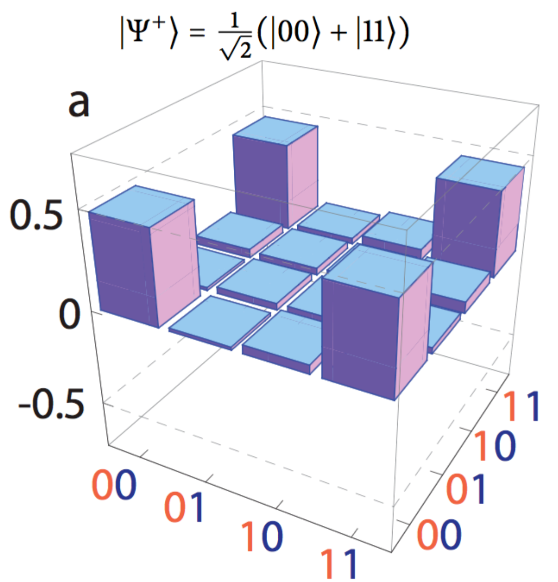
Swiss National Science Foundation
Université de Genève

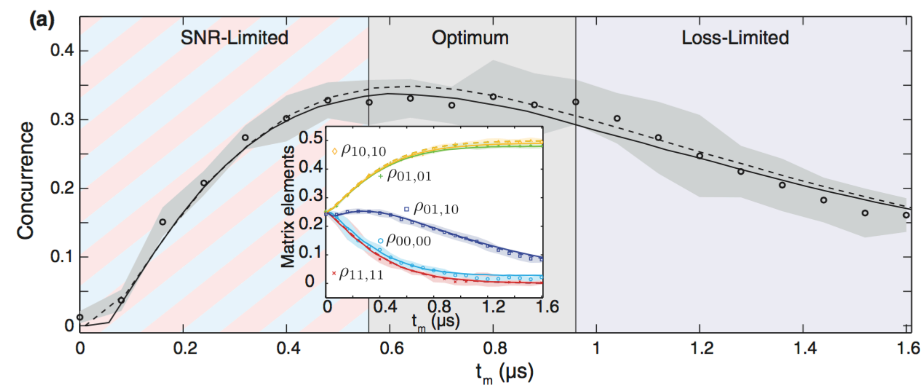
Energetic-based witnesses
to certify the presence of entanglement for QIP

arXiv:1807.02487

Cyril Elouard, Alexia Auffèves and Géraldine Haack



Tomography of a Bell state
(Schoelkopf lab, Yale)



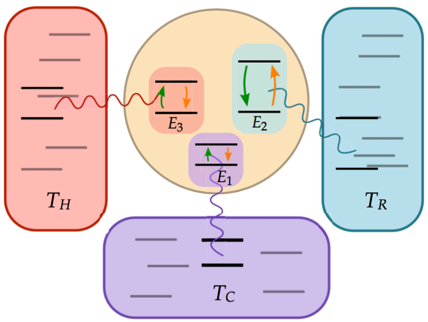
Time-dependent concurrence
(Siddiqi's lab, Berkeley)

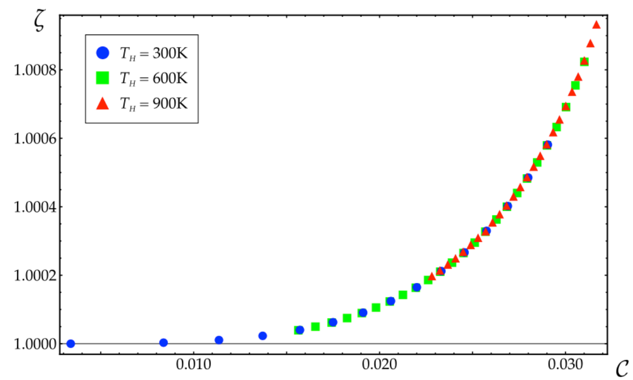
New witnesses for Q. correlations?

Thermodynamics of quantum systems : Measurement, engines and control
KITP 09.07.2018

Quantum correlations vs. thermodynamics

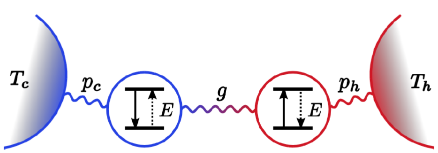
- Enhance efficiency of thermal machines

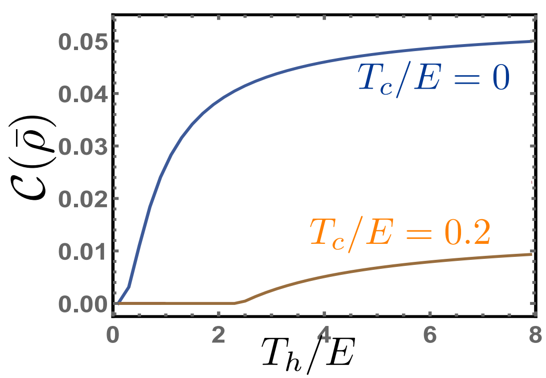




Linden et al., PRL 105 (2010)
Levy, Kosloff, PRL 108 (2012)
Chen et al., EPL 97 (2012)
Venturelli et al., PRL 110 (2013)
Kosloff, Levy, Ann. R. Phys. Chem. 65 (2014)
Correa et al., Sc. Reports 4 (2014)
N. Brunner et al., PRE 89 (2014)
Hofer et al., PRB 94 (2016)
Maslennikov et al., arXiv:1702.08672 (2017)

- Generate entanglement using thermal resources



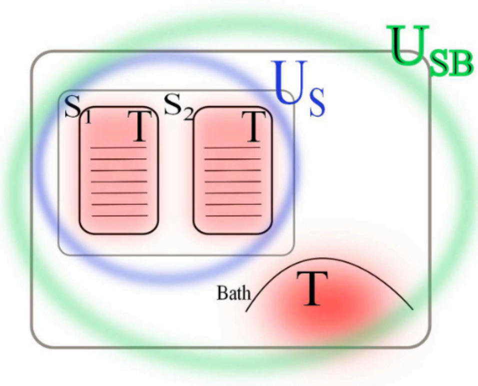


M. Plenio, S.F. Hulega, PRL 88 (2002)
V. Eisler, Z. Zimboras, PRA 71 (2005)
L. Hartmann et al., NJP 9 (2007)
L. Quiroga et al., PRA 75 (2007)
N. Linden et al., PRL 105 (2010)
B. Bellomo et a., NJP 15 (2013)
N. Brunner et al., PRE 89 (2014)
J. B. Brask et al., NJP 17 (2015)
D. Boyanovsky et al., PRA 96 (2017)
A. Tavakoli et al., Quantum2 (2018)
B. Tacchino et al., PRL 120 (2018)
B. Hewgill et al., arXiv:1806.10512 (2018)

Quantum correlations vs. thermodynamics

- Investigate the thermodynamic cost of creating Q. correlations

Resource theory approach
Connection with Circuit QED exp.



Huber et al., NJP 17 (2015)
Bruschi et al., PRE 91 (2015)
Friis et al., PRE 93 (2016)

- Energy as an entanglement witness

Thermal state entanglement in harmonic lattices
Non-interacting bosonic gas

Anders et al., NJP 8 (2006)
Anders, PRA 77 (2008)

Entanglement generation and certification for QIP

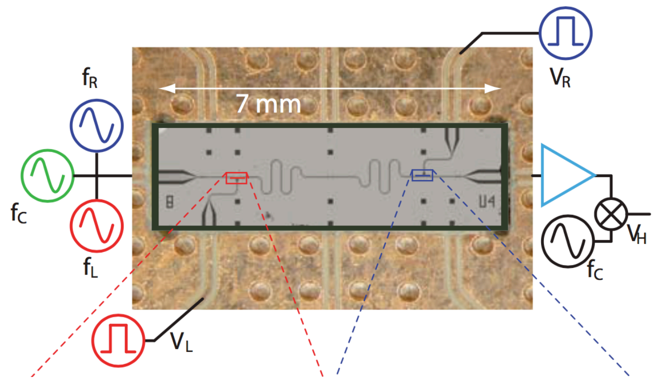
- Logical two-qubit gates

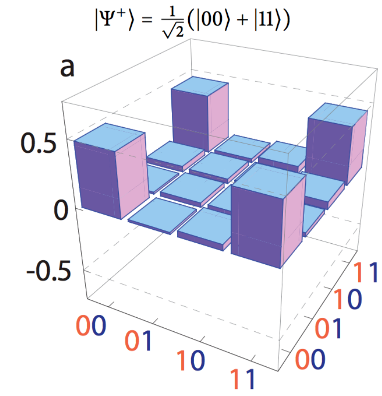
- Measurement-induced entanglement

Entanglement generation and certification for QIP

- Logical two-qubit gates

$$\mathrm{CPhase} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$





Schoelkopf lab, Yale

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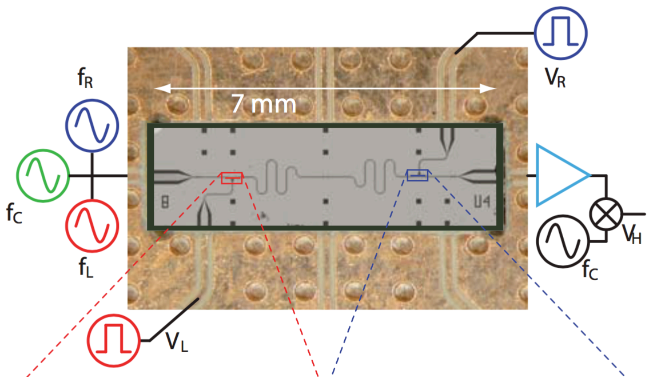
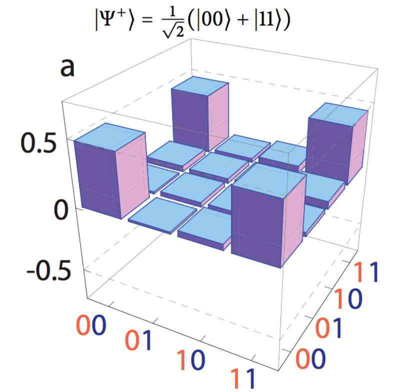
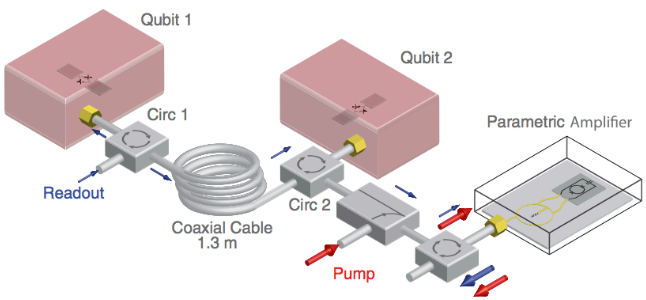


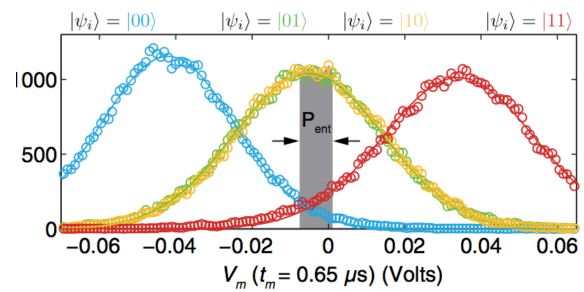


Schoelkopf lab, Yale

- Measurement-induced entanglement

$$\hat{P}_{1/2} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$





Riste et al., Nature 502 (2013)
Roch et al., PRL 112 (2014)

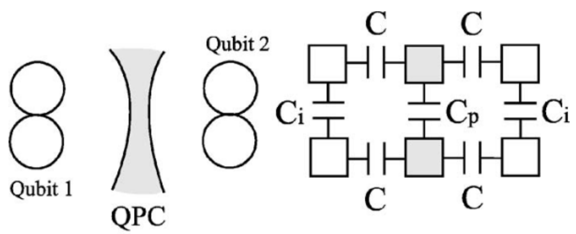
$$|gg\rangle \xrightarrow{\mathcal{H}^{\otimes 2}} \frac{1}{2}\left(|ee\rangle + |eg\rangle + |ge\rangle + |gg\rangle\right) \xrightarrow{\hat{P}_{1/2}} \begin{cases} |ee\rangle \\ \frac{1}{\sqrt{2}}\left(|eg\rangle + |ge\rangle\right) \\ |gg\rangle \end{cases}$$

Parity measurement : History

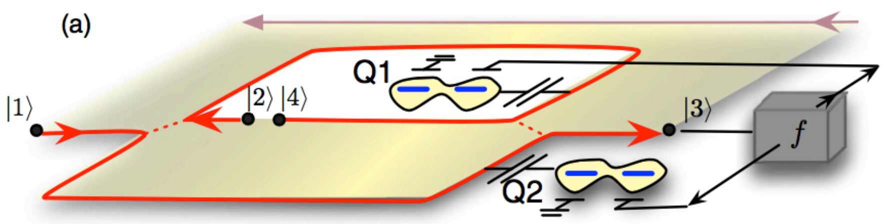
- Deterministic way to generate entanglement

  Beenakker, DiVicenzo, Emary, Kindermann et al., PRL 93 (2004)
  Engel, Loss, Science 309 (2005)
  Trauzettel, Jordan, Beenakker, Büttiker, PRB 73 (2006)
  Zilberberg, Braunecker, Loss, PRA 77 (2008)

- Solid-state proposals with double-quantum dot charge qubits



Trauzettel et al., PRB 73 (2006)



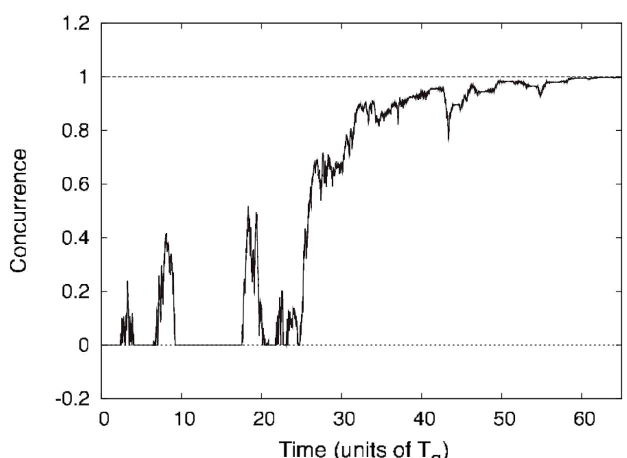
Haack, Förster, Büttiker, PRB 82 (2010)
Meyer zu Rheda, Haack, Romito, PRB 90 (2014)

- Cavity QED proposals

  Lalumière, Gambetta, Blais, PRA 81 (2010)
  Tornberg, Johansson, PRA 82 (2010)

- Entanglement genesis from continuous measurement

  Williams and Jordan, PRA 78 (2008)
  Tornberg, Johansson, PRA 82 (2010)
  S. Hofer, Vasilyev, Aspelmeyer, Hammerer, PRL 111 (2013)
  Meyer zu Rheda, Haack, Romito, PRB 90 (2014)



