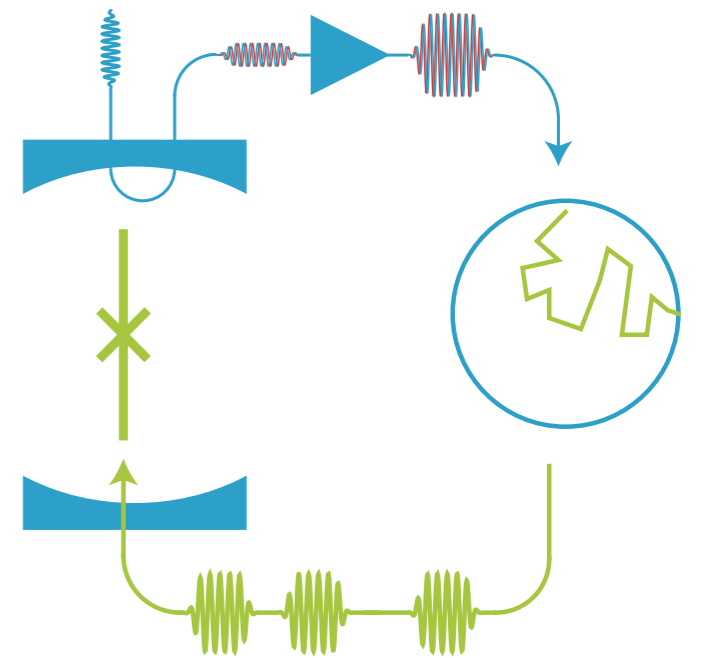
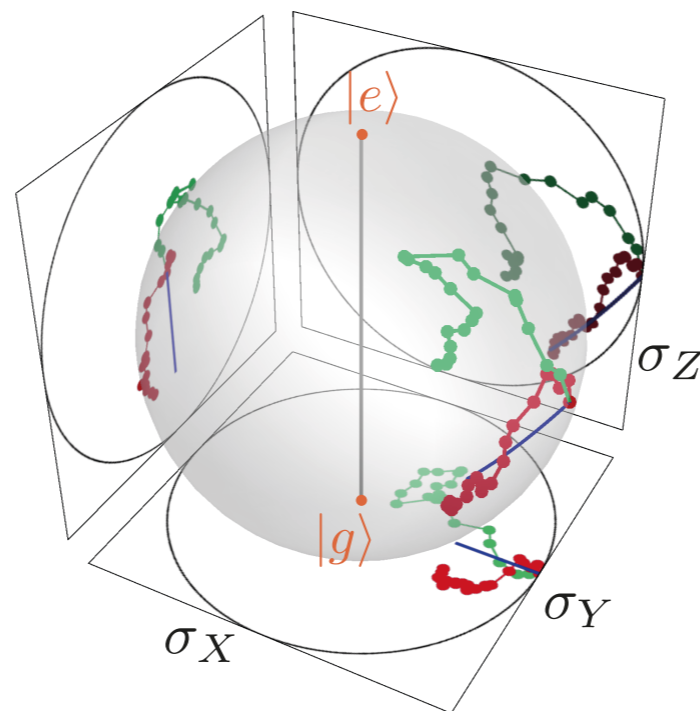
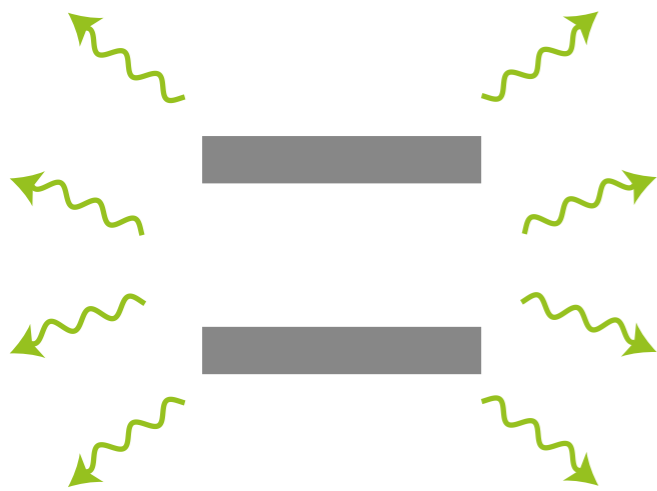


# Quantum trajectories and feedback in circuit-QED

**Benjamin Huard**

Ecole Normale Supérieure de Lyon, France



# Quantum laws of evolution

## Closed system

$$\partial_t |\psi\rangle = -\frac{i}{\hbar} H |\psi\rangle \qquad \partial_t \rho = -\frac{i}{\hbar} [H, \rho]$$



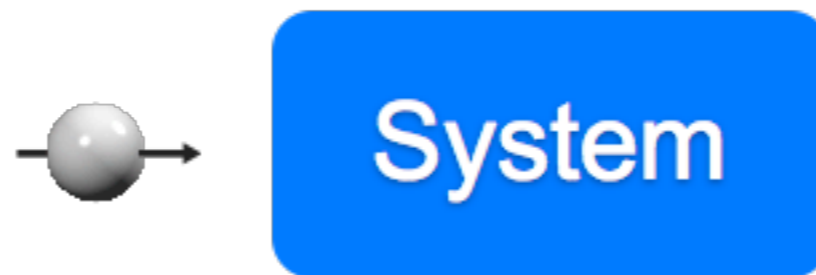
System



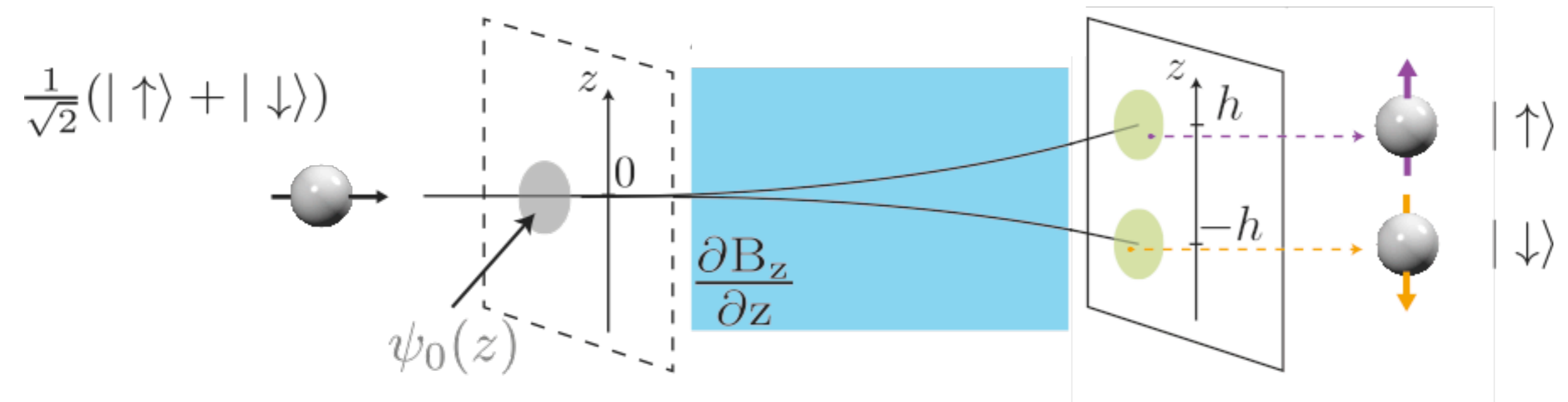
# Quantum laws of evolution

## Closed system

$$\partial_t |\psi\rangle = -\frac{i}{\hbar} H |\psi\rangle \qquad \partial_t \rho = -\frac{i}{\hbar} [H, \rho]$$



## Quantum measurement



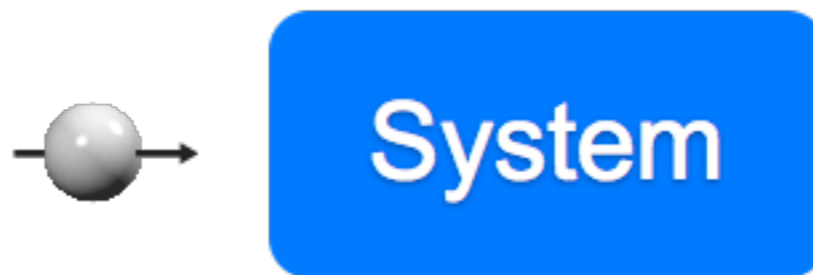
Possible outcomes

$z \approx h$	with proba	$p = \langle \uparrow   \rho   \uparrow \rangle$	and new state is	$ \uparrow\rangle$
$z \approx -h$		$p = \langle \downarrow   \rho   \downarrow \rangle$		$ \downarrow\rangle$

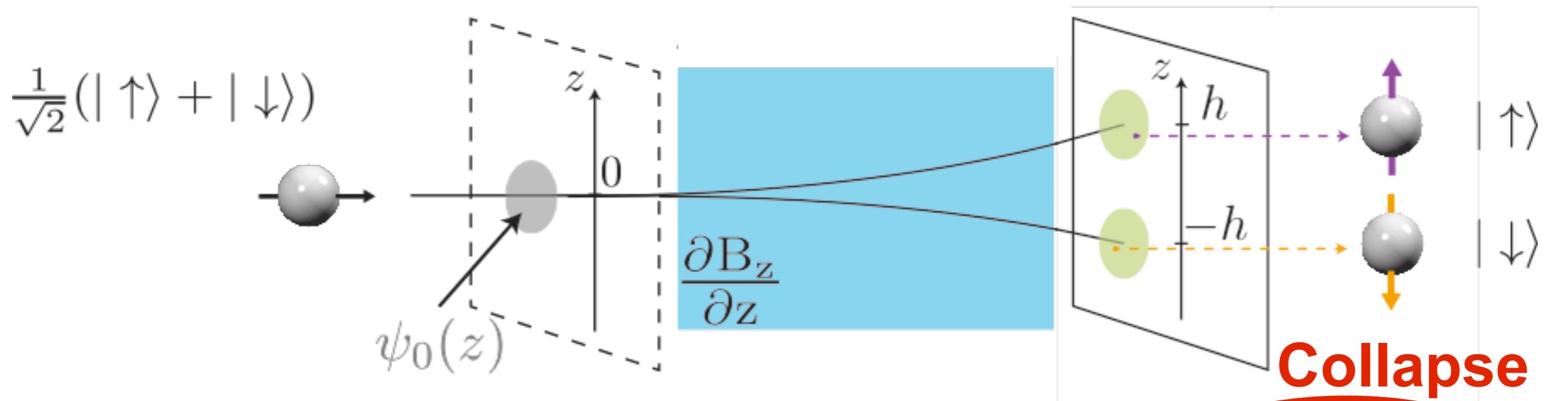
# Quantum laws of evolution

## Closed system

$$\partial_t |\psi\rangle = -\frac{i}{\hbar} H |\psi\rangle \qquad \partial_t \rho = -\frac{i}{\hbar} [H, \rho]$$



## Quantum measurement

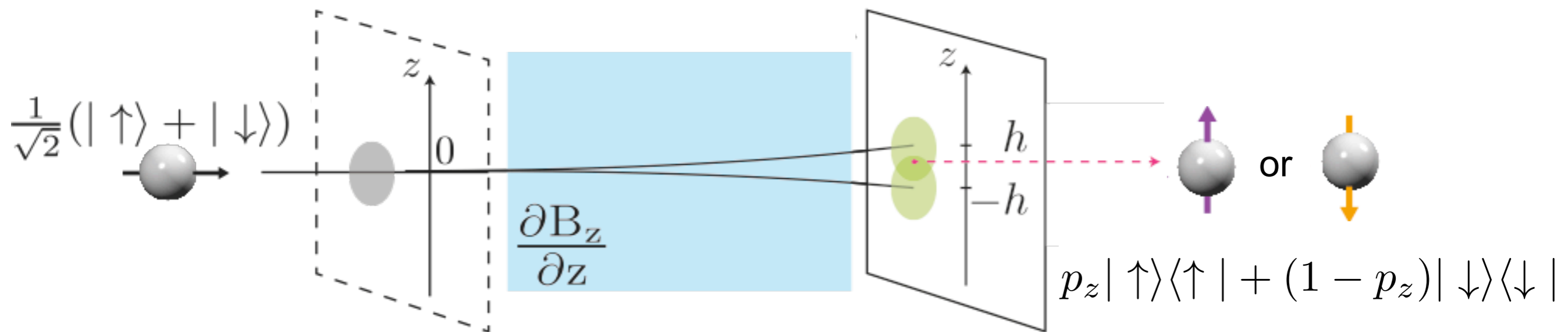


Possible outcomes

$z \approx h$	with proba	$p = \langle \uparrow   \rho   \uparrow \rangle$	and new state is	$ \uparrow\rangle$
$z \approx -h$		$p = \langle \downarrow   \rho   \downarrow \rangle$		$ \downarrow\rangle$

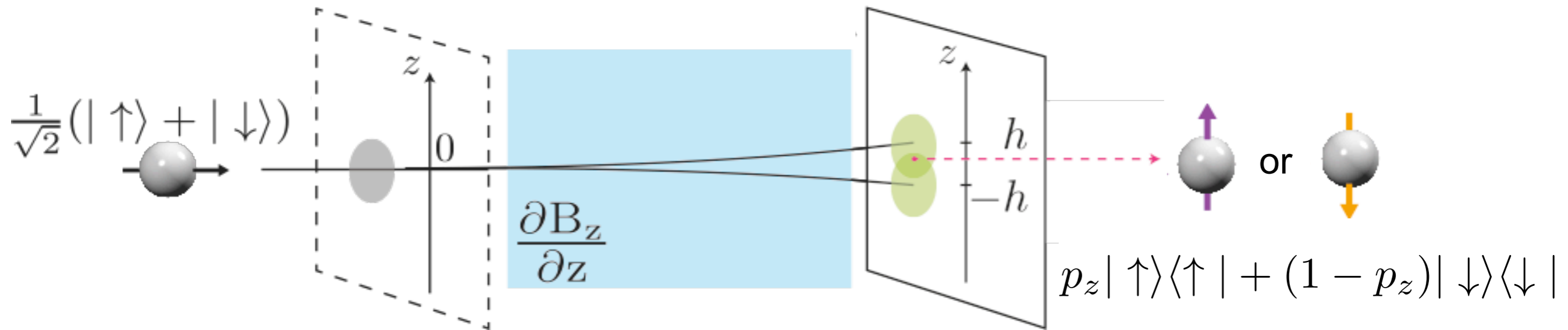
# Generalized measurement

if position fluctuations are mostly classical

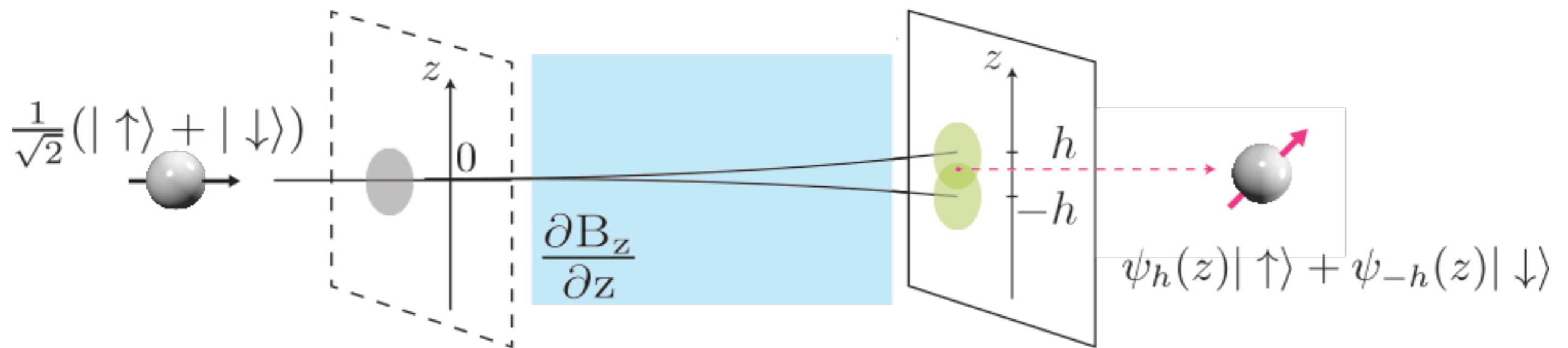


# Generalized measurement

if position fluctuations are mostly classical

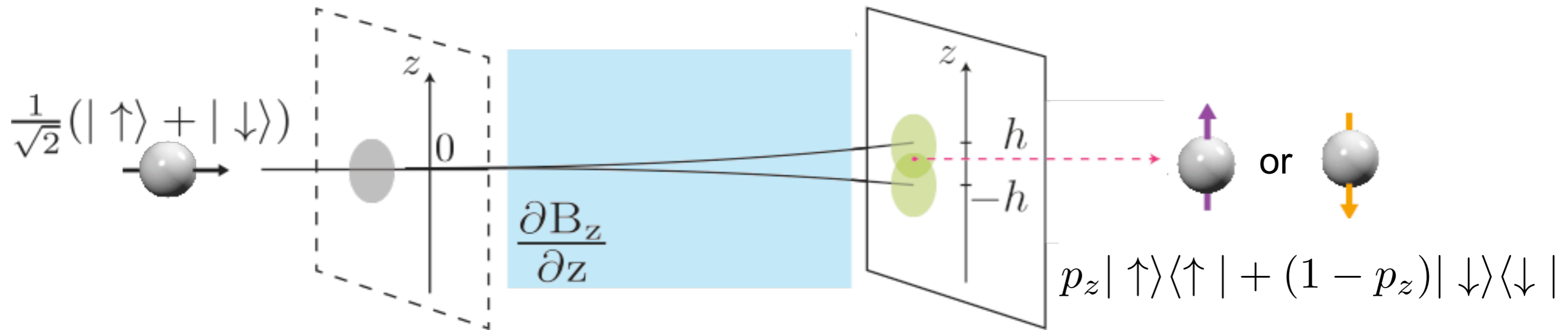


if zero point fluctuations only

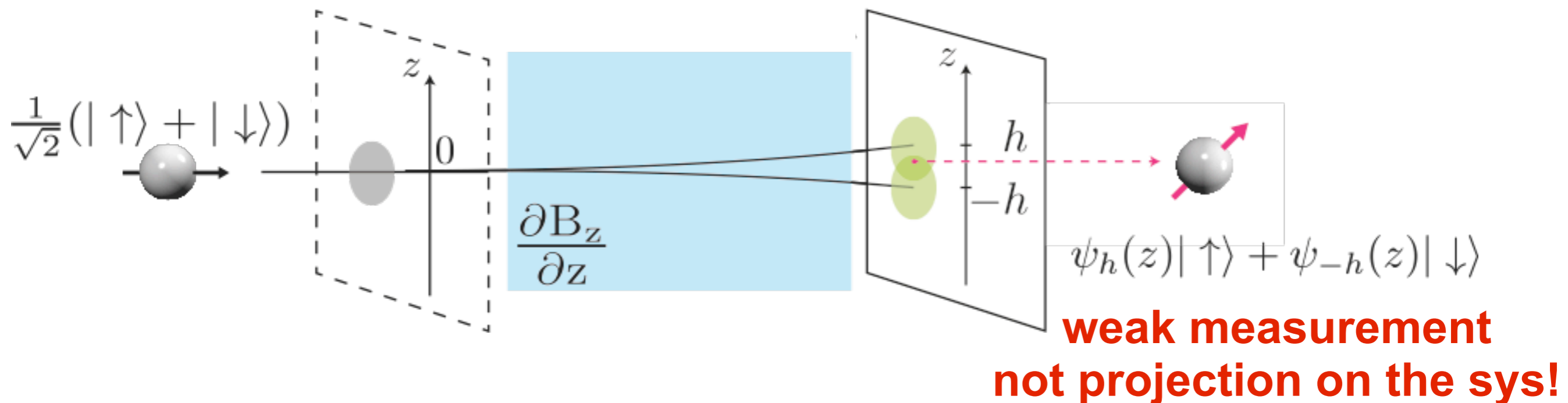


# Generalized measurement

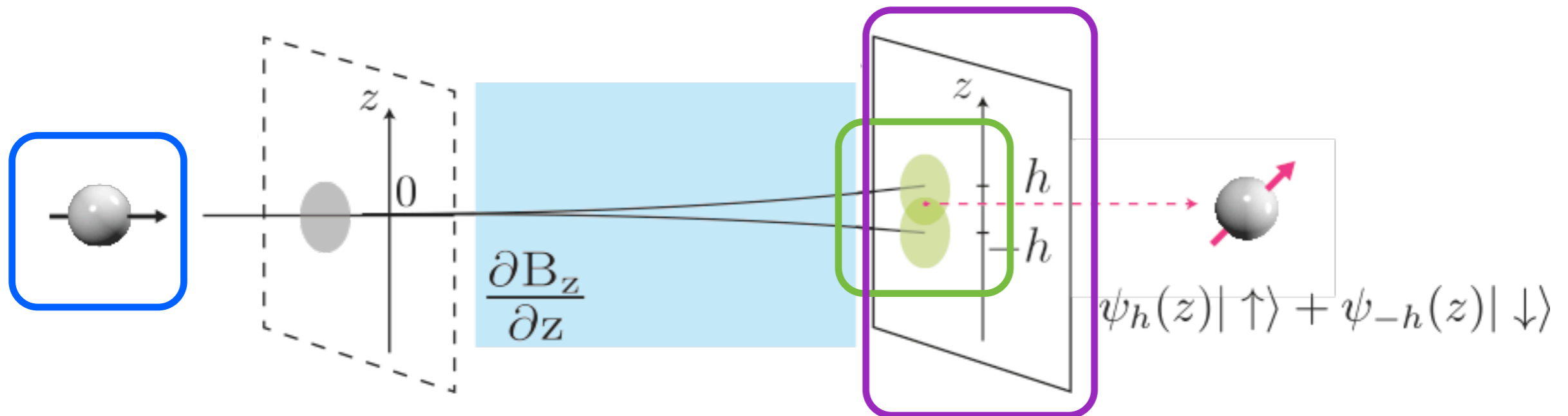
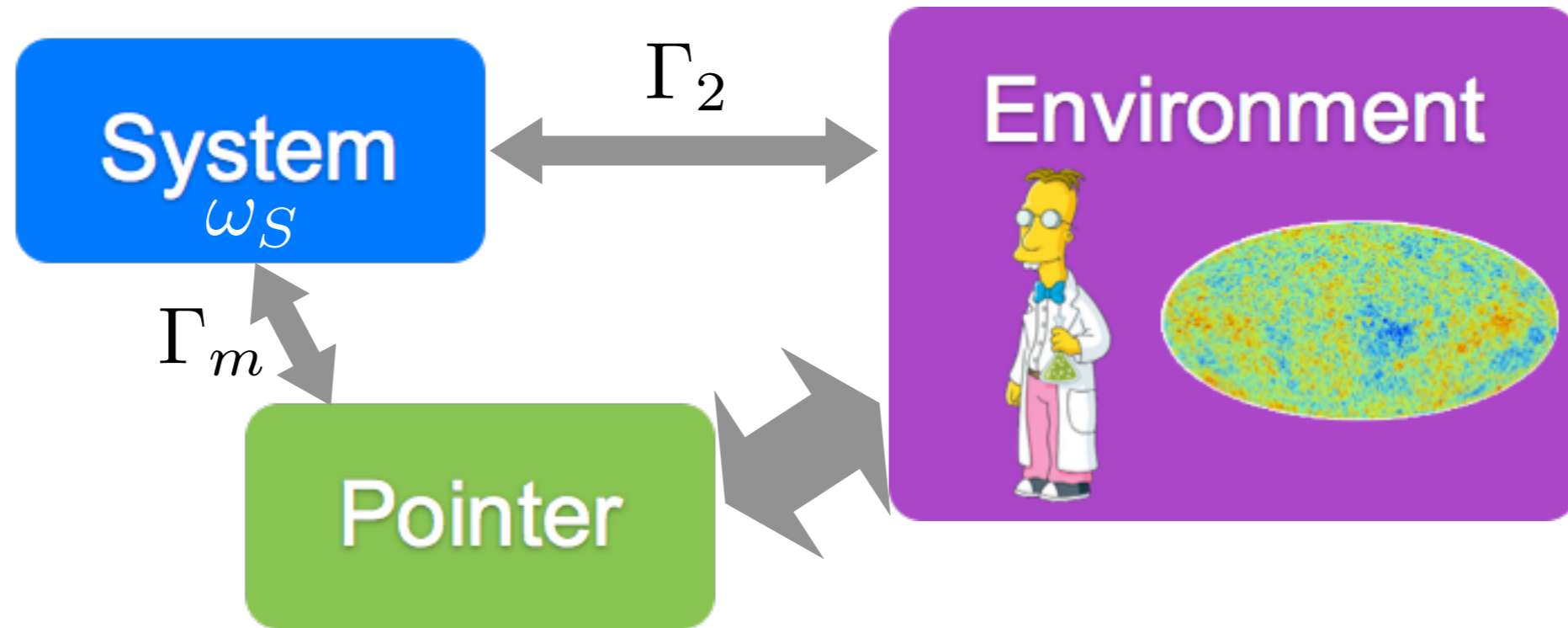
if position fluctuations are mostly classical



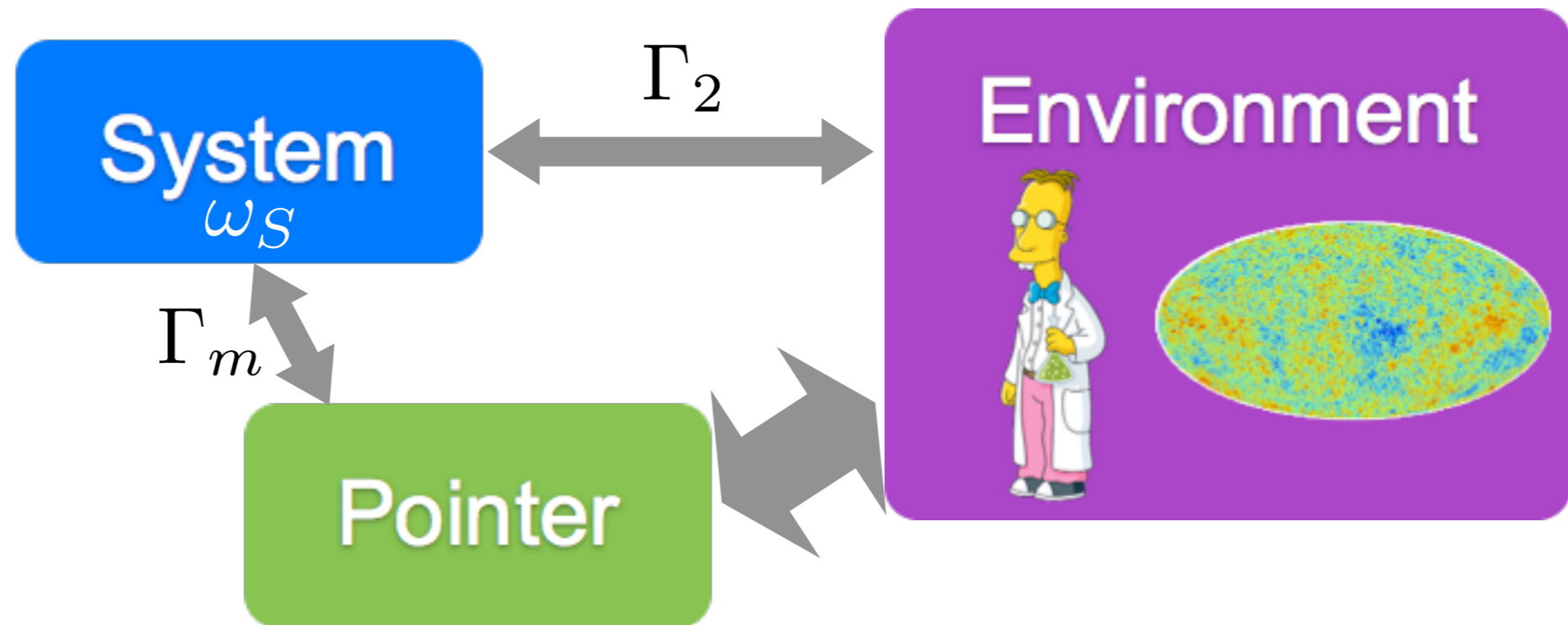
if zero point fluctuations only



# Generalized measurement

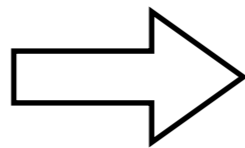


# Observing measurement backaction



traditional detectors

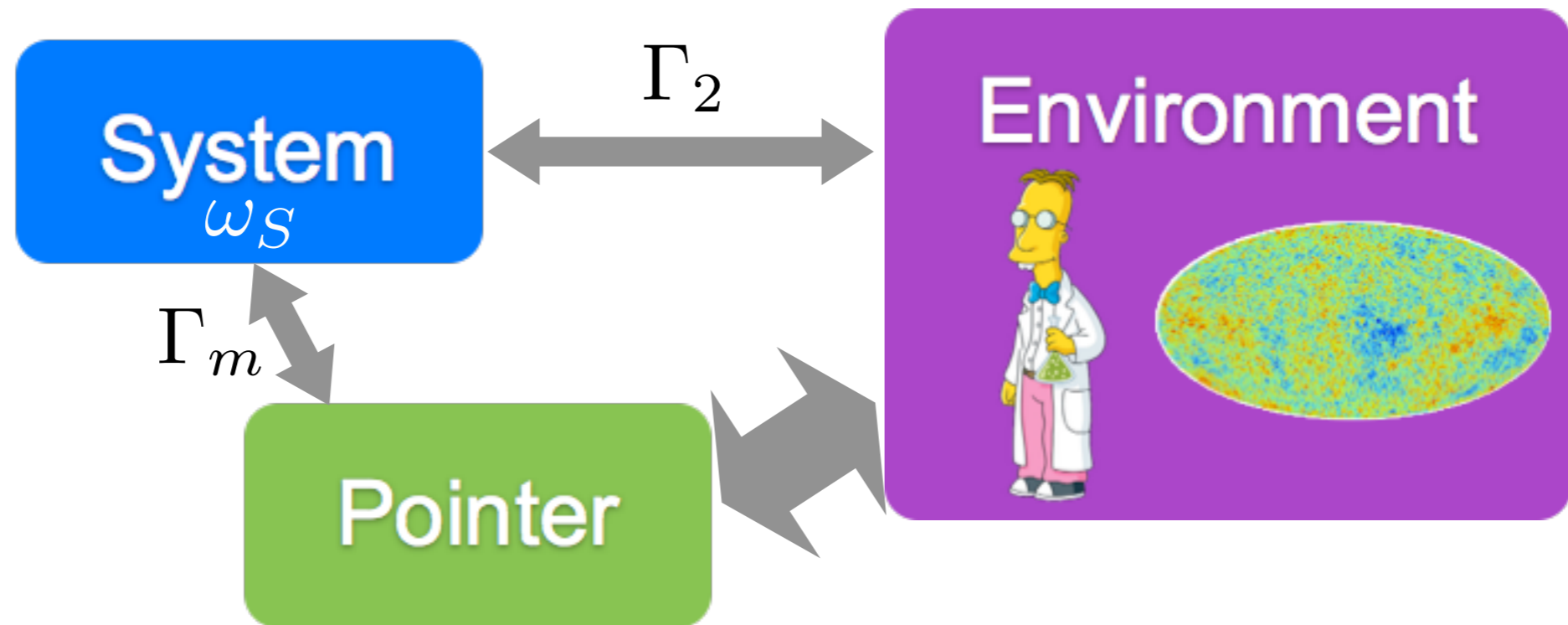
$$\Gamma_m \ll \Gamma_2 \ll \omega_S$$



average on large ensemble of  
systems (in time or space)

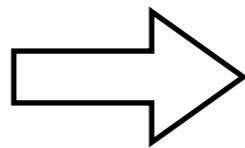
**no visible measurement backaction**

# Observing measurement backaction



traditional detectors

$$\Gamma_m \ll \Gamma_2 \ll \omega_S$$

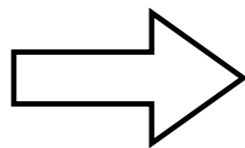


average on large ensemble of systems (in time or space)

**no visible measurement backaction**

quantum limited detectors

$$\Gamma_2 \lesssim \Gamma_m, \omega_S$$



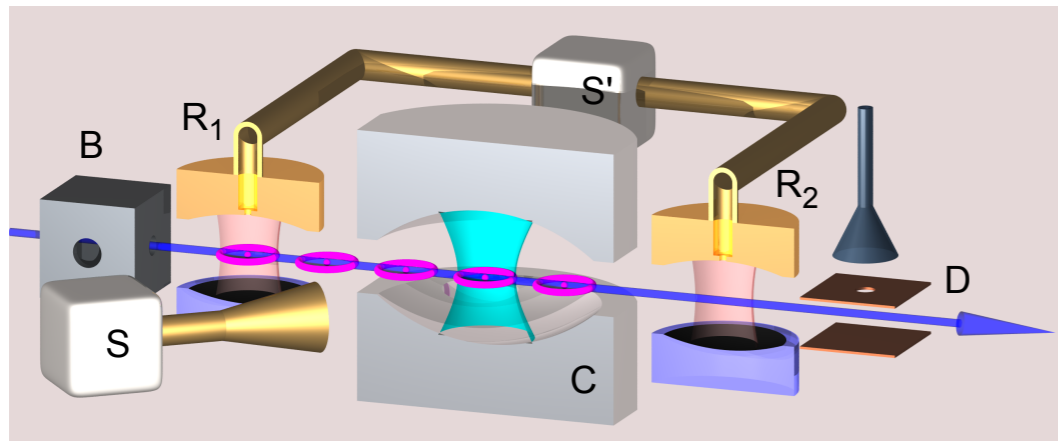
evolution of single realizations depends on outcomes

**visible measurement backaction**



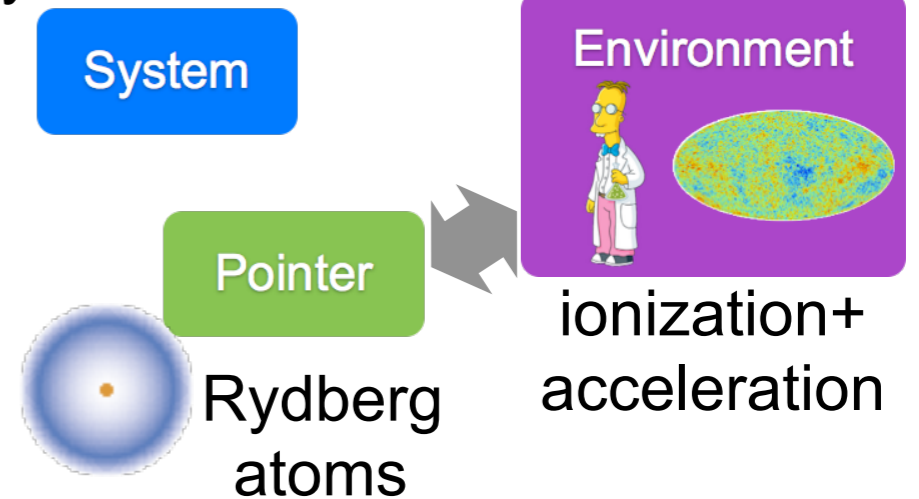
# Quantum trajectories already measured in...

## Rydberg atoms

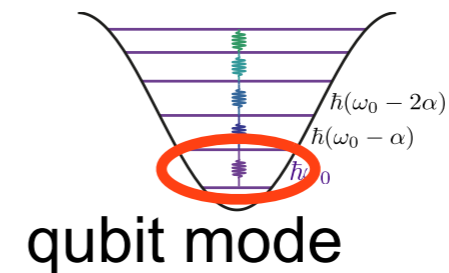
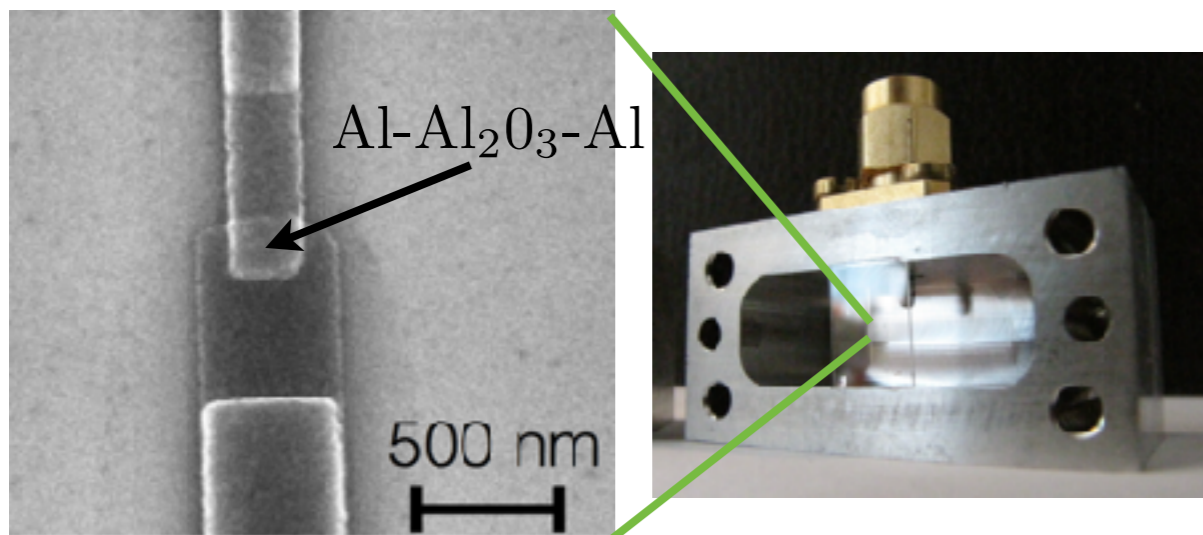


[pic from Haroche group, College de France Paris]

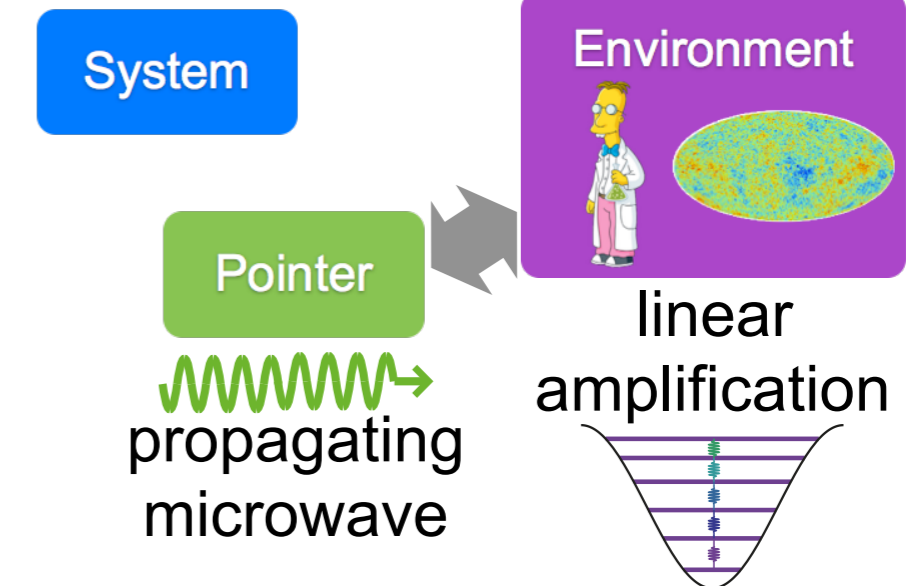
## Cavity mode



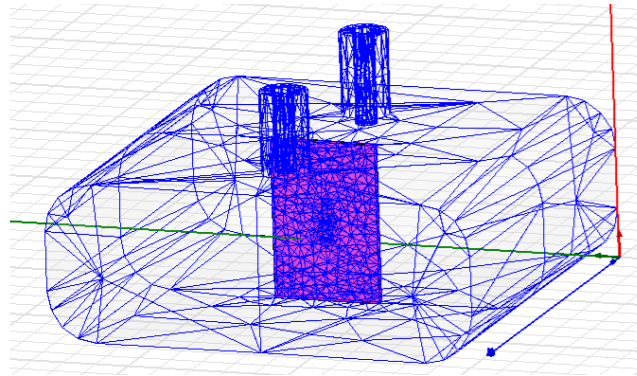
## Superconducting circuits



## qubit mode



# Quantum trajectories and feedback in circuit-QED



## Introduction to circuit-QED

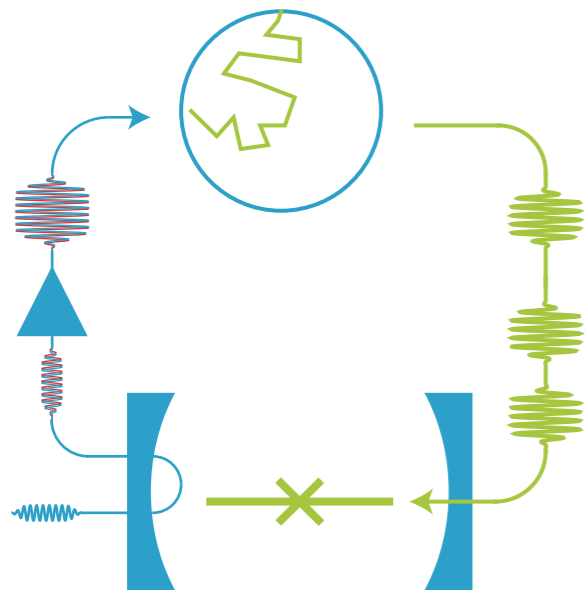
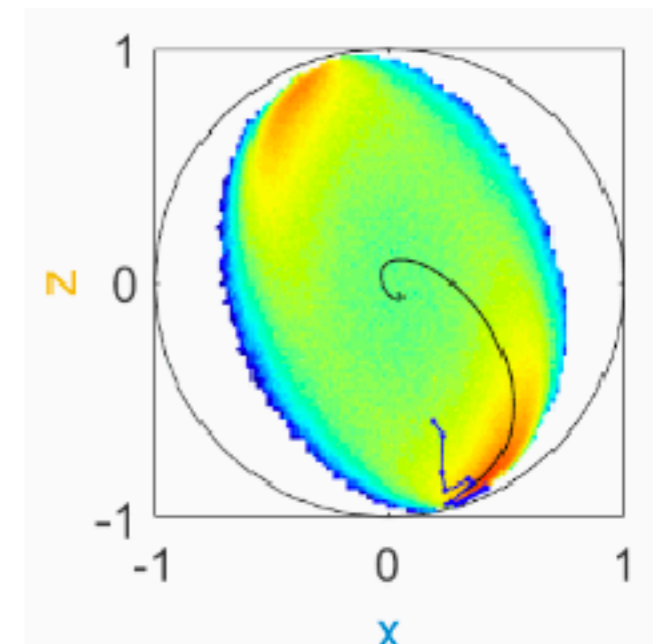
## Quantum trajectories in circuit-QED

dispersive measurement

fluorescence measurement

both simultaneously

generating entanglement



## Measurement based feedback

dispersive case

fluorescence case

# Quantum trajectories and feedback in circuit-QED

## Introduction to circuit-QED

### Quantum trajectories in circuit-QED

dispersive measurement

fluorescence measurement

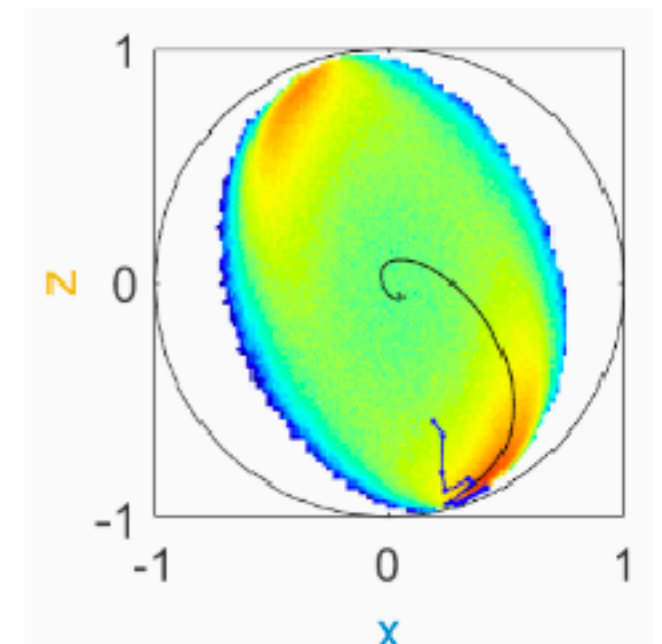
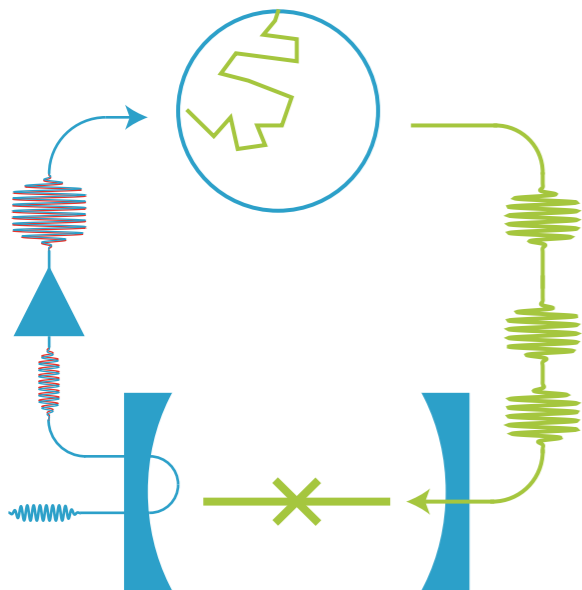
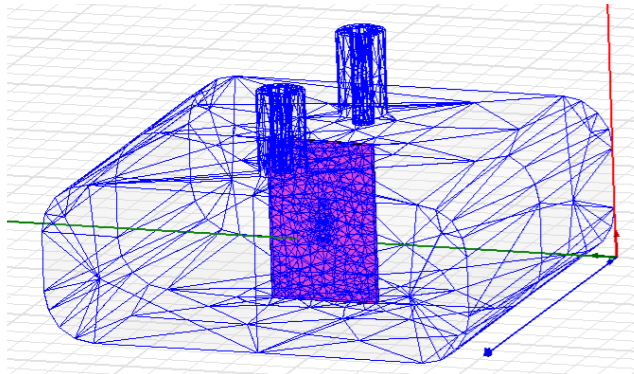
both simultaneously

generating entanglement

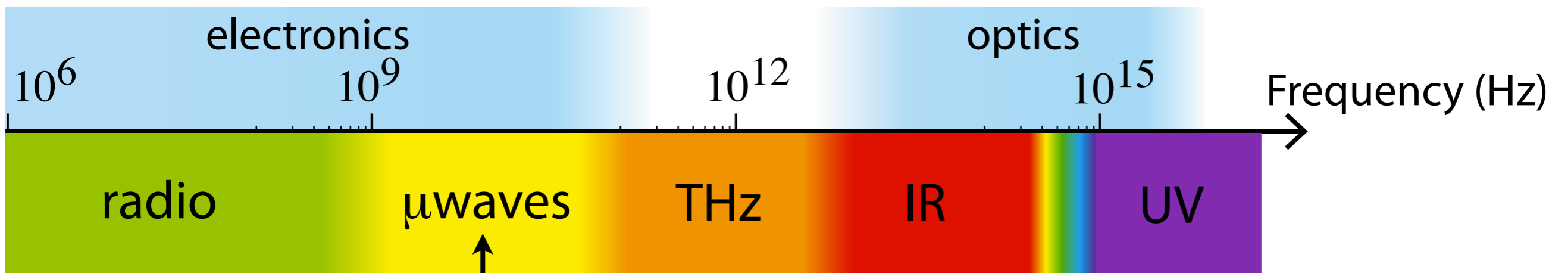
### Measurement based feedback

dispersive case

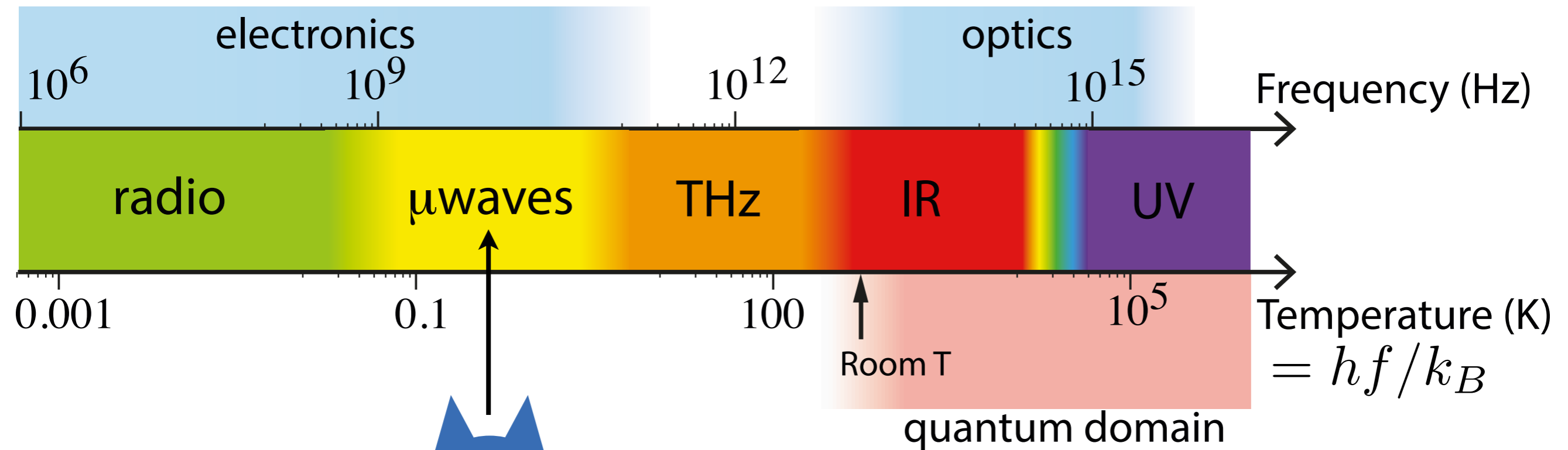
fluorescence case



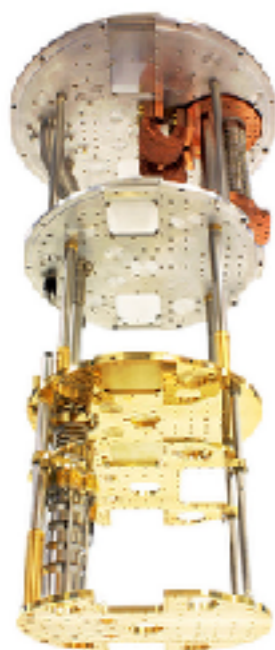
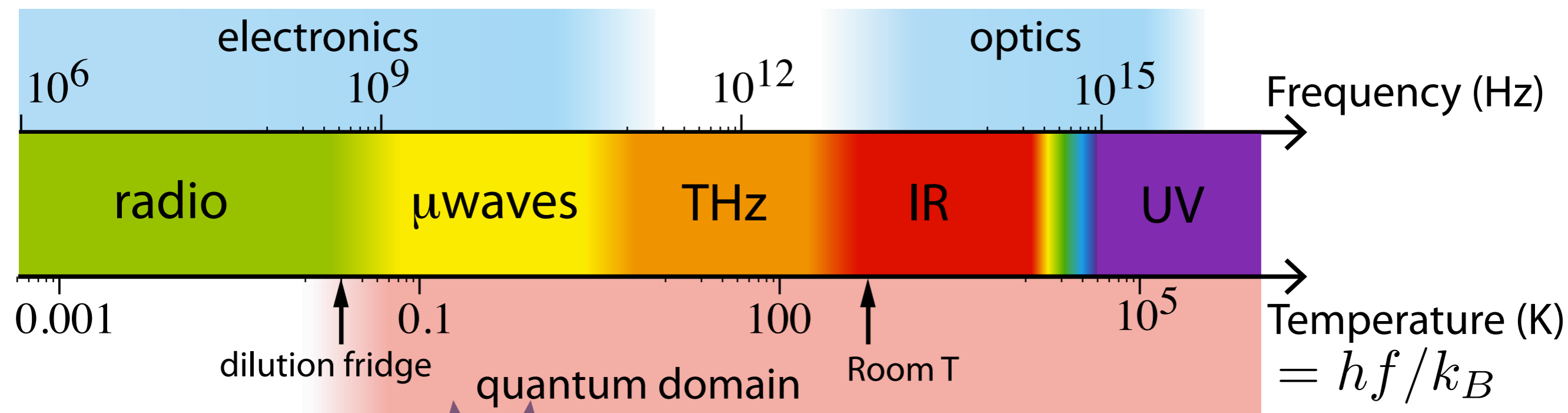
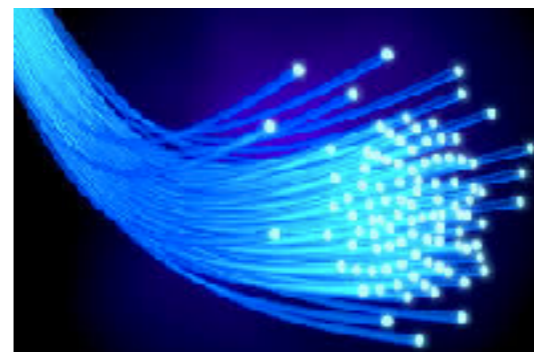
# Microwave quantum optics



# Microwave quantum optics



# Microwave quantum optics

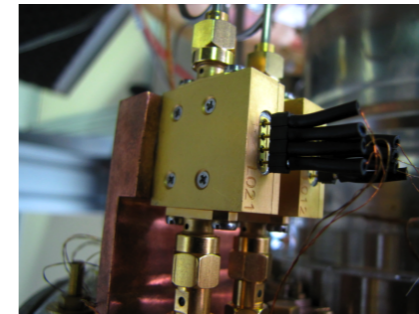




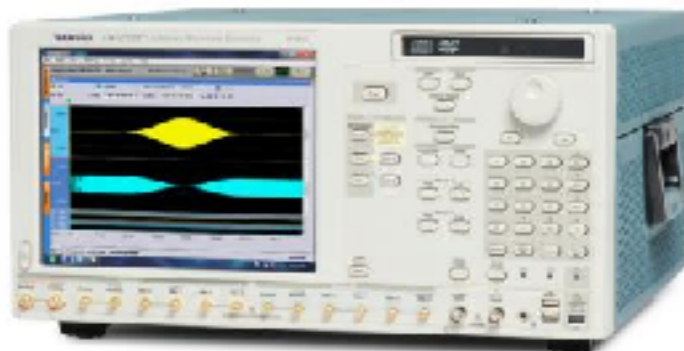
# Commercial toolbox for microwave optics



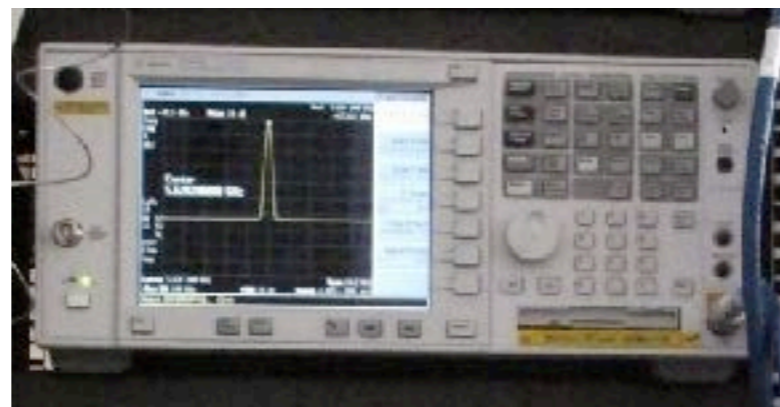
Microwave components



microwave sources

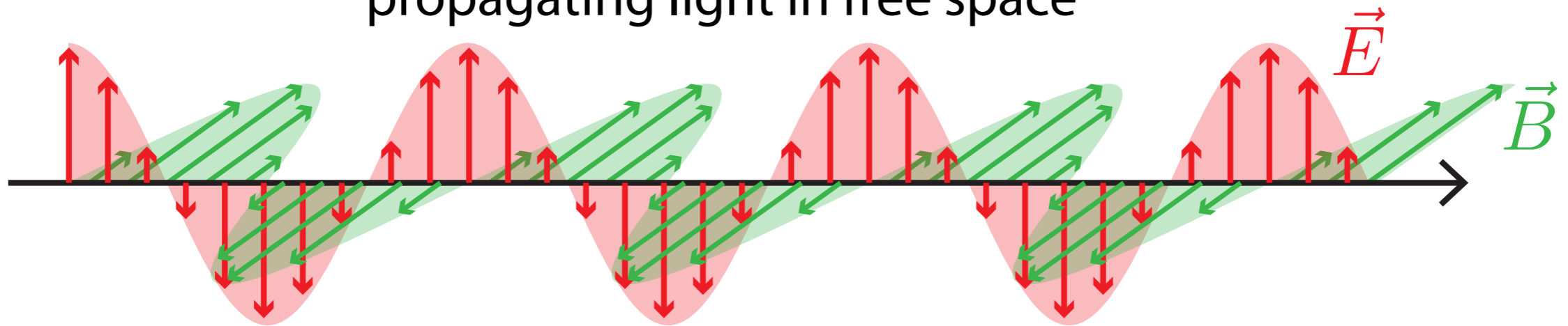


microwave detectors

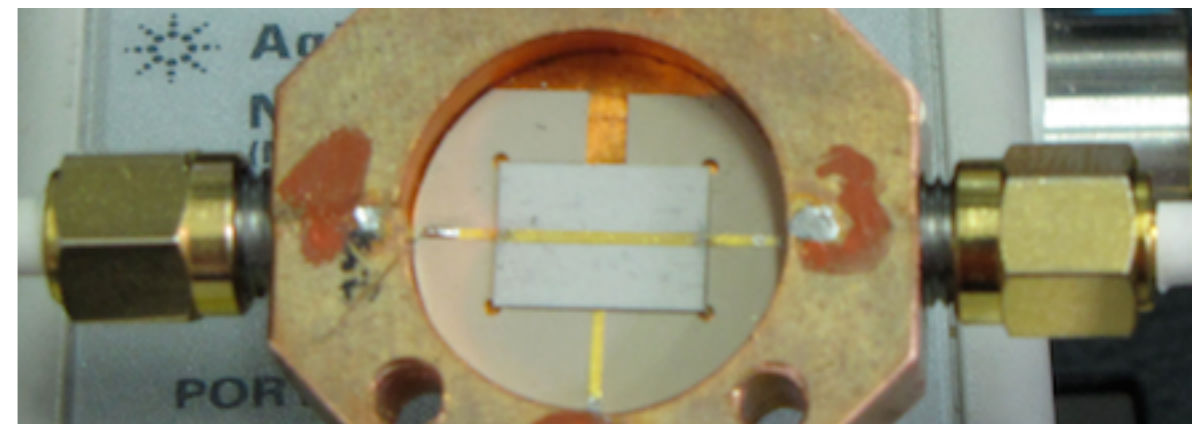
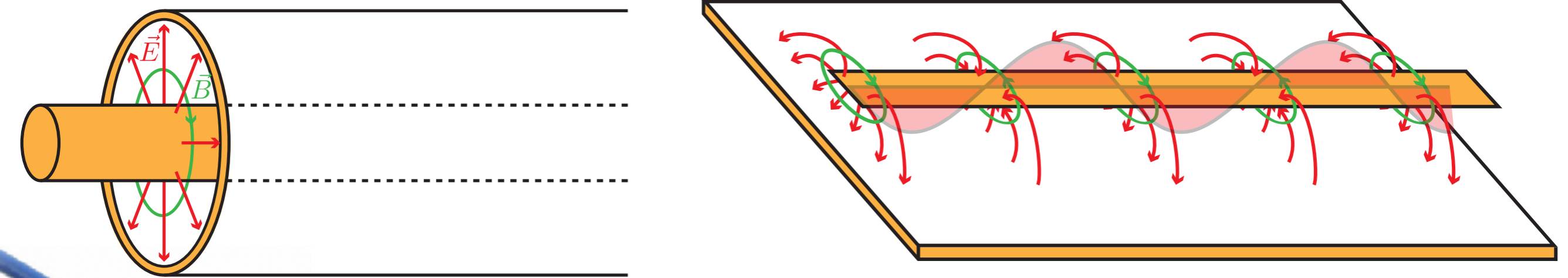


# Optics with circuits?

propagating light in free space



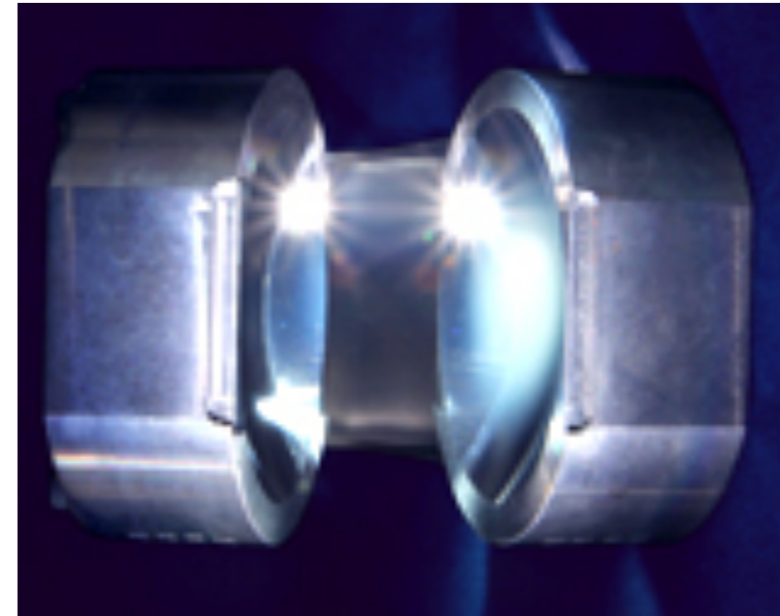
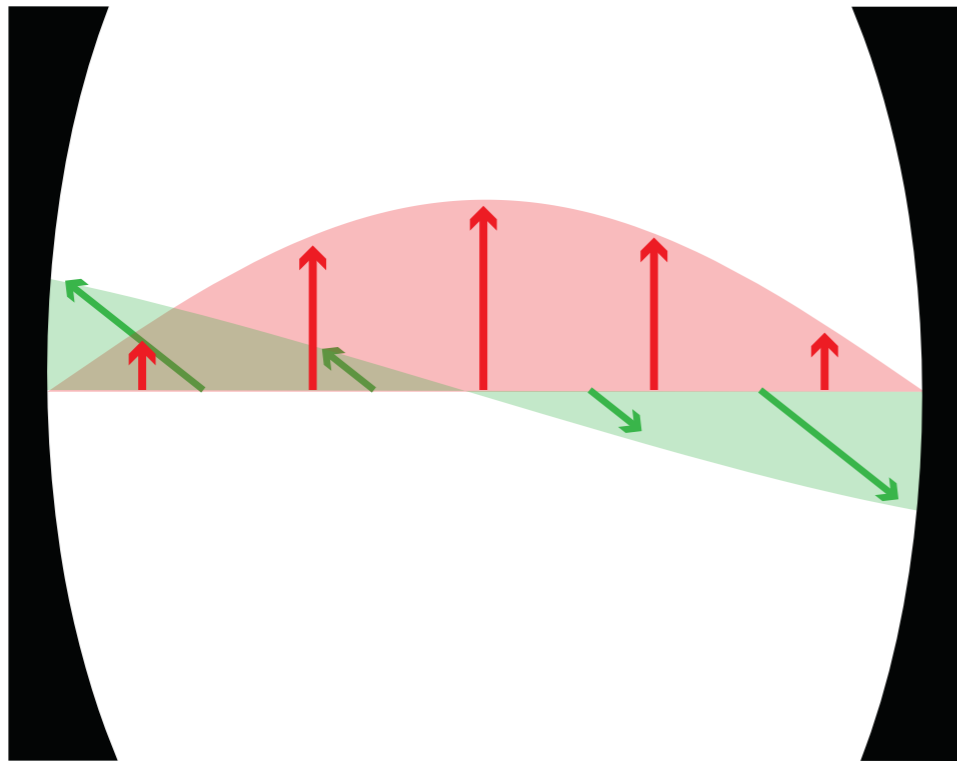
propagating light in waveguides





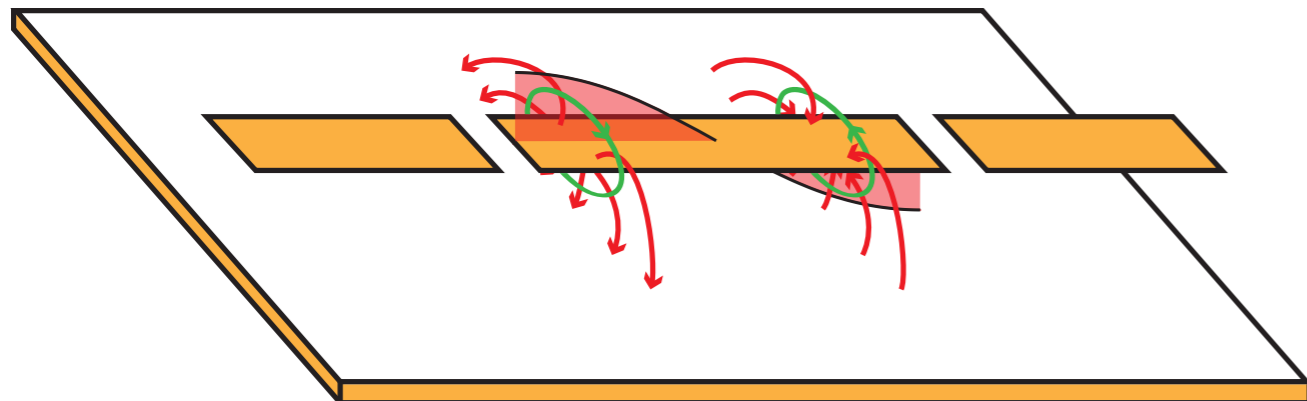
# Optics with circuits?

## Confined light mode between mirrors

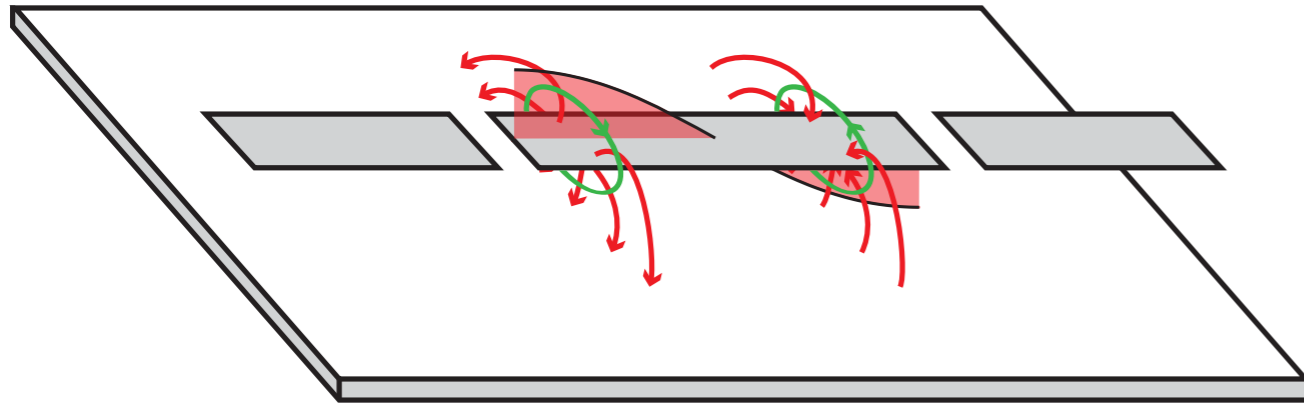


(CQED, LKB, Paris)

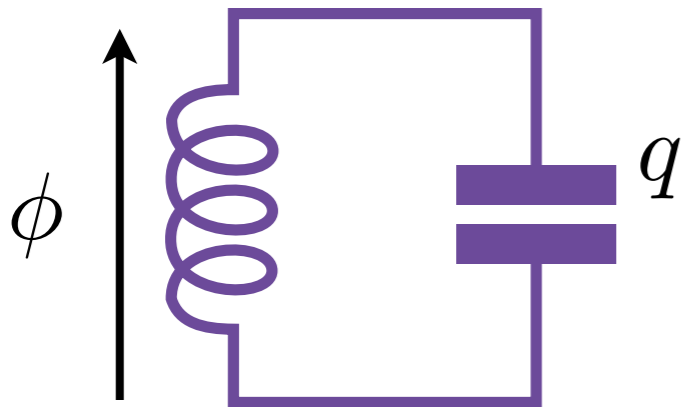
## Confined light mode in waveguide



# Superconducting circuits



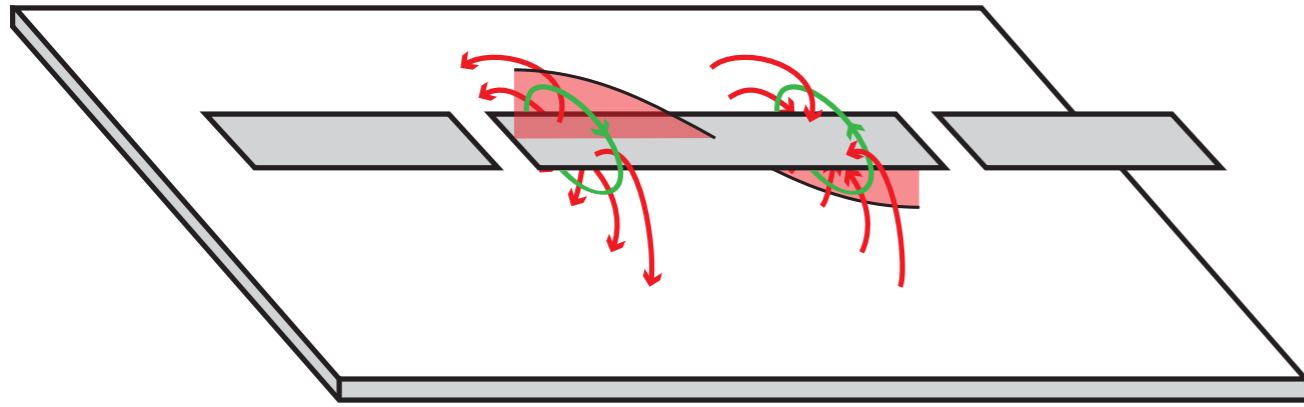
dissipationless LC circuit



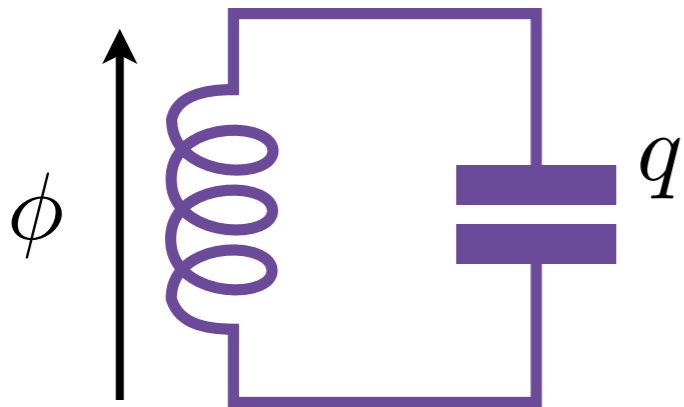
$$\hat{H} = \frac{\hat{q}^2}{2C} + \frac{\hat{\phi}^2}{2L}$$

$$[\hat{\phi}, \hat{q}] = i\hbar$$

# Superconducting circuits



dissipationless LC circuit



$$\hat{H} = \frac{\hat{q}^2}{2C} + \frac{\hat{\phi}^2}{2L}$$

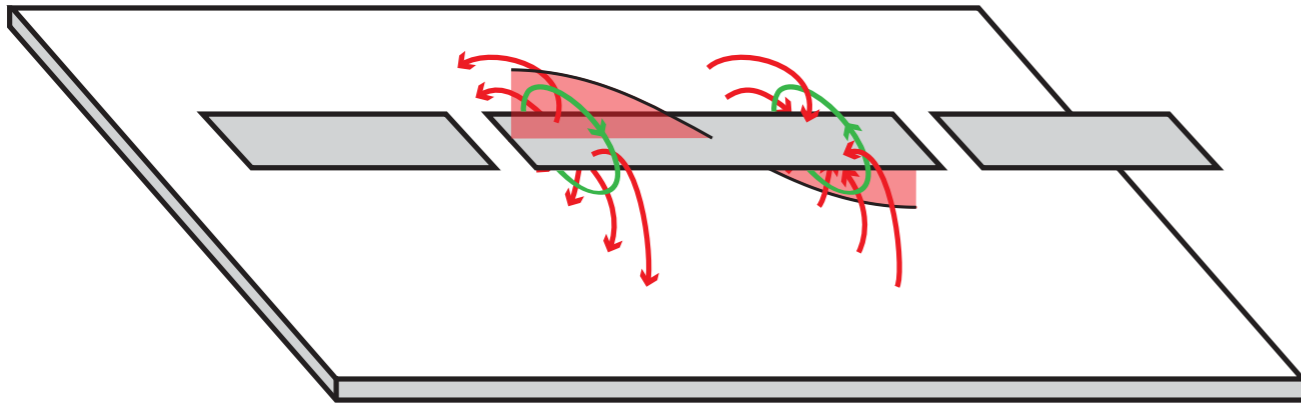
$$[\hat{\phi}, \hat{q}] = i\hbar$$



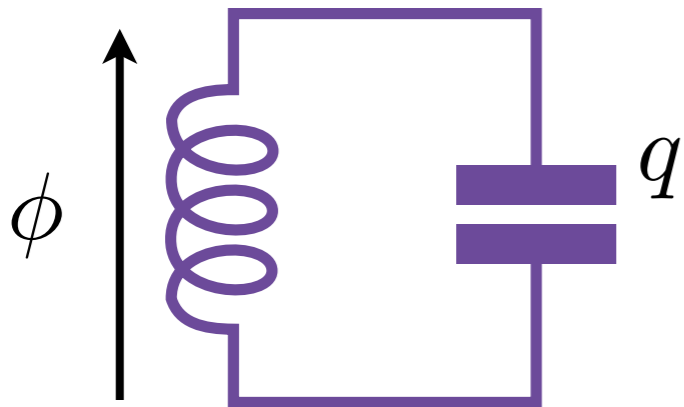
$$\hat{H} = \frac{k\hat{X}^2}{2} + \frac{\hat{P}^2}{2m}$$

$$[\hat{X}, \hat{P}] = i\hbar$$

# Superconducting circuits



dissipationless LC circuit



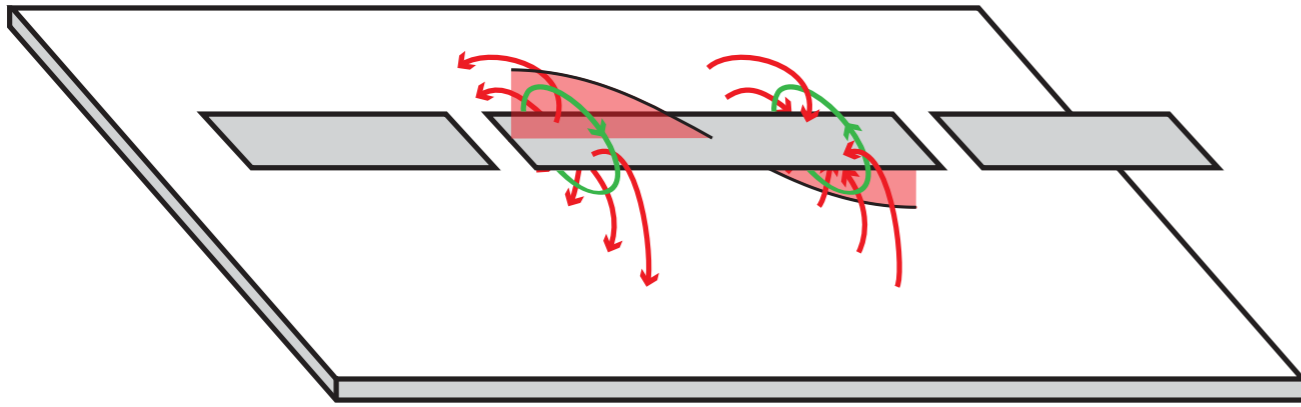
$$\hat{H} = \frac{\hat{q}^2}{2C} + \frac{\hat{\phi}^2}{2L}$$

$$[\hat{\phi}, \hat{q}] = i\hbar$$

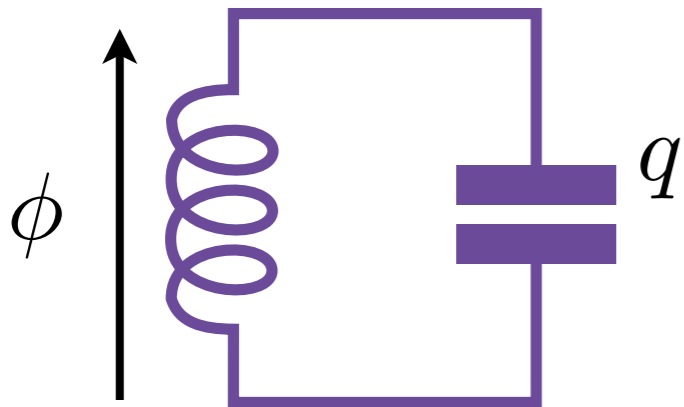
Heisenberg inequality

$$\delta\phi\delta q \geq \frac{\hbar}{2}$$

# Superconducting circuits



dissipationless LC circuit



$$\hat{H} = \frac{\hat{q}^2}{2C} + \frac{\hat{\phi}^2}{2L}$$

$$[\hat{\phi}, \hat{q}] = i\hbar$$

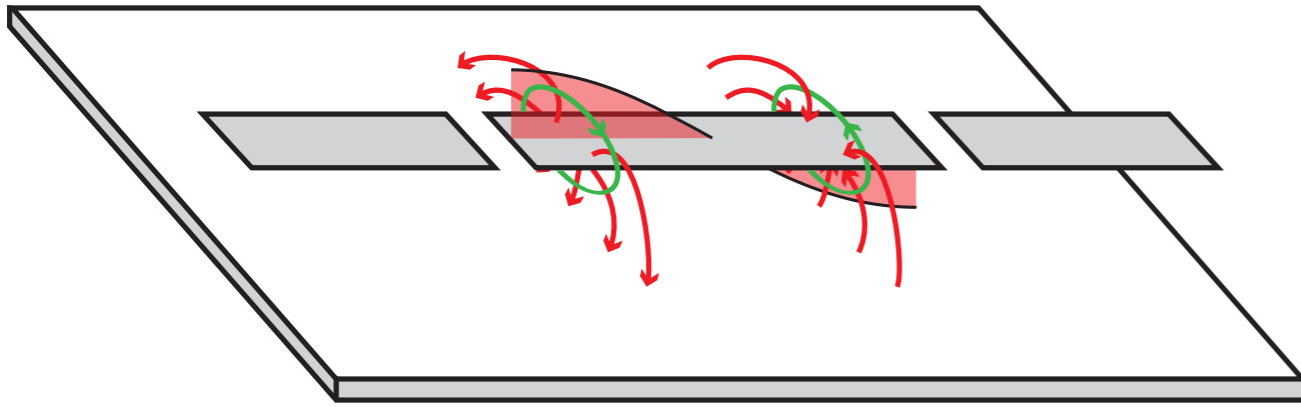
$$\omega_0 = \frac{1}{\sqrt{LC}} \quad Z_0 = \sqrt{\frac{L}{C}}$$

$$\phi_{ZPF} = \sqrt{\frac{\hbar Z_0}{2}} \quad q_{ZPF} = \sqrt{\frac{\hbar}{2Z_0}}$$

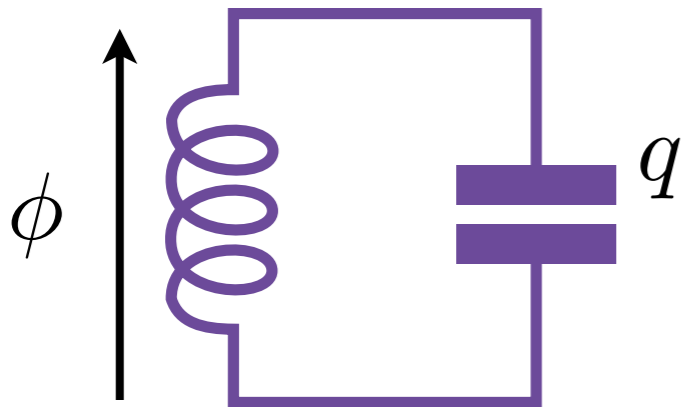
Heisenberg inequality

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# Superconducting circuits



dissipationless LC circuit



$$\hat{H} = \frac{\hat{q}^2}{2C} + \frac{\hat{\phi}^2}{2L}$$

$$[\hat{\phi}, \hat{q}] = i\hbar$$

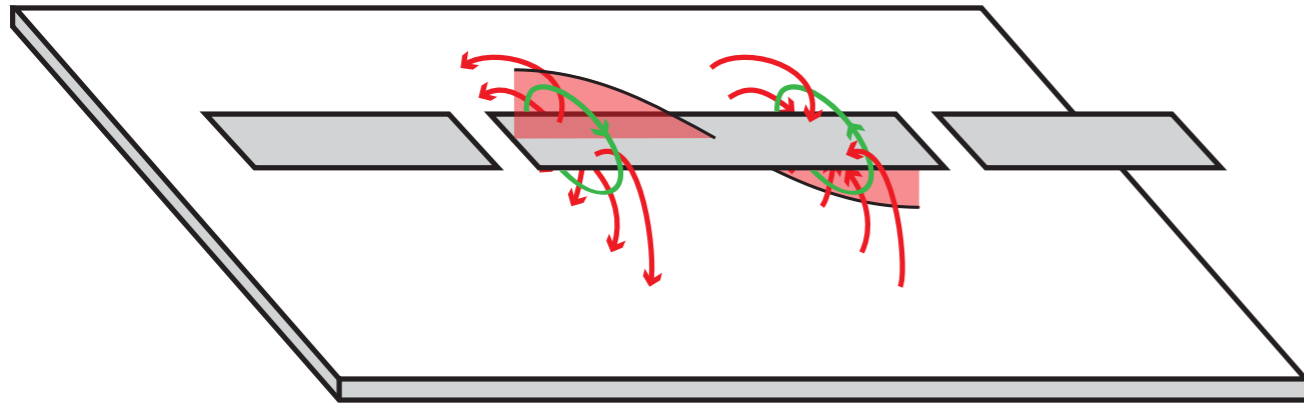
$$\omega_0 = \frac{1}{\sqrt{LC}} \quad Z_0 = \sqrt{\frac{L}{C}}$$

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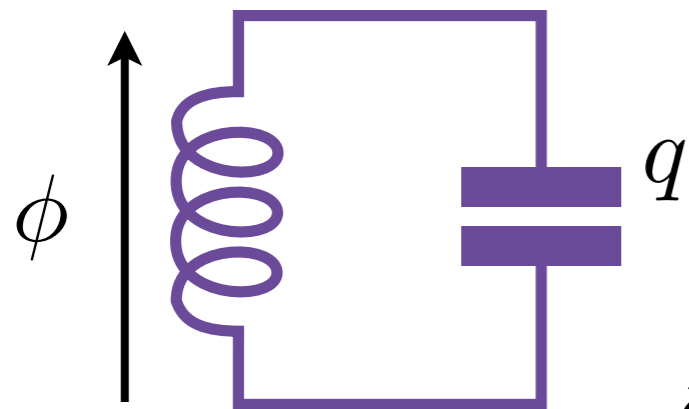
$$\hat{a} = \frac{\hat{\phi}/\phi_{ZPF} + i\hat{q}/q_{ZPF}}{2}$$

$$[\hat{a}, \hat{a}^\dagger] = 1$$

# Superconducting circuits



dissipationless LC circuit...



