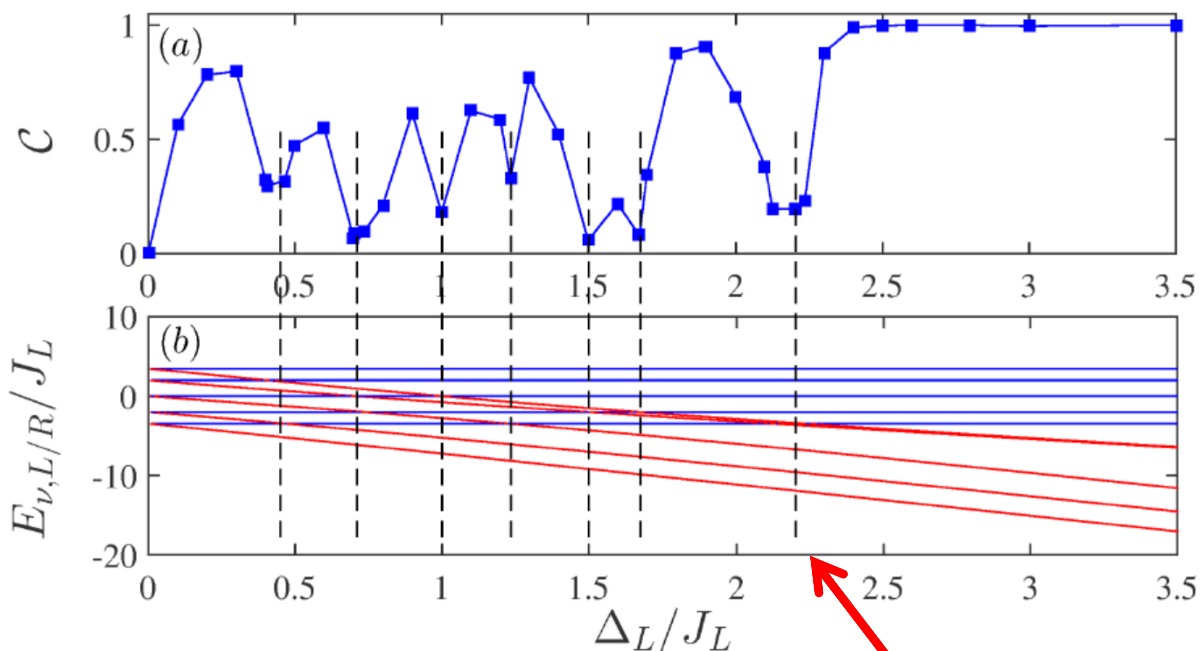


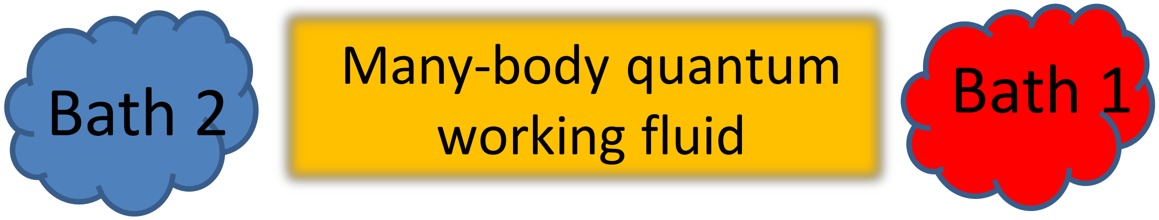
Spectrum of right half of the chain

Spectrum of left half of the chain

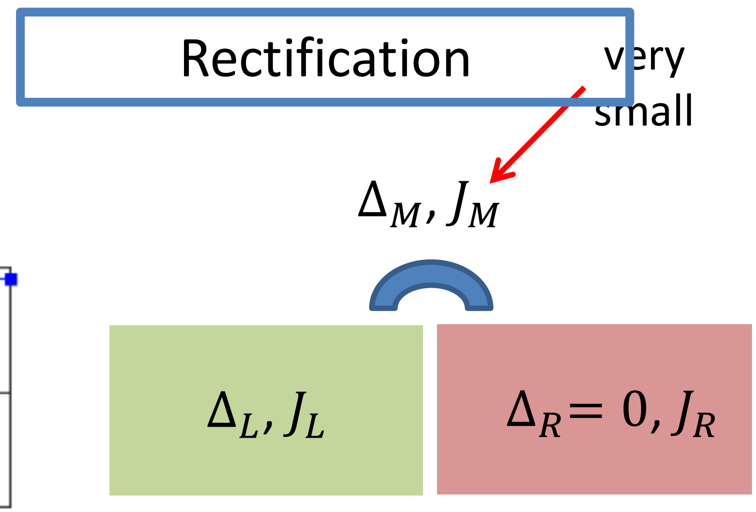
Beyond this point the contrast is always large