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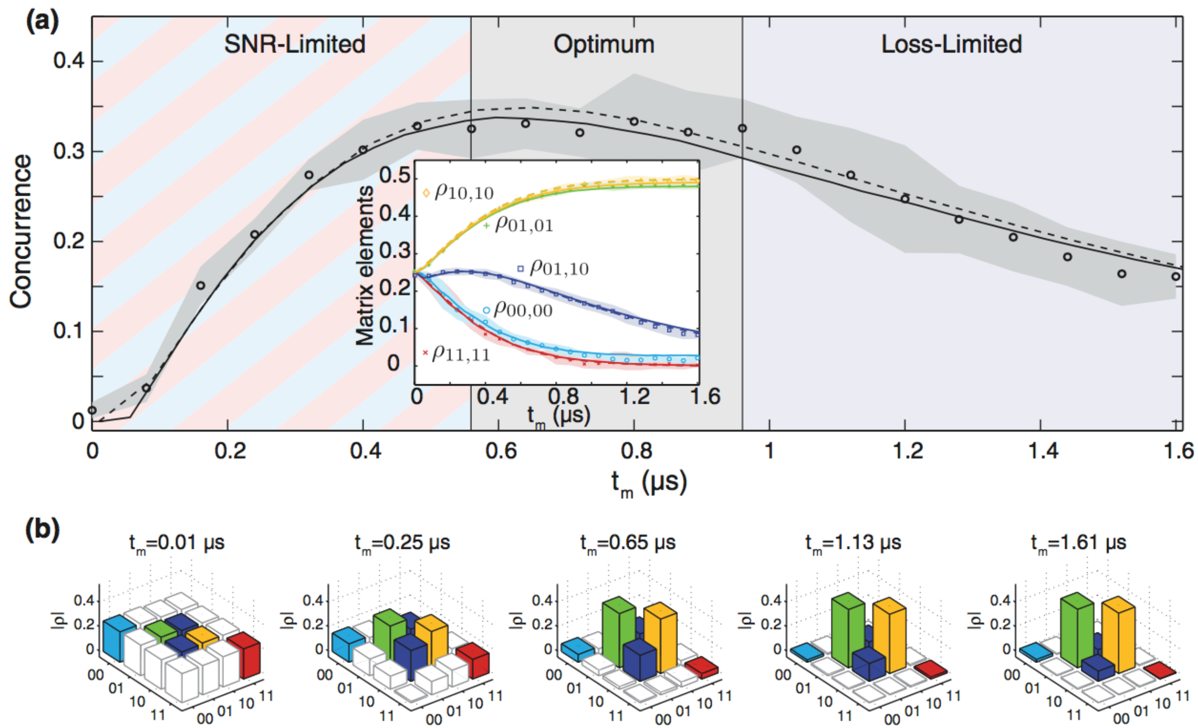

Roch et al., PRL 112 (2014)

- Weak continuous measurements
- Generates separable and entangled states
- Time-resolved measurement outcome at the level of a single trajectory

Parity measurement

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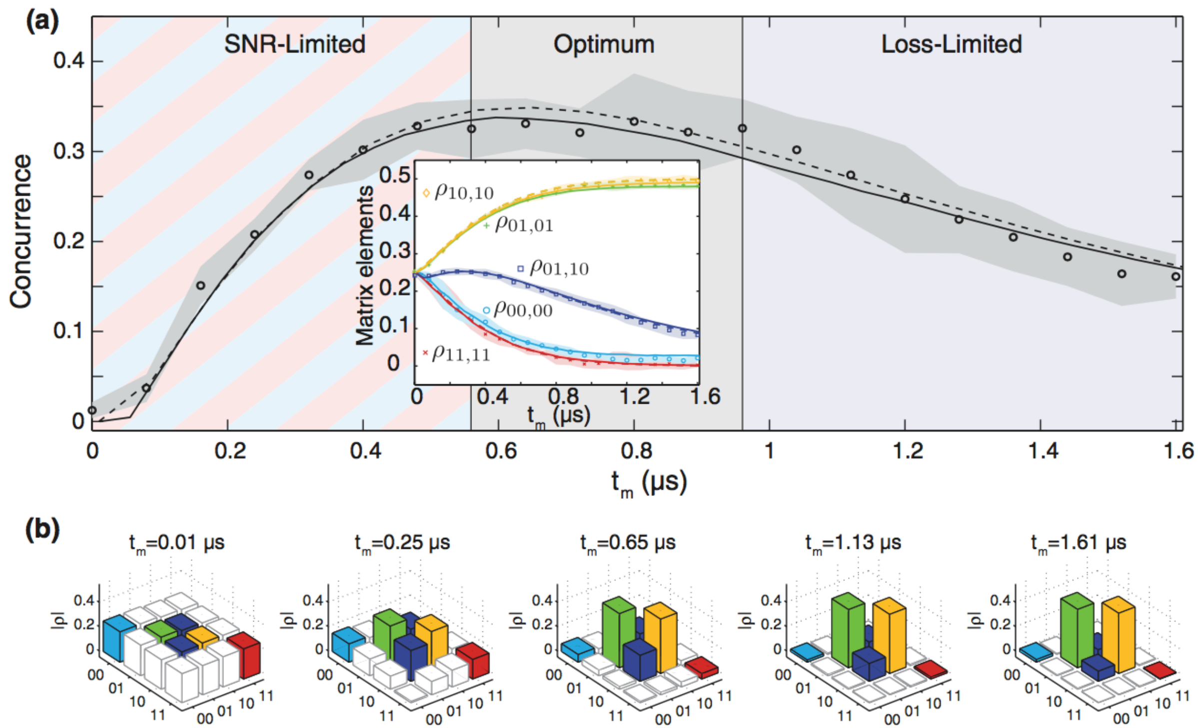

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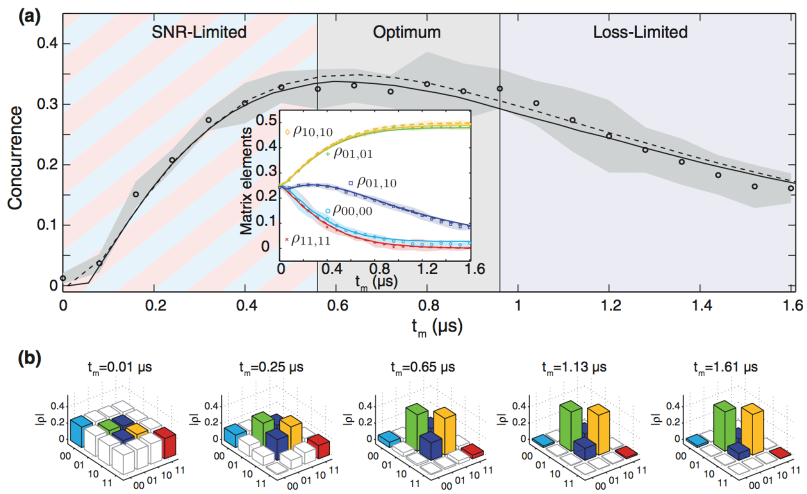
Energetic counterpart of measurement-induced entanglement genesis?
Can we derive energetic witnesses, an alternative to tomography?

Outline



Energetic counterpart of measurement-induced entanglement genesis?
Can we derive energetic witnesses, an alternative to tomography?

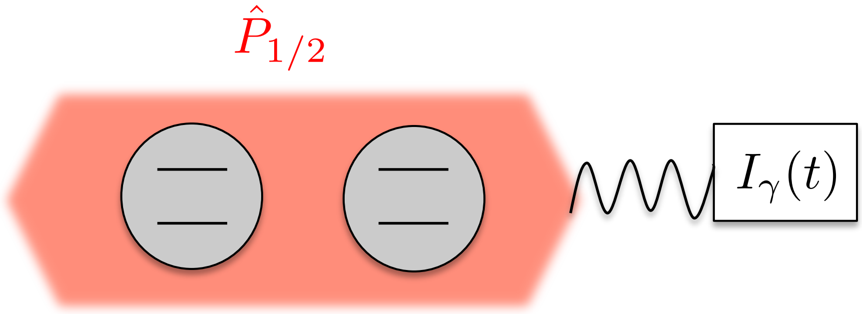
I. Derivation of quantum trajectories induced by a half-parity measurement

II. Application of stochastic thermodynamics

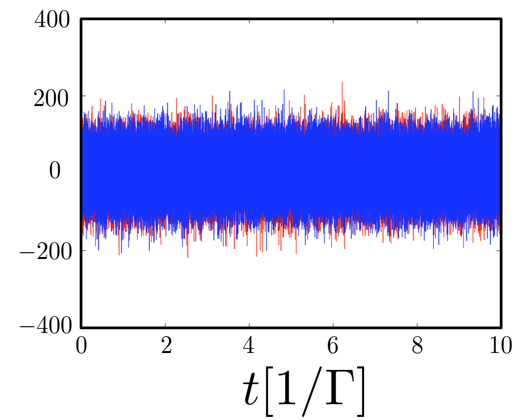
III. Beyond average stochastic quantities -> fluctuations!

IV. Energetic bounds for the entanglement genesis rate and energetic witnesses

Quantum trajectories in a half-parity meas.



$$\hat{H}_s = \epsilon\left(\hat{\sigma}_z^{(1)} \otimes \mathbf{1} + \mathbf{1} \otimes \hat{\sigma}_z^{(2)}\right)$$



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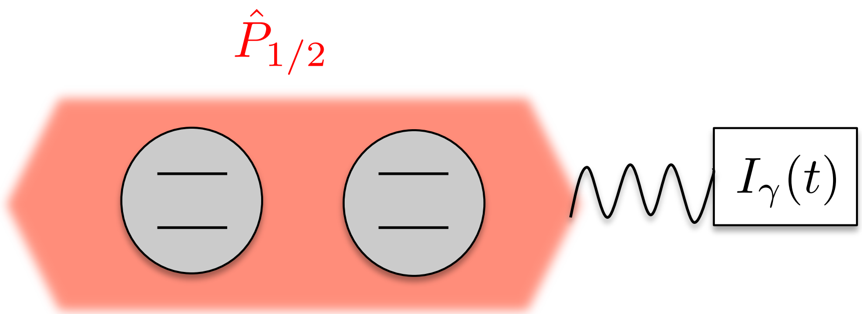

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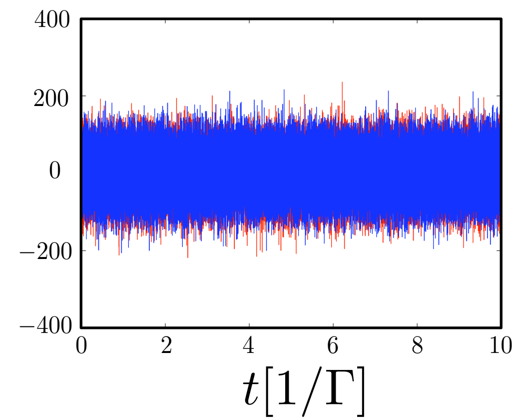
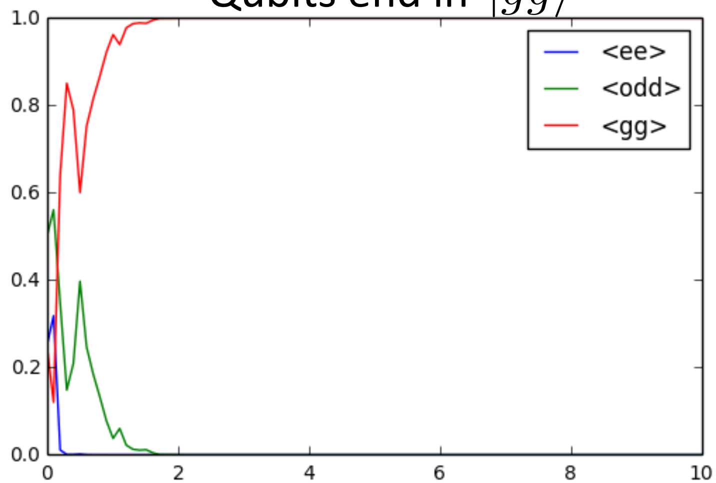
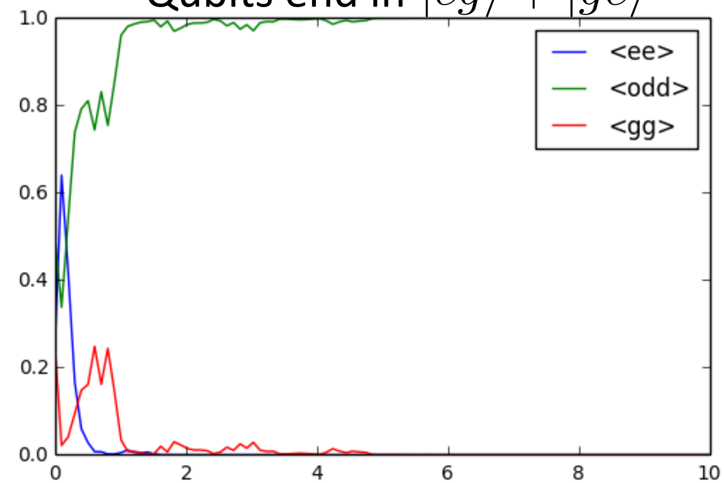


Qubits end in $|gg\rangle$



Qubits end in $|eg\rangle + |ge\rangle$



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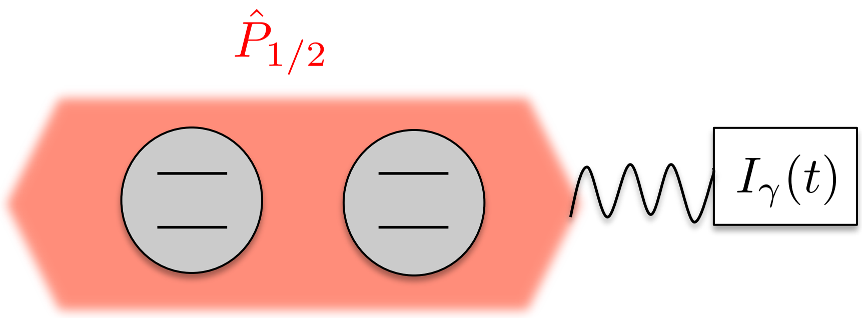

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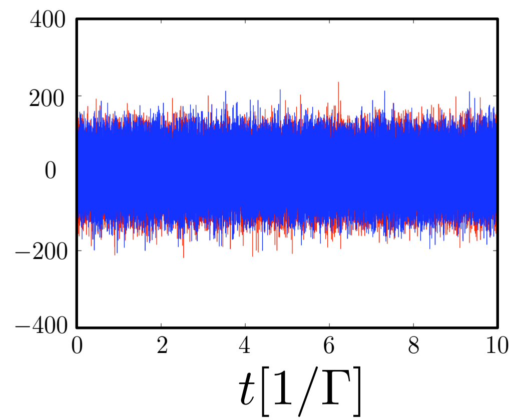


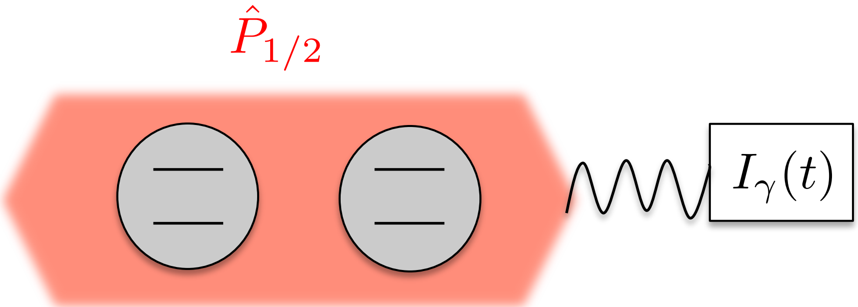
- Measurement record: $$I_\gamma(t) = \langle \hat{P}_{1/2}(t) \rangle_\gamma + \frac{dW_\gamma(t)}{2\sqrt{\Gamma} dt}$$
- Measurement outcome: $$J_\gamma(t) = \frac{1}{t} \int_0^t I_\gamma(\tau) d\tau$$
- Γ : measurement-induced dephasing rate
- $dW_\gamma(t)$: Wiener increment (stochastic variable)

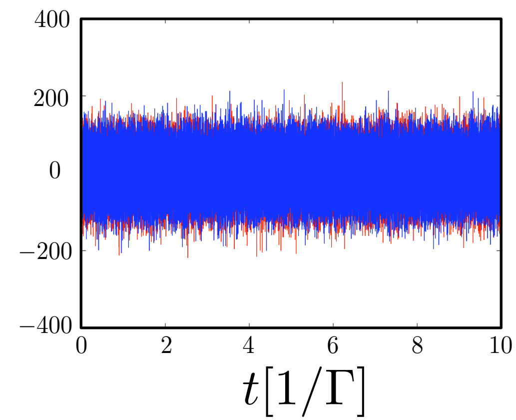
$$\langle\langle dW_\gamma(t) \rangle\rangle = 0$$

$$\langle\langle dW_\gamma(t)^2 \rangle\rangle = dt$$

Assuming the initial state is pure, each trajectory γ is made of a sequence of pure states

Quantum trajectories in a half-parity meas.