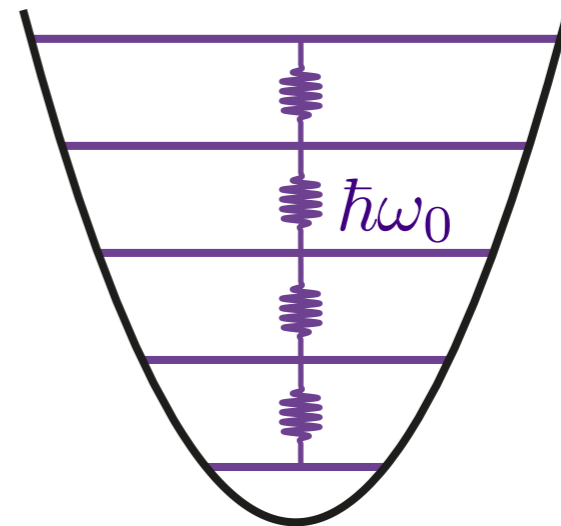
$$\hat{a} = \frac{\hat{\phi}/\phi_{ZPF} + i\hat{q}/q_{ZPF}}{2}$$

$$\hat{H} = \frac{\hat{q}^2}{2C} + \frac{\hat{\phi}^2}{2L}$$

$$[\hat{\phi}, \hat{q}] = i\hbar$$



...canonically quantized

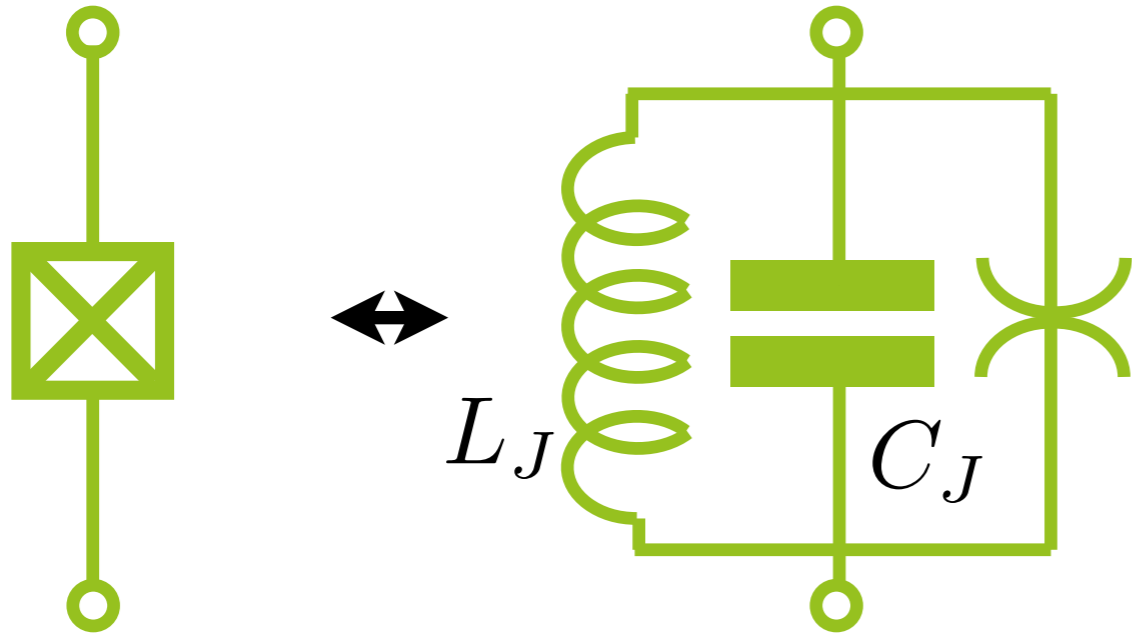


$$\hat{H} = \hbar\omega_0 \left( \hat{a}^\dagger \hat{a} + \frac{1}{2} \right)$$

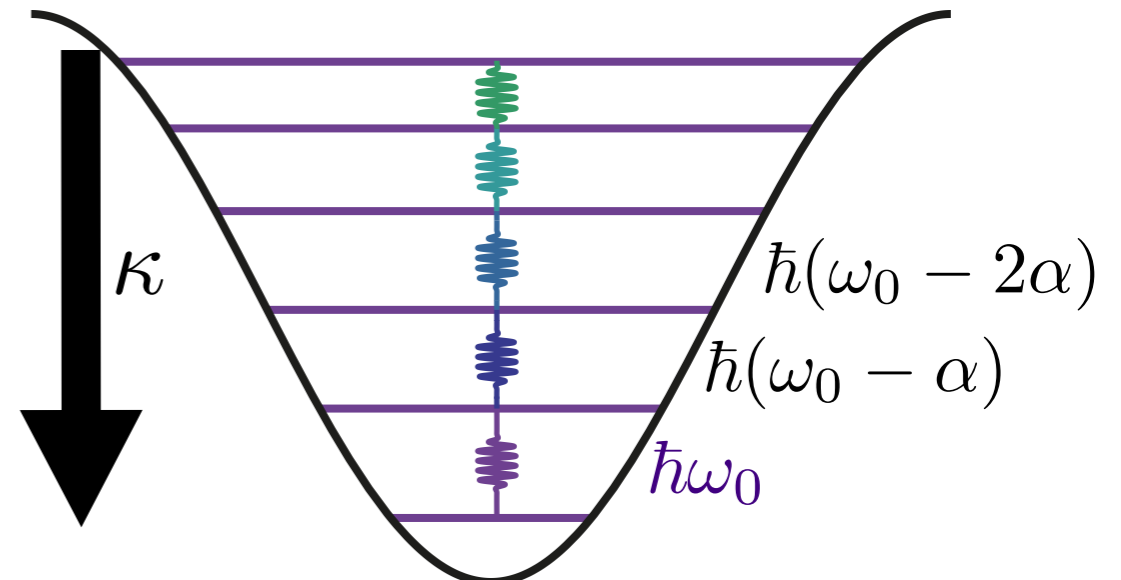
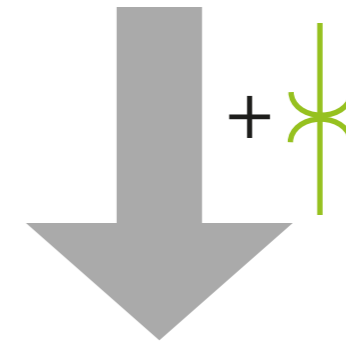
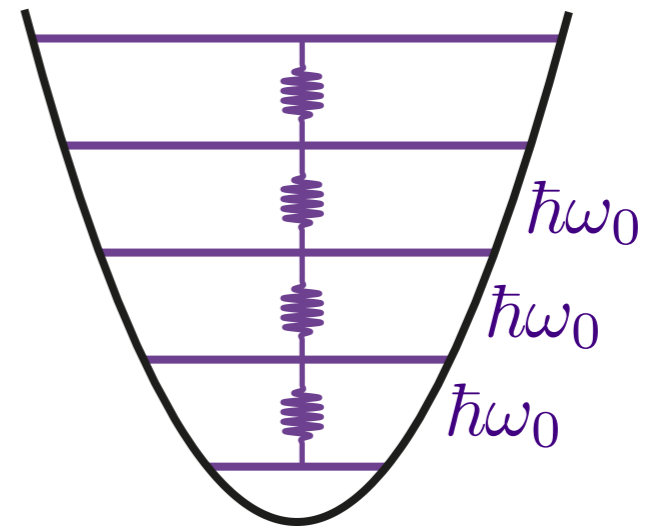


# Superconducting circuits

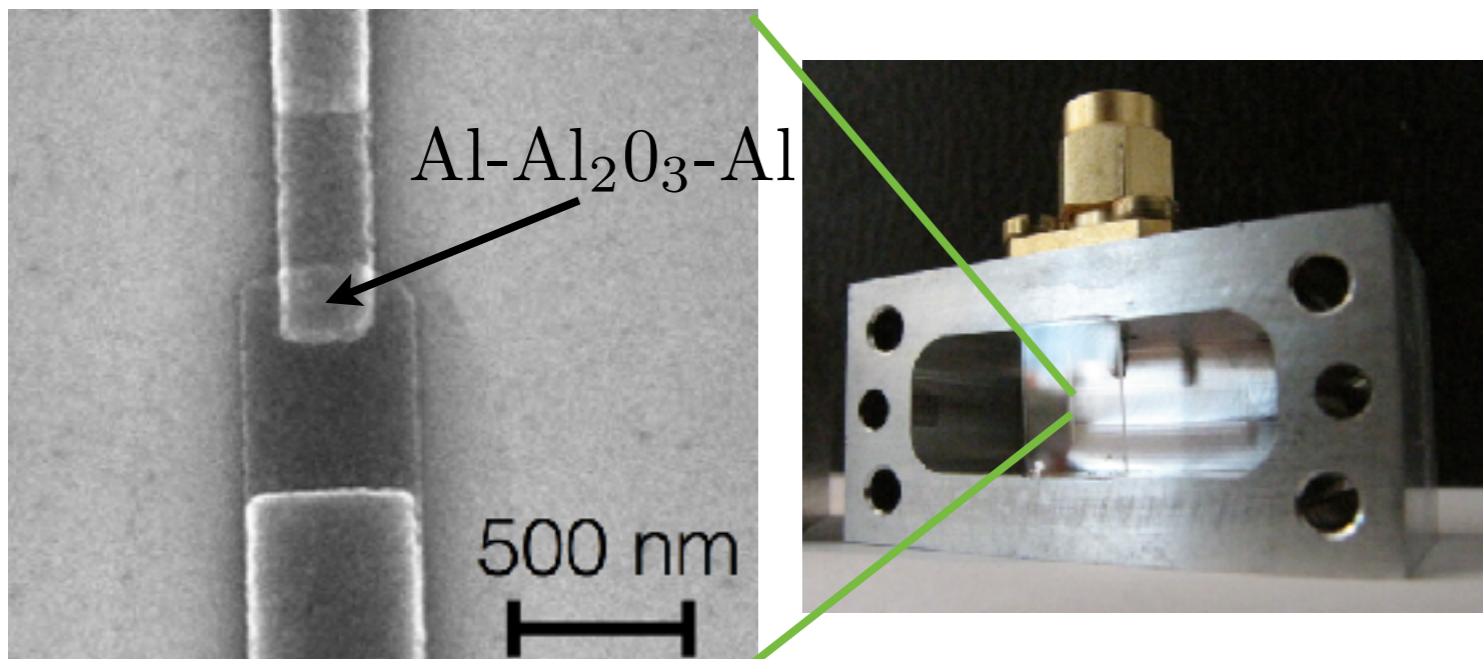
dissipation-less **non linear** LC circuit



$$\hat{H} = \frac{\hat{q}^2}{2C_J} - E_J \cos \frac{\hat{\phi}}{\hbar/2e} = \frac{\hat{q}^2}{2C_J} + \frac{\hat{\phi}^2}{2L_J} + H_{\text{non-lin}}(\hat{\phi})$$

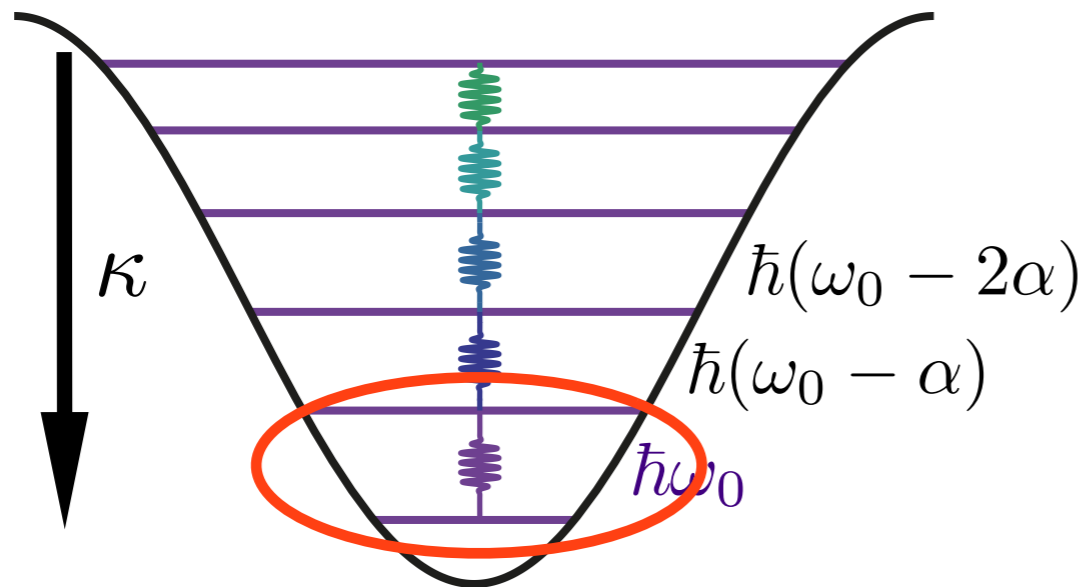


transitions observed in 1980's [Berkeley & Saclay]  
strong coupling regime of CQED in 2004 [Yale]





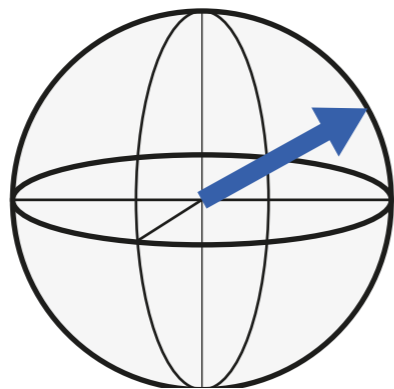
# Non-linear superconducting circuits



Strongly anharmonic

$$\alpha \gg \kappa$$

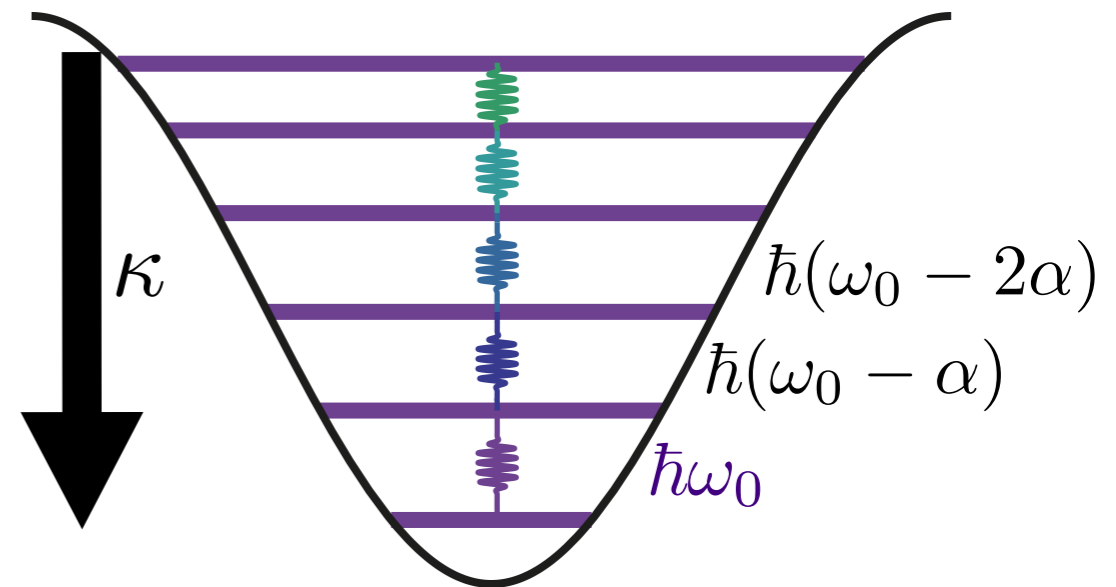
qubit  $\hbar\omega\hat{\sigma}_z/2$



First Rabi oscillations in 1999 [NEC group]

Quantrium in 2002 [Quantronics group]

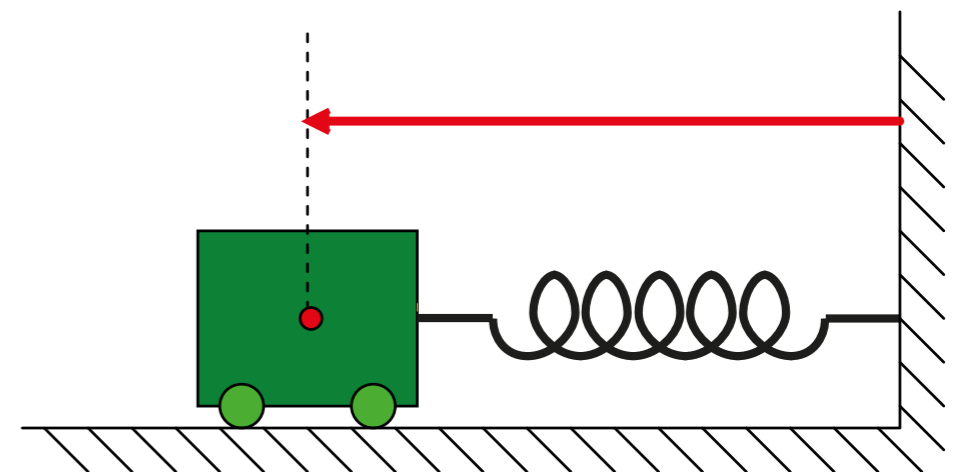
Charge qubit, phase qubit [Grenoble & others], flux qubit, transmon [Yale, ETH & others], fluxonium, Xmon...



Weakly anharmonic

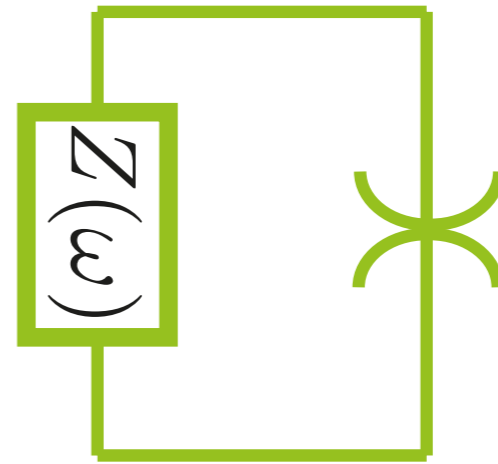
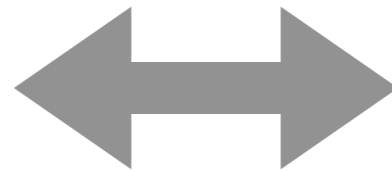
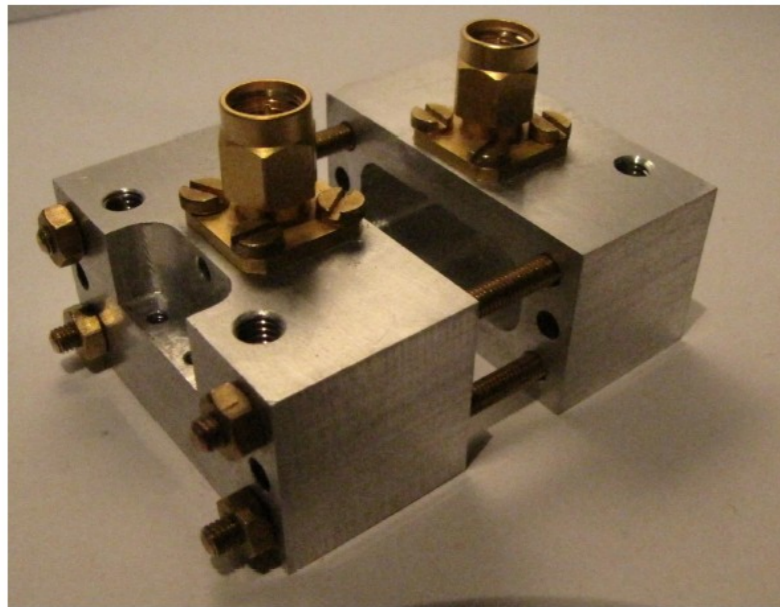
$$\alpha \ll \kappa$$

oscillator  $\hbar\omega\hat{a}^\dagger\hat{a}$



Parametric amplifiers & squeezing  
in 1980's [Bell Labs]

# Black Box Quantization



BBQ Recipe [Nigg et al., PRL 2012 (Yale)]

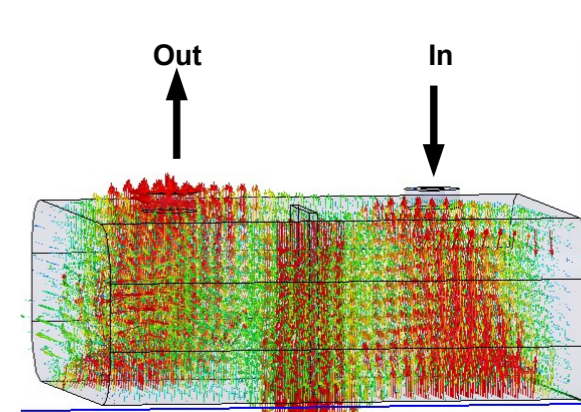
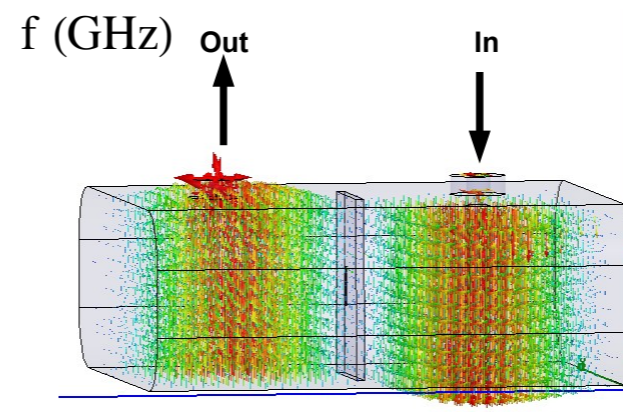
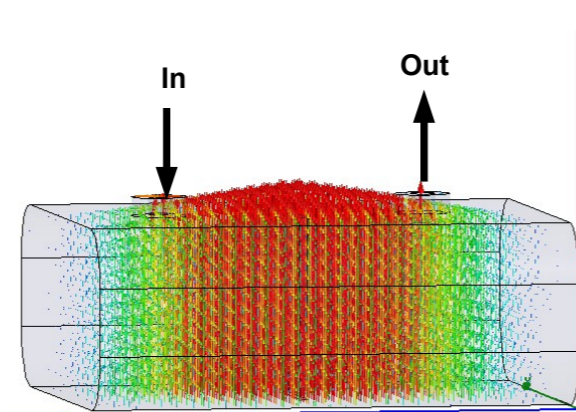
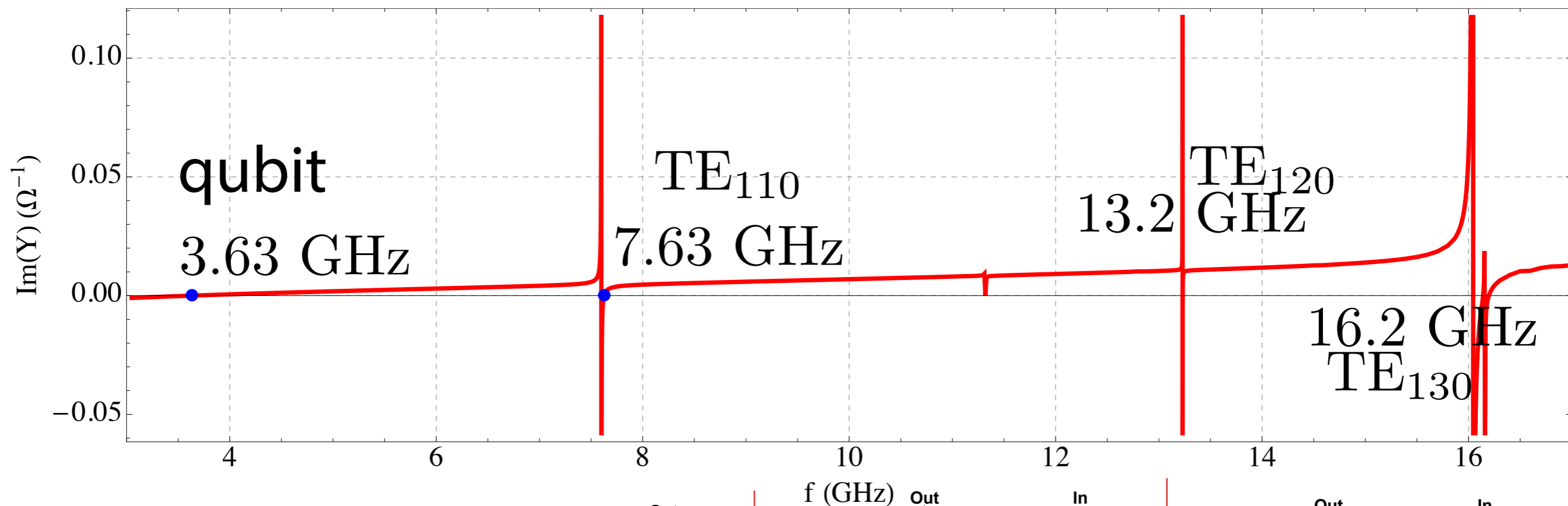
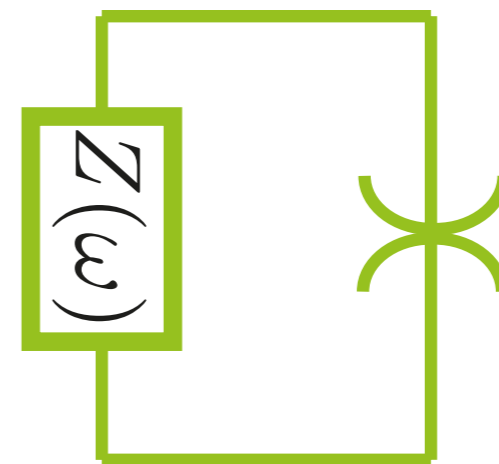
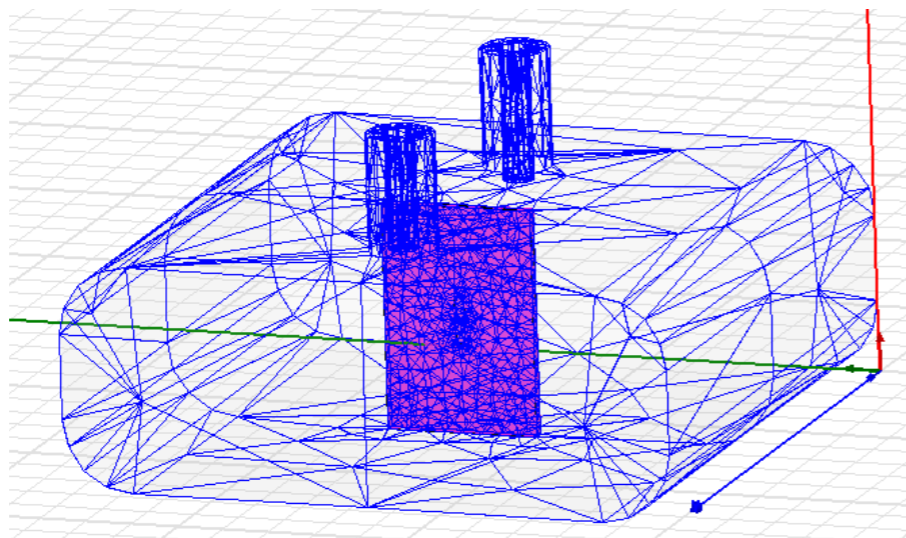
$$\hat{H} = \sum_k \hbar\omega_k + \frac{1}{2} \sum_{k,l} \hbar\chi_{kl} \hat{n}_k \hat{n}_l$$

self-Kerr  $\chi_{kk} = -\frac{e^2}{2L_J} Z_k^2$

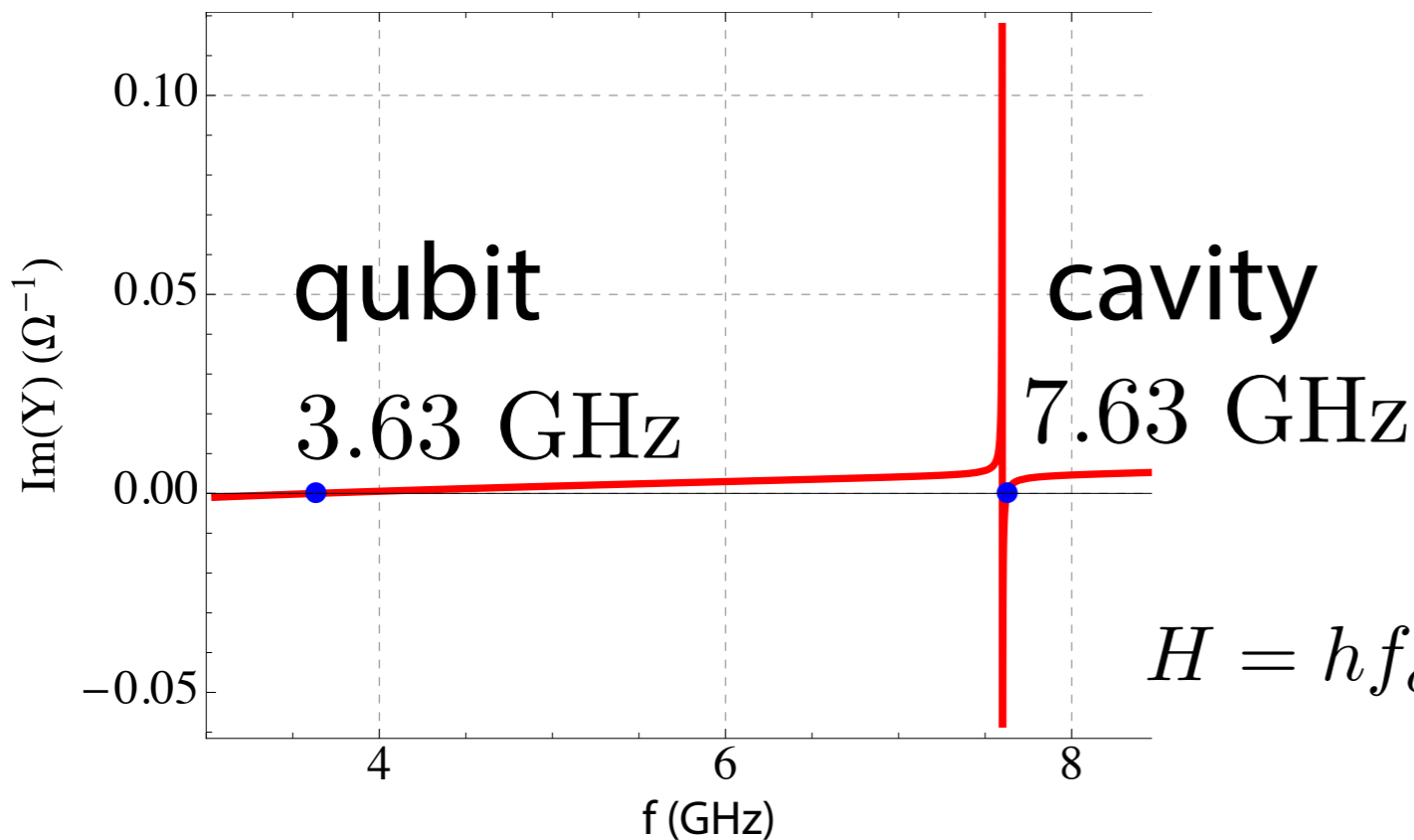
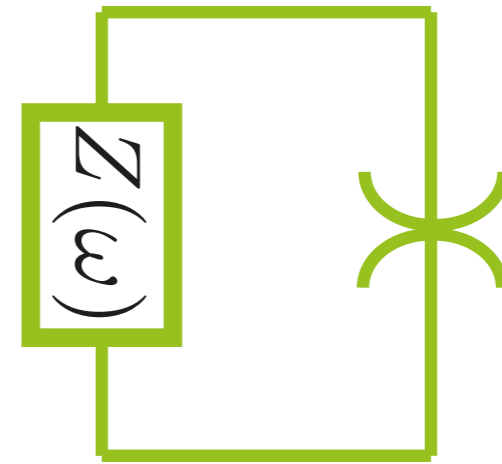
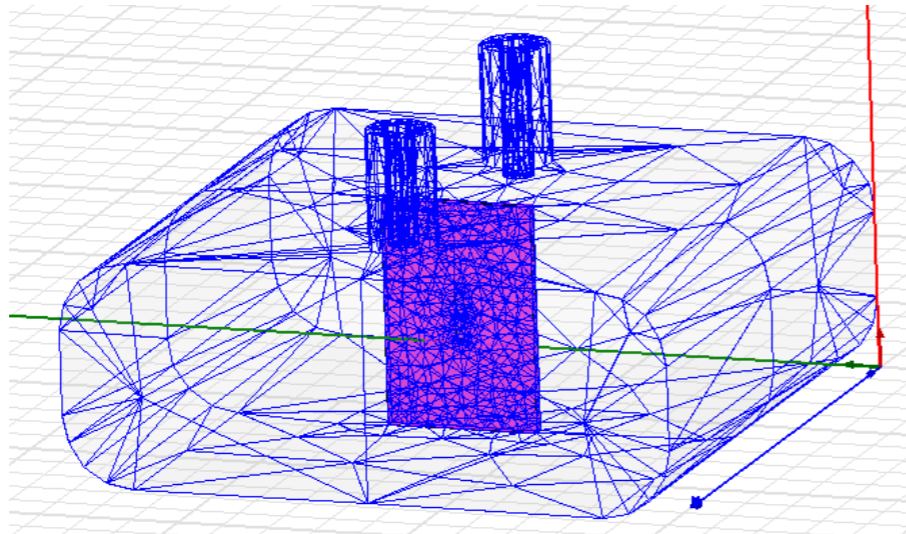
cross-Kerr  $\chi_{kl} = -2\sqrt{\chi_{kk}\chi_{ll}}$

$$Z_k = \frac{2}{\omega_k \text{Im} Y'(\omega_k)}$$

# Black Box Quantization



# Black Box Quantization



$$\chi_{cq}^{(thy)} / 2\pi = -1.4 \text{ MHz}$$

$$\chi_{qq}^{(thy)} / 4\pi = -150 \text{ MHz}$$

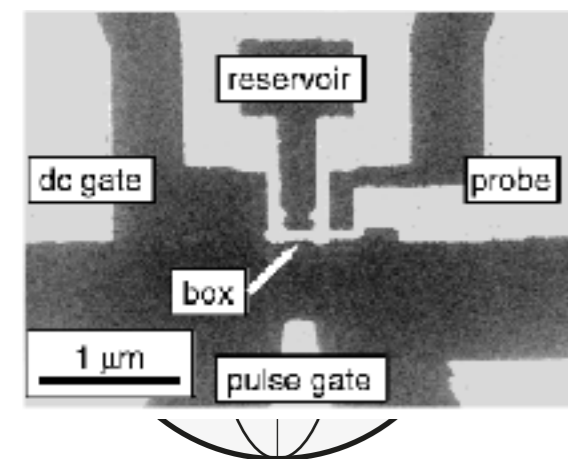
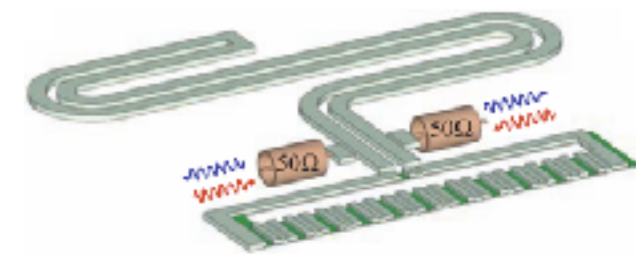
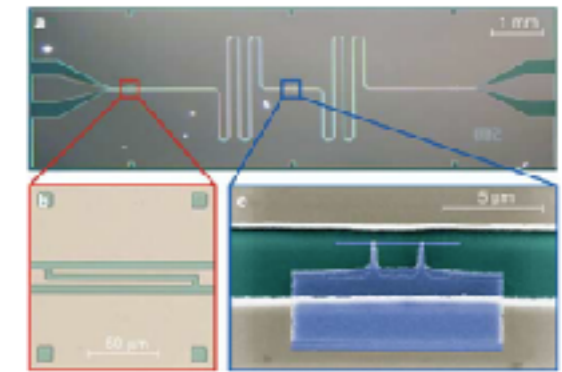
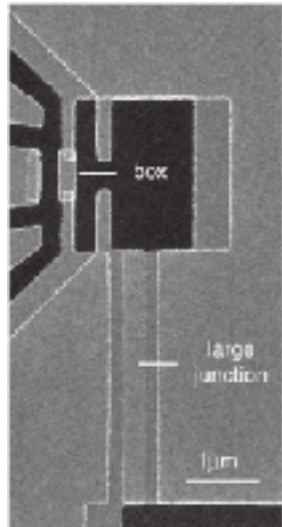
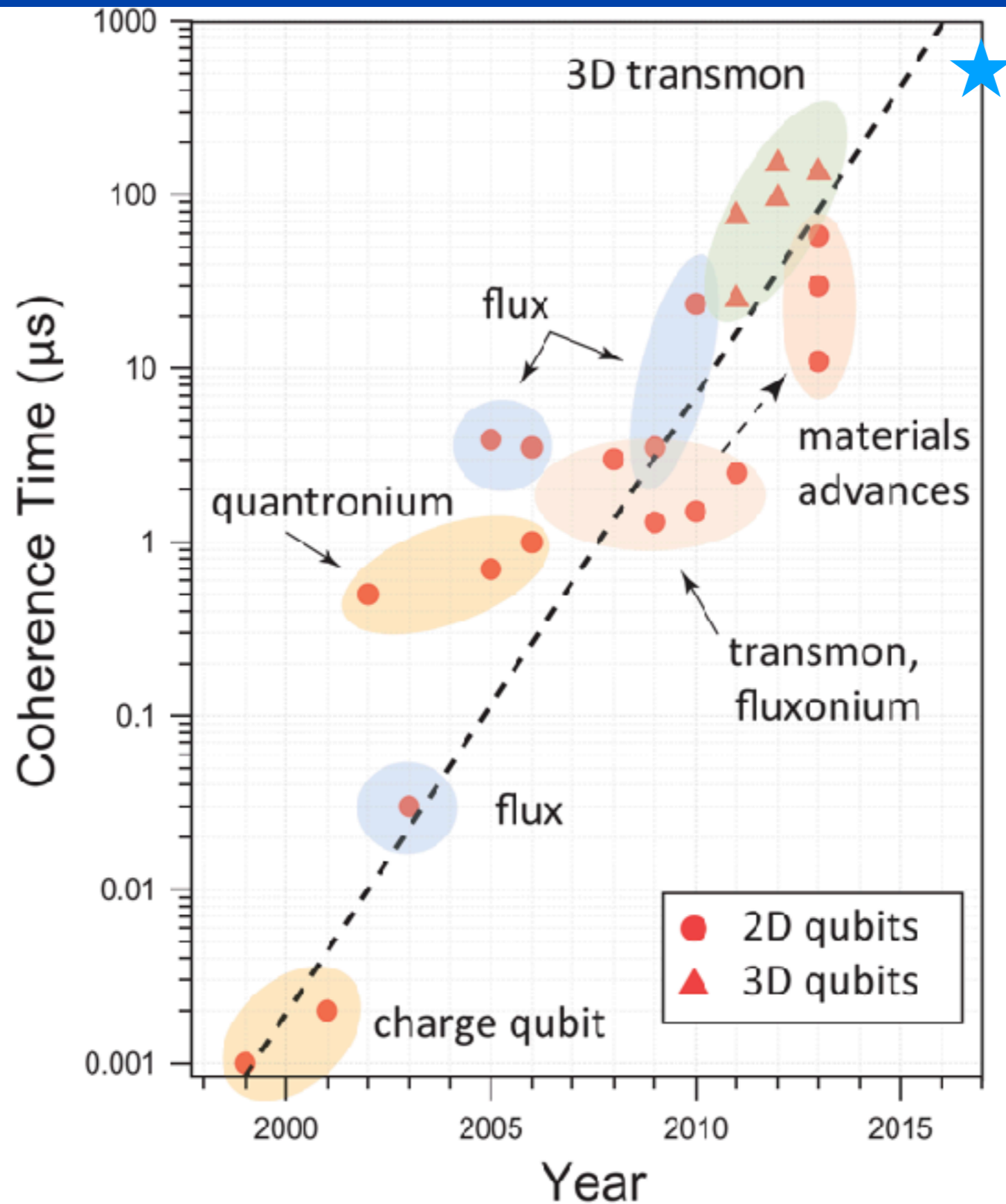
$$H = hf_c a^\dagger a + h(f_q + \chi_{qq} n_q) n_q + h\chi_{cq} n_q a^\dagger a$$

$$H = hf_c a^\dagger a + hf_q \frac{\sigma_Z}{2} + h\chi_{cq} \frac{\sigma_Z}{2} a^\dagger a$$

Fast approach using participation of nonlinearities in modes pyEPR on Github



# Superconducting qubits



# Gate-based quantum computing with superconducting circuits

<https://www.research.ibm.com/ibm-q/>



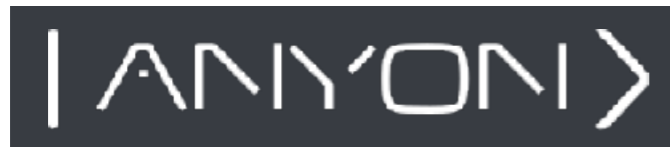
2013 50 qubits



2014



2014



Computing with Manufactured Qubits™

2014



2015



2015



2015



2015



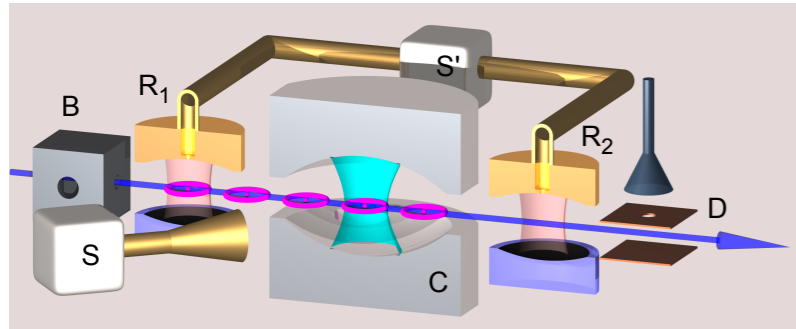
2017





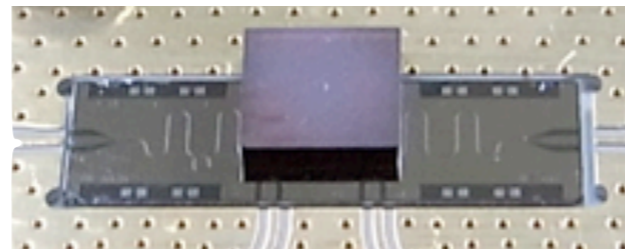
# Superconducting circuits with non linear systems

Rydberg atoms



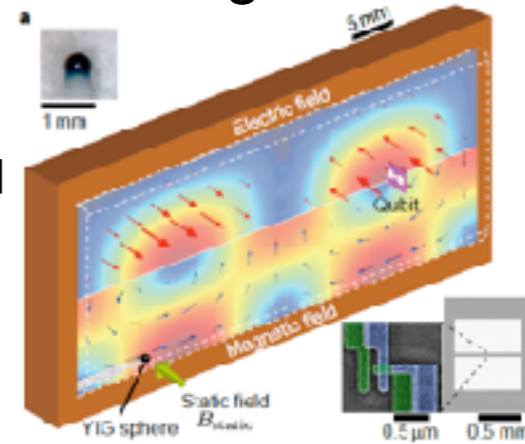
[pic from CQED group, College de France Paris]

NV centers



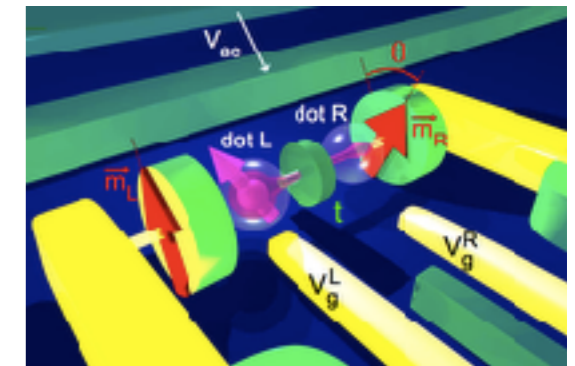
[pic from Quantronics group, CEA Saclay]

Ferromagnetic magnons



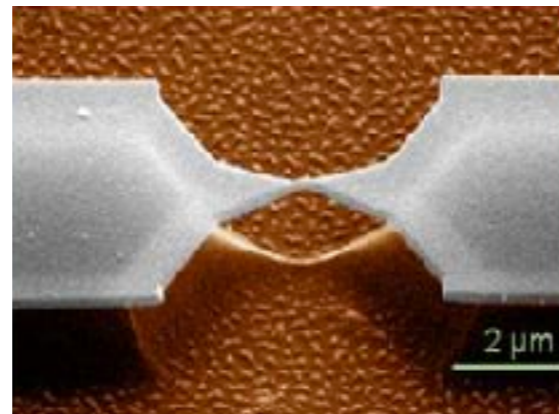
[pic from Nakamura-Usami group, Univ. Tokyo]

Spins in CNT



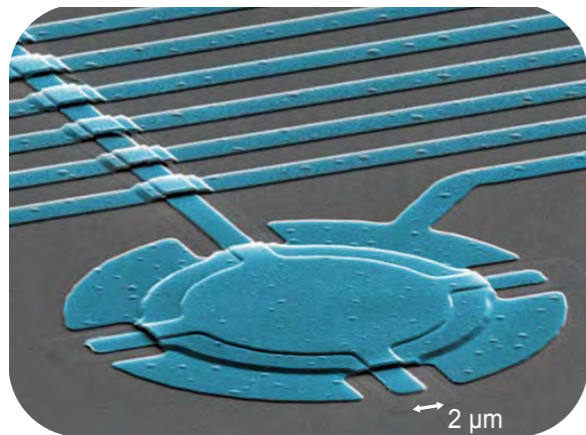
[pic from Kontos group, ENS Paris]

Andreev Bound States



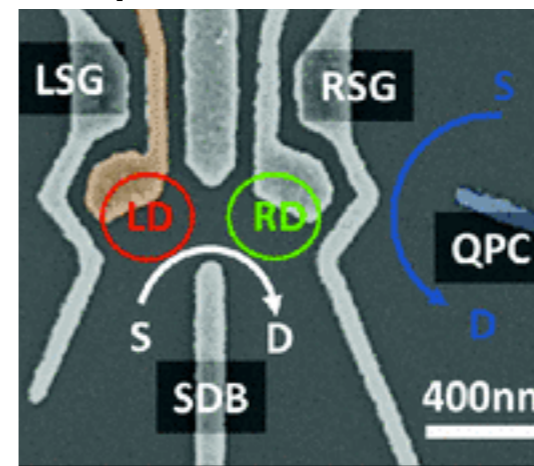
[pic from Quantronics group, CEA Saclay]

Metallic membrane



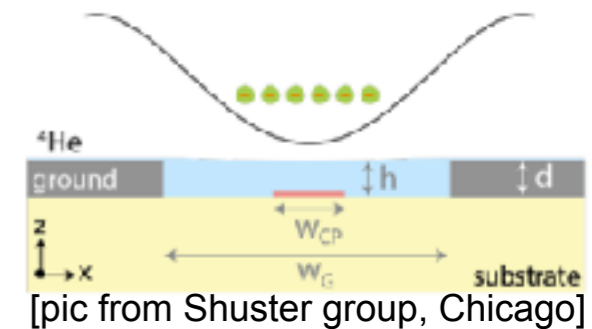
[pic from Lehnert group, JILA Boulder]

Semiconductor quantum dots



[pic from Wallraff group, ETH Zurich]

Electrons on <sup>4</sup>He



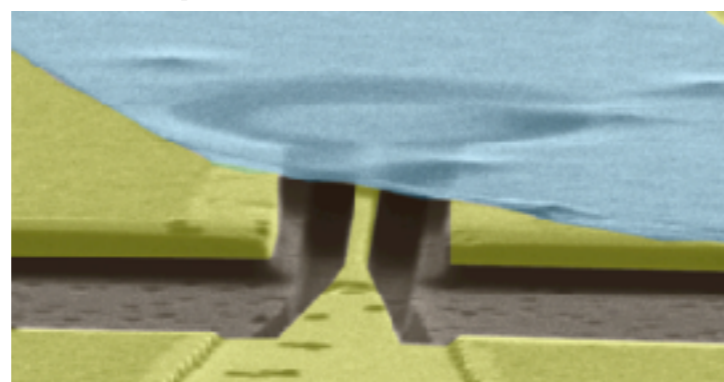
[pic from Shuster group, Chicago]

Propagating phonons



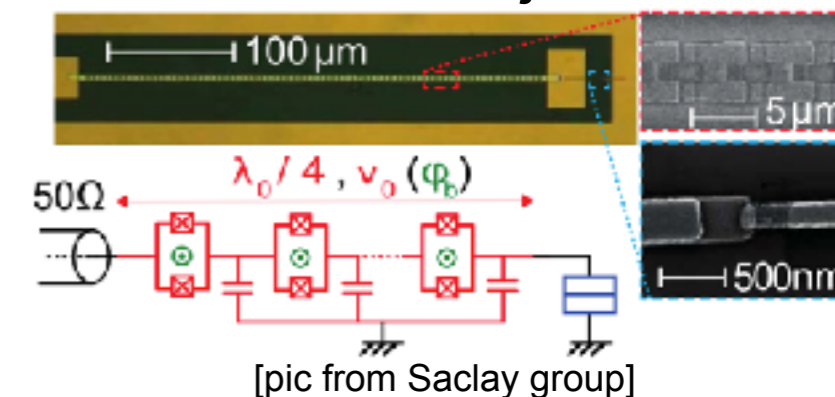
[pic from Delsing group, Chalmers UT]

Graphene membrane



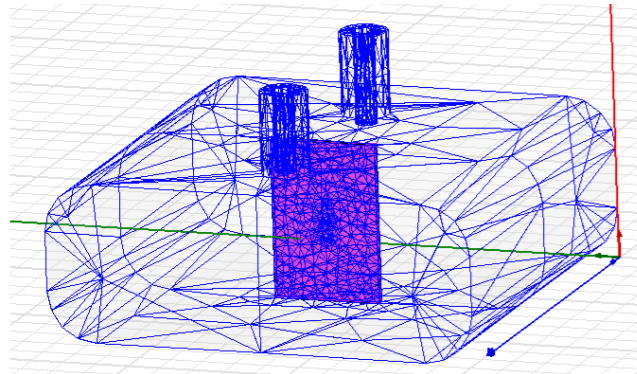
[pic from Steele group, TU Delft]

DC biased junction



[pic from Saclay group]

# Quantum trajectories and feedback in circuit-QED



## Introduction to circuit-QED

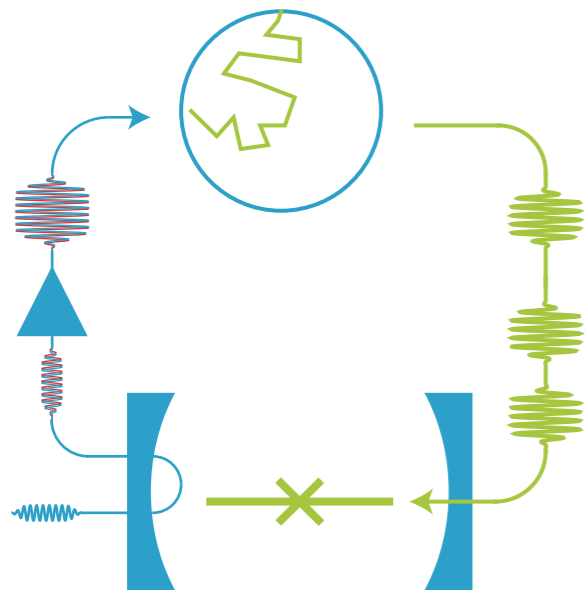
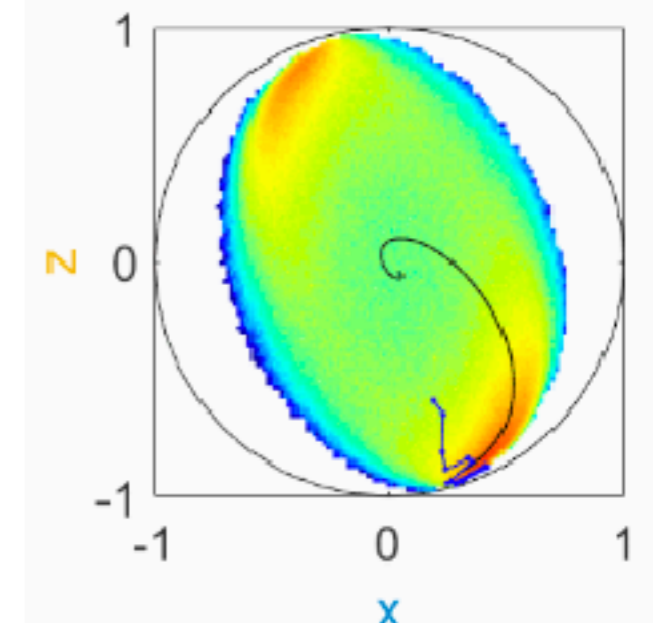
### Quantum trajectories in circuit-QED

dispersive measurement

fluorescence measurement

both simultaneously

generating entanglement



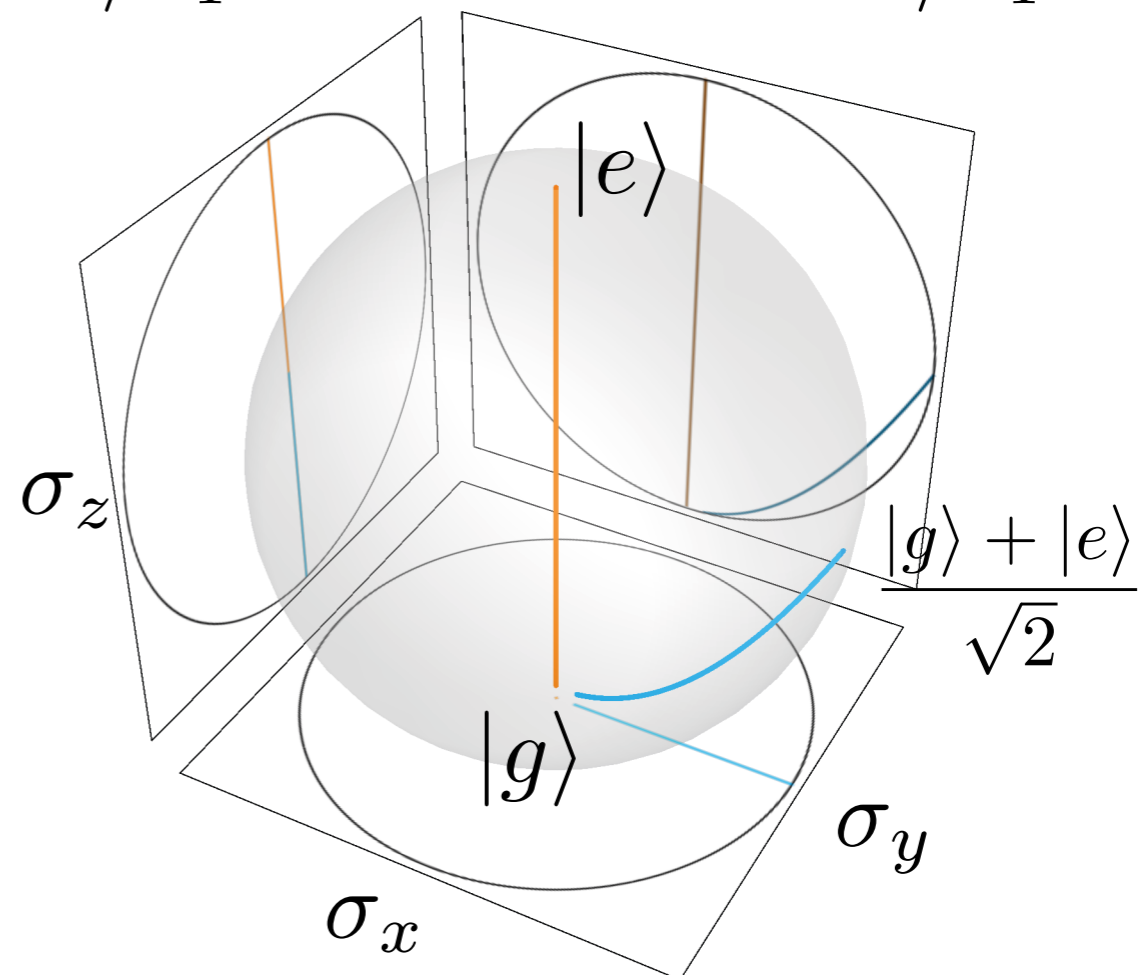
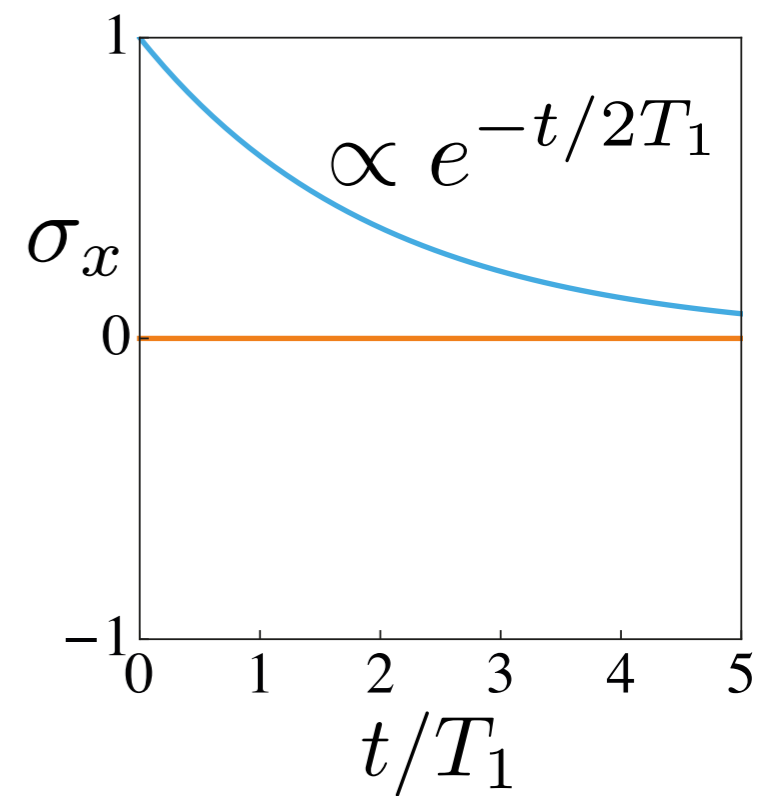
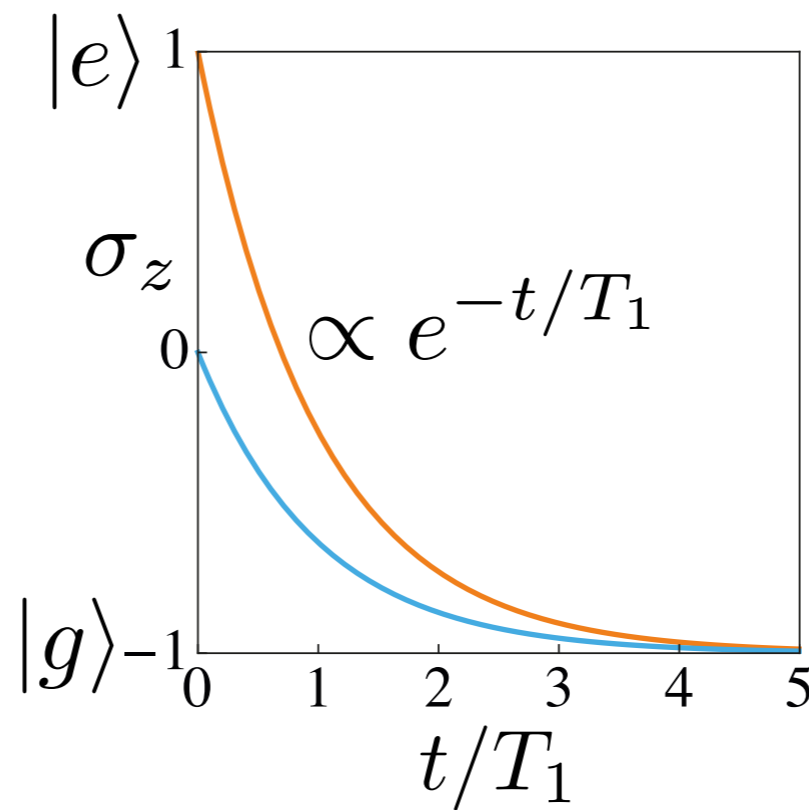
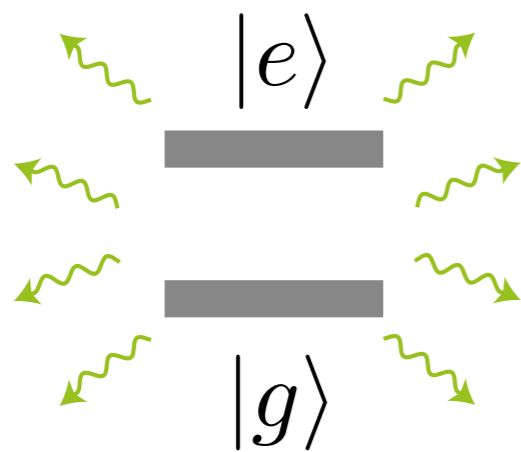
## Measurement based feedback

dispersive case

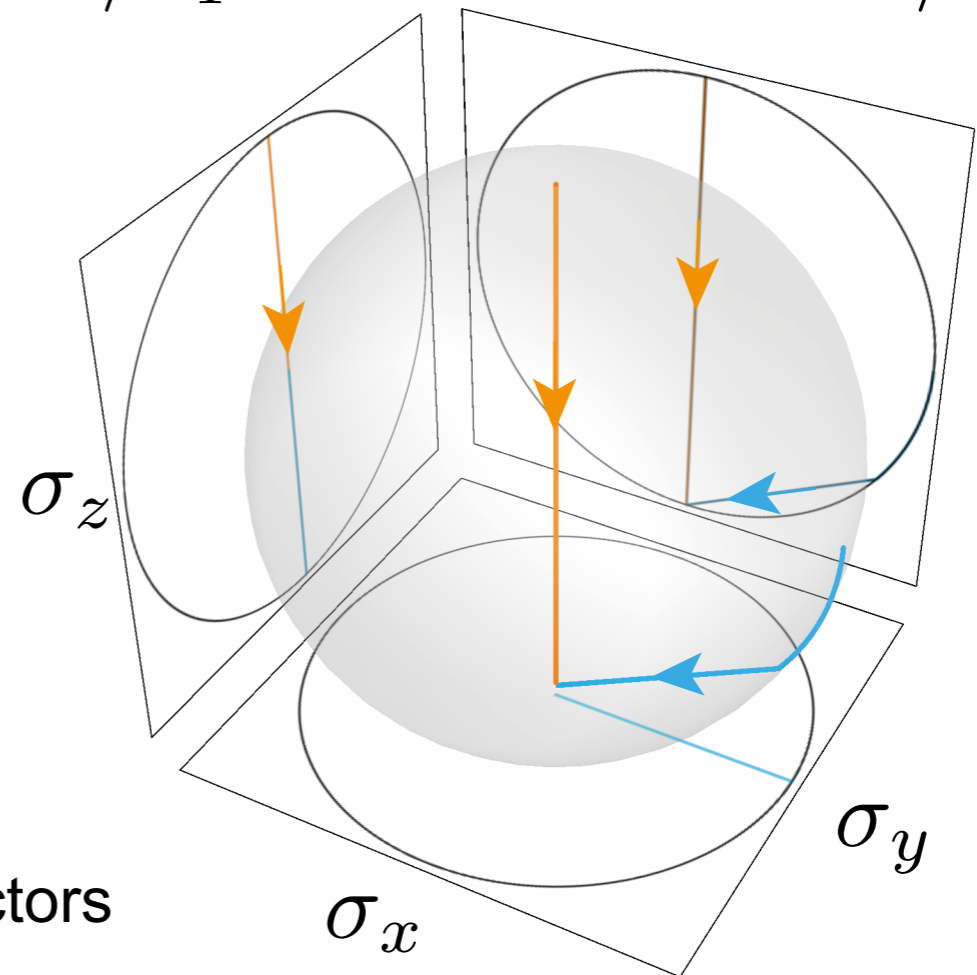
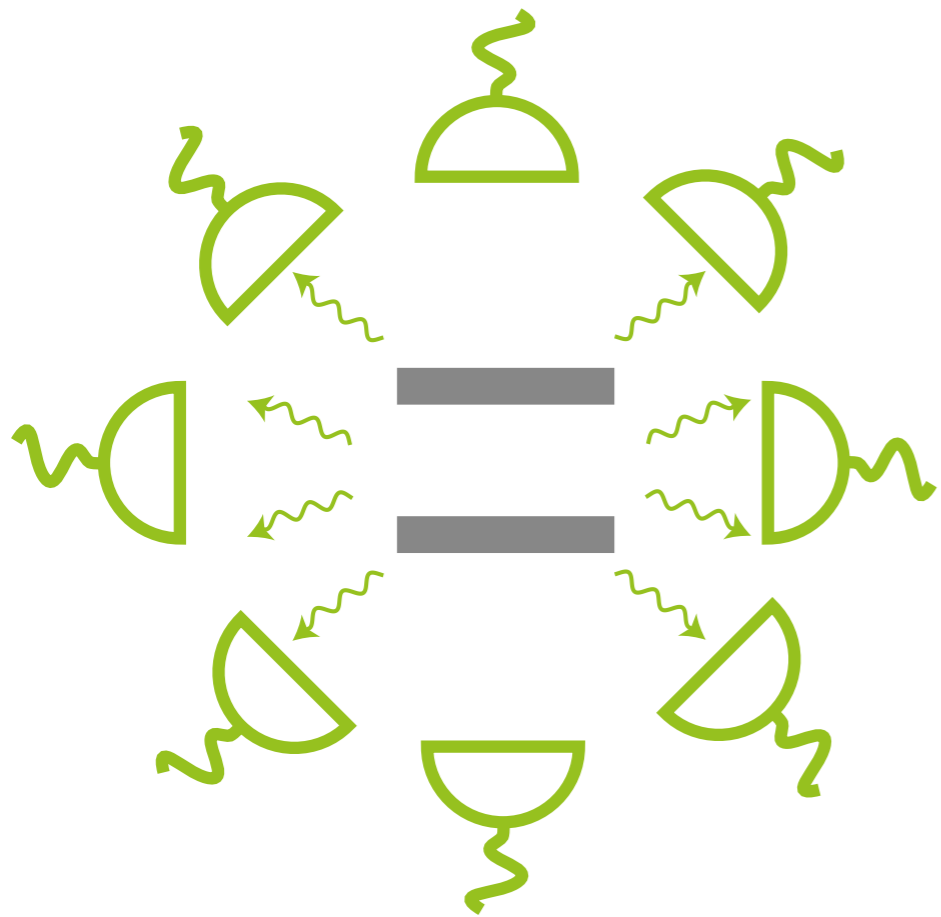
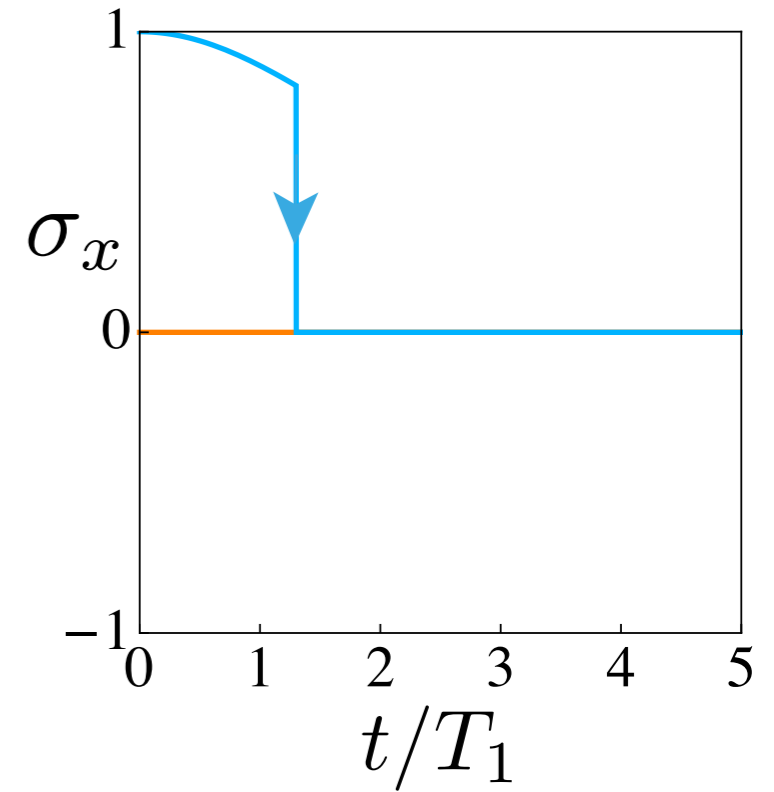
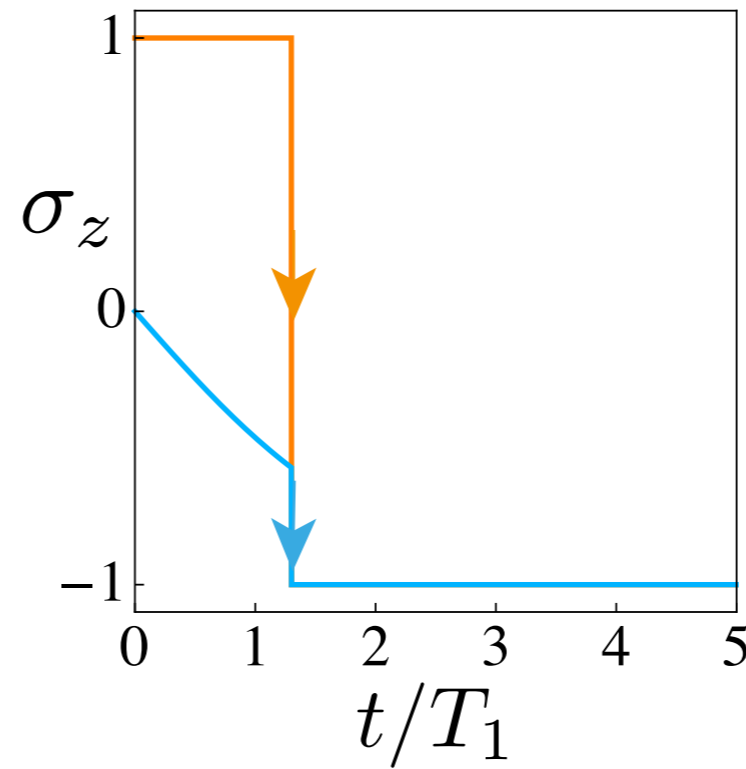
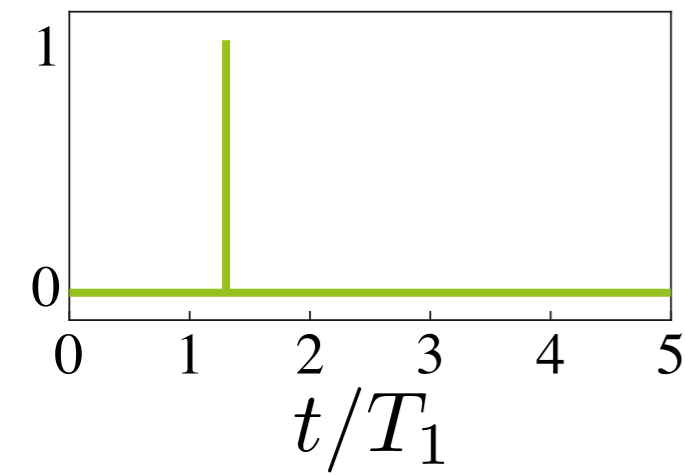
fluorescence case



# Ideal quantum jump of an atom



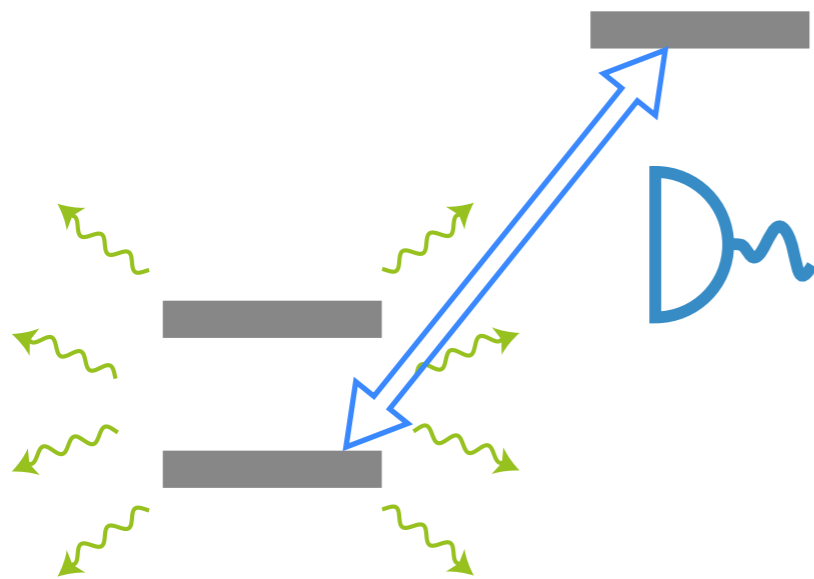
# Ideal quantum jump of an atom



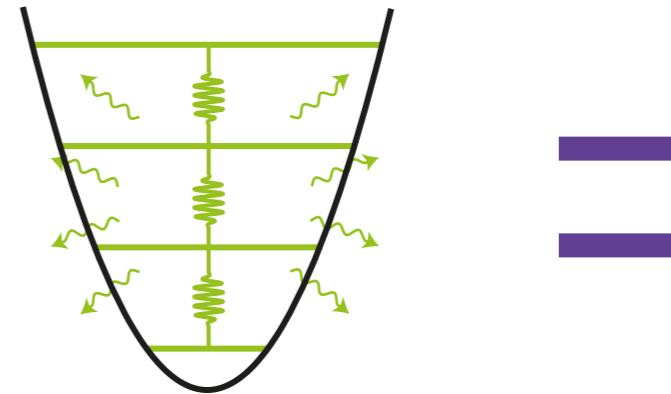
Note: purity of state is 1 only for perfect detectors

# Ideal quantum jump

hard to collect  $\longrightarrow$  use an ancillary detector

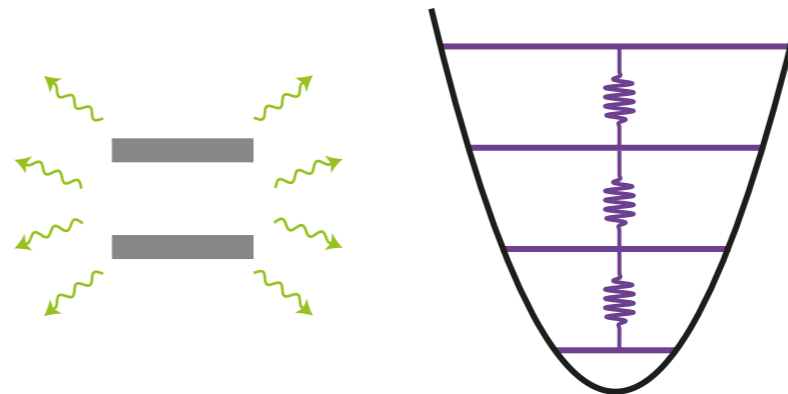


since 1986 in trapped ions  
 [Wineland group, Boulder  
 Dehmelt groupe, Seattle  
 Toschek group, Hambourg]



$$H_{\text{coupl}} = \hbar\chi a^\dagger a \frac{\sigma_Z}{2}$$

Rydberg atom probing cavity jumps  
 [Haroche group, Paris (2007)]

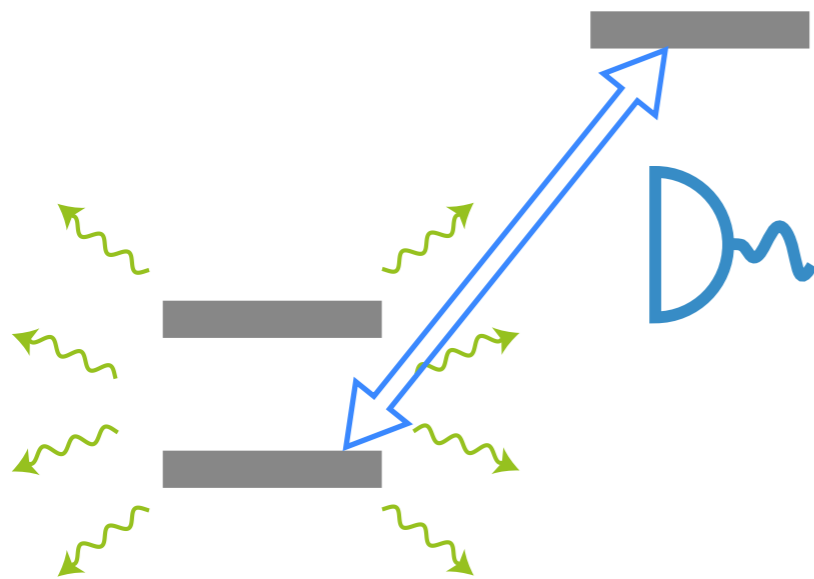


$$H_{\text{coupl}} = \hbar\chi a^\dagger a \frac{\sigma_Z}{2}$$

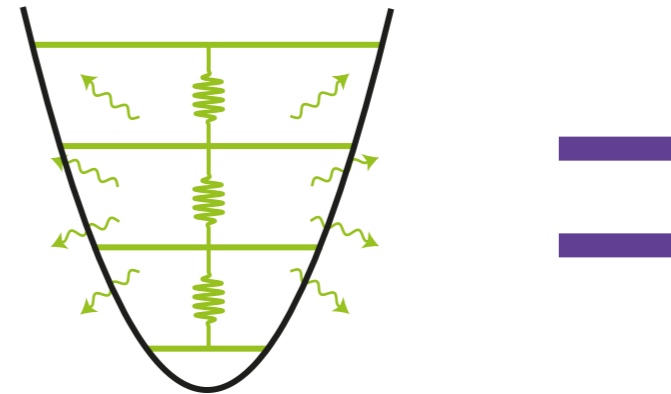
Cavity probing qubit jumps  
 [Siddiqi group, Berkeley (2011)]

# Ideal quantum jump

hard to collect  $\longrightarrow$  use an ancillary detector

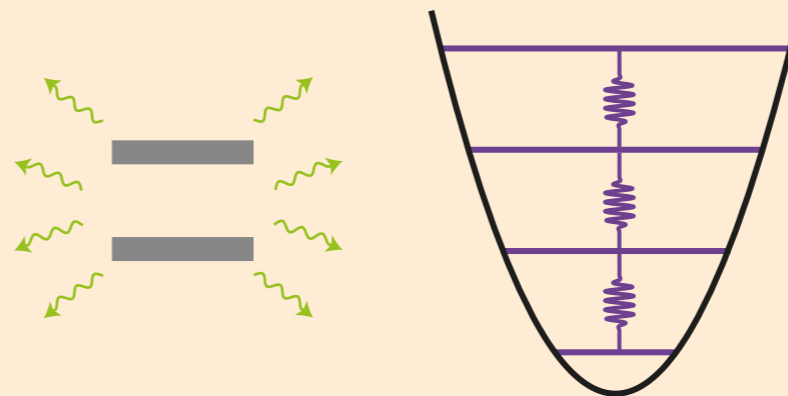


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$$H_{\text{coupl}} = \hbar\chi a^\dagger a \frac{\sigma_Z}{2}$$

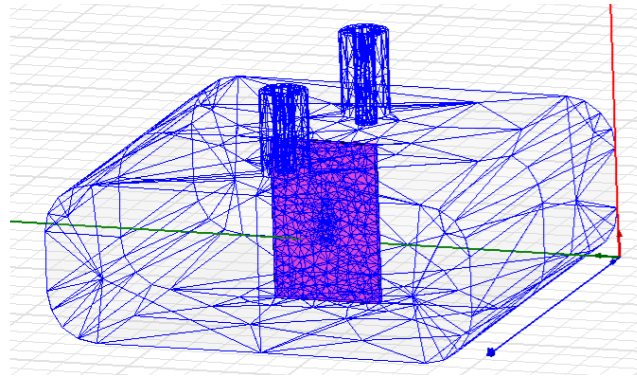
Rydberg atom probing cavity jumps  
 [Haroche group, Paris (2007)]



$$H_{\text{coupl}} = \hbar\chi a^\dagger a \frac{\sigma_Z}{2}$$

Cavity probing qubit jumps  
 [Siddiqi group, Berkeley (2011)]