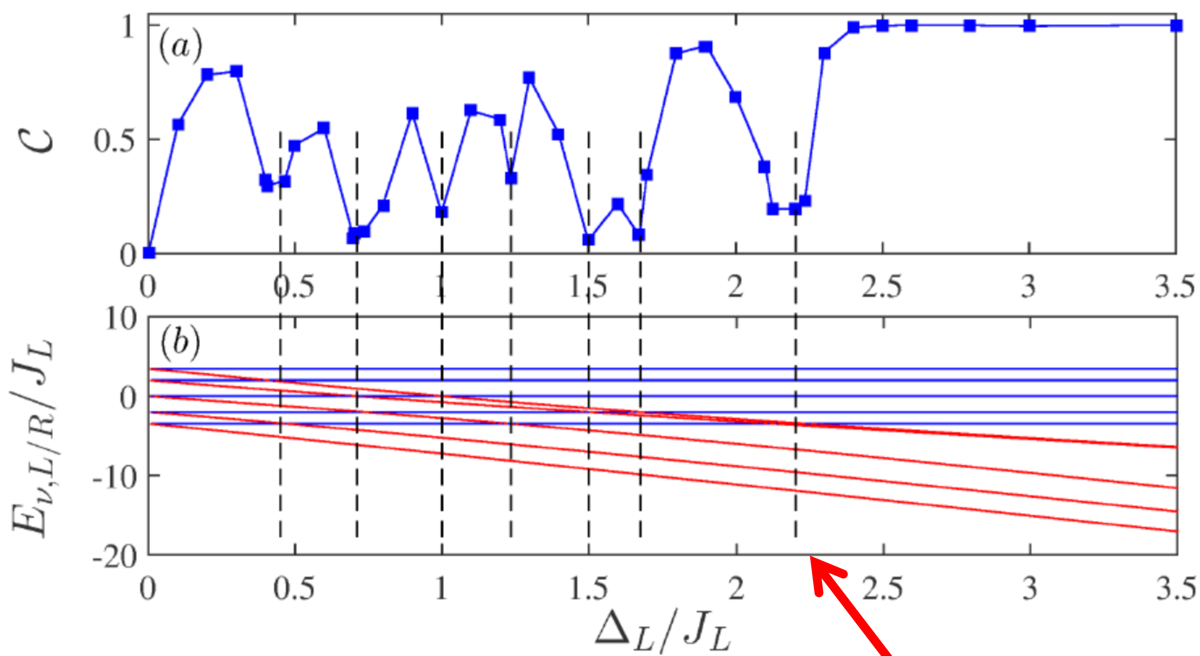
For large systems the **critical value** is

$$\Delta_{L,c} = J_R + \sqrt{J_R^2 + J_L^2}.$$



We test this in detail on a small system of 10 spins





Spectrum of right half of the chain

Spectrum of left half of the chain

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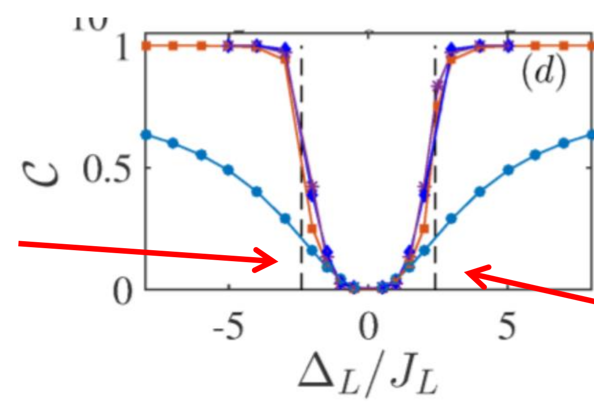
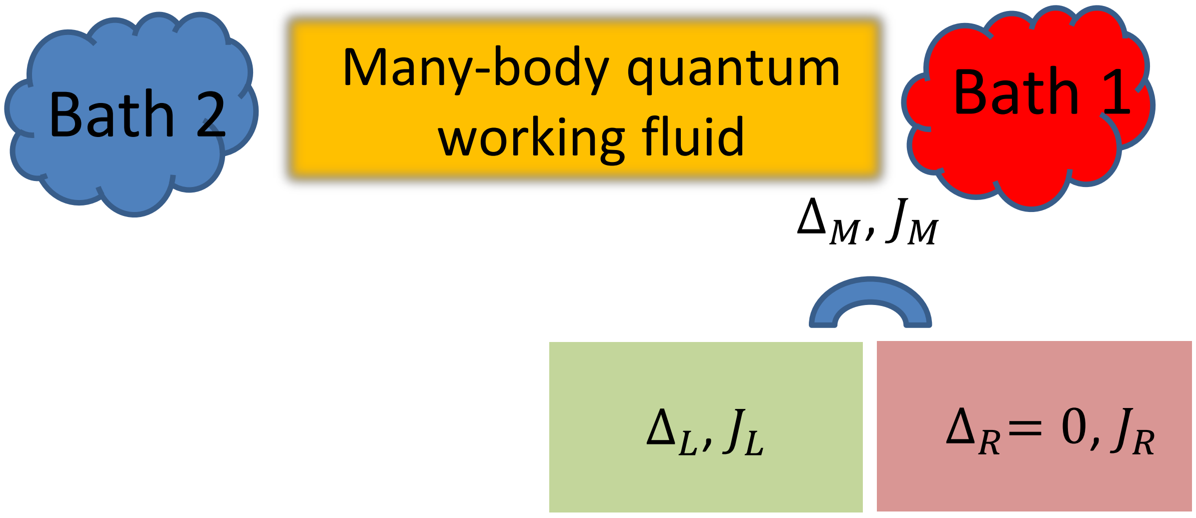