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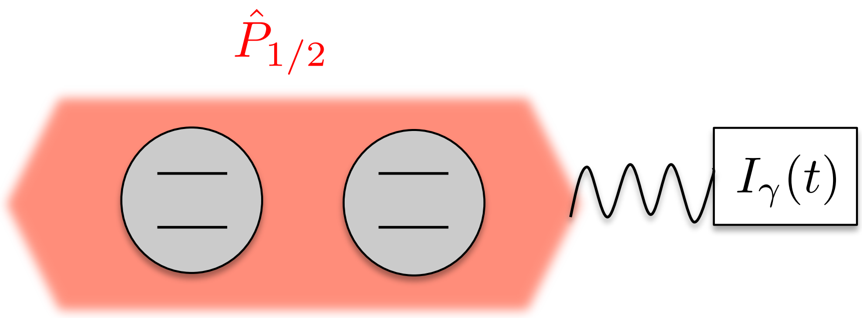
$dW_\gamma(t)$: Wiener increment (stochastic variable) $\langle\langle dW_\gamma(t) \rangle\rangle = 0$ $\langle\langle dW_\gamma(t)^2 \rangle\rangle = dt$

- Stochastic Schrödinger equation can be solved at any time $[\hat{H}_s, \hat{P}_{1/2}] = 0$

$$|\psi(J_\gamma, t)\rangle = \frac{1}{N_\gamma(t)}\left(e^{(-i\epsilon+2\Gamma J_\gamma)\hat{P}_{1/2}t - \Gamma\hat{P}_{1/2}^2 t}\right)|\psi(0)\rangle$$

Quantum trajectories in a half-parity meas.



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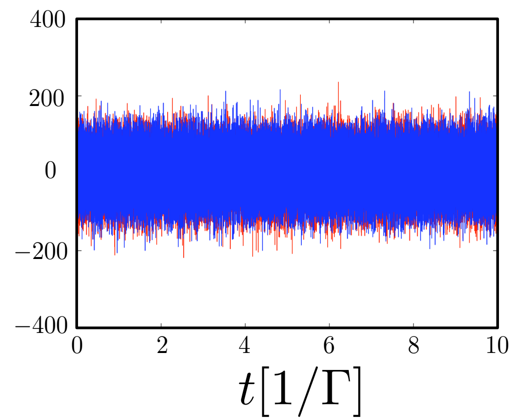

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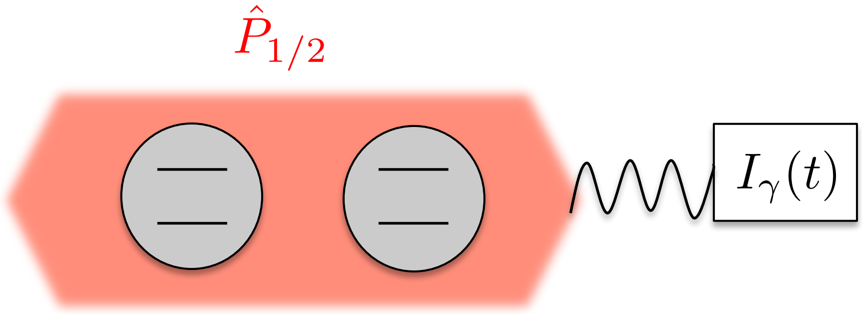

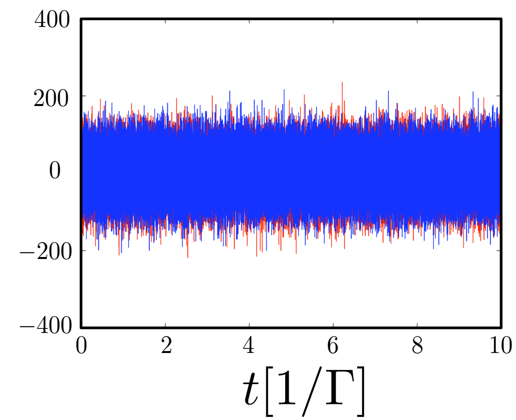

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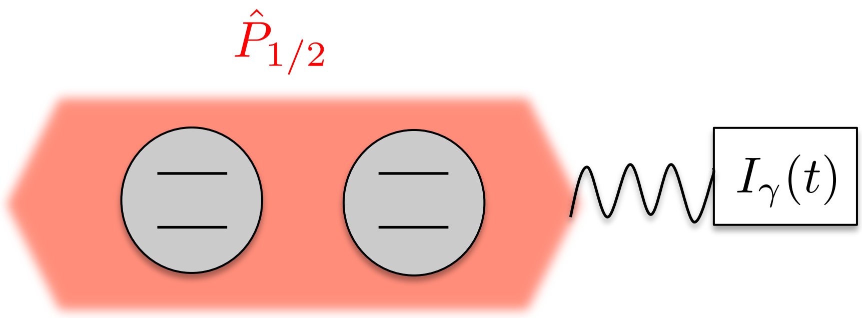
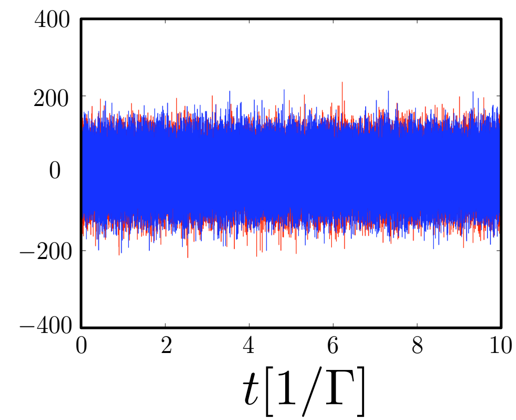
Quantum trajectories in a half-parity meas.





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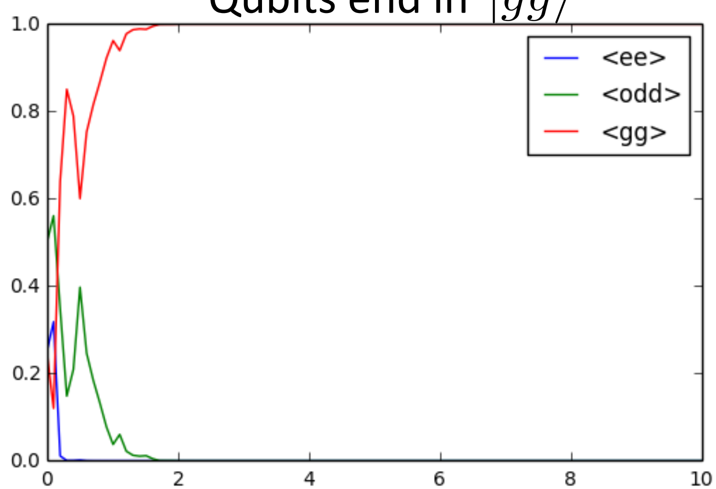

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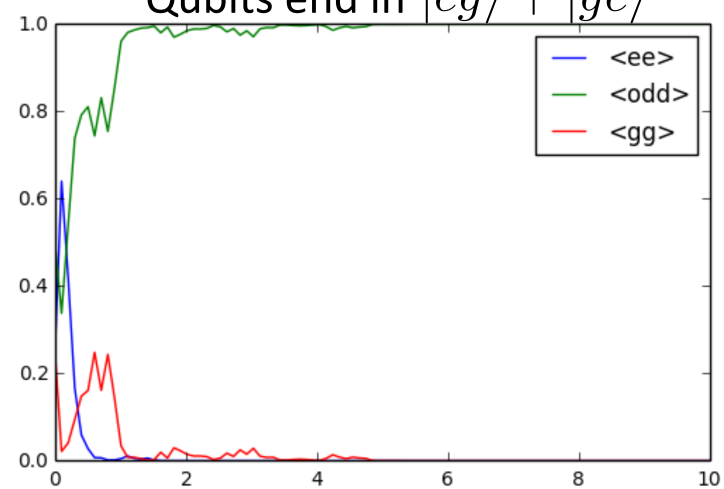
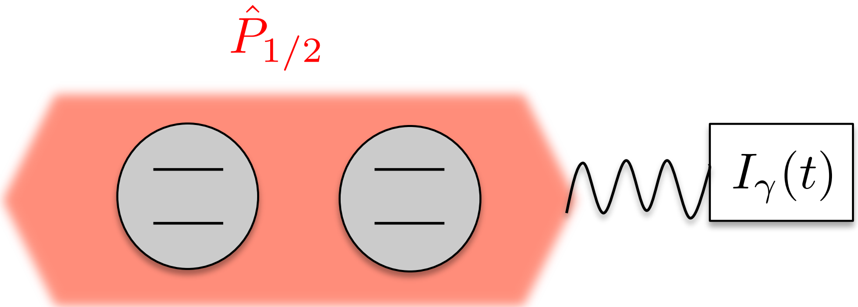


Quantum trajectories in a half-parity meas.



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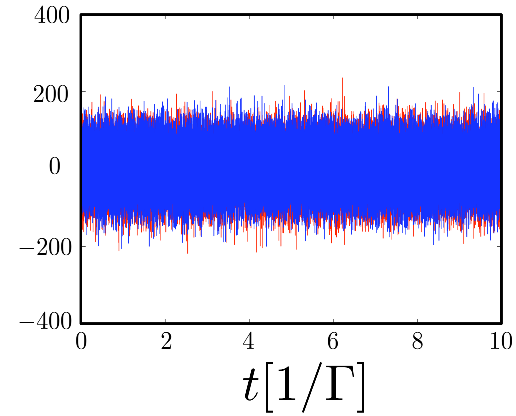
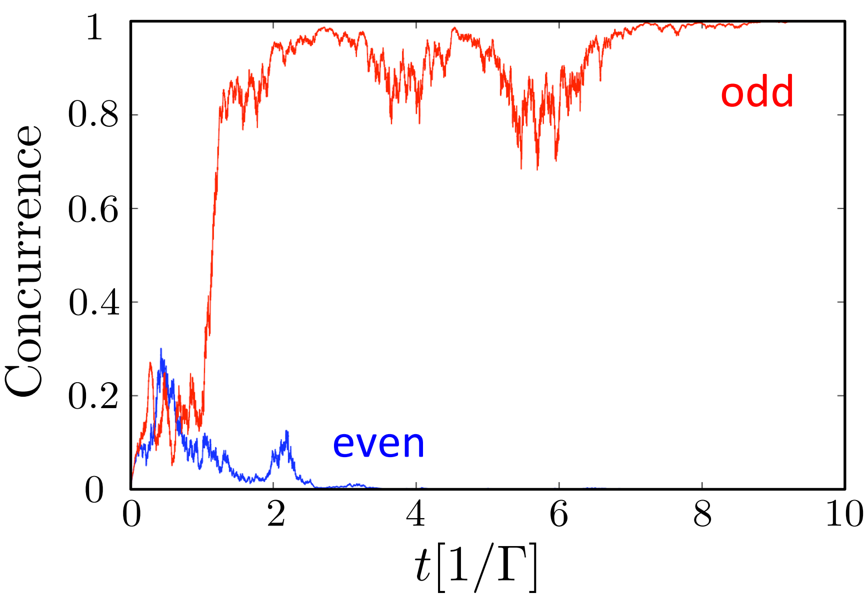
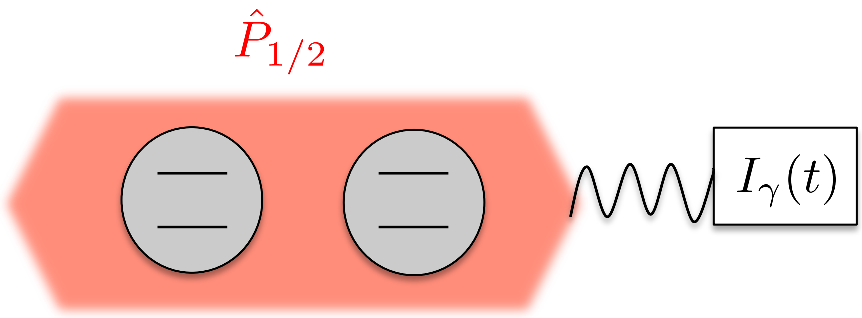




Single realization

Post-selection after several runs of the experiment



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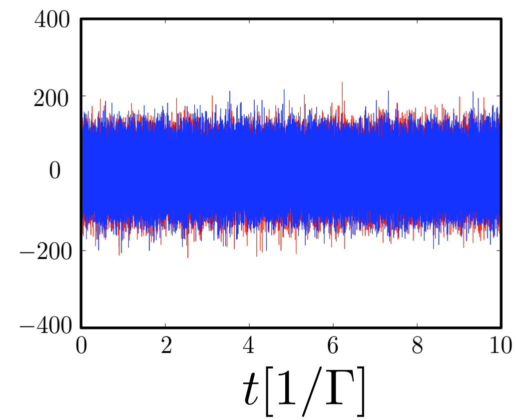