# Quantum trajectories and feedback in circuit-QED



## Introduction to circuit-QED

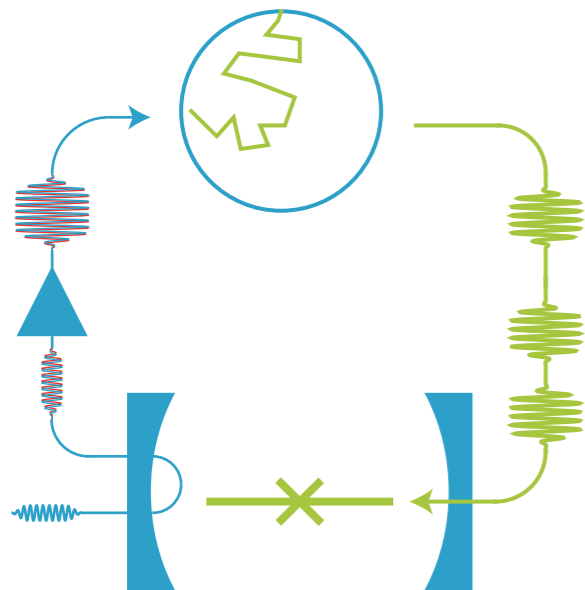
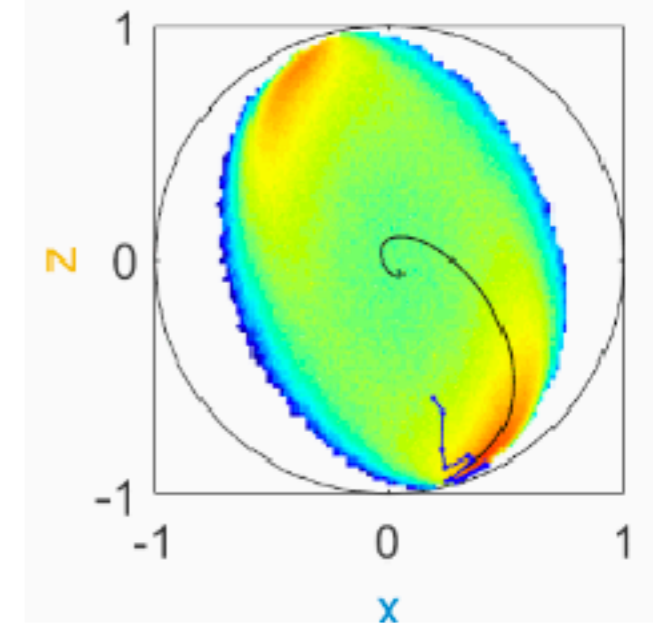
## Quantum trajectories in circuit-QED

dispersive measurement

fluorescence measurement

both simultaneously

generating entanglement



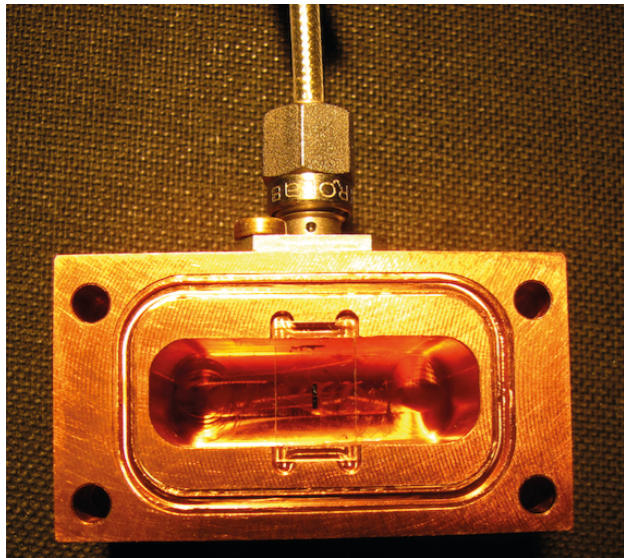
## Measurement based feedback

dispersive case

fluorescence case

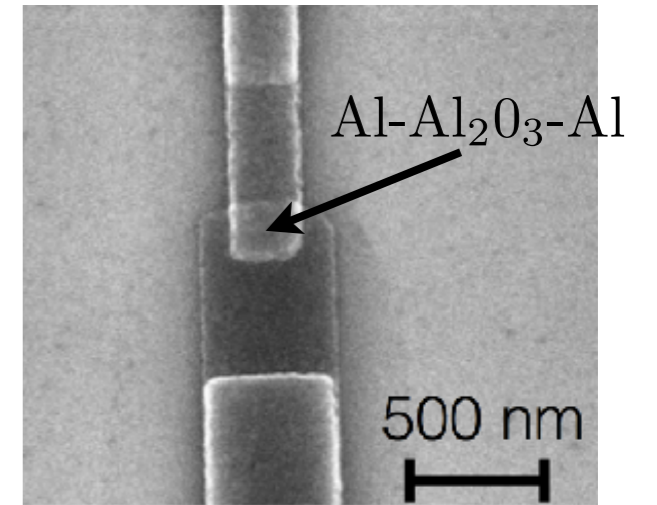


# 3D transmon architecture



$$H_{\text{disp}} = -h\chi \frac{\sigma_z}{2} a^\dagger a$$

$$\chi = 4 \text{ MHz}$$

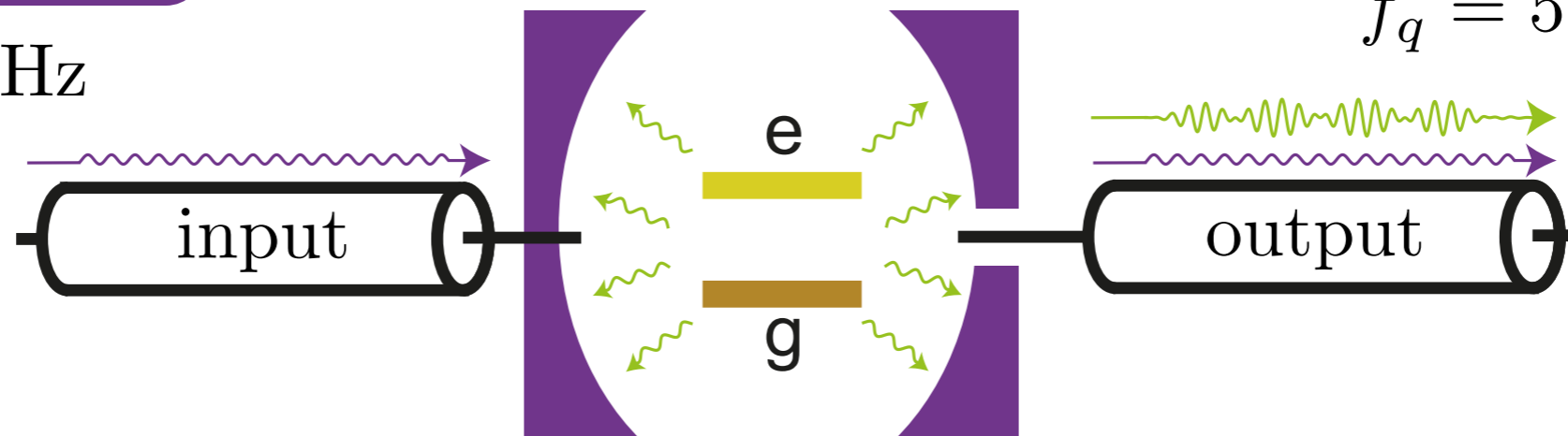


$$H_c = hf_c \left( a^\dagger a + \frac{1}{2} \right)$$

$$f_c = 7.8 \text{ GHz}$$

$$H_q = hf_q \frac{\sigma_z}{2}$$

$$f_q = 5.4 \text{ GHz}$$

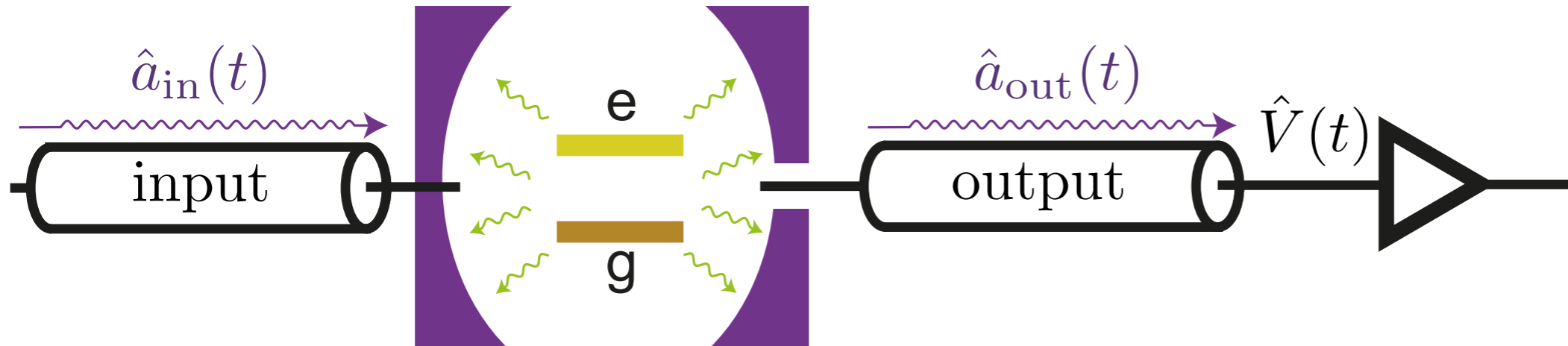


Dispersive Hamiltonian

$$H = hf_c \left( a^\dagger a + \frac{1}{2} \right) - h\frac{\chi}{2} \sigma_z a^\dagger a + hf_q \frac{\sigma_z}{2}$$

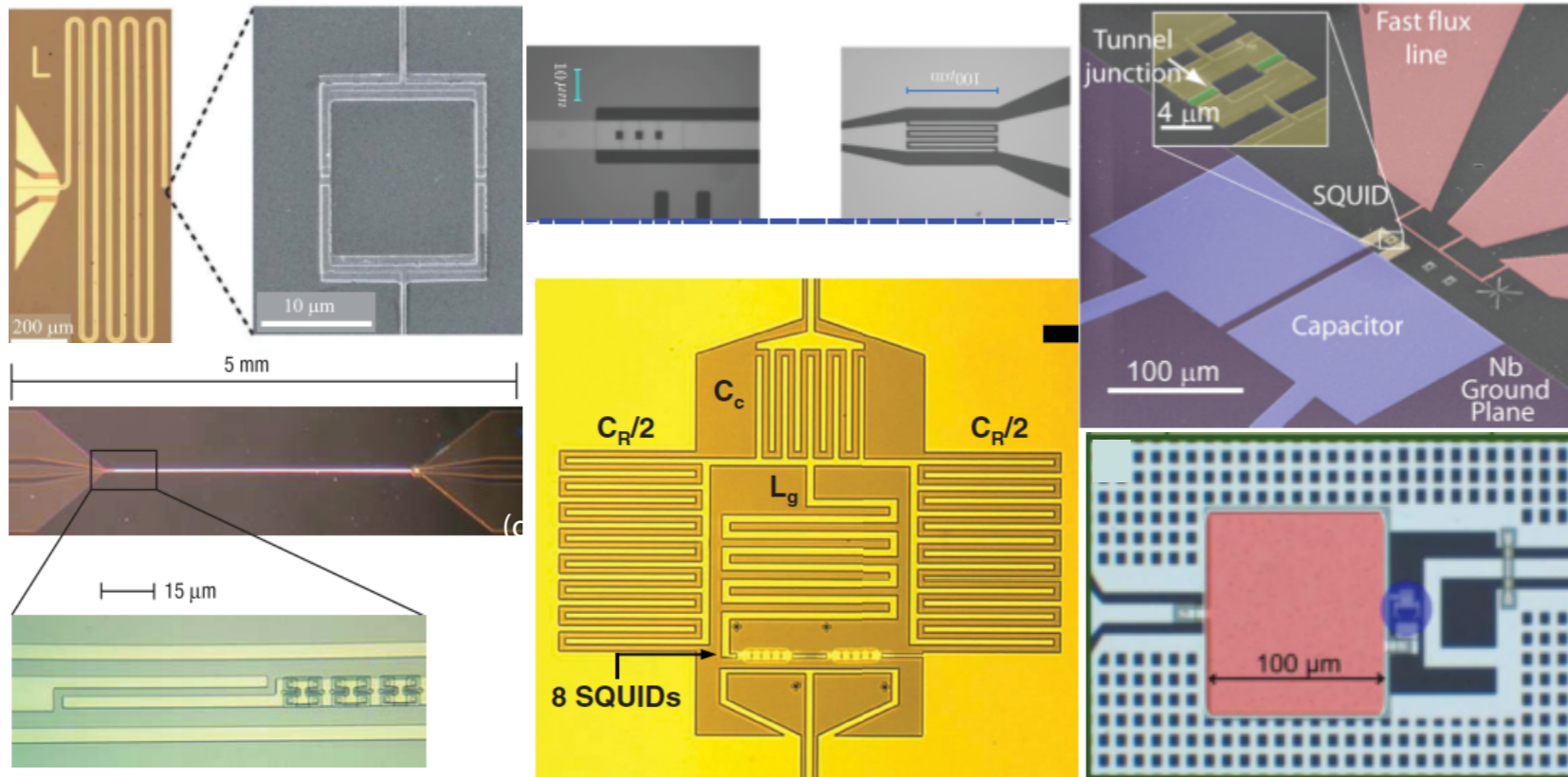
# Dispersive Measurement

$$H = hf_q \frac{\sigma_z}{2} + h(f_c - \frac{\chi}{2} \sigma_z) a^\dagger a$$



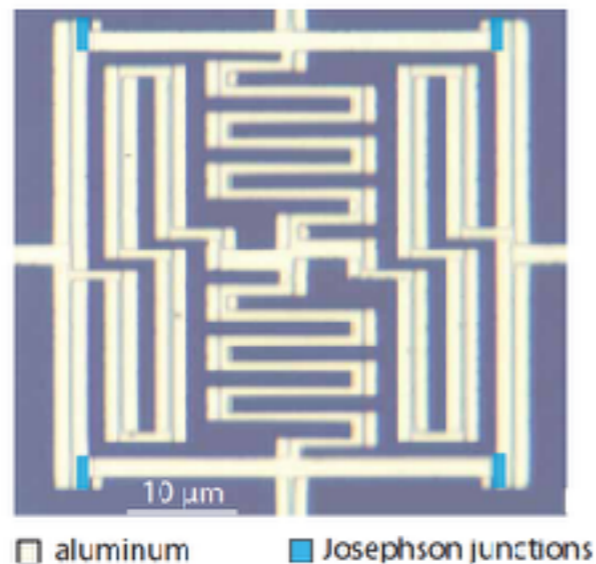
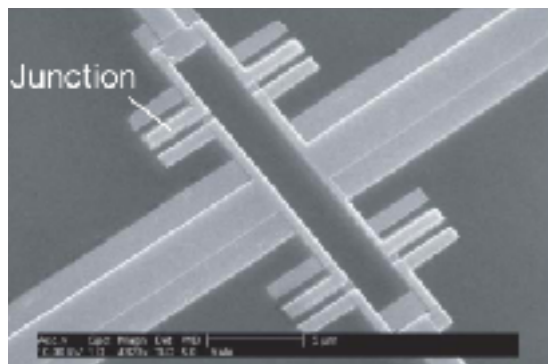
# Quantum limited amplifiers

## Degenerate amplifiers



- (Bell Labs, 1989)
- (NEC Tokyo, 2008)
- (Boulder, 2008)
- (Yale, 2009)
- (Zurich, 2011)
- (Berkeley, 2011)
- (Santa Barbara, 2013)
- (Saclay, 2014)

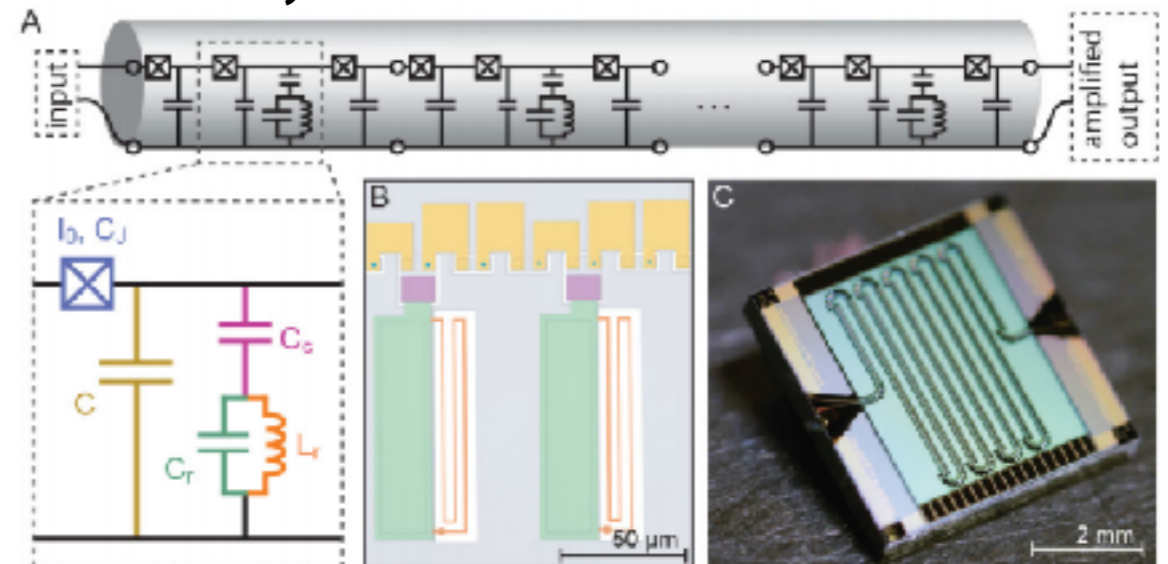
## Non degenerate amplifiers



(Yale, 2010)

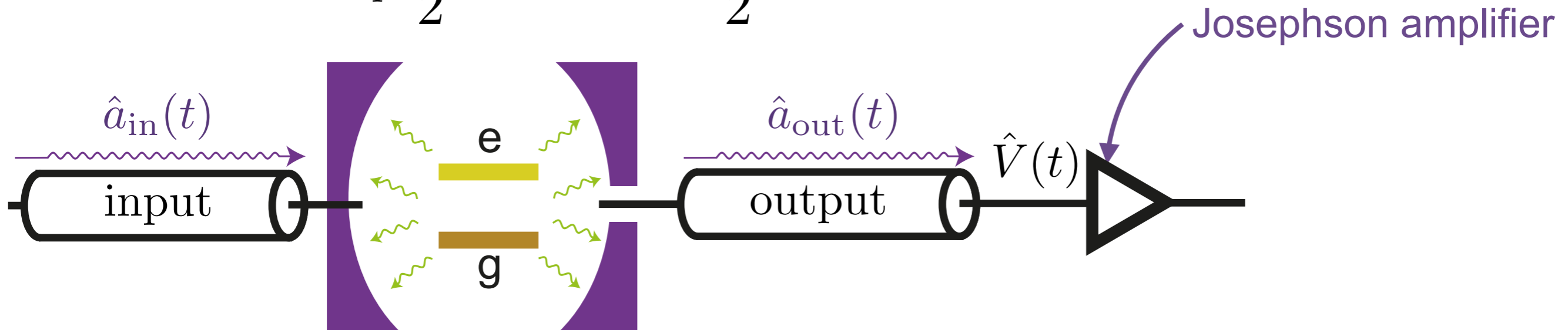
(ENS Paris, 2012)

## Traveling wave amplifier (Berkeley, MIT 2015)



# Dispersive Measurement

$$H = hf_q \frac{\sigma_z}{2} + h(f_c - \frac{\chi}{2} \sigma_z) a^\dagger a$$



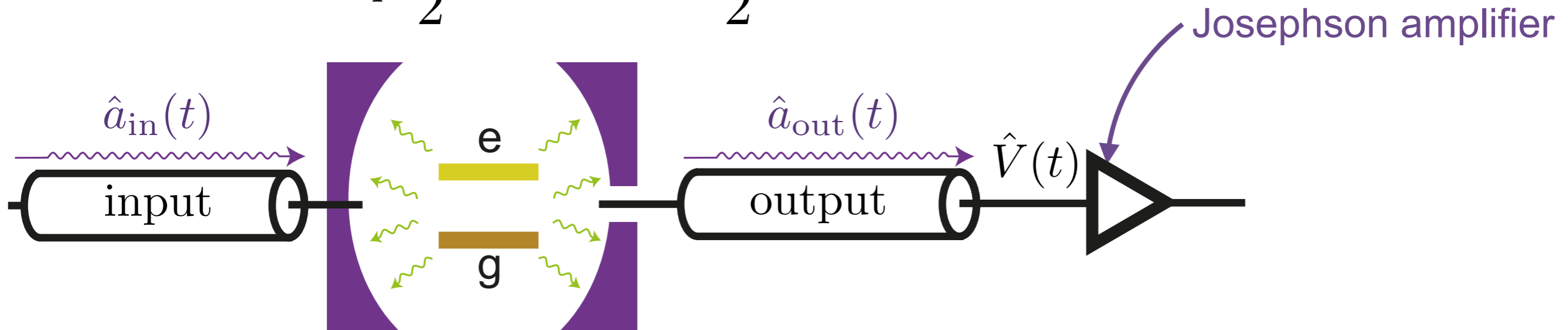
Classically  $V(t) = I(t) \cos(2\pi f_c t) + Q(t) \sin(2\pi f_c t)$

$$I_t \rightarrow \hat{I}_t \propto \frac{\hat{a}_{out} + \hat{a}_{out}^\dagger}{2} = \text{Re}(\hat{a}_{out})$$

$$Q_t \rightarrow \hat{Q}_t \propto \frac{\hat{a}_{out} - \hat{a}_{out}^\dagger}{2i} = \text{Im}(\hat{a}_{out})$$

# Dispersive Measurement

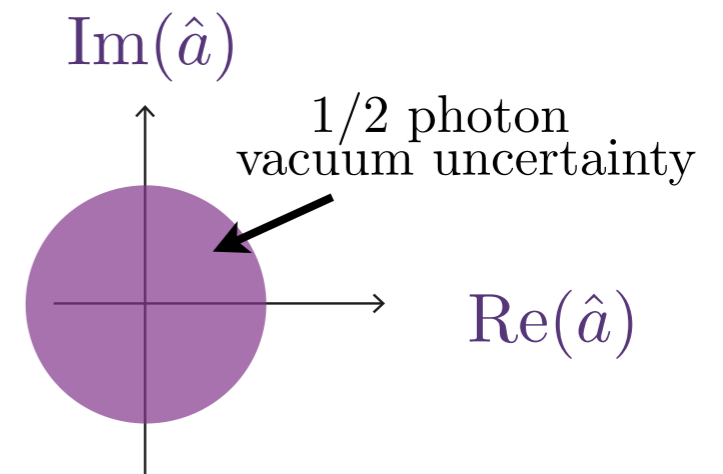
$$H = hf_q \frac{\sigma_z}{2} + h(f_c - \frac{\chi}{2} \sigma_z) a^\dagger a$$



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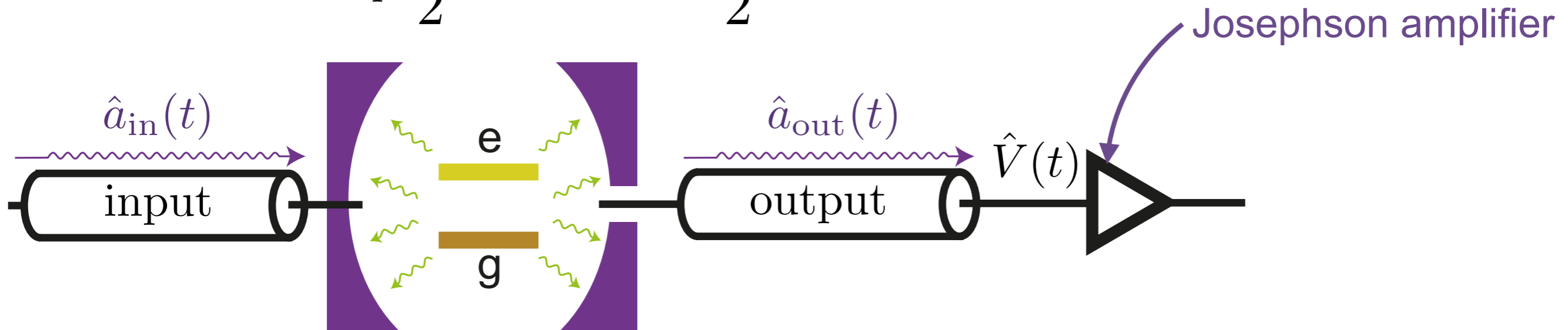


Zero-point fluctuations  $|0\rangle$



# Dispersive Measurement

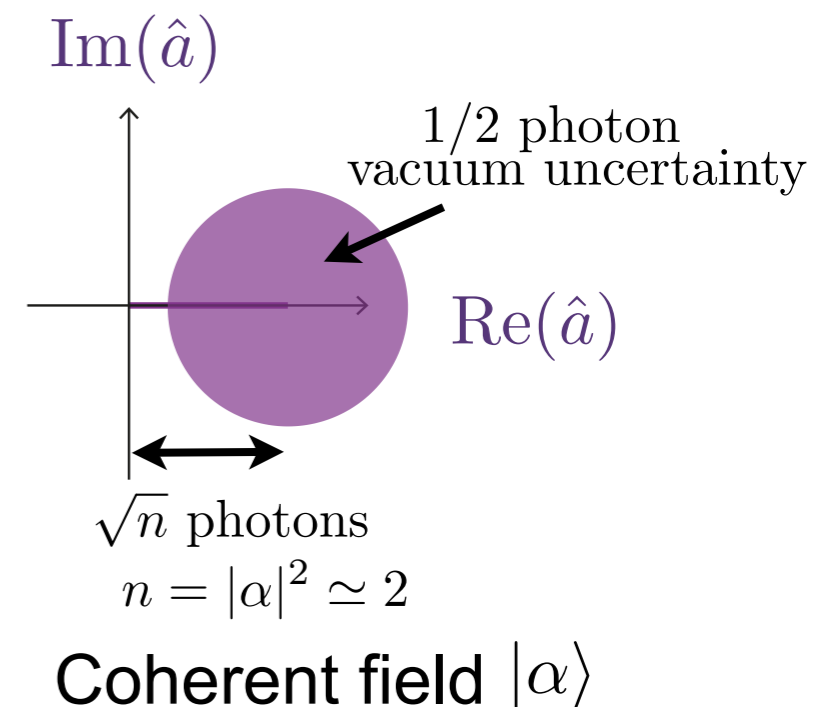
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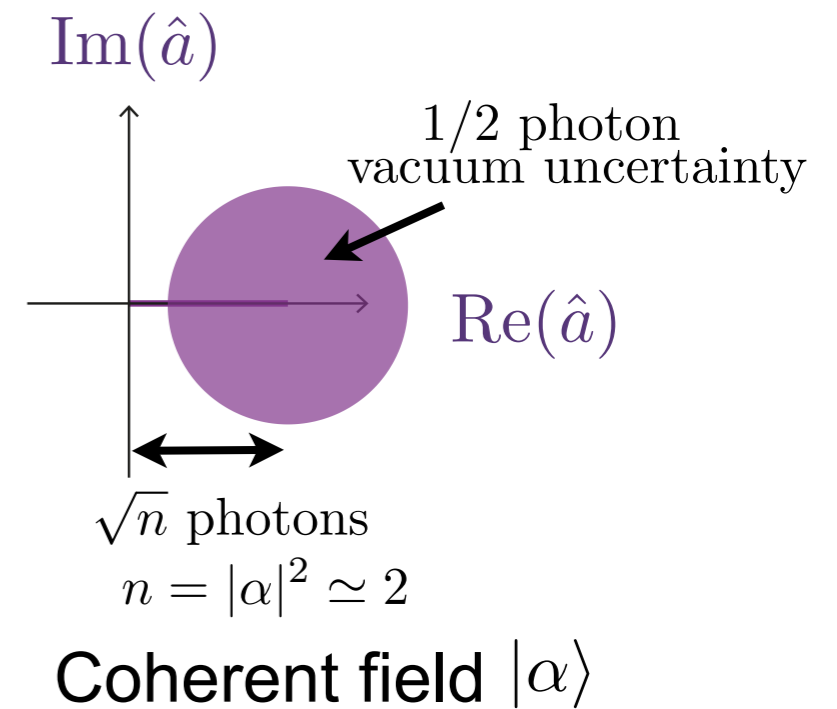
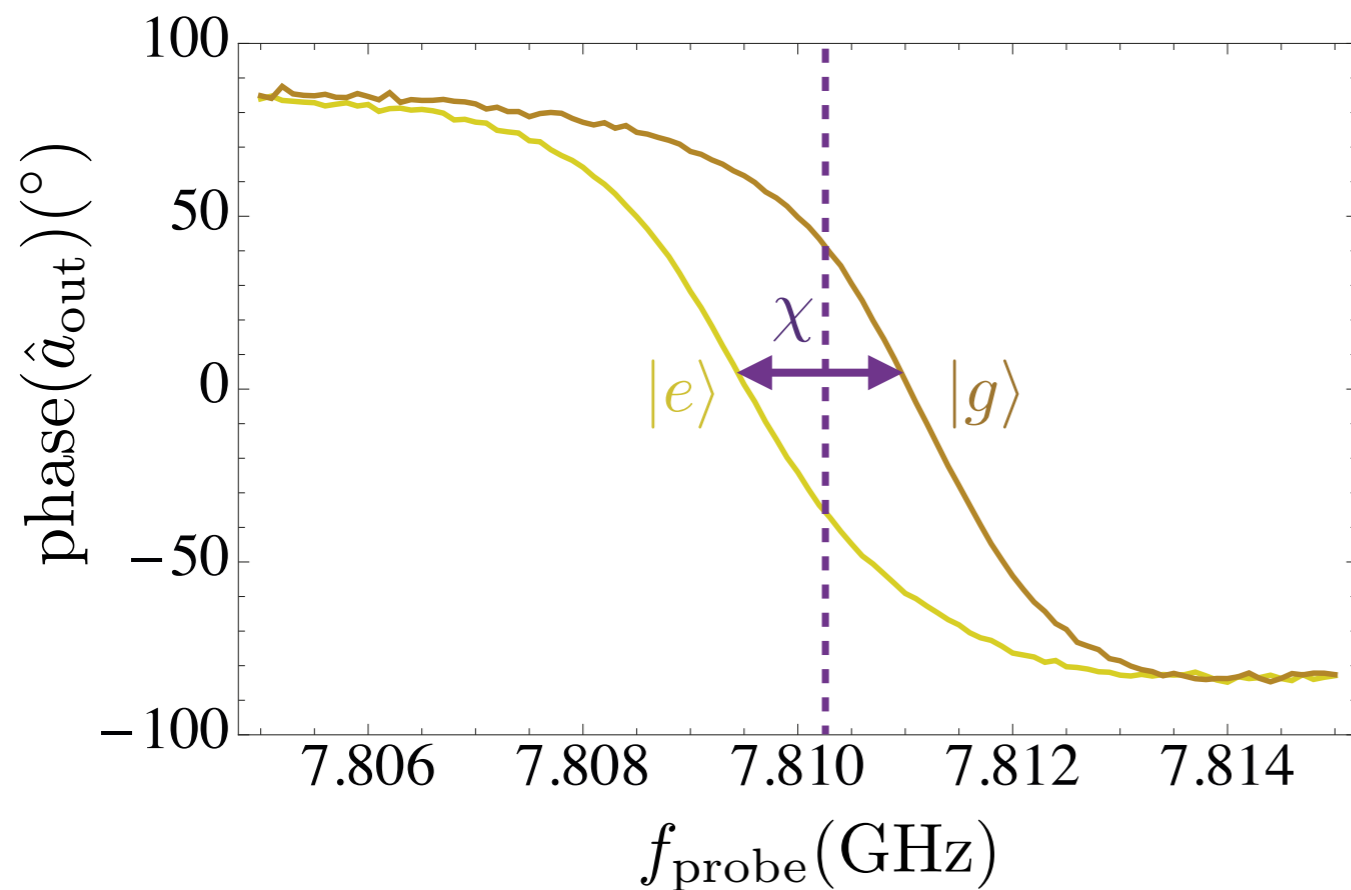
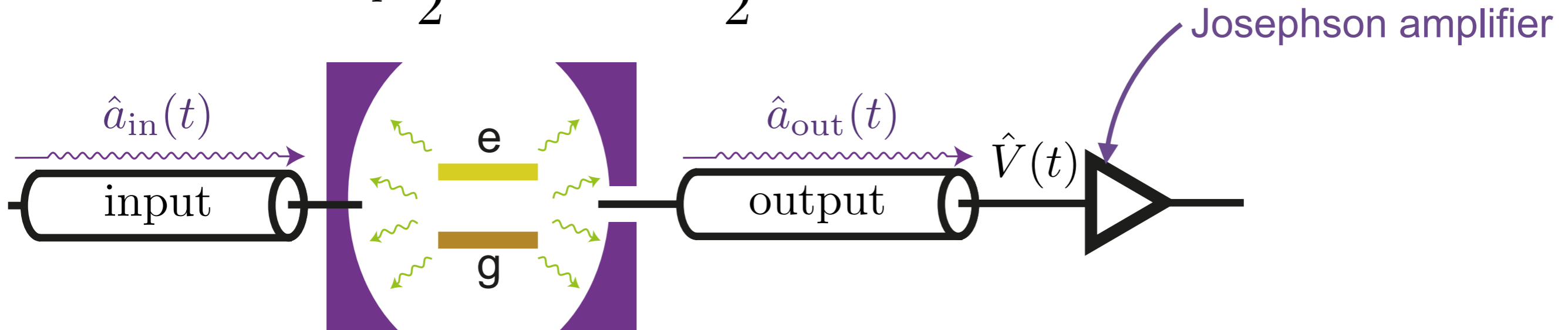
$$Q_t \rightarrow \hat{Q}_t \propto \frac{\hat{a}_{\text{out}} - \hat{a}_{\text{out}}^\dagger}{2i} = \text{Im}(\hat{a}_{\text{out}})$$





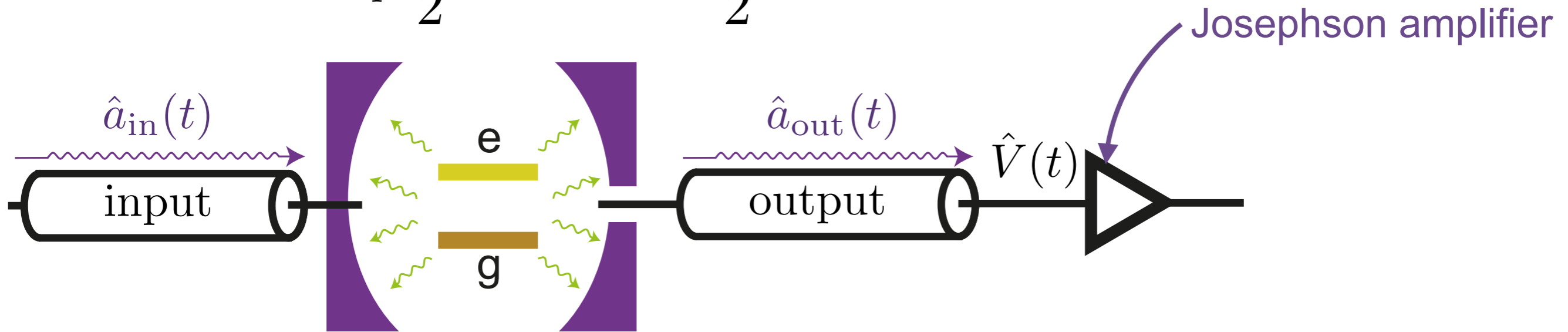
# Dispersive Measurement

$$H = hf_q \frac{\sigma_z}{2} + h(f_c - \frac{\chi}{2} \sigma_z) a^\dagger a$$



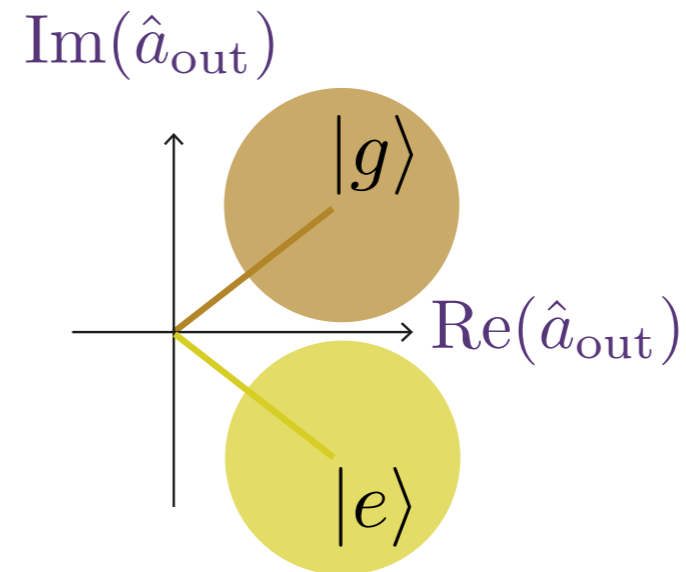
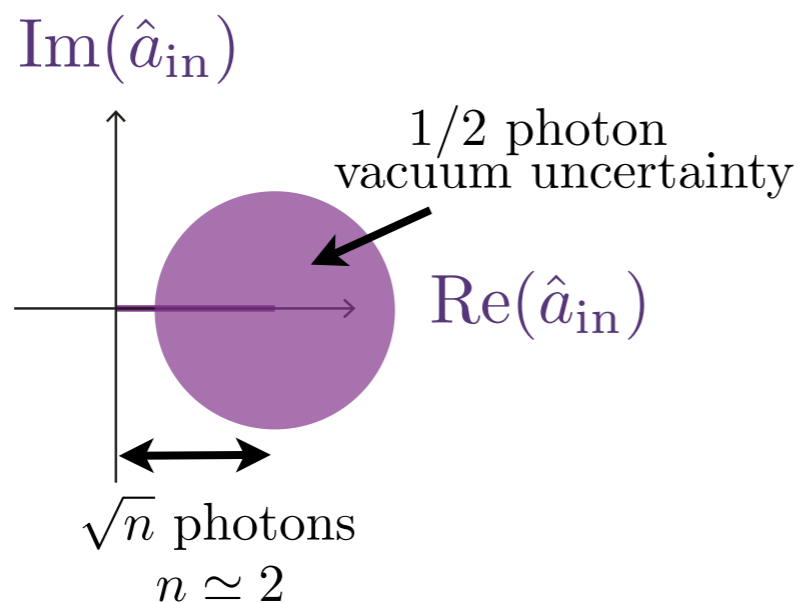
# Dispersive Measurement

$$H = hf_q \frac{\sigma_z}{2} + h(f_c - \frac{\chi}{2} \sigma_z) a^\dagger a$$



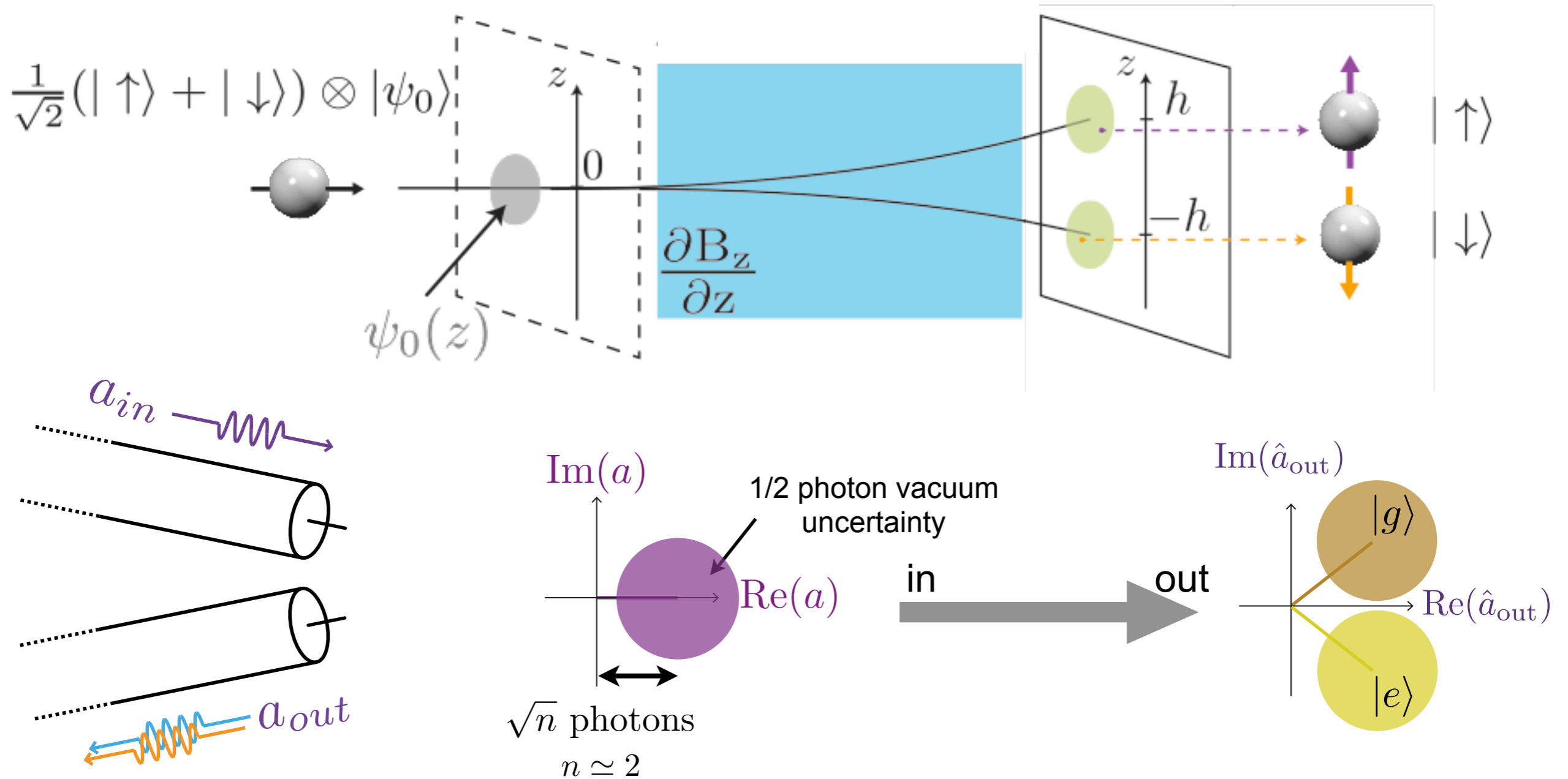
field going in ...

... field coming out



measuring  $\text{Im}(\hat{a}_{out})$   $\longrightarrow$  Strong QND measurement

# Dispersive measurement

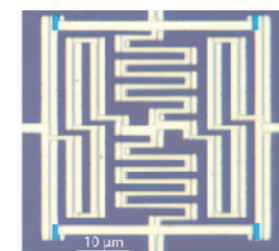


Phase encodes qubit state



measurement uses a non-degenerate quantum limited amplifier

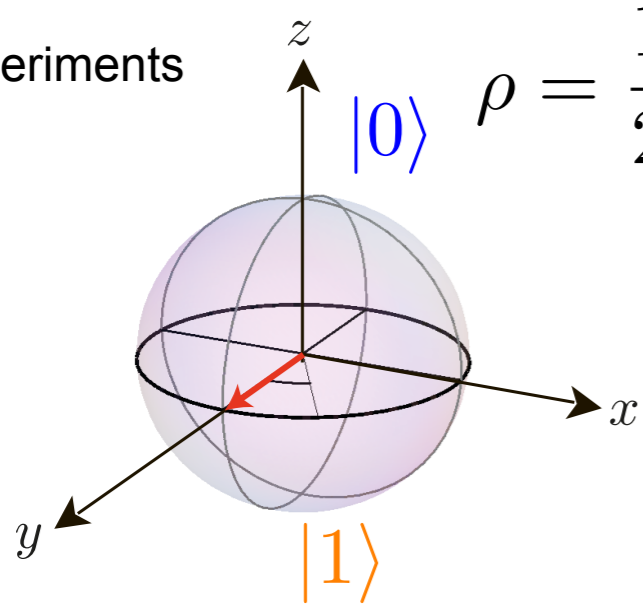
[Roch et al., PRL 2012]



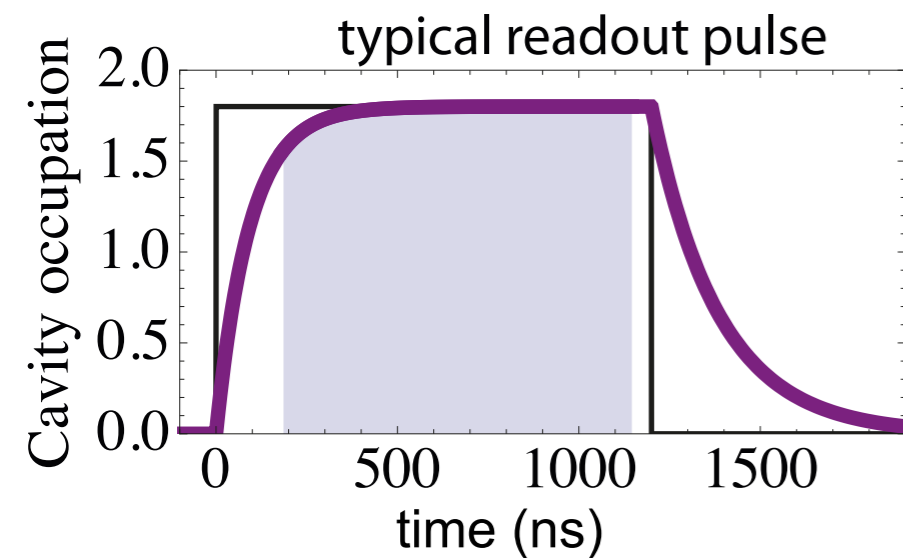
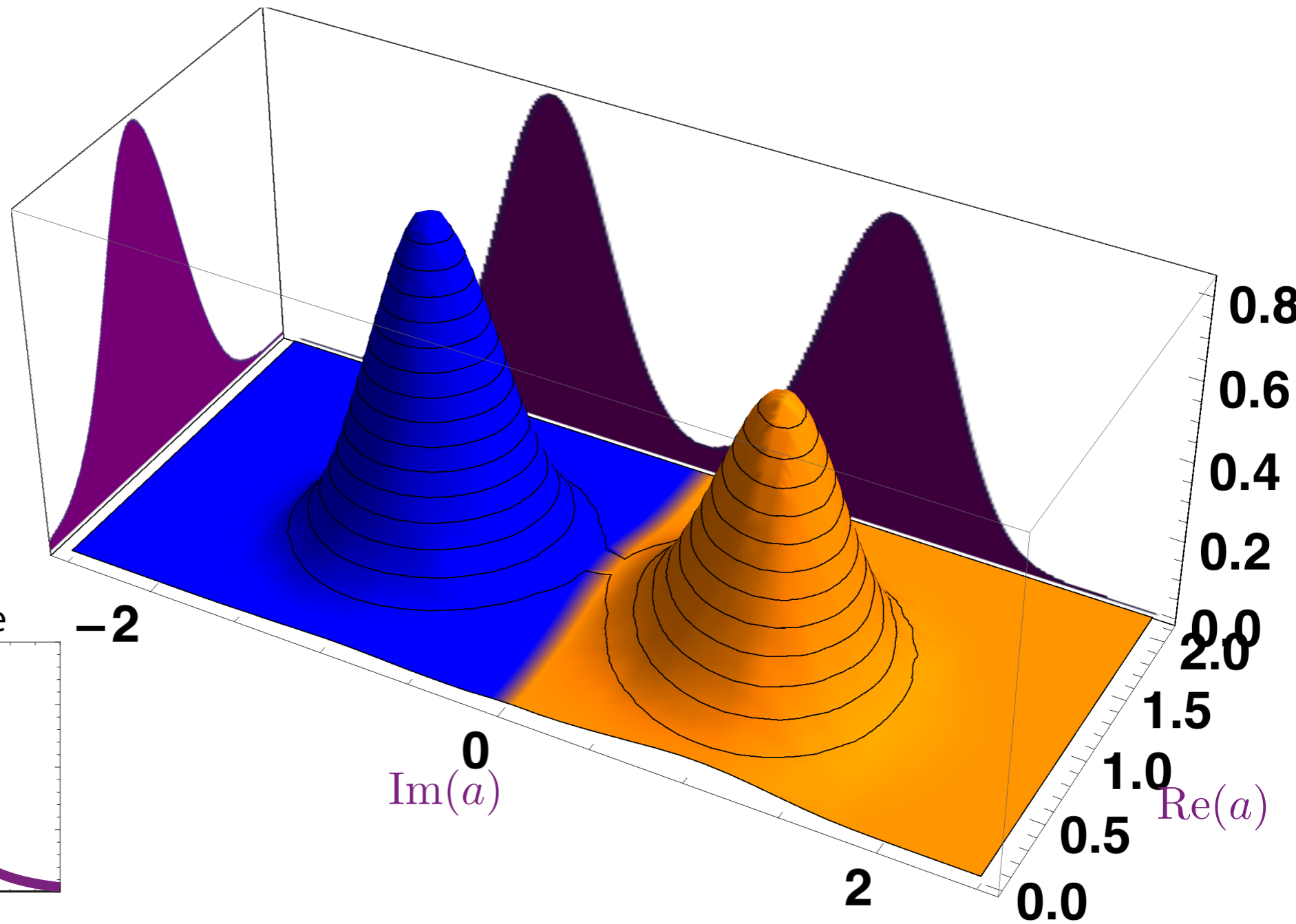
aluminum Josephson junctions

# Single shot qubit state readout

$10^6$  experiments

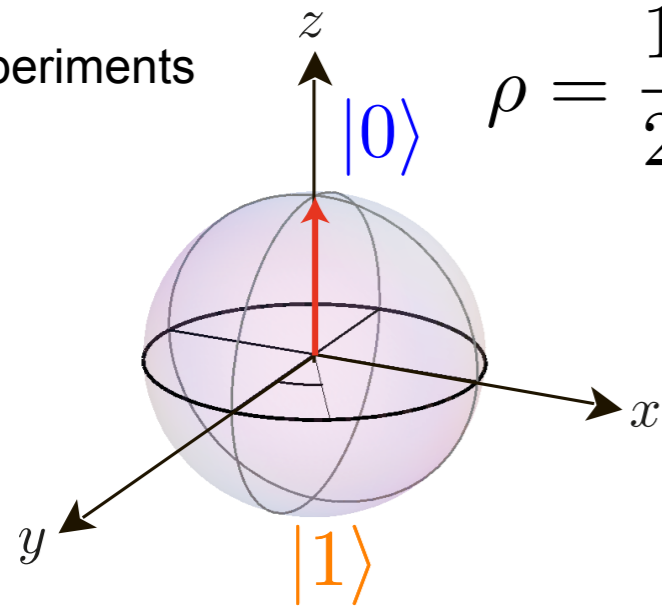


$$\rho = \frac{1}{2}(\mathbf{1} + x\sigma_x + y\sigma_y + z\sigma_z)$$



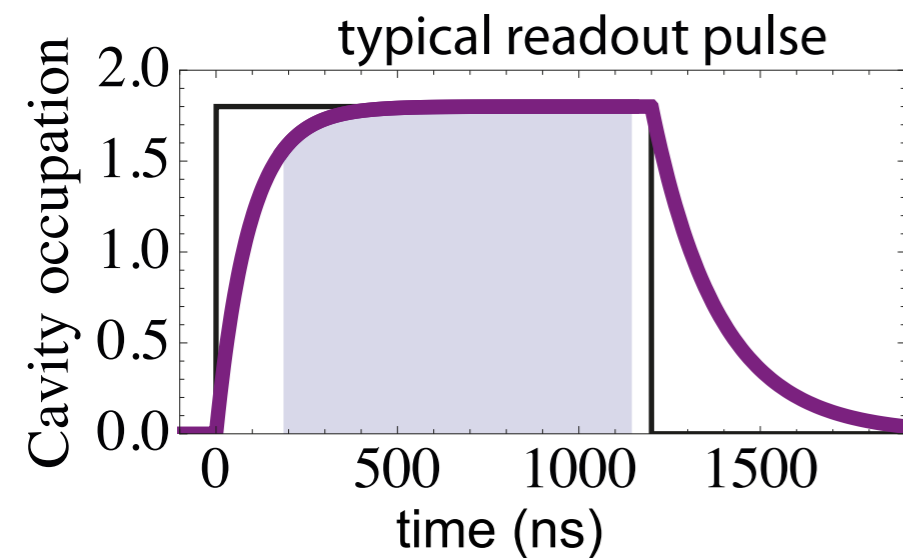
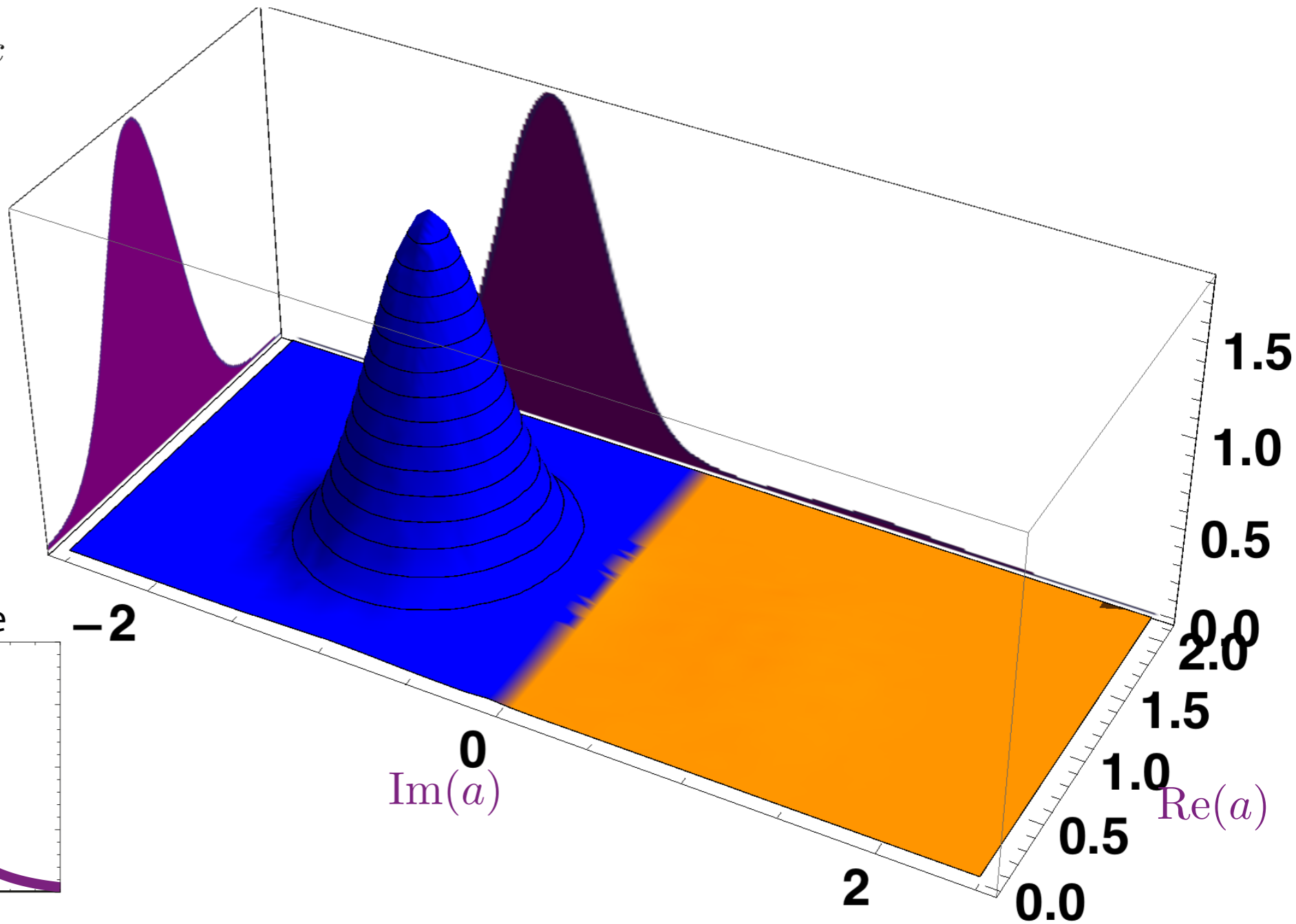
# Single shot qubit state readout

10<sup>6</sup> experiments



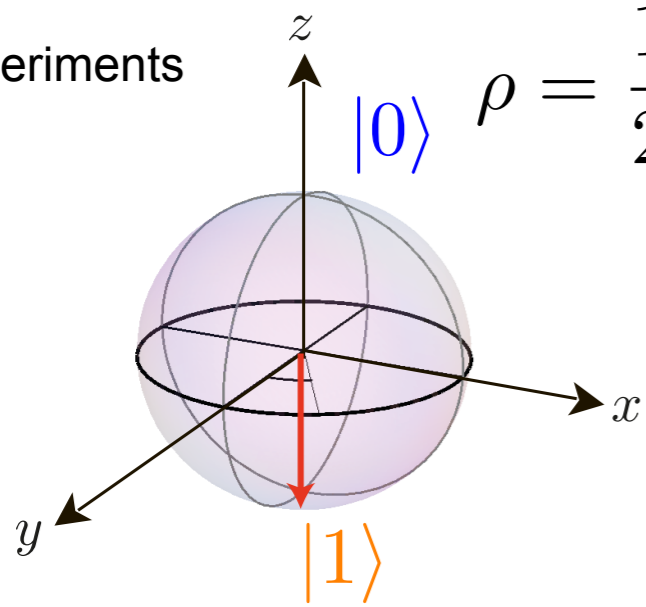
$$\rho = \frac{1}{2}(\mathbf{1} + x\sigma_x + y\sigma_y + z\sigma_z)$$

$$P(\text{Im}(a) > 0) = 2.5\%$$



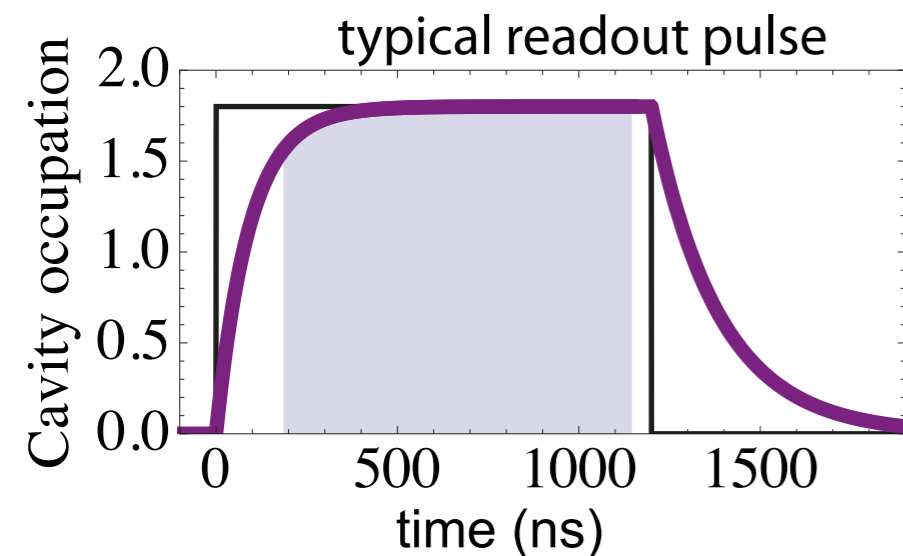
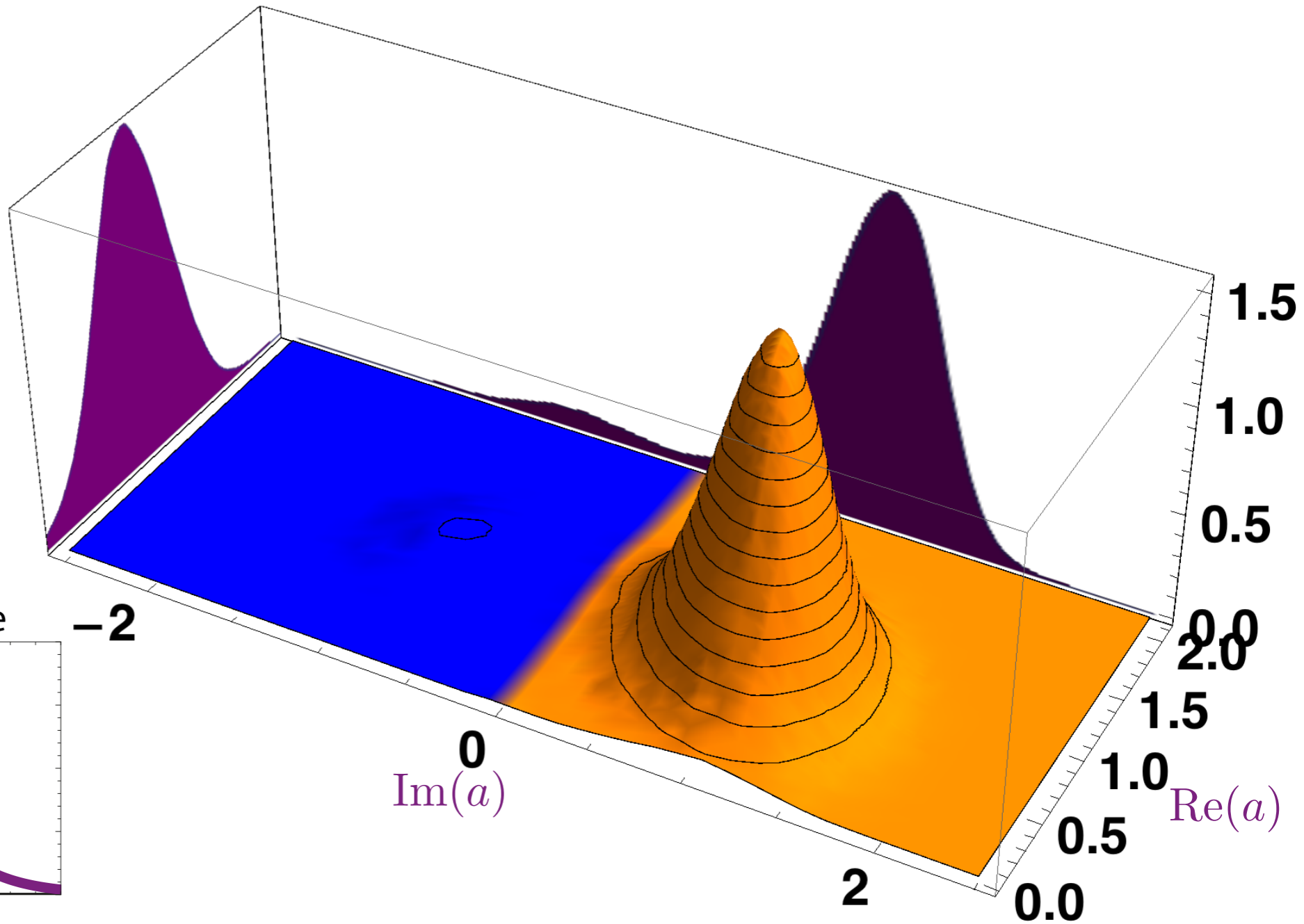
# Single shot qubit state readout

$10^6$  experiments



$$\rho = \frac{1}{2}(\mathbf{1} + x\sigma_x + y\sigma_y + z\sigma_z)$$

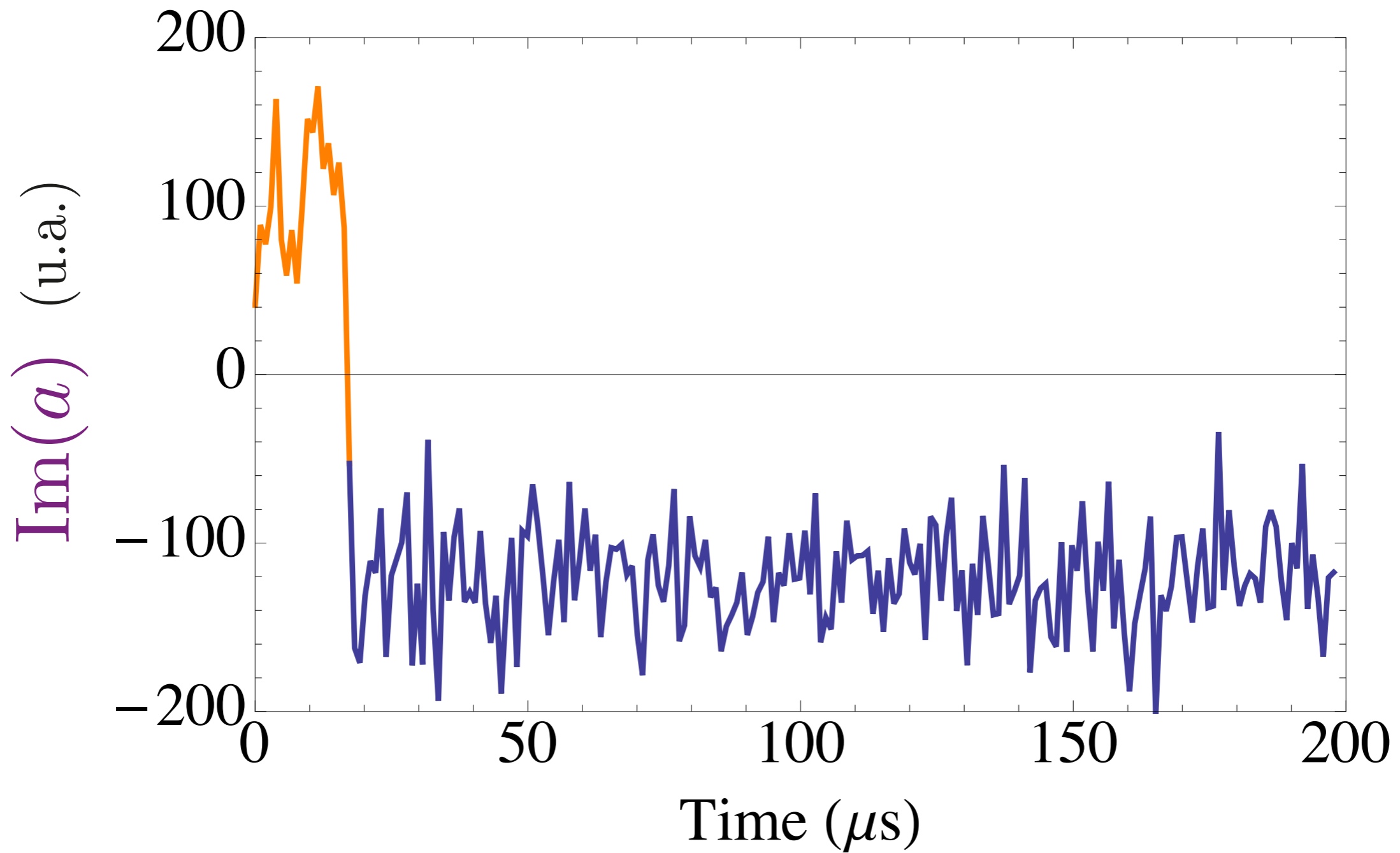
$P(\text{Im}(a) > 0) = 93.5\%$





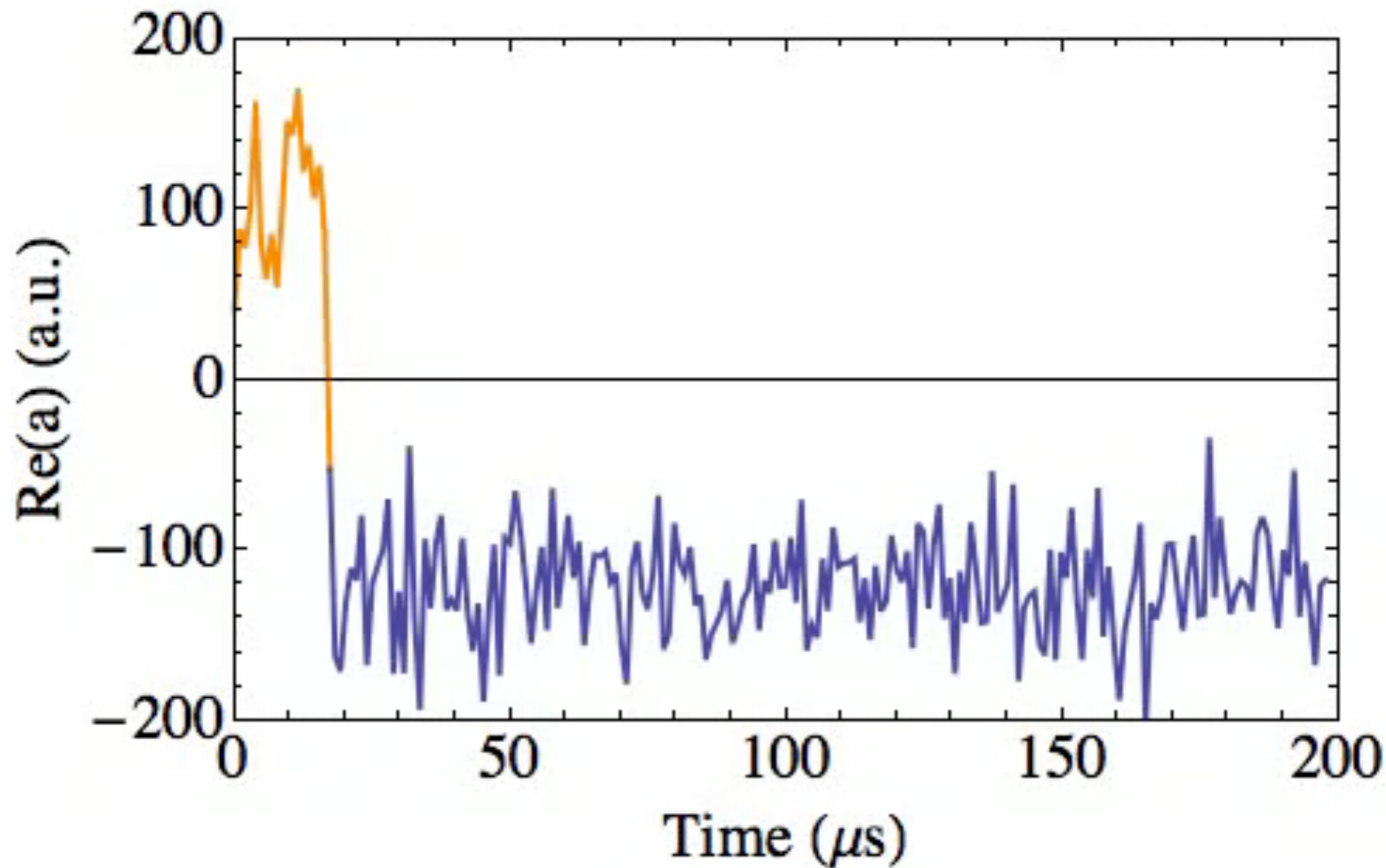
# Quantum jumps

prepare  $|1\rangle$  and continuous measurement at 1.8 photons



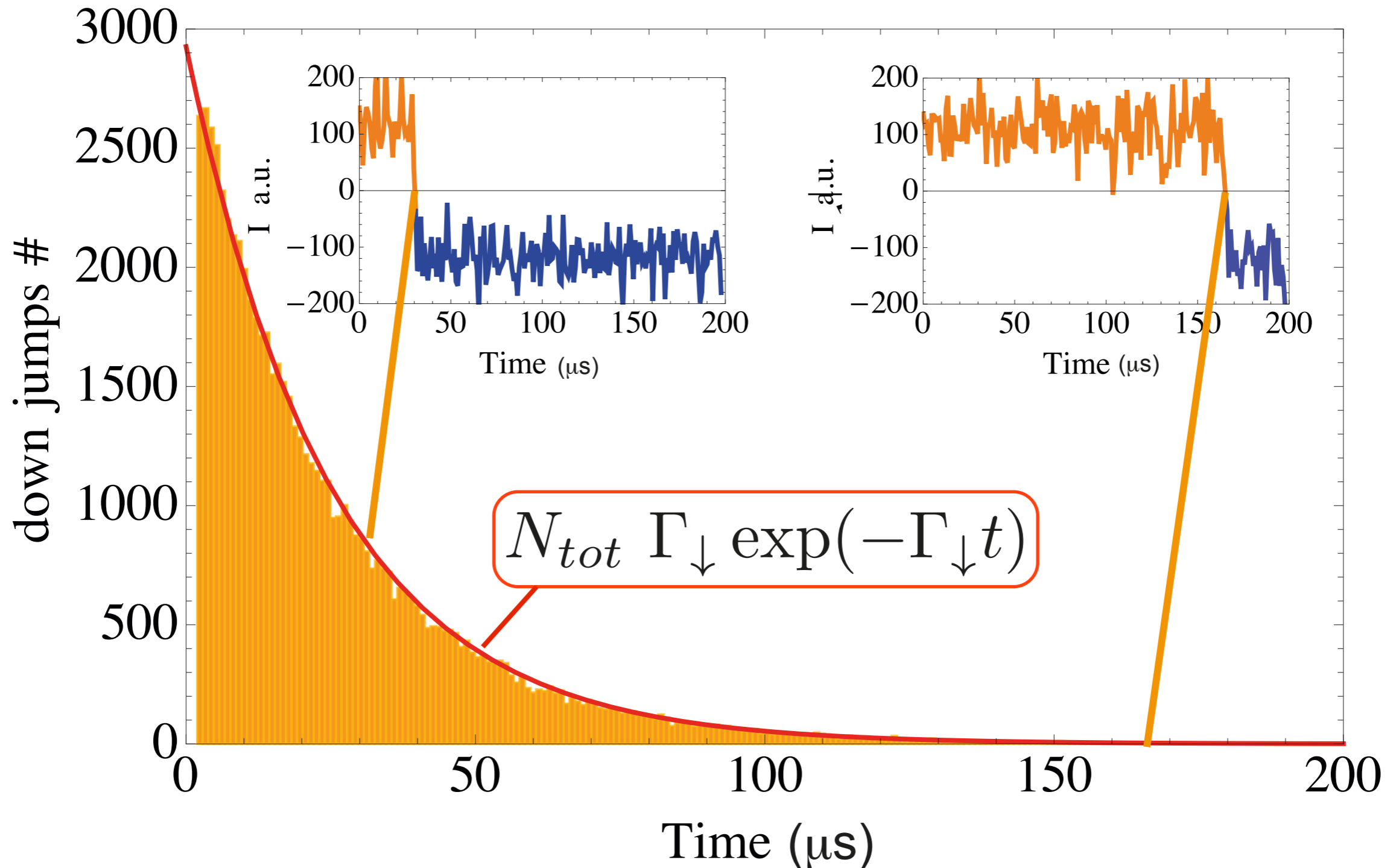
# Quantum jumps

prepare  $|1\rangle$  and continuous measurement at 1.8 photons



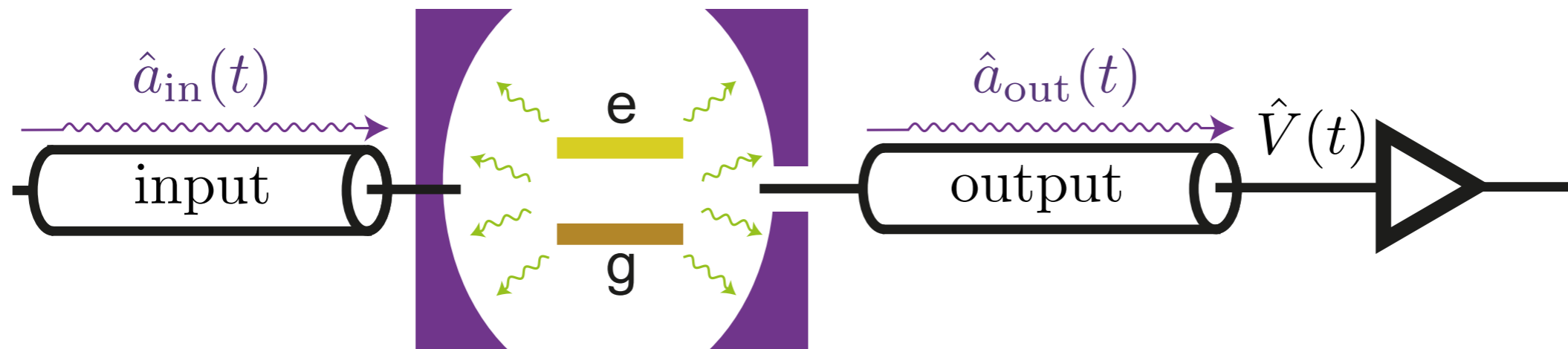
# Quantum jumps

continuous measurement at 1.8 photons

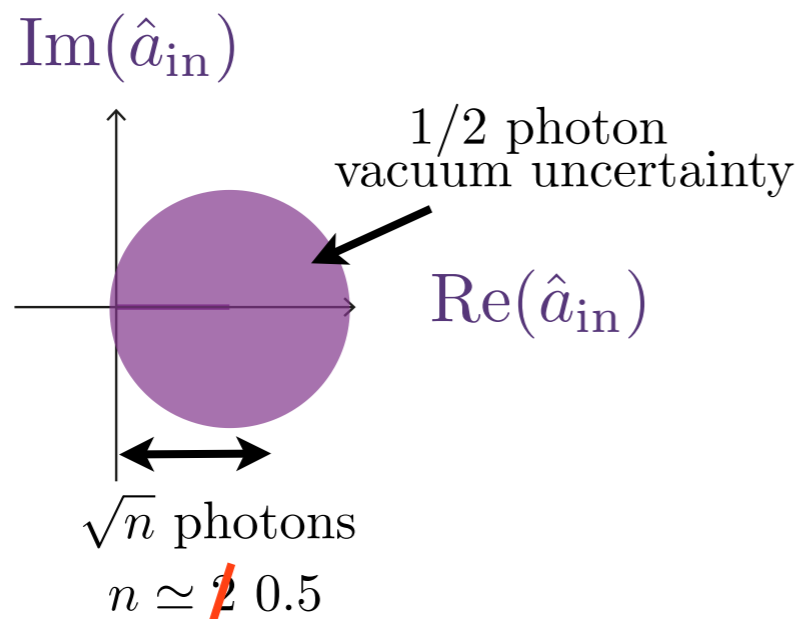


$$\frac{1}{\Gamma_{\downarrow}} \simeq T_1 = 26 \mu\text{s}$$

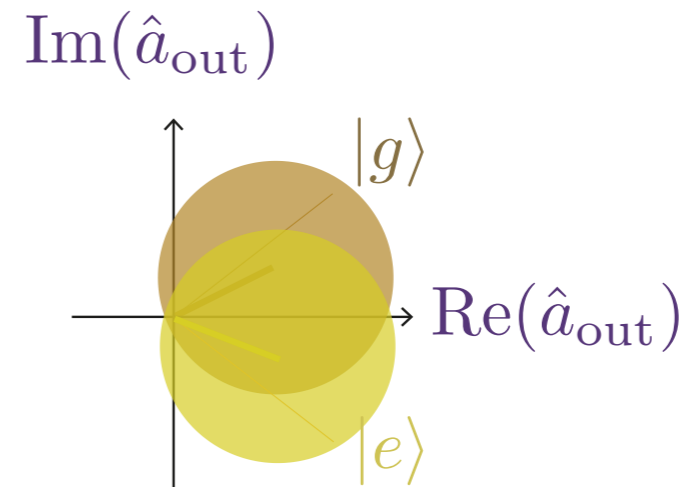
# Weak measurement



field going in ...