Small system with large J_M

$1 + \sqrt{2}$

Rectification

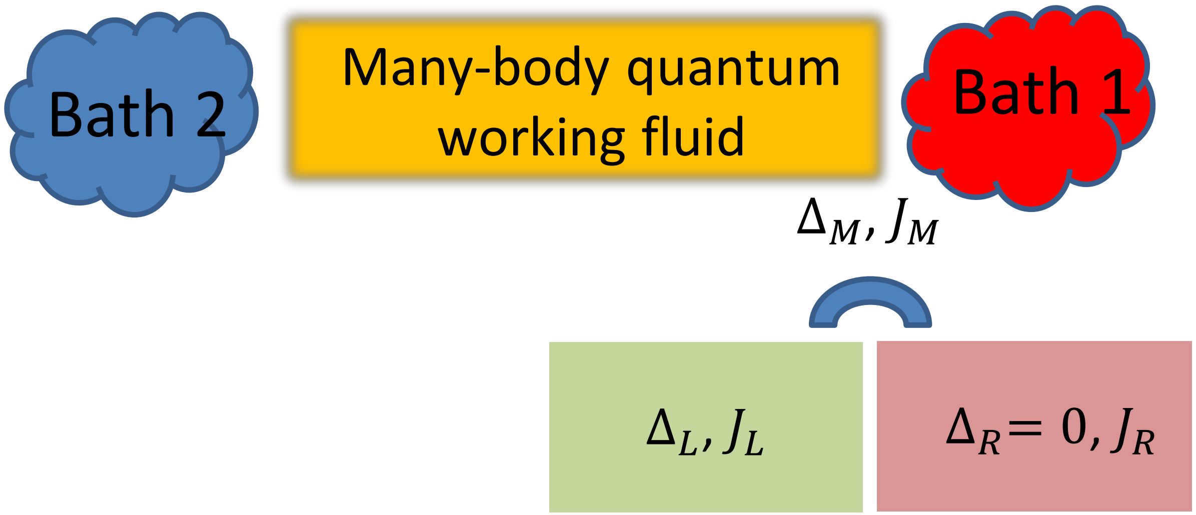


Ljubotina et al. Nat Comm 2017
Mascarenhas et al Quantum 2017
Biella et al. PRB 2016
Ponomarev et al. PRL 2011
…

$\bigotimes_n \left(|\uparrow\rangle_n \langle\uparrow| + |\downarrow\rangle_n \langle\downarrow| \right) / 2$ $\bigotimes_n |\downarrow\rangle_n \langle\downarrow|$

We prepare two long chains in either the infinite temperature state or fully polarized state. We then connect them and measure the time t^* it takes for the spin at the interface to change from $\left\langle \sigma^z_{L/2}(0) \right\rangle = -1$ to $\left\langle \sigma^z_{L/2}(t^*) \right\rangle = -0.99$

Rectification



Ljubotina et al. Nat Comm 2017
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...

$\bigotimes_n \left(|\uparrow\rangle_n\langle\uparrow| + |\downarrow\rangle_n\langle\downarrow|\right)/2$ $\bigotimes_n |\downarrow\rangle_n\langle\downarrow|$

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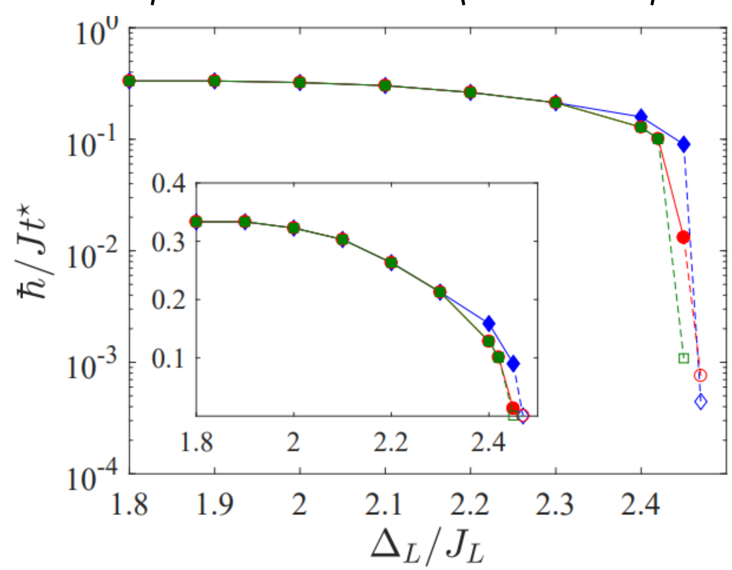
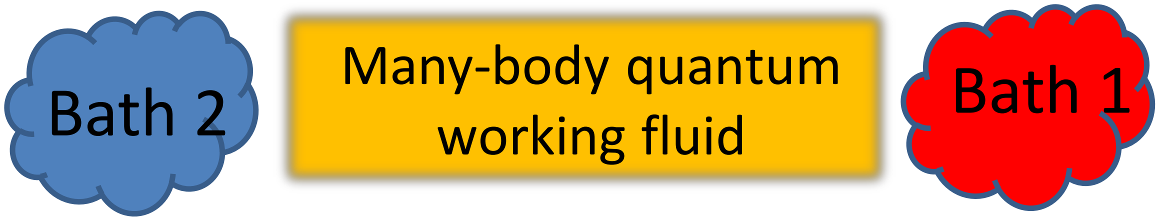
N=20
N=50
N=100



$1/t^*$ decreases abruptly as Δ_L/J_L approaches $1+\sqrt{2}$



Rectification

We have found that thanks to the interplay between baths and interactions not only it is possible to rectify currents but in the thermodynamic limit the system is

