# Quantum dynamics and correlations in disordered interacting systems

KITP June 2018

John Goold

Trinity College Dublin
Coláiste na Tríonóide, Baile Átha Cliath
The University of Dublin

# Talk Outline

**Part 0: Precursor - Background**

**Part 1: Total Correlations and integrability breaking**

**Part 2: Some results on transport both transient and steady state**

# Some background

## Thermalisation and equilibration in closed quantum systems


A. Polkovnikov, K. Sengupta, A. Silva, and M. Vengalattore, Rev. Mod. Phys. 83, 863 (2011)

J. Eisert, M. Friesdorf, and C. Gogolin, Nat. Phys. 11, 124 (2015)

F. Borgonovi, F. Izrailev, L. Santos and V. Zelevinsky, Phys. Rep 626 1 (2016)

L. D'Alessio, Y. Kafri, A. Polkovnikov, M. Rigol, Adv. Phys, 65, 239 (2016)


## Many-body localisation


R. Nandkishore and D. Huse, Ann. Rev. Cond. Mat. Phys 6 15 (2015)

E. Altman and R. Vosk, Annu. Rev. Condens. Matter Phys. 6 383 (2015)

D. A. Abanin, E. Altman, I. Bloch, M. Serbyn https://arxiv.org/abs/1804.11065

# Ergodicity

$$\lim_{T \to \infty} \frac{1}{T} \int_0^T dt \delta(\mathcal{X} - \mathcal{X}(t)) = \rho_{mc}(E)$$

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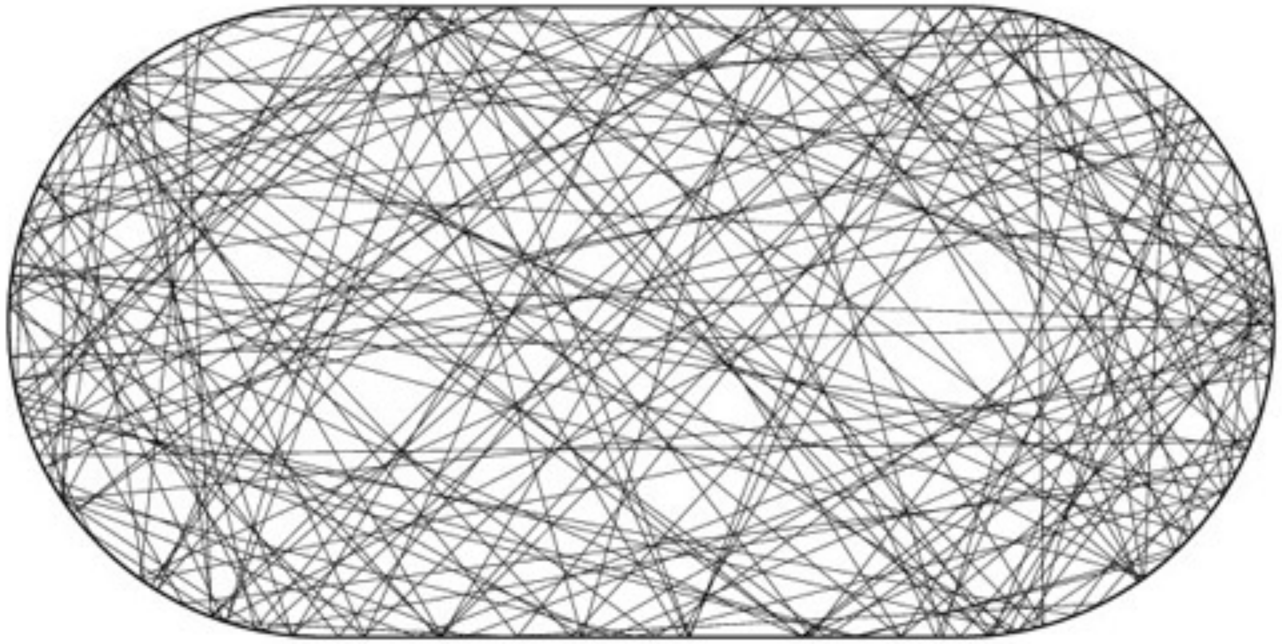