... field coming out



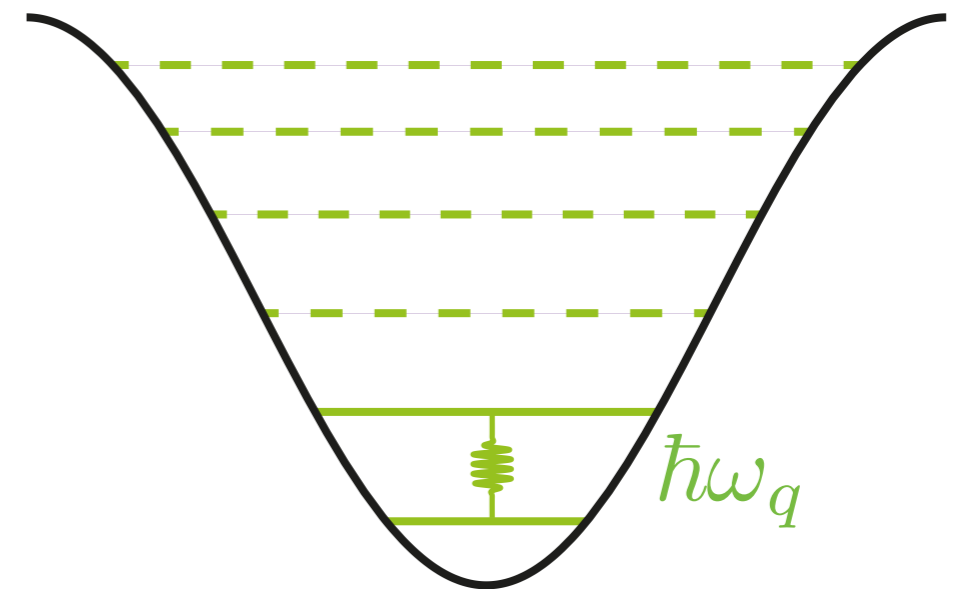
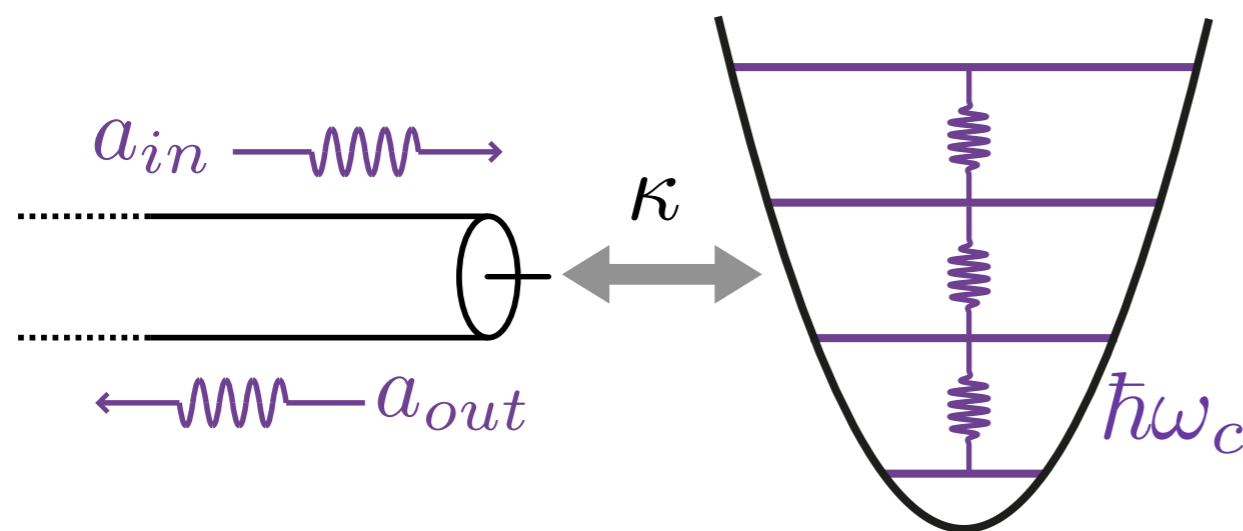
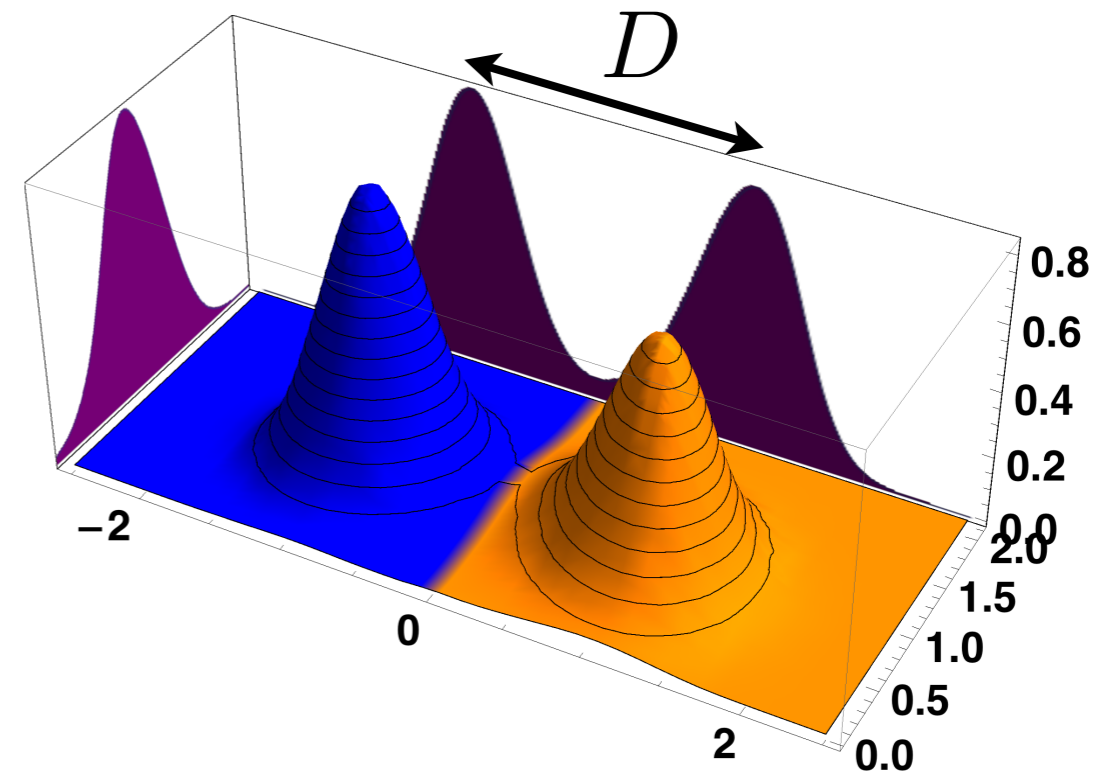
measuring  $\text{Im}(\hat{a}_{out})$   $\longrightarrow$  Weak QND measurement

# Measurement rate

$$\Gamma_m = 2n_{ph} \frac{\chi^2}{\kappa} = \kappa \frac{D^2}{2}$$

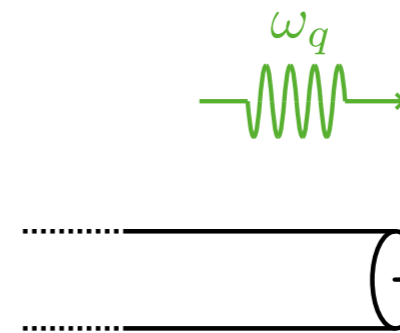
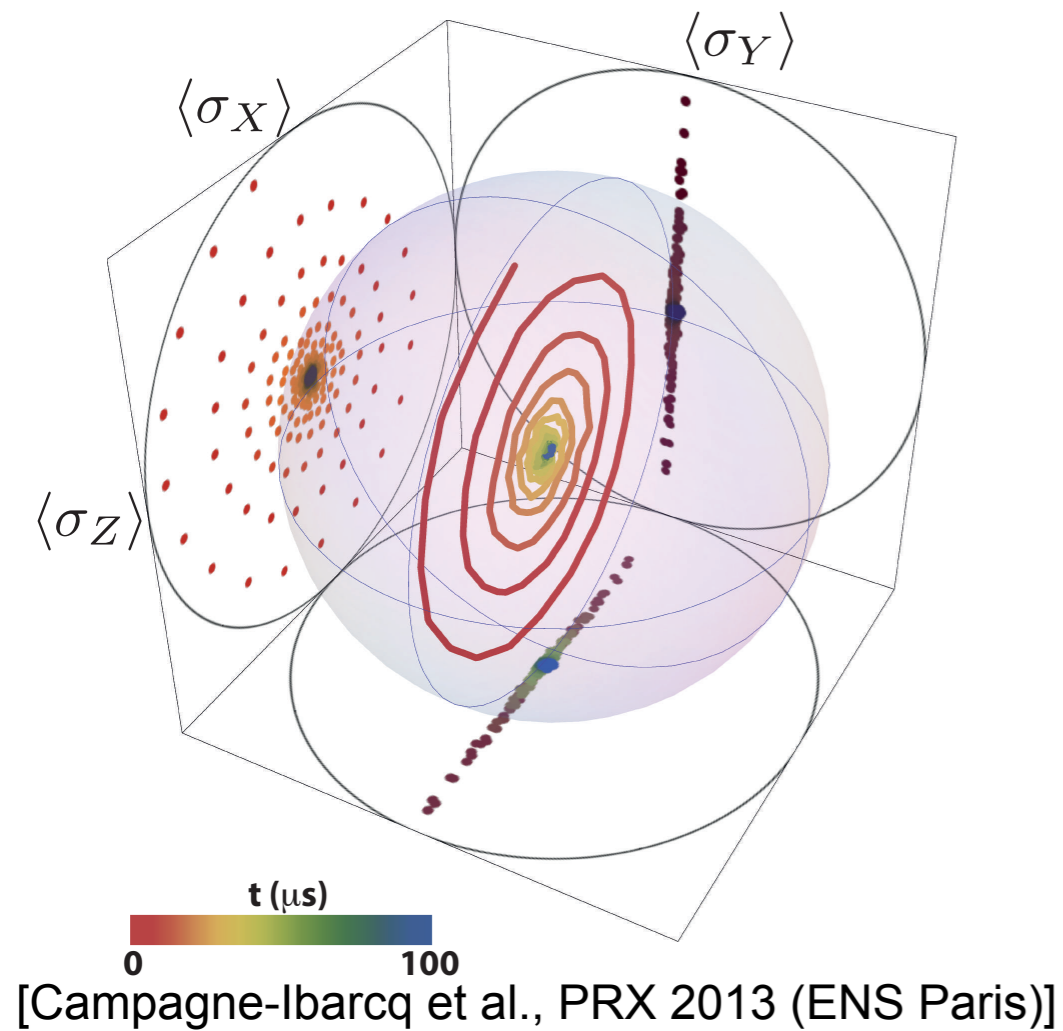
Acquired bits per second  $\frac{\Gamma_m}{\ln(2)}$

if unread, extra dephasing  $\Gamma_m$



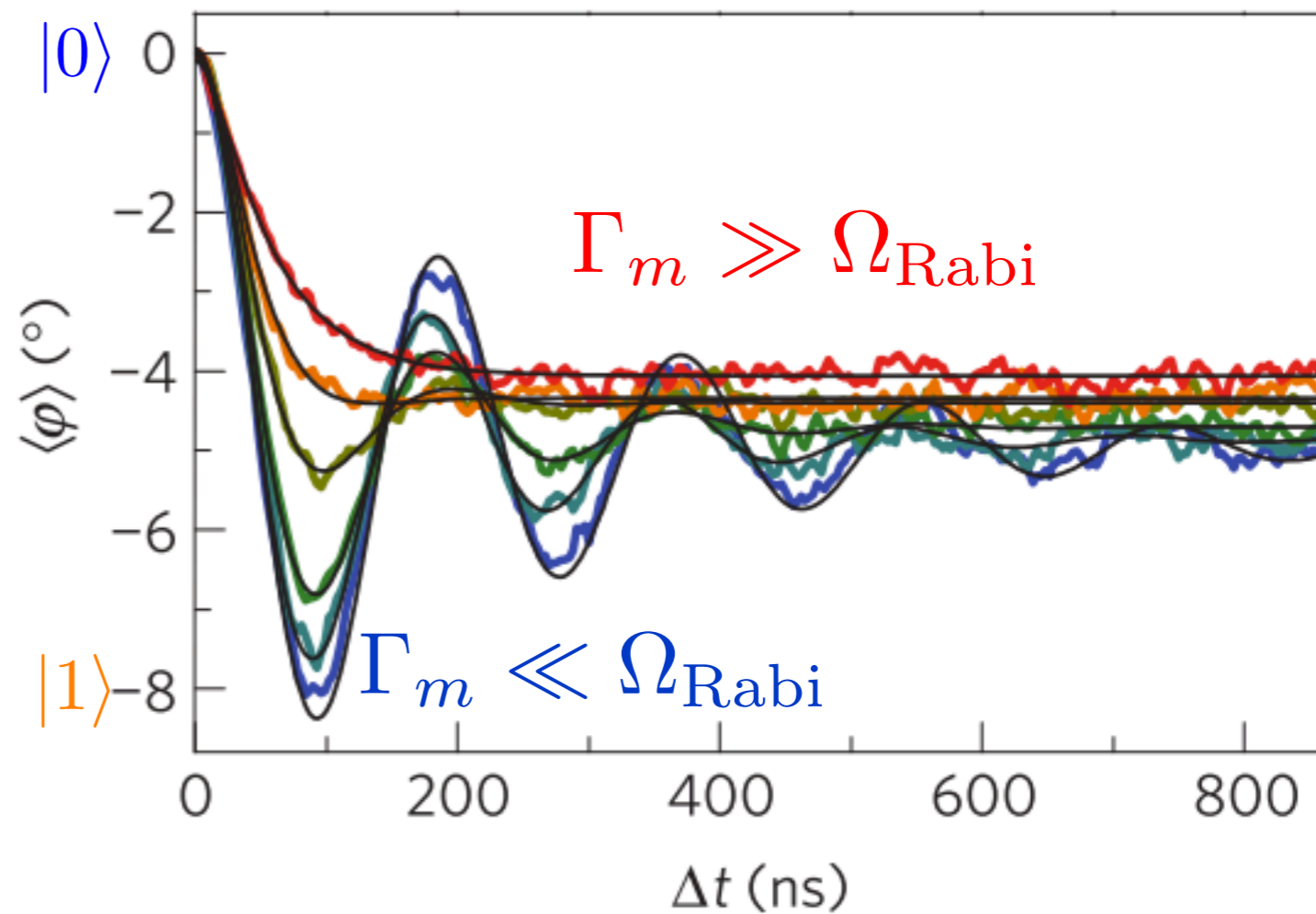
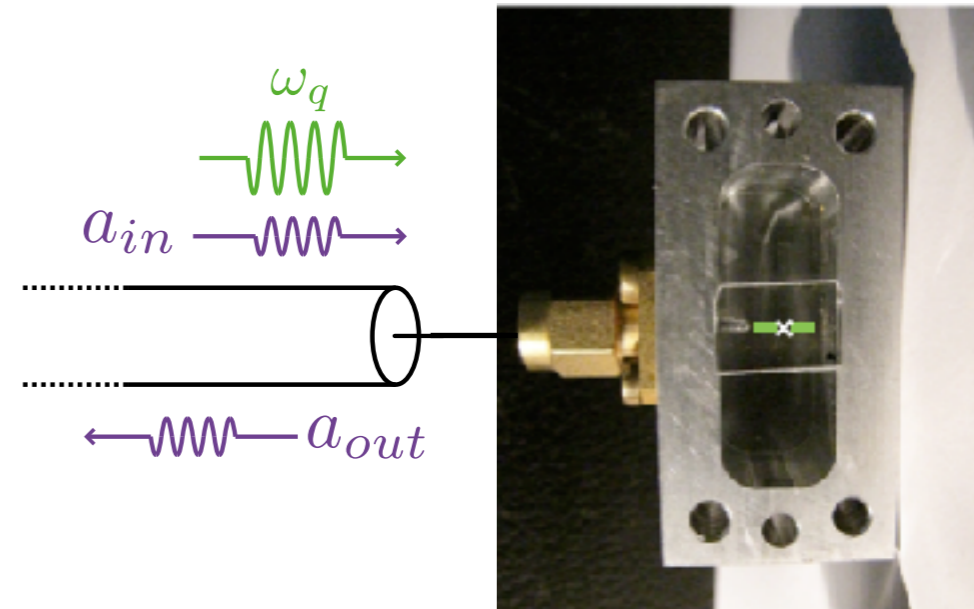
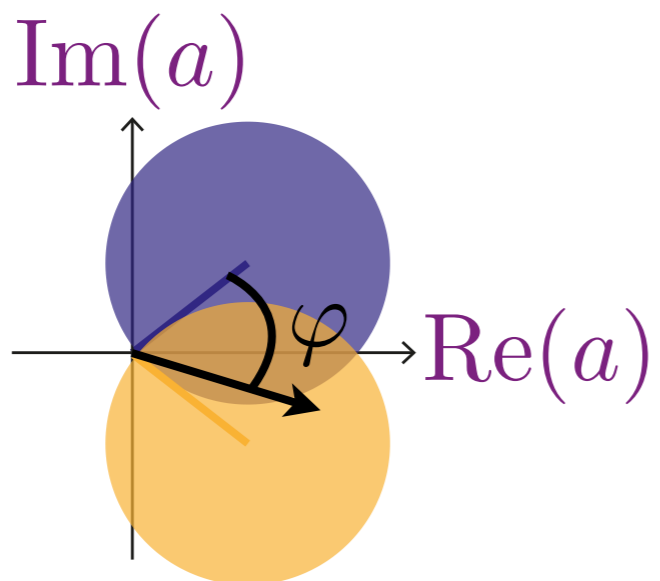
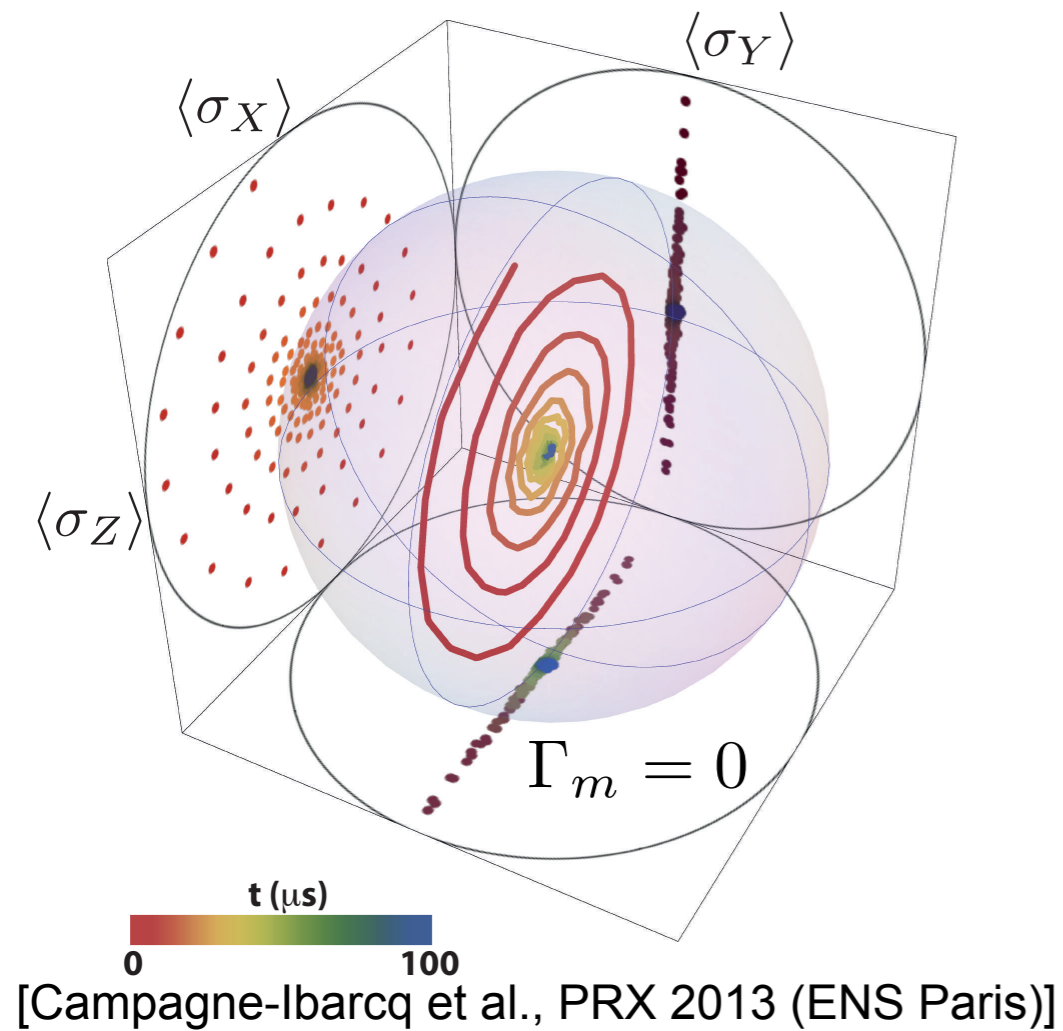
$$H_{\text{coupl}} = \hbar\chi a^\dagger a \frac{\sigma_z}{2}$$

# Zeno effect on Rabi oscillations





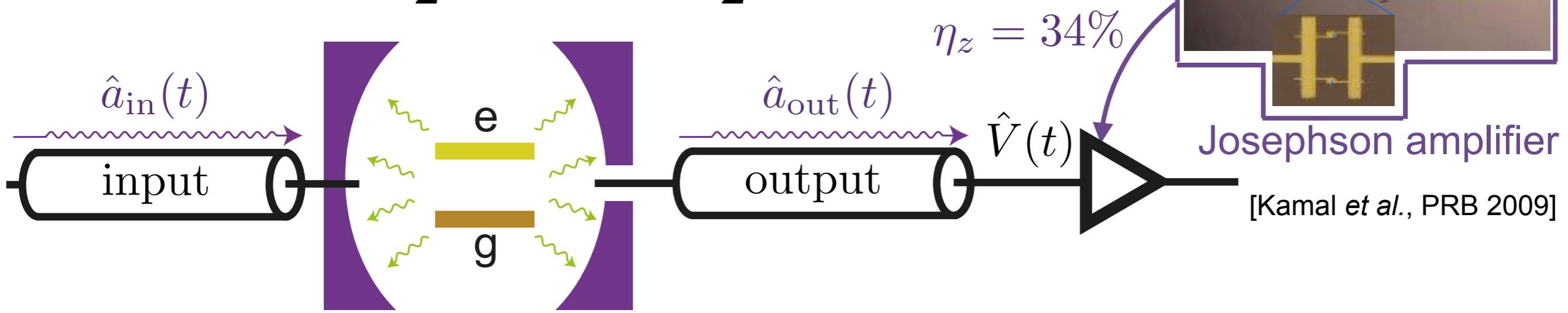
# Zeno effect on Rabi oscillations



[Palacios-Laloy et al., Nat. Phys. 2010 (Saclay)]

# Dispersive Measurement

$$H = hf_q \frac{\sigma_z}{2} + h(f_c - \frac{\chi}{2} \sigma_z) a^\dagger a$$



jump operator  $L_z = \sqrt{\frac{\Gamma_d}{2}} \sigma_z$

$$dw_t = \underbrace{\sqrt{2\eta_z \Gamma_d} \langle \sigma_z \rangle_{\rho_t} dt}_{\text{average outcome}} + \underbrace{dW_{t,3}}_{\text{noise (Wiener)}}$$

$\{dw_t\}$   $\xrightarrow{\text{stochastic master equation}}$   $\rho_t^A$

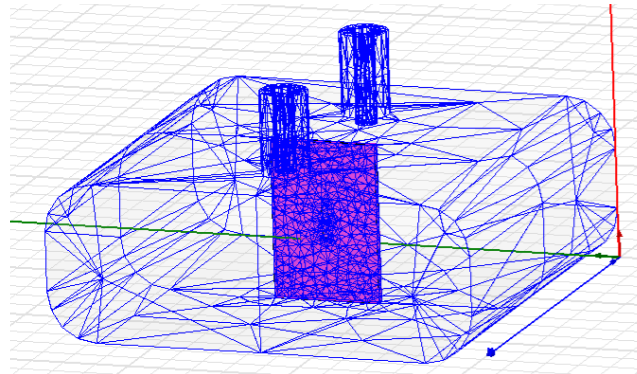
**we will trajectories see later**



Wiener Process  
 $\mathbb{E}(dW_{t,i}) = 0$   
 $dW_{t,i}^2 = dt$

[Murch et al., Nature 2013]  
 [Hatridge et al., Science 2013]

# Quantum trajectories and feedback in circuit-QED



## Introduction to circuit-QED

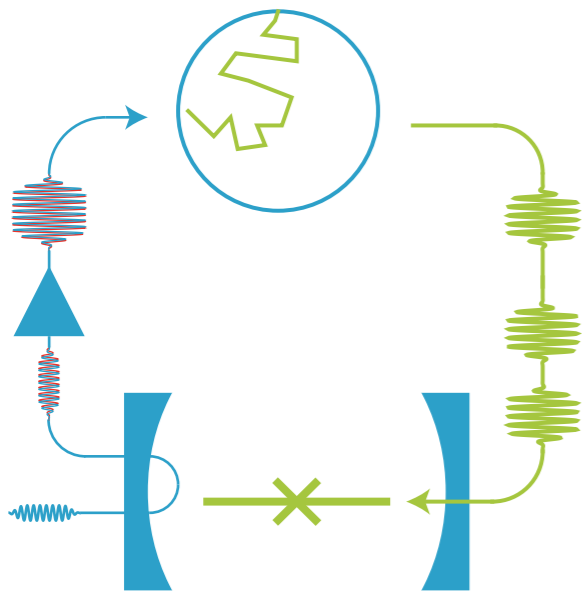
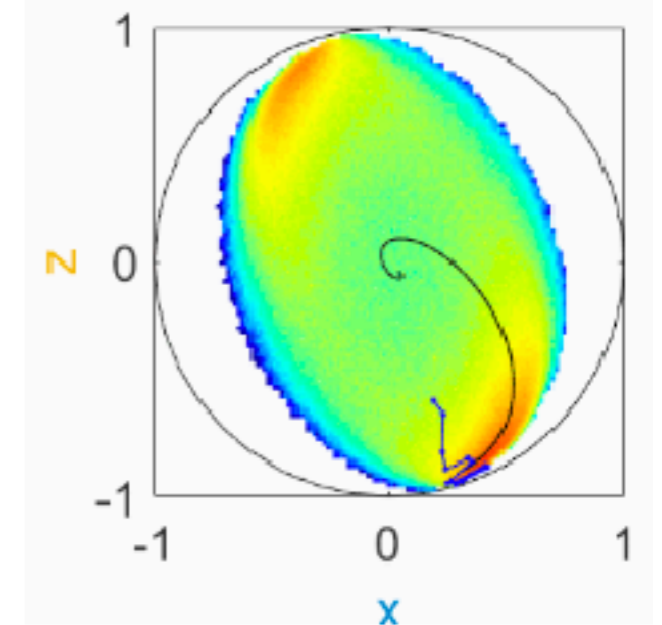
## Quantum trajectories in circuit-QED

dispersive measurement

fluorescence measurement

both simultaneously

generating entanglement



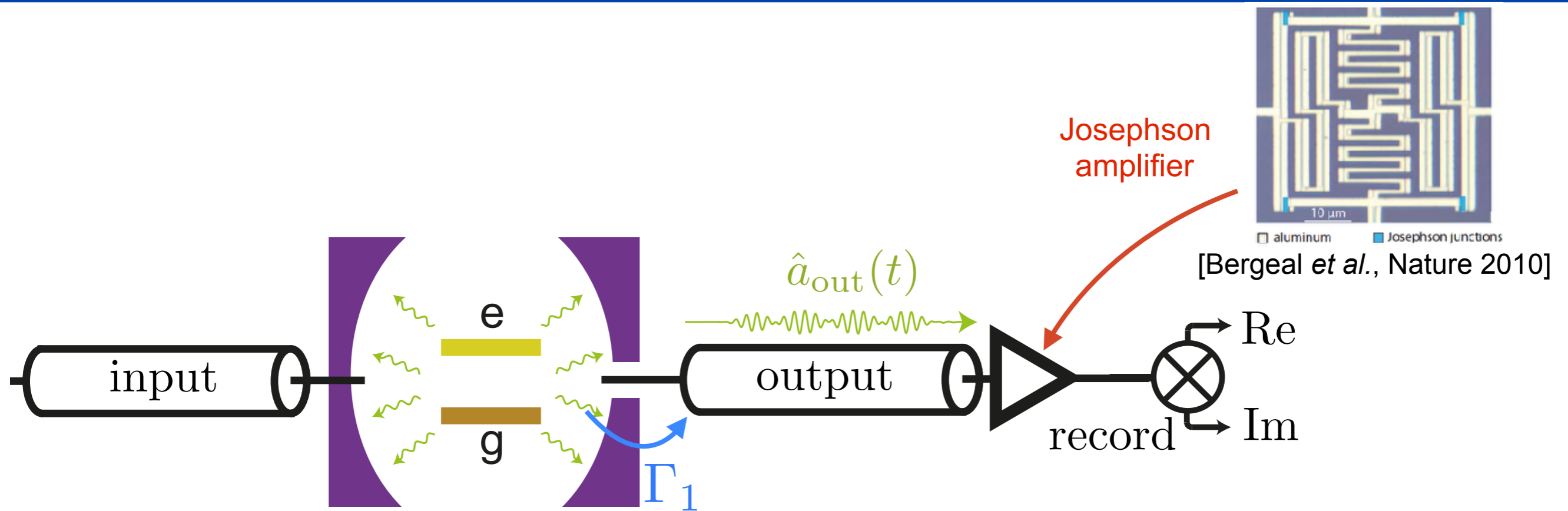
## Measurement based feedback

dispersive case

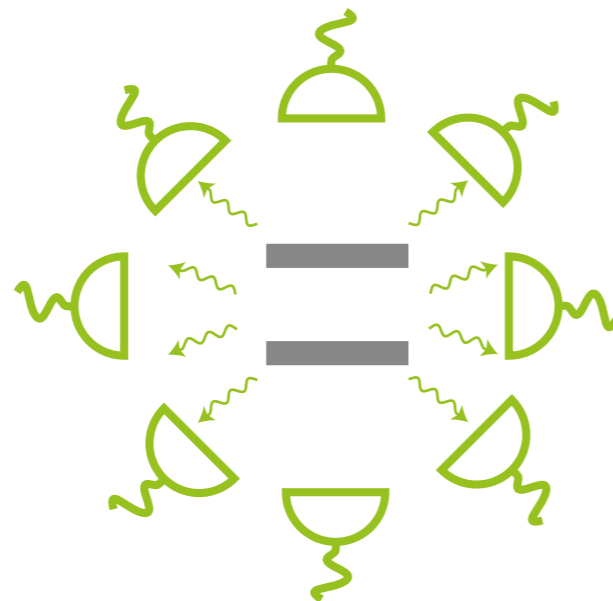
fluorescence case



# Fluorescence Measurement

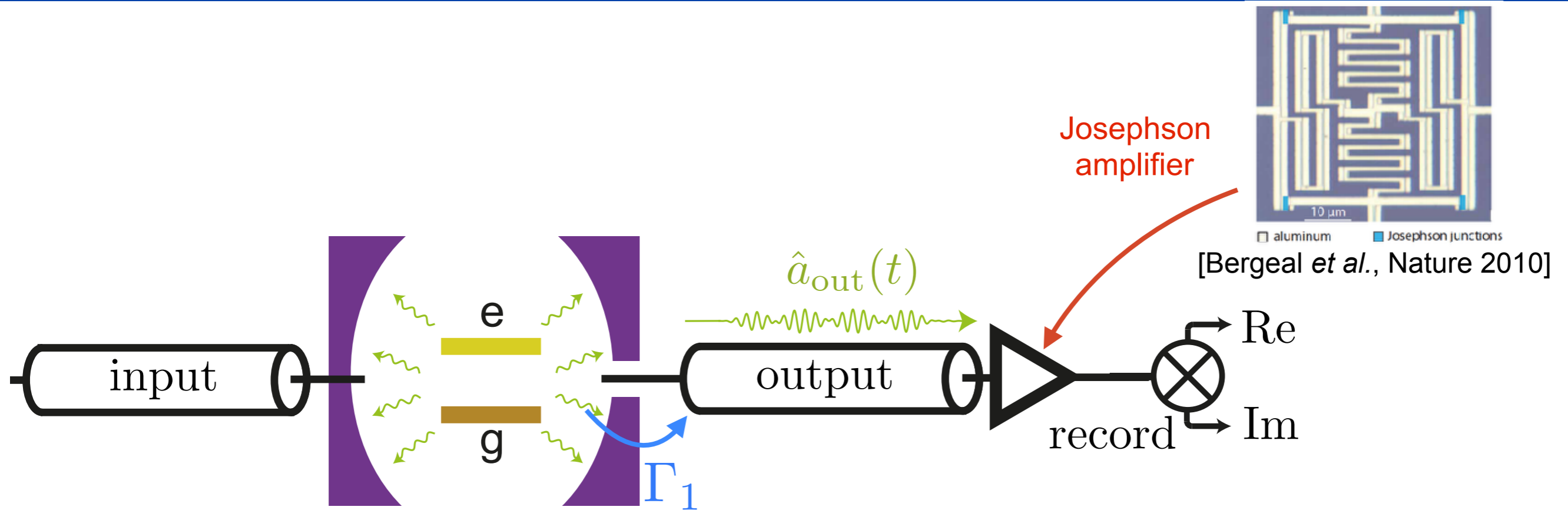


similar to our  
gedanken experiment





# Fluorescence Measurement



mean signal

$$\langle \hat{a}_{out} \rangle \propto \sqrt{\Gamma_1} \langle \sigma_- \rangle$$

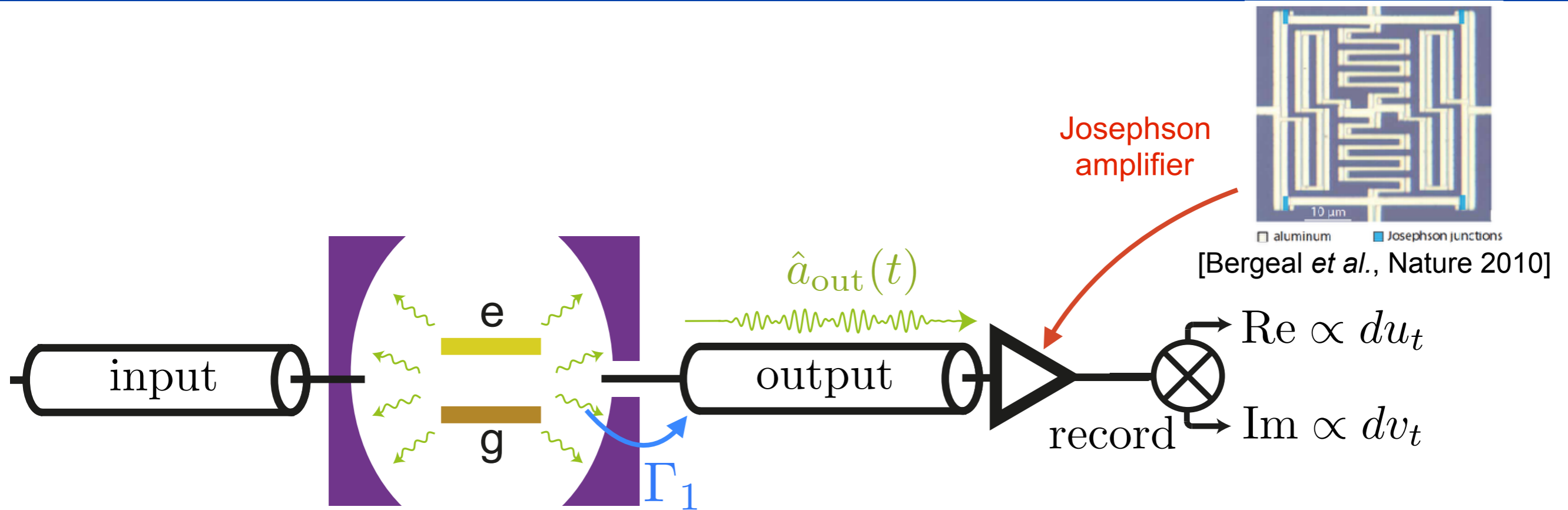


jump operator  $\propto \sigma_- = |g\rangle \langle e| = \frac{\sigma_x - i\sigma_y}{2}$

$$\Gamma_1 = (12.5 \mu\text{s})^{-1}$$



# Fluorescence Measurement

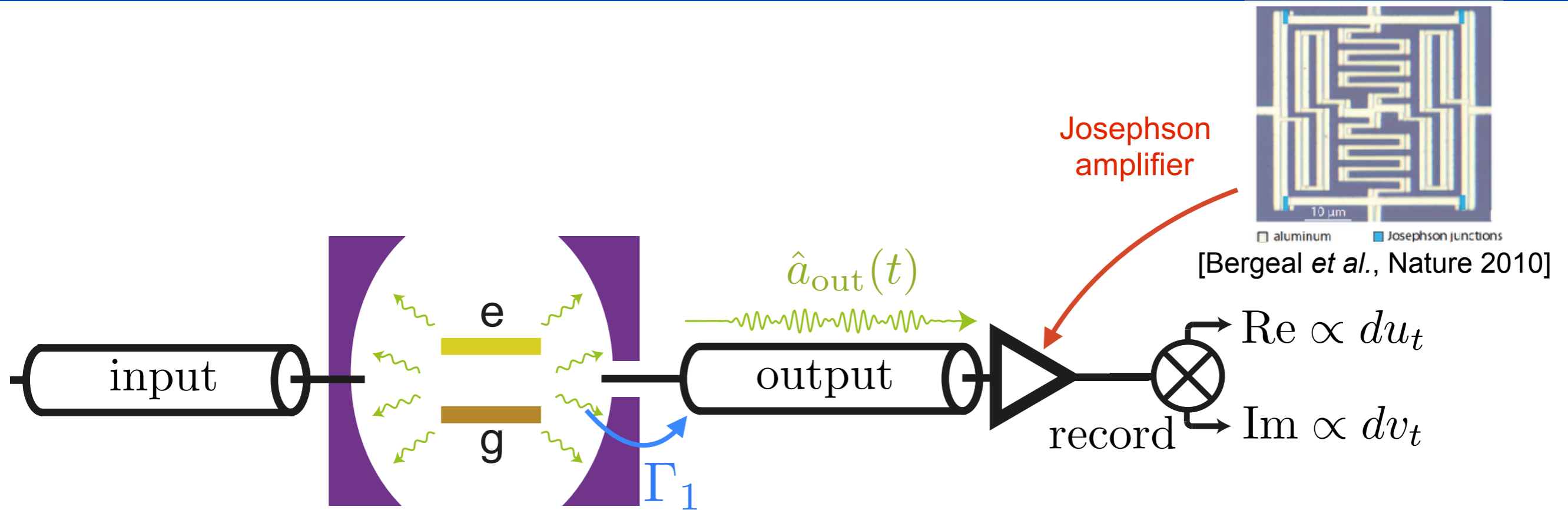


$$du_t = \sqrt{\frac{\eta\Gamma_1}{2}} \langle \sigma_X \rangle_{\rho_t} dt + dW_{t,1}$$

$$dv_t = \sqrt{\frac{\eta\Gamma_1}{2}} \langle \sigma_Y \rangle_{\rho_t} dt + dW_{t,2}$$



# Fluorescence Measurement



$$du_t = \sqrt{\frac{\eta\Gamma_1}{2}} \langle \sigma_X \rangle_{\rho_t} dt + dW_{t,1}$$

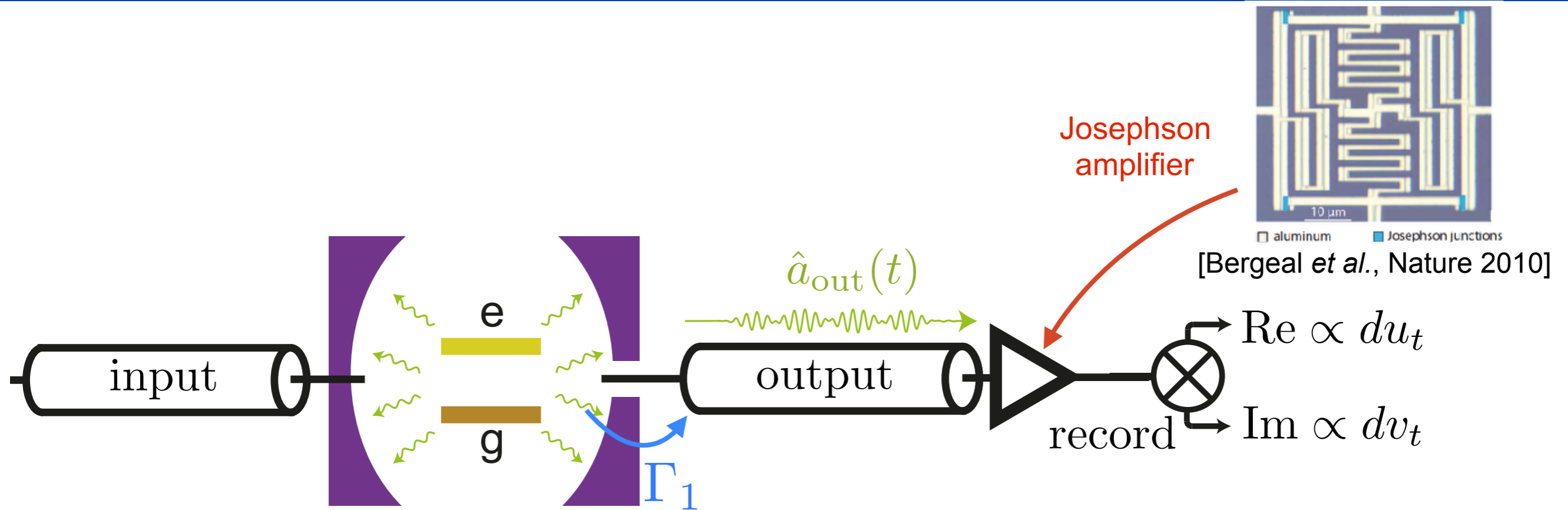
$$dv_t = \sqrt{\frac{\eta\Gamma_1}{2}} \langle \sigma_Y \rangle_{\rho_t} dt + dW_{t,2}$$

average outcome

noise (Wiener)



# Fluorescence Measurement



$$du_t = \sqrt{\frac{\eta\Gamma_1}{2}} \langle \sigma_X \rangle_{\rho_t} dt + dW_{t,1}$$

$$dv_t = \sqrt{\frac{\eta\Gamma_1}{2}} \langle \sigma_Y \rangle_{\rho_t} dt + dW_{t,2}$$

average outcome

noise (Wiener)



$\{du_t, dv_t\}$   $\xrightarrow{\text{stochastic master equation}}$   $\rho_t^B$

[Campagne-Ibarcq et al., PRX 2016]  
 [Naghiloo et al., Nat. Comm. 2016]  
 [Ficheux et al., Nat. Comm. 2018]

# Quantum Trajectories - SME

Stochastic Master Equation for a continuous and weak measurement

$$d\rho_t = -\frac{i}{\hbar}[H, \rho_t]dt + \sum_{i=1}^m \mathcal{D}_i(\rho_t)dt$$

Decoherence  $\mathcal{D}_i(\rho_t) = L_i \rho_t L_i^\dagger - \frac{1}{2} \rho_t L_i^\dagger L_i - \frac{1}{2} L_i^\dagger L_i \rho_t$



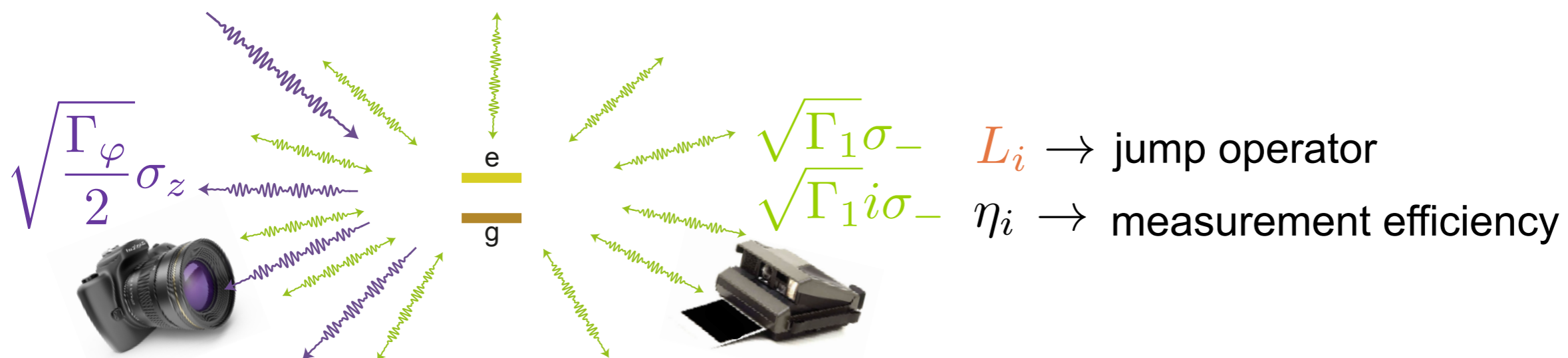
# Quantum Trajectories - SME

Stochastic Master Equation for a continuous and weak measurement

$$d\rho_t = -\frac{i}{\hbar}[H, \rho_t]dt + \sum_{i=1}^m \mathcal{D}_i(\rho_t)dt + \sum_{i=1}^m \sqrt{\eta_i} \mathcal{M}_i(\rho_t) dW_{t,i}$$

Decoherence  $\mathcal{D}_i(\rho_t) = L_i \rho_t L_i^\dagger - \frac{1}{2} \rho_t L_i^\dagger L_i - \frac{1}{2} L_i^\dagger L_i \rho_t$

Innovation  $\mathcal{M}_i(\rho_t) = L_i \rho_t + \rho_t L_i^\dagger - \text{Tr}(L_i \rho_t + \rho_t L_i^\dagger) \rho_t$



# Quantum Trajectories - SME

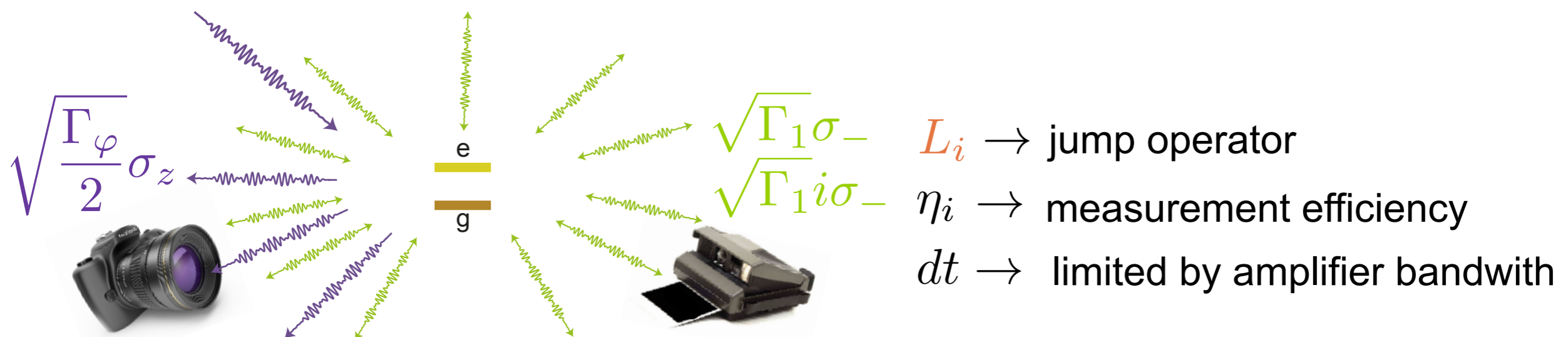
Stochastic Master Equation for a continuous and weak measurement

$$d\rho_t = -\frac{i}{\hbar}[H, \rho_t]dt + \sum_{i=1}^m \mathcal{D}_i(\rho_t)dt + \sum_{i=1}^m \sqrt{\eta_i} \mathcal{M}_i(\rho_t) dW_{t,i}$$

Decoherence  $\mathcal{D}_i(\rho_t) = L_i \rho_t L_i^\dagger - \frac{1}{2} \rho_t L_i^\dagger L_i - \frac{1}{2} L_i^\dagger L_i \rho_t$

Innovation  $\mathcal{M}_i(\rho_t) = L_i \rho_t + \rho_t L_i^\dagger - \text{Tr}(L_i \rho_t + \rho_t L_i^\dagger) \rho_t$

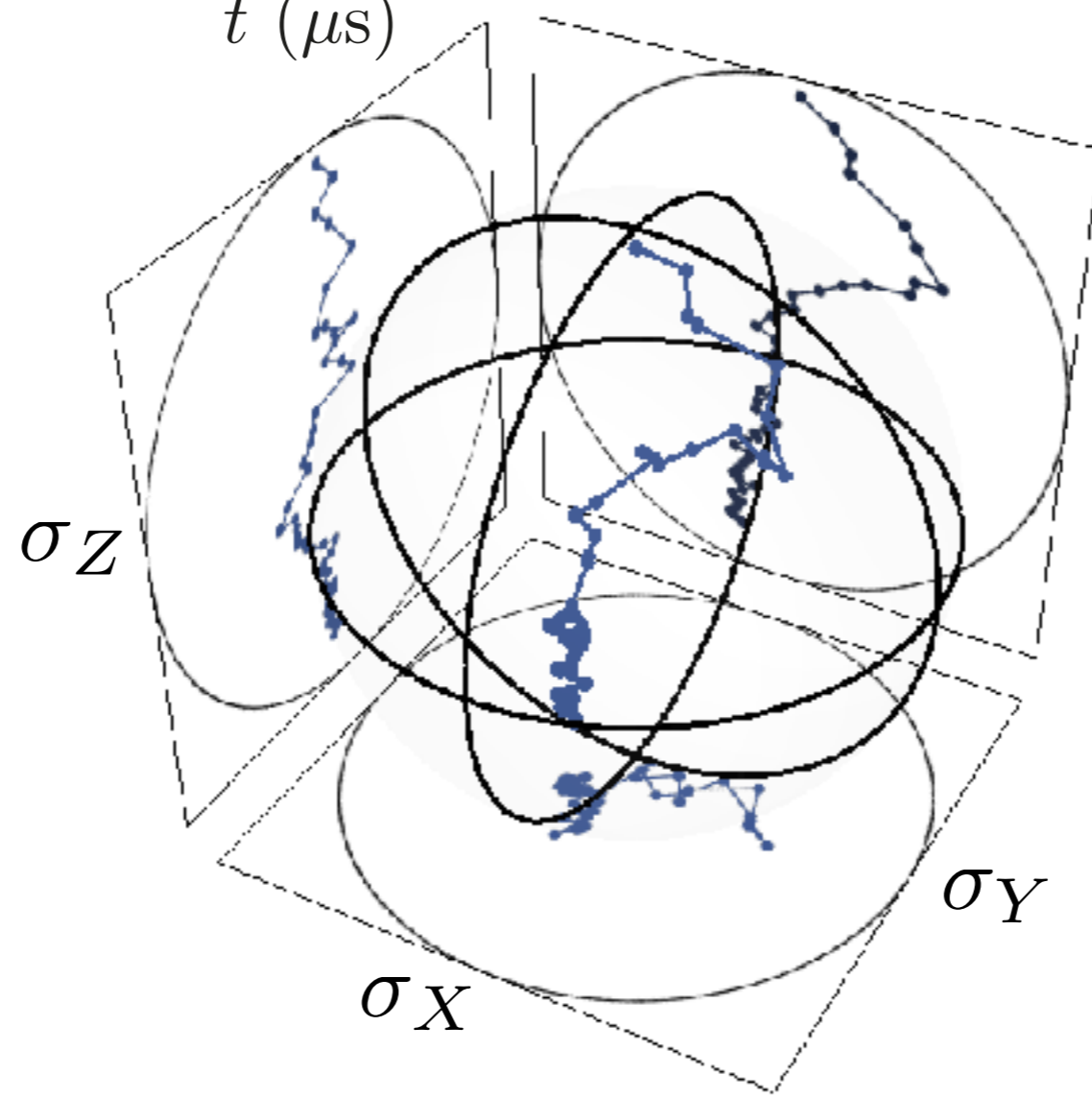
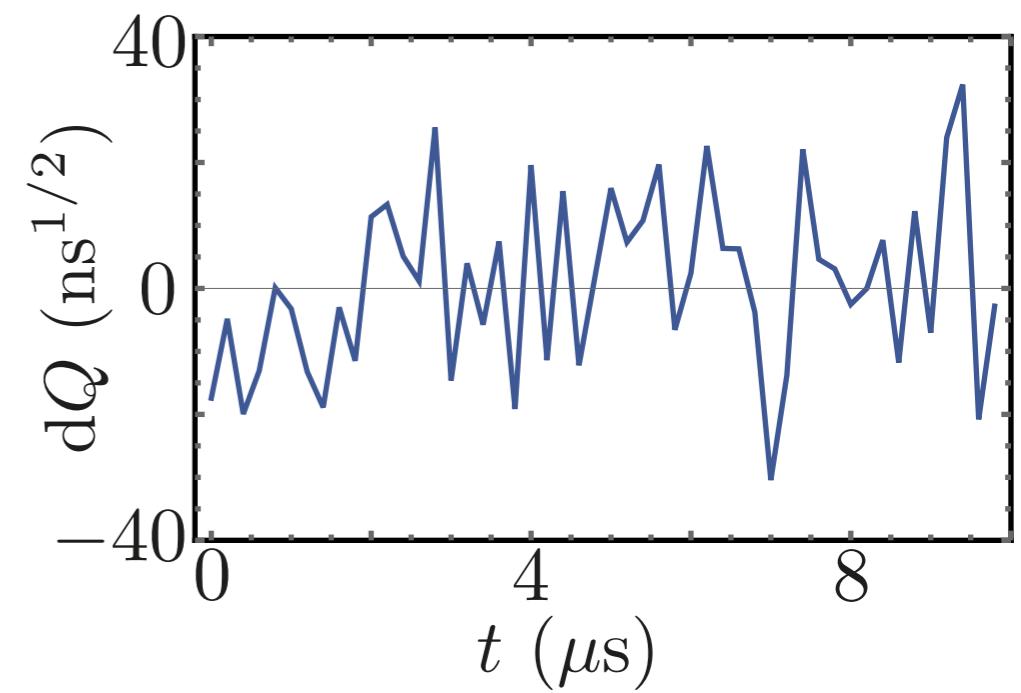
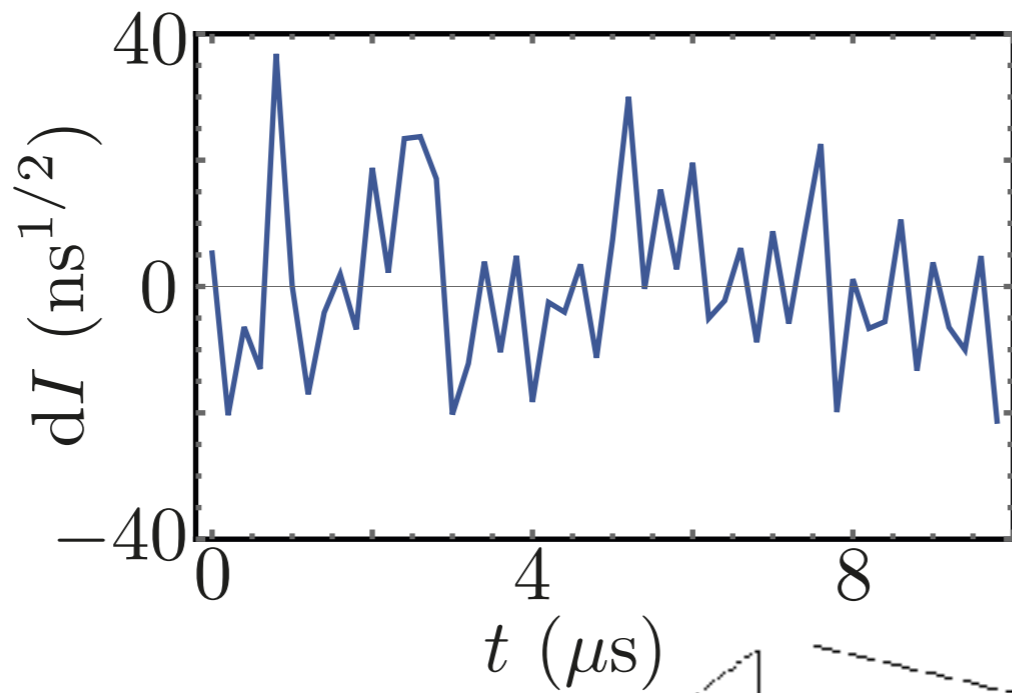
Measurement records  $dy_t^i = \sqrt{\eta_i} \text{Tr}(L_i \rho_t + \rho_t L_i^\dagger) dt + dW_{t,i}$





# Quantum trajectory

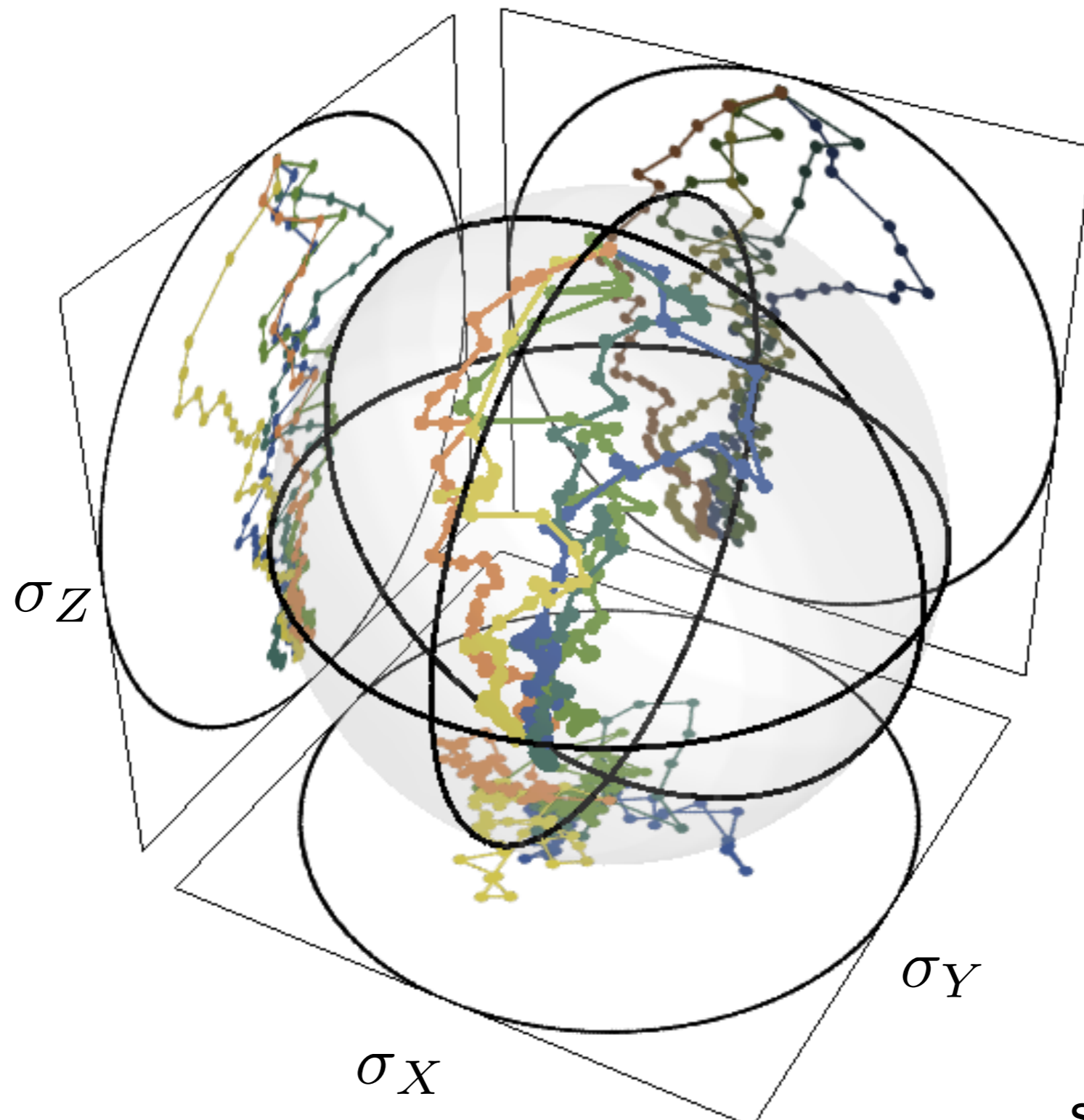
corrected  
for  
JPC low-  
pass filter



start from  $|e\rangle$   
at  $t = 0$

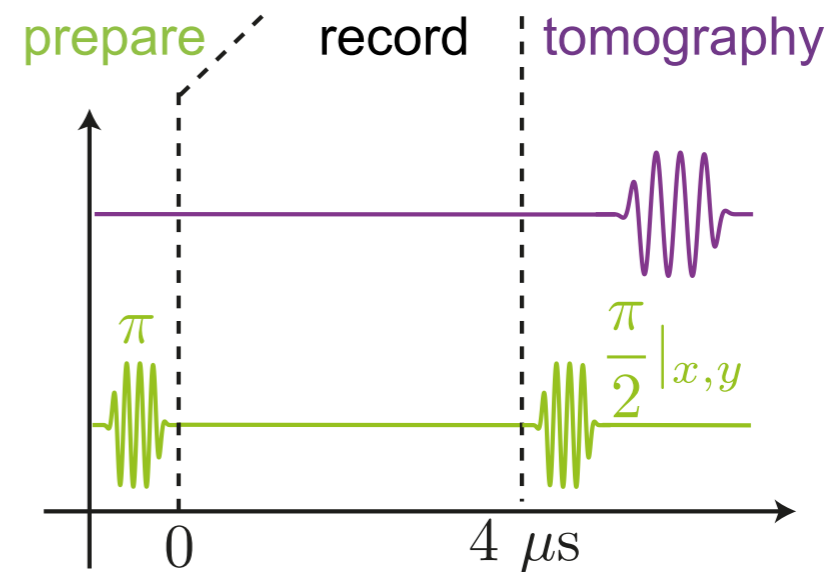
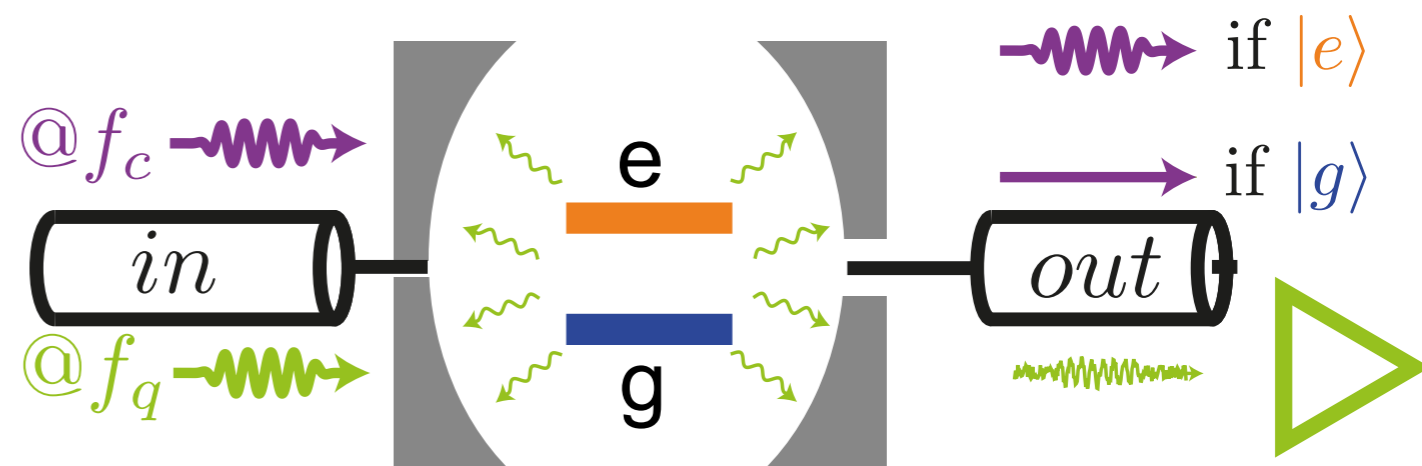
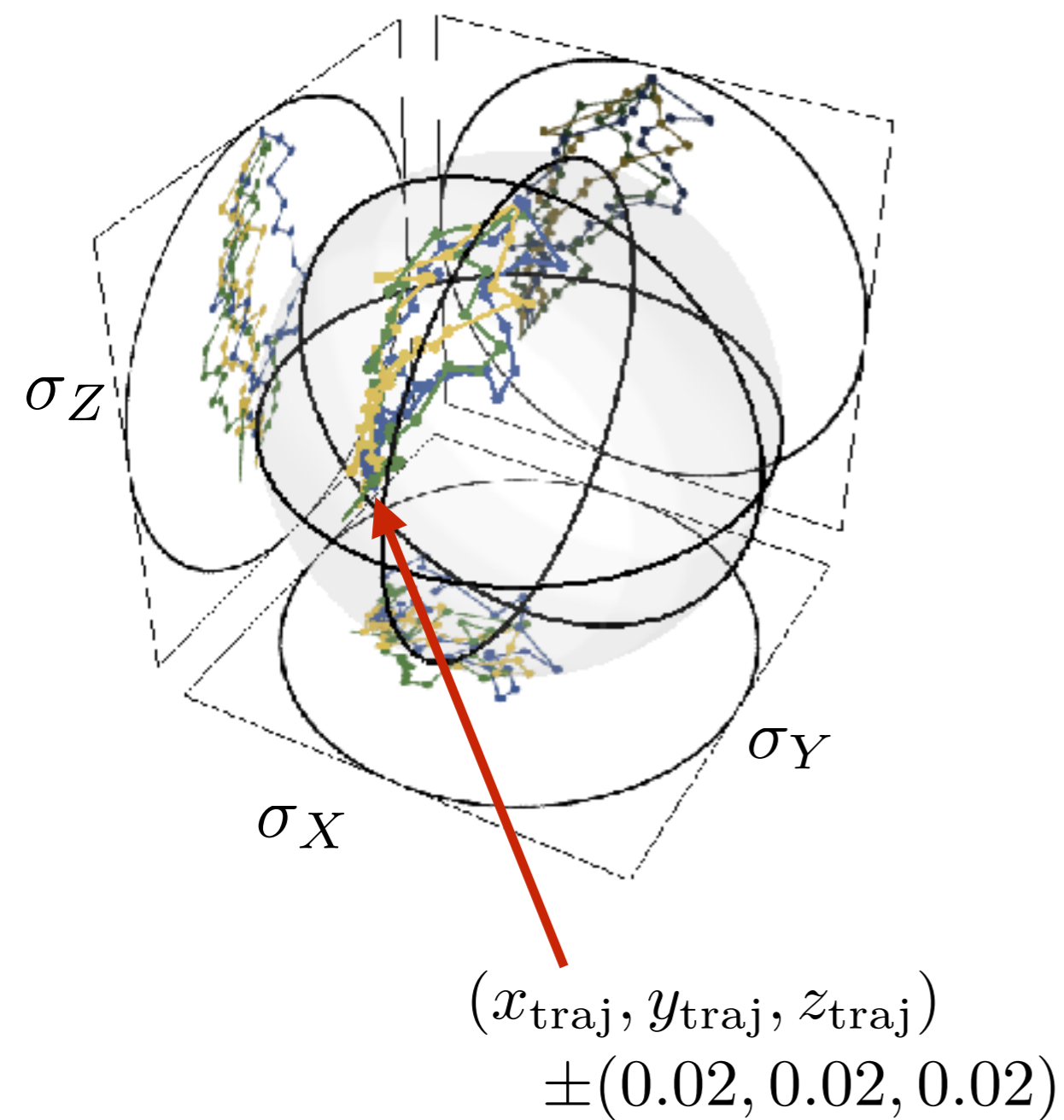
# 5 Quantum trajectories

$$T_{\text{traj}} = 10 \mu\text{s}$$
$$T_1 = 4 \mu\text{s}$$

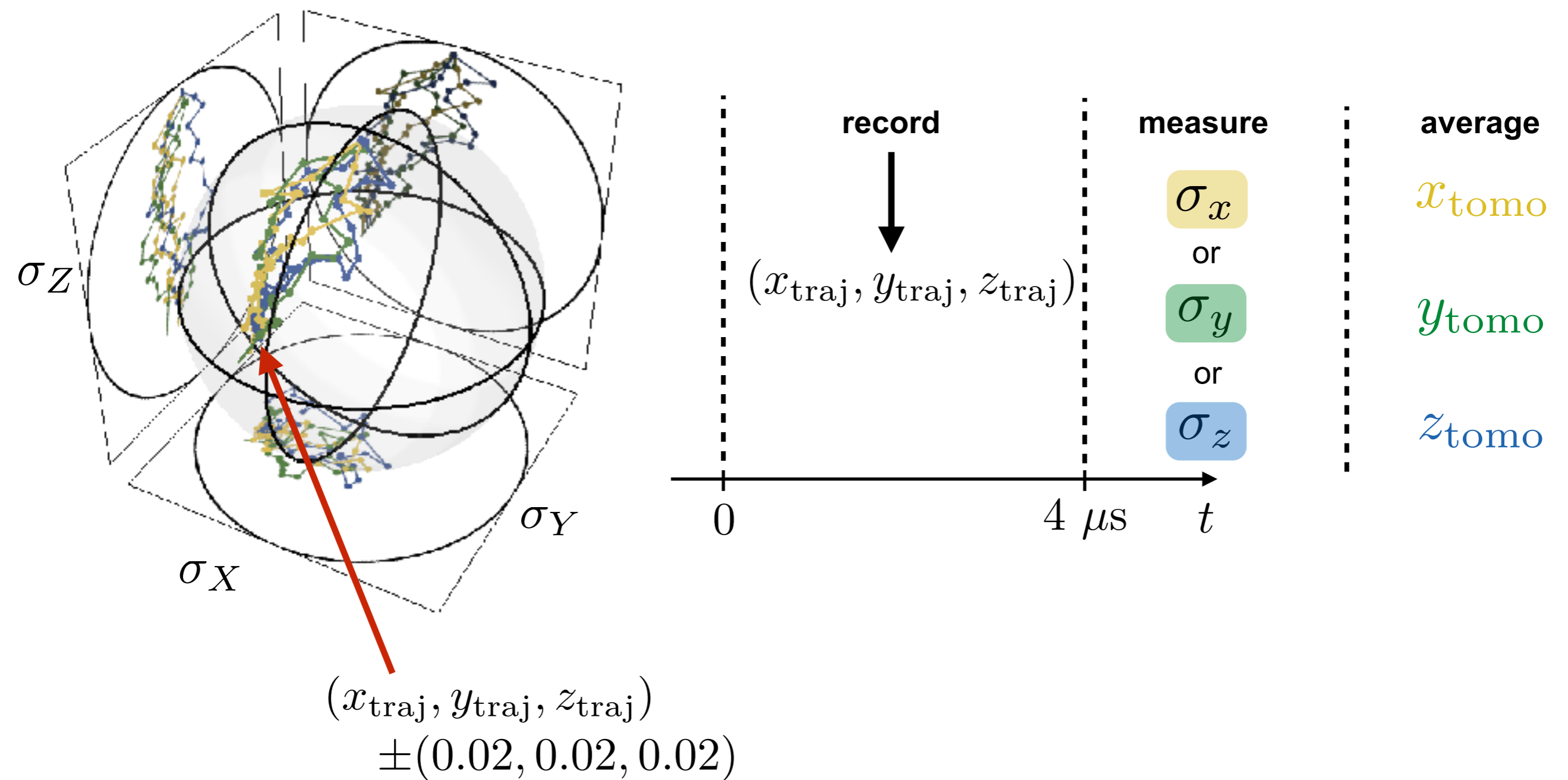


start from  $|e\rangle$   
at  $t = 0$

# Trajectories vs tomography

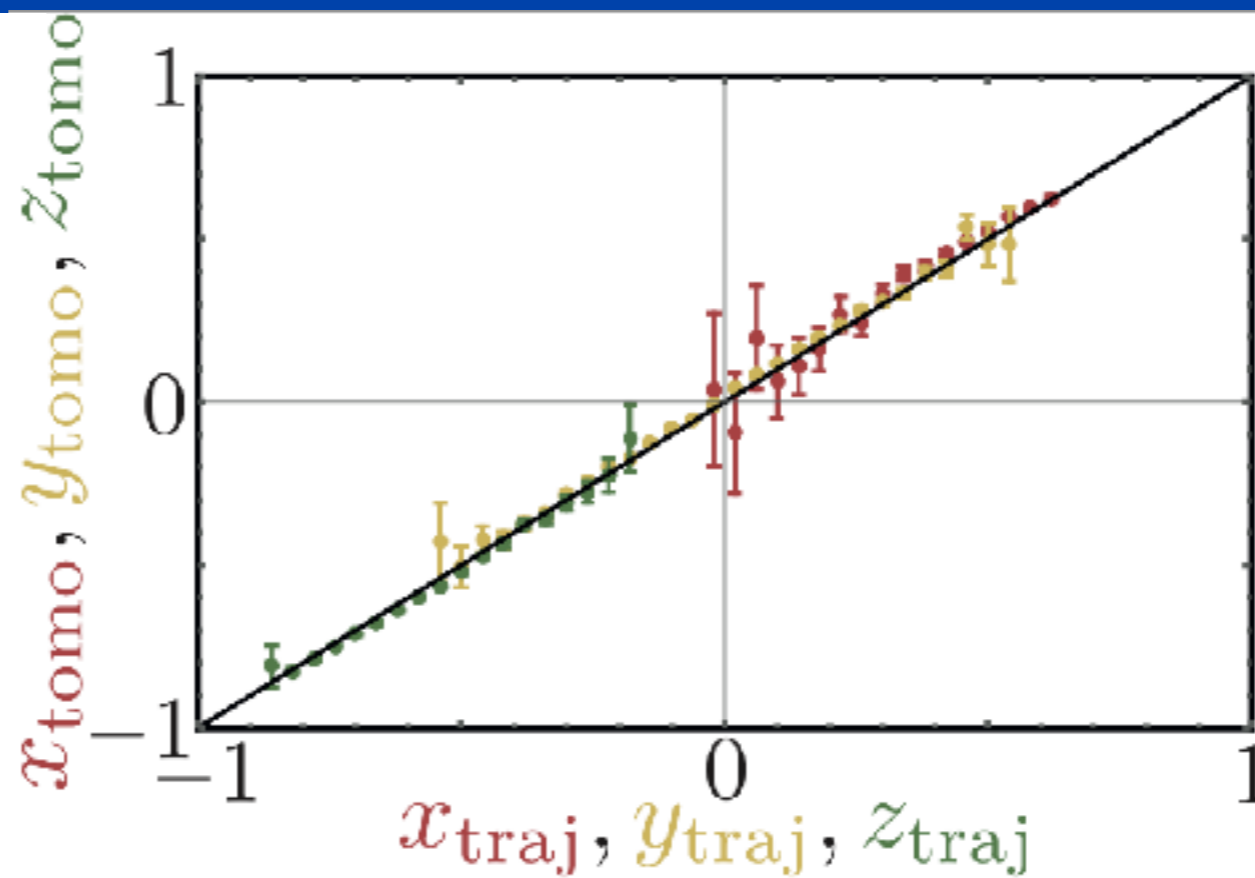


# Trajectories vs tomography



# Trajectories vs tomography

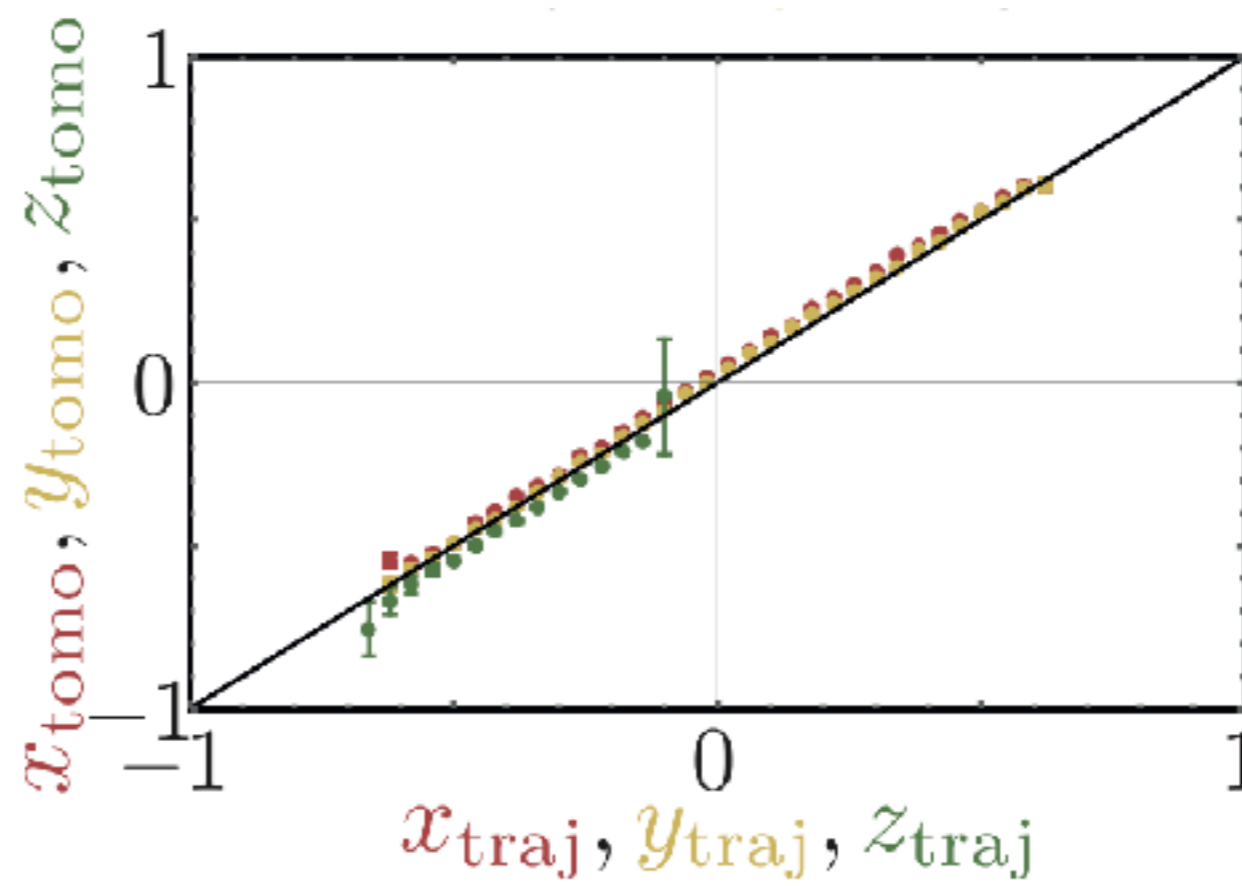
from  $\frac{|g\rangle + |e\rangle}{\sqrt{2}}$



after  $4 \mu\text{s}$

$\eta = 24\%$

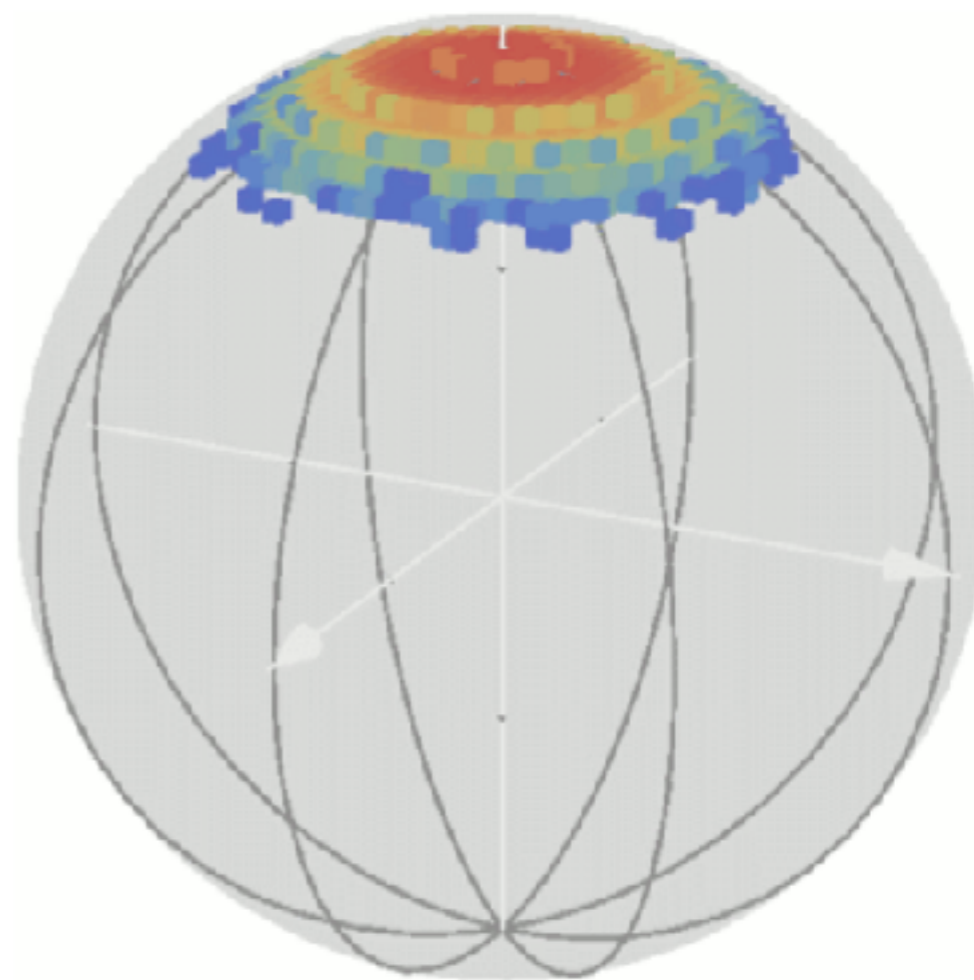
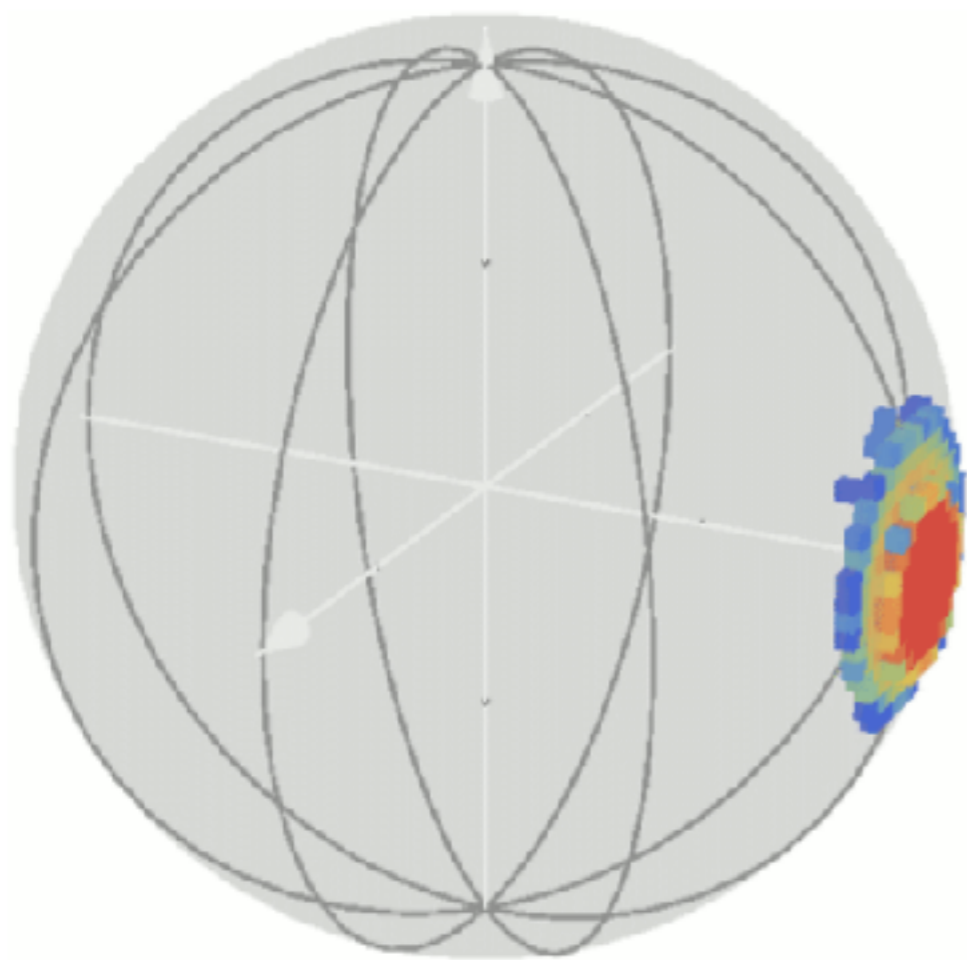
from  $|e\rangle$



# Statistics of relaxation trajectories

start in  $|+x\rangle = \frac{|g\rangle + |e\rangle}{\sqrt{2}}$

start in  $|e\rangle$



$10^6$  experiments

$\mathbb{P}(\rho_t)$

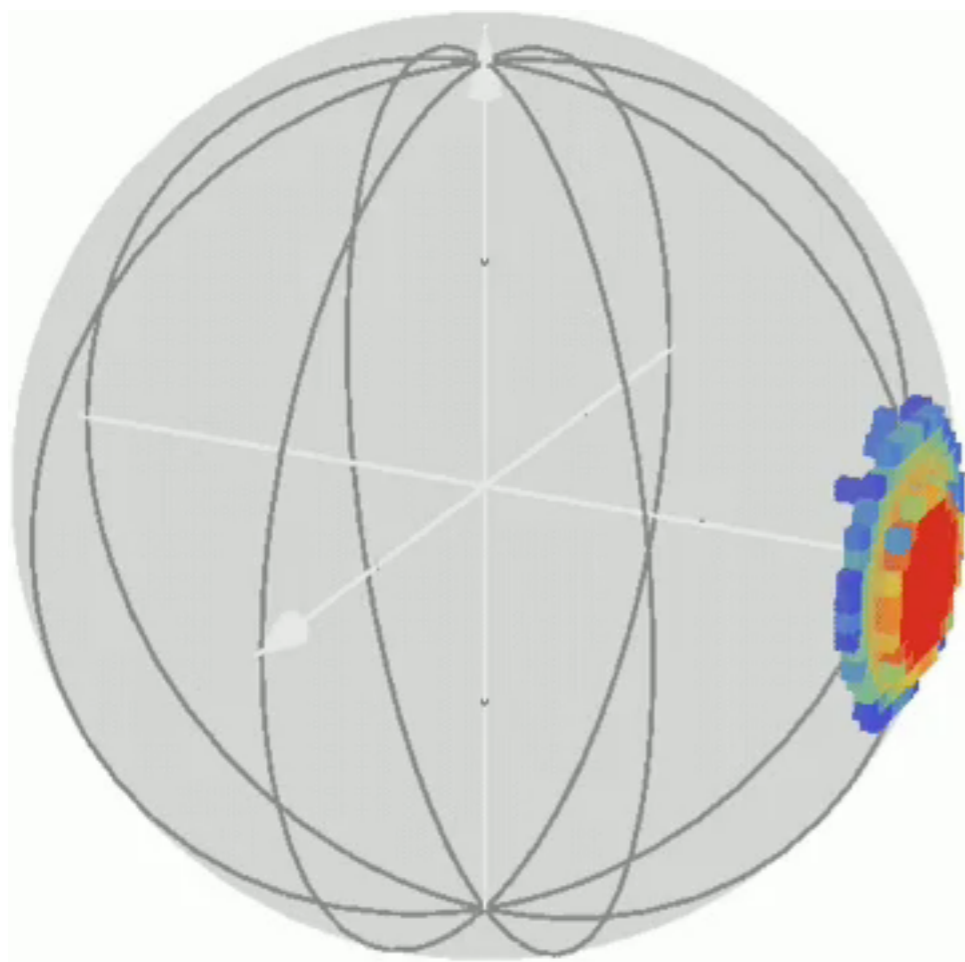
trajectories  
per pixel



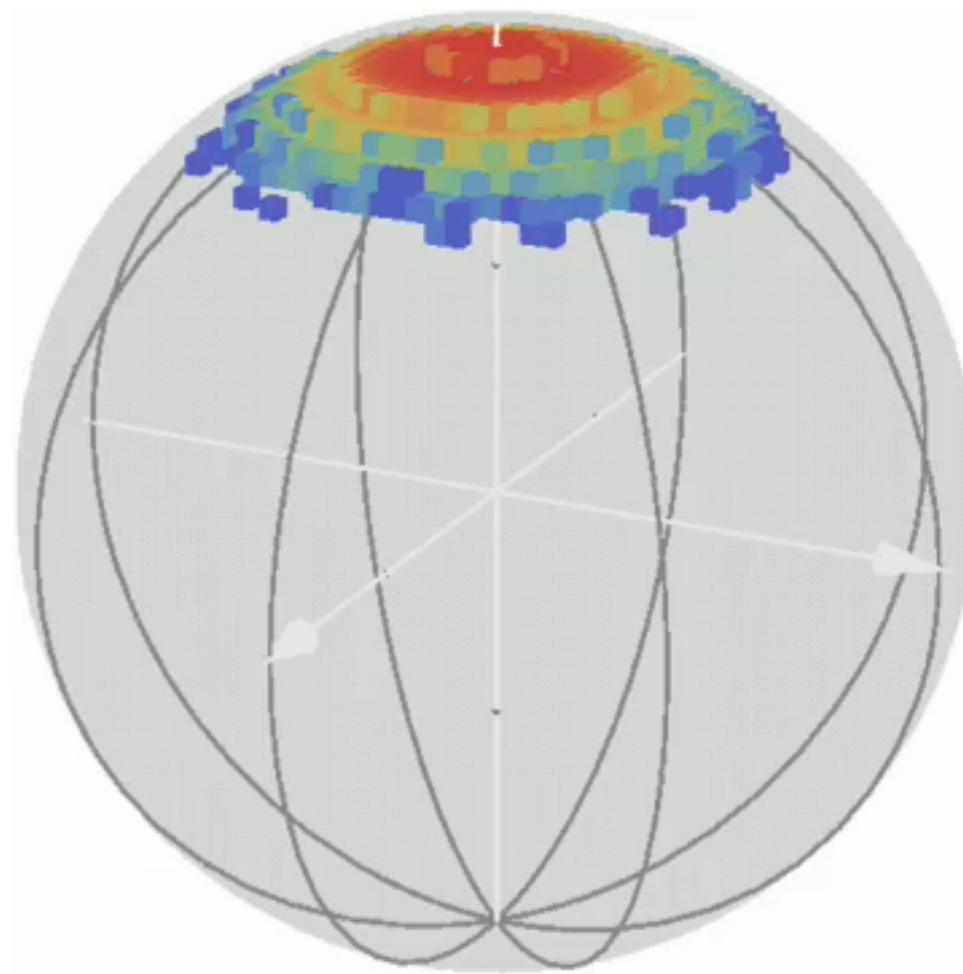


# Statistics of relaxation trajectories

start in  $|+x\rangle = \frac{|g\rangle + |e\rangle}{\sqrt{2}}$



start in  $|e\rangle$



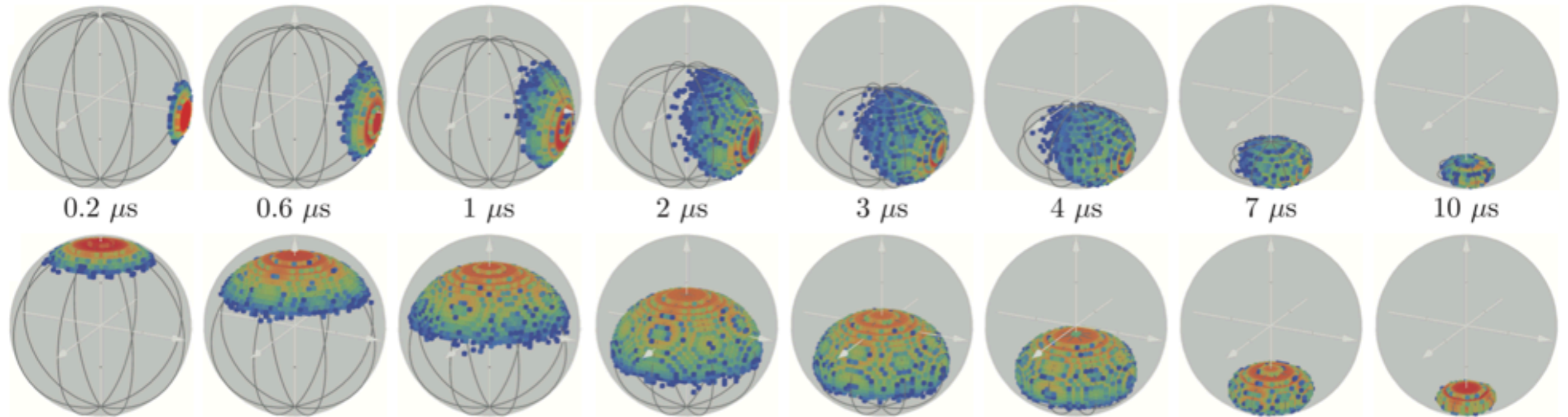
$10^6$  experiments

$\mathbb{P}(\rho_t)$

trajectories  
per pixel



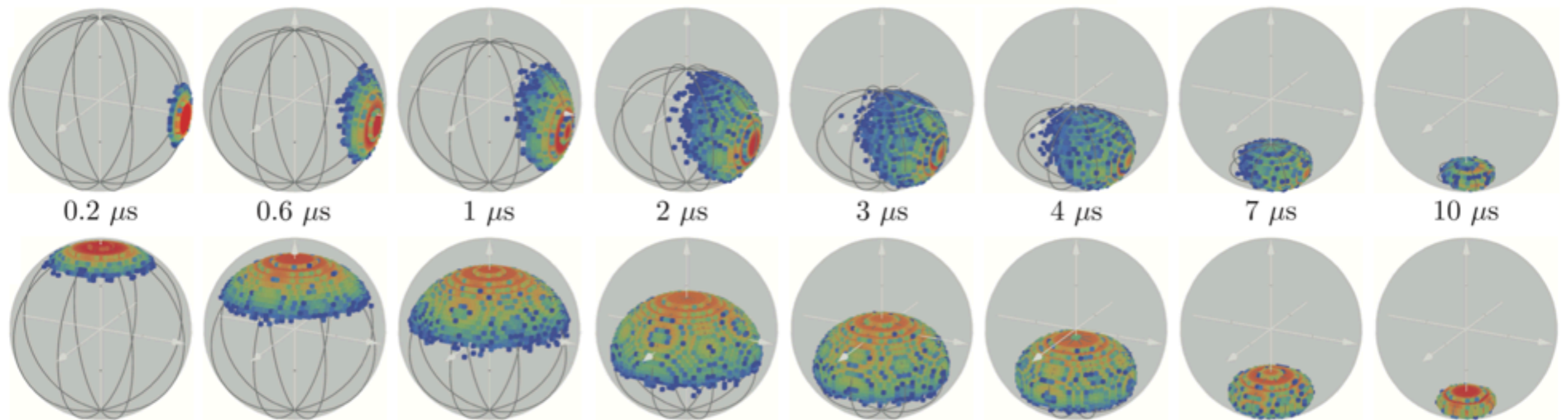
# Statistics of trajectories



[Campagne-Ibarcq *et al.*, PRX 2016]