- diffusive in forward bias
- Insulating in reverse bias

when the interaction exceeds

$$\Delta_{L,c} = J_R + \sqrt{J_R^2 + J_L^2}$$

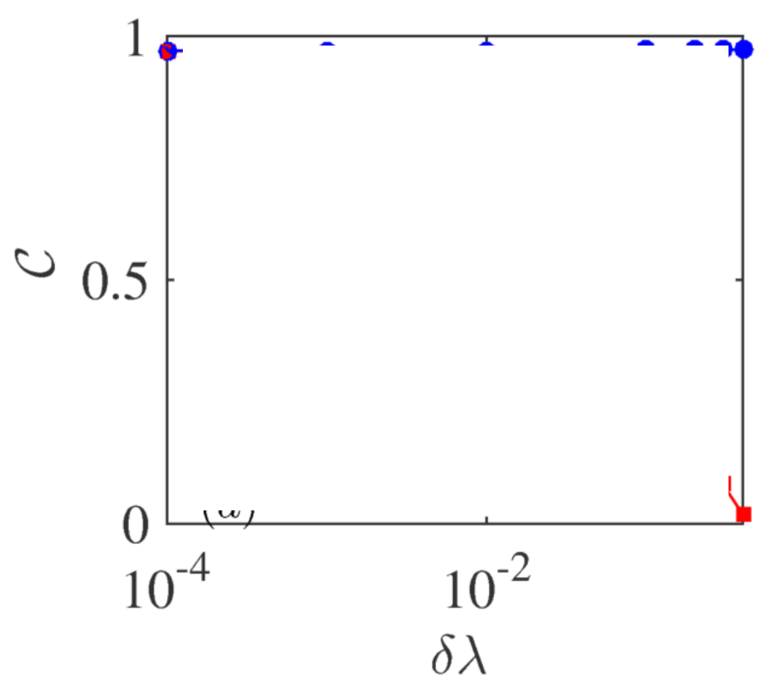
resulting in a perfect diode

Bath 2 | Many-body quantum working fluid | Bath 1

Rectification

What about the stability of the effect?

We add a deviation of the bath parameter λ only on one bath and check how large the contrast remains.



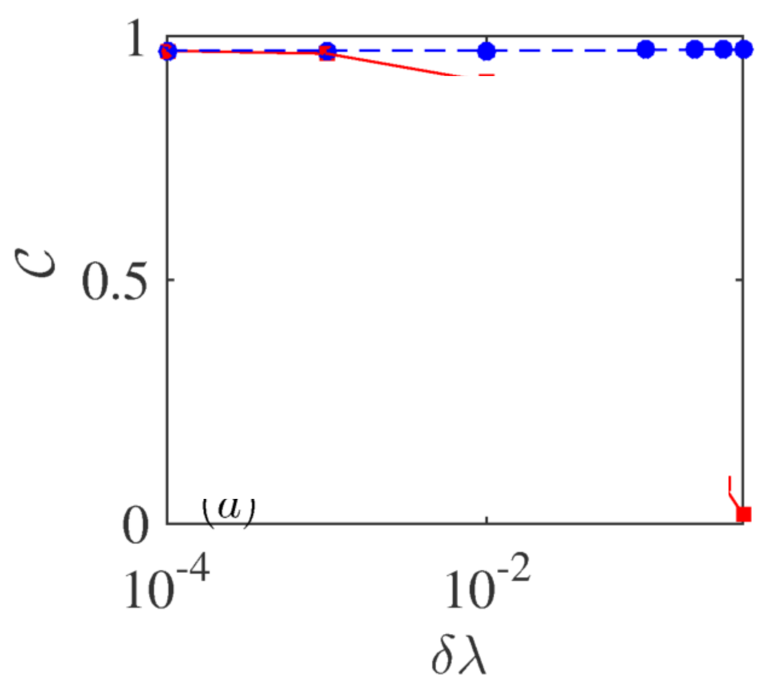
Deviation of the bath parameter λ from the ideal scenario of either **fully polarized** or from **infinite temperature**.

Bath 2 | Many-body quantum working fluid | Bath 1

Rectification

What about the stability of the effect?

We add a deviation of the bath parameter λ only on one bath and check how large the contrast remains.



Modifying λ for the **infinite temperature** portion

Deviation of the bath parameter λ from the ideal scenario of either **fully polarized** or from **infinite temperature**.

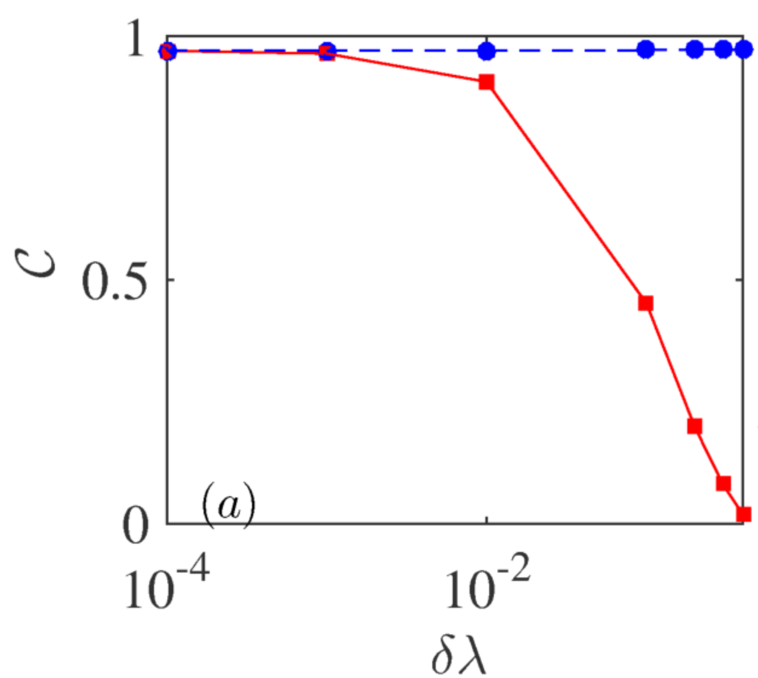
Bath 2

Many-body quantum working fluid

Bath 1

Rectification

What about the stability of the effect?
We add a deviation of the bath parameter λ only on one bath and check how large the contrast remains.