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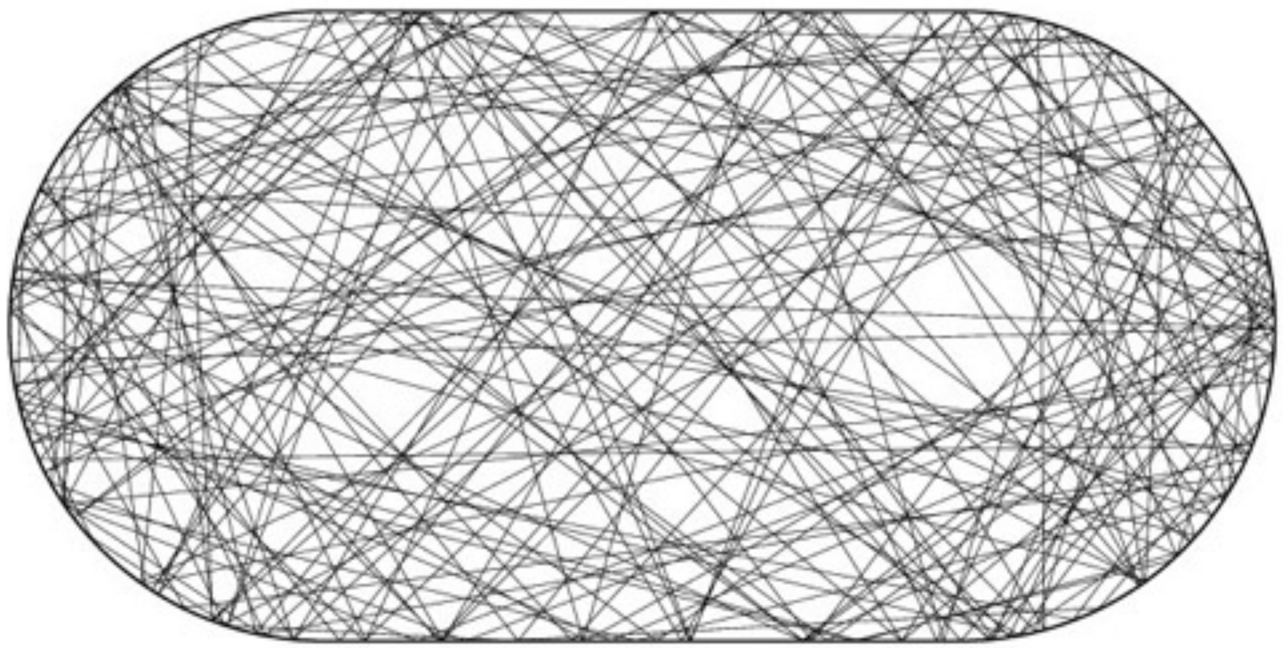