[Jordan *et al.* Quant. Studies: Math and Found. 2016]

# Statistics of trajectories



[Campagne-Ibarcq *et al.*, PRX 2016]

[Jordan *et al.* Quant. Studies: Math and Found. 2016]

equation of the  
spheroid

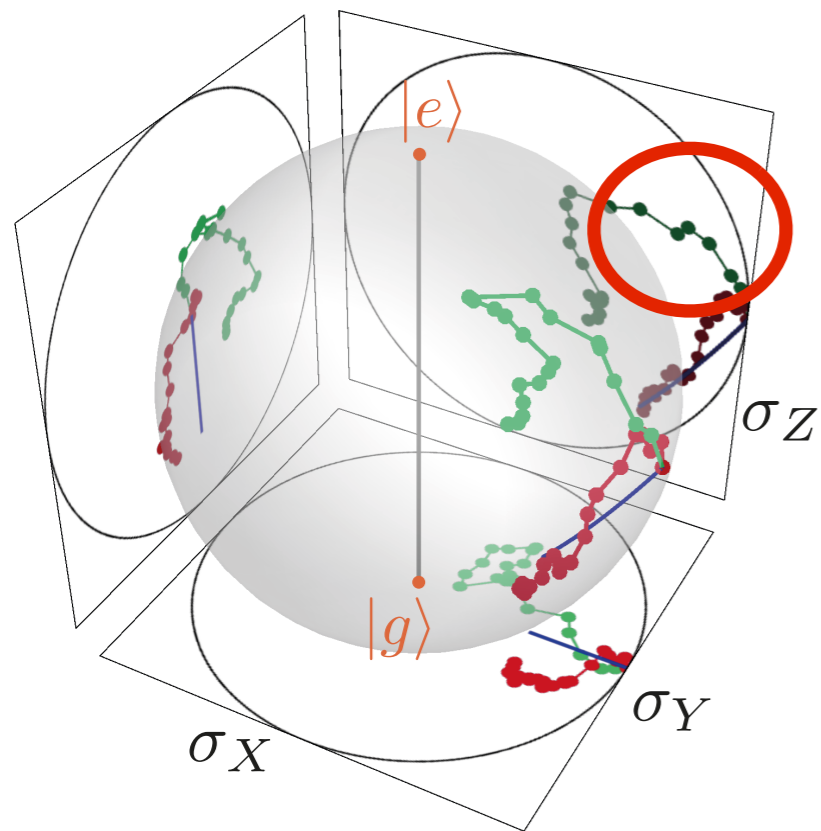
$$\alpha(x^2 + y^2) + \alpha^2 \left( z + 1 - \frac{1}{\alpha} \right)^2 = 1$$

parameter  $\alpha(t) = \eta + [\alpha(0) - \eta]e^{\Gamma_1 t}$

[A.Sarlette and P.Rouchon, Communications in Mathematical Physics 2016]



# Counterintuitive trajectories

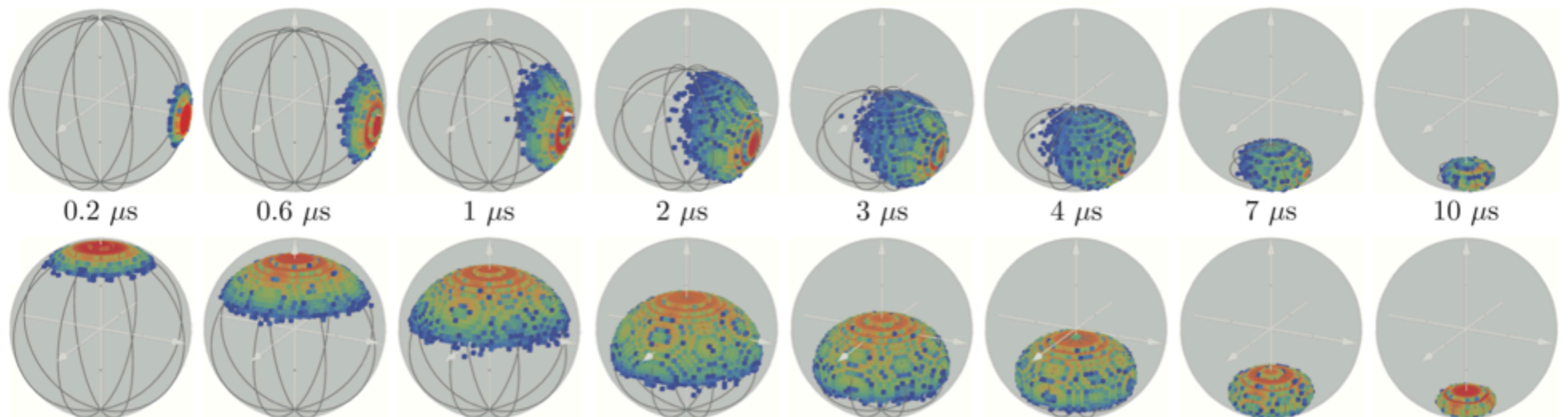


Energy expectation can **increase** due to the backaction of measuring spontaneously emitted photons

[Bolund and Mölmer, PRA 2014]

Exploiting the energy transfer of measurement process

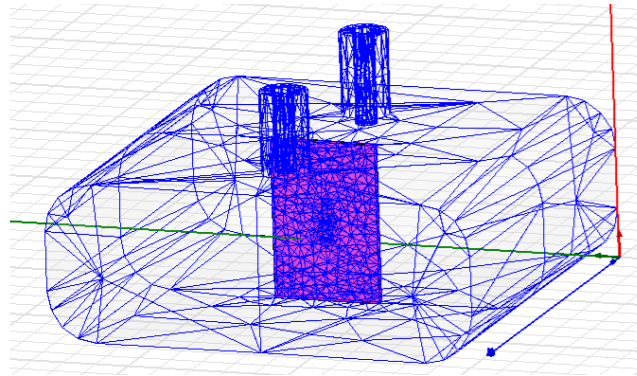
[Elouard, herrera-Marti, BH, Auffèves PRL 2017]



[Campagne-Ibarcq *et al.*, PRX 2016]

[Jordan *et al.* Quant. Studies: Math and Found. 2016]

# Quantum trajectories and feedback in circuit-QED



## Introduction to circuit-QED

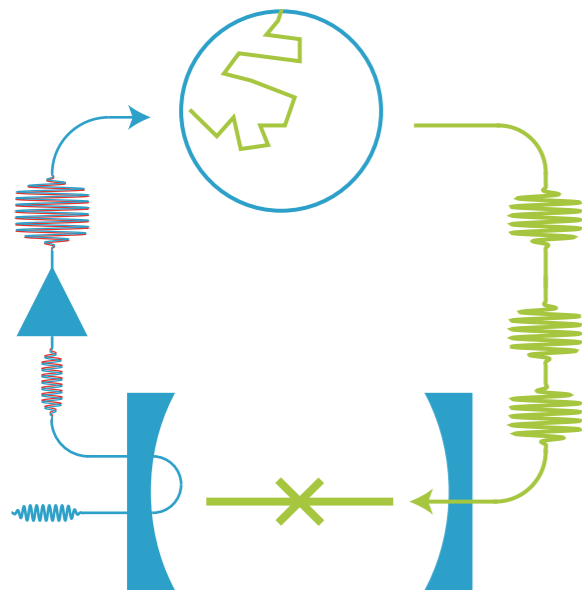
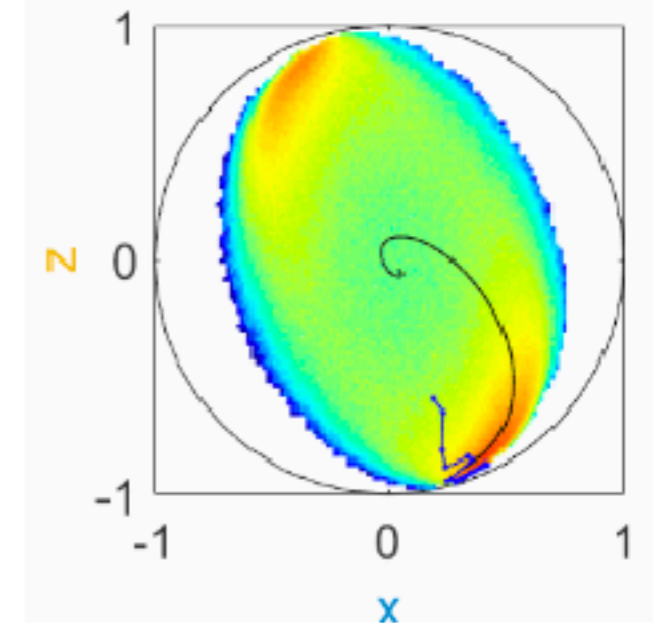
## Quantum trajectories in circuit-QED

dispersive measurement

fluorescence measurement

**both simultaneously**

generating entanglement



## Measurement based feedback

dispersive case

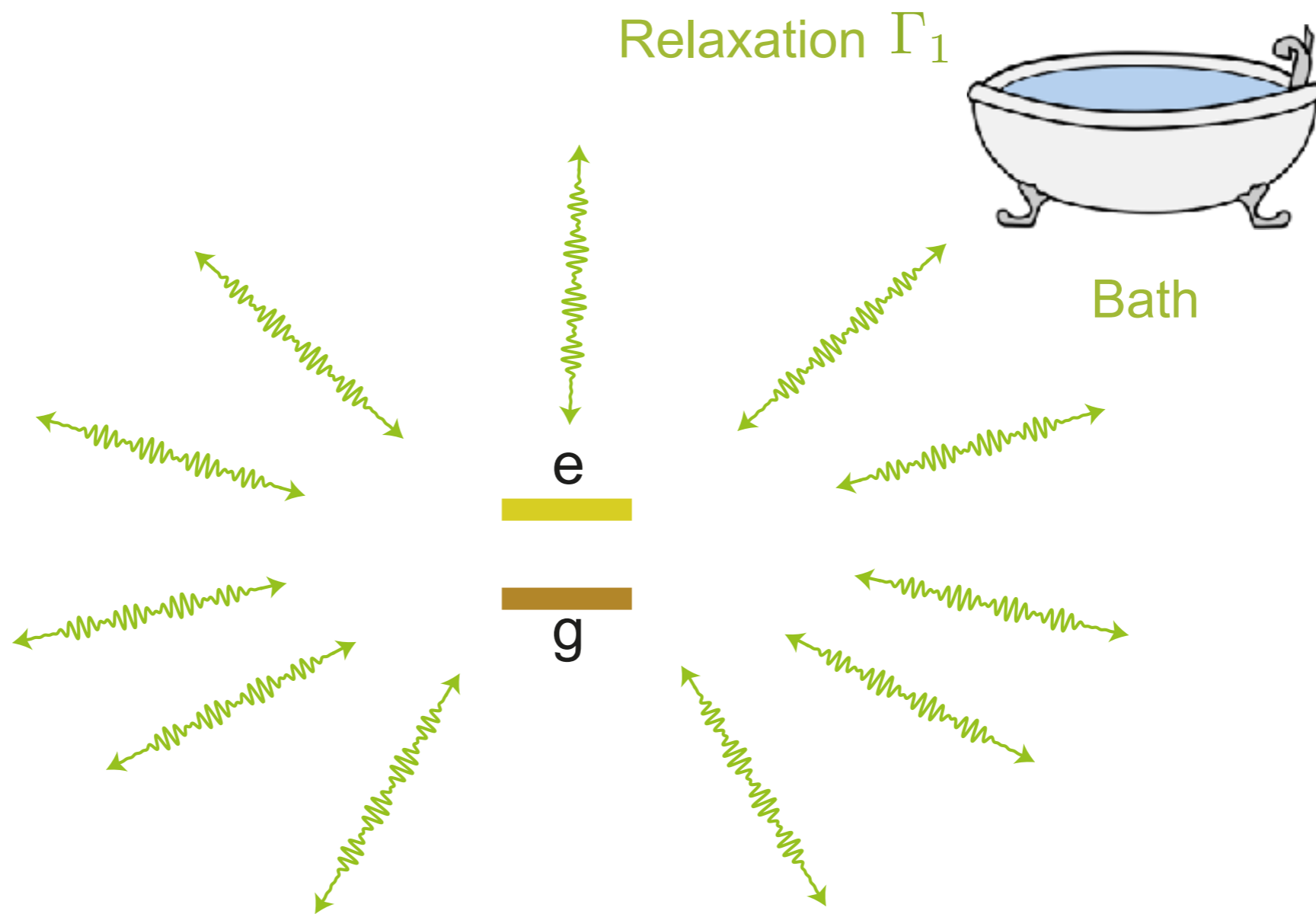
fluorescence case

# Decoherence channels of a qubit

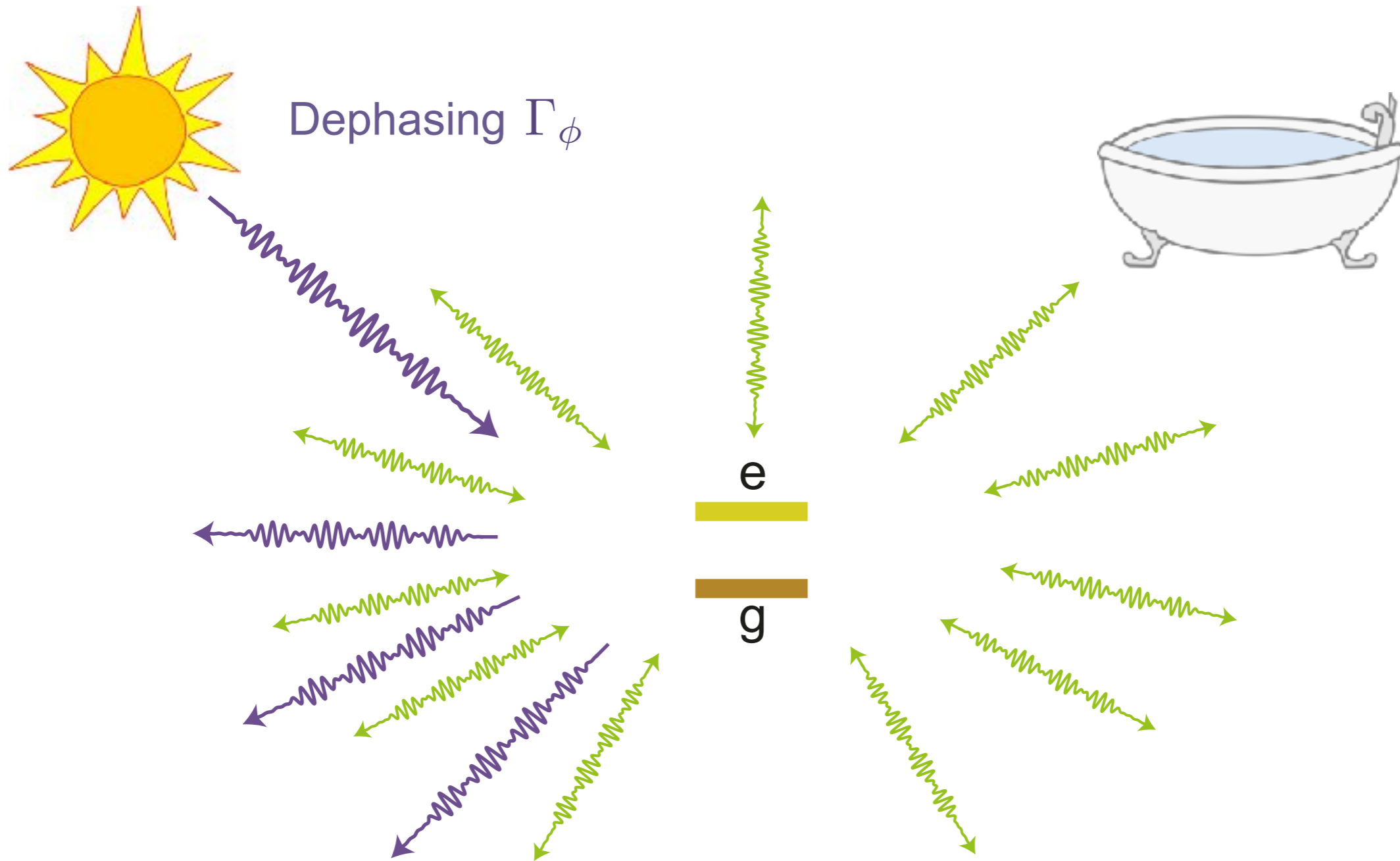




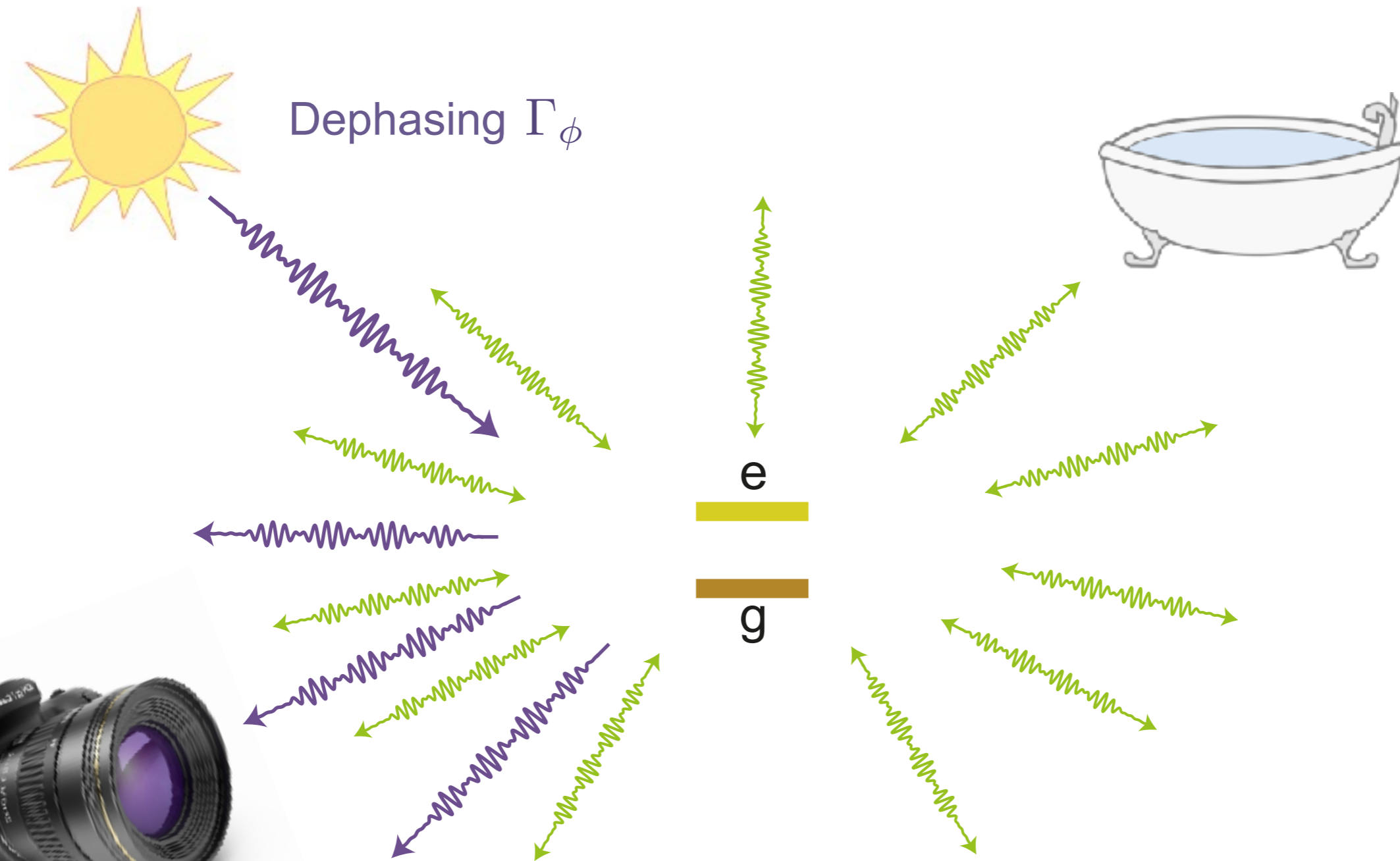
# Decoherence channels of a qubit



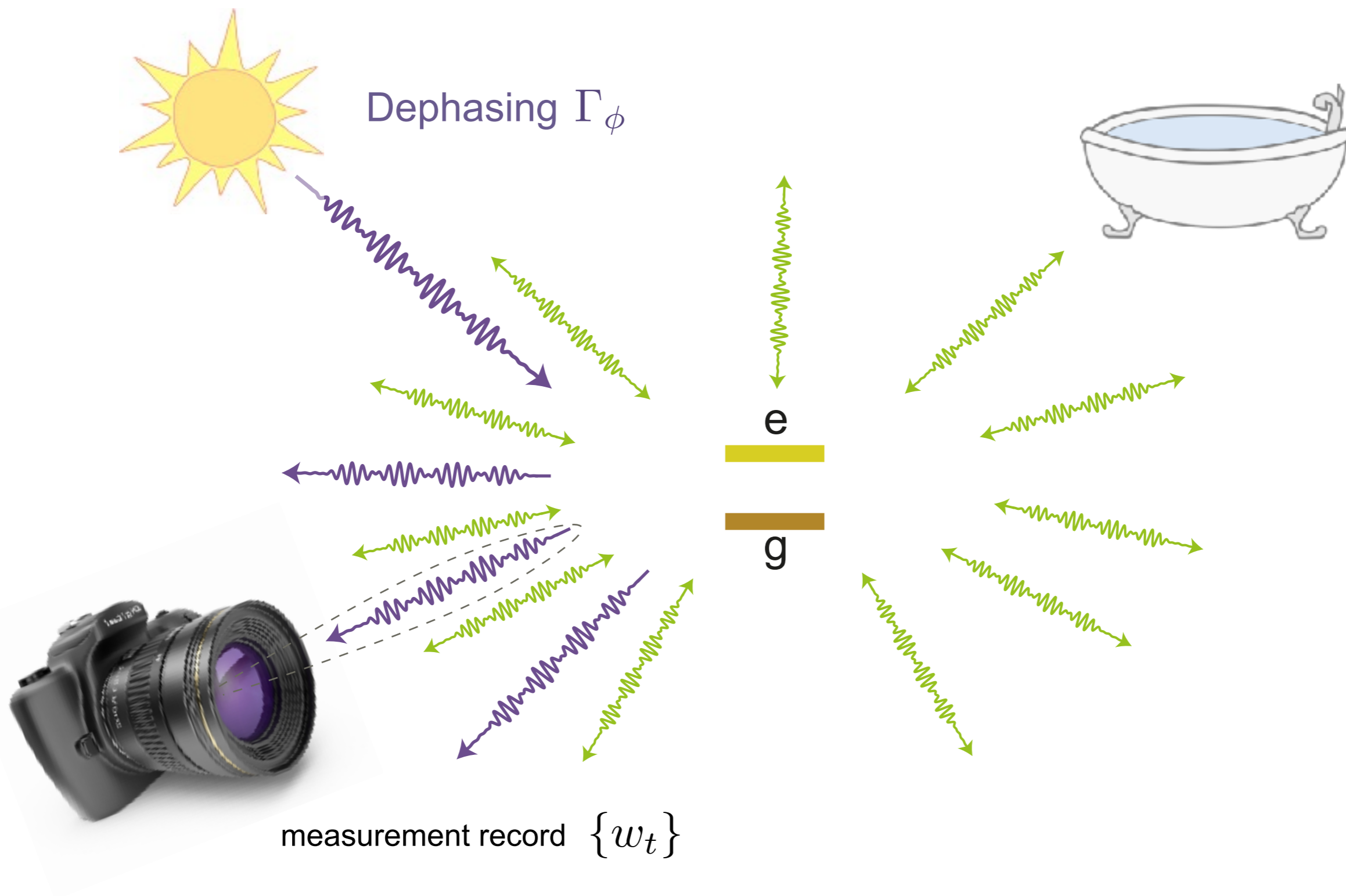
# Decoherence channels of a qubit



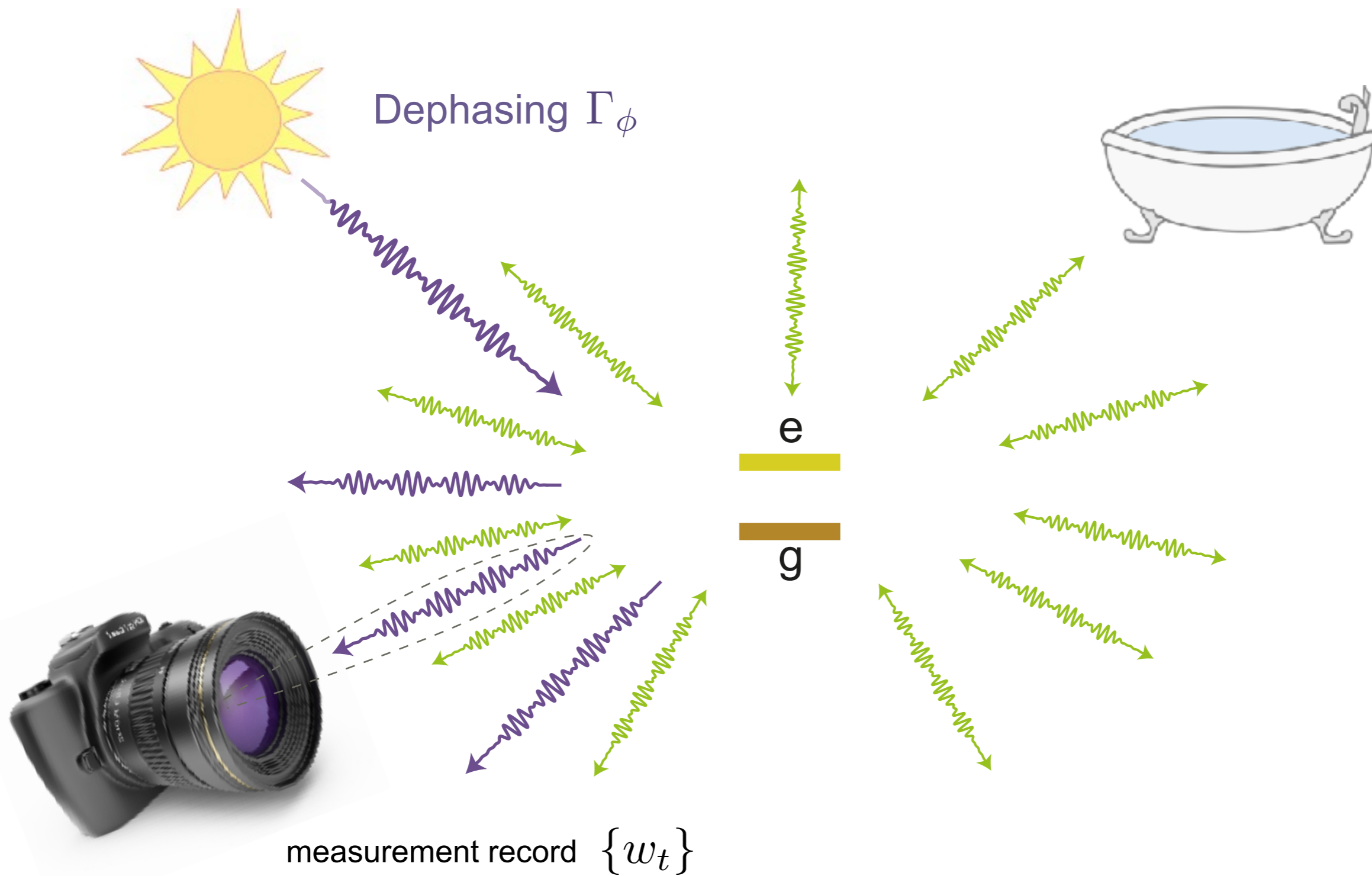
# Measurement of decoherence channels of a qubit



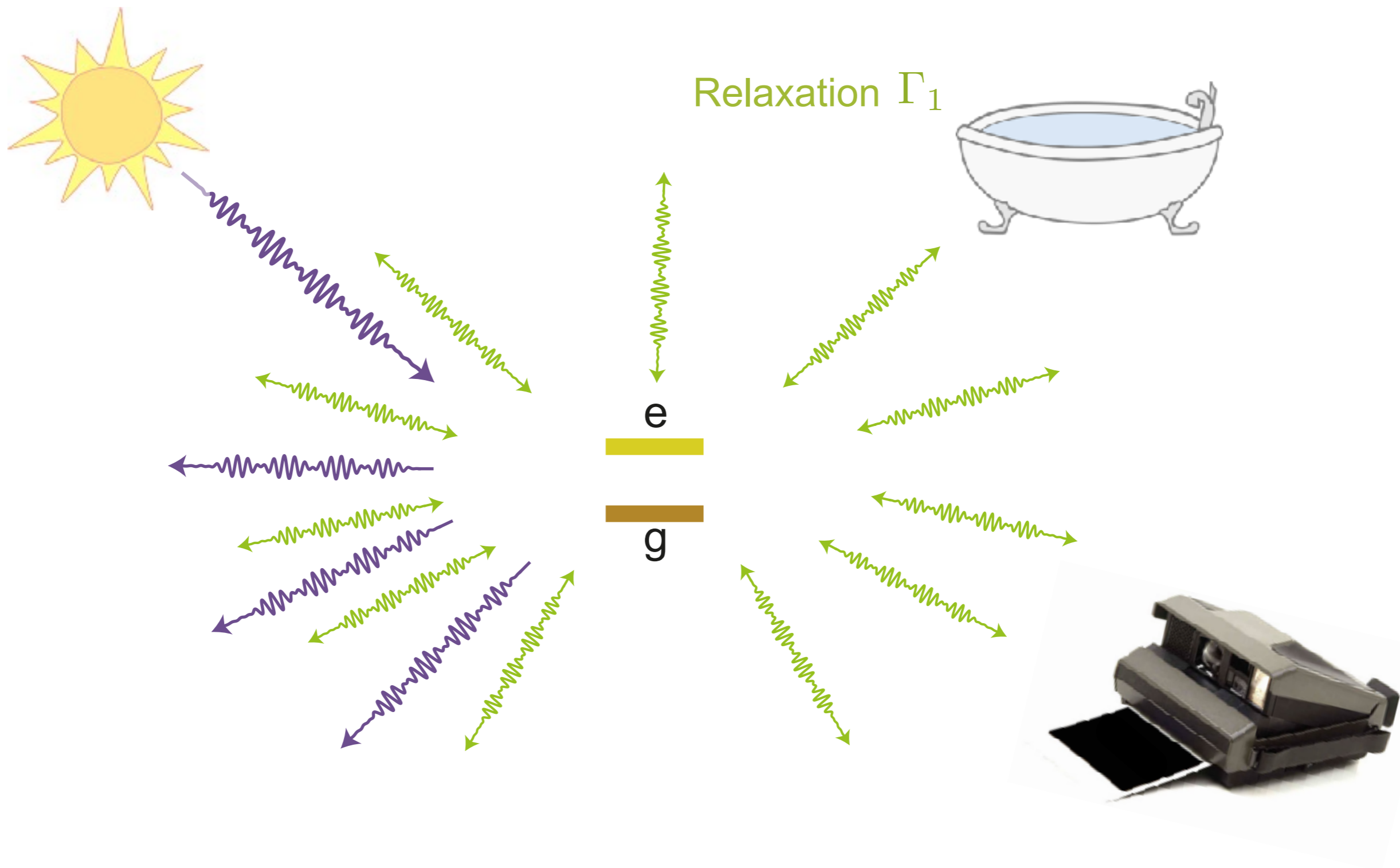
# Measurement of decoherence channels of a qubit



# Measurement of decoherence channels of a qubit

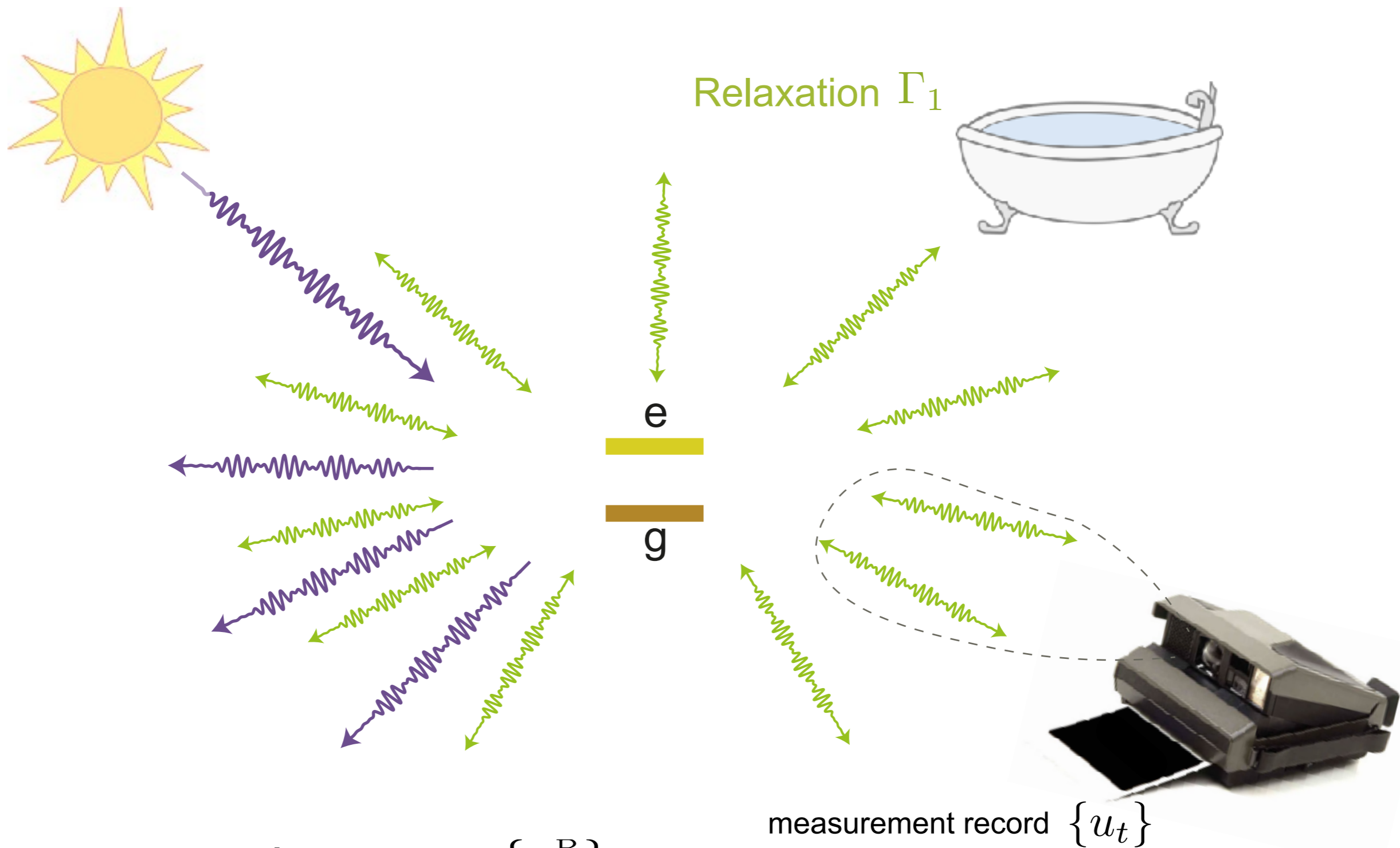


# Measurement of decoherence channels of a qubit





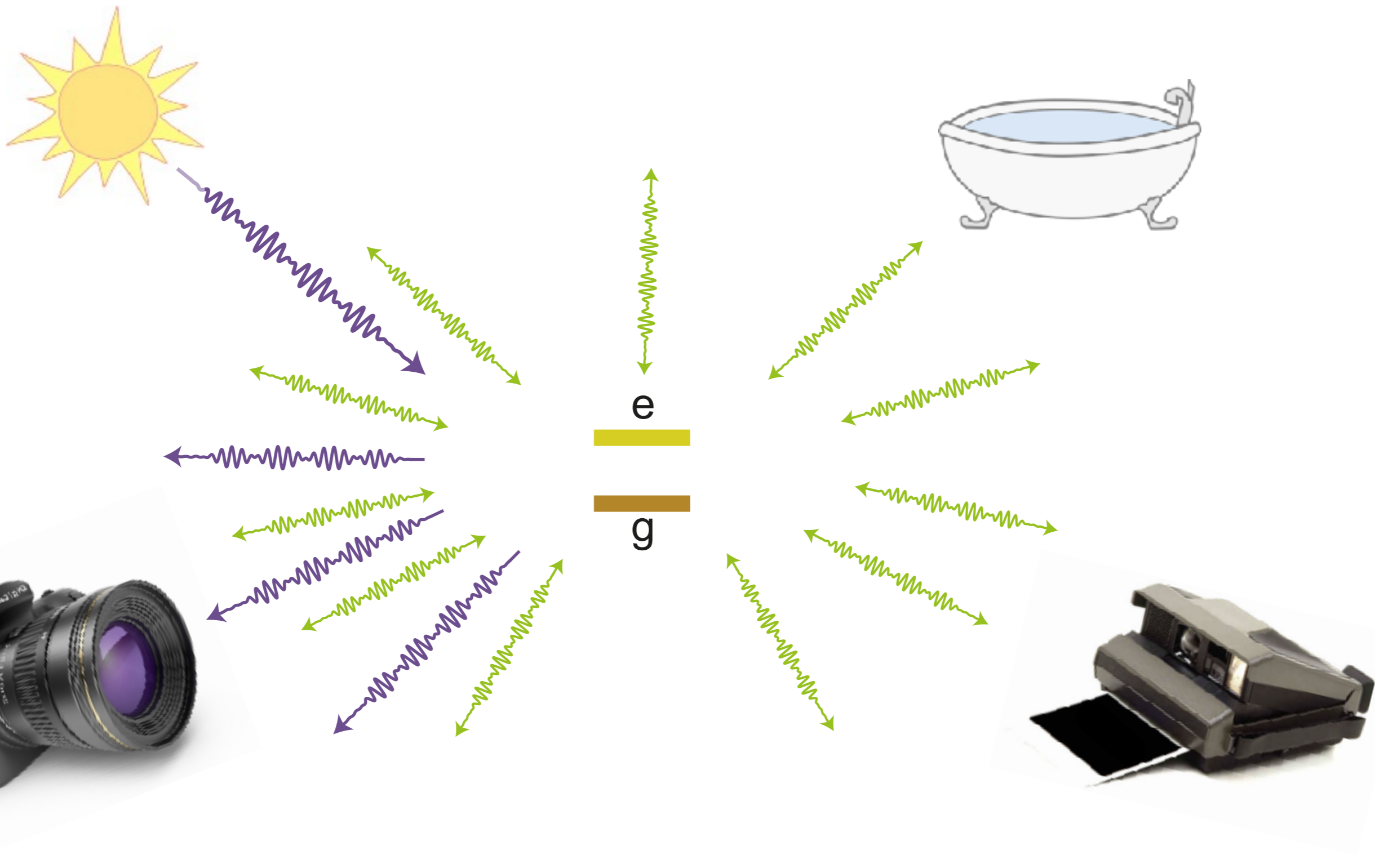
# Measurement of decoherence channels of a qubit



Quantum trajectory =  $\{\rho_t^B\}$

measurement B

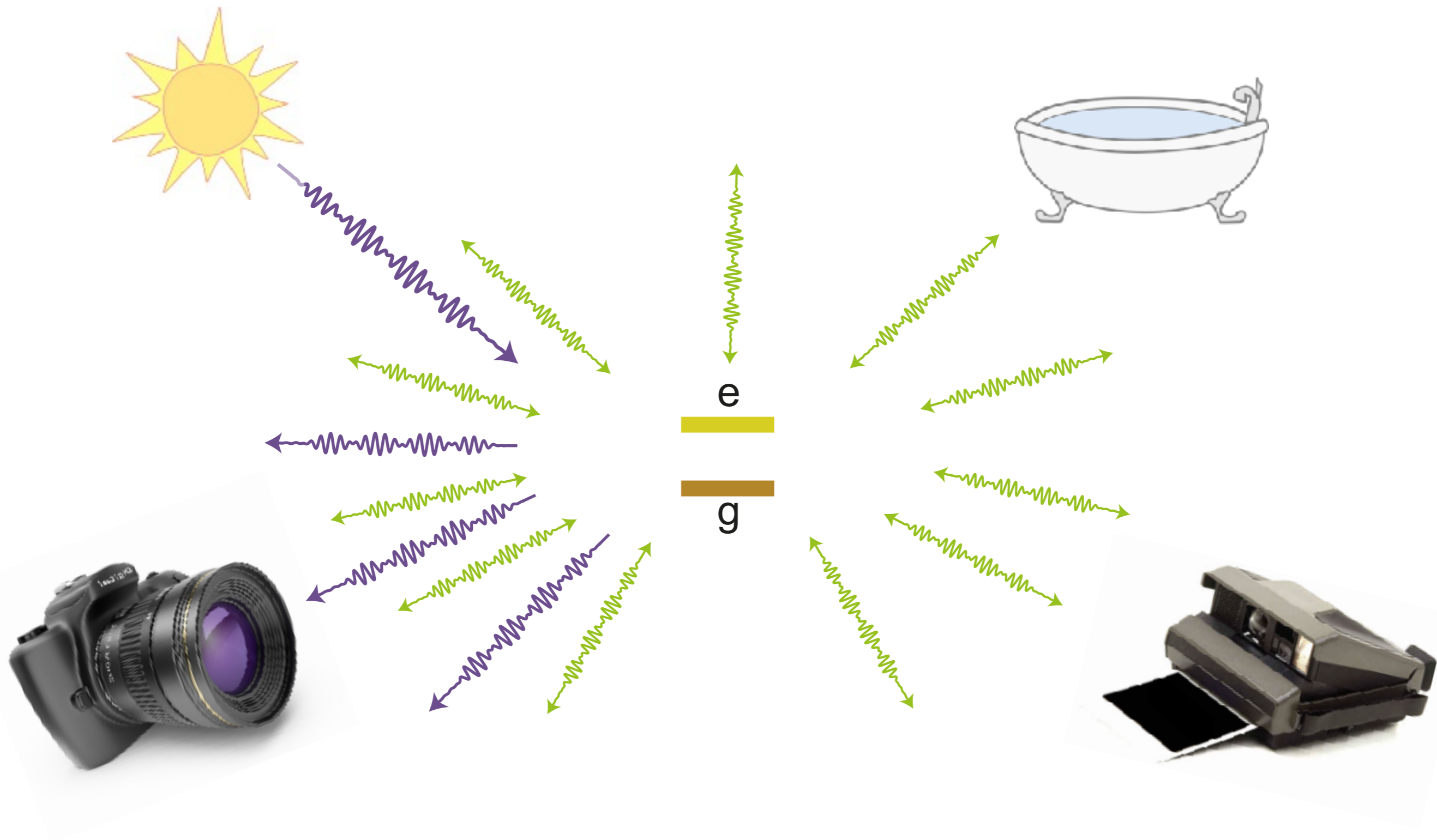
# Measurement of decoherence channels of a qubit



measurement A

measurement B

# Measurement of decoherence channels of a qubit



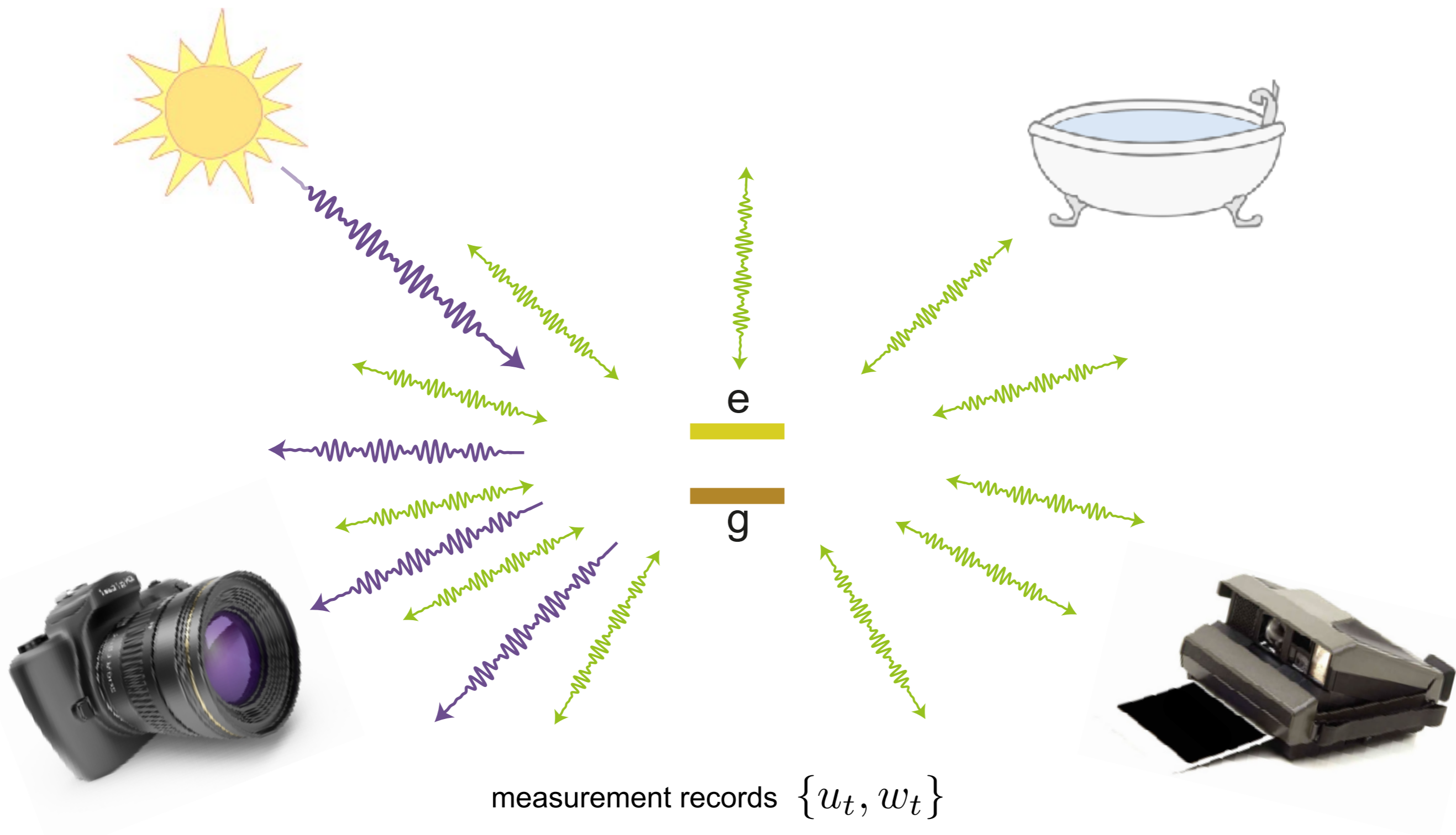
measurement A



Incompatible measurements

measurement B

# Measurement of decoherence channels of a qubit



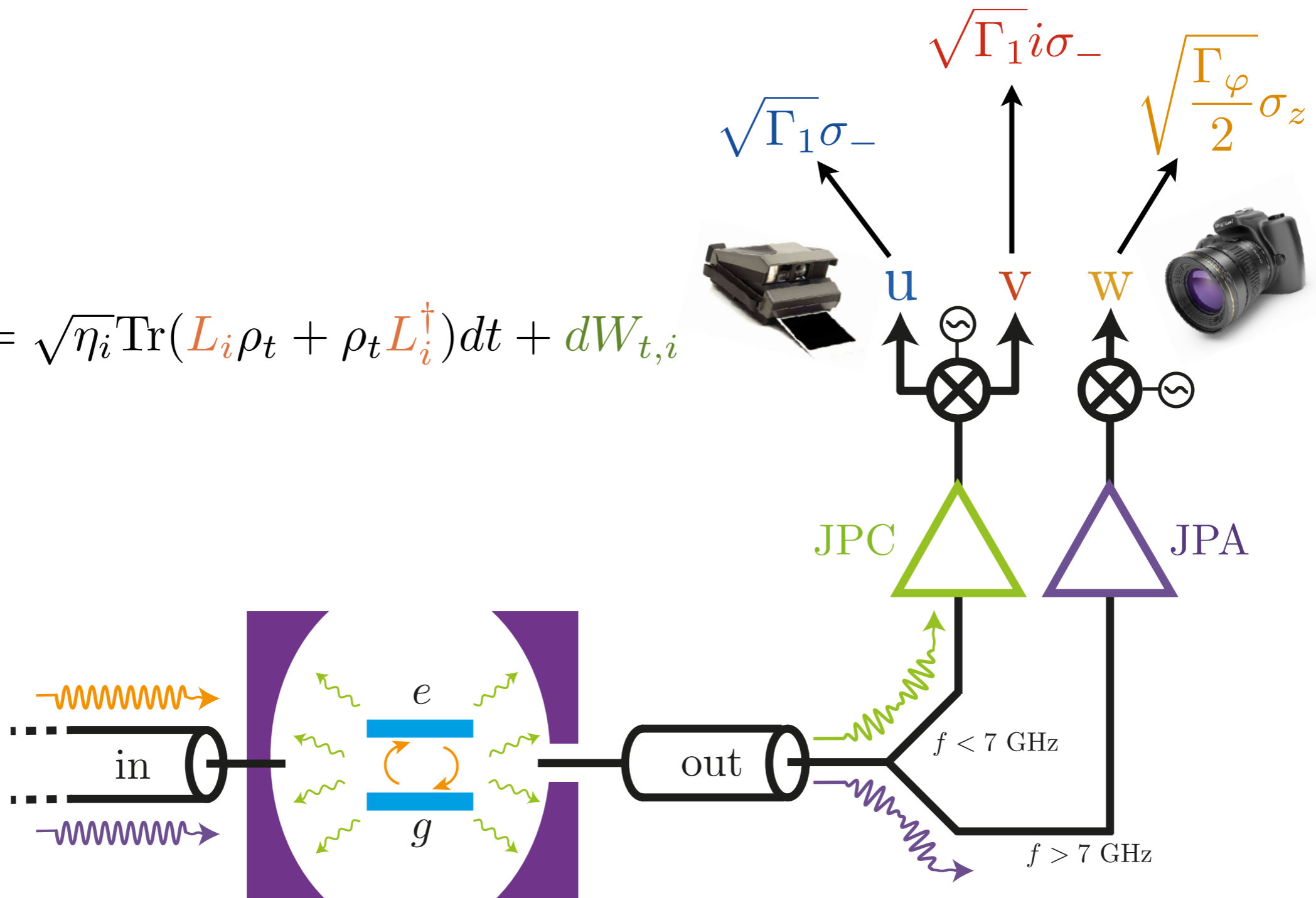
measurement A

$$\text{Quantum trajectory} = \{\rho_t^{A+B}\}$$

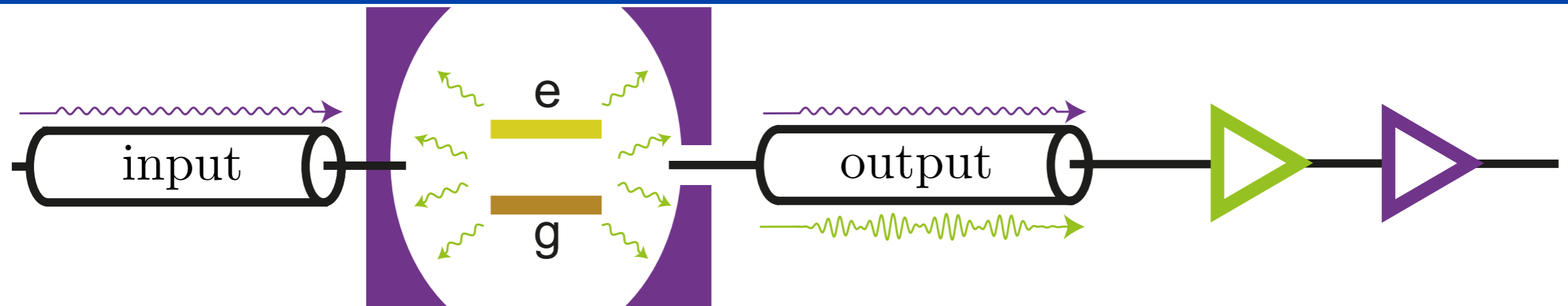
measurement B

# Measurement setup

$$dy_t^i = \sqrt{\eta_i} \text{Tr}(L_i \rho_t + \rho_t L_i^\dagger) dt + dW_{t,i}$$



# Records of simultaneous X, Y and Z



full  
measurement  
records

$$\begin{aligned}
 du_t &= \sqrt{\eta_{\text{fluor}} \Gamma_1 / 2} \langle \sigma_X \rangle_{\rho_t} dt + dW_{t,1} \\
 dv_t &= \sqrt{\eta_{\text{fluor}} \Gamma_1 / 2} \langle \sigma_Y \rangle_{\rho_t} dt + dW_{t,2} \\
 dw_t &= \sqrt{2\eta_{\text{disp}} \Gamma_d} \langle \sigma_Z \rangle_{\rho_t} dt + dW_{t,3}
 \end{aligned}$$

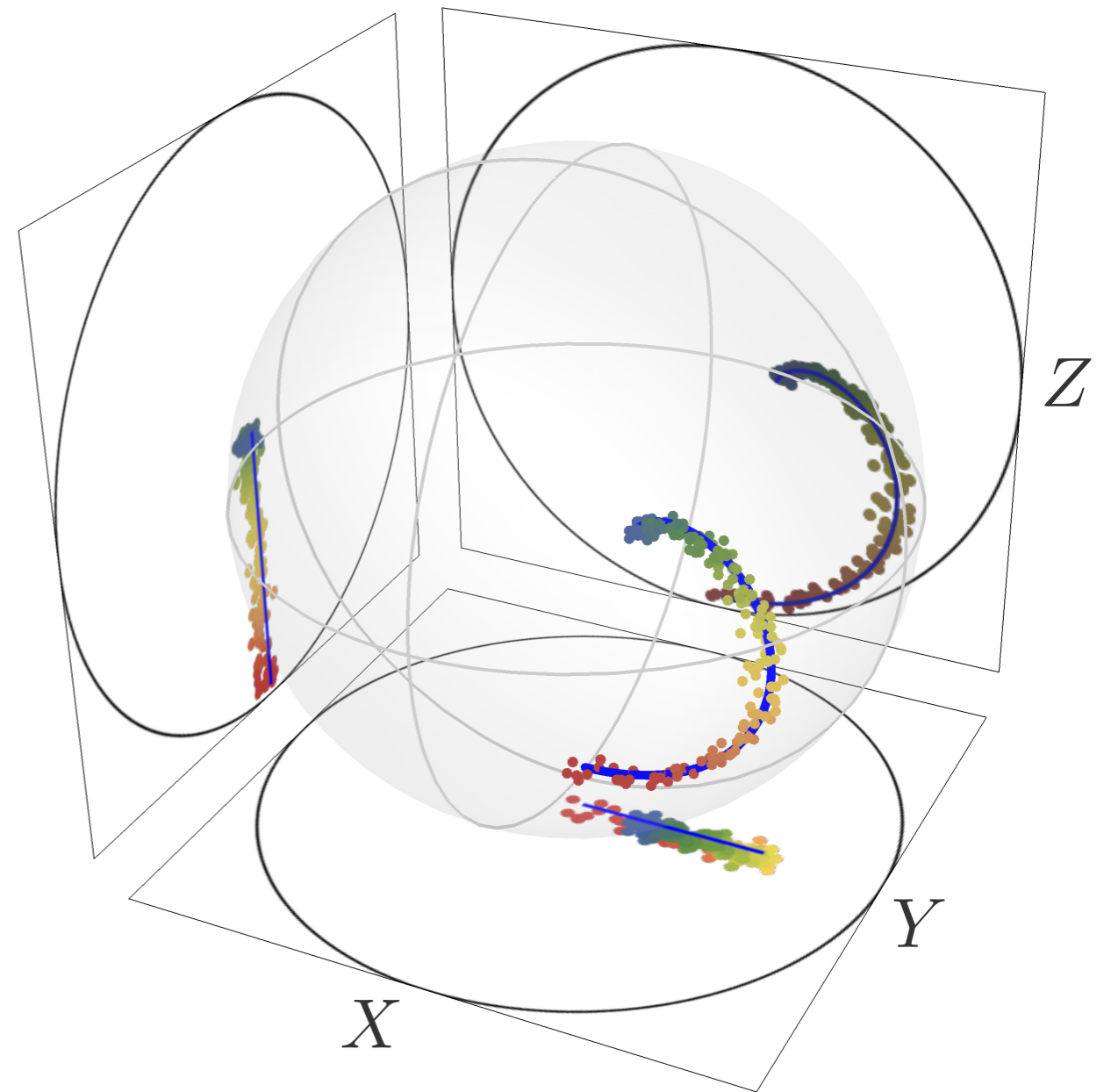
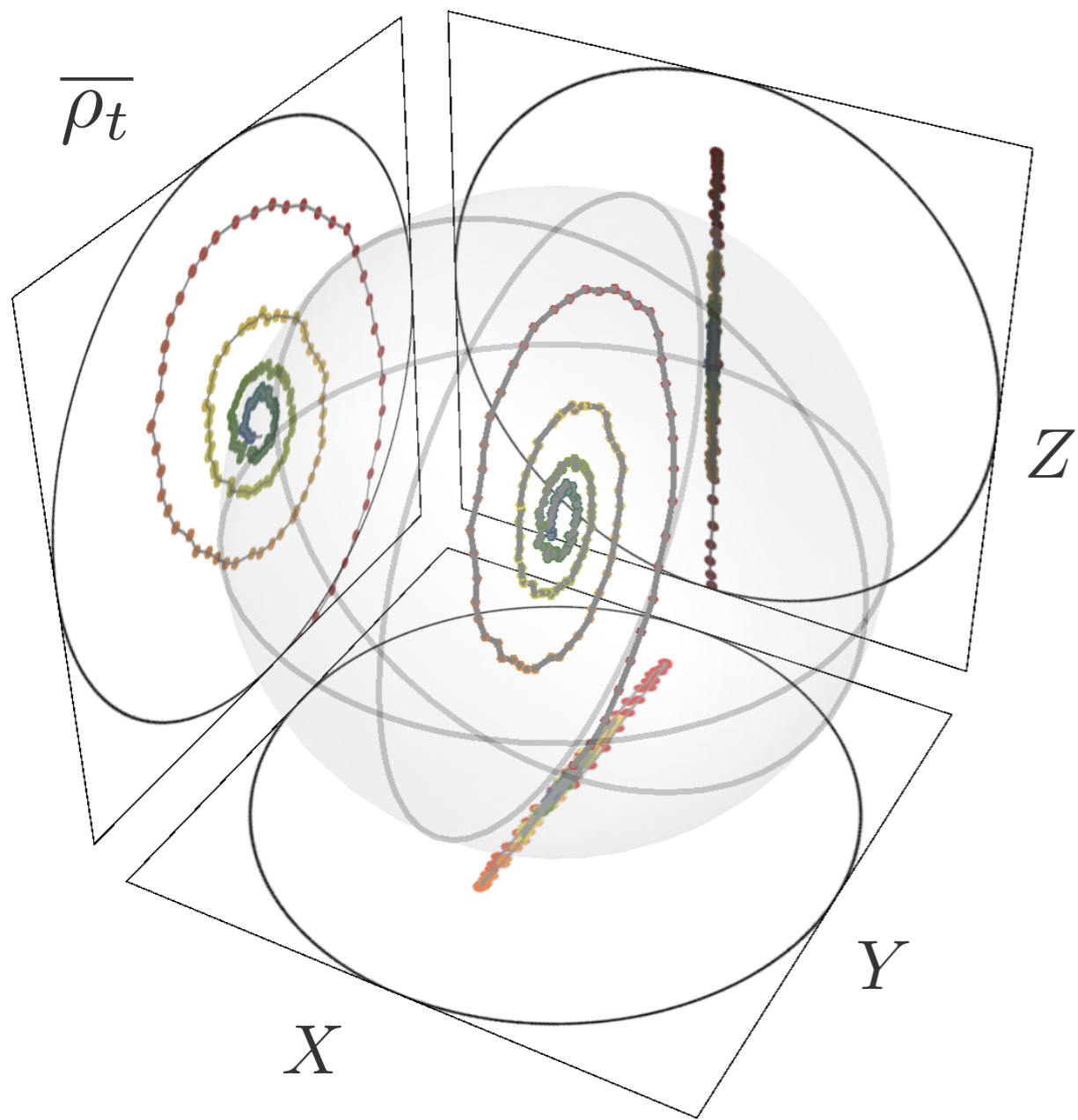
average outcome

noise  
(Wiener)

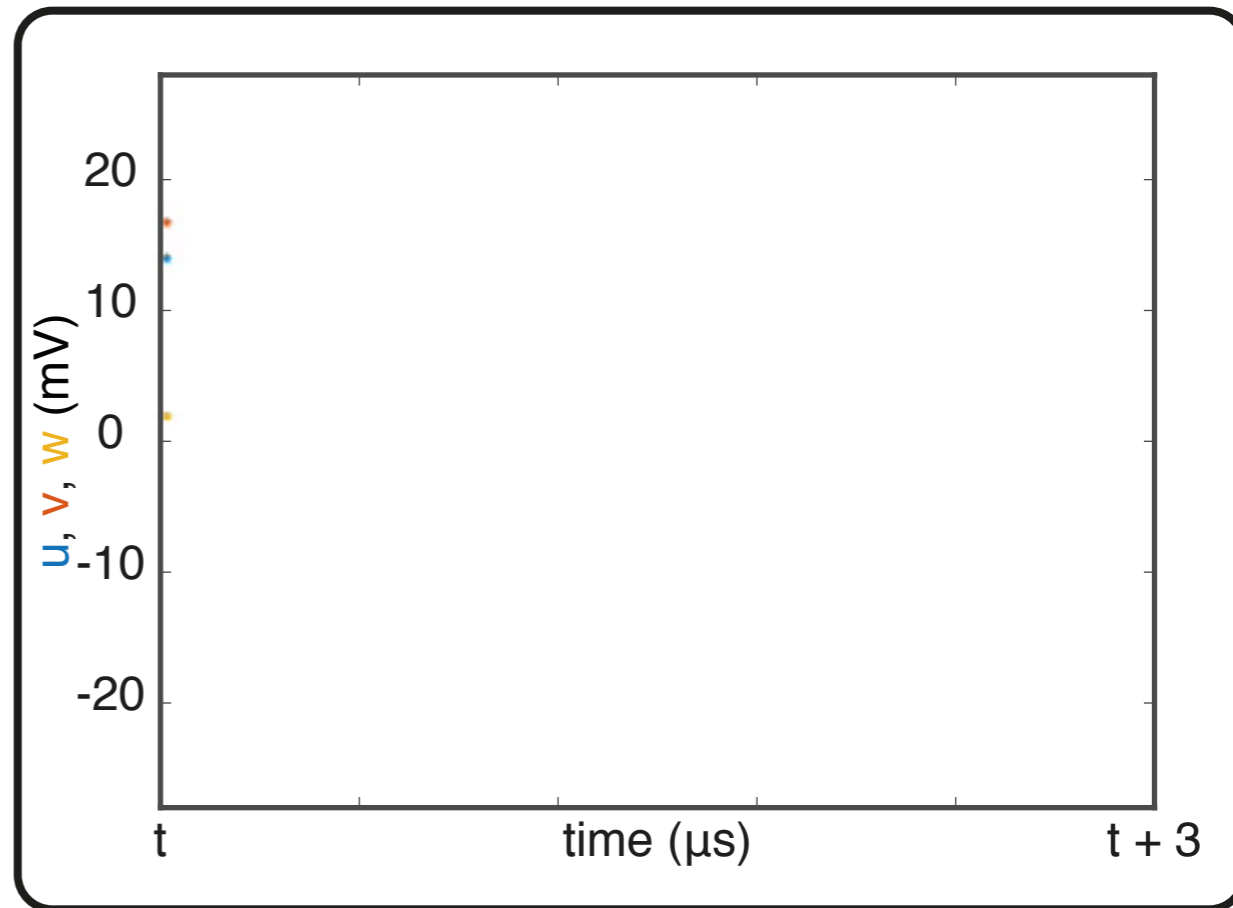
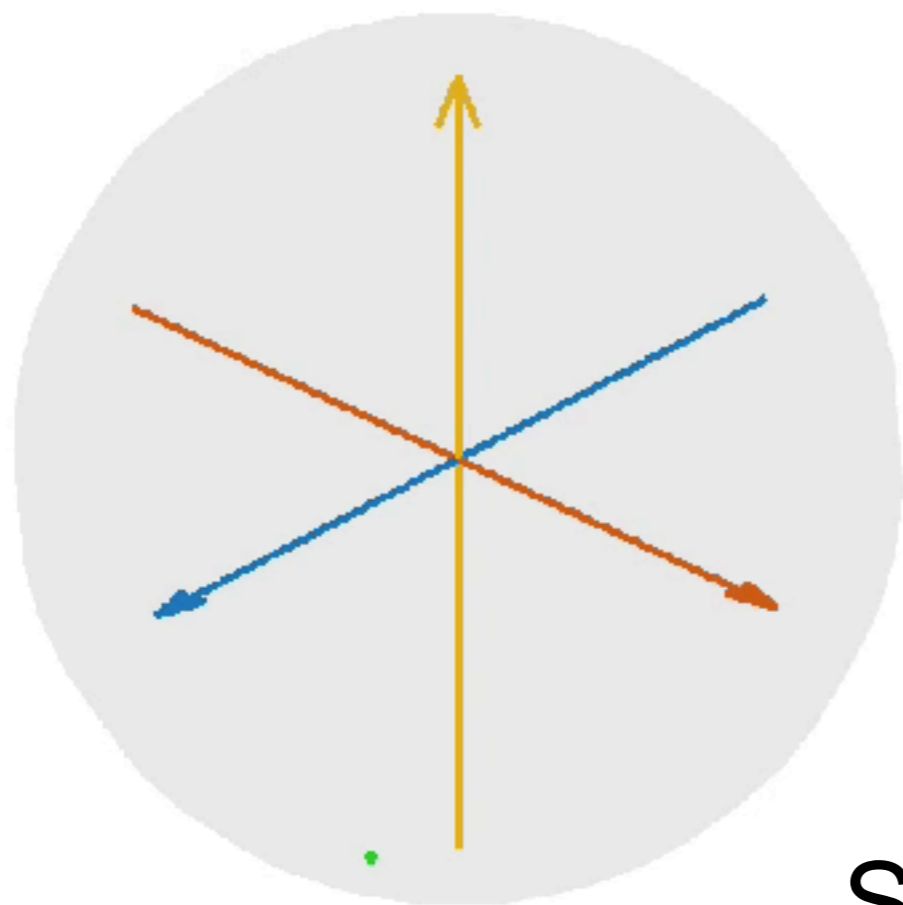
raw averaging directly gives Bloch vector



# Average trajectory

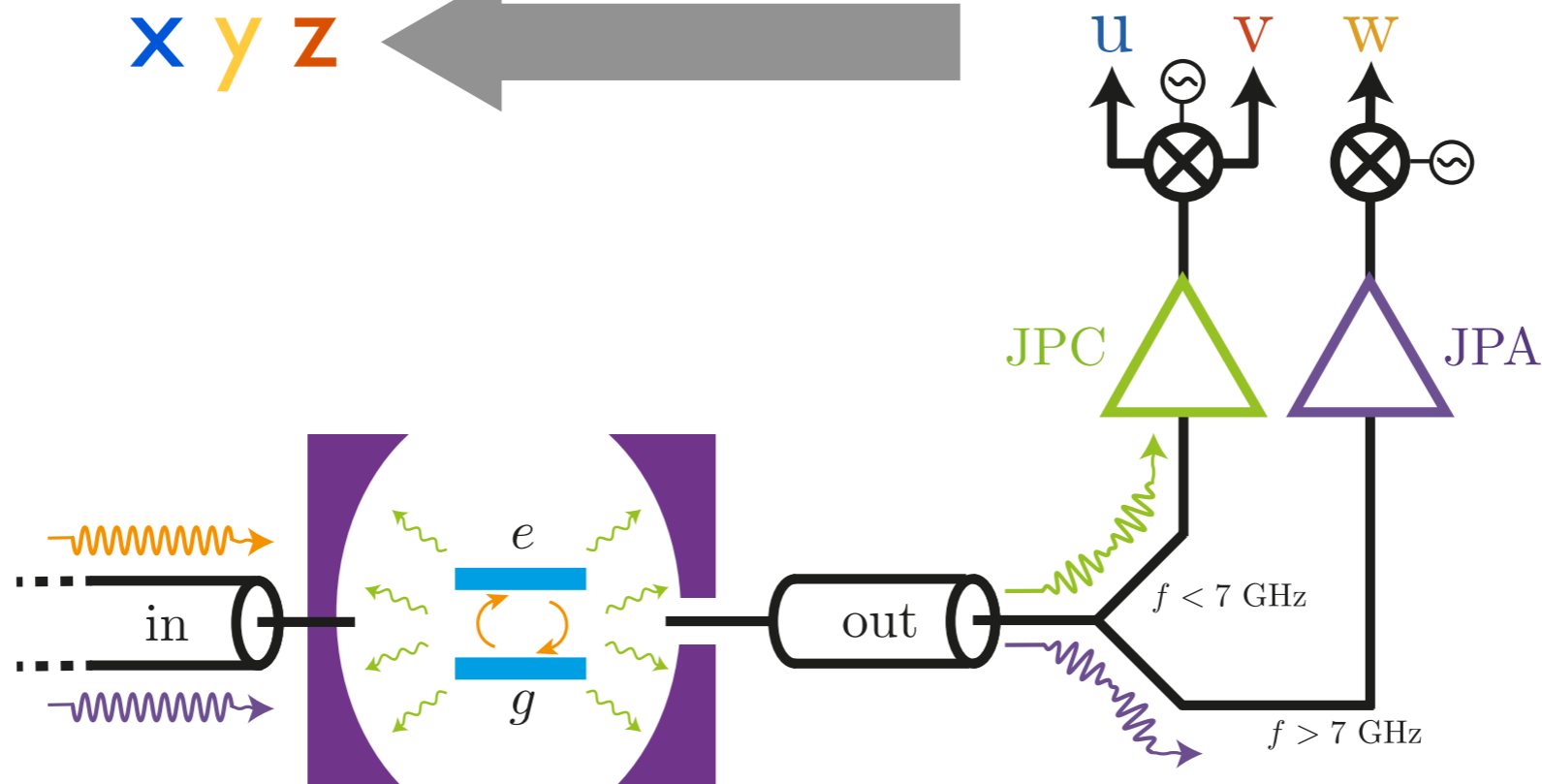


# One quantum trajectory



SME

x y z



$$\eta_{\text{fluo}} = 14 \%$$

$$\eta_{\text{disp}} = 34 \%$$

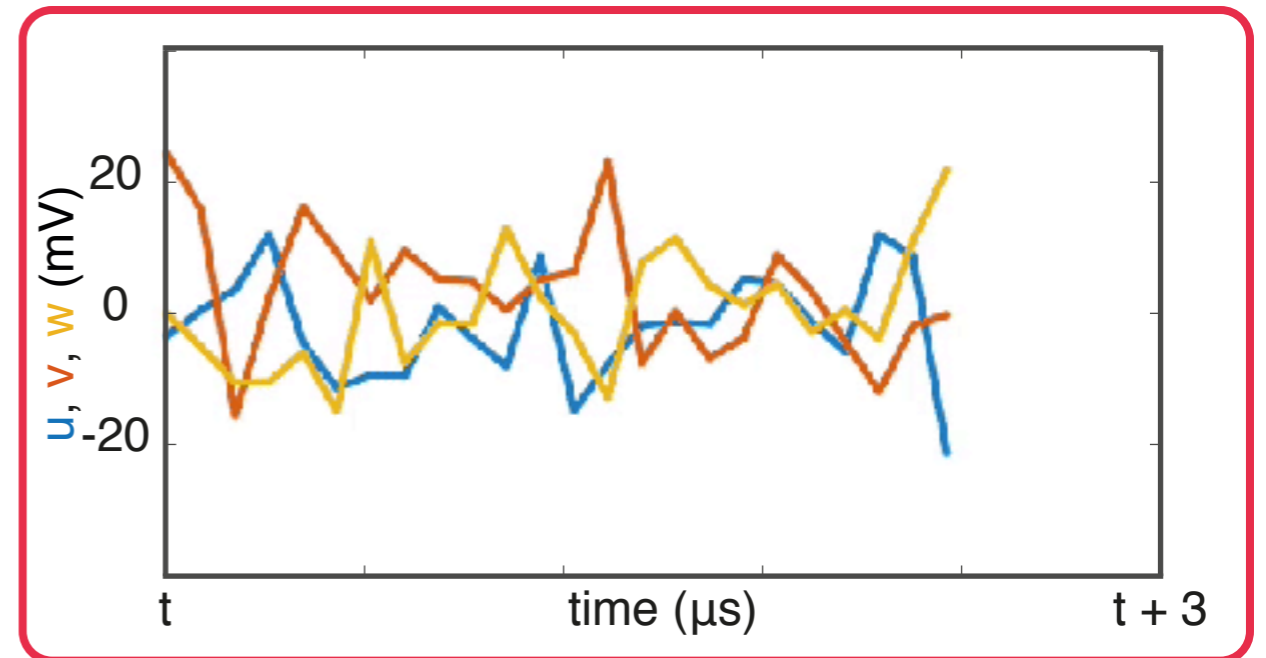
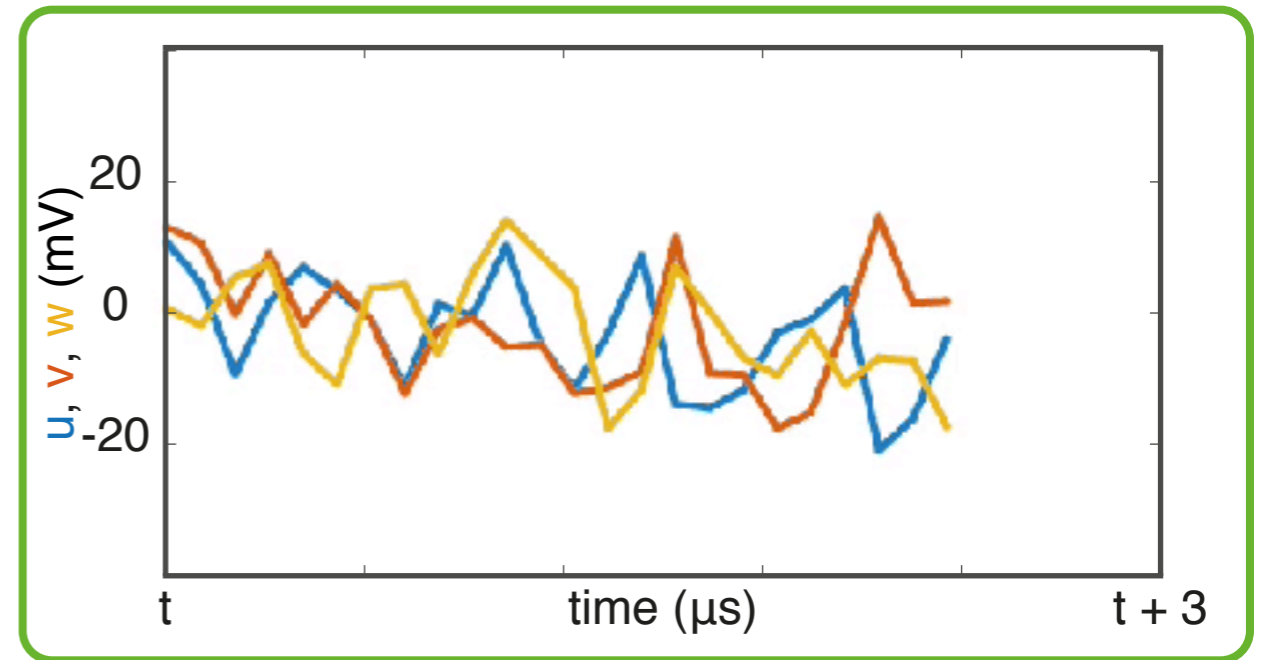
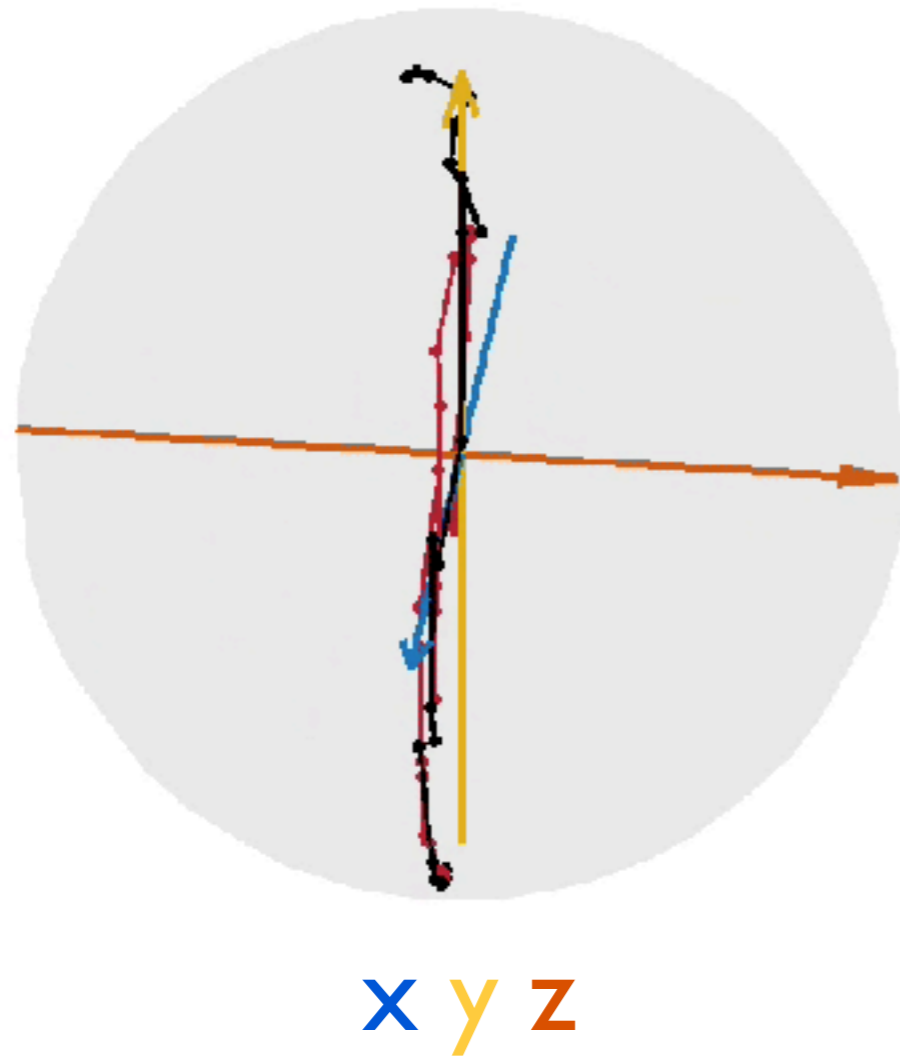
$$T_1 = 15.0 \mu\text{s}$$