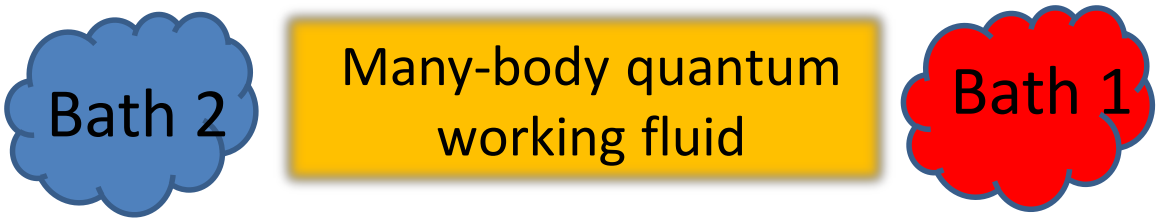


Modifying λ for the **infinite temperature** portion

Modifying λ for the **fully polarized** portion

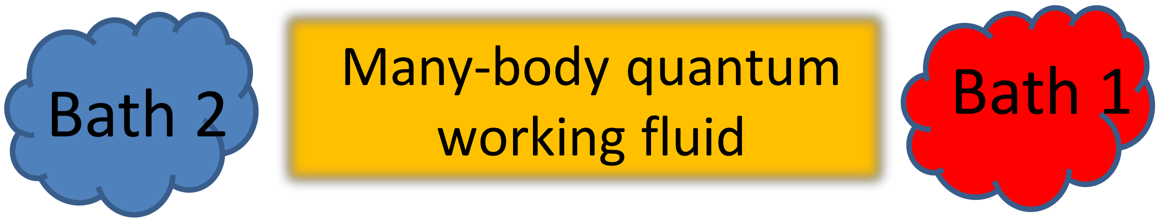
Deviation of the bath parameter λ from the ideal scenario of either **fully polarized** or from **infinite temperature**.

The contrast is very stable when the portion close to infinite temperature is changed to a lower T.
Not as stable for the portion fully polarized.



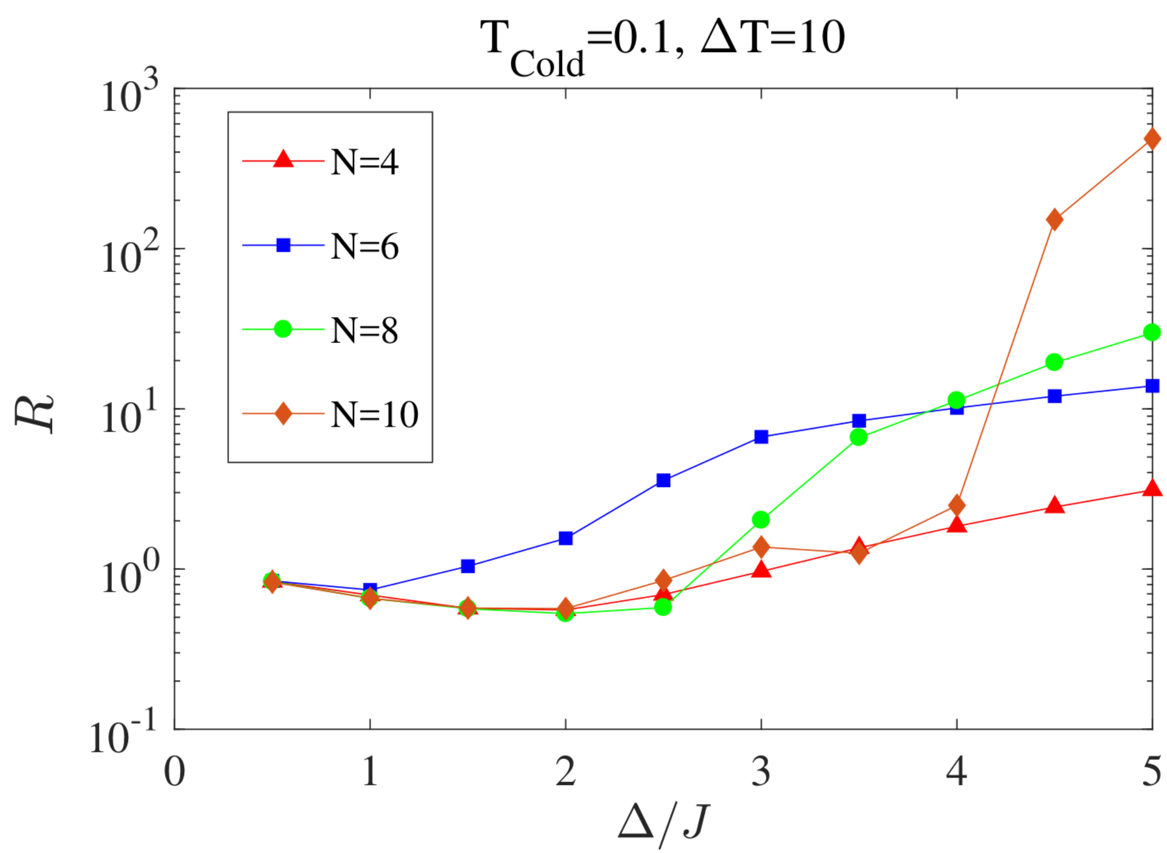
Rectification

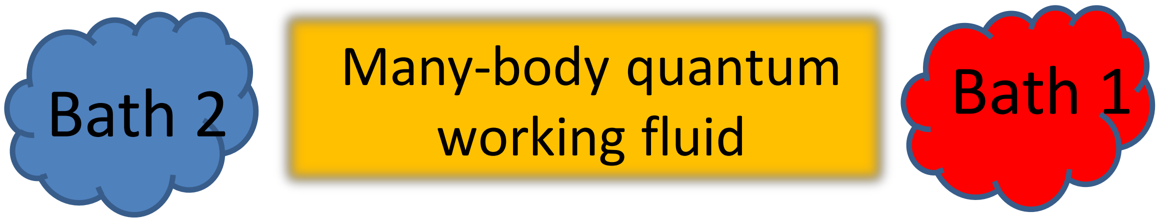
What about thermal baths?