$\mathcal{X}(t)$

phase space trajectory

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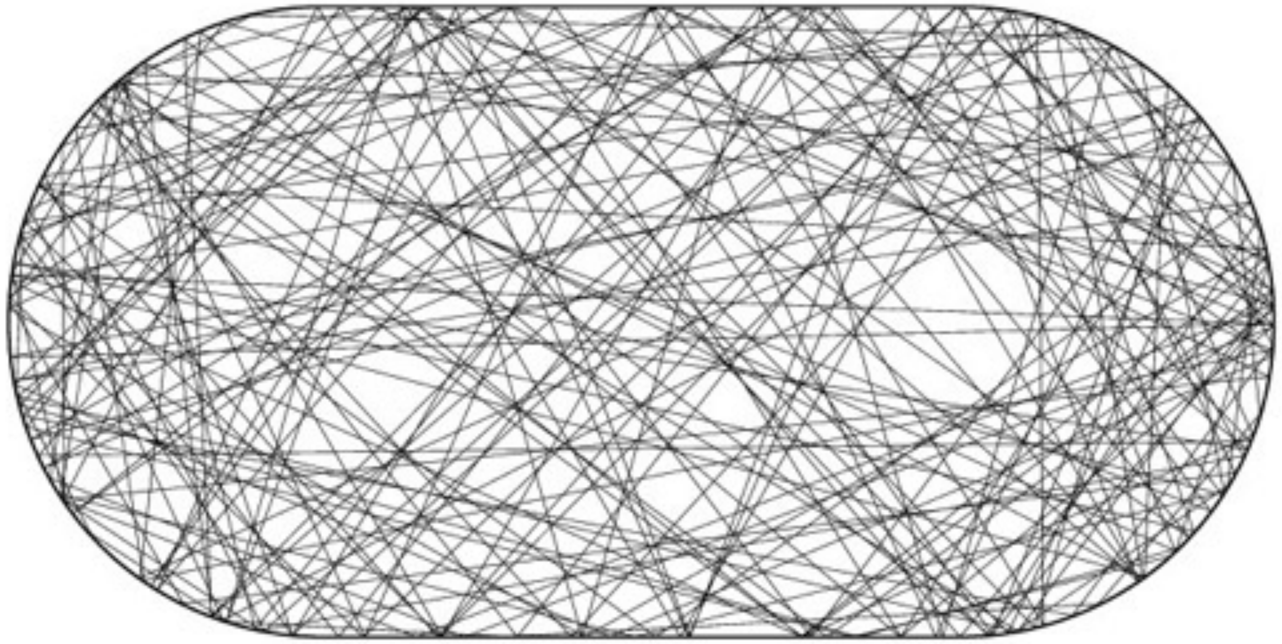
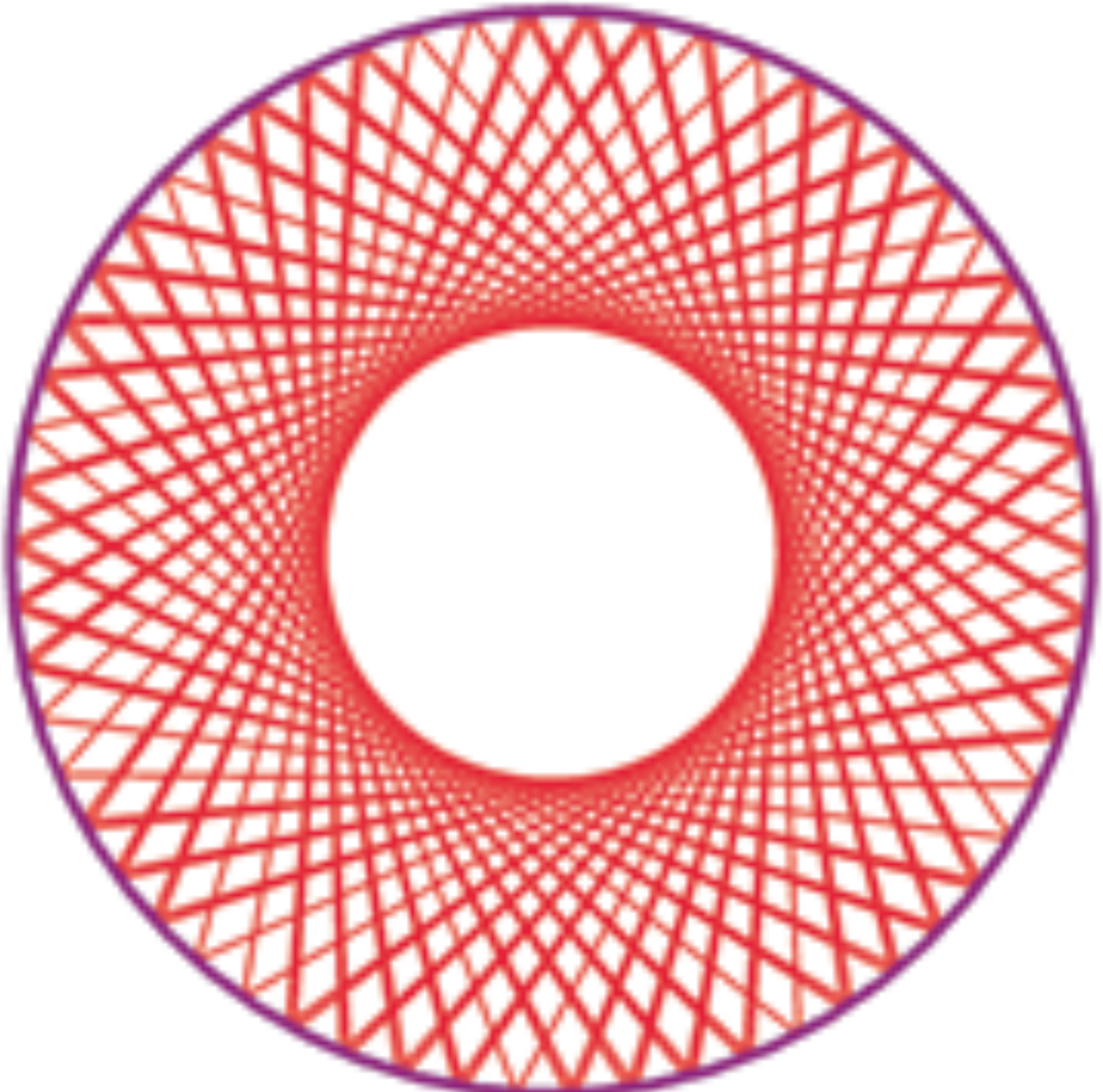


$\mathcal{X}(t)$

phase space trajectory

**uniformly covers all energy surface for all initial conditions**



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# Quantum Ergodicity

Let $\mathcal{H}(E)$ be set of eigenstates with energies in $[E, E+\delta E]$

$$\rho_{mc}(E) = \sum_{\alpha \in \mathcal{H}(E)} \frac{1}{\mathcal{N}} |\phi_\alpha\rangle\langle\phi_\alpha|$$

Given an initial state made out of states in the microcanonical energy shell
- when will long time average look like MC ensemble ?