$$T_2 = 11.2 \mu\text{s}$$

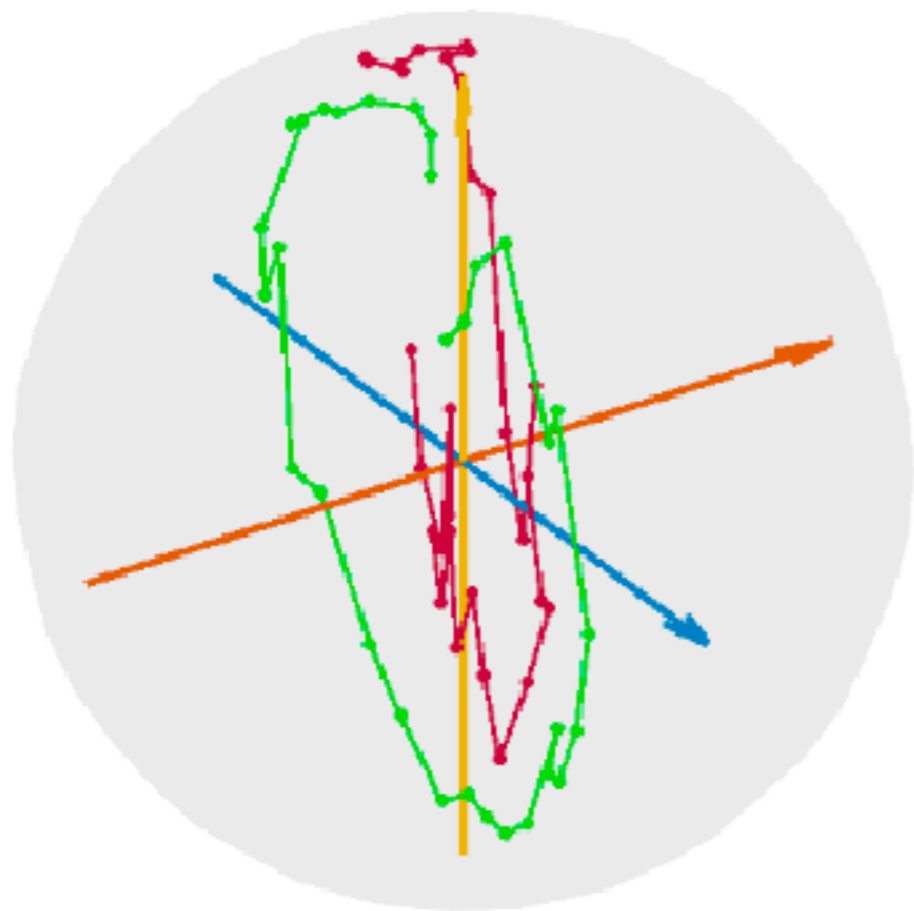
$$T_d = 0.9 \mu\text{s}$$

$$T_R = 5.2 \mu\text{s}$$

# Two quantum trajectories



# Control trajectories by tomography



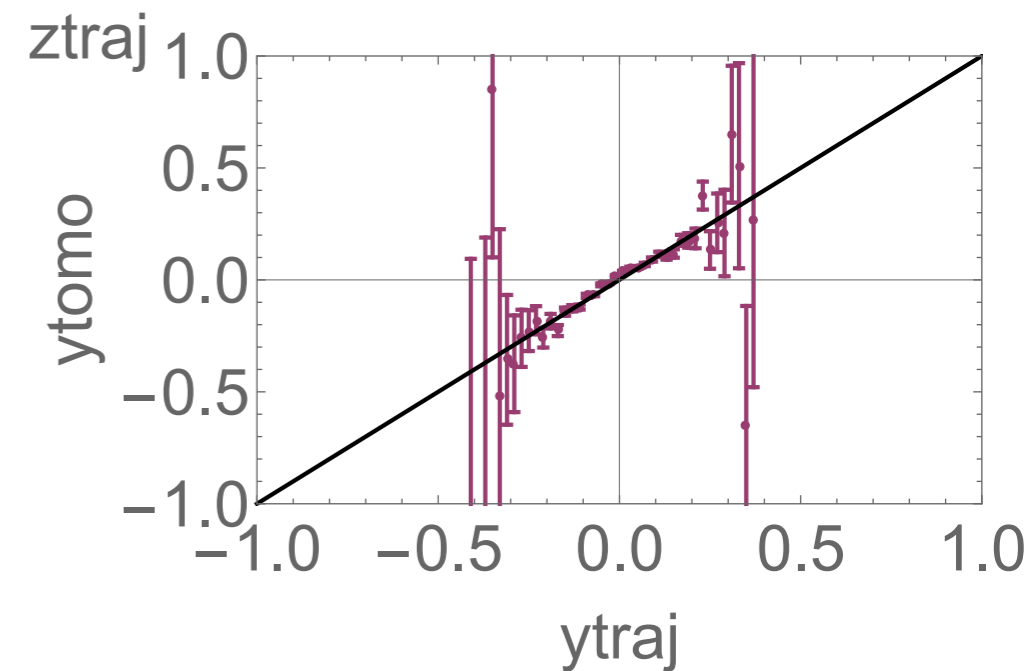
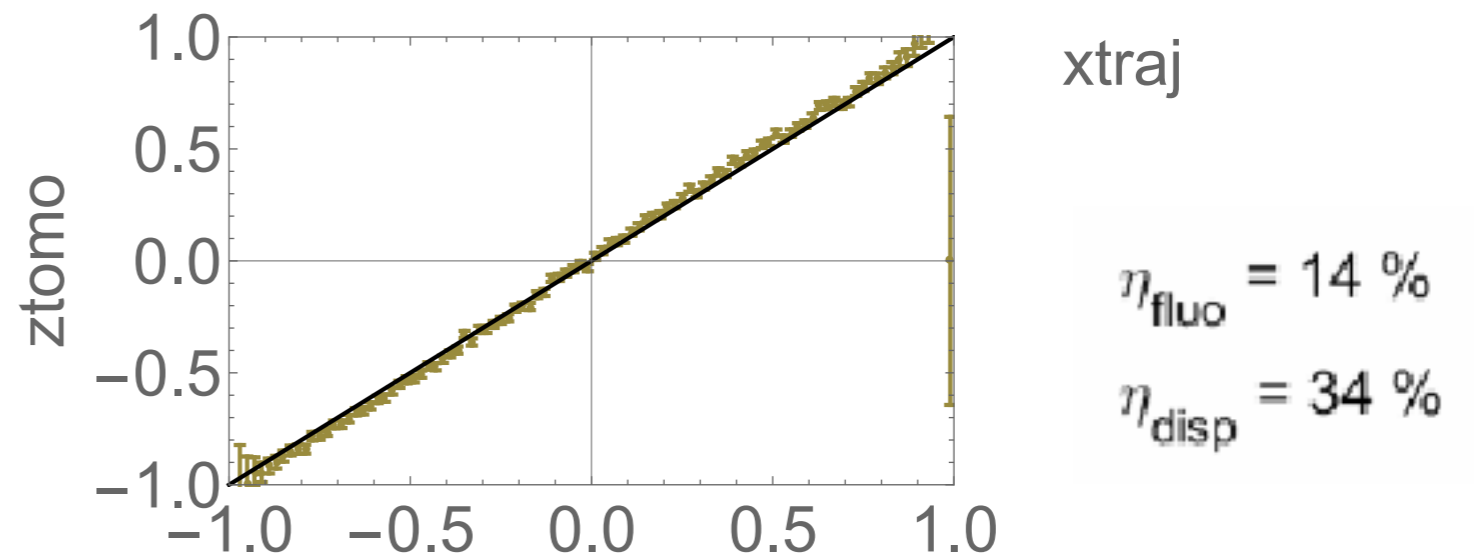
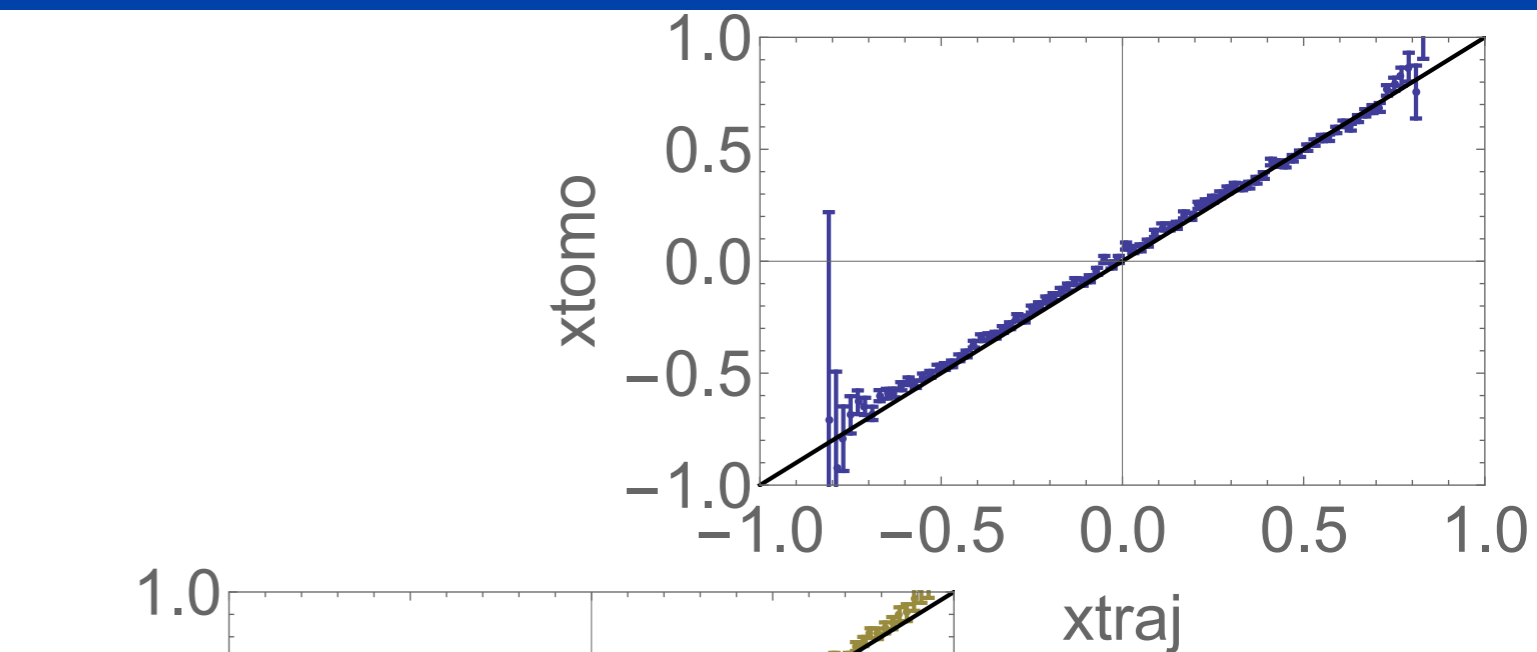
x y z

$$T_1 = 15.0 \mu\text{s}$$

$$T_2 = 11.2 \mu\text{s}$$

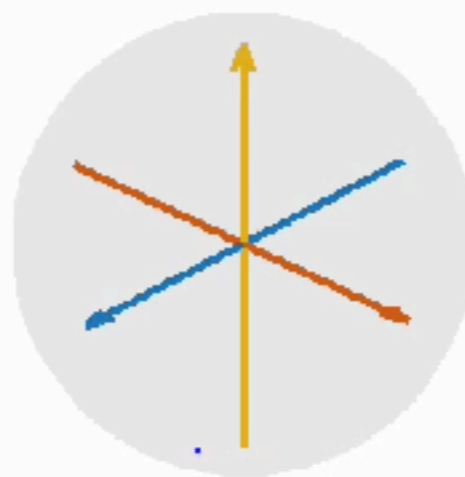
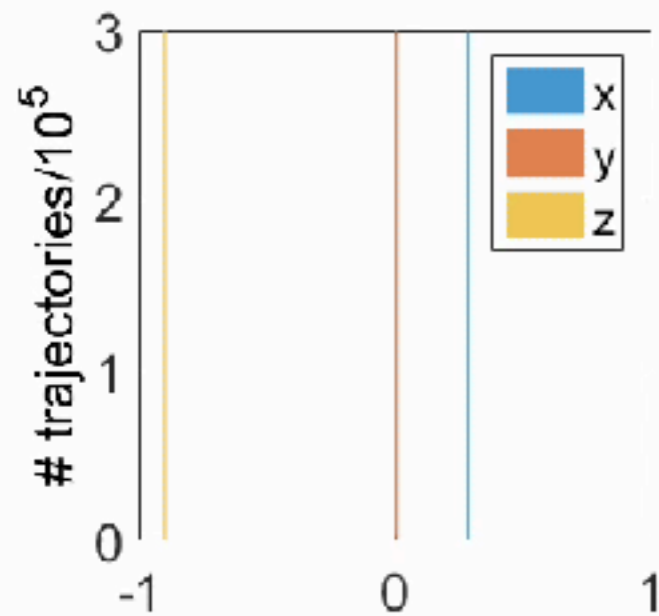
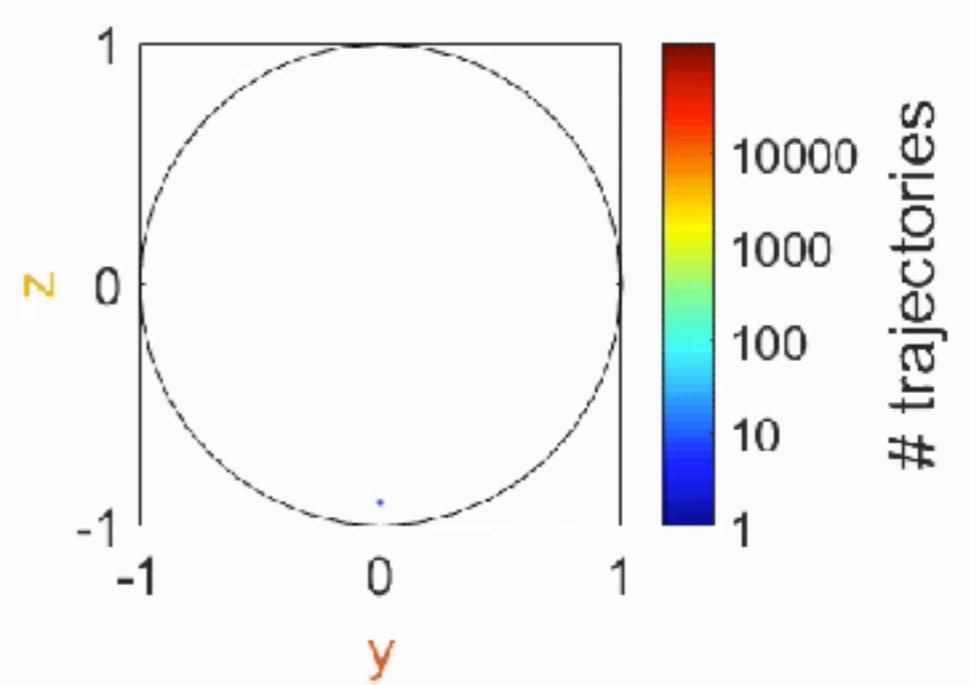
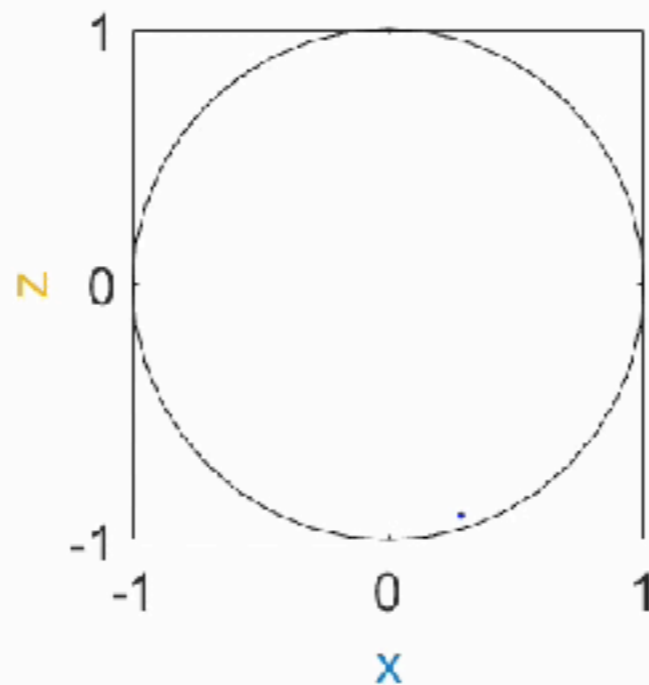
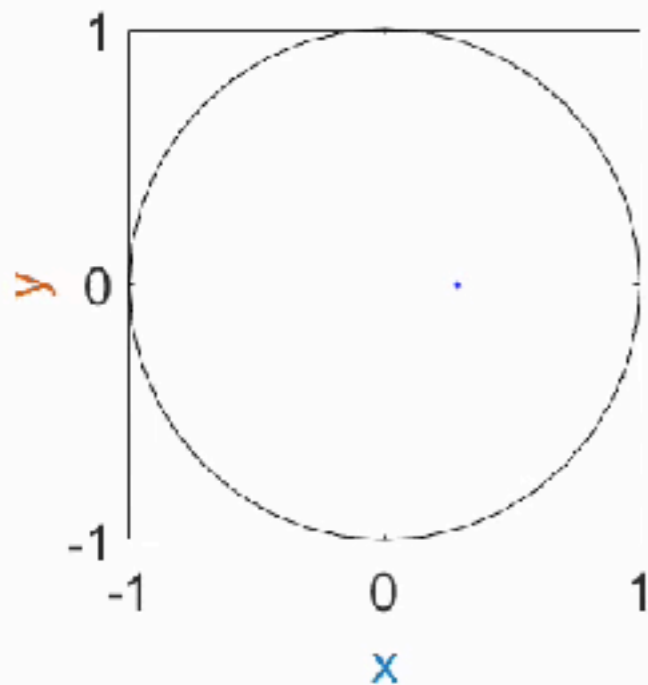
$$T_d = 0.9 \mu\text{s}$$

$$T_R = 5.2 \mu\text{s}$$



# statistics in the Zeno regime

$\sigma_Z$  measurement only



$t = 0.1 \mu\text{s}$

$\eta_{\text{fluo}} = 0 \%$

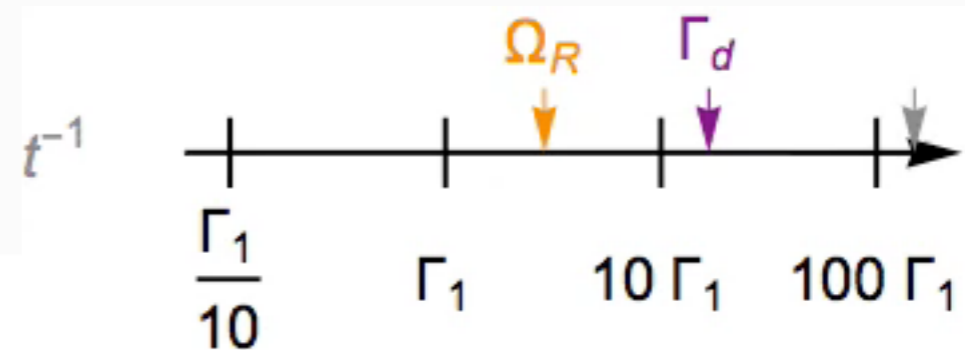
$\eta_{\text{disp}} = 34 \%$

$T_1 = 15.0 \mu\text{s}$

$T_2 = 11.2 \mu\text{s}$

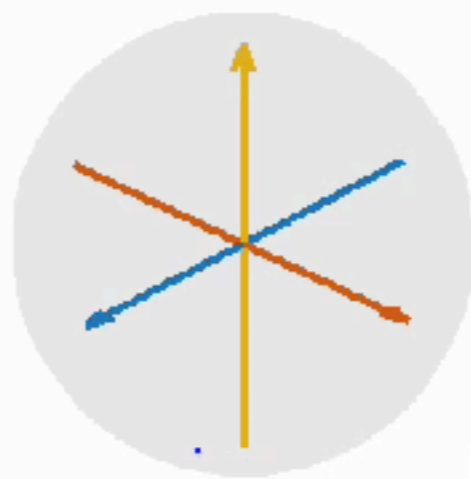
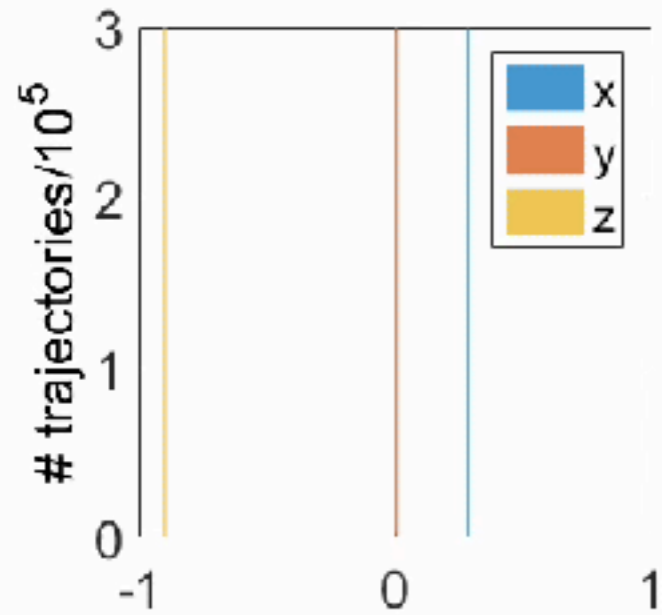
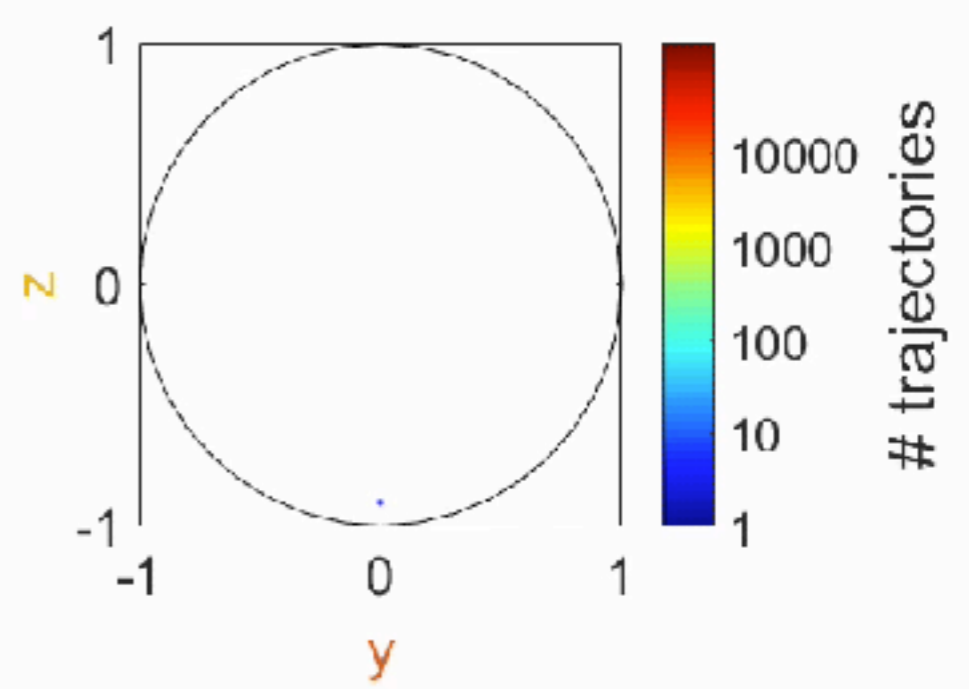
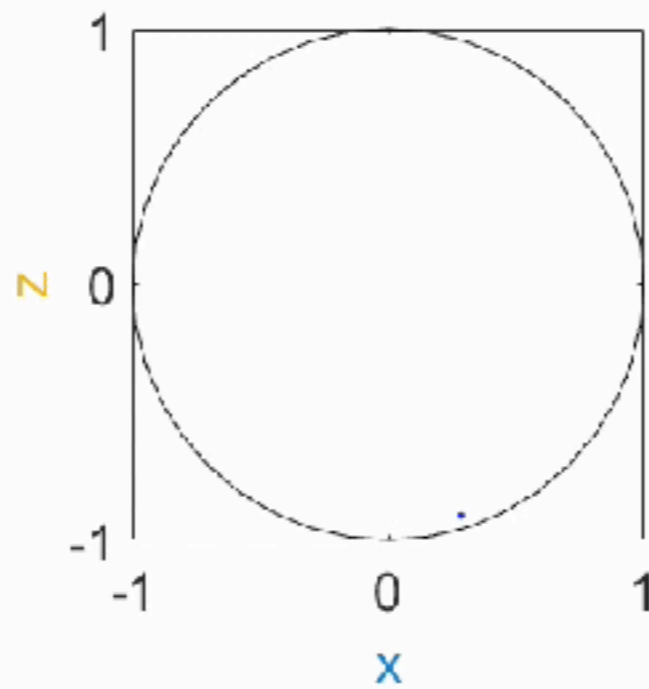
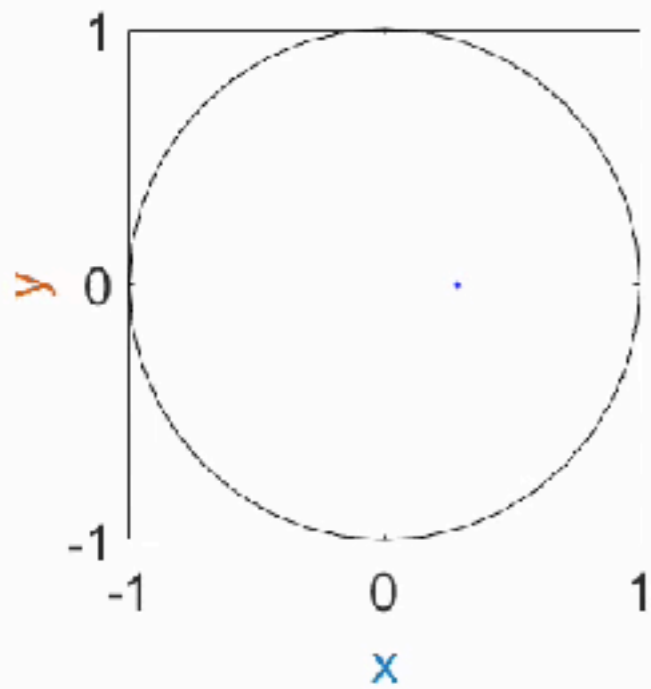
$T_d = 0.9 \mu\text{s}$

$T_R = 5.2 \mu\text{s}$



# statistics in the Zeno regime

$\sigma_-$  measurement only



$t = 0.1 \mu\text{s}$

$\eta_{\text{fluo}} = 14 \%$

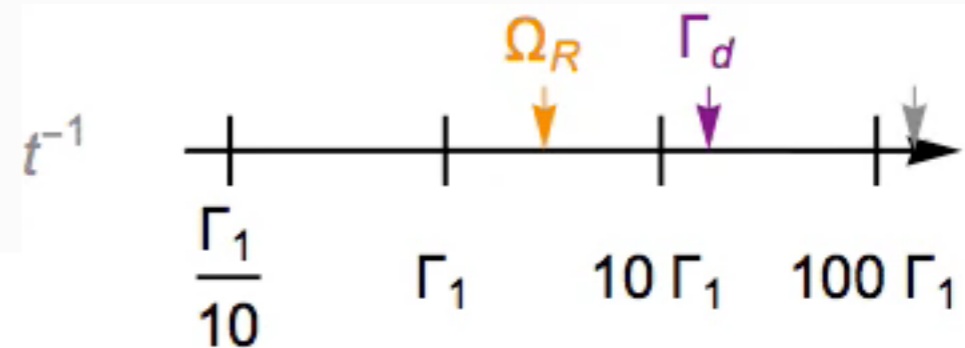
$\eta_{\text{disp}} = 0 \%$

$T_1 = 15.0 \mu\text{s}$

$T_2 = 11.2 \mu\text{s}$

$T_d = 0.9 \mu\text{s}$

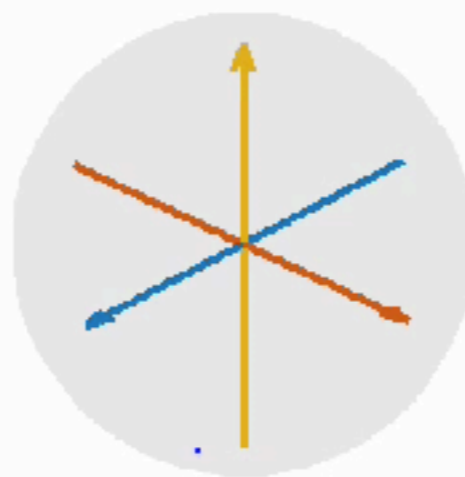
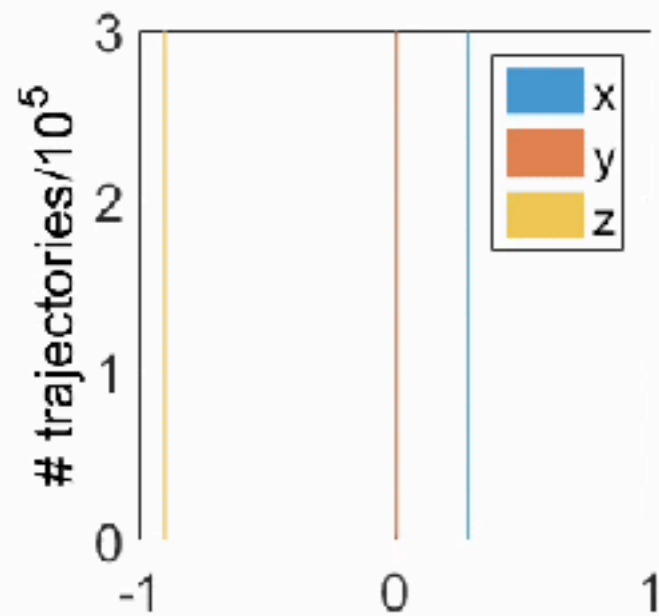
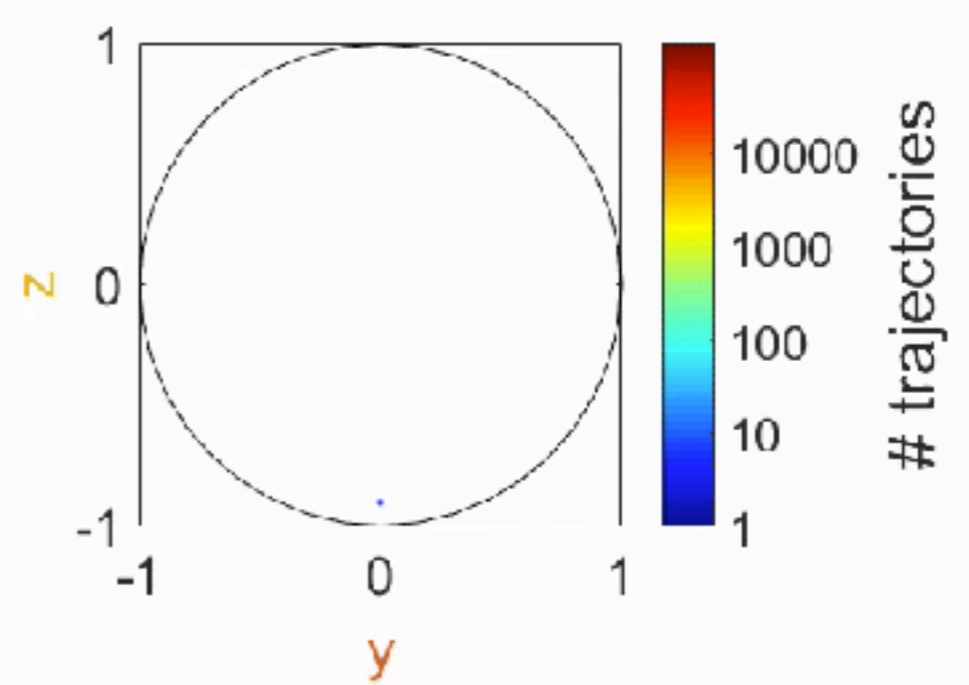
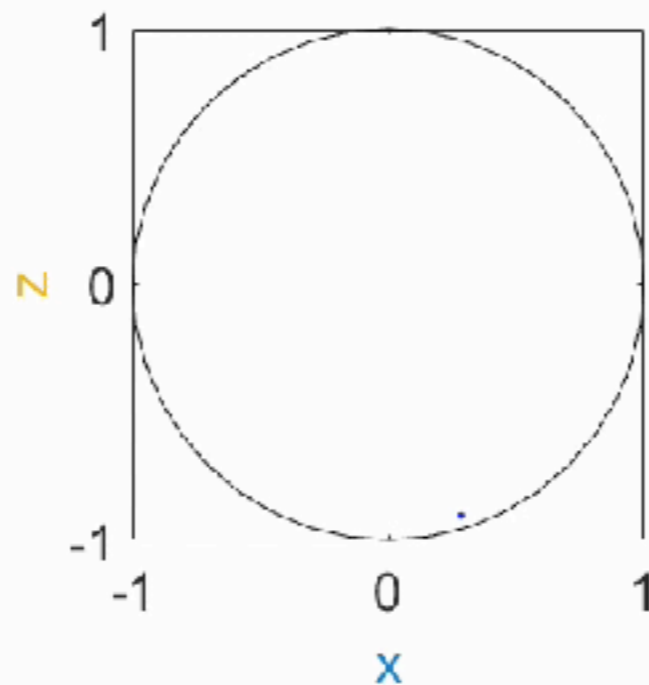
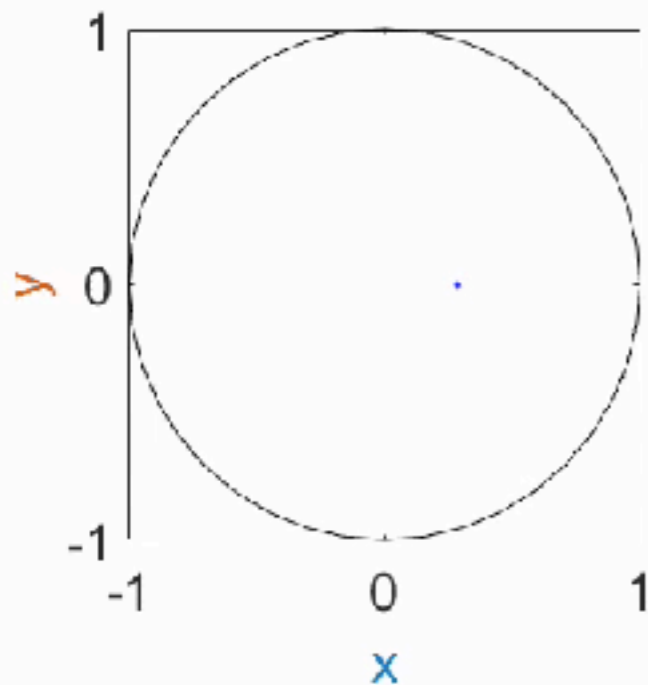
$T_R = 5.2 \mu\text{s}$





# statistics in the Zeno regime

$\sigma_x$  and  $\sigma_z$  measurements at the same time



$t = 0.1 \mu\text{s}$

$\eta_{\text{fluo}} = 14 \%$

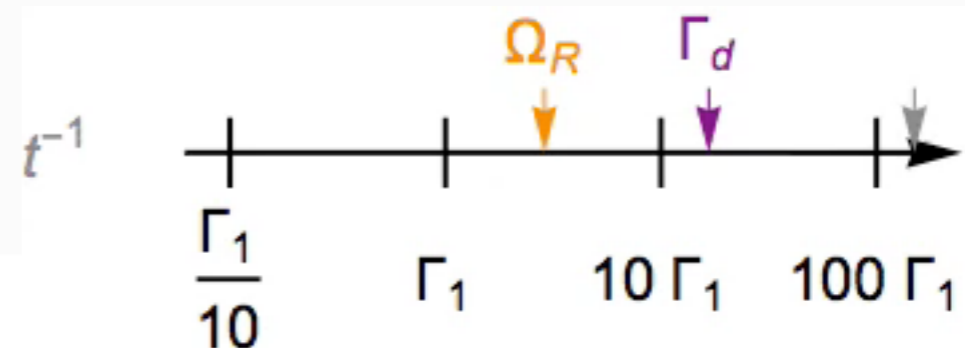
$\eta_{\text{disp}} = 34 \%$

$T_1 = 15.0 \mu\text{s}$

$T_2 = 11.2 \mu\text{s}$

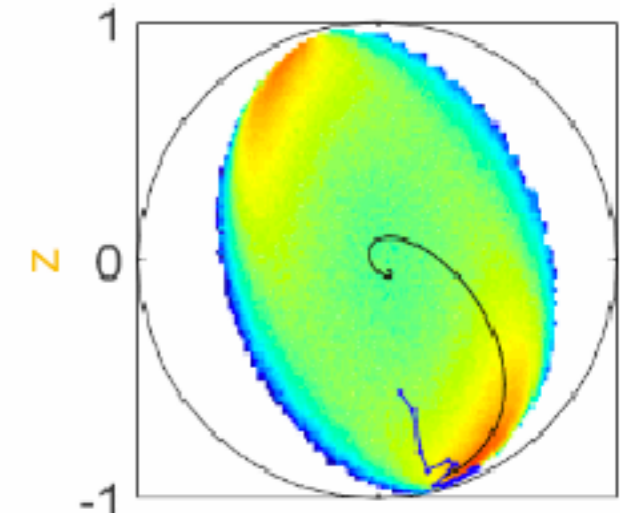
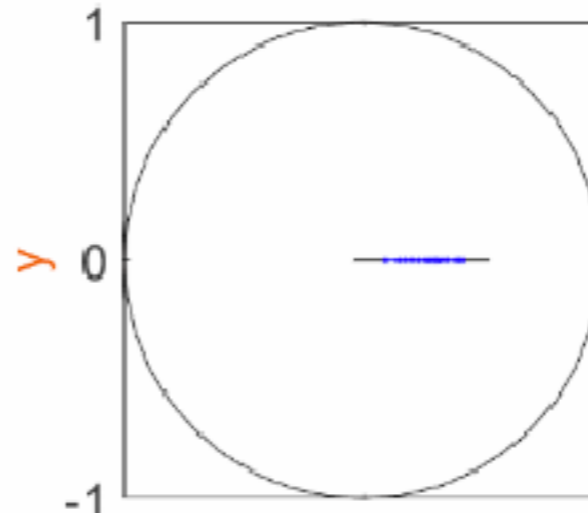
$T_d = 0.9 \mu\text{s}$

$T_R = 5.2 \mu\text{s}$

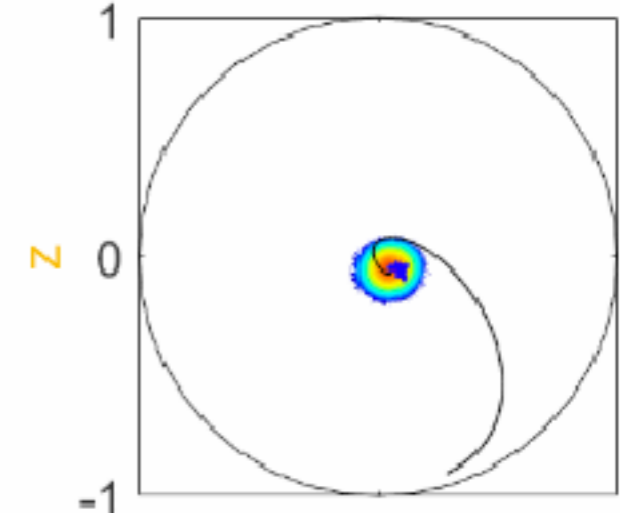
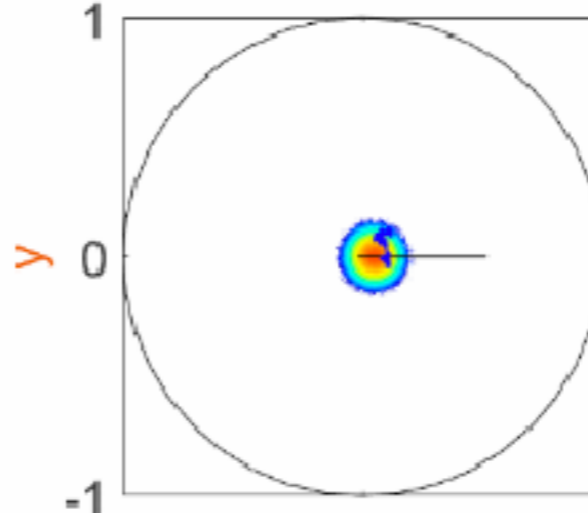


# statistics in the Zeno regime

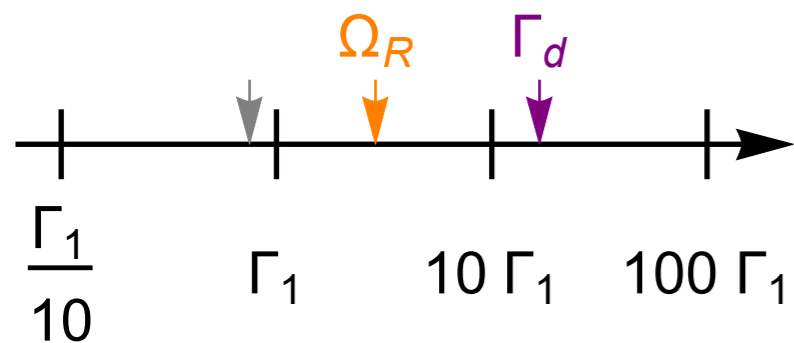
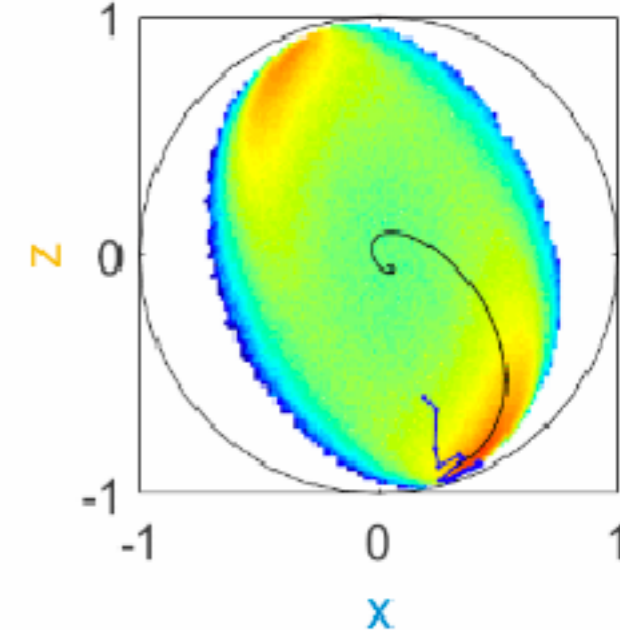
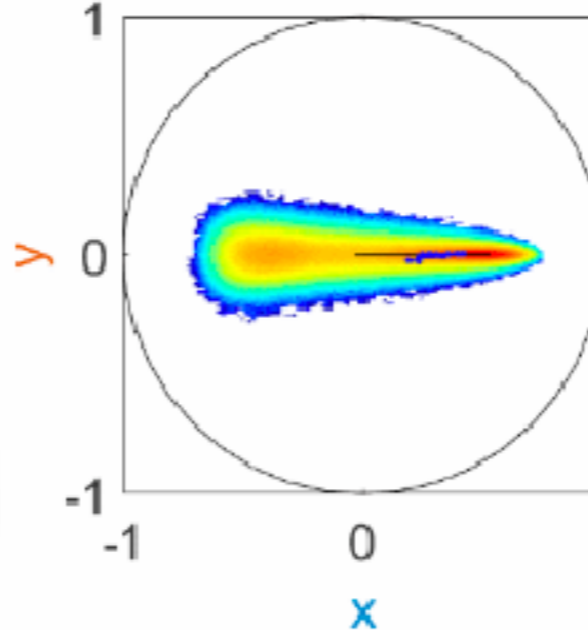
$\sigma_Z$  measurement only



$\sigma_-$  measurement only



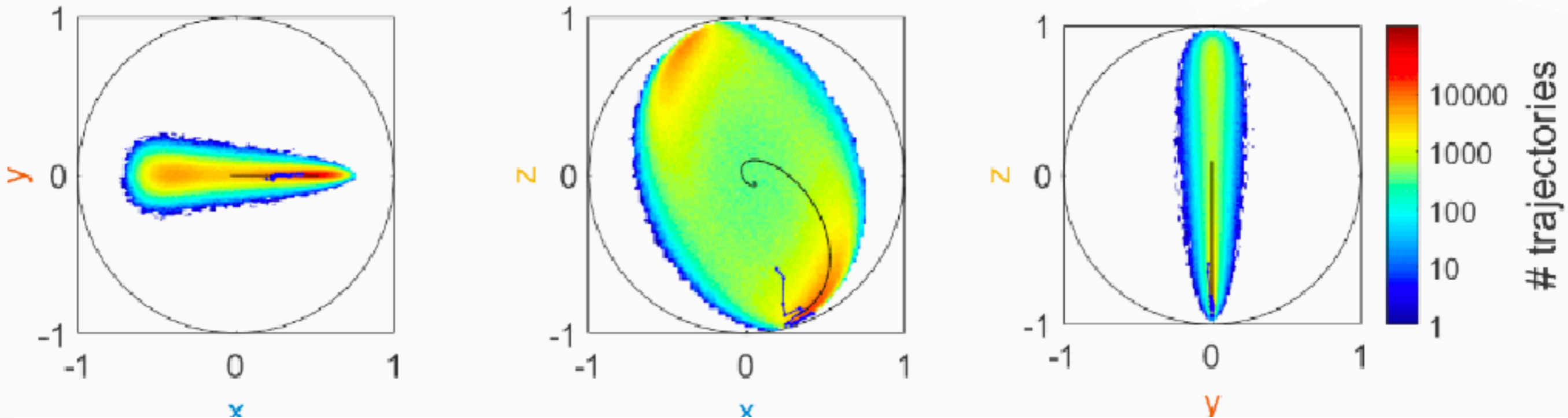
$\sigma_-$  and  $\sigma_Z$  measurements



# statistics in the Zeno regime

$\sigma_x$  and  $\sigma_z$  measurements at the same time

[Ficheux *et al.*, Nat. Comm. 2018]

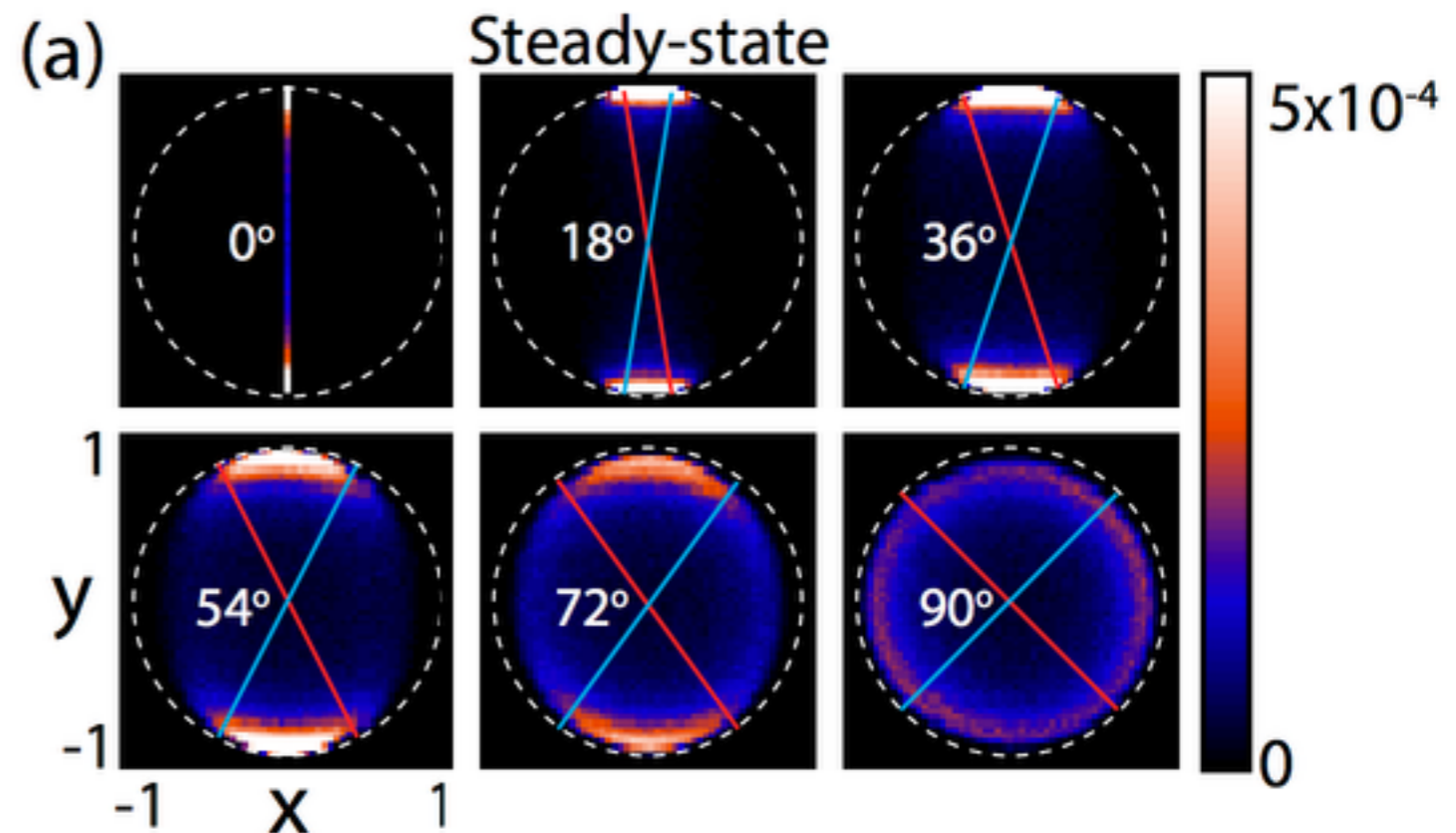


differs from the case of

$$\sigma_x \text{ and } \sigma_y$$

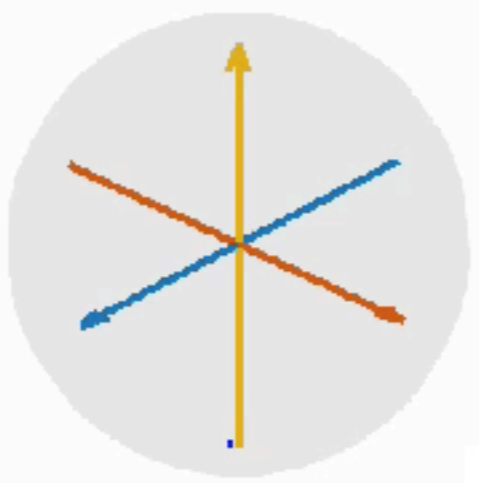
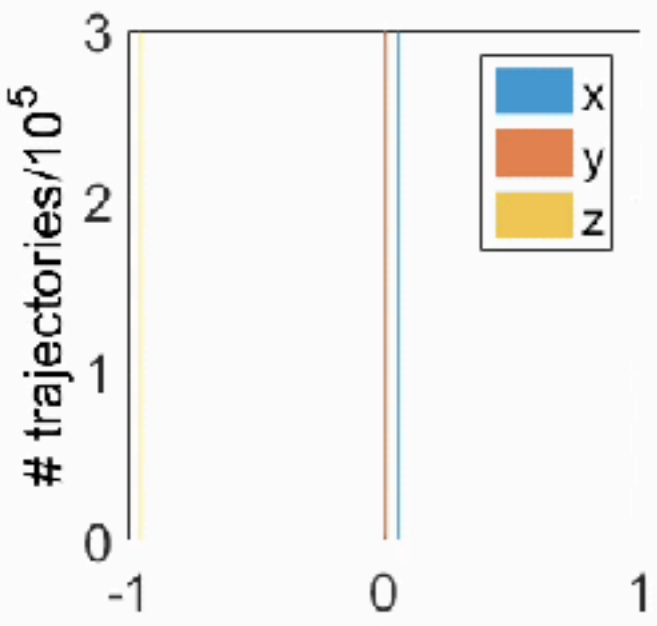
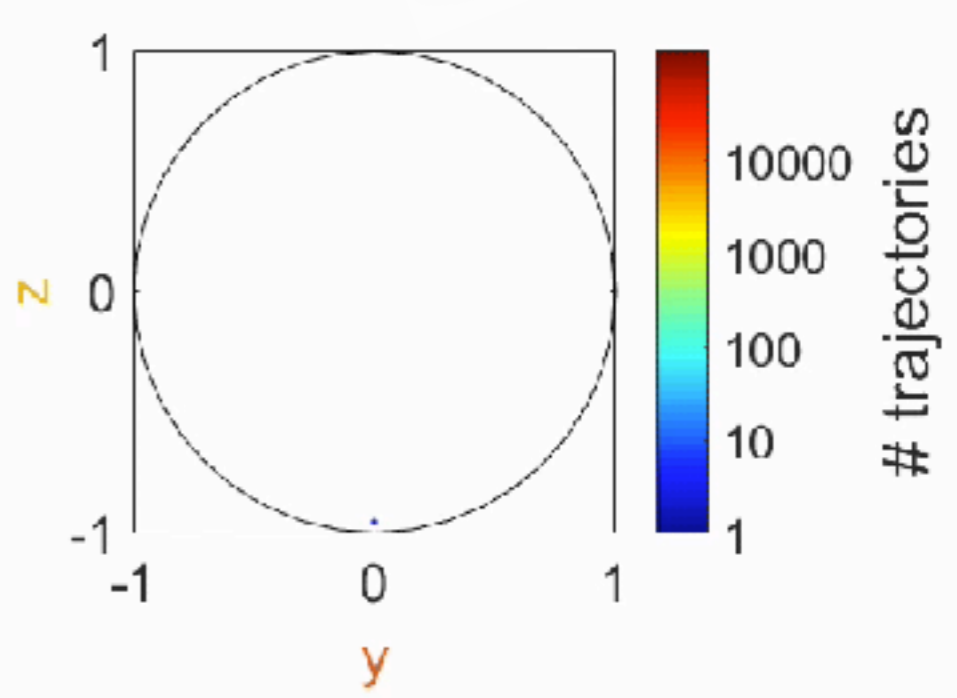
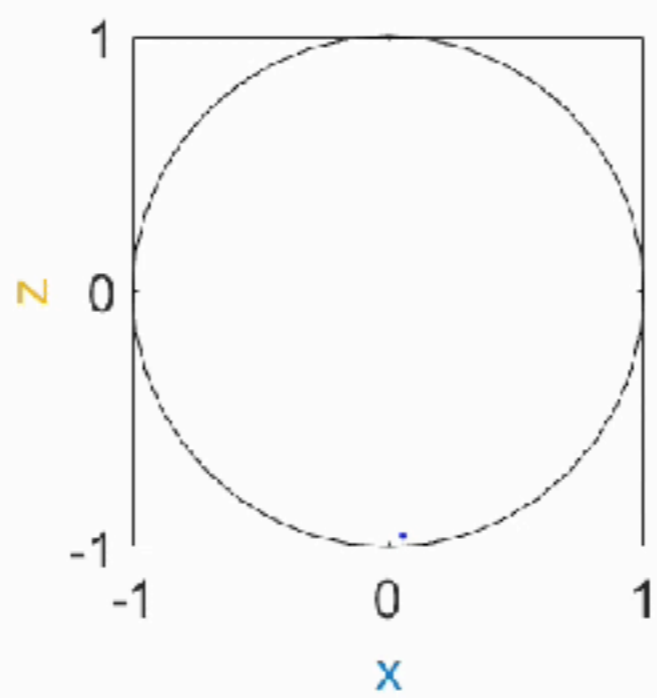
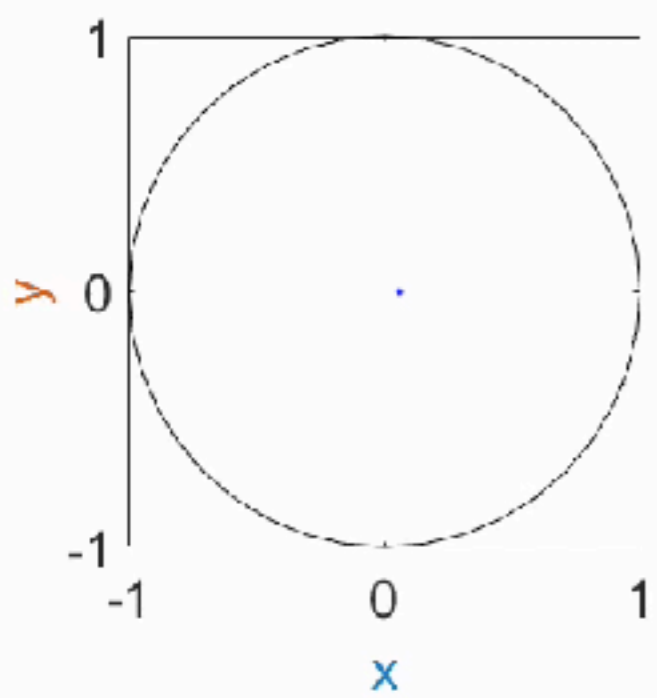
measurement

[Hacohen-Gourgy *et al.*, Nature 2016]



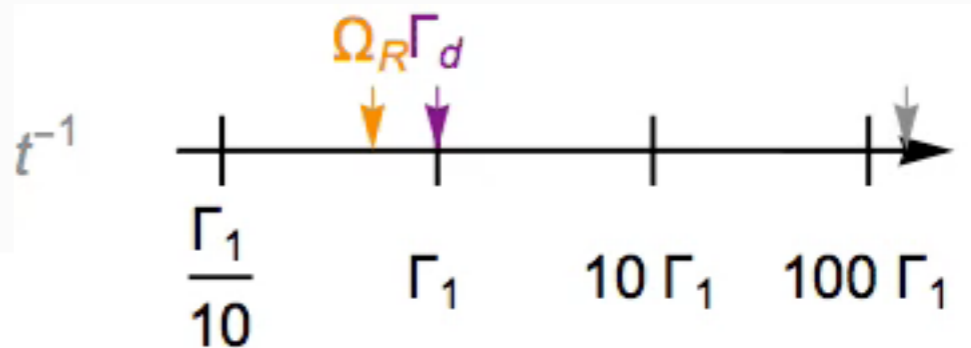
# statistics with weak measurement and slow rotation

$\sigma_Z$  measurement only



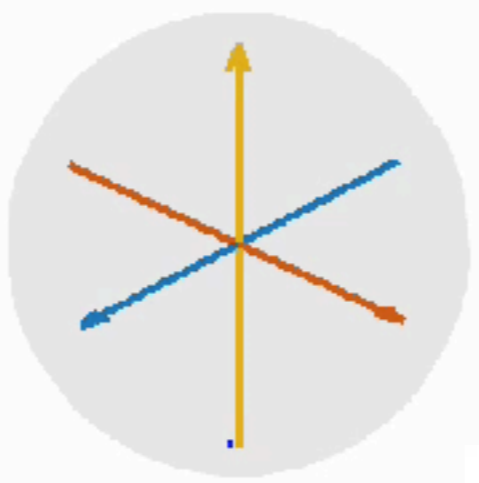
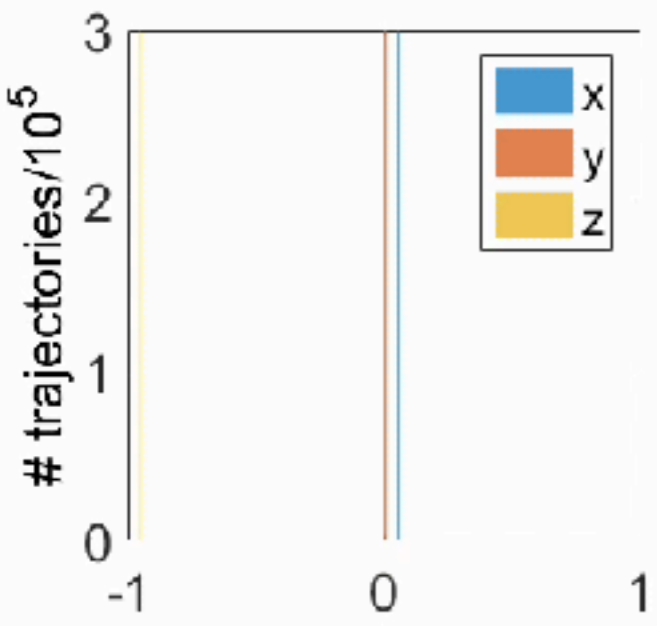
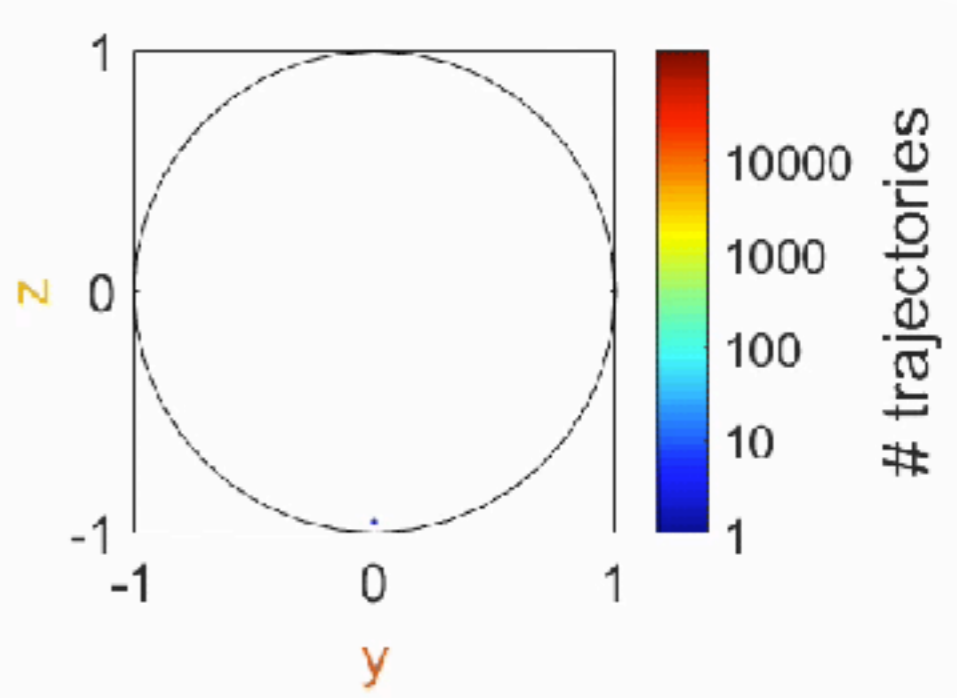
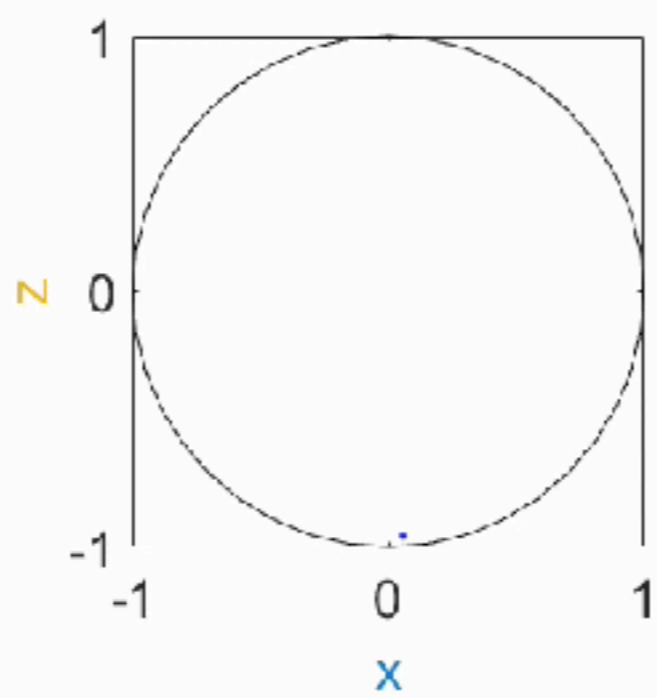
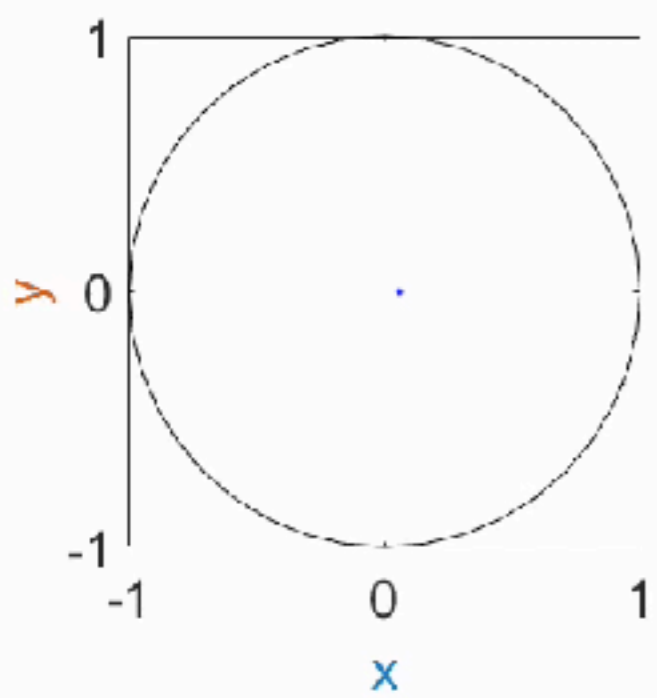
$t = 0.1 \mu\text{s}$   
 $\eta_{\text{fluo}} = 0 \%$   
 $\eta_{\text{disp}} = 34 \%$

$T_1 = 15.0 \mu\text{s}$   
 $T_2 = 11.2 \mu\text{s}$   
 $T_d = 15.0 \mu\text{s}$   
 $T_R = 30.0 \mu\text{s}$

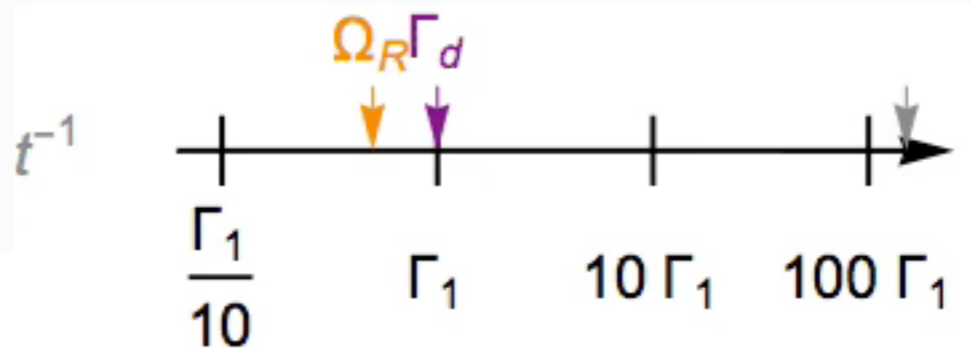


# statistics with weak measurement and slow rotation

$\sigma_-$  measurement only



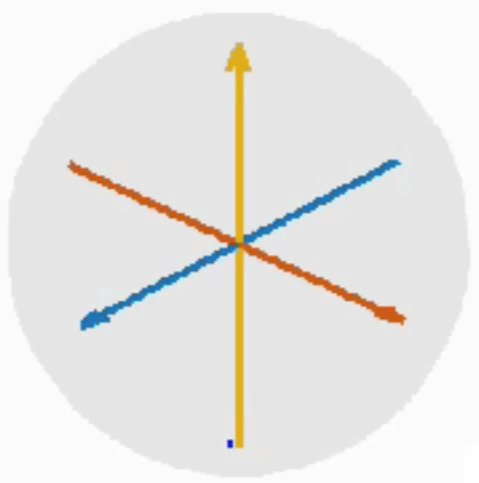
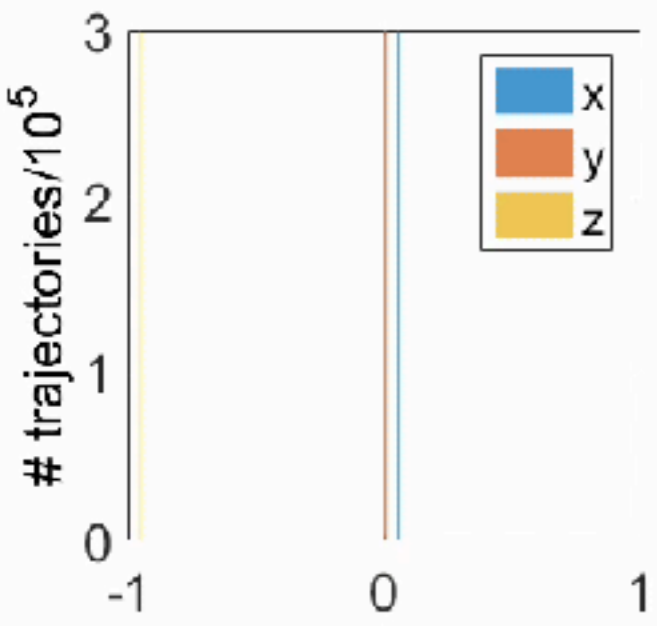
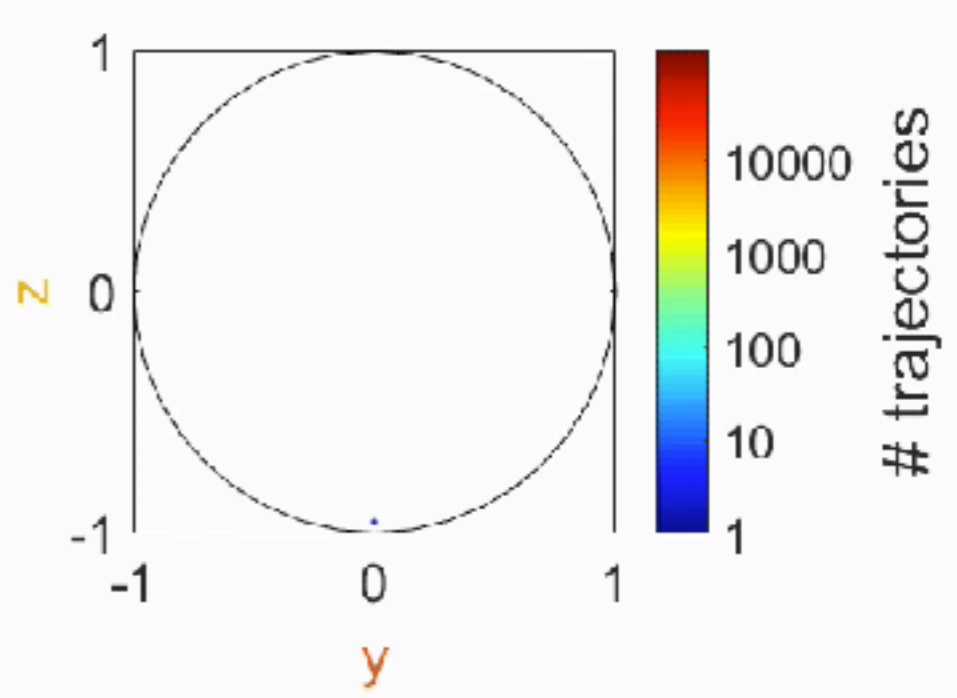
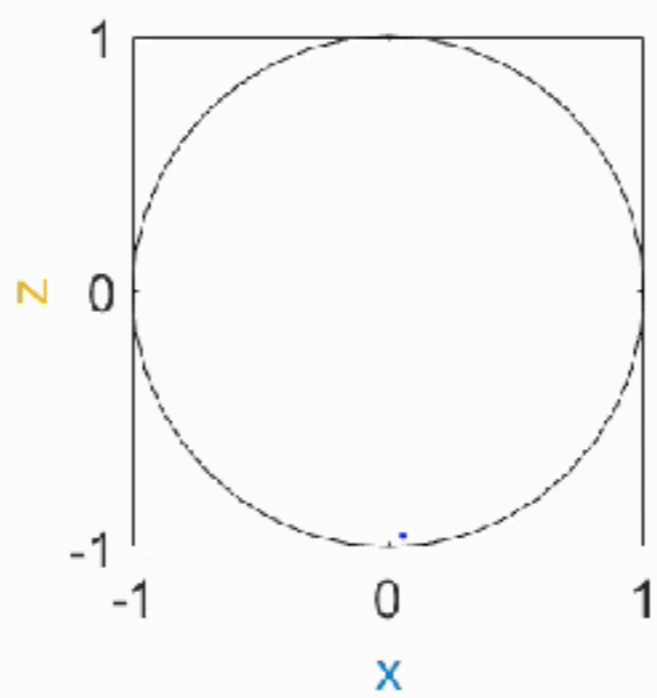
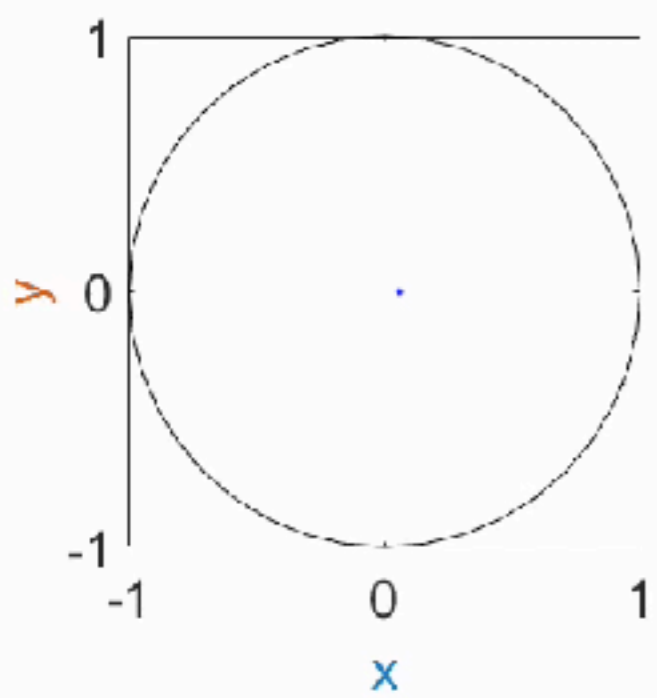
$t = 0.1 \mu\text{s}$   
 $\eta_{\text{fluo}} = 14 \%$   
 $\eta_{\text{disp}} = 0 \%$   
 $T_1 = 15.0 \mu\text{s}$   
 $T_2 = 11.2 \mu\text{s}$   
 $T_d = 15.0 \mu\text{s}$   
 $T_R = 30.0 \mu\text{s}$





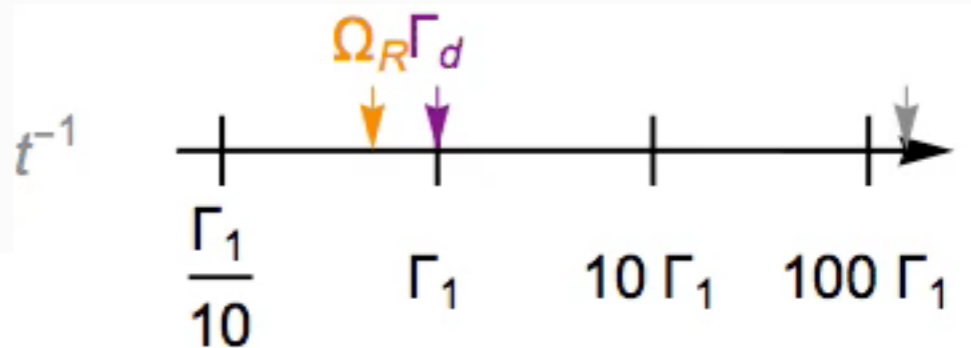
# statistics with weak measurement and slow rotation

$\sigma_x$  and  $\sigma_z$  measurements at the same time



$t = 0.1 \mu\text{s}$   
 $\eta_{\text{fluo}} = 14 \%$   
 $\eta_{\text{disp}} = 34 \%$

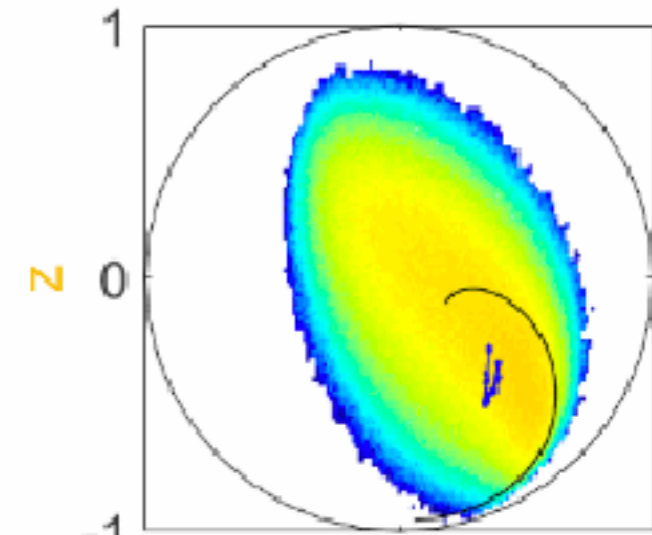
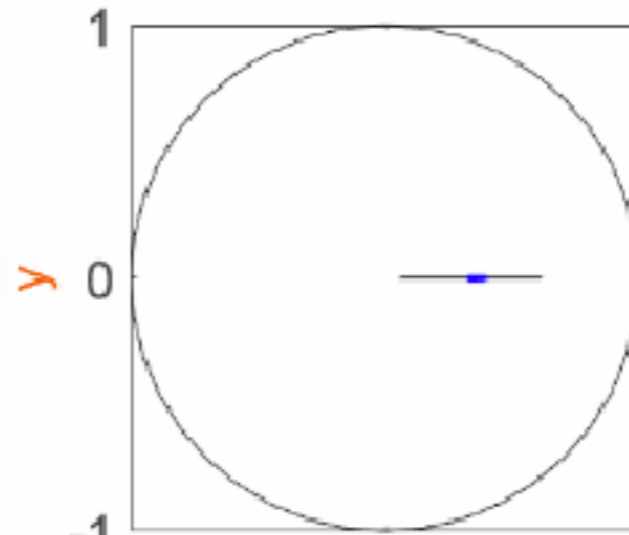
$T_1 = 15.0 \mu\text{s}$   
 $T_2 = 11.2 \mu\text{s}$   
 $T_d = 15.0 \mu\text{s}$   
 $T_R = 30.0 \mu\text{s}$



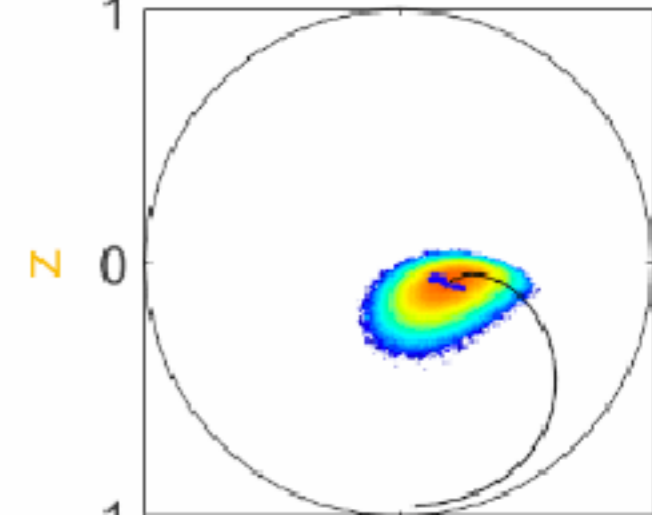
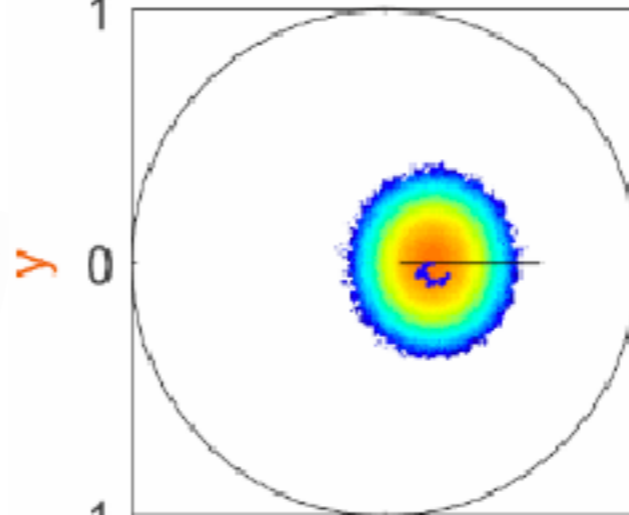


# statistics with weak measurement and slow rotation

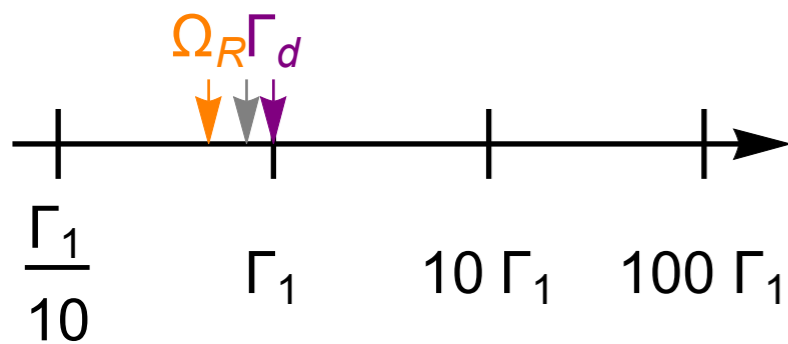
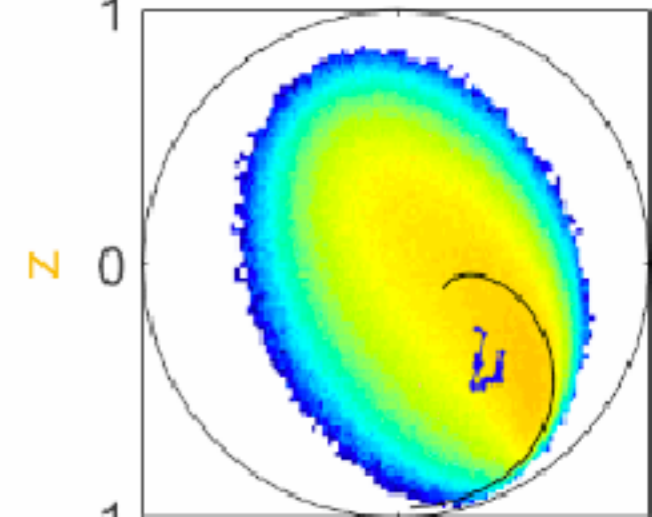
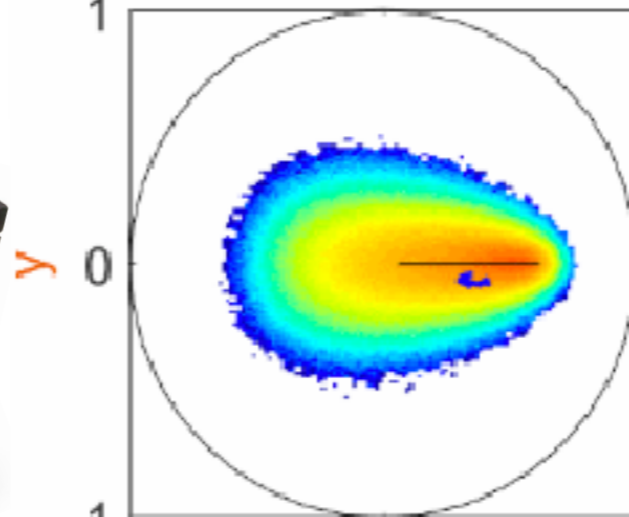
$\sigma_Z$  measurement only