Rectification

What about thermal baths?
We use global Lindblad baths, for which we do exact diagonalization, and we still see strong rectification as the anisotropy Δ increases.





Experimental realizations?

Solid state: difficult to find materials with such large anisotropy and of course difficult to grow them on top of others.

Bath 2

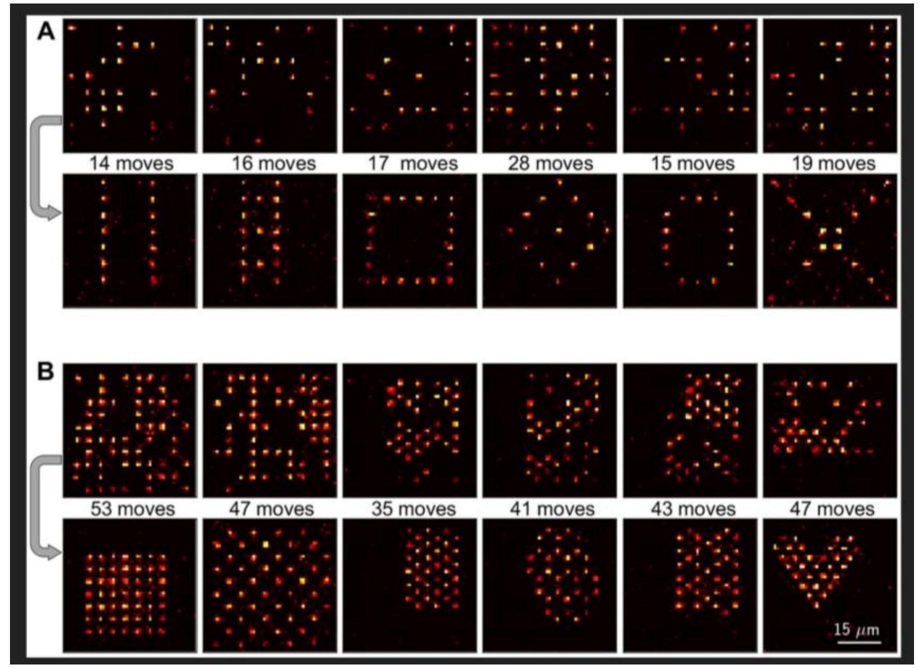
Many-body quantum working fluid

Bath 1

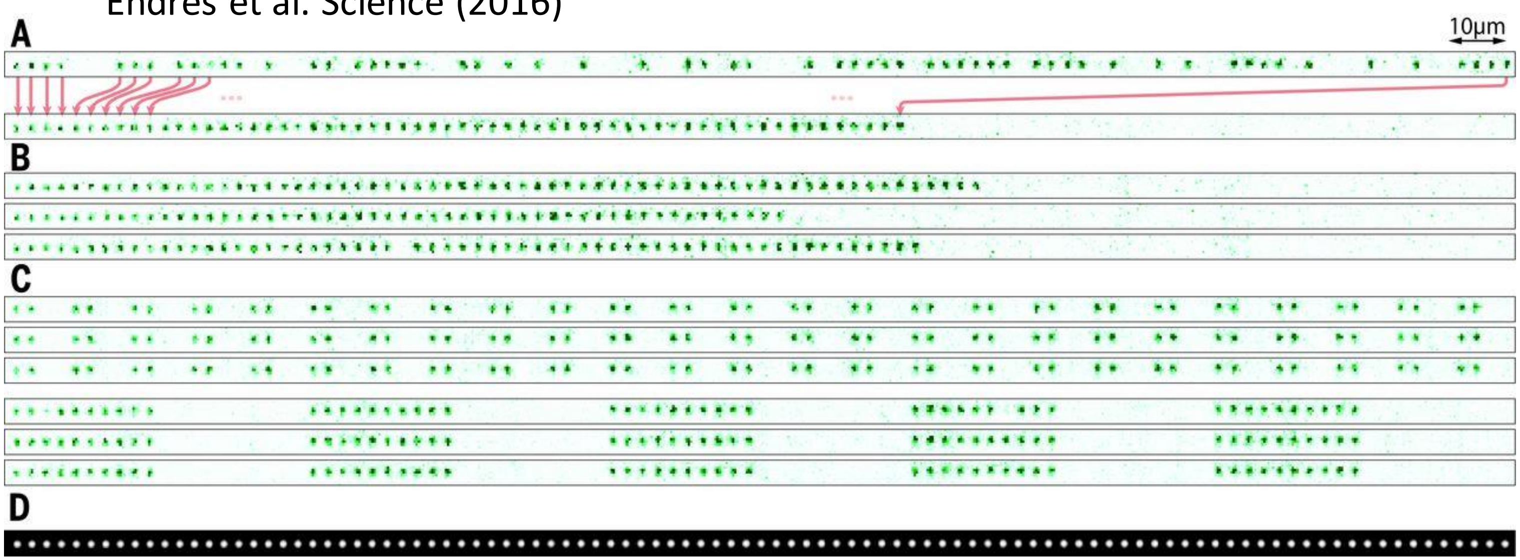
Experimental realizations?