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Given an initial state made out of states in the microcanonical energy shell
- when will long time average look like MC ensemble ?

$$|\Psi_0\rangle = \sum_{\alpha \in \mathcal{H}(E)} c_\alpha |\phi_\alpha\rangle \quad \Rightarrow \quad \omega = \sum_\alpha |c_\alpha|^2 |\phi_\alpha\rangle\langle\phi_\alpha|$$

**Initial state** → **long time-average (diagonal ensemble)**

# Equilibration

$$|\psi(t)\rangle = \sum_m C_m e^{-iE_m t}|m\rangle \qquad \hat{H}|m\rangle = E_m|m\rangle \qquad C_m = \langle m|\psi_0\rangle$$

Generic time evolution of observable:

$$O(t) = \langle\psi(t)|\hat{O}|\psi(t)\rangle = \sum_m |C_m|^2 O_{mm} + \sum_{m,n\neq m} C_m^* C_n e^{i(E_m - E_n)t} O_{mn}$$

Long time limit $\quad O(t\to\infty) \approx \sum_m |C_m|^2 O_{mm}$ Diagonal ensemble

# Eigenstate Thermalisation Hypothesis

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$$\langle n|\hat{O}|m\rangle = O(\bar{E})\delta_{nm} + e^{-S(\bar{E})/2} R_{nm} f(\omega, \bar{E})$$

Deutsch, Srednicki, Rigol & many authors

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Deutsch, Srednicki, Rigol & many authors

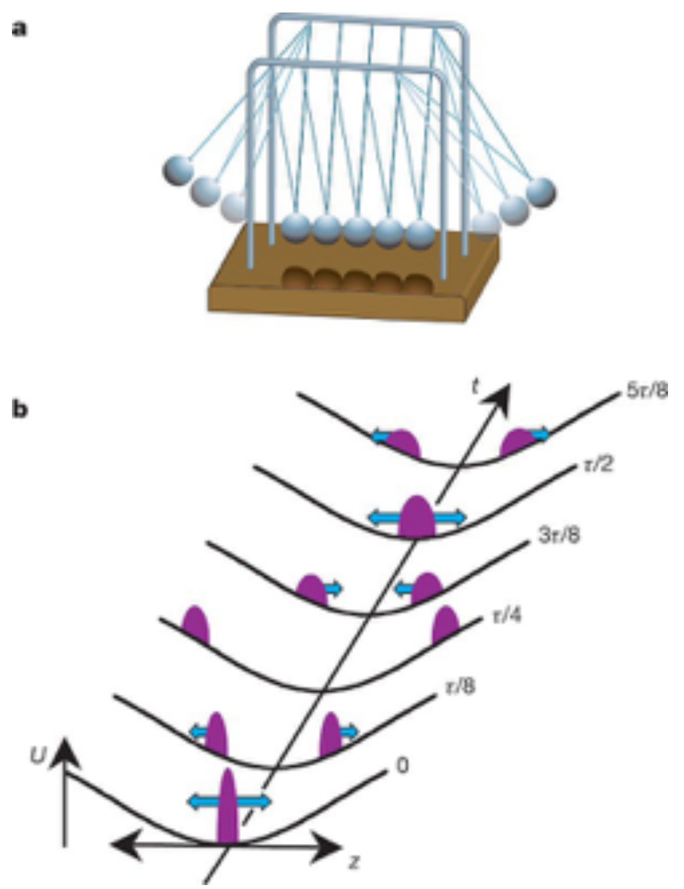
$$\langle n|\hat{O}|n\rangle \approx \langle m|\hat{O}|m\rangle \approx O(E) \qquad \langle n|\hat{O}|m\rangle \to 0$$