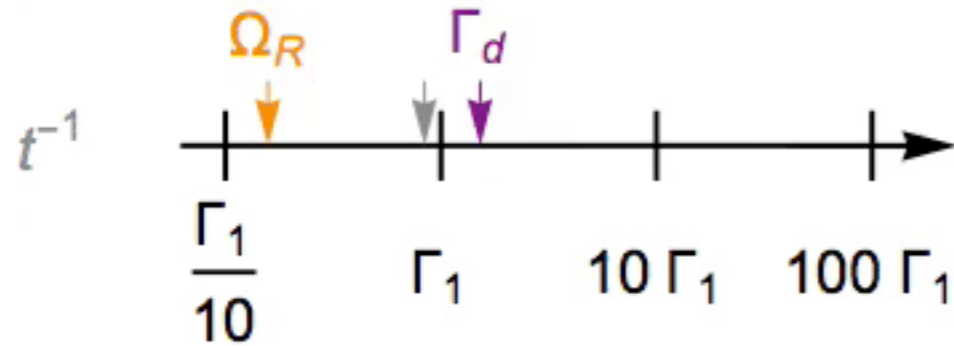
$\sigma_-$  measurement only



$\sigma_-$  and  $\sigma_Z$  measurements

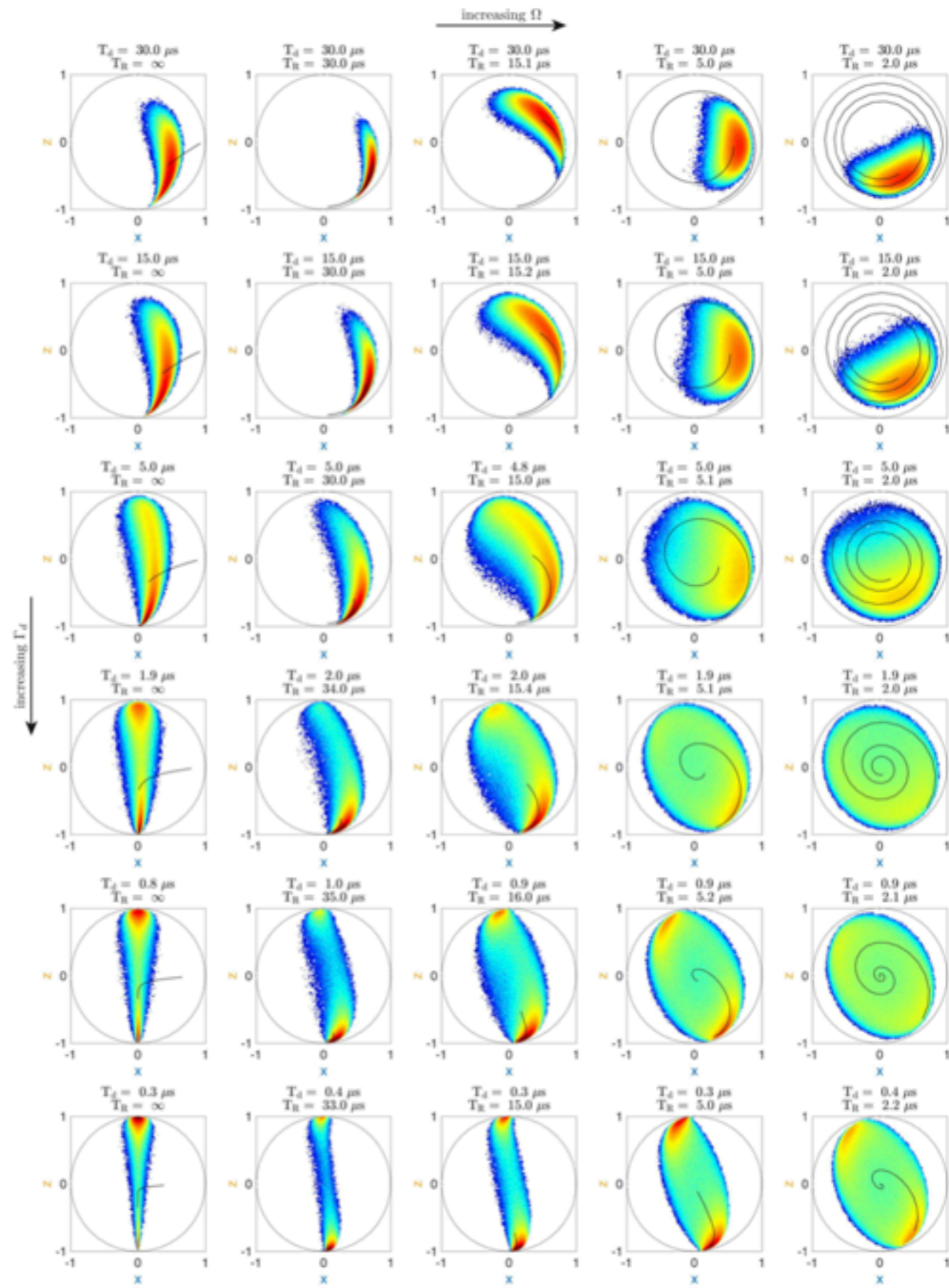


# Any configuration

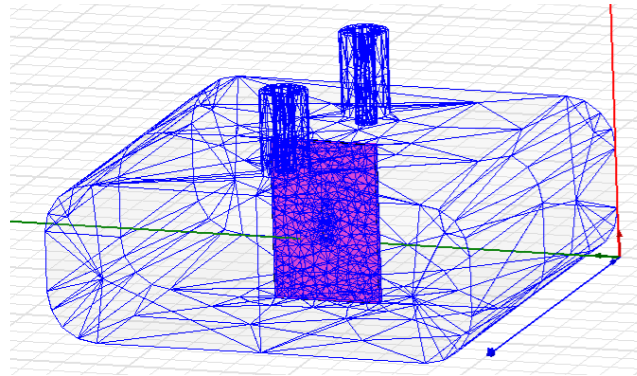


<http://www.physinfo.fr>

[Ficheux *et al.*, Nat. Comm. 2018]



# Quantum trajectories and feedback in circuit-QED



## Introduction to circuit-QED

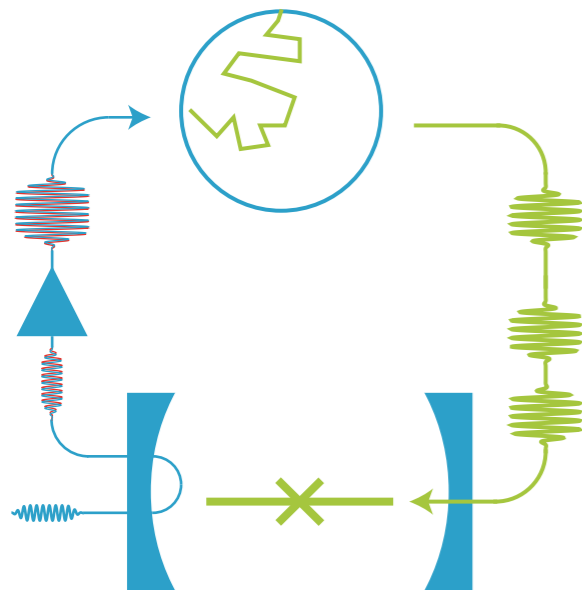
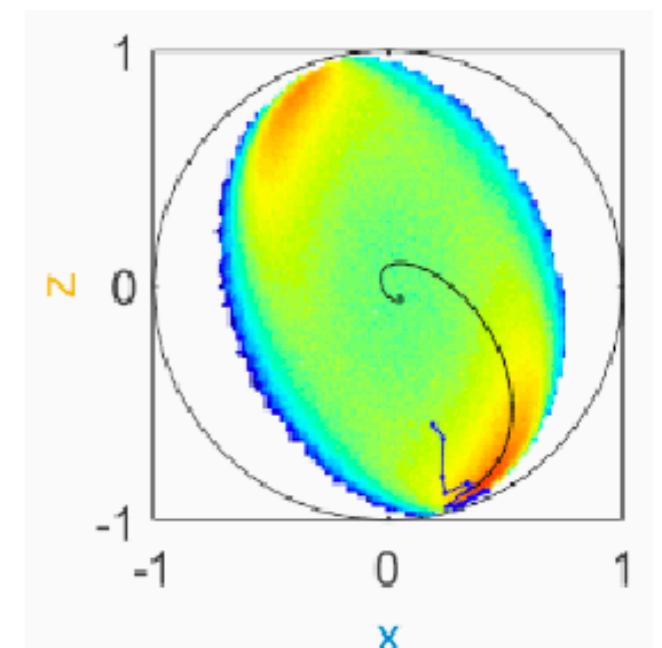
## Quantum trajectories in circuit-QED

dispersive measurement

fluorescence measurement

both simultaneously

generating entanglement

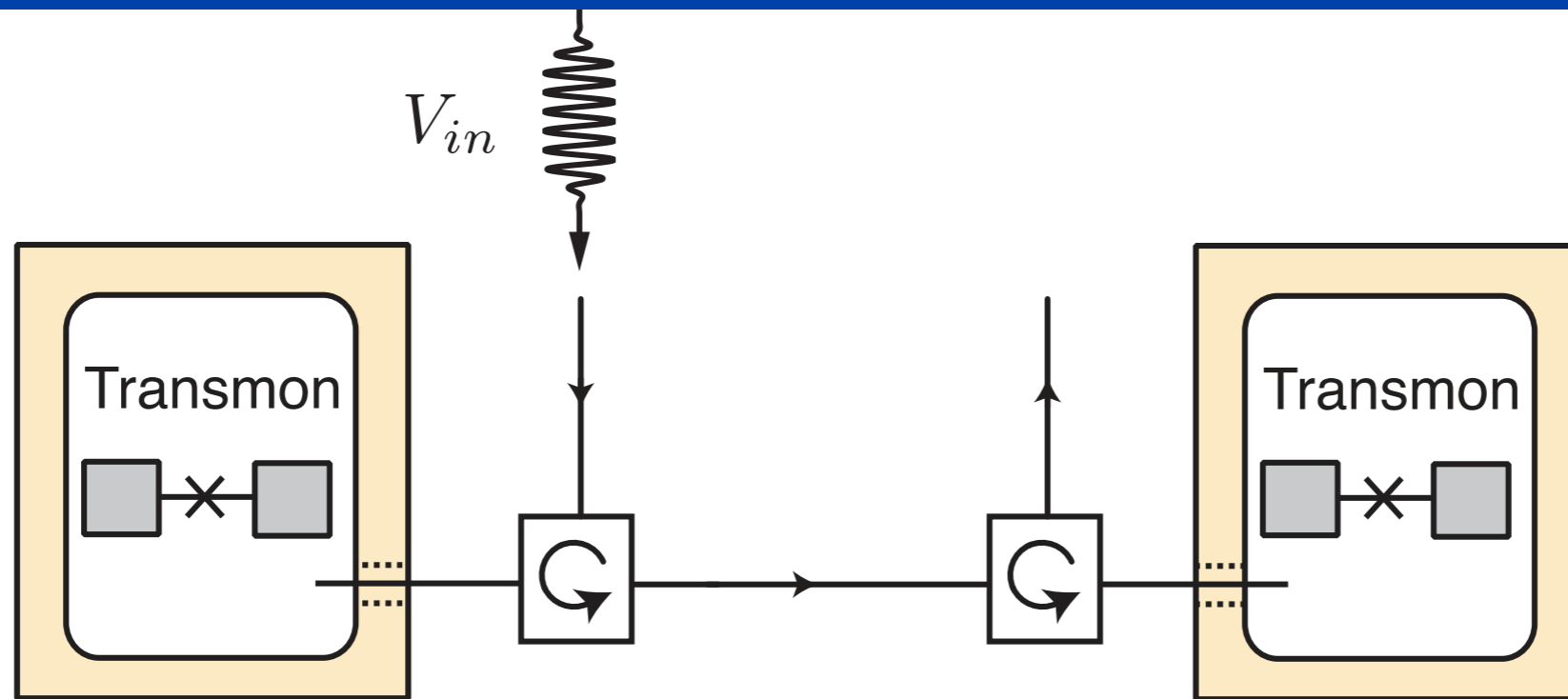


## Measurement based feedback

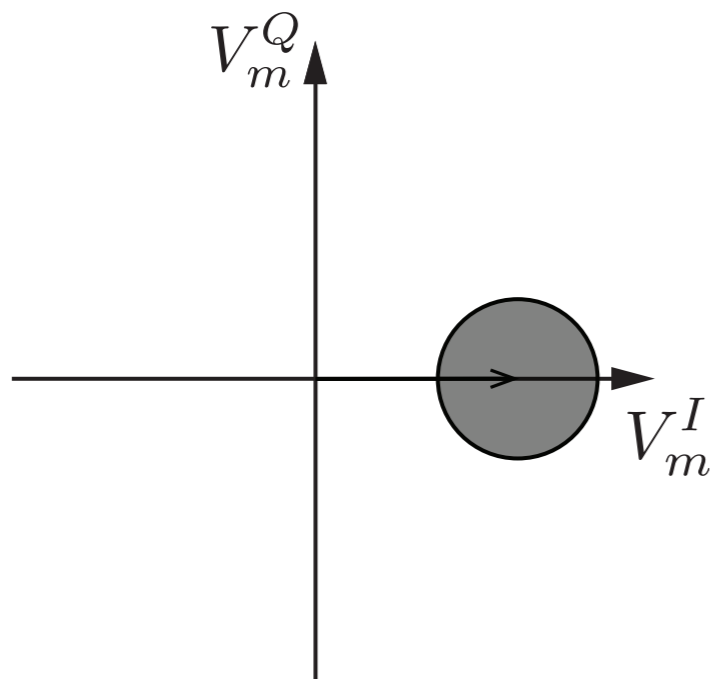
dispersive case

fluorescence case

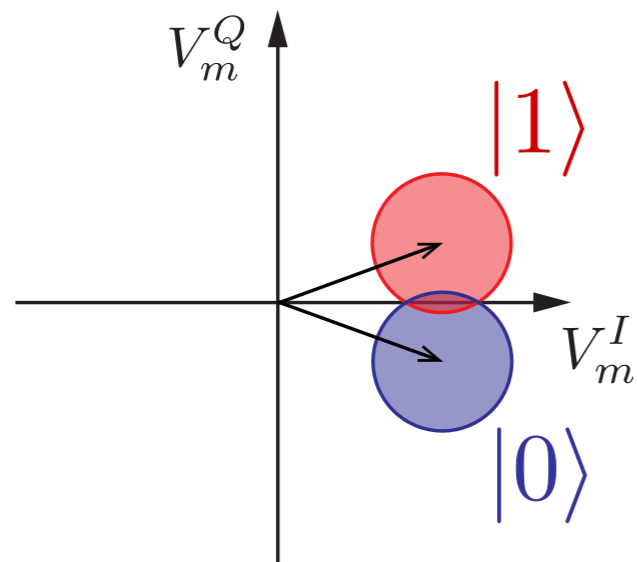
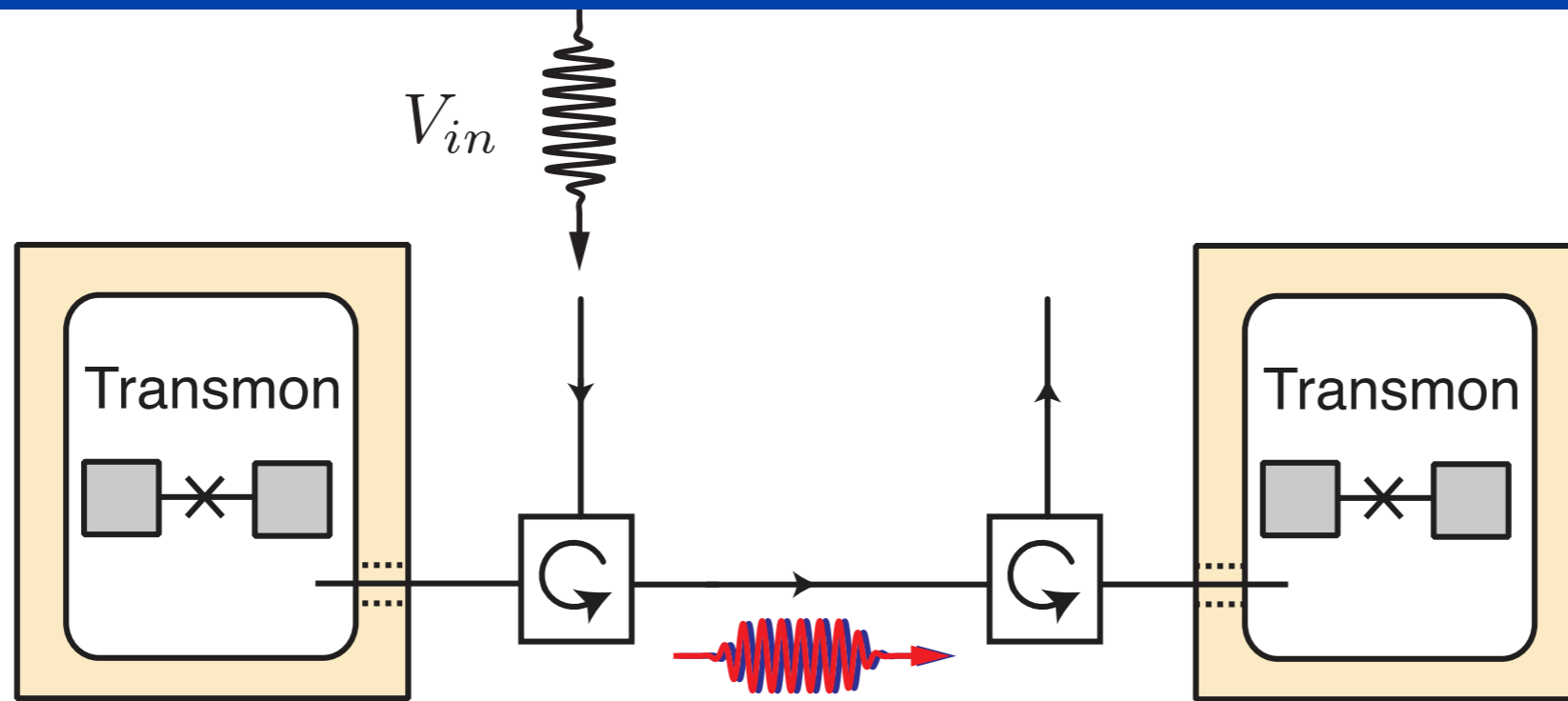
# Cascade approach to entanglement generation



[Risté et al., Nature 2013 (Delft)]  
[Roch et al., PRL 2014 (Berkeley)]  
[Chantasri et al., PRX 2016  
(Rochester+Berkeley)]  
[Dickel et al., PRB 2018 (Delft)]

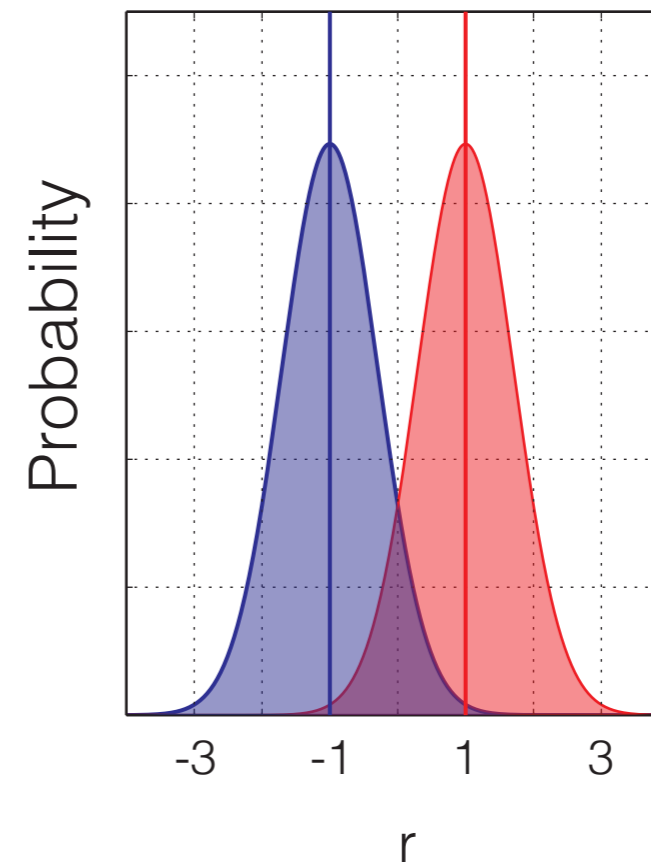


# Cascade approach to entanglement generation

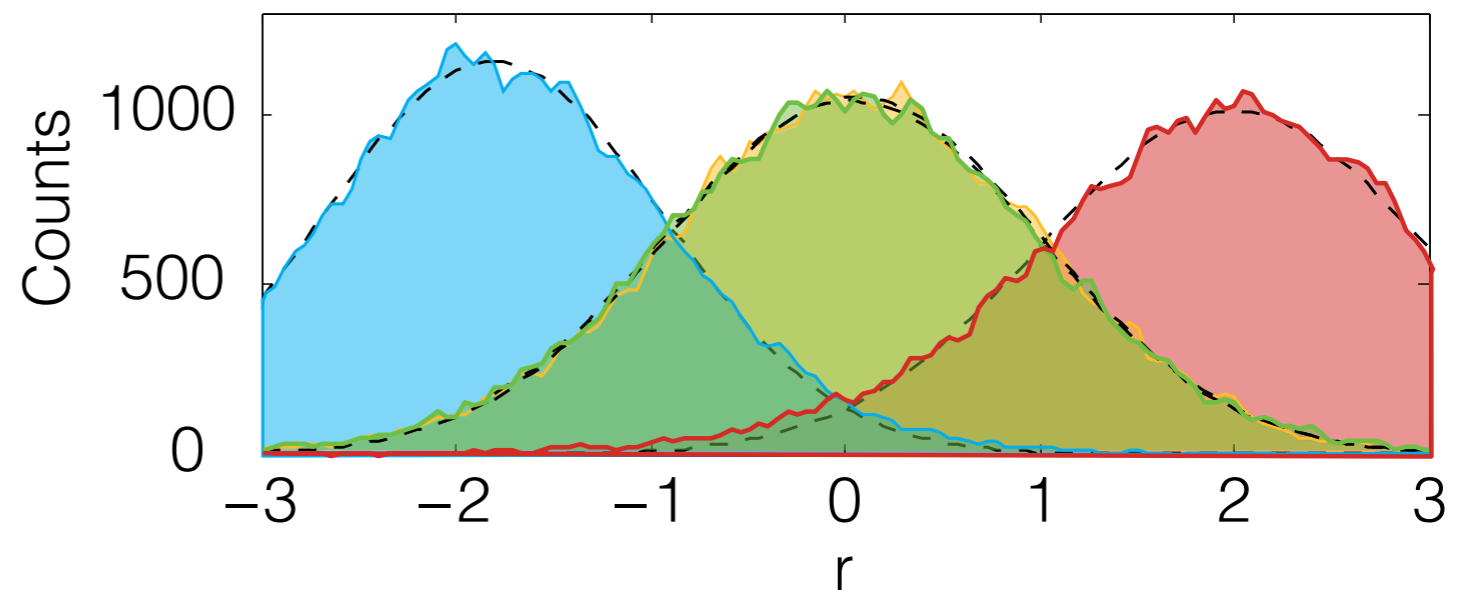
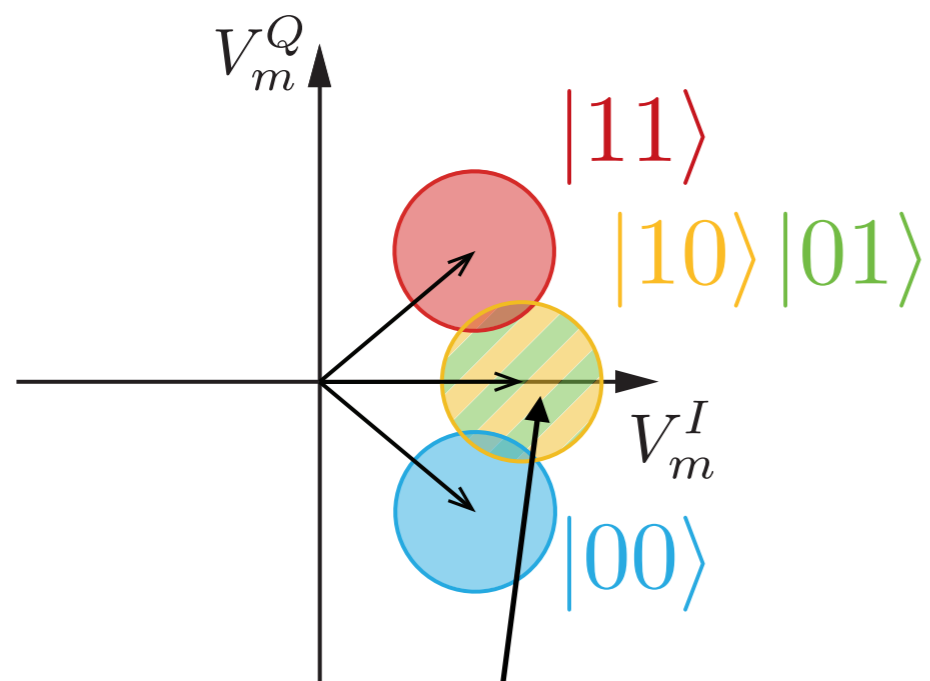
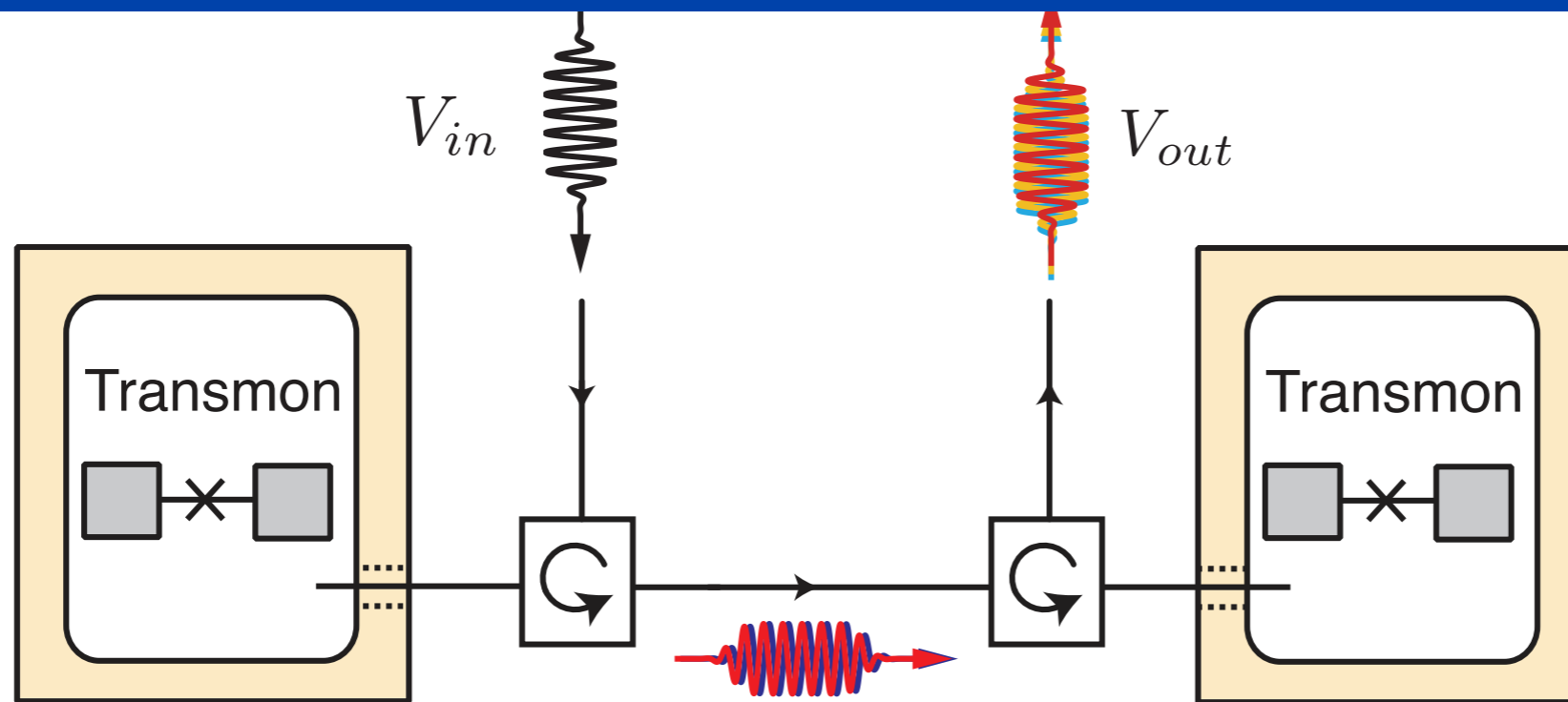


$$V_m = \frac{1}{\Delta t} \int_0^{\Delta t} V_{out}(t) dt$$

$$r = 2V_m^Q / \Delta V$$



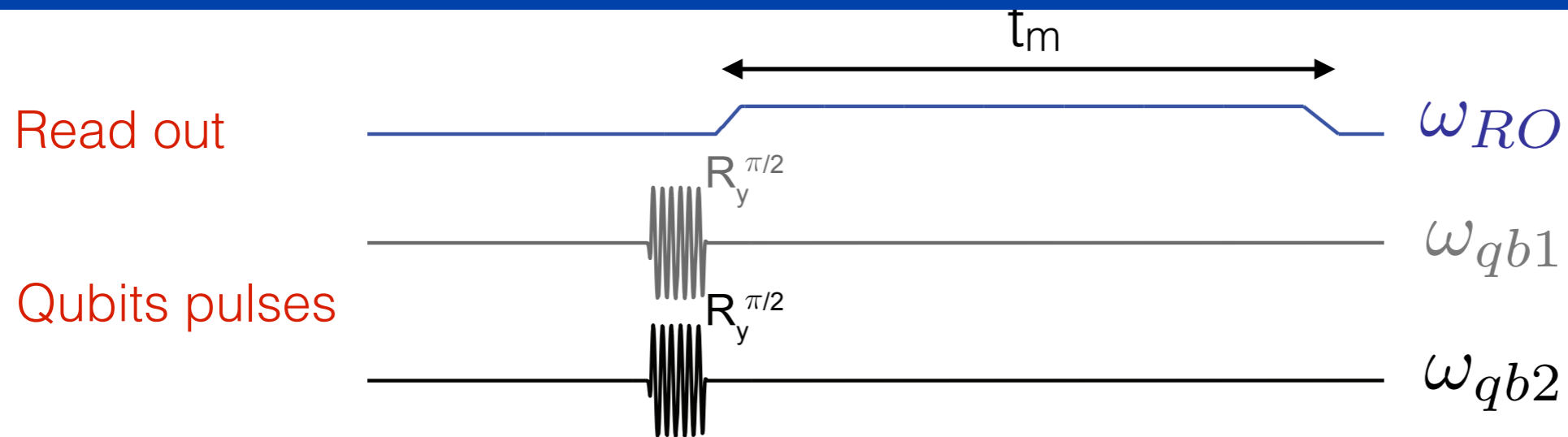
# Cascade approach to entanglement generation



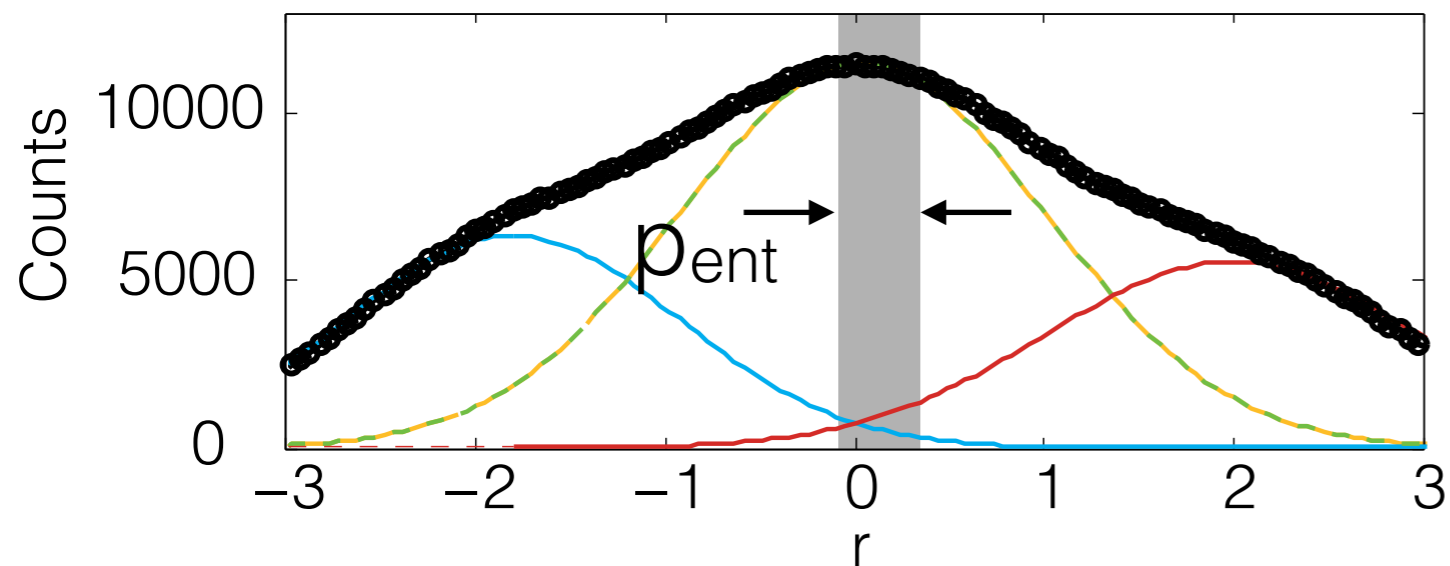
outcome here  $\longrightarrow$  projects the qubit pair onto  $\text{span}(|10\rangle, |01\rangle)$



# Measurement induced entanglement

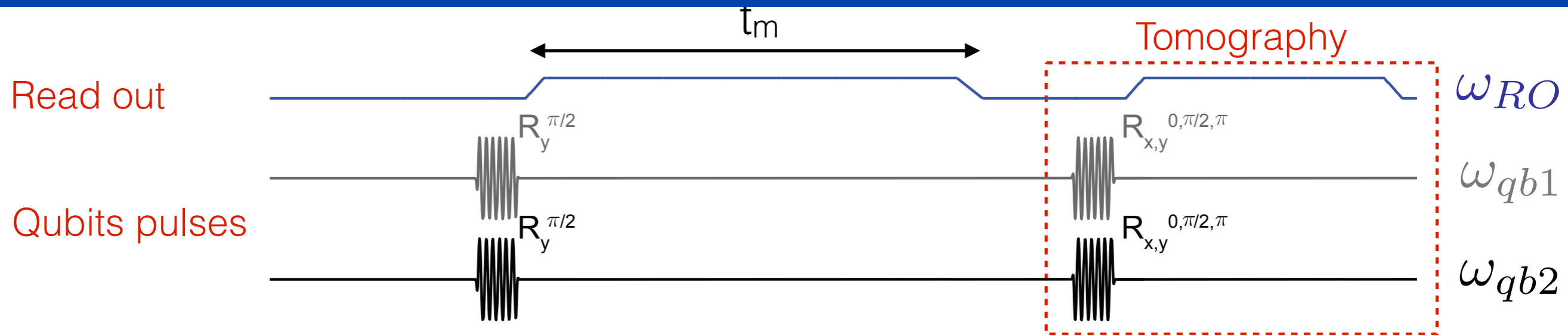


$$|\psi_i\rangle = (|00\rangle + |01\rangle + |10\rangle + |11\rangle) / 2$$

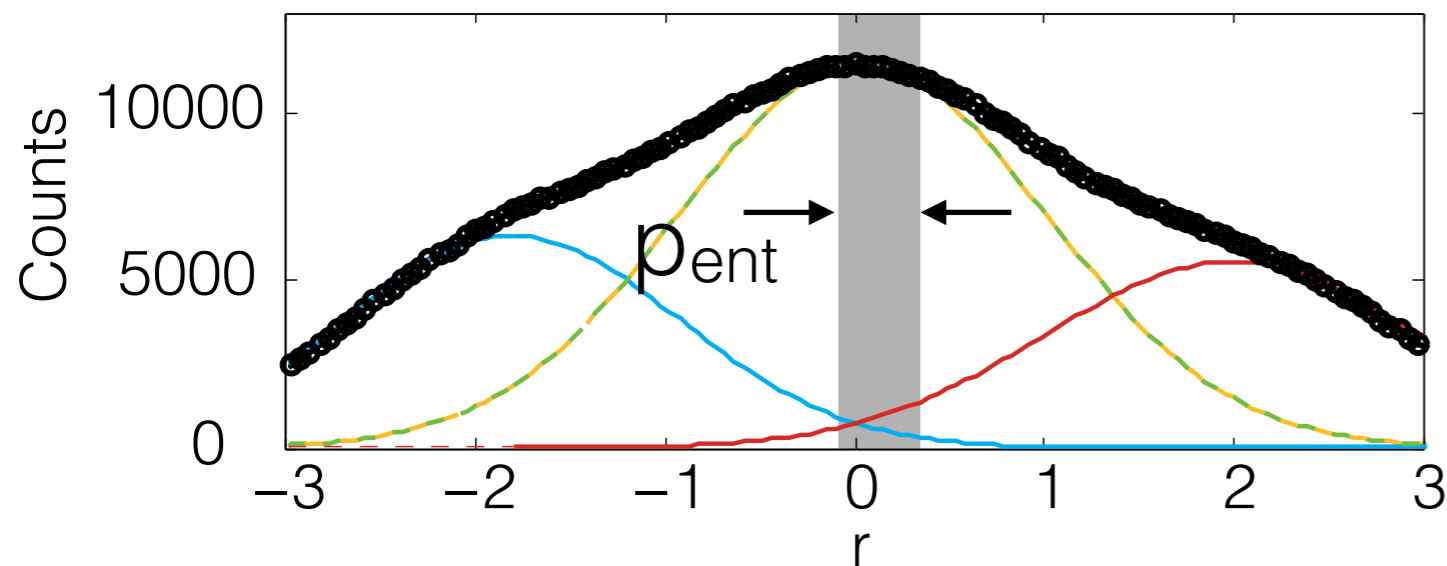


$$\rho_{ent} = 10\%$$

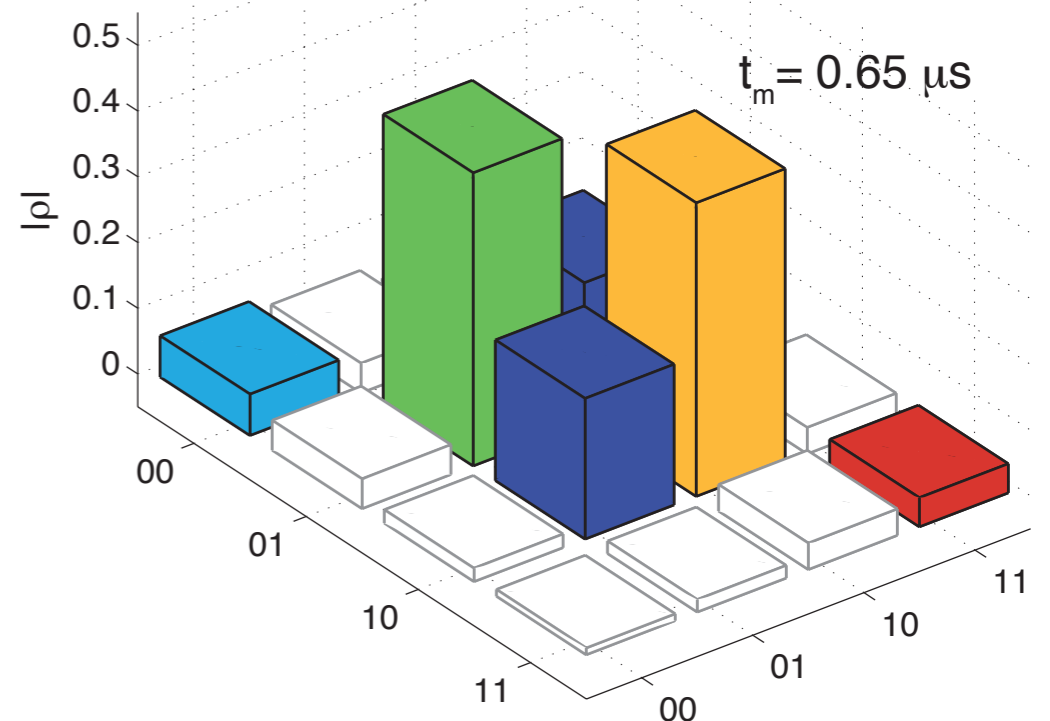
# Measurement induced entanglement



$$|\psi_i\rangle = (|00\rangle + |01\rangle + |10\rangle + |11\rangle) / 2$$

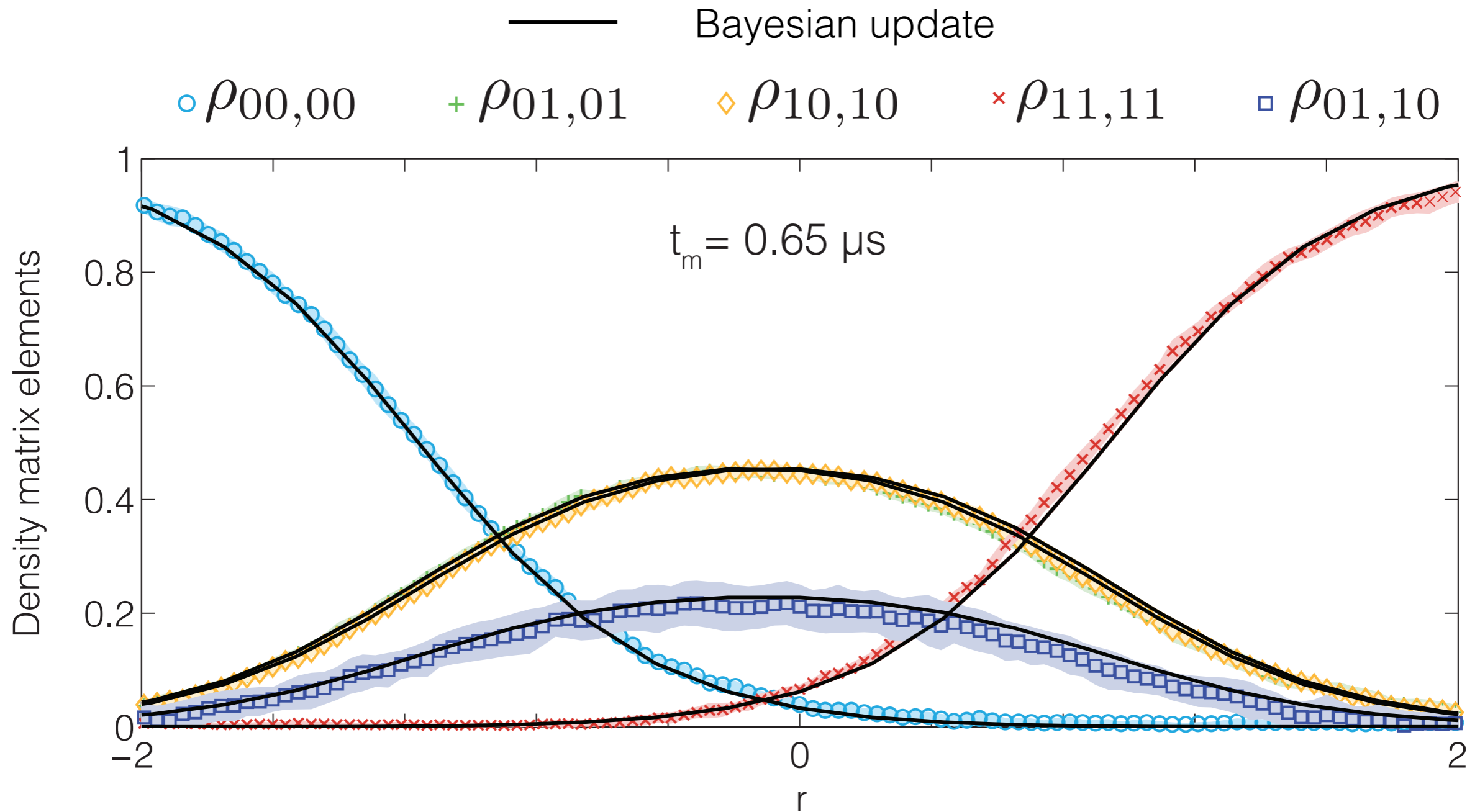


$$\rho_{ent} = 10\%$$



$$\mathcal{C} = 0.35$$

# Conditional tomography



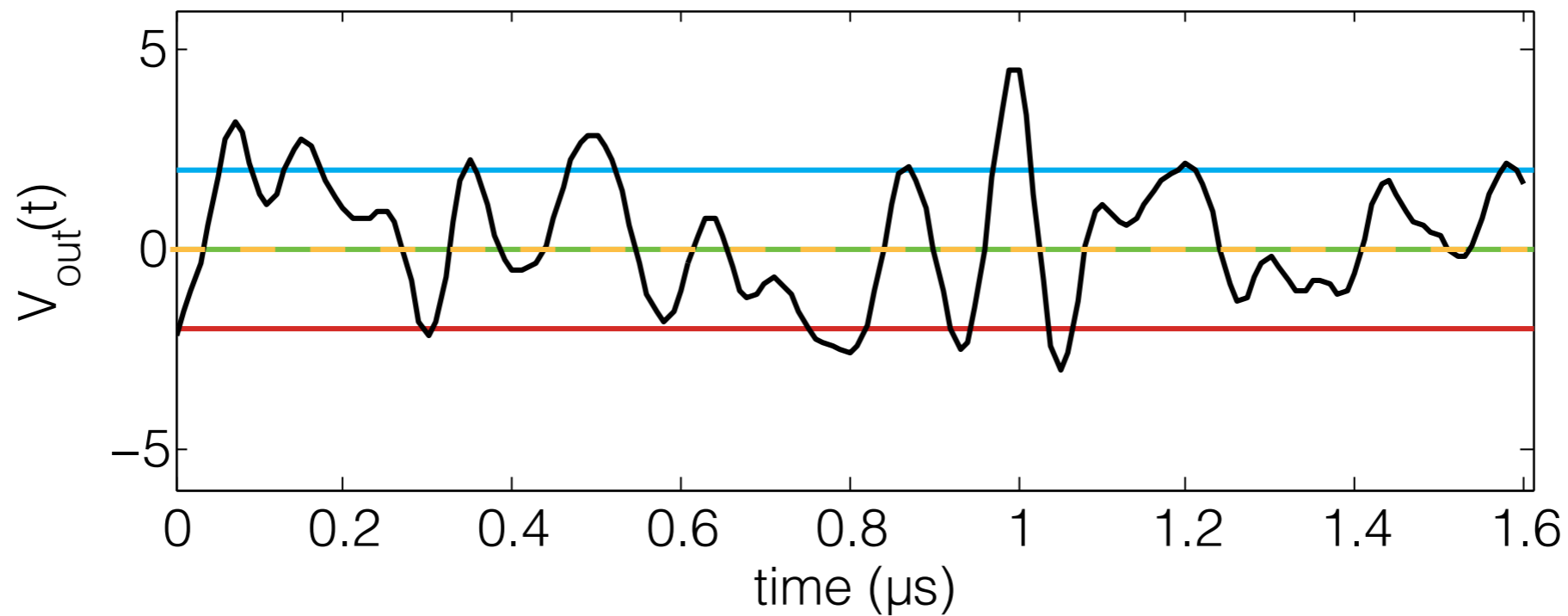
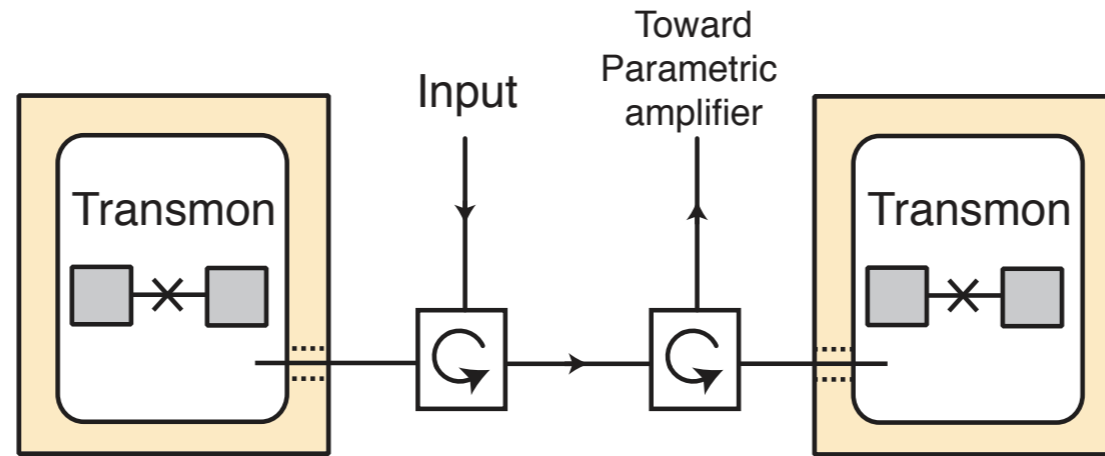
Bayes rule:

$$p(|ij\rangle|r) = \frac{p(|ij\rangle)p(r||ij\rangle)}{p(r)}$$



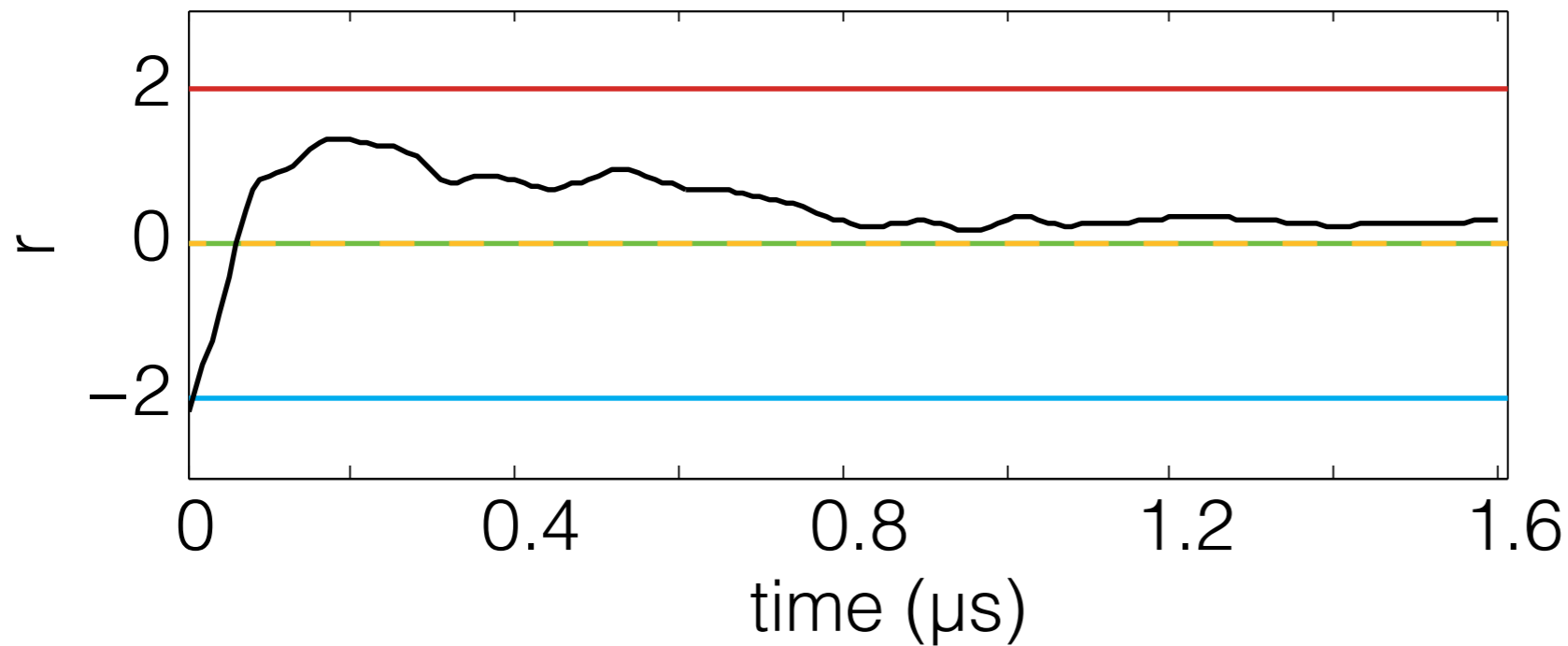
Mapping of  $r$  onto  $\rho$

# Quantum trajectories for 2 qubits

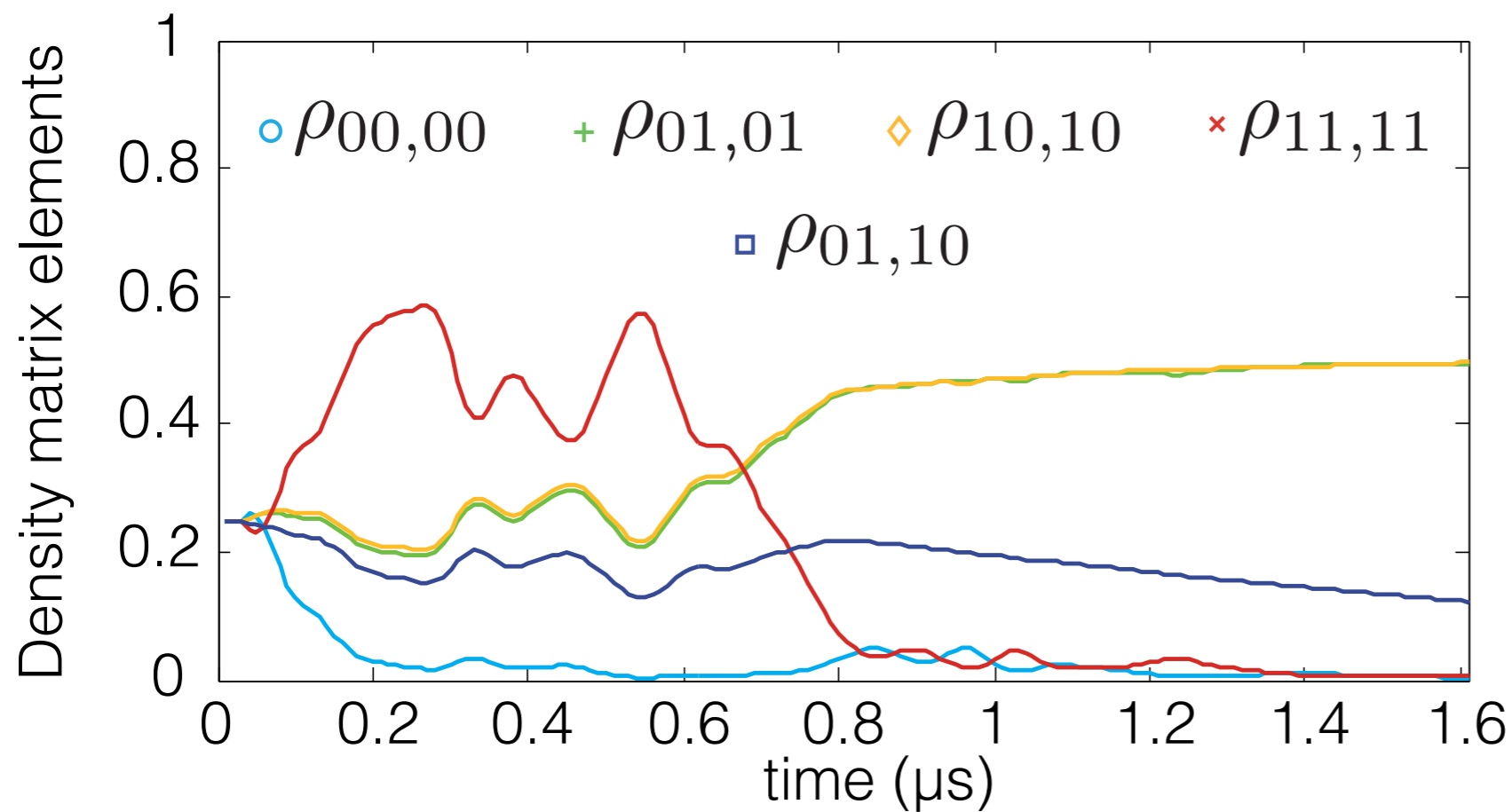


Single time trace

# Quantum trajectories for 2 qubits

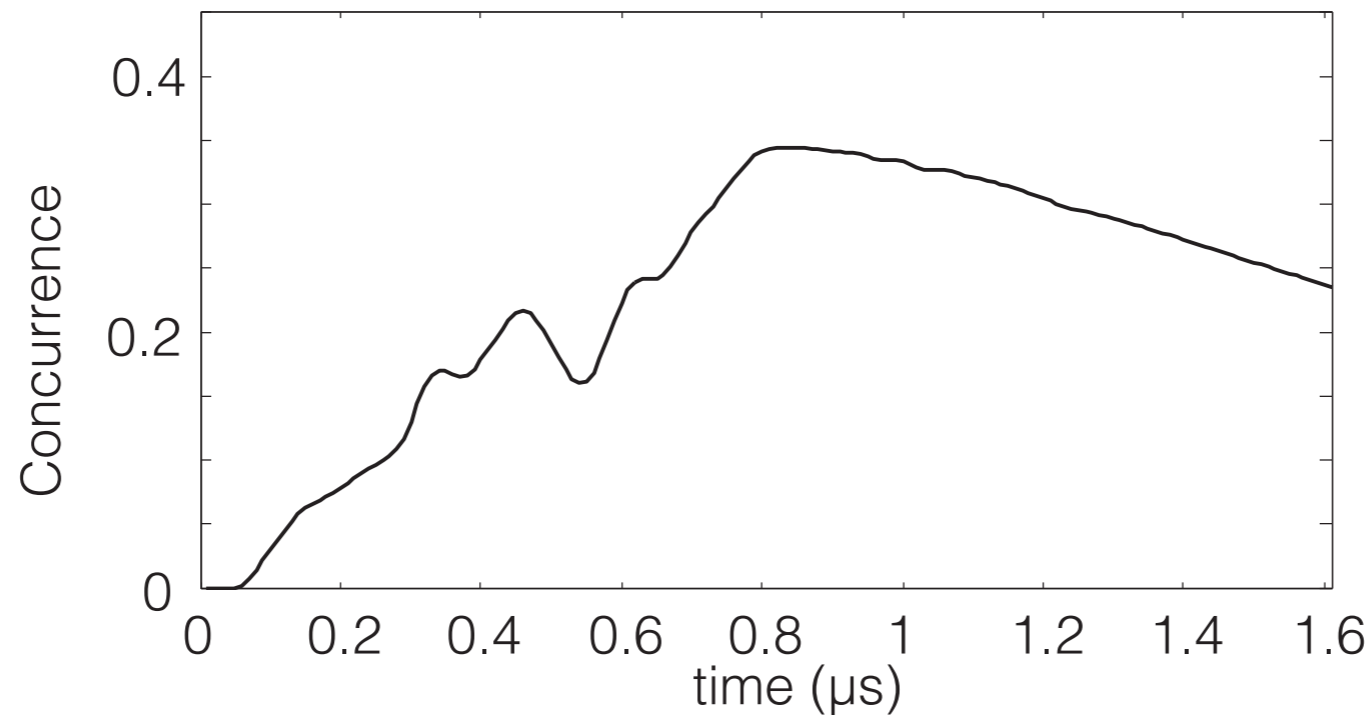
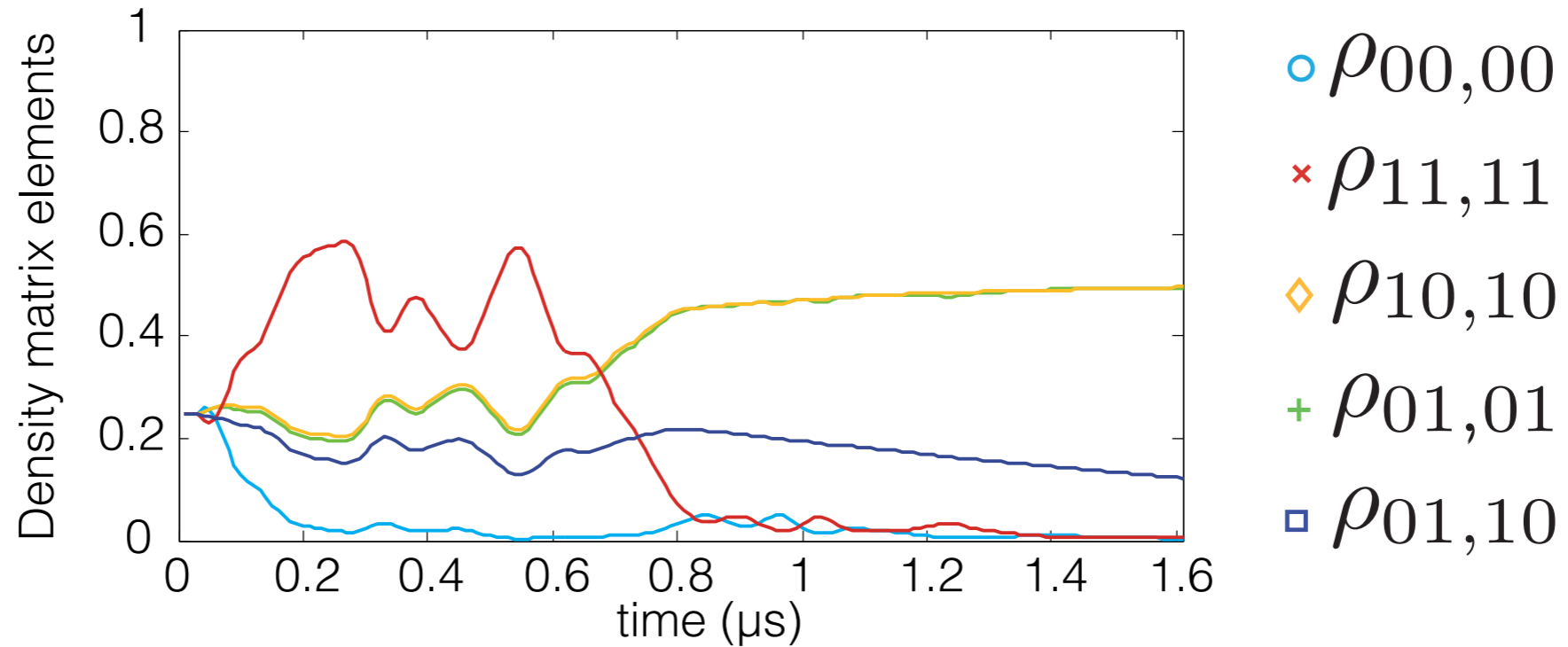


for each point:



$$p(|ij\rangle|r) = \frac{p(|ij\rangle)p(r||ij\rangle)}{p(r)}$$

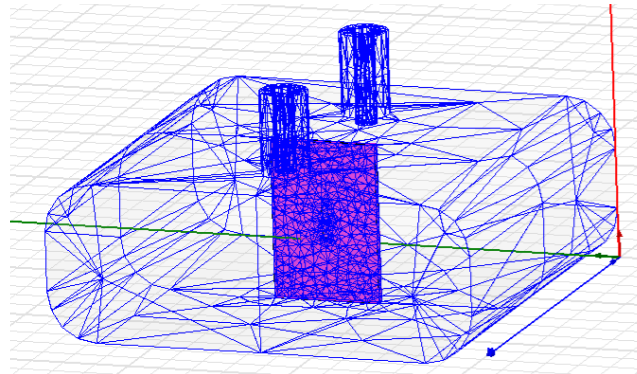
# Quantum trajectories for 2 qubits



$$\mathcal{C} = \max(0, |\rho_{01,10}| - \sqrt{\rho_{00,00}\rho_{11,11}})$$



# Quantum trajectories and feedback in circuit-QED



## Introduction to circuit-QED

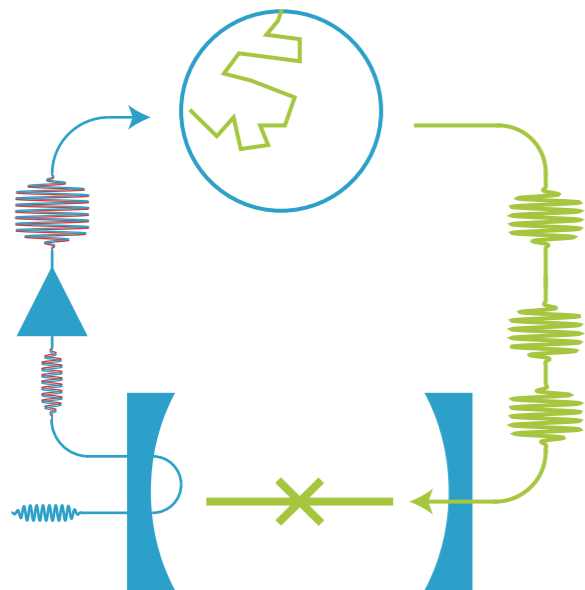
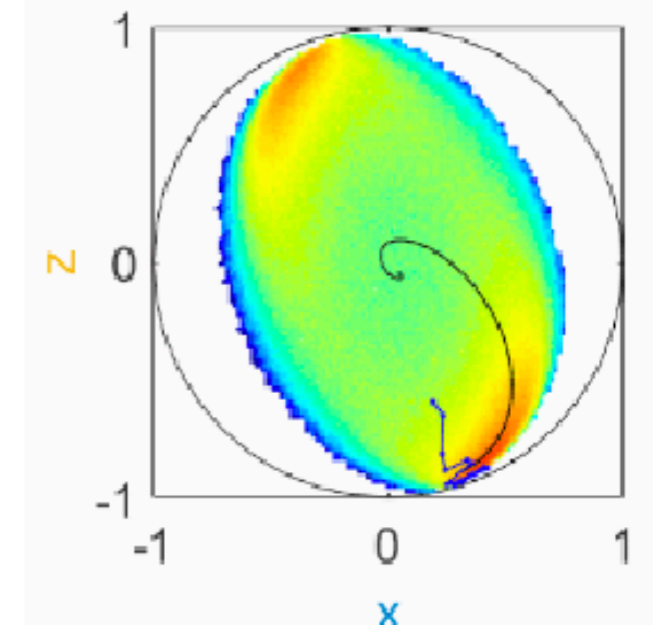
## Quantum trajectories in circuit-QED

dispersive measurement

fluorescence measurement

both simultaneously

generating entanglement



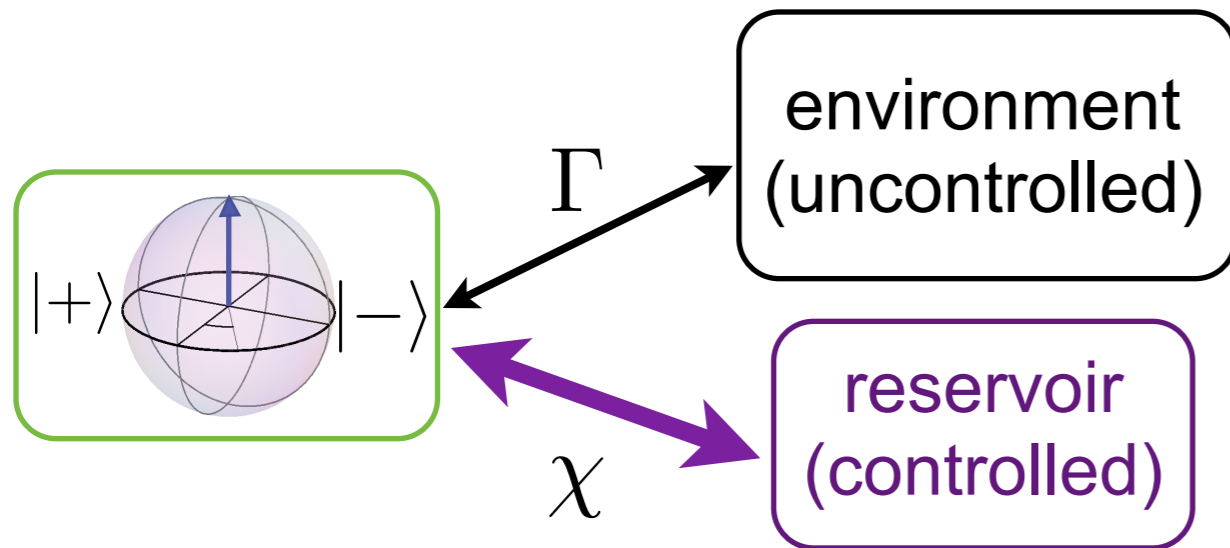
## Measurement based feedback

dispersive case

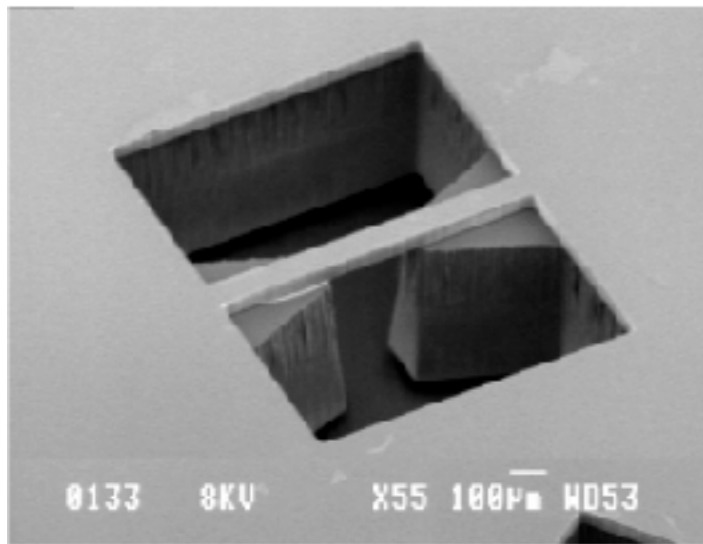
fluorescence case

# How to preserve an arbitrary state?

Reservoir engineering  $\chi \gg \Gamma$



Side band cooling of a mirror

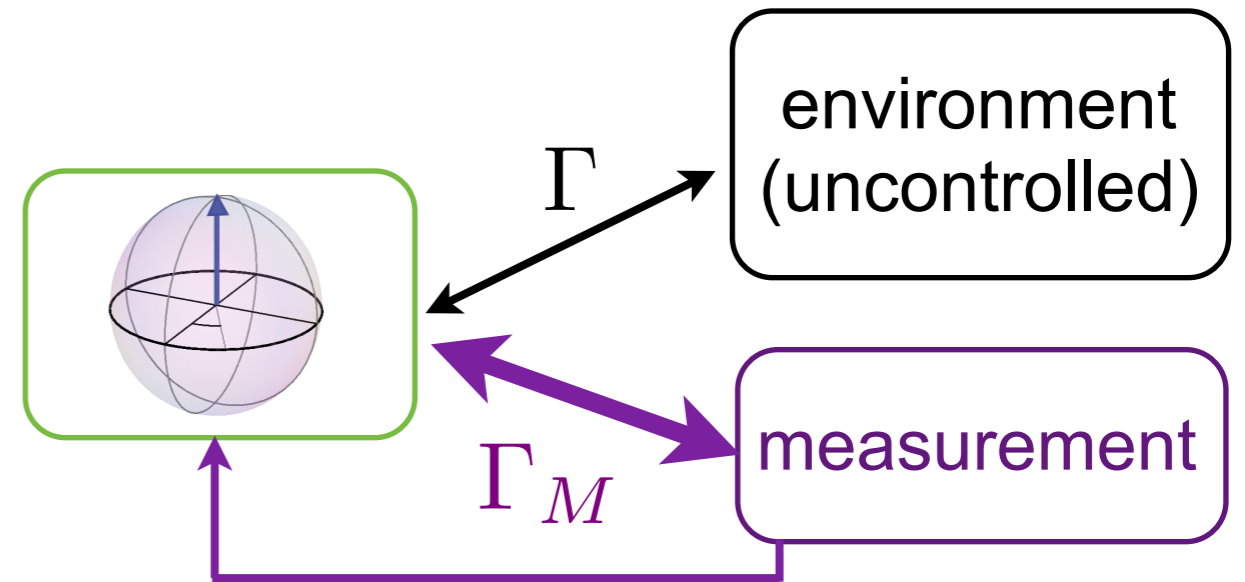


(Pinard et al., LKB, 1999)

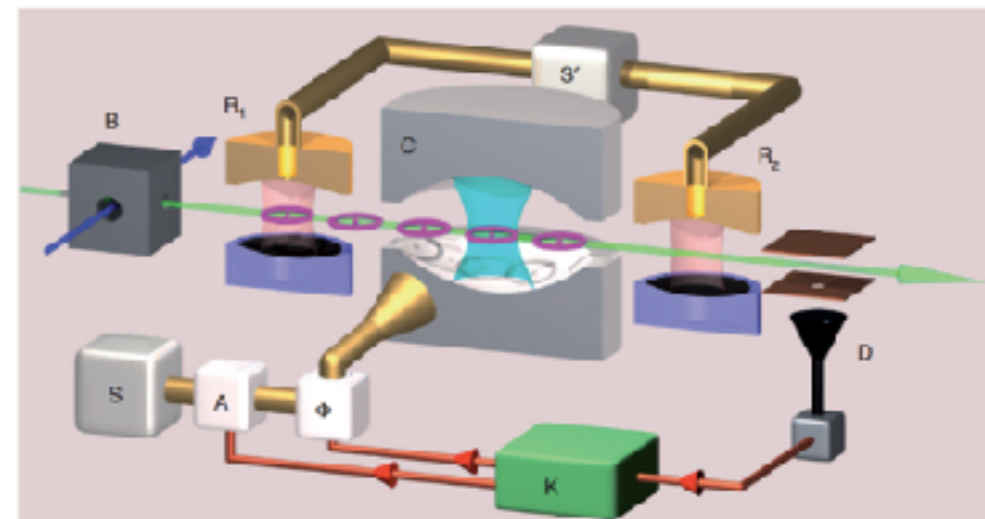
$$\frac{1}{\sqrt{2}}(|g\rangle + |e\rangle) \quad (\text{Siddiqi group, Berkeley, 2012})$$

Measurement based feedback

$$\Gamma_M \gg \Gamma$$



Stabilization of Fock states

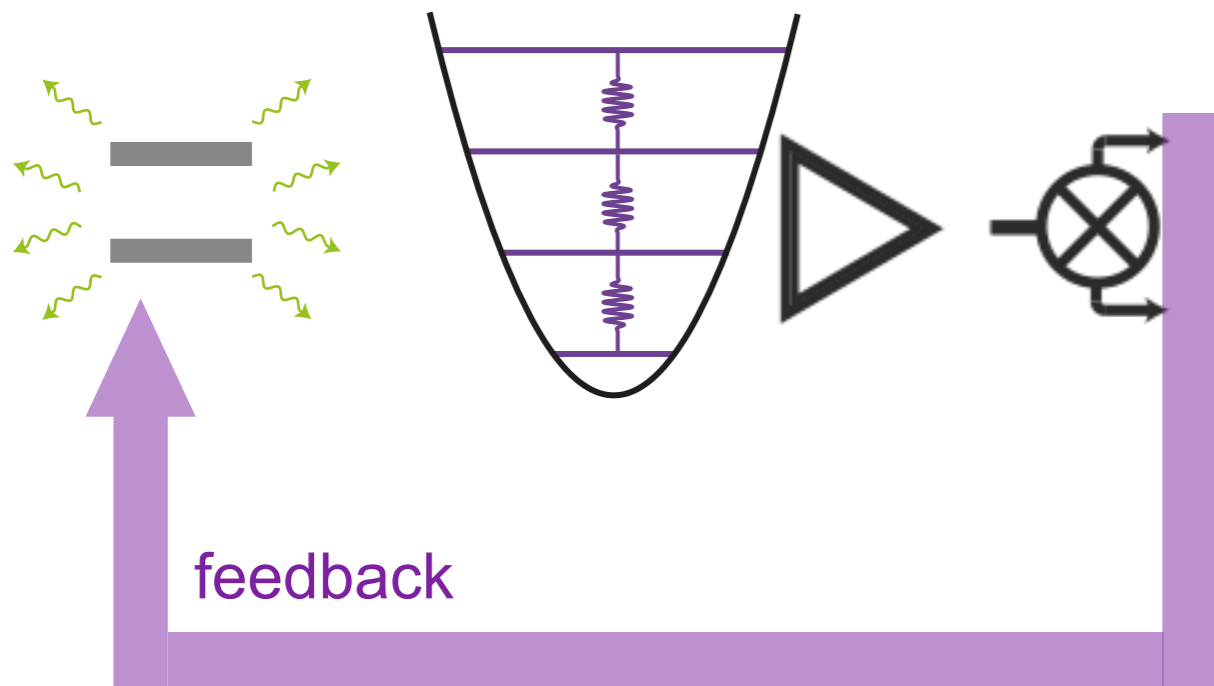


(Haroche, LKB, 2011)

And in circuit QED?