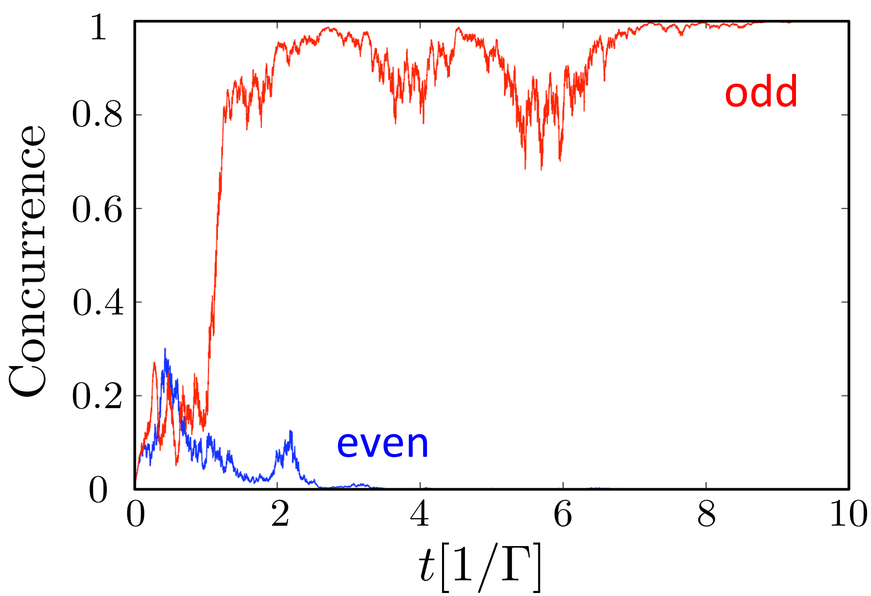

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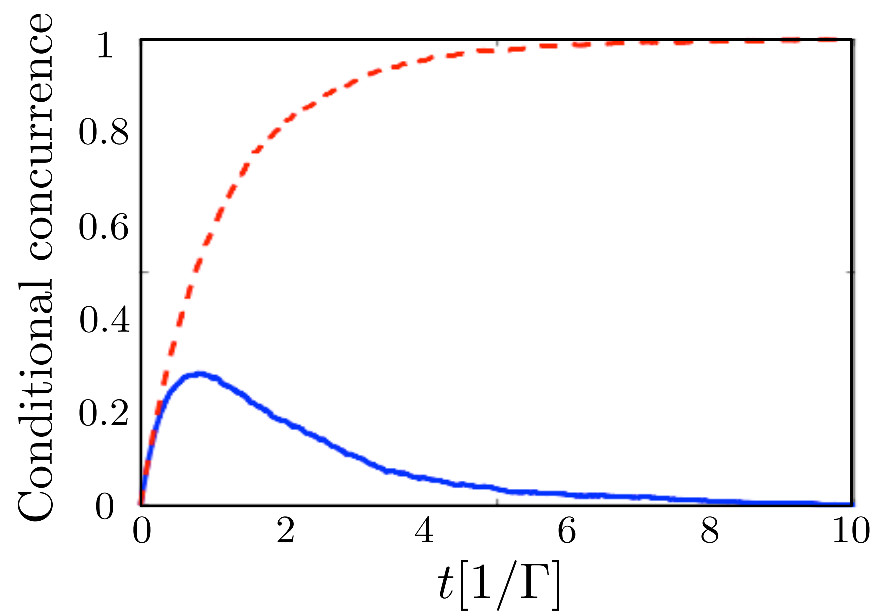
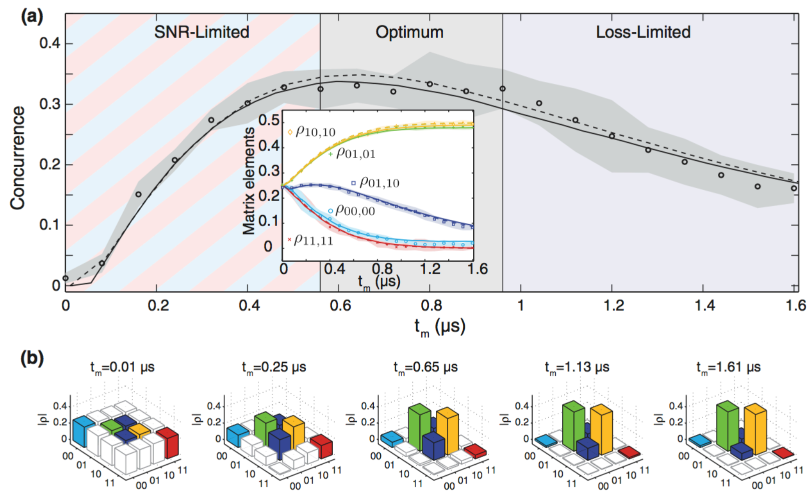


Postselection over 800 trajectories

Outline



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Energetics of entanglement genesis

- Internal energy of two qubits along a given trajectory γ

$$U_\gamma(t) = \langle \psi_\gamma(t) | \hat{H}_s | \psi_\gamma(t) \rangle .$$

$$U(0) = 0 \qquad \text{given} \qquad |\psi(0)\rangle = \frac{1}{2} \left(|e\rangle + |g\rangle \right) \otimes \left(|e\rangle + |g\rangle \right) \ .$$

- Change of internal energy

$$\Delta U_\gamma(t) = \epsilon \frac{e^{-2\Gamma t} \sinh(4\Gamma t J_\gamma)}{1 + e^{-2\Gamma t} \cosh(4\Gamma t J_\gamma)} \quad = Q_\gamma(t)$$

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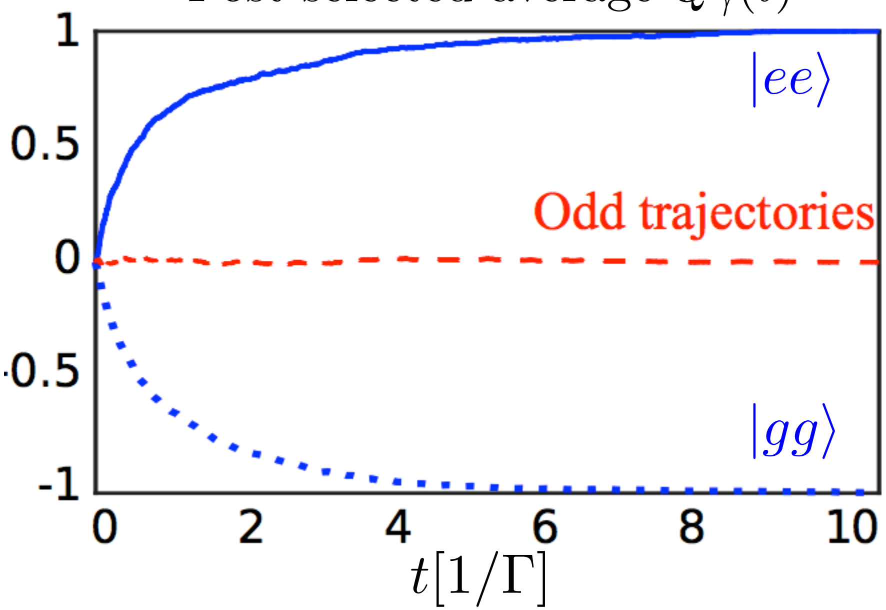
$$\Delta U_\gamma(t) = \epsilon \frac{e^{-2\Gamma t}\sinh(4\Gamma t J_\gamma)}{1 + e^{-2\Gamma t}\cosh(4\Gamma t J_\gamma)} \quad = Q_\gamma(t)$$

Post-selected average $Q_\gamma(t)$



$$\hat{P}_{1/2} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Energetics of entanglement genesis

- Internal energy of two qubits along a given trajectory

$$U_\gamma(t) = \langle \psi_\gamma(t) | \hat{H}_s | \psi_\gamma(t) \rangle.$$

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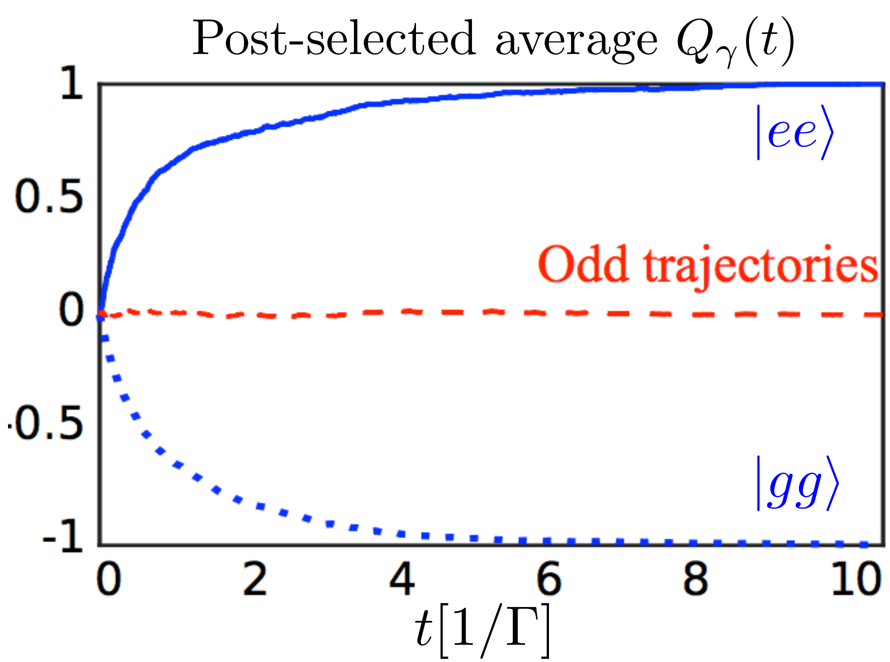

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See:

J.J. Alonso et al., PRL 116 (2016)
Elouard et al., NPJ QI 2 (2017)

Talks by Cyril and Alexia

Comparison with concurrence

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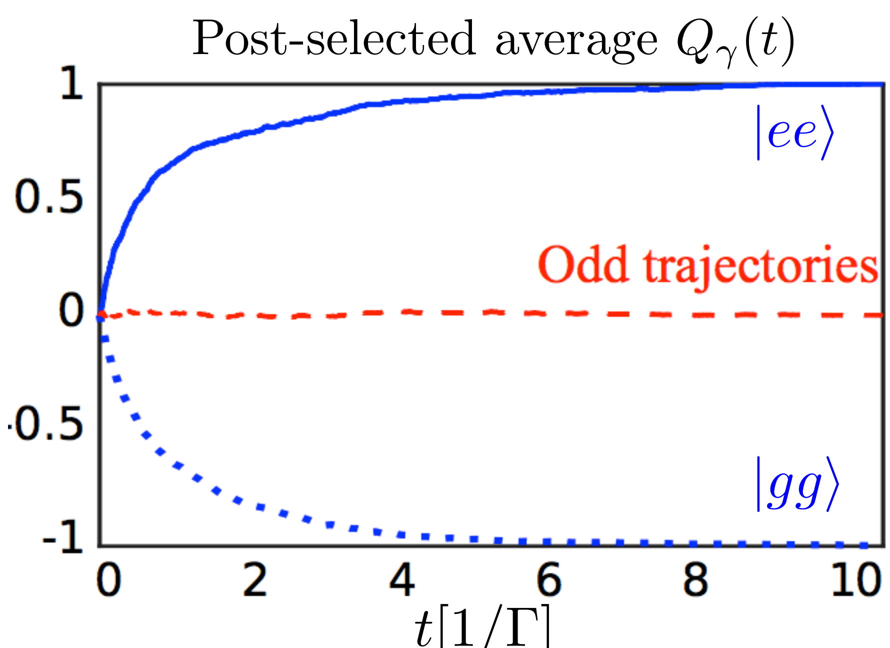
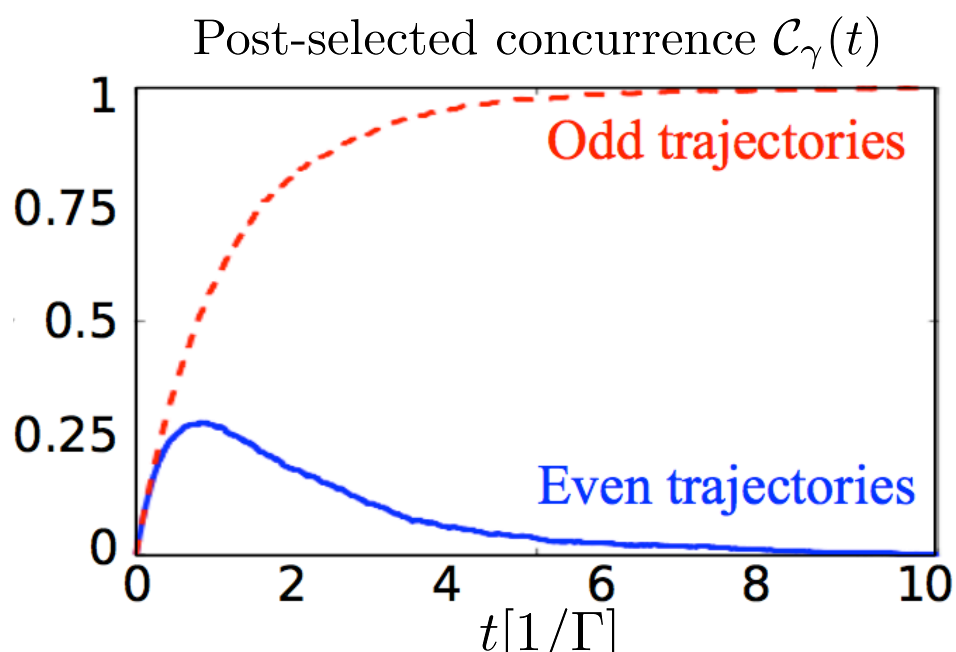
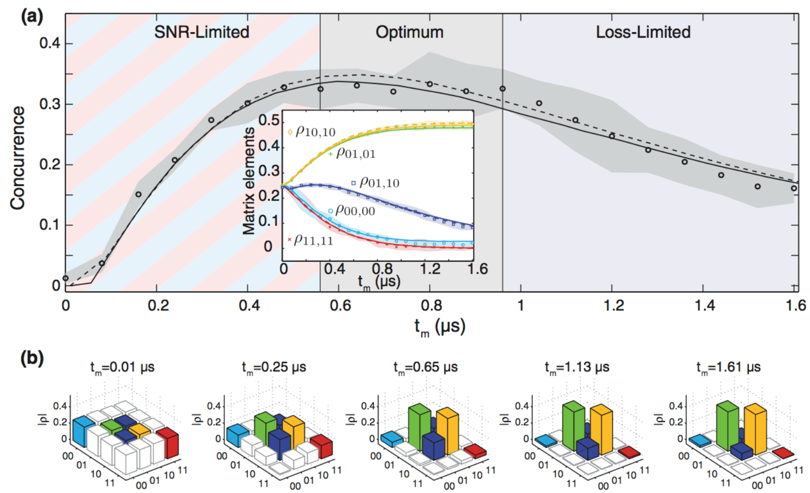




Outline



Energetic counterpart of measurement-induced entanglement genesis?
Can we derive energetic witnesses, an alternative to tomography?

I. Derivation of quantum trajectories induced by a half-parity measurement

II. Application of stochastic thermodynamics

III. Beyond average stochastic quantities -> fluctuations!

IV. Energetic bounds for the entanglement genesis rate and energetic witnesses

Beyond average stochastic energetic quantities

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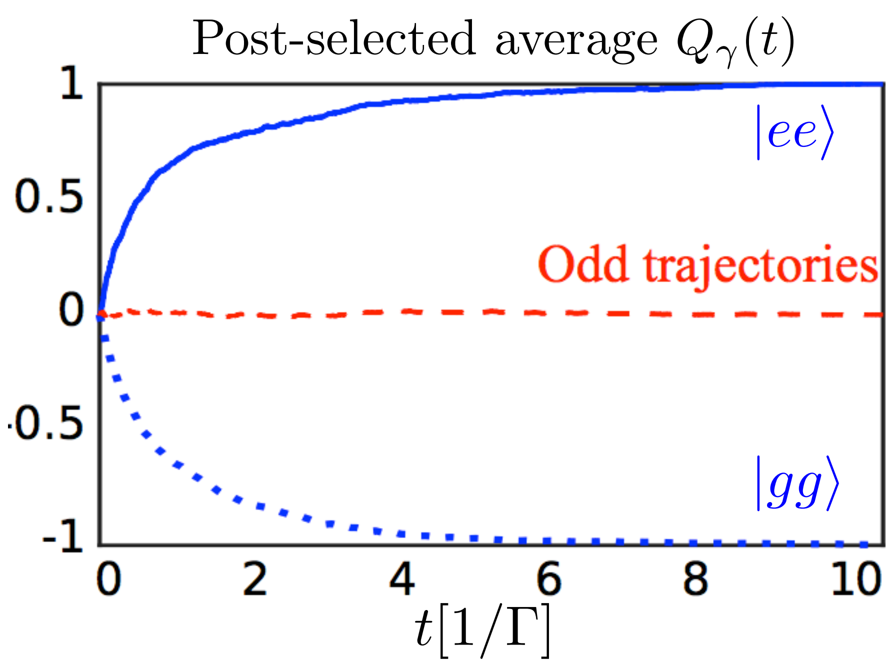