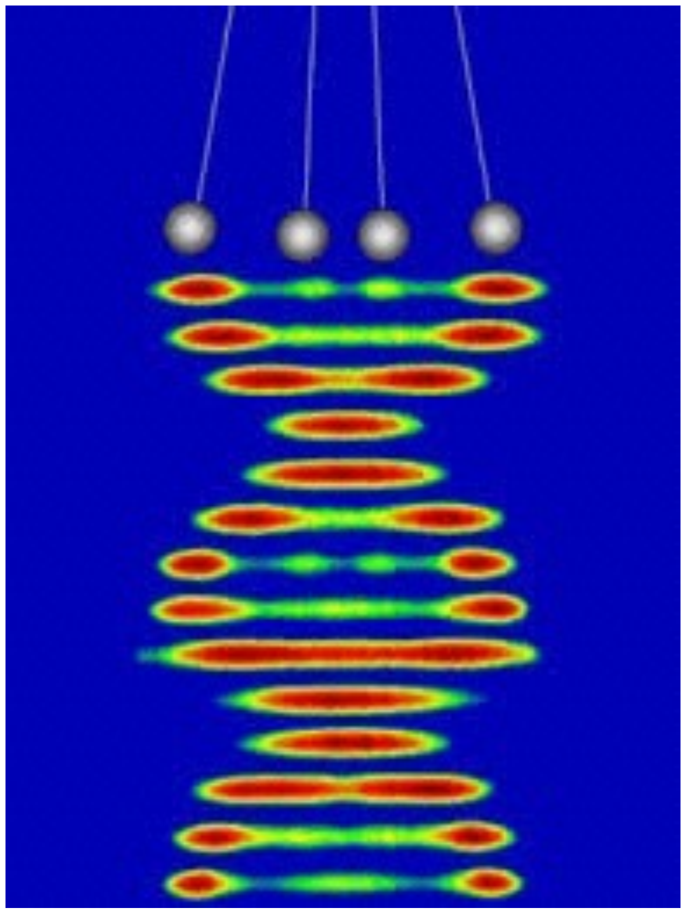
$$\langle \mathcal{O}(t \to \infty)\rangle = \mathcal{O}(E) = \mathcal{O}(T)$$