Rydberg atoms controlled by optical tweezers

Barredo et al. Science (2016)



Endres et al. Science (2016)



Bath 2

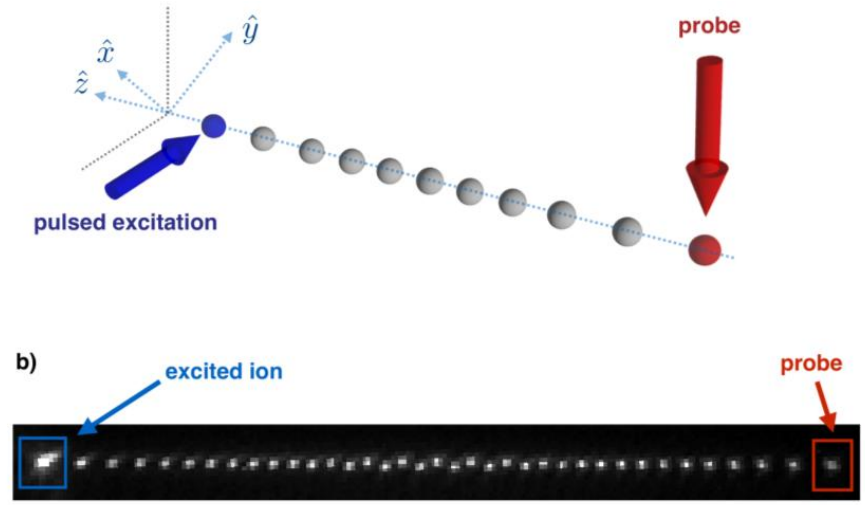
Many-body quantum working fluid

Bath 1

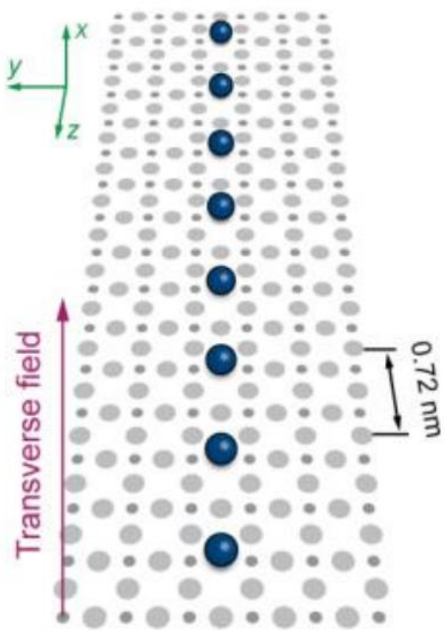
Experimental realizations?

Trapped ions

Ramm et al. NJP (2014)