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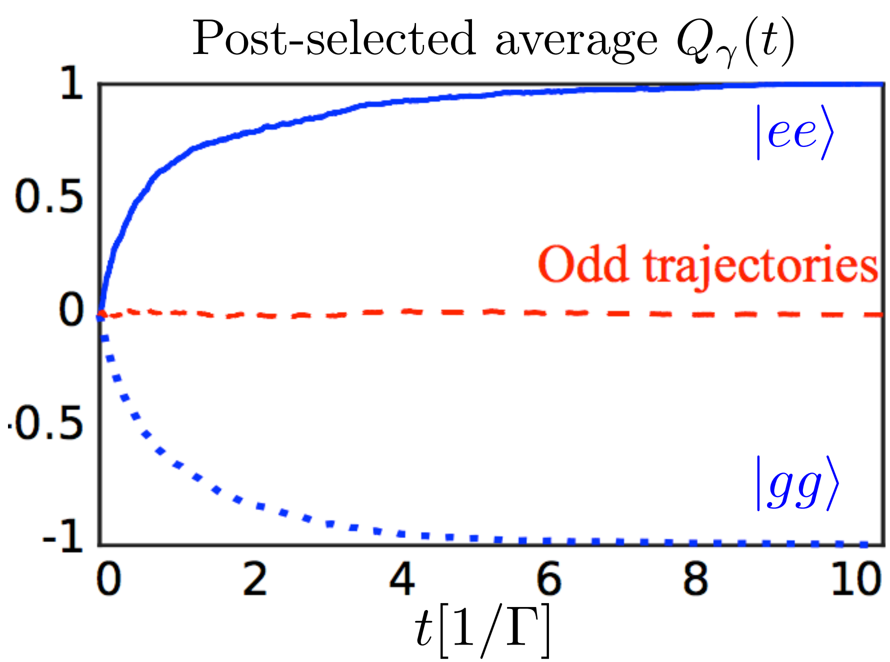

Infinitesimal increment of heat exchange

$$Q_\gamma(t) = \int_0^t \delta Q_\gamma(t') \, .$$

$$\delta Q_\gamma(t) \equiv d(U_\gamma(t))$$

$$\sigma_\gamma(t) = \sqrt{\langle \delta Q_\gamma(t)^2 \rangle_{I_\gamma(t)}}$$

Comparison with concurrence

Infinitesimal increment of heat exchange

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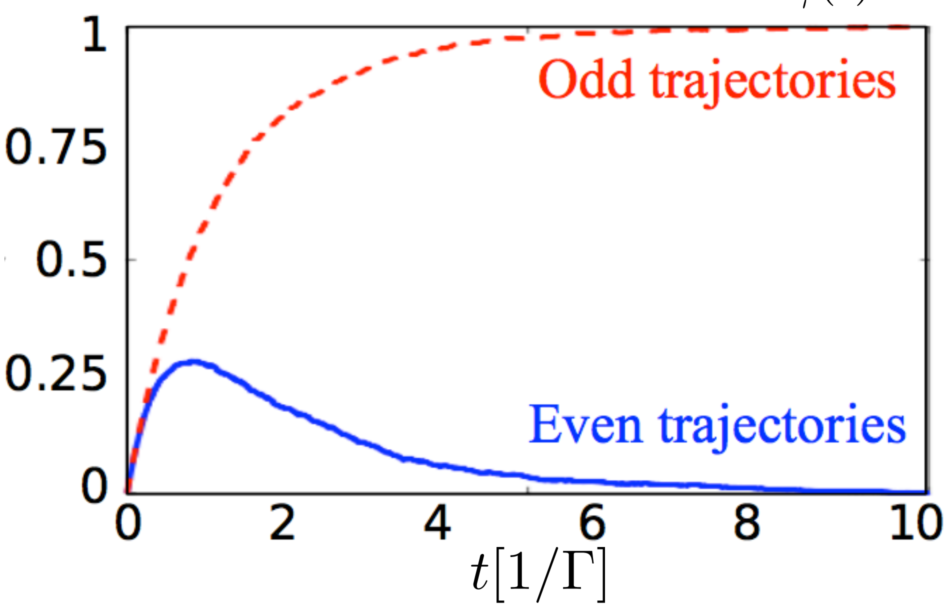
Post-selected concurrence $\mathcal{C}_\gamma(t)$



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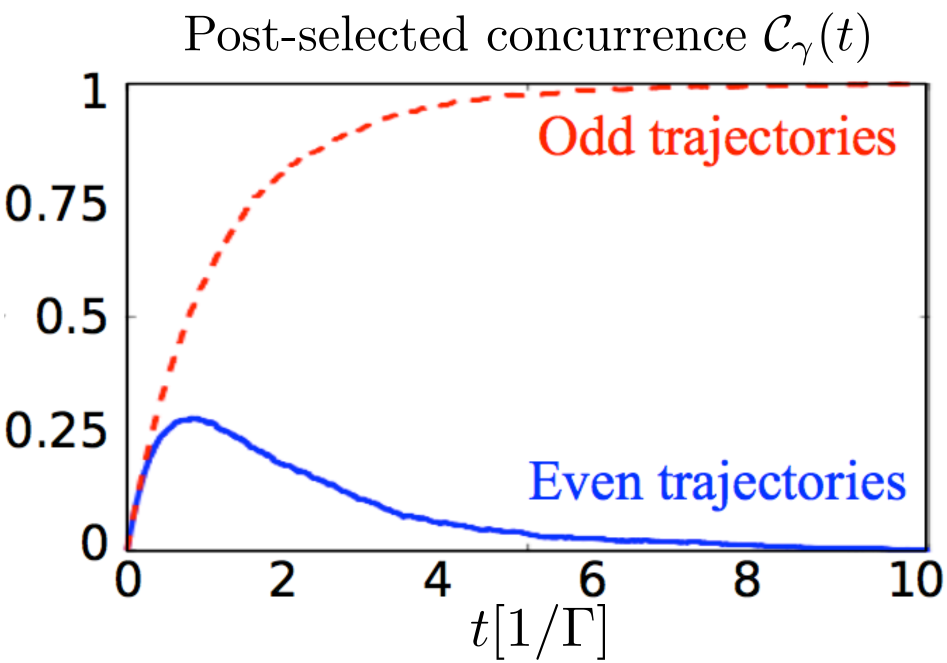

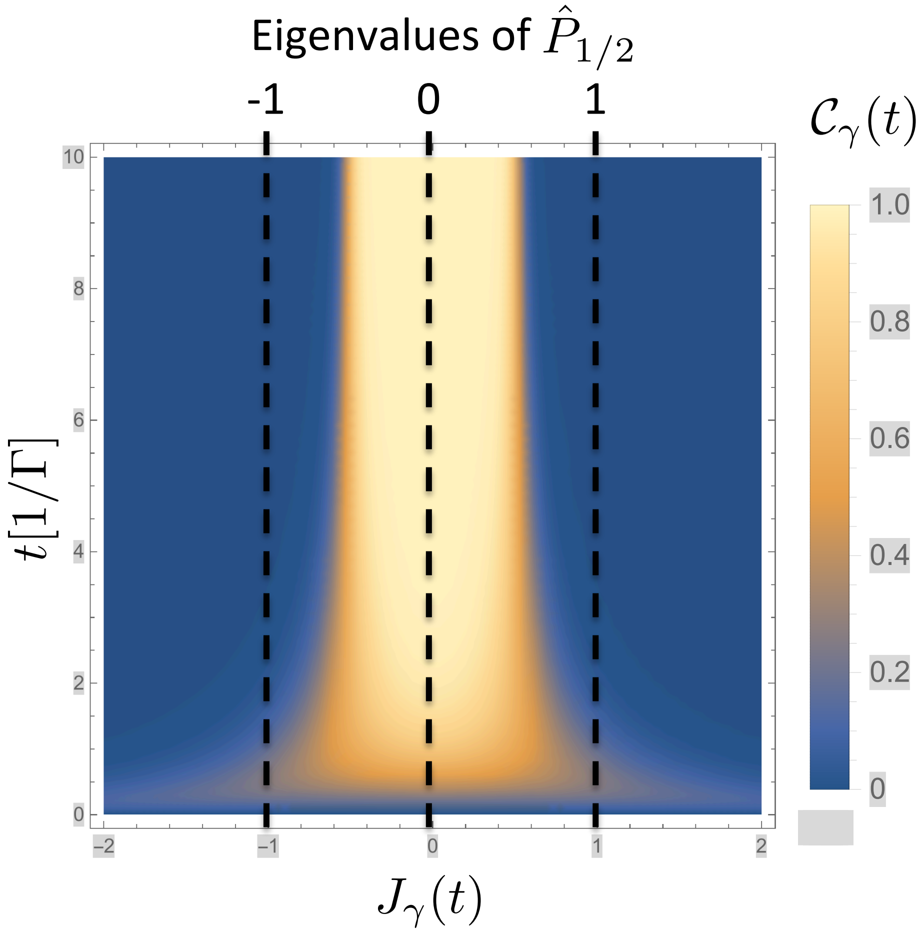

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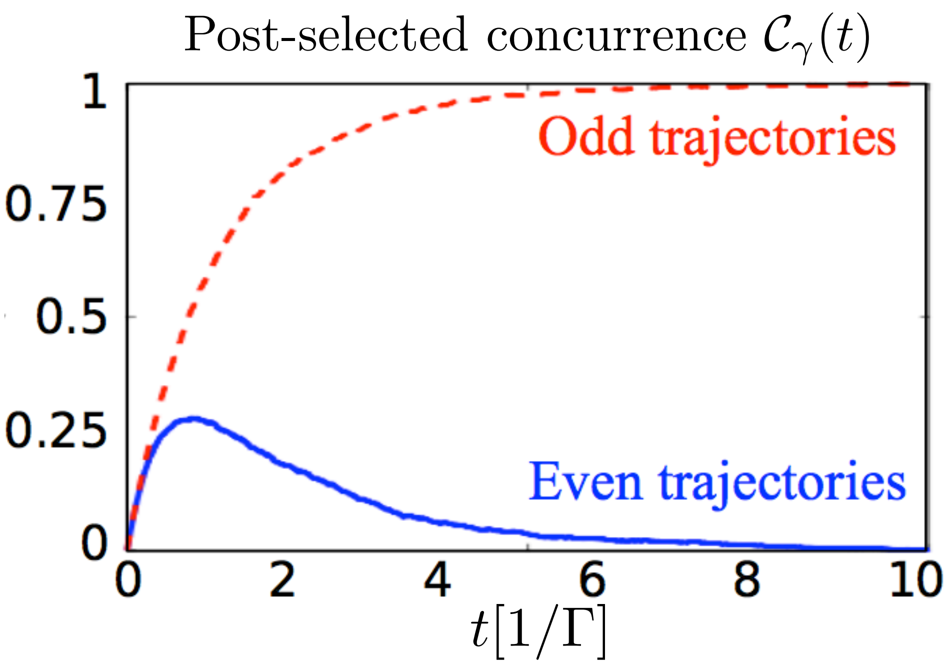

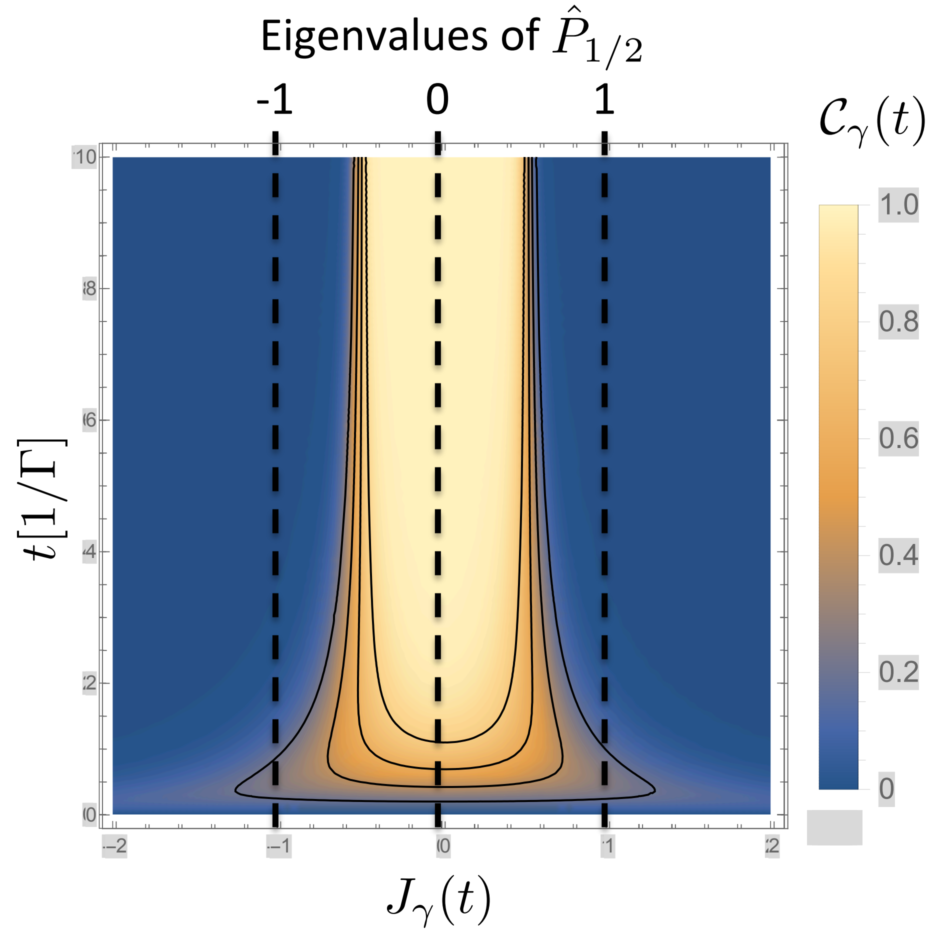

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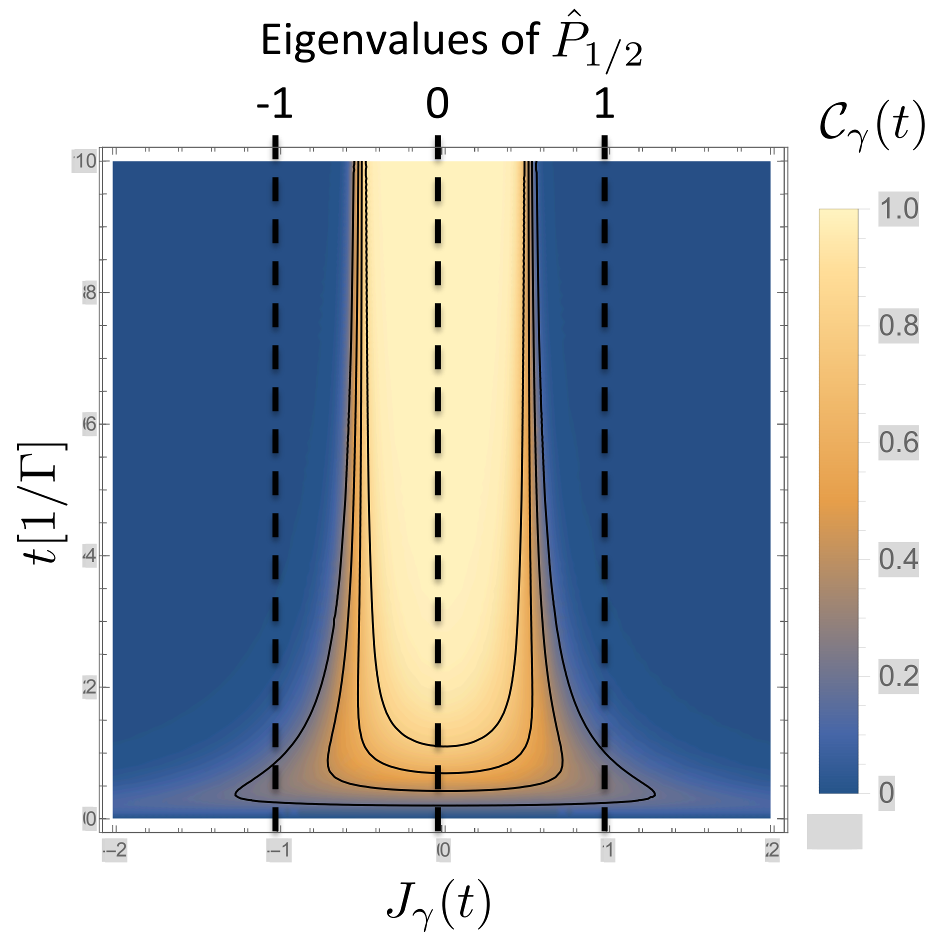