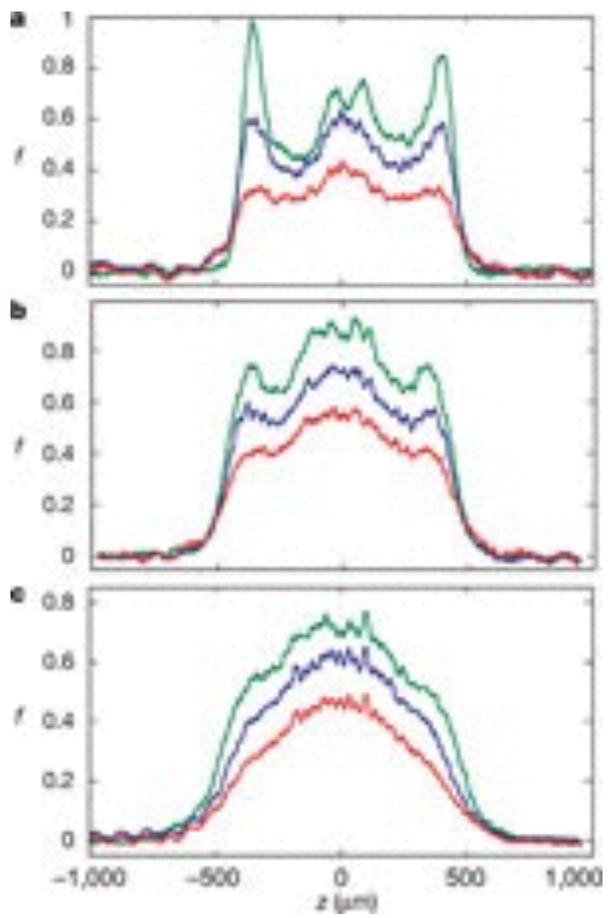
$$E = \langle \Psi_0|H|\Psi_0\rangle$$

$$E = \langle H\rangle_T$$

# Non-ergodic systems







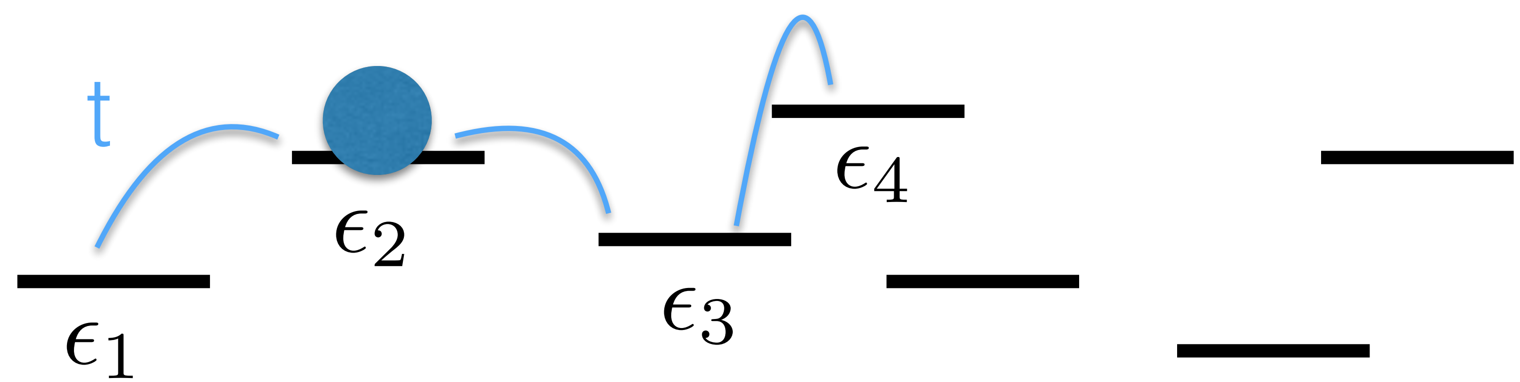
A quantum newtons cradle
Weiss Group 2006

No thermalisation !

# Anderson Localization

$$\epsilon_i \in [-\epsilon \;\; \epsilon]$$

One quantum particle
in 1D disordered crystal



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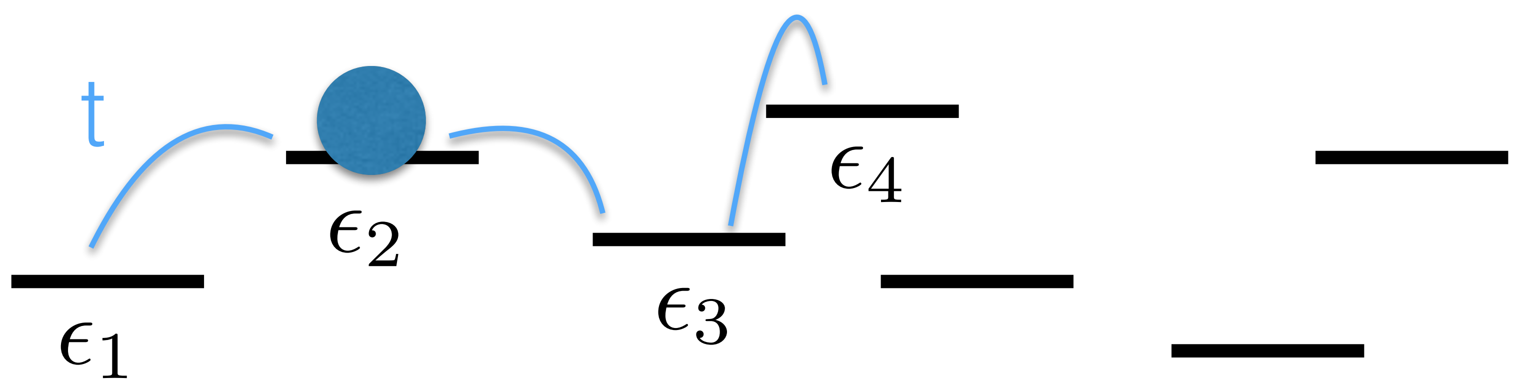


Anderson Hamiltonian

$$H = -t\sum_i (c_i^\dagger c_{i+1} + h.c) + \sum_i \epsilon_i c_i^\dagger c_i$$

PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

**Absence of Diffusion in Certain Random Lattices**

P. W. ANDERSON
*Bell Telephone Laboratories, Murray Hill, New Jersey*
(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.

# Anderson Localization

$$\epsilon_i \in [-\epsilon \ \epsilon]$$

## LOCAL MOMENTS AND LOCALIZED STATES

Nobel Lecture, 8 December, 1977

by
PHILIP W. ANDERSON
Bell Telephone Laboratories, Inc, Murray Hill, New Jersey, and Princeton University, Princeton, New Jersey, USA

I was cited for work both. in the field of magnetism and in that of disordered systems, and I would like to describe here one development in each held which was specifically mentioned in that citation. The two theories I will discuss differed sharply in some ways. The theory of local moments in metals was, in a sense, easy: it was the condensation into a simple mathematical model of ideas which. were very much in the air at the time, and it had rapid and permanent acceptance because of its timeliness and its relative simplicity. What mathematical difficulty it contained has been almost fully- cleared up within the past few years.