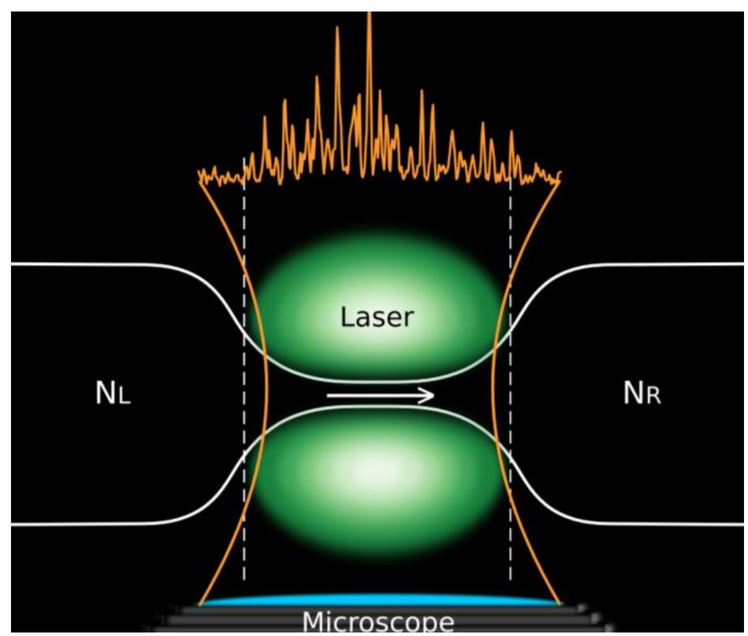
XXZ with surface adatoms

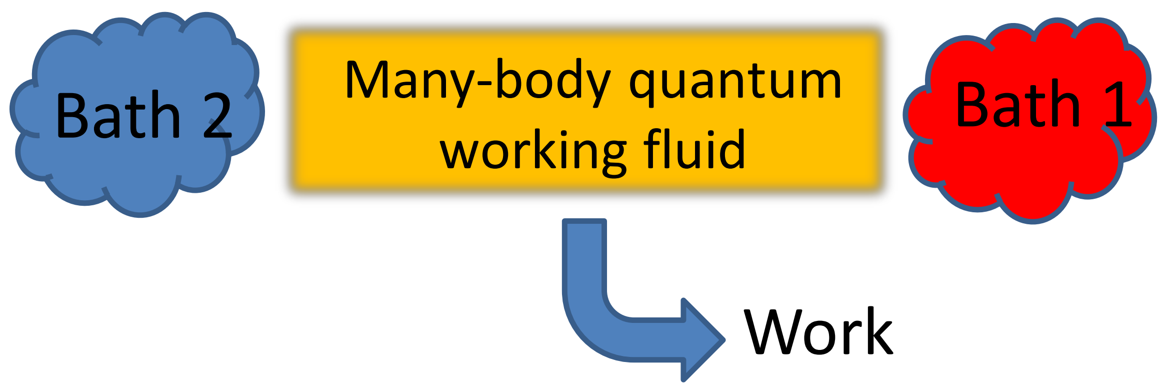


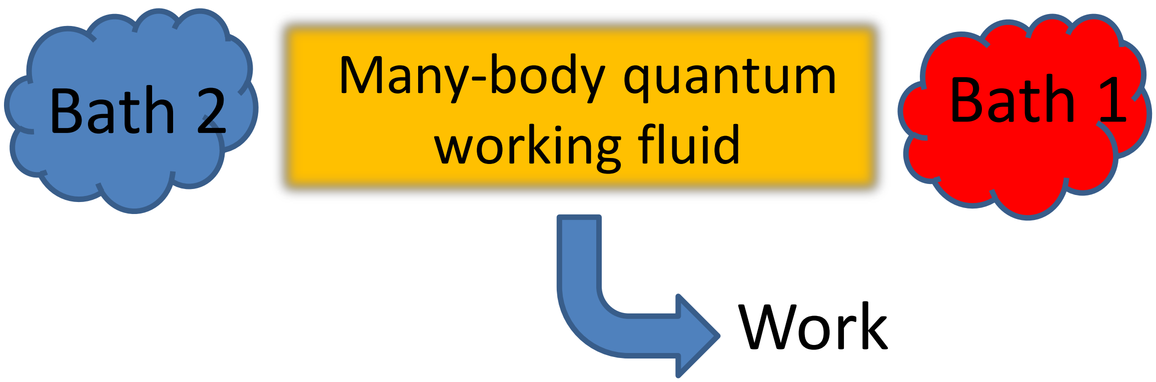
Toskovic et al.
Nat. Phys. (2016)

Ultracold atoms

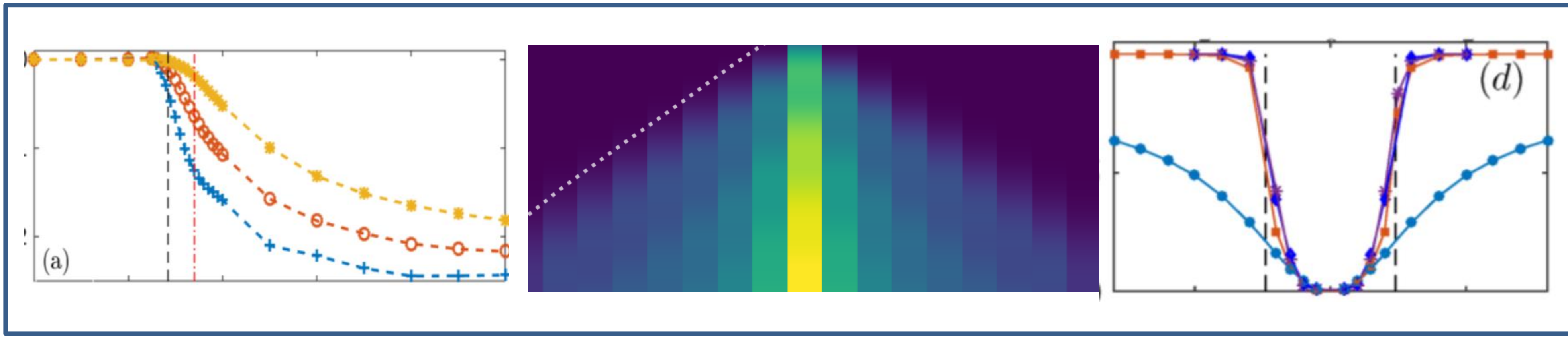
Brantut, Esslinger




Bath 2
Many-body quantum working fluid
Bath 1
Work



Well … this will be for another time☺

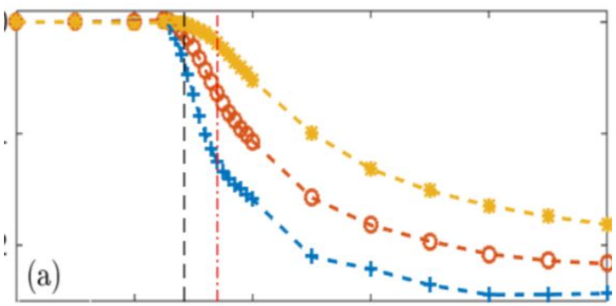


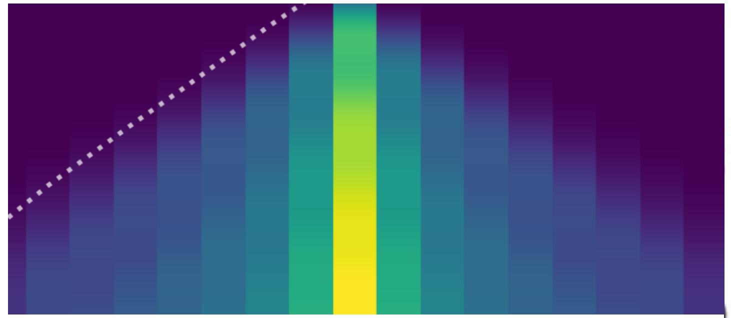
CONCLUSIONS

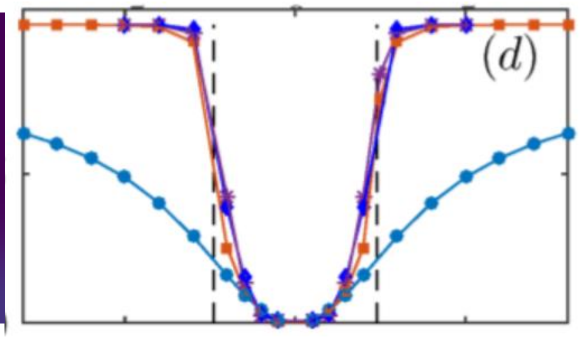
- Many-body open quantum systems are the best!
  - Emerging properties
  - Phase transitions
  - …
- Implementation of Redfield
- Perfect diode

OUTLOOK

- We need to understand them better
- We need to find ways to understand them better!
- Extract work
- …







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