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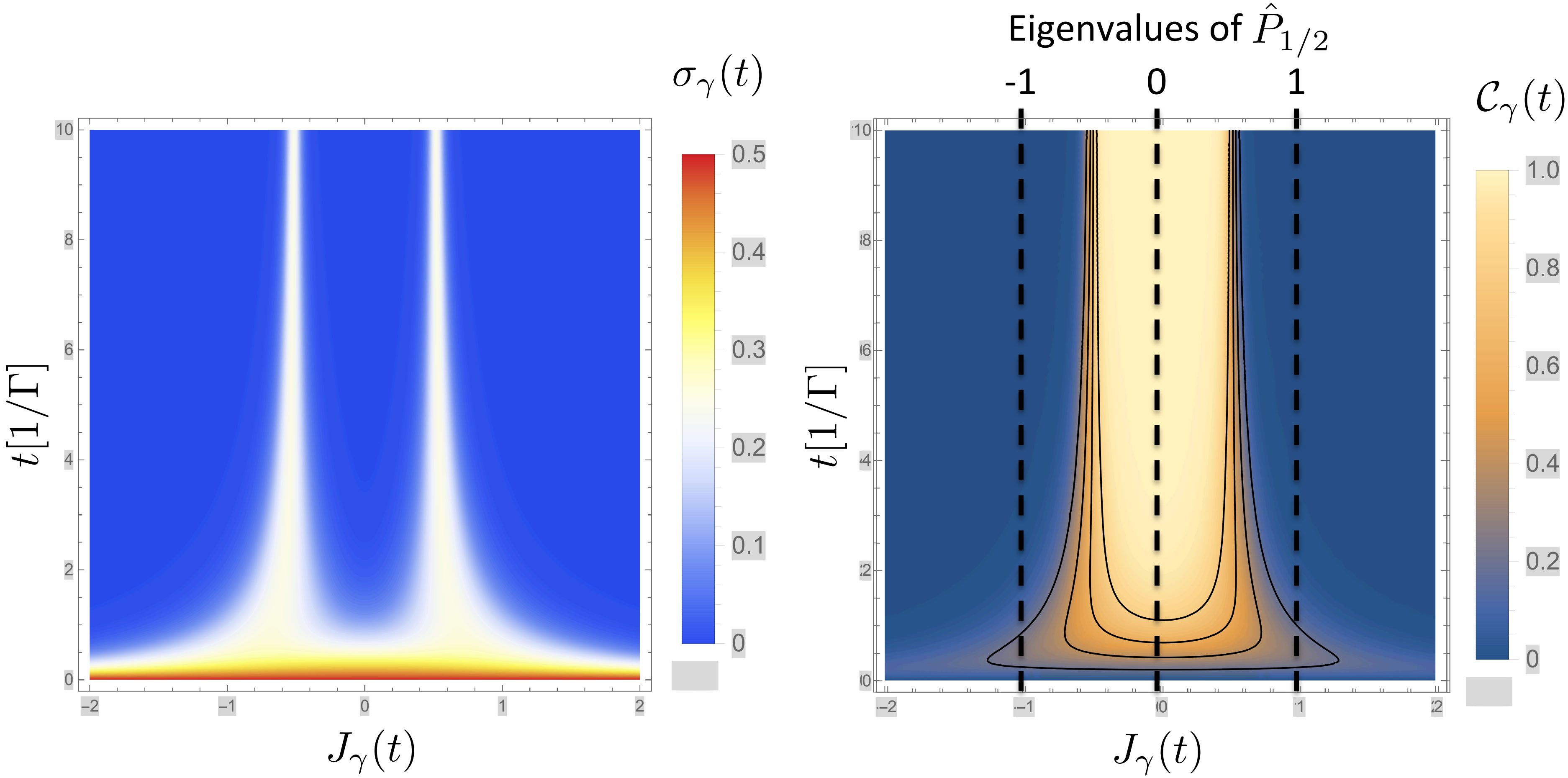

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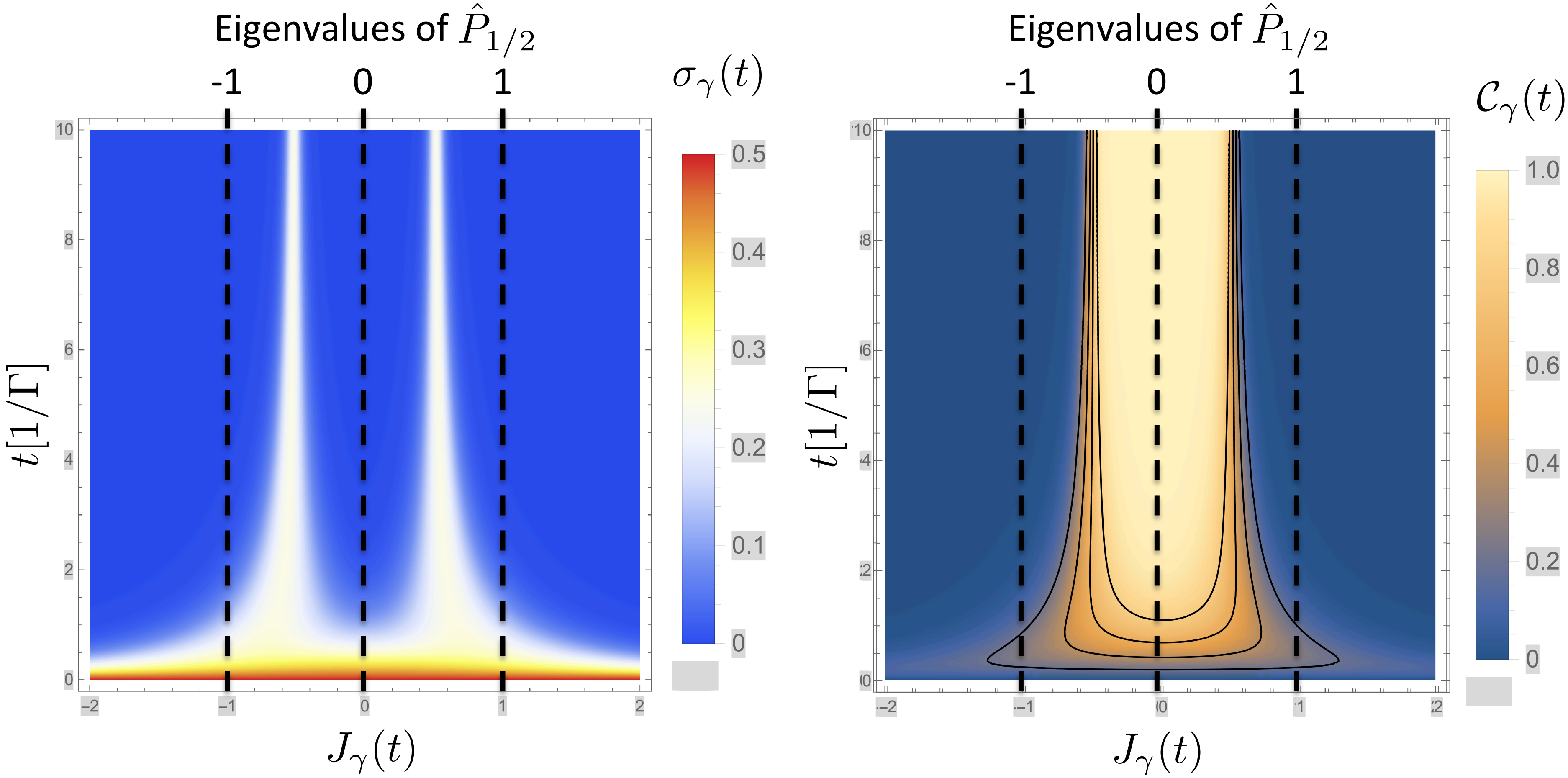

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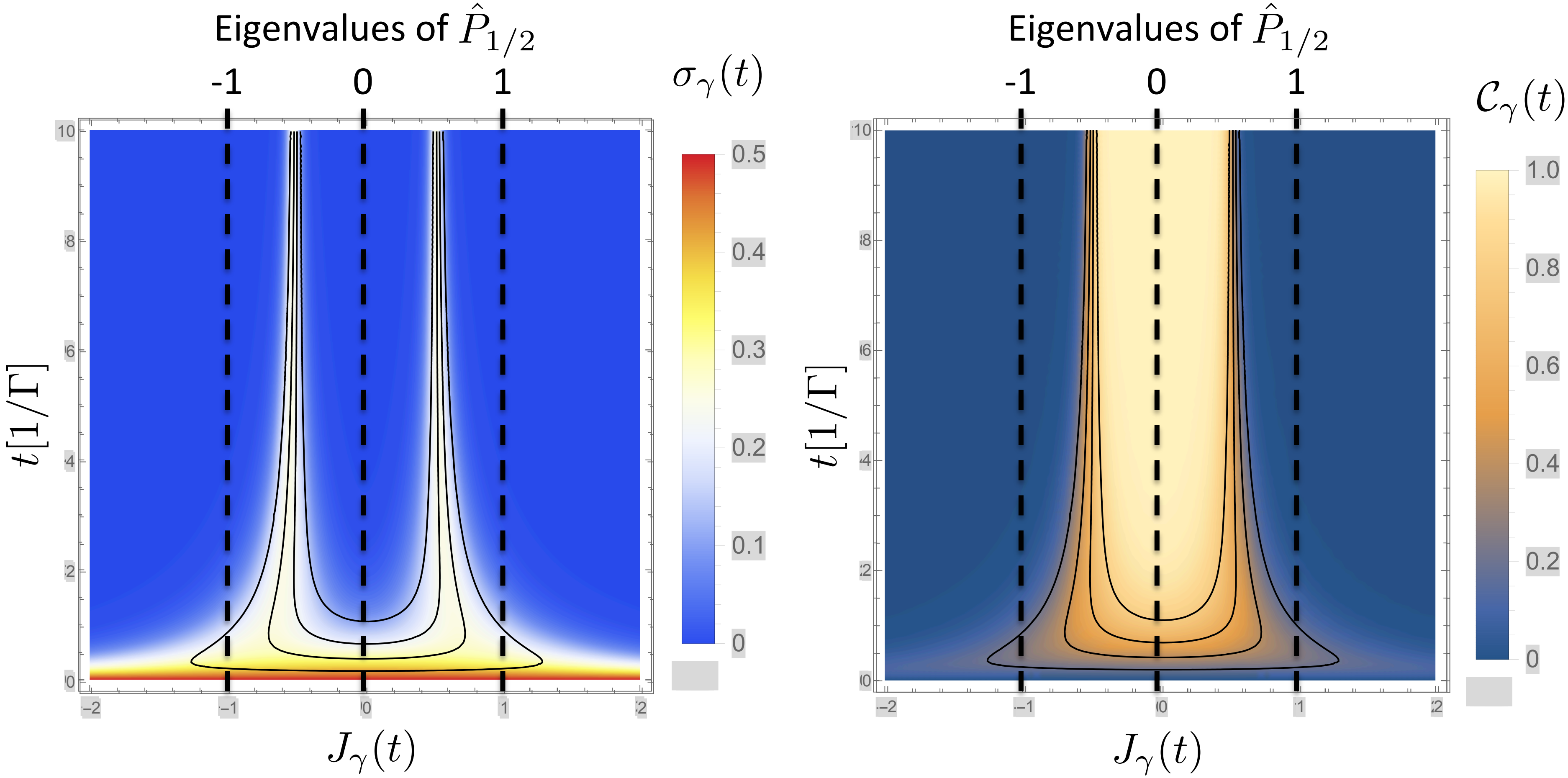

Comparison with concurrence

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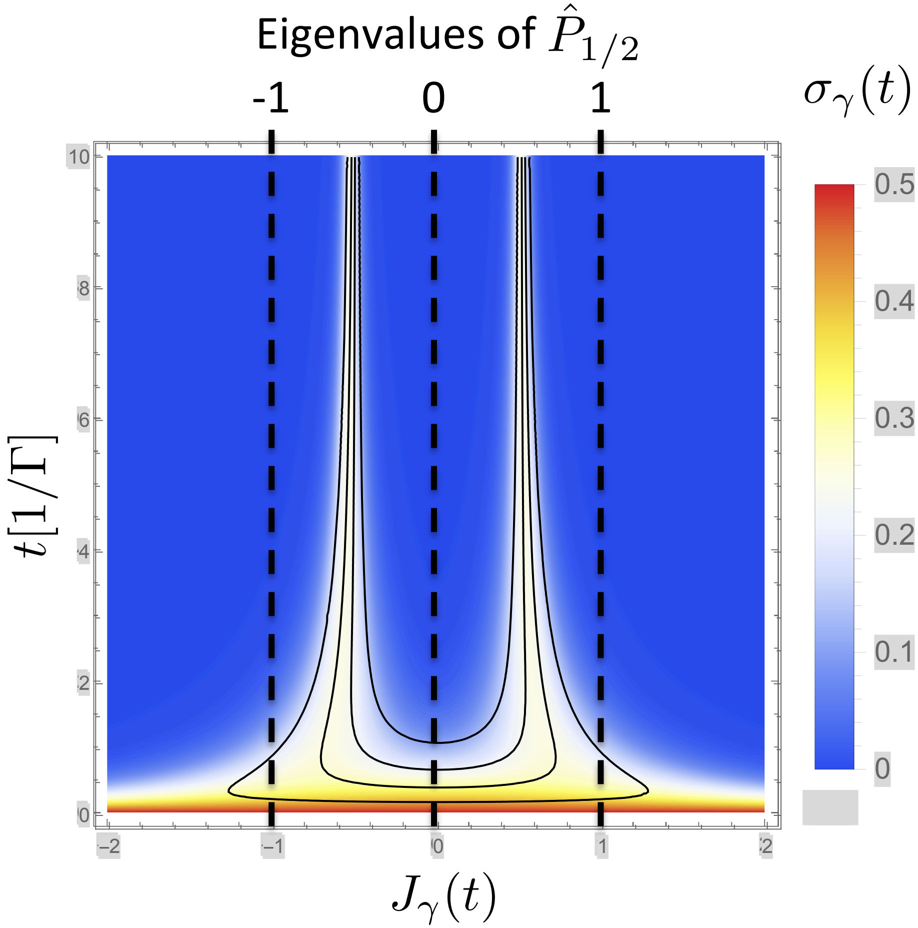
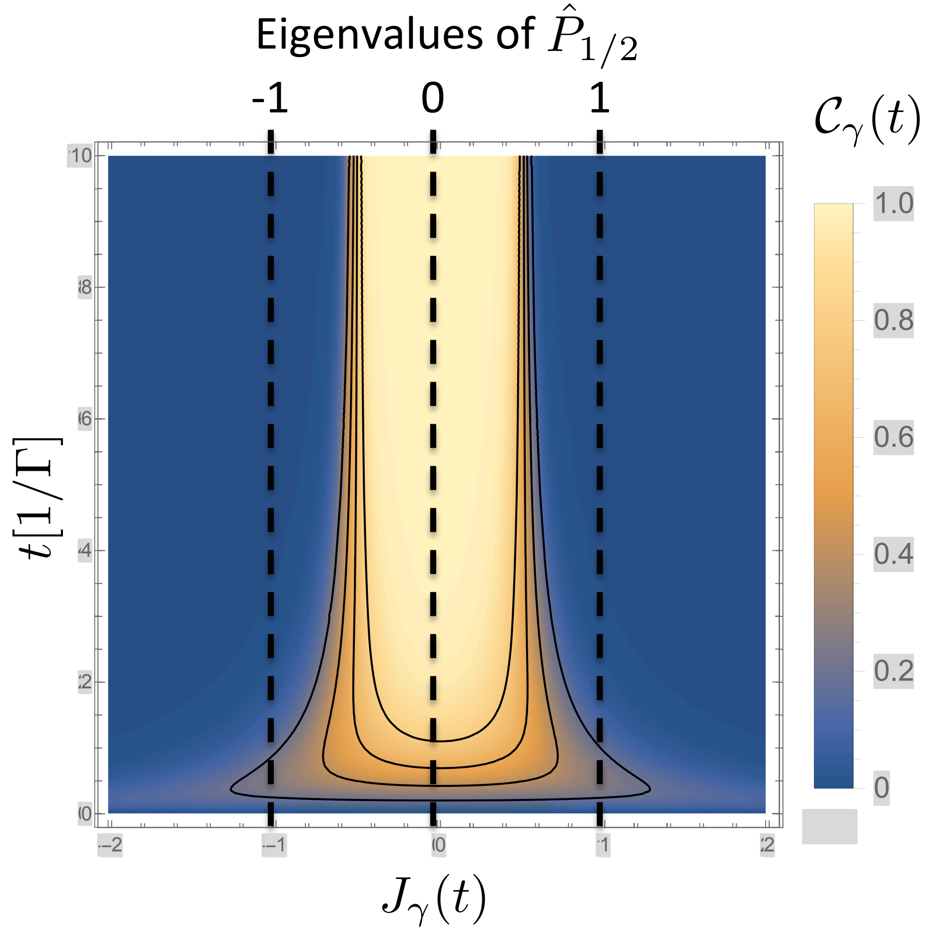
$$\sigma_\gamma(t) = \sqrt{\langle \delta Q_\gamma(t)^2 \rangle_{I_\gamma(t)}}$$