Localization was a different matter: very few believed it at the time, and even fewer saw its importance; among those who failed to fully understand it at first was certainly its author. It has yet to receive adequate mathematical treatment, and one has to resort to the indignity of numerical simulations to settle even the simplest questions about it. Only now, and through primarily Sir Nevill Mott's efforts, is it beginning to gain general acceptance.


1958

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# Are interactions relevant ?

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$$H = -t\sum_i (c_i^\dagger c_{i+1} + h.c) + \sum_i \epsilon_i c_i^\dagger c_i \; + \lambda \sum_{i,j} v(|i-j|) n_i n_j$$

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18 *D. M. Basko, I. L. Aleiner, and B. L. Altshuler*

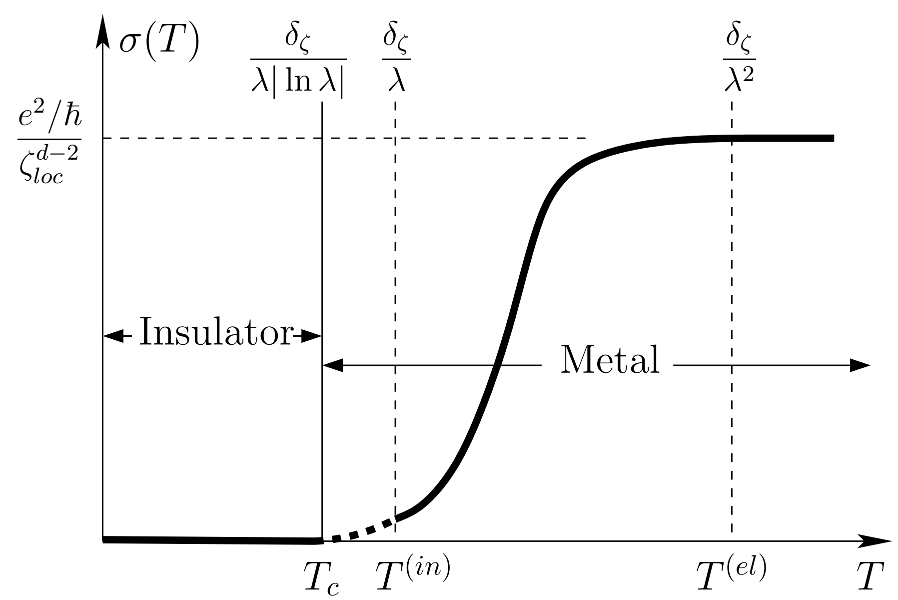



FIG. 5. Schematic temperature dependence of the dc conductivity $\sigma(T)$. Below the point of the many-body metal-insulator transition, $T < T_c$, no inelastic relaxation occurs and $\sigma(T) = 0$. Temperature interval $T \gg T^{(in)} > T_c$ corresponds to the developed metallic phase, where Eq. (21) is valid. At $T \gg T^{(el)}$ the high-temperature metallic perturbation theory (Altshuler and Aronov, 1985) is valid, and the conductivity is given by the Drude formula.

insulator) or all excitations (band insulator). In both cases the conductivity re-

*Basko, Aleiner and Altshuler*

**Annals of Physics 321, 1126 (2006)**

Transition to localized phase!

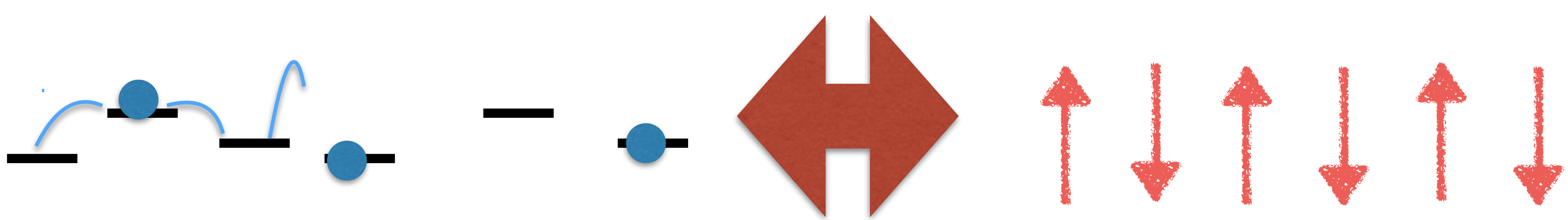
**MBL**

# Toy Model

$$H_{XXZ} = \sum_{i=1}^{N-1} \left[\alpha\left(\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y\right) + \Delta \sigma_i^z \sigma_{i+1}^z\right]$$