# Measurement based feedback

based on dispersive measurement



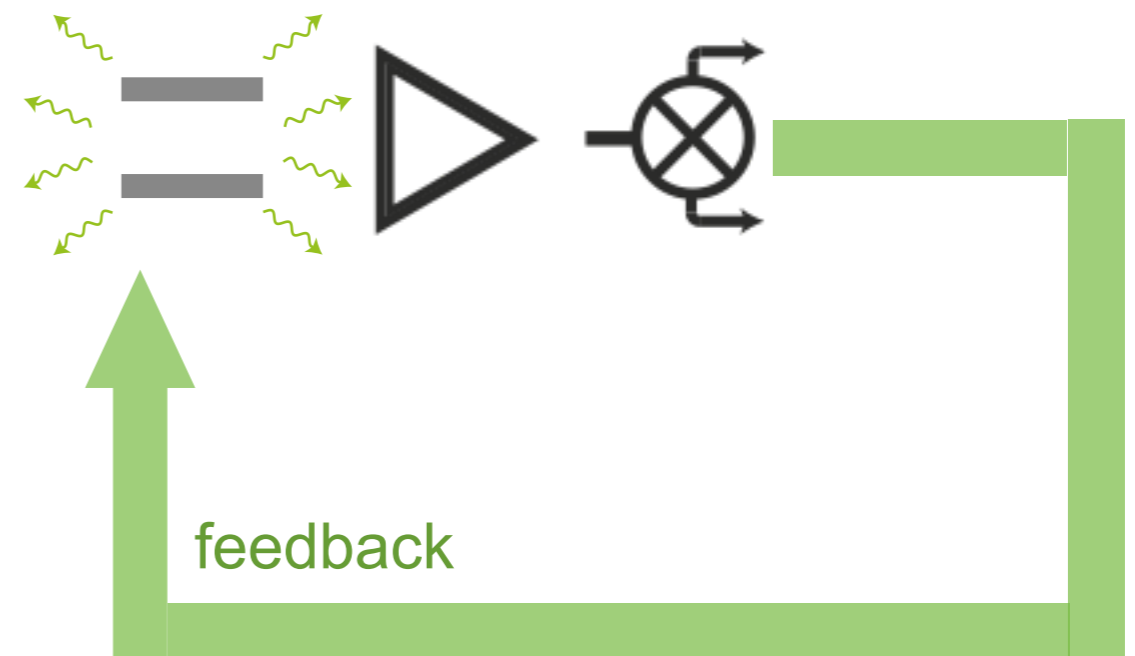
[Vijay et al., Nature 2012 (Berkeley)]

[Ristè et al., PRL 2012 (Delft)]

[Campagne-Ibarcq et al., PRX 2013 (ENS Paris)]

...now standard technique

based on fluorescence



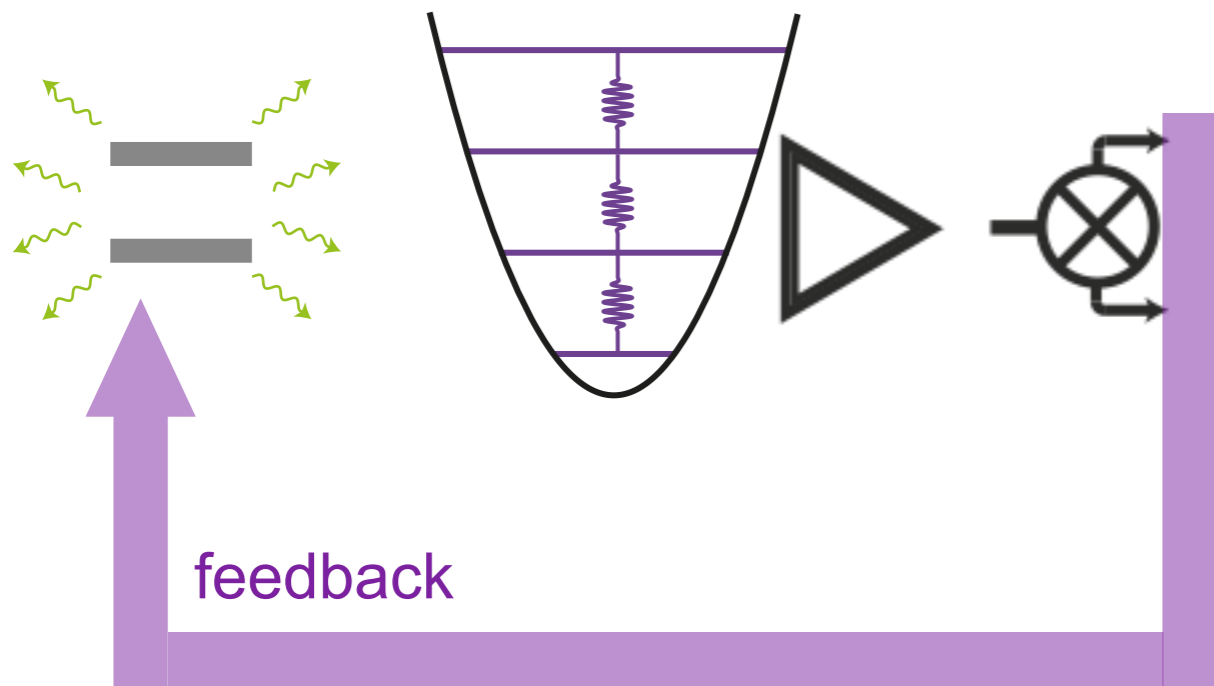
compatible with dispersive feedback

converges in  $T_1$  for any state

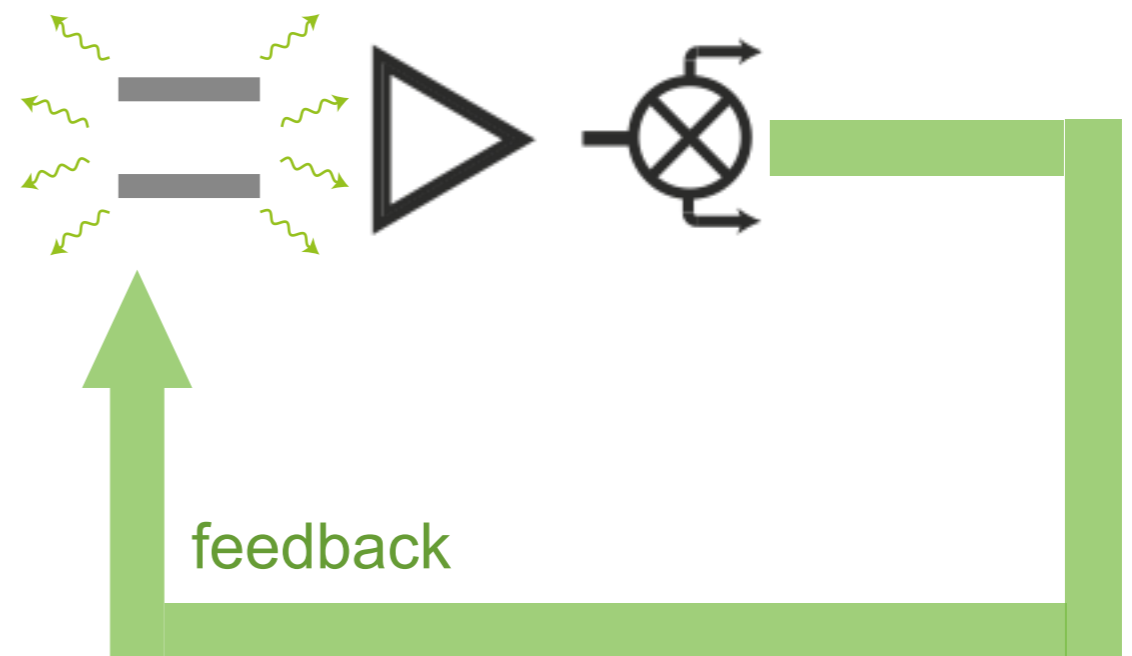
here continuous feedback qualitatively more efficient than stroboscopic

# Measurement based feedback

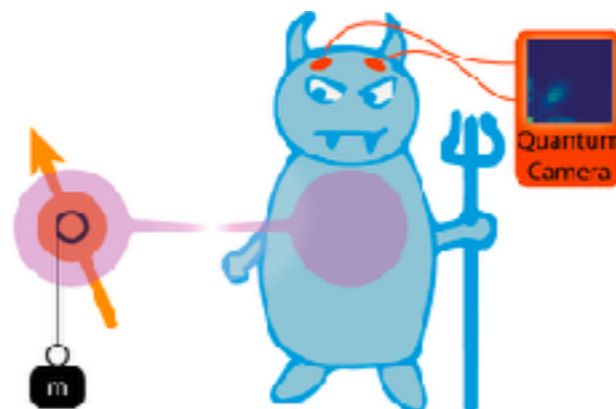
based on dispersive measurement



based on fluorescence



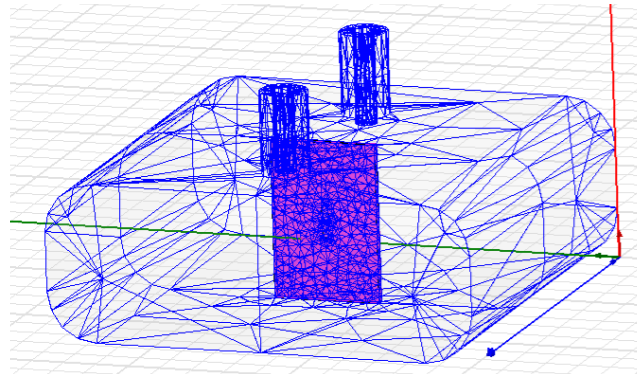
Quantum Maxwell demon



[Cottet et al., PNAS 2017]

see review on Maxwell demons in  
[Cottet and BH, arXiv:1805.01224]

# Quantum trajectories and feedback in circuit-QED



## Introduction to circuit-QED

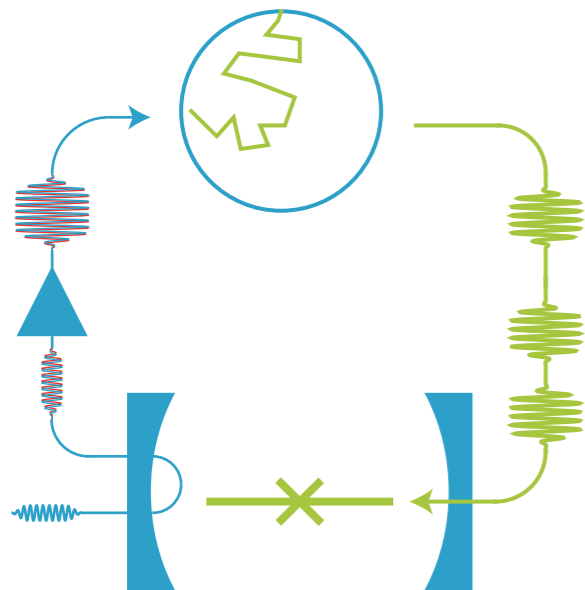
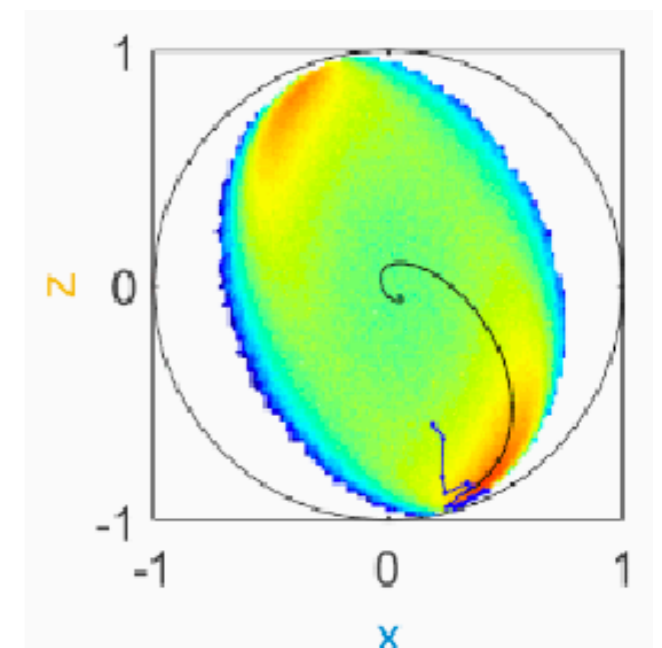
## Quantum trajectories in circuit-QED

dispersive measurement

fluorescence measurement

both simultaneously

generating entanglement



## Measurement based feedback

dispersive case

fluorescence case

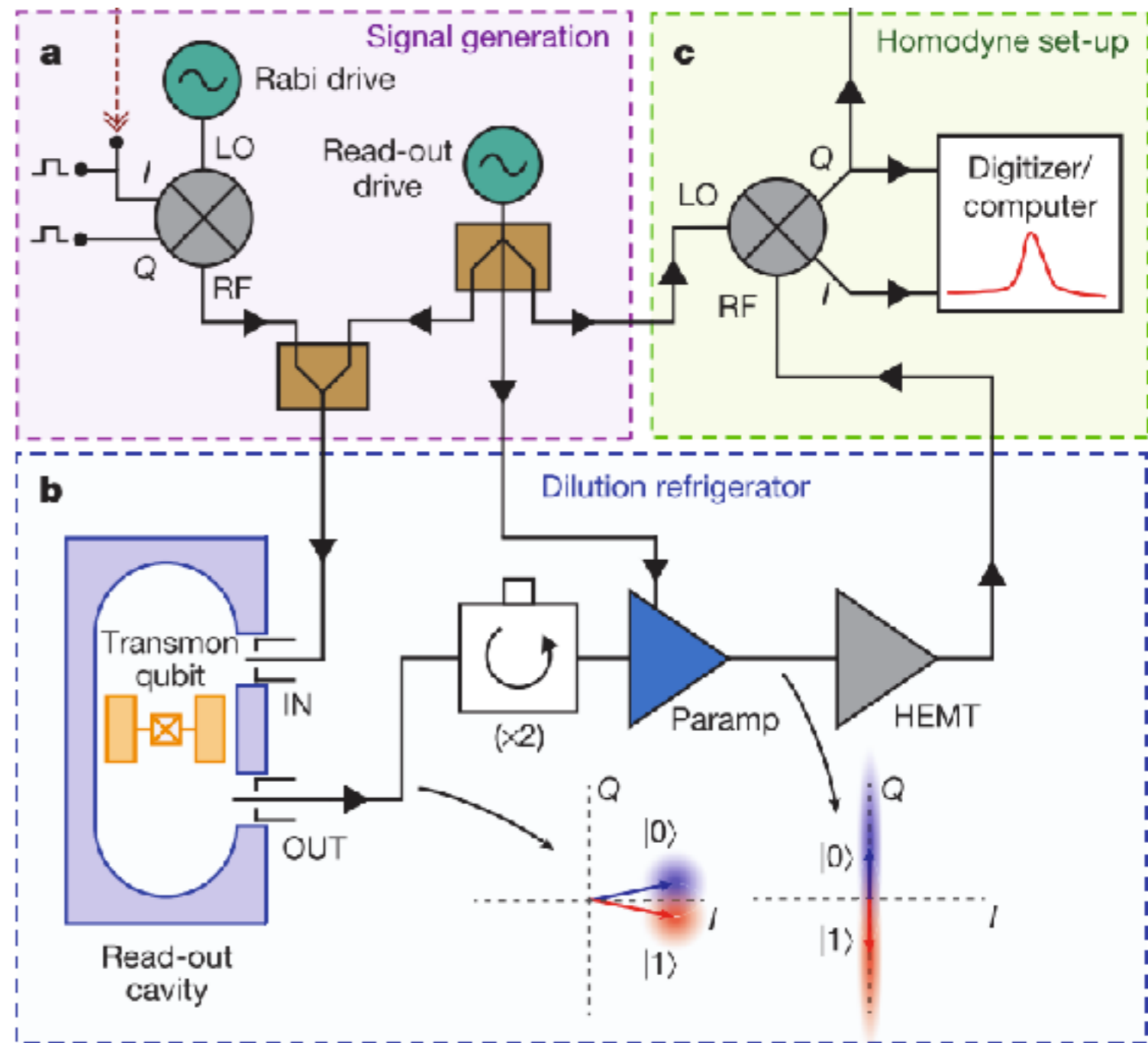
# Analog feedback

Stabilize Rabi oscillations against decoherence

$$H = \hbar \frac{\Omega(t)}{2} \sigma_X$$

$$Q_{out}(t) = \cos(\Omega_0 t + \theta)$$

goal is  $\theta = 0$





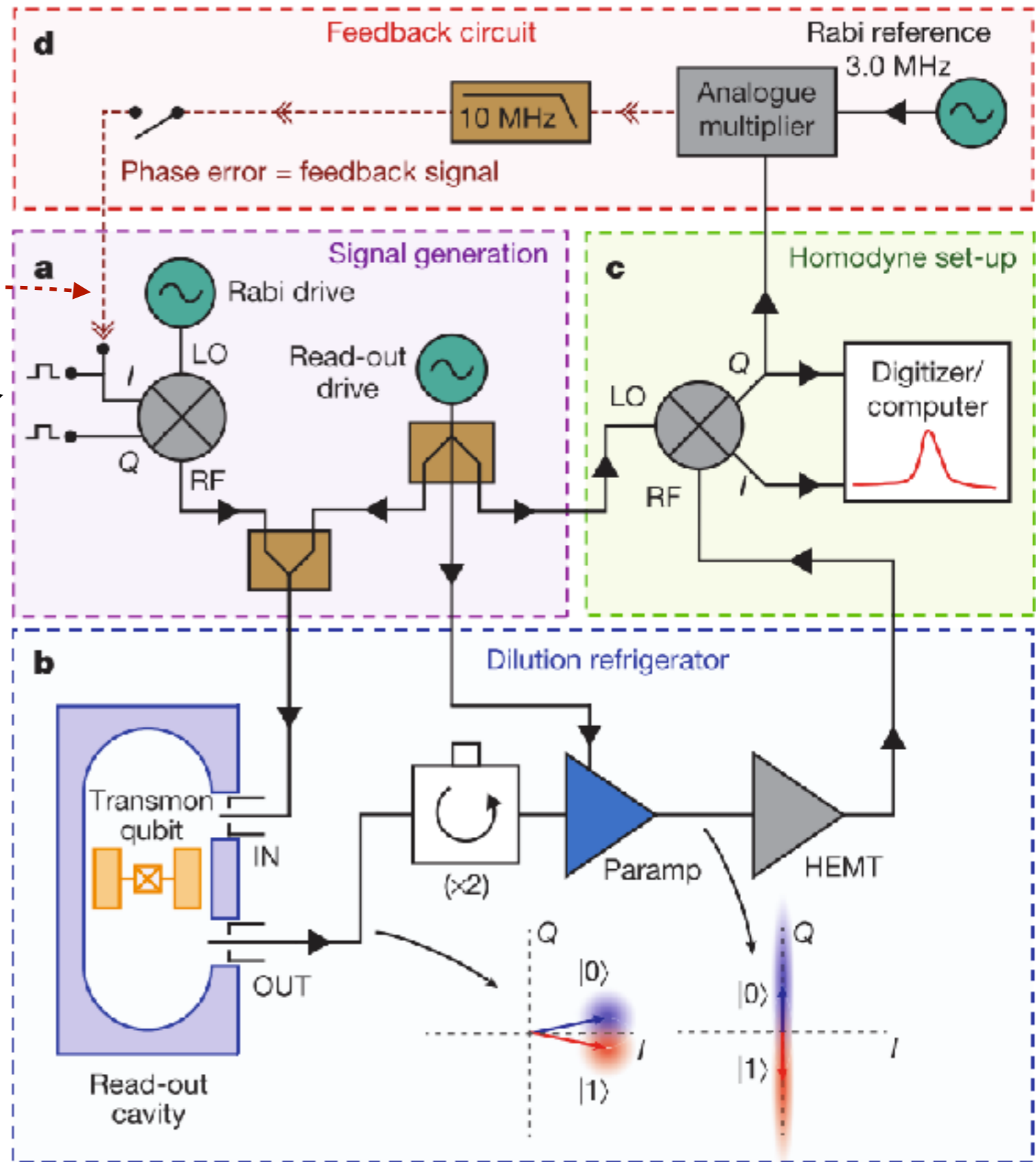
# Analog feedback

Stabilize Rabi oscillations against decoherence

$$H = \hbar \frac{\Omega(t)}{2} \sigma_X$$

$$\Omega(t) = \Omega_0 + 2\lambda \sin(\Omega_0 t) Q_{out}(t)$$

$$Q_{out}(t) = \cos(\Omega_0 t + \theta)$$



# Analog feedback

Stabilize Rabi oscillations against decoherence

$$H = \hbar \frac{\Omega(t)}{2} \sigma_X$$

$$\Omega(t) = \Omega_0 + 2\lambda \sin(\Omega_0 t) Q_{out}(t)$$

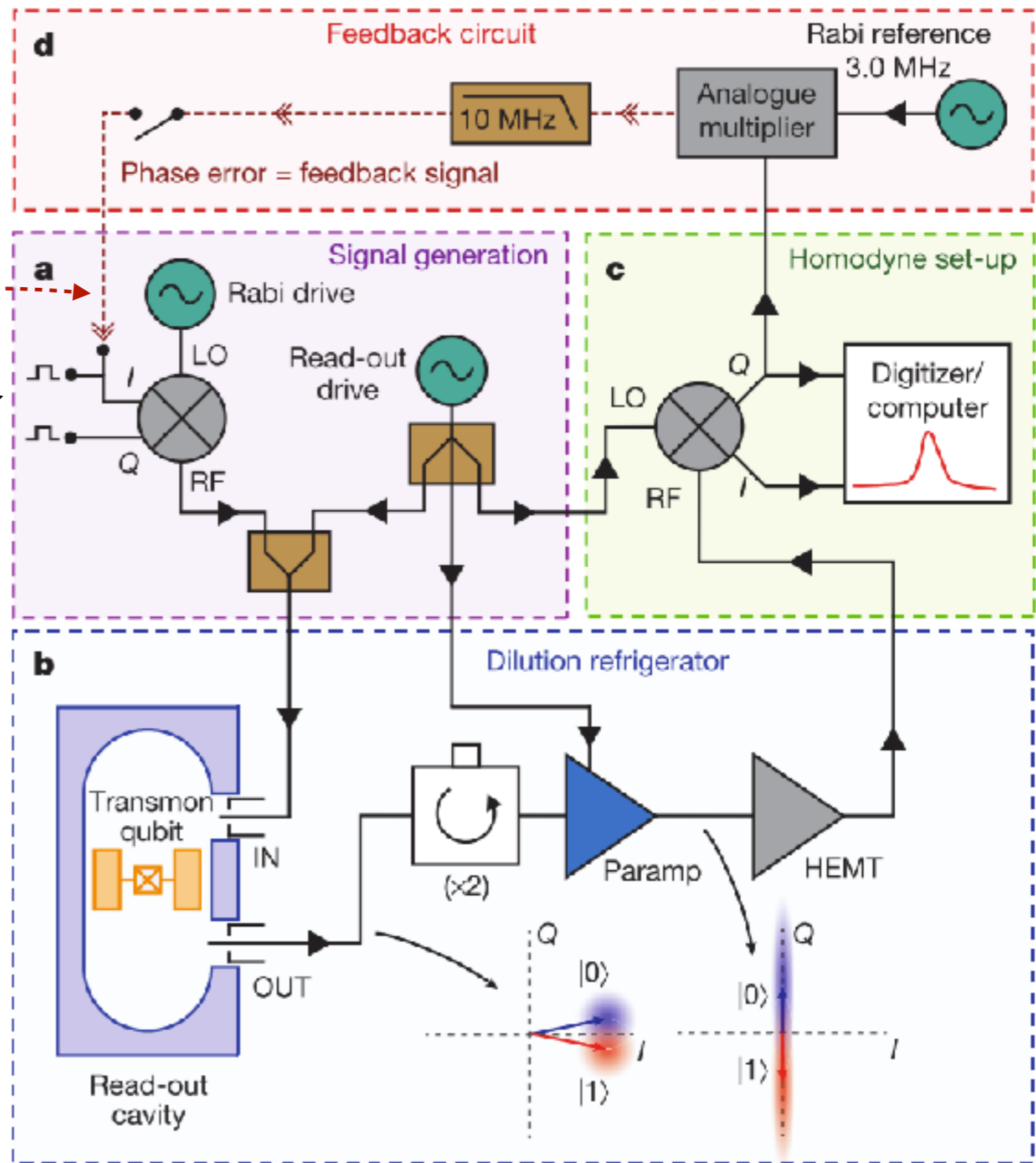
$$Q_{out}(t) = \cos(\Omega_0 t + \theta)$$

~~$$\Omega(t) = \Omega_0 + \lambda \sin(\theta) + \lambda \sin(2\Omega_0 t + \theta)$$~~

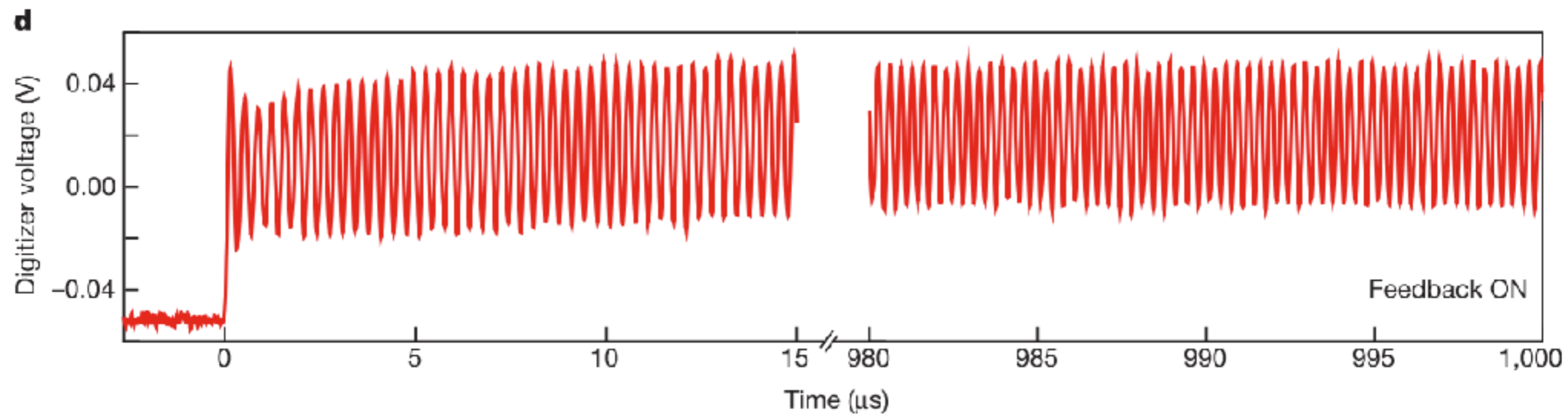
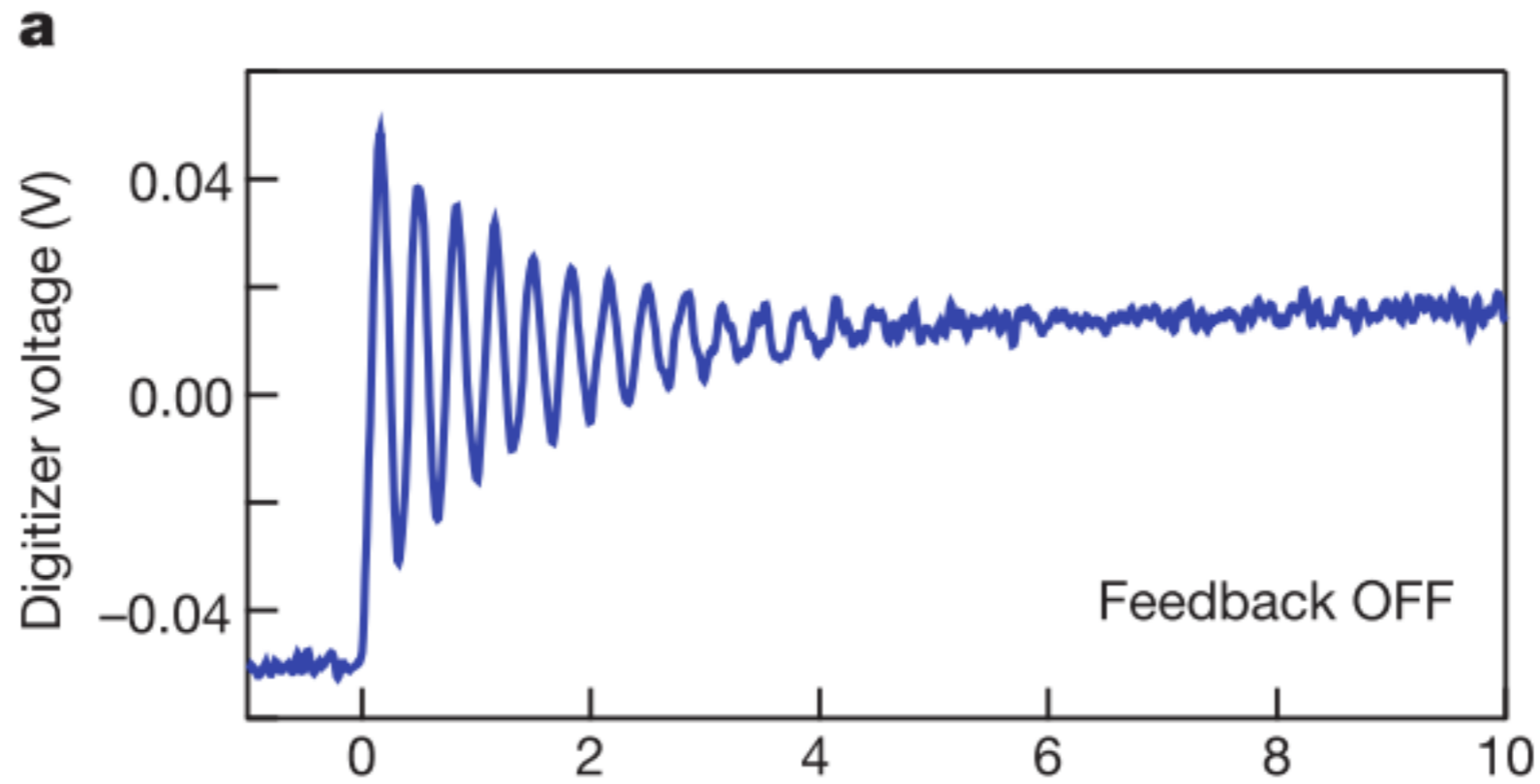
$$\lambda < 0$$

$\theta < 0 \Rightarrow$  late so rotation accelerates

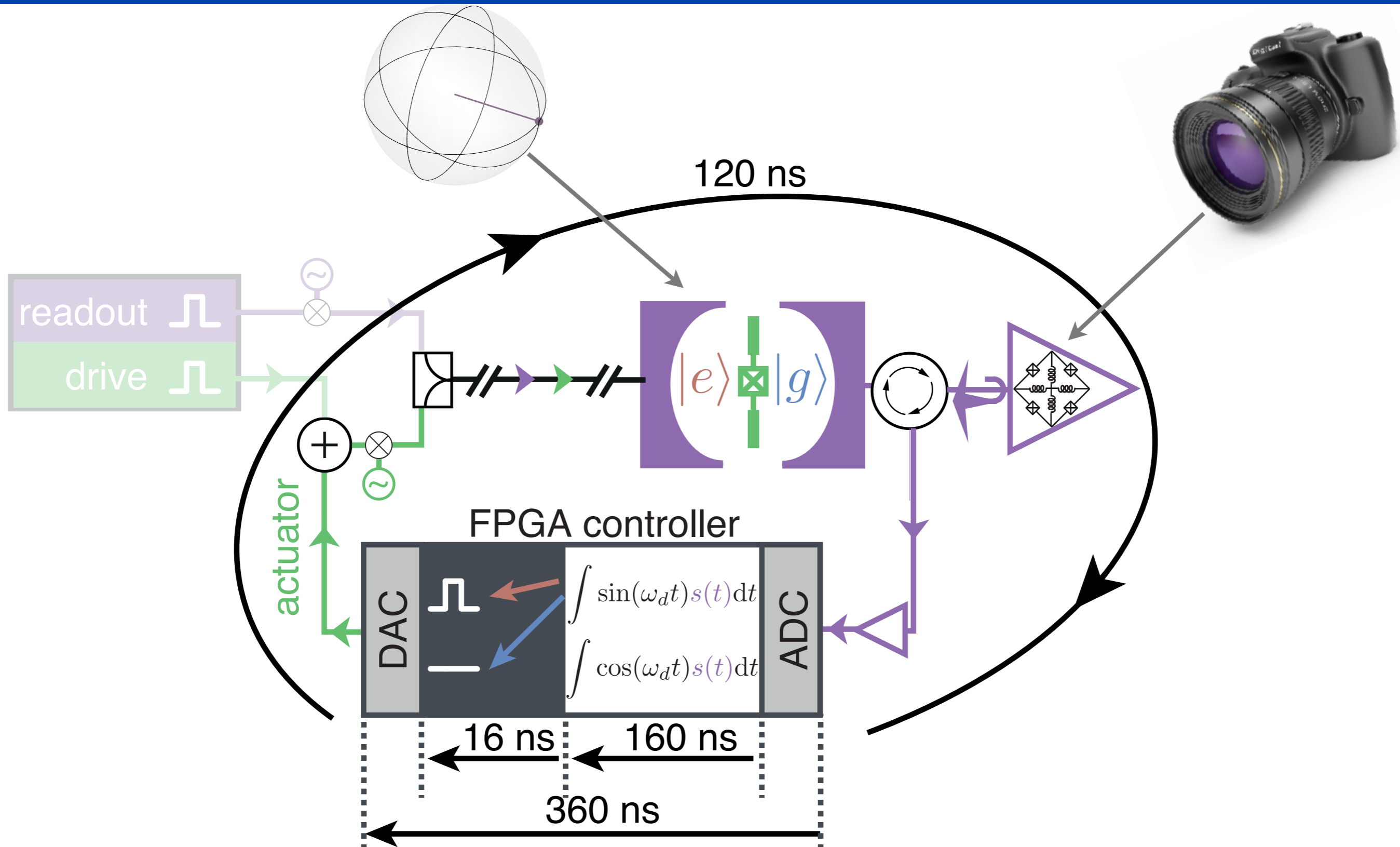
$\theta > 0 \Rightarrow$  in advance so rotation slows down



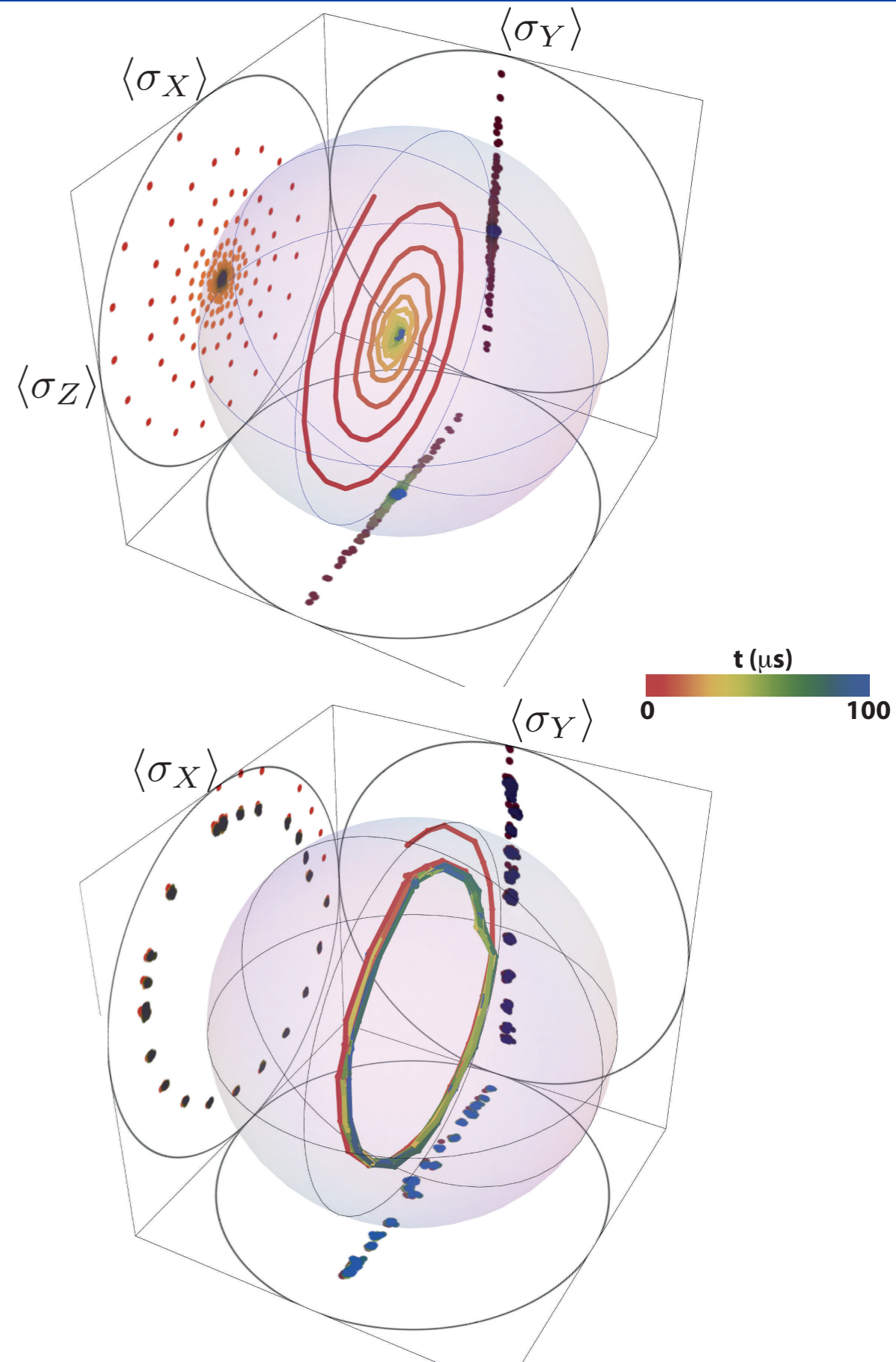
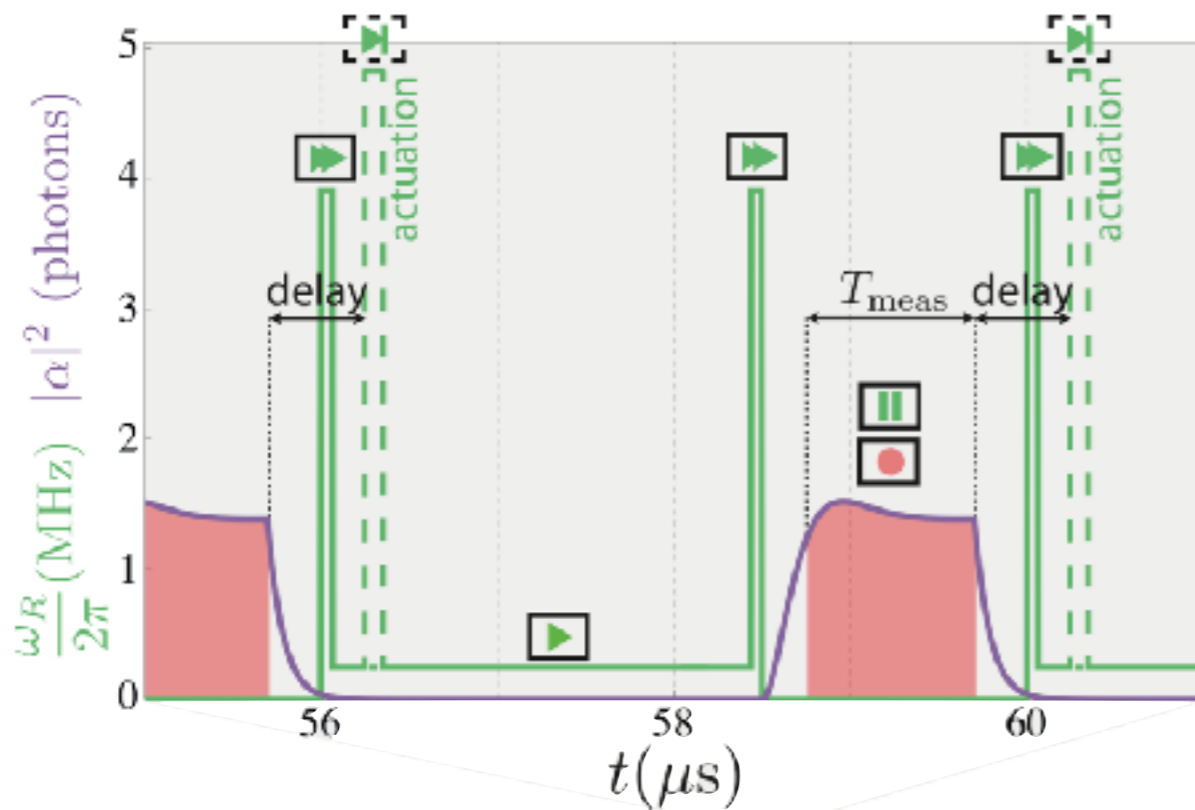
# Analog feedback



# Digital feedback and variable measurement strength



# Digital feedback



## Stabilization of Rabi and Ramsey oscillations

[Campagne-Ibarcq et al., PRX 2013 (ENS Paris)]

77% average Bloch vector length  
vs 45% in constant measurement strength  
[Vijay et al., Nature 2012 (Berkeley)]

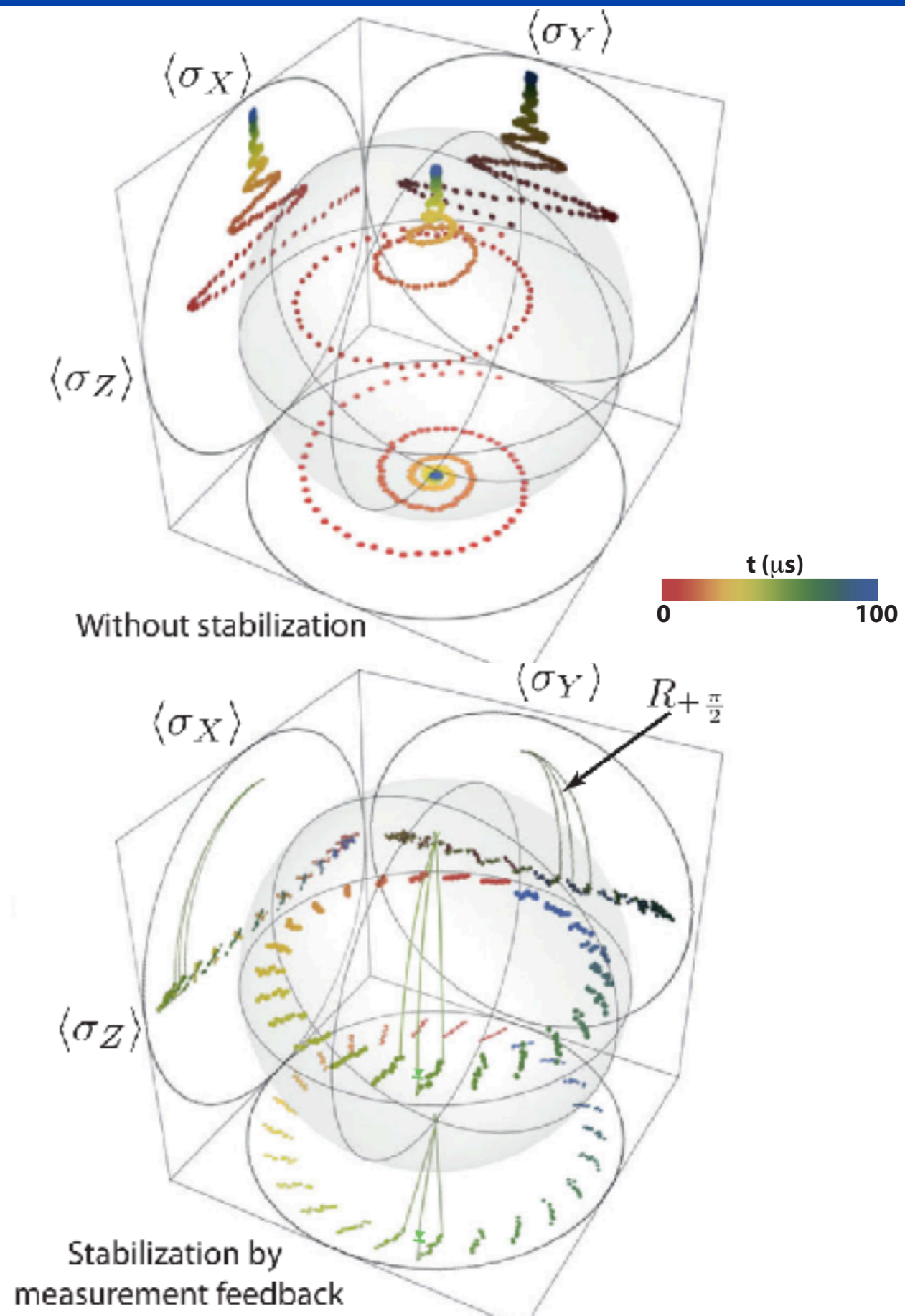
## Reset by feedback

[Ristè et al., PRL 2012 (Delft)]



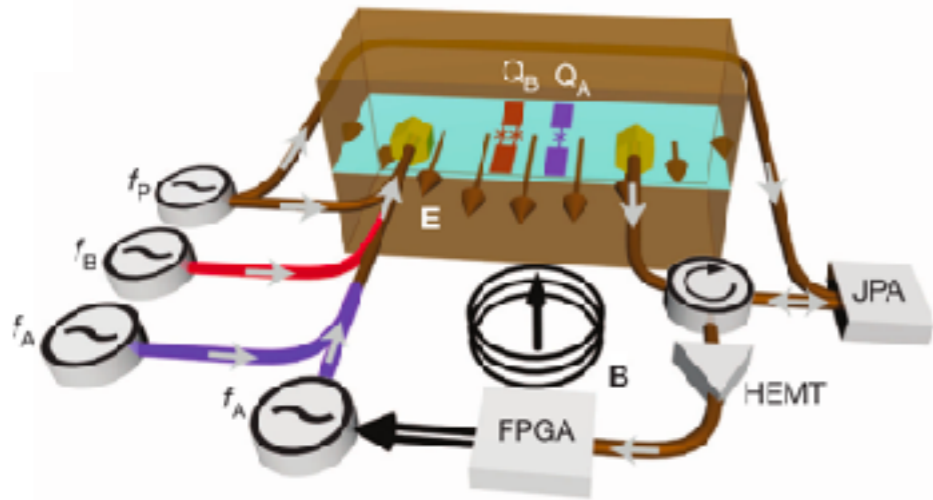
# Digital feedback for Ramsey oscillations

Stabilization of Rabi and Ramsey oscillations  
[Campagne-Ibarcq et al., PRX 2013 (ENS Paris)]





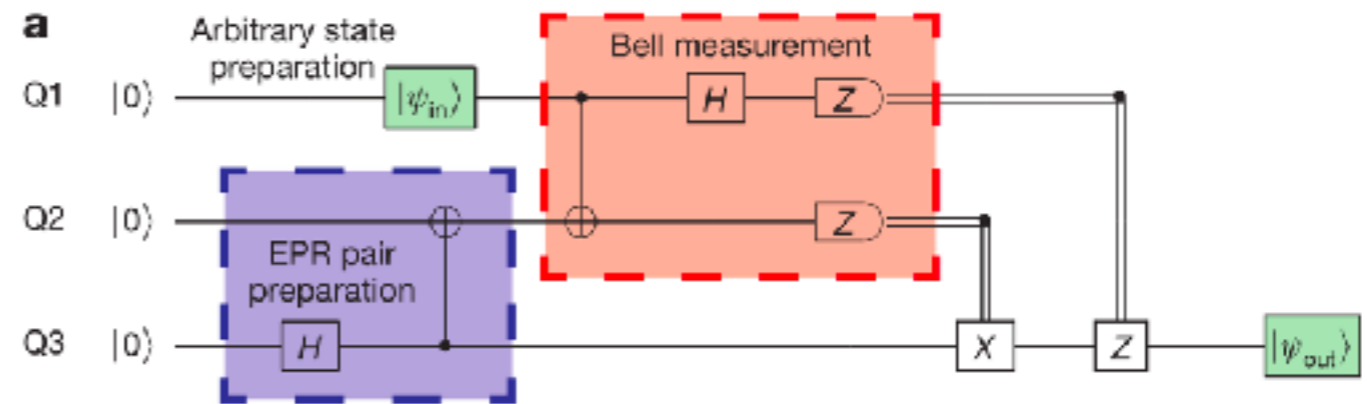
# Other applications of dispersive measurement feedback



## Entanglement stabilization

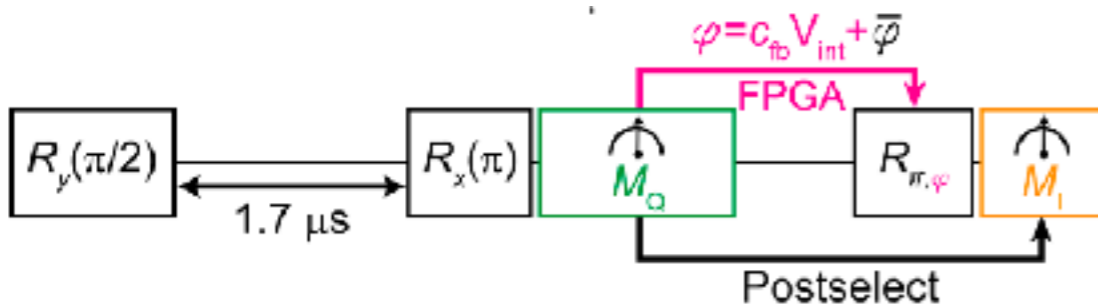
[Risté et al., Nature 2013 (Delft)]

[Liu et al., PRX 2016 (Yale)]



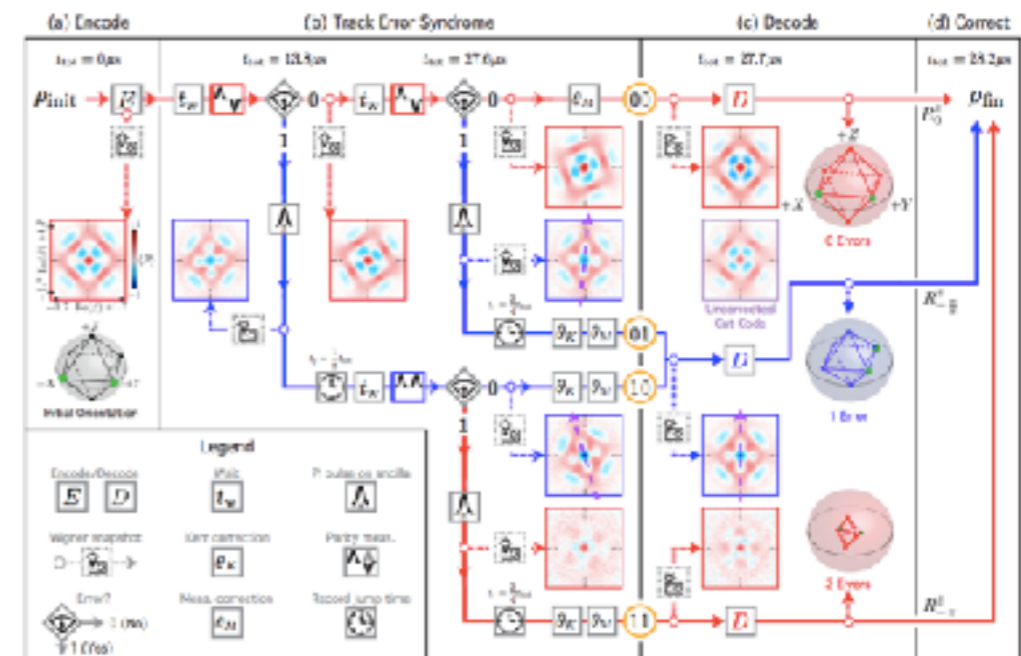
## Teleportation using feed forward

[Steffen et al., Nature 2013 (Zurich)]



## Canceling the dephasing induced by the measurement

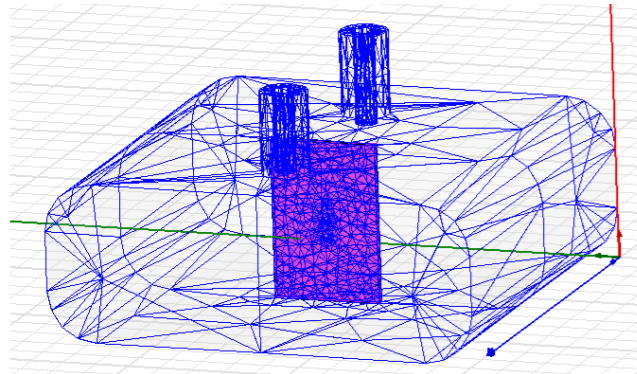
[de Lange et al., PRL 2014 (Delft)]



## Parity measurement for quantum error correction

[Offek et al., Nature 2016 (Yale)]

# Quantum trajectories and feedback in circuit-QED



## Introduction to circuit-QED

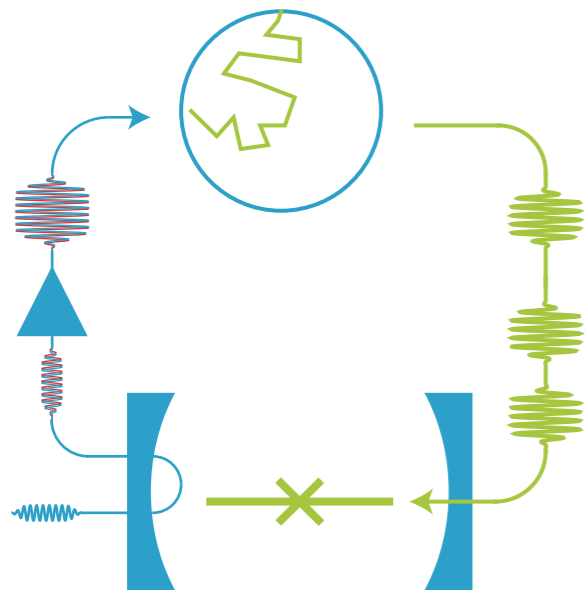
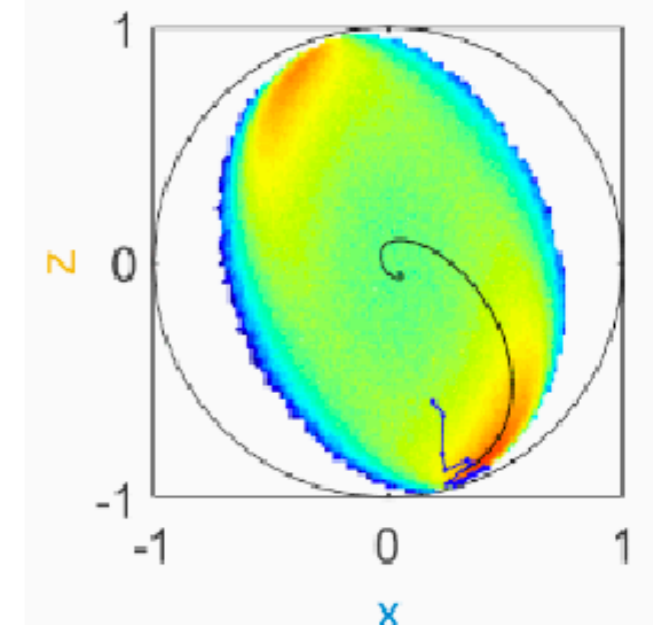
## Quantum trajectories in circuit-QED

dispersive measurement

fluorescence measurement

both simultaneously

generating entanglement



## Measurement based feedback

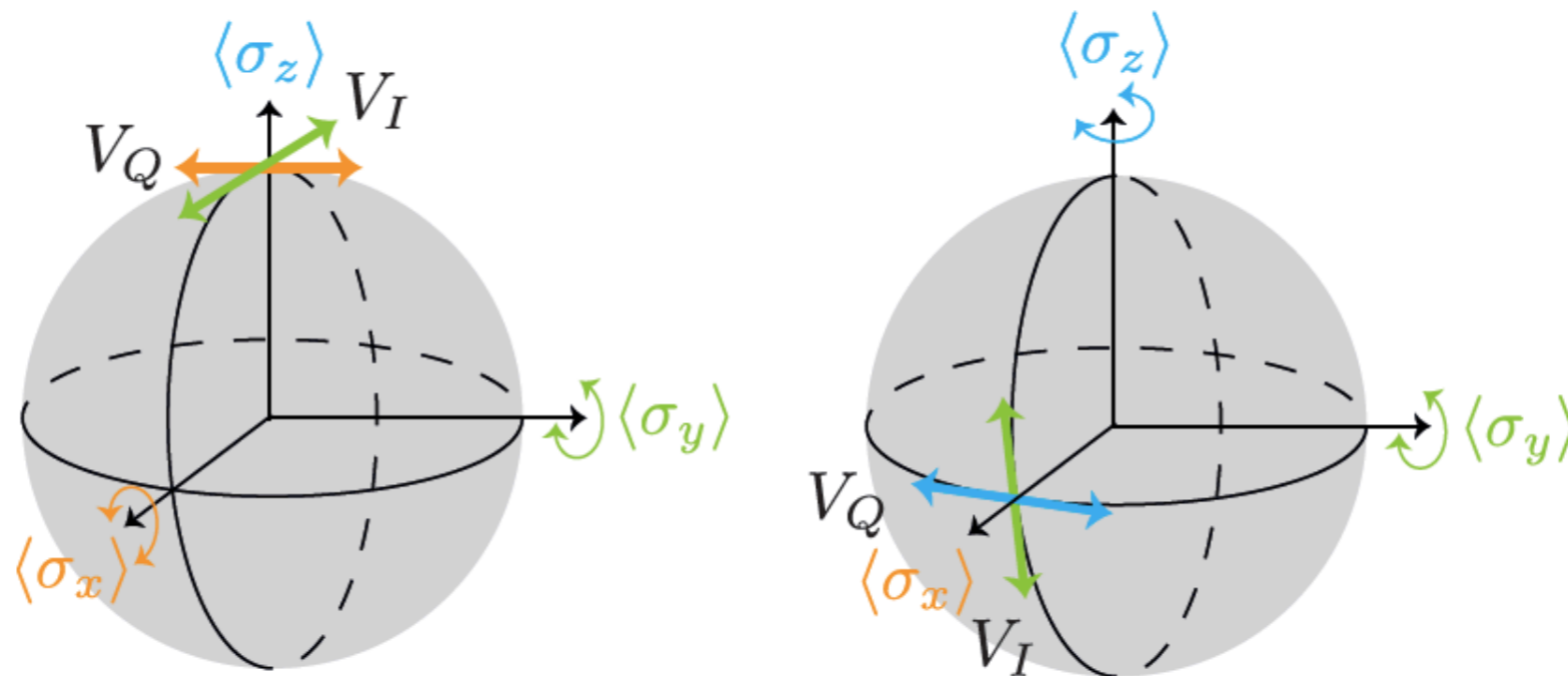
dispersive case

fluorescence case

# Fluorescence based feedback

$$\text{stabilize target } \cos \frac{\theta}{2} |e\rangle + \sin \frac{\theta}{2} e^{i\varphi} |g\rangle$$

compensate stochastic kicks due to fluorescence?



use 3 rotation axes and Markovian feedback

[Campagne-Ibarcq *et al.*, PRL (2016)]

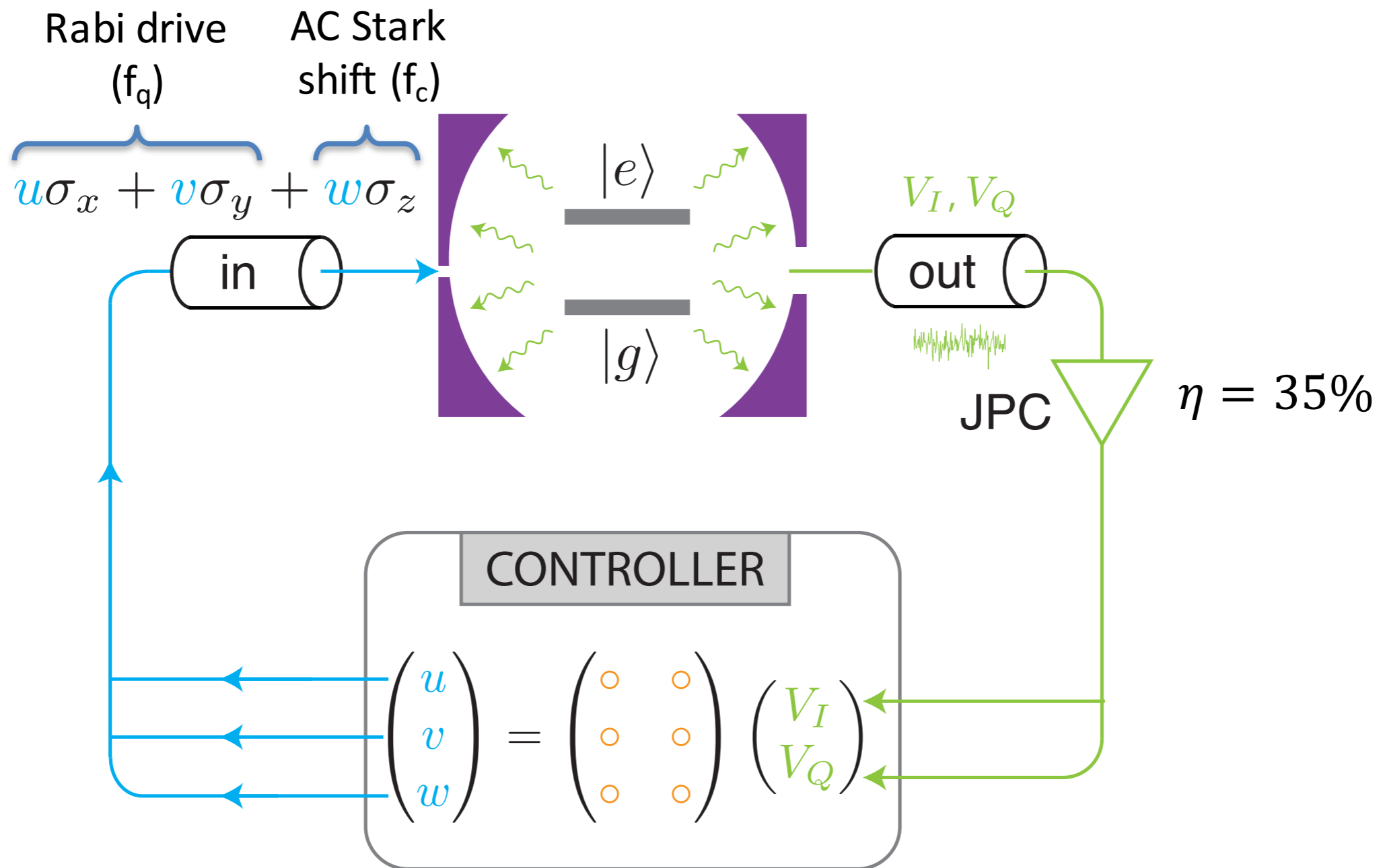
## previous proposals

south hemisphere only [Hofmann, Mahler, Hess, PRA (1998)]

every state but equator [Wang, Wiseman, PRA (2001)]

# Fluorescence based feedback

stabilize target  $\cos \frac{\theta}{2} |e\rangle + \sin \frac{\theta}{2} e^{i\varphi} |g\rangle$

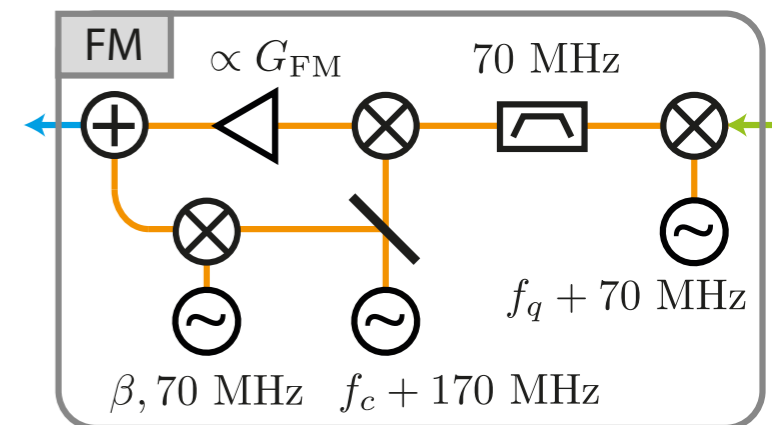
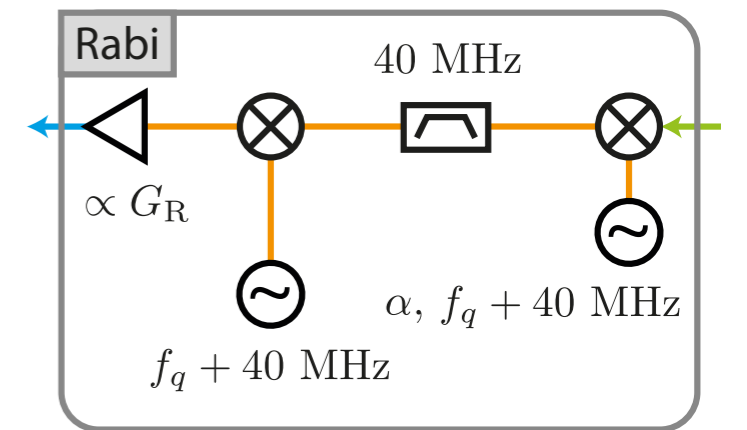
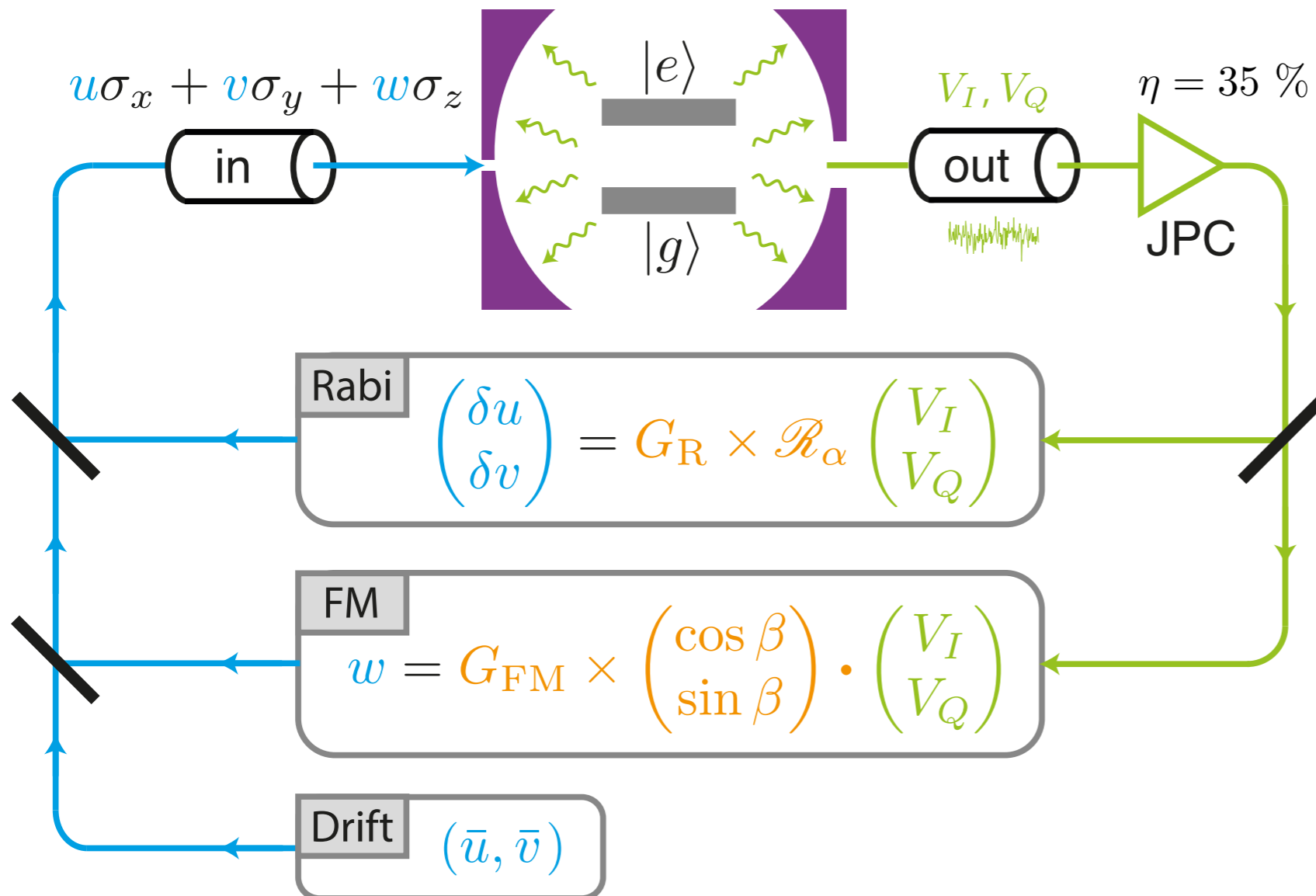


multi inputs and multi output Markovian feedback

# Fluorescence based feedback

stabilize target  $\cos \frac{\theta}{2} |e\rangle + \sin \frac{\theta}{2} e^{i\varphi} |g\rangle$

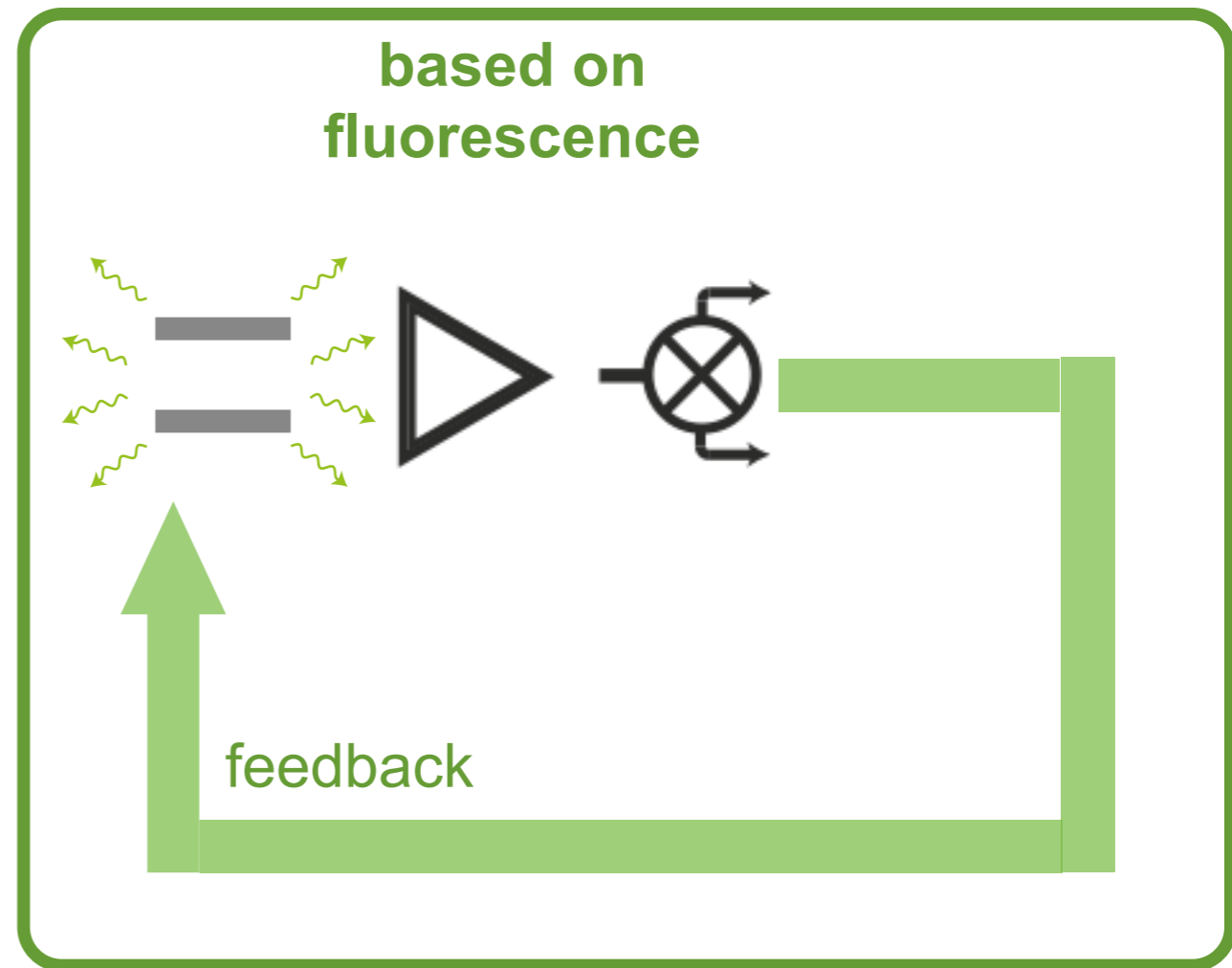
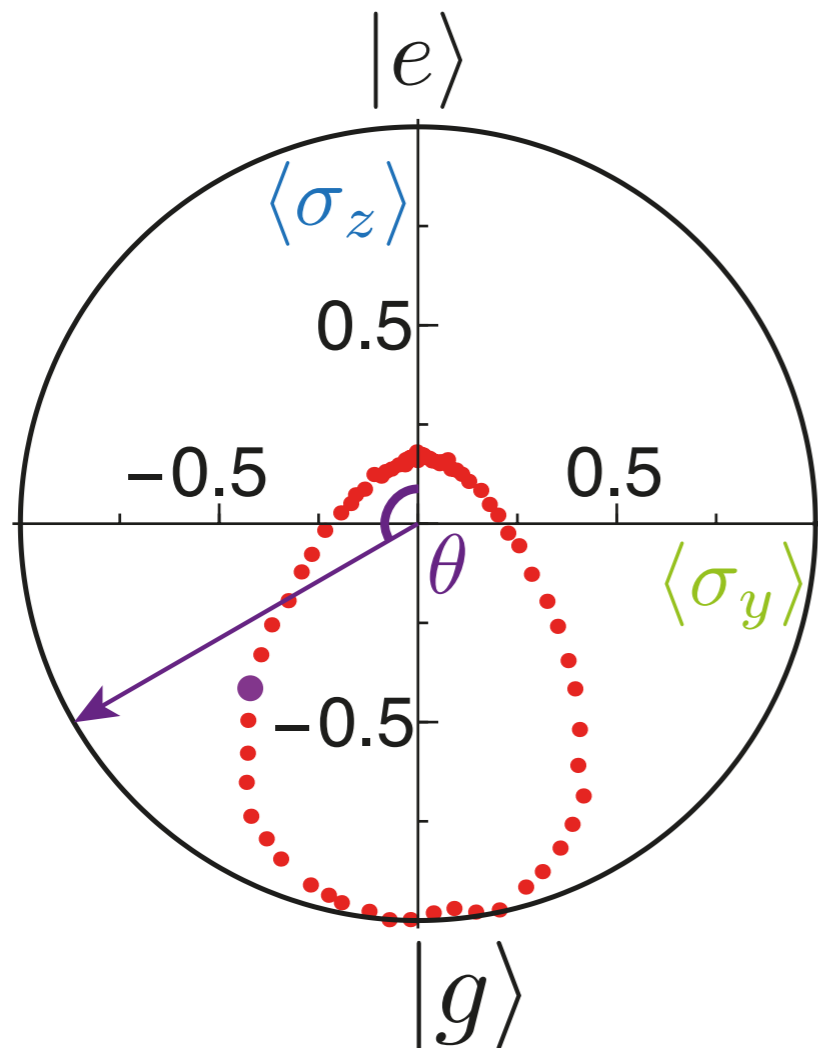
$$\begin{cases} G_R = \sqrt{\frac{\gamma_1}{8\eta}} (1 + \cos \theta), & \alpha = \pi/2 \\ G_{FM} = \sqrt{\frac{\gamma_1}{8\eta}} \sin \theta, & \beta = \varphi - \pi/2 \\ -\frac{\bar{u}}{\sin \varphi} = \frac{\bar{v}}{\cos \varphi} = \frac{\gamma_1}{8\eta} (\cos \theta - \eta) \sin \theta \end{cases}$$



# Fluorescence based feedback

$$\text{stabilize target } \cos \frac{\theta}{2} |e\rangle + \sin \frac{\theta}{2} e^{i\varphi} |g\rangle$$

Stabilization of any state



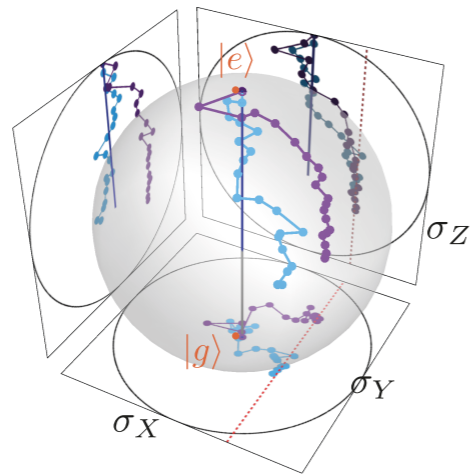
continuous measurement based feedback  
with **multi inputs and multi outputs**  
in the quantum regime



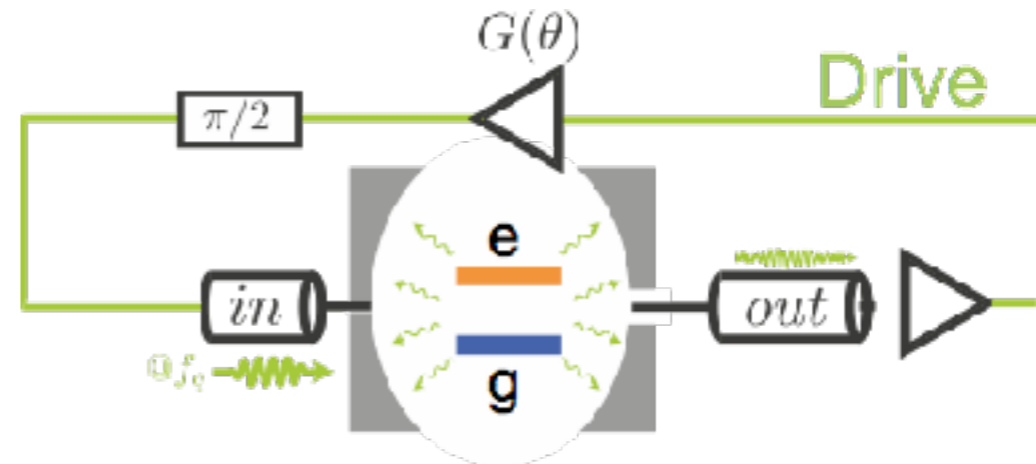
# Conclusion

**Superconducting circuits are a testbed for quantum measurement backaction**

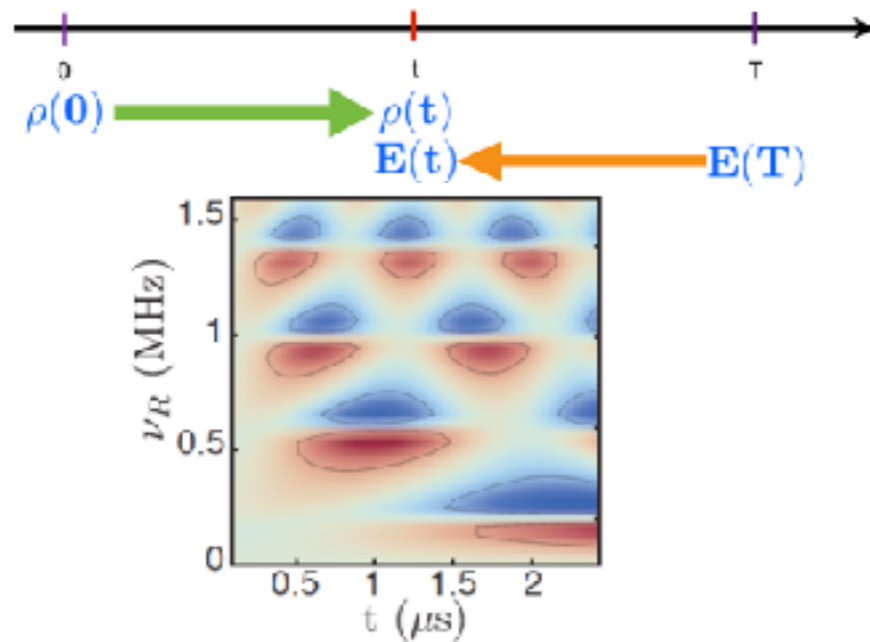
quantum trajectories



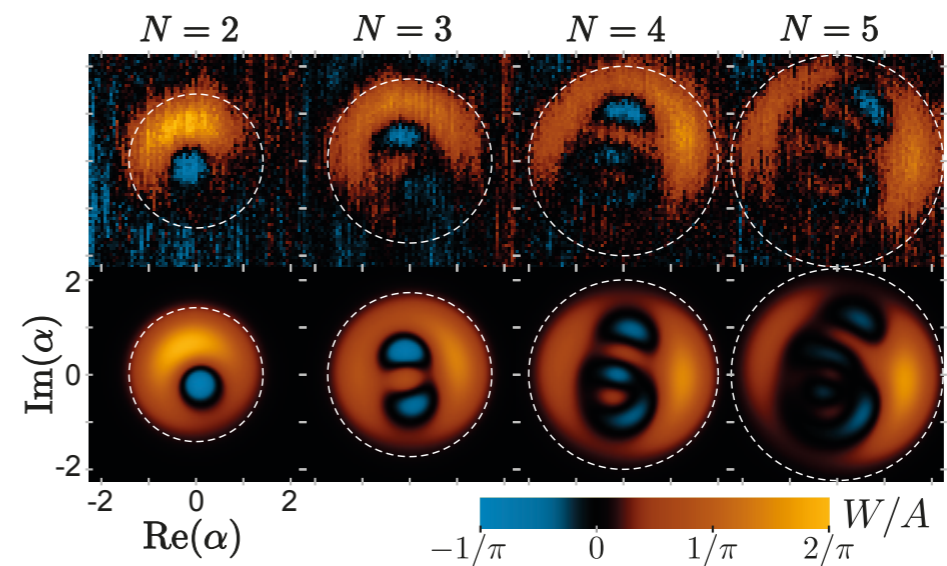
feedback



post-selection and quantum states



quantum Zeno dynamics



[Campagne-Ibarcq et al., PRL 2013]

[Bretheau et al., Science 2016]

also: autonomous feedback, quantum smoothing,  
parameter estimation, **link with thermodynamics...**





Philippe  
Campagne-Ibarcq  
(now at Yale)



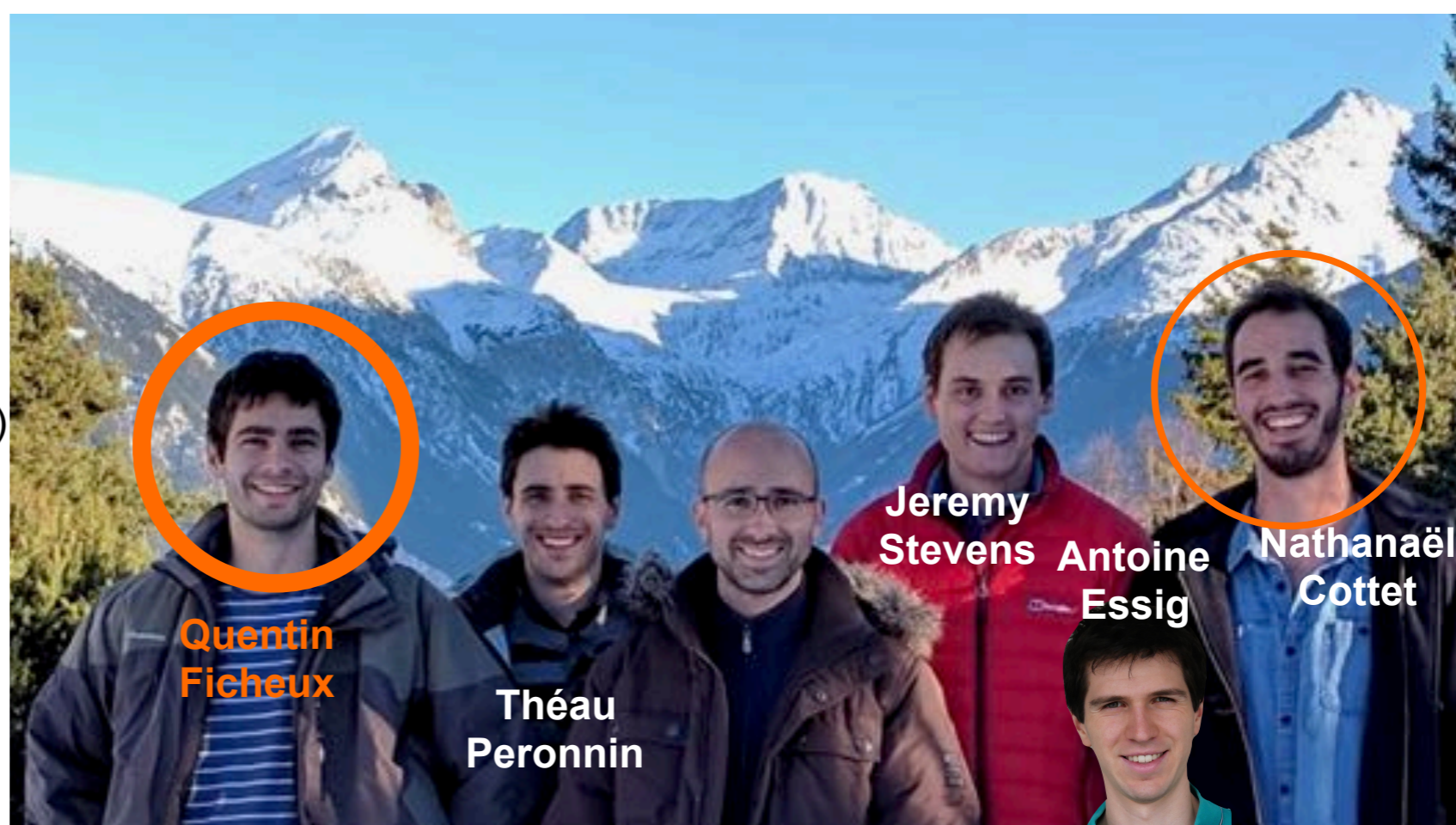
Sébastien  
Jezouin  
(now at Sherbrooke)



Landry  
Bretheau  
(now at  
Polytechnique)



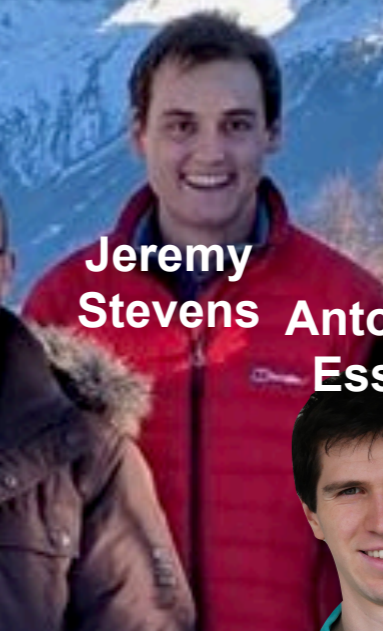
Emmanuel  
Flurin  
(now at Saclay)



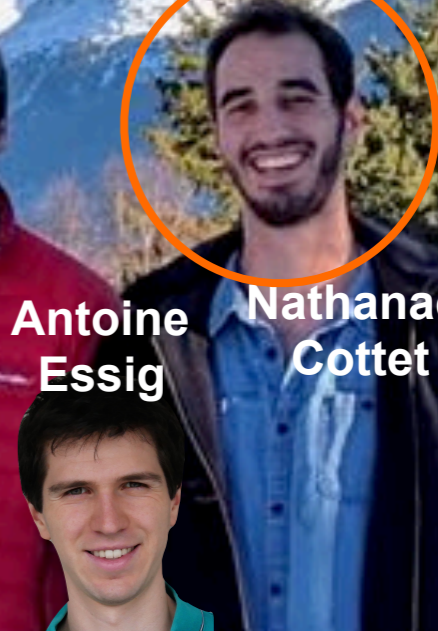
Quentin  
Ficheux



Théau  
Peronnin



Jeremy  
Stevens



Antoine  
Essig



Nathanaël  
Cottet



Nicolas  
Roch  
(now at Grenoble)



François  
Mallet



Pierre Six  
(now at  
Mines Paris)



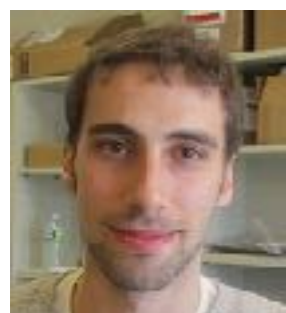
Pierre  
Rouchon



Mazyar  
Mirrahimi



Alain  
Sarlette



Zaki  
Leghtas



Michel Devoret  
(Yale University)



Alexia  
Auffèves  
(Grenoble)



Cyril  
Elouard  
(Rochester)



Areeya  
Chantasri  
(Griffith)

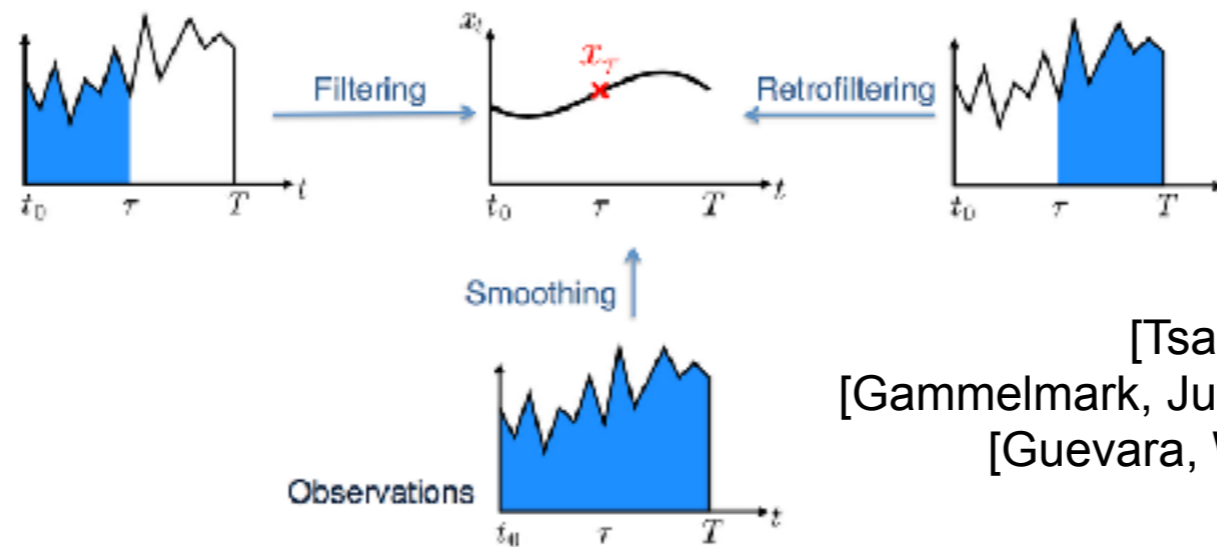


Andrew  
Jordan  
(Rochester)



# Perspectives

quantum smoothing



[Tsang, PRL 2009]  
[Gammelmark, Julsgaard, Mølmer, PRL 2013]  
[Guevara, Wiseman, PRL 2015]

statistics of postselected outcomes

higher dimension

# POVM for fluorescence

$$M_{(V_{\text{Re}}, V_{\text{Im}})}^\dagger M_{(V_{\text{Re}}, V_{\text{Im}})} = \frac{e^{-(V_{\text{Re}}^2 + V_{\text{Im}}^2)/V_0^2}}{\pi} (\mathbf{1} + \gamma_{1b} \delta t [V_{\text{Re}} \sigma_x - V_{\text{Im}} \sigma_y]/V_0)$$



# Particle Quantum Filter

$$\rho(t + dt) = \frac{K_{dy_t}^\eta(\rho(t))}{\text{Tr}[K_{dy_t}^\eta(\rho(t))]}$$

$$dy_t = (dW_{I_t}, dW_{Q_t})$$

~ probability of measuring  $dy_t$

## Partial Kraus map

$$K_{dy_t}^\eta : \rho(t) \mapsto M_{dy_t} \rho(t) M_{dy_t}^\dagger + (1 - \eta) \gamma_1 \sigma_- \rho(t) \sigma_+ dt$$

$$\text{with } M_{dy_t}^\eta = \text{I} - (iH + \frac{1}{2} \gamma_1 (\sigma_+ \sigma_- + \sigma_- \sigma_+)) dt + \sqrt{\frac{\eta \gamma_1}{2}} (\sigma_- dW_{I_t} + i \sigma_- dW_{Q_t})$$

# Particle Quantum Filter

$$\rho(t + dt) = \frac{K_{dy_t}^\eta(\rho(t))}{\text{Tr}[K_{dy_t}^\eta(\rho(t))]} \quad \left. \vphantom{\rho(t + dt)} \right\} \text{propagate}$$
$$P(t + dt) = P(t) \times \text{Tr}[K_{dy_t}^\eta(\rho(t))]$$

$P(t)$  probability of having measured the record  $\{dy_0, dy_{dt} \dots dy_t\}$

→ depends on  $\eta$

→ comparing  $P^{\eta_1}(t)$  and  $P^{\eta_2}(t)$ , **more likely**  $\eta$

**Problem**  $P^\eta(t) \simeq 0$



# Particle Quantum Filter

$$\rho_i(t + dt) = \frac{K_{dy_t}^{\eta_i}(\rho(t))}{\text{Tr}[K_{dy_t}^{\eta_i}(\rho_i(t))]}$$
$$\pi_i(t + dt) = \frac{\pi_i(t) \times \text{Tr}[K_{dy_t}^{\eta_i}(\rho_i(t))]}{\sum_j \pi_j(t) \times \text{Tr}[K_{dy_t}^{\eta_j}(\rho_j(t))]}$$

} propagate

# Particle Quantum Filter

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