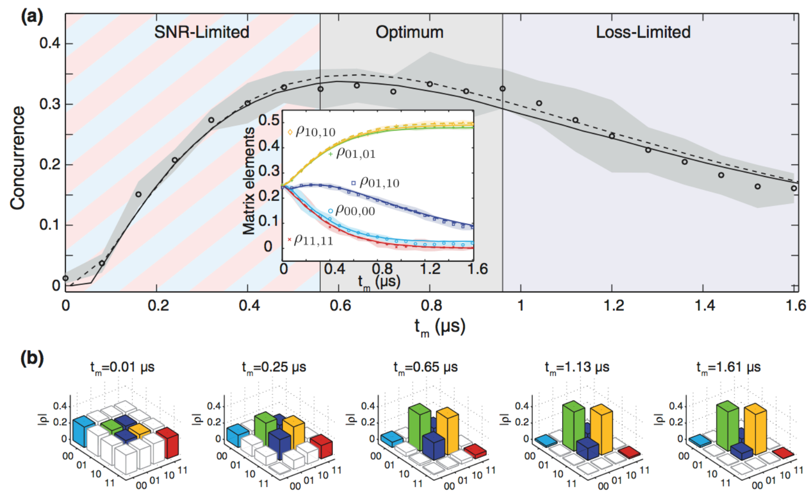
Energetic hallmark

$$\frac{d\mathcal{C}_\gamma(t)}{dt} \neq 0 \Leftrightarrow \sigma_\gamma(t) \neq 0$$





Outline



Energetic counterpart of measurement-induced entanglement genesis?
Can we derive energetic witnesses, an alternative to tomography?

I. Derivation of quantum trajectories induced by a half-parity measurement

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Single-trajectory energetic bounds

- Entanglement genesis rate $\left\langle \frac{d\mathcal{C}_\gamma(t)}{dt} \right\rangle_{I_\gamma(t)}$

$$\left\langle \frac{d\mathcal{C}_\gamma(t)}{dt} \right\rangle_{I_\gamma(t)} = 2\Gamma \frac{e^{-2\Gamma t} + e^{-2\Gamma t}\cosh(4\Gamma t J_\gamma)}{(1 + e^{-2\Gamma t}\cosh(4\Gamma t J_\gamma))^2} - 4\Gamma \frac{Q_\gamma}{\epsilon} \mathcal{C}_\gamma \langle \hat{P}_{1/2}(t) \rangle_\gamma$$

$$\frac{\sigma_\gamma(t)}{2\sqrt{\Gamma dt}} = \frac{e^{-4\Gamma t} + e^{-2\Gamma t}\cosh(4\Gamma t J_\gamma)}{(1 + e^{-2\Gamma t}\cosh(4\Gamma t J_\gamma))^2} . \qquad \mathcal{C}_\gamma(t) \in [0,1]$$

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- Energetic bounds for a given trajectory γ

$$2\Gamma \left[\frac{\sigma_\gamma(t)}{2\sqrt{\Gamma dt}} - 2\left(\frac{Q_\gamma(t)}{\epsilon} \right)^2 \right] \leq \left\langle \frac{d\mathcal{C}_\gamma(t)}{dt} \right\rangle_{I_\gamma(t)} \leq 4\Gamma \frac{\sigma_\gamma(t)}{2\sqrt{\Gamma dt}}$$

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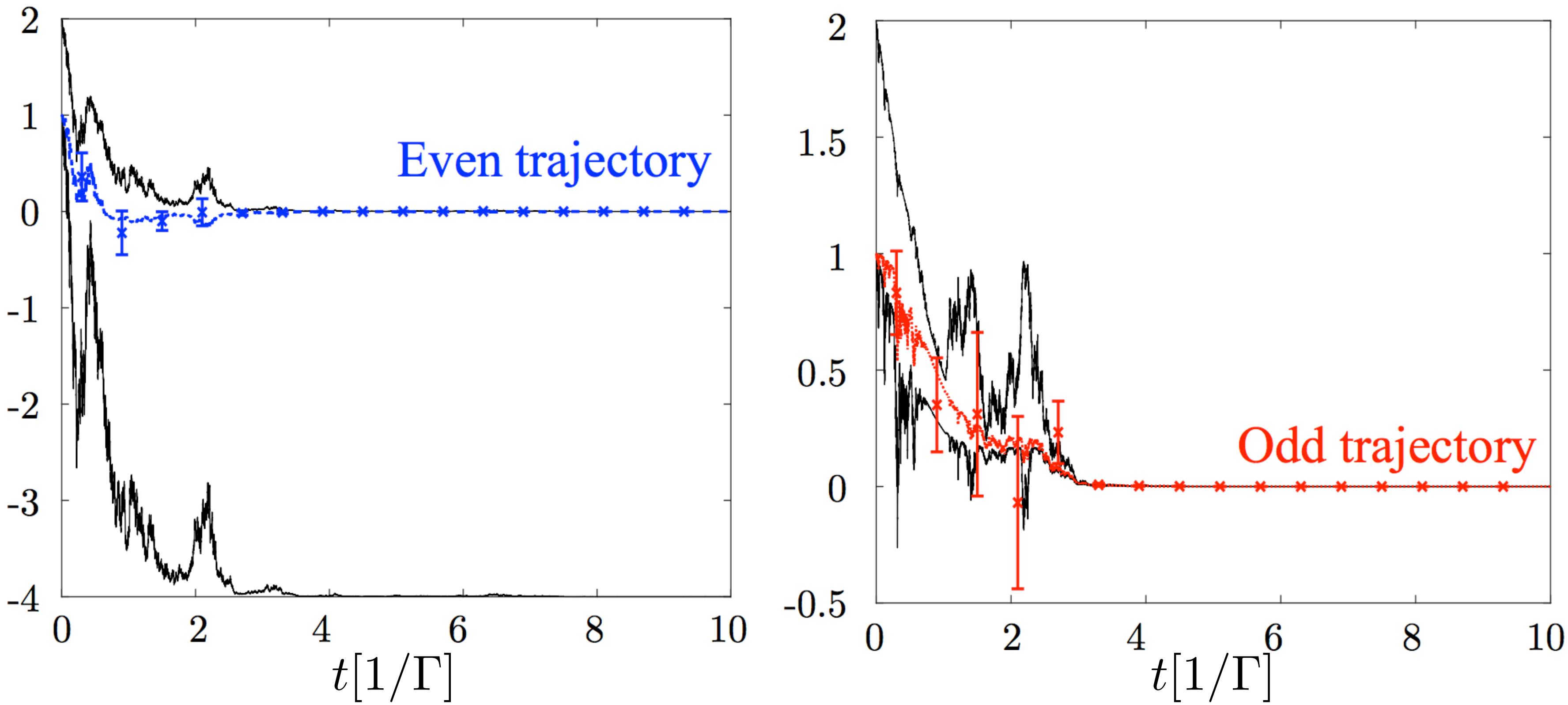
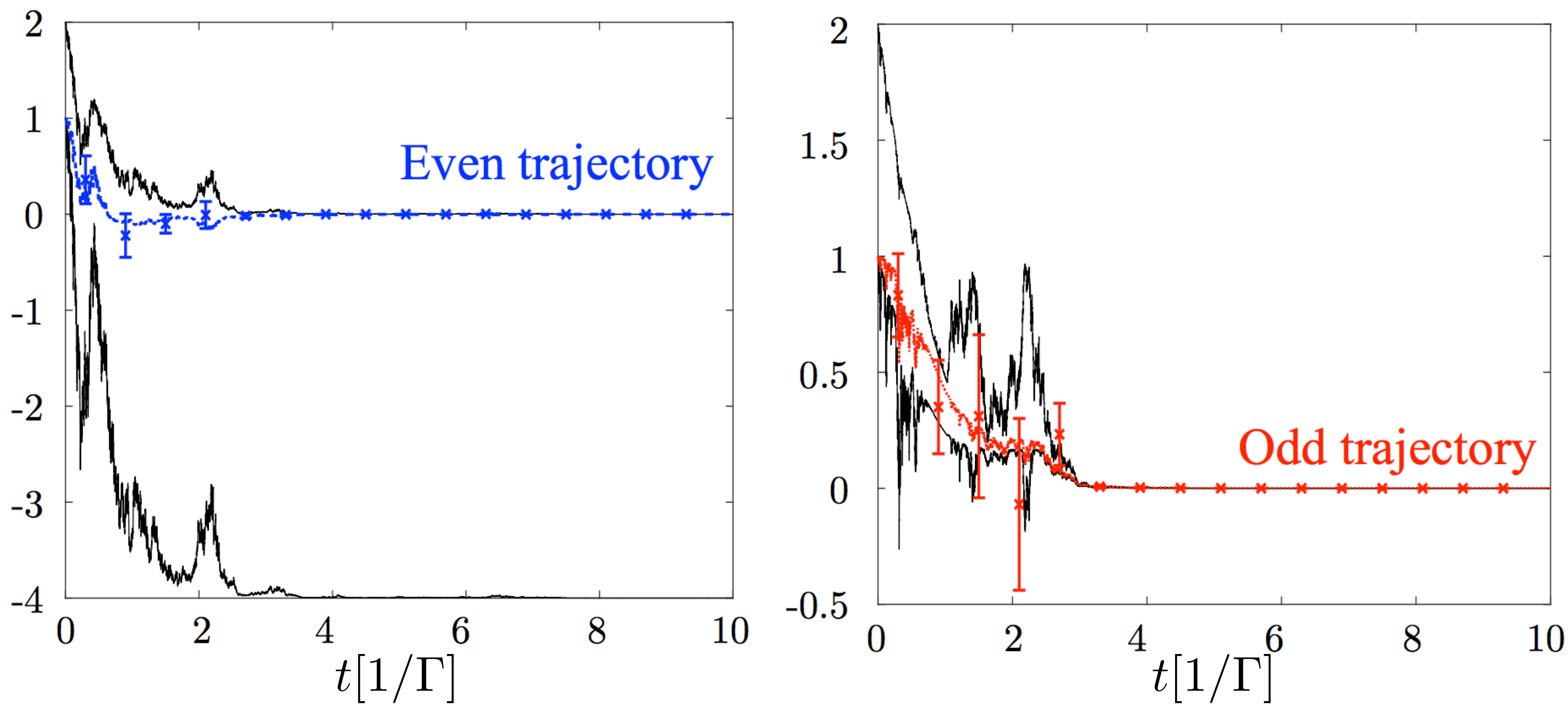


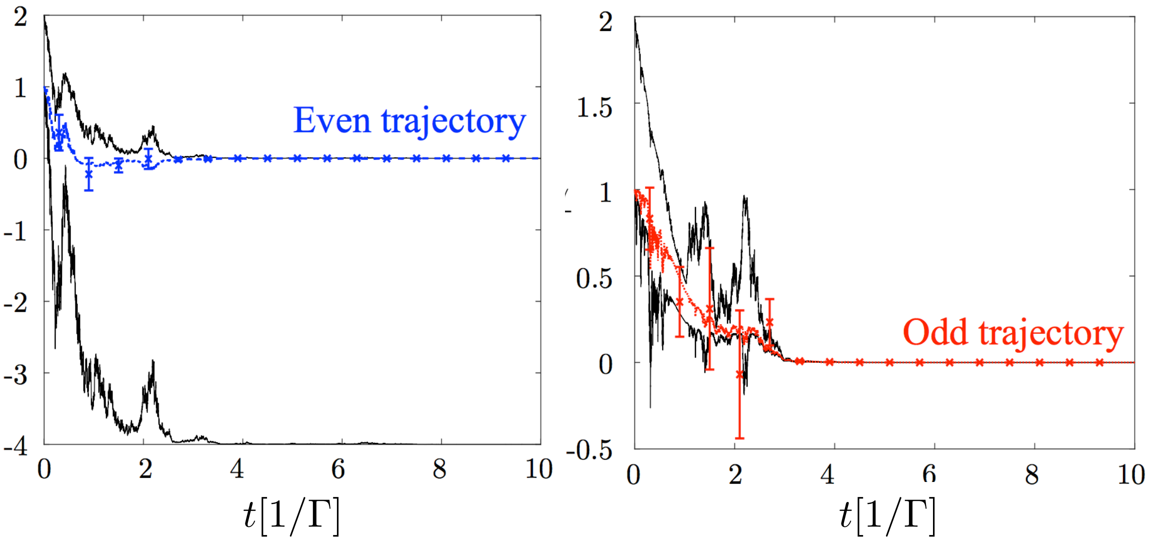
Energetic witness from the lower bound

$$2\Gamma\left[\frac{\sigma_\gamma(t)}{2\sqrt{\Gamma dt}} - 2\left(\frac{Q_\gamma(t)}{\epsilon}\right)^2\right] \leq \left\langle \frac{d\mathcal{C}_\gamma(t)}{dt} \right\rangle_{I_\gamma(t)} \leq 4\Gamma\frac{\sigma_\gamma(t)}{2\sqrt{\Gamma dt}}$$



$$\left\langle \frac{d\mathcal{C}_\gamma(t)}{dt} \right\rangle_{I_\gamma(t)} \geq 0 \quad \Rightarrow \quad \sigma_\gamma(t) - \frac{2Q_\gamma^2(t)}{\epsilon^2} \geq 0$$

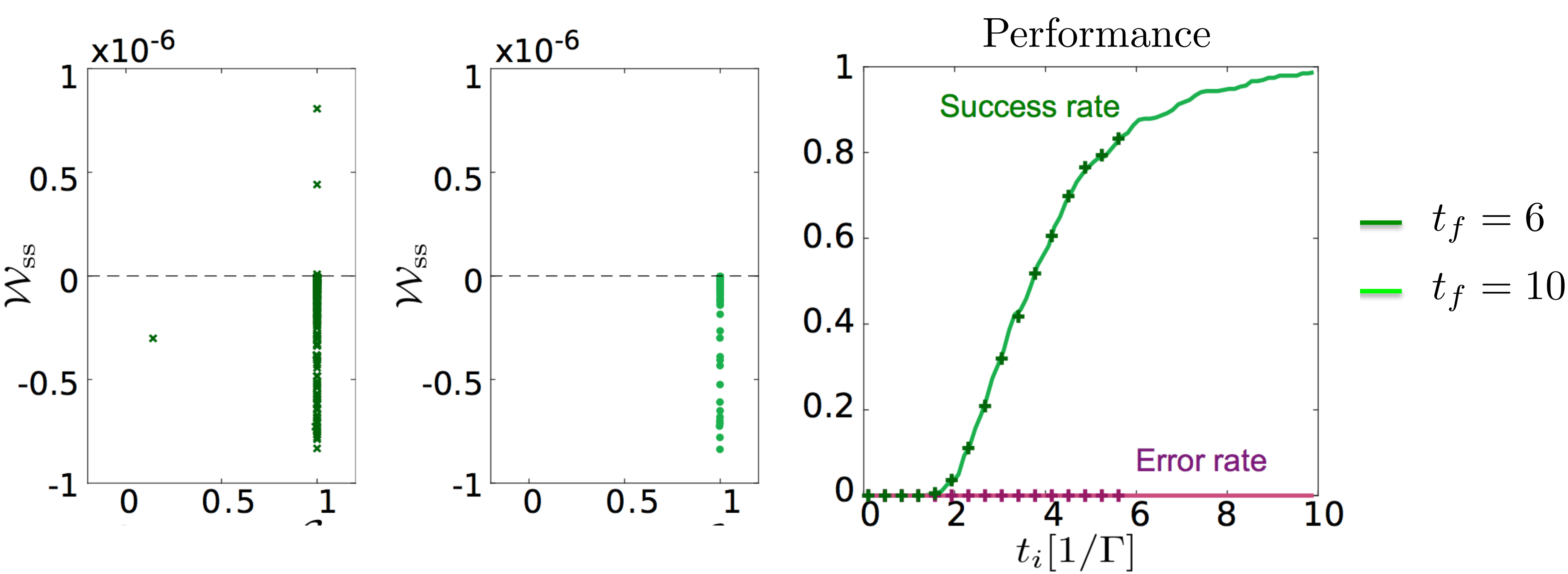
Single-shot energetic witness



$$\mathcal{W}_{ss} = \int_{t_i}^{t_f} \left(\frac{2Q_\gamma^2(t)}{\epsilon^2} - \delta Q_\gamma^2(t) \right) dt$$

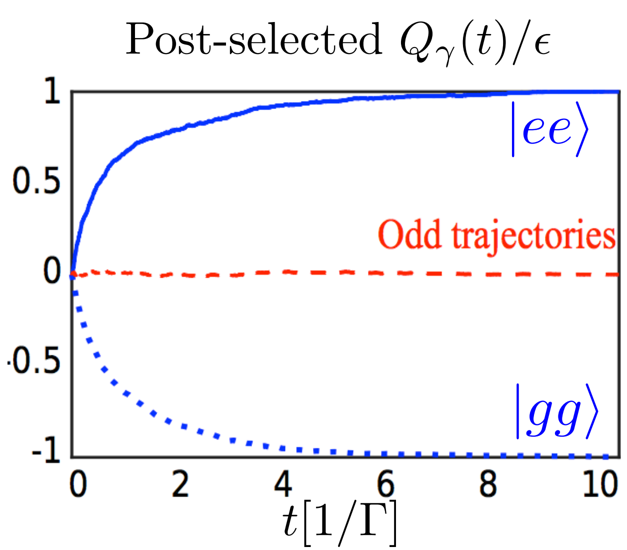
if $\mathcal{W} > 0$, no entanglement

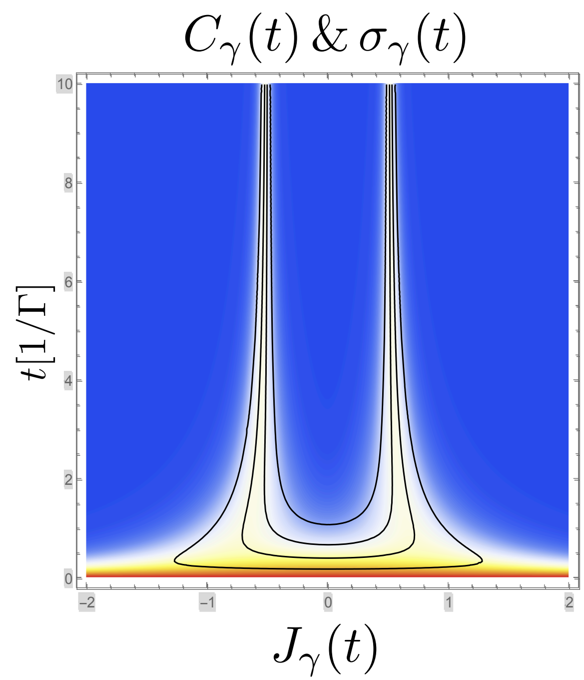
if $\mathcal{W} < 0$, entanglement

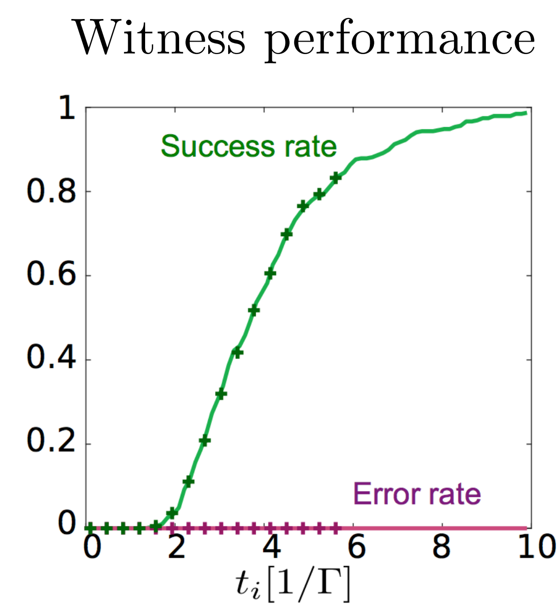


Conclusions & Outlooks

- Weak continuous measurement of the parity is a paradigmatic setup to investigate entanglement genesis
- Importance of the fluctuations to witness genuine quantum properties
- Model-dependent first evidence of an energetic hallmark of entanglement genesis
- Energetic witness: an alternative to tomography?







arXiv:1807.02487

Conclusions & Outlooks

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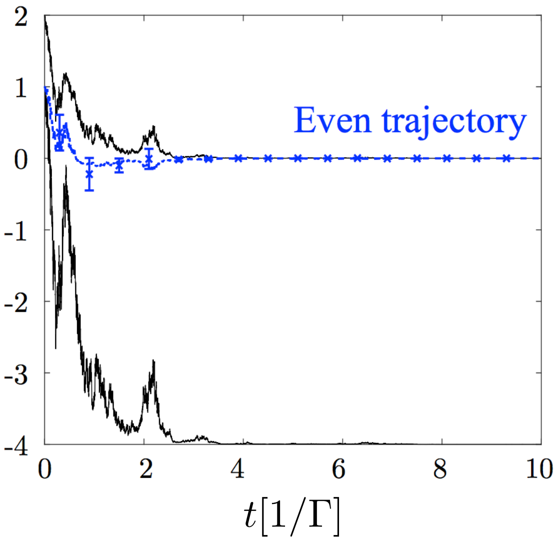
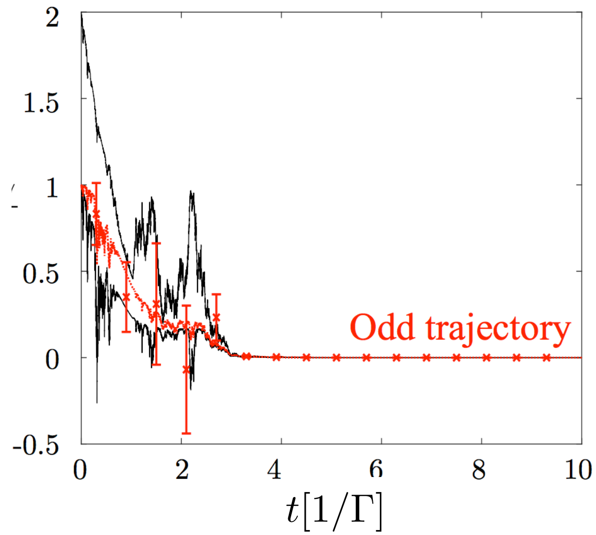
- Entropy consideration?
- Model-independent approach? Resource theory approach?
- Experiments

See talks from I. Siddiqi, B. Huard, K. Murch, F. Giazotto, ….

Energetic witness from the lower bound





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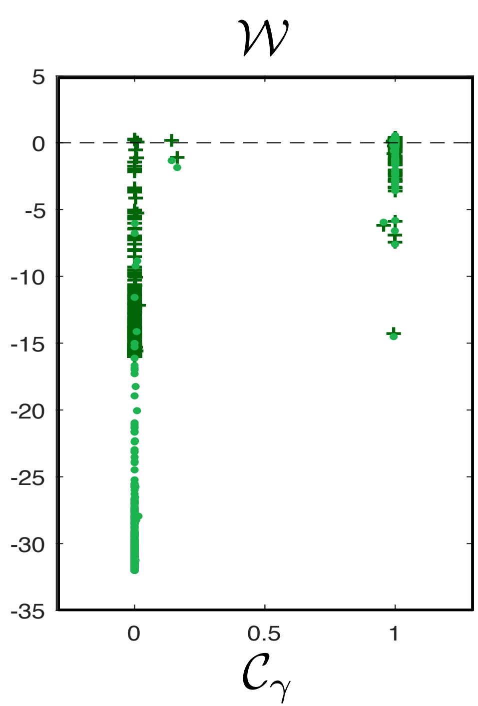
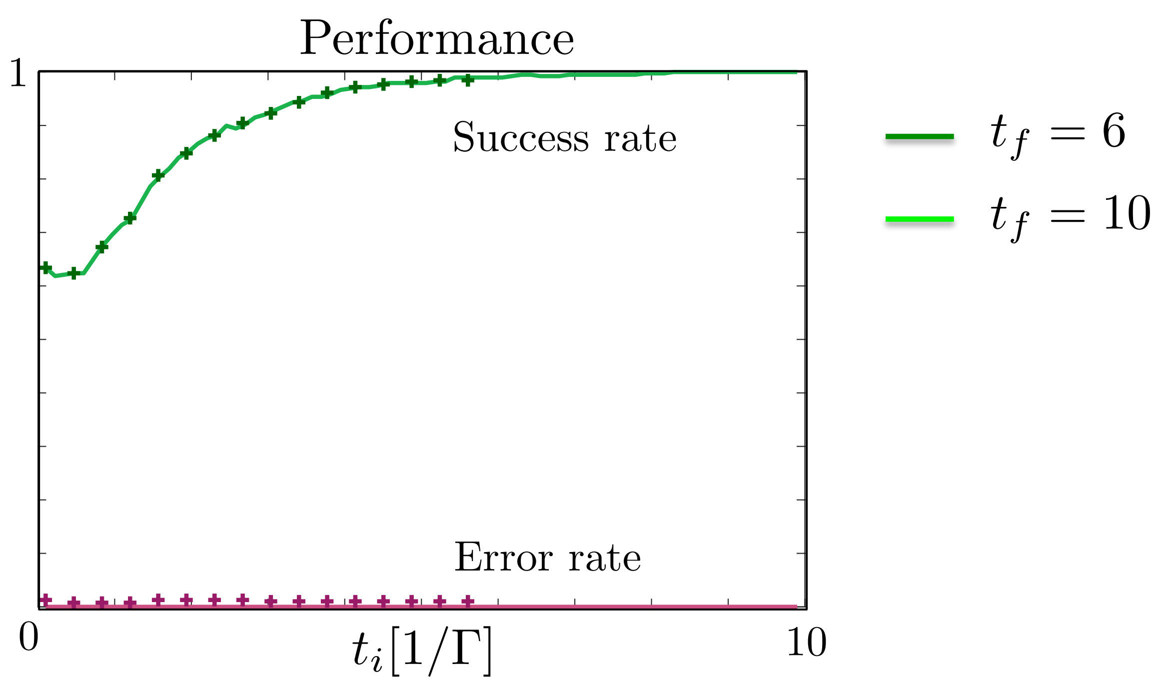
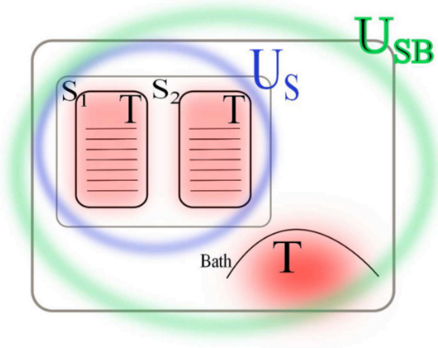


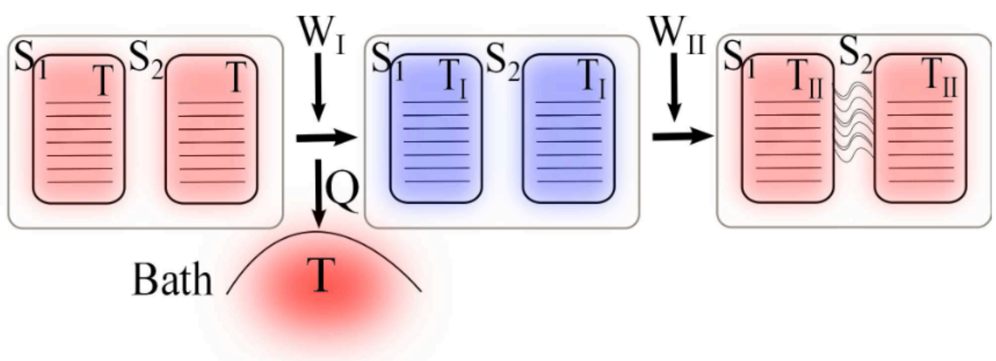


Quantum correlations vs. thermodynamics
"Thermodynamic cost of creating Q. correlations"

Resource theory approach



Huber et al., NJP 17 (2015)
Bruschi et al., PRE 91 (2015)
Friis et al., PRE 93 (2016)



Connection with realistic implementation of CPhase gate in Circuit QED!

Trajectories and infinitesimal heat exchange

- Stochastic Schrödinger equation

$$d|\psi_\gamma(t)\rangle = \left[-i\hat{H}_s dt - \frac{\Gamma}{2} dt(\hat{P}_{1/2} - \langle \hat{P}_{1/2}(t)\rangle_\gamma)^2 + \sqrt{\Gamma}\, dW_\gamma(t)(\hat{P}_{1/2} - \langle \hat{P}_{1/2}(t)\rangle_\gamma)\right] |\psi_\gamma(t)\rangle.$$

- Solution

$$|\psi(J_\gamma, t)\rangle = \frac{1}{N_\gamma(t)} \left(e^{-i\epsilon t - \Gamma\, t + \sqrt{\Gamma} J_\gamma(t)} |ee\rangle + |eg\rangle + e^{-i\epsilon t/2} |ge\rangle + e^{-i\Gamma\, t - \sqrt{\Gamma} J_\gamma(t)} |gg\rangle \right).$$

$$N_\gamma(t) = \sqrt{(1 + e^{-2\Gamma t} \cosh(4\Gamma t J_\gamma))/2}$$

- Infinitesimal heat exchange increment during time interval dt

$$\delta Q_\gamma(t) \equiv d\left(\langle\psi_\gamma(t)|\hat{H}_s|\psi_\gamma(t)\rangle\right)$$

$$= \left(d\langle\psi_\gamma(t)|\right)\hat{H}_s|\psi_\gamma(t)\rangle + \langle\psi_\gamma(t)|\hat{H}_s\left(d|\psi_\gamma(t)\rangle\right)$$

$$+\left(d\langle\psi_\gamma(t)|\right)\hat{H}_s\left(d|\psi_\gamma(t)\rangle\right)$$