The general Hamiltonian can be mapped into a model of spinless fermions by means of the Jordan-Wigner transformation

$$= 4 \sum_{i=1}^{N-1} \left[\frac{\alpha}{2}\left(c_i^\dagger c_{i+1} + c_i c_{i+1}^\dagger\right) + \Delta\left(n_i - \frac{1}{2}\right)\left(n_{i+1} - \frac{1}{2}\right)\right]$$



# XXZ

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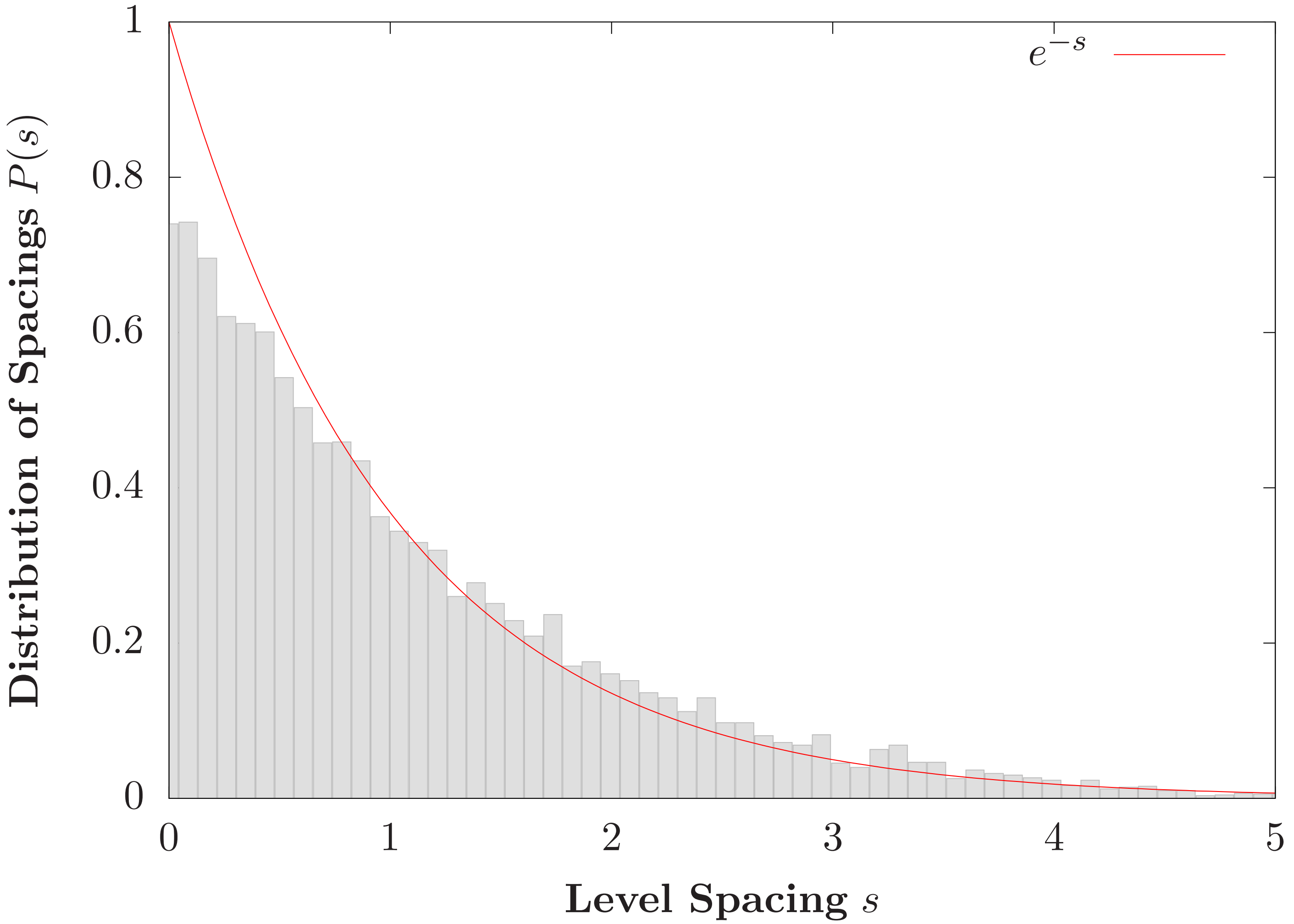



**Distribution of spacings, s, of neighbouring energy levels P(s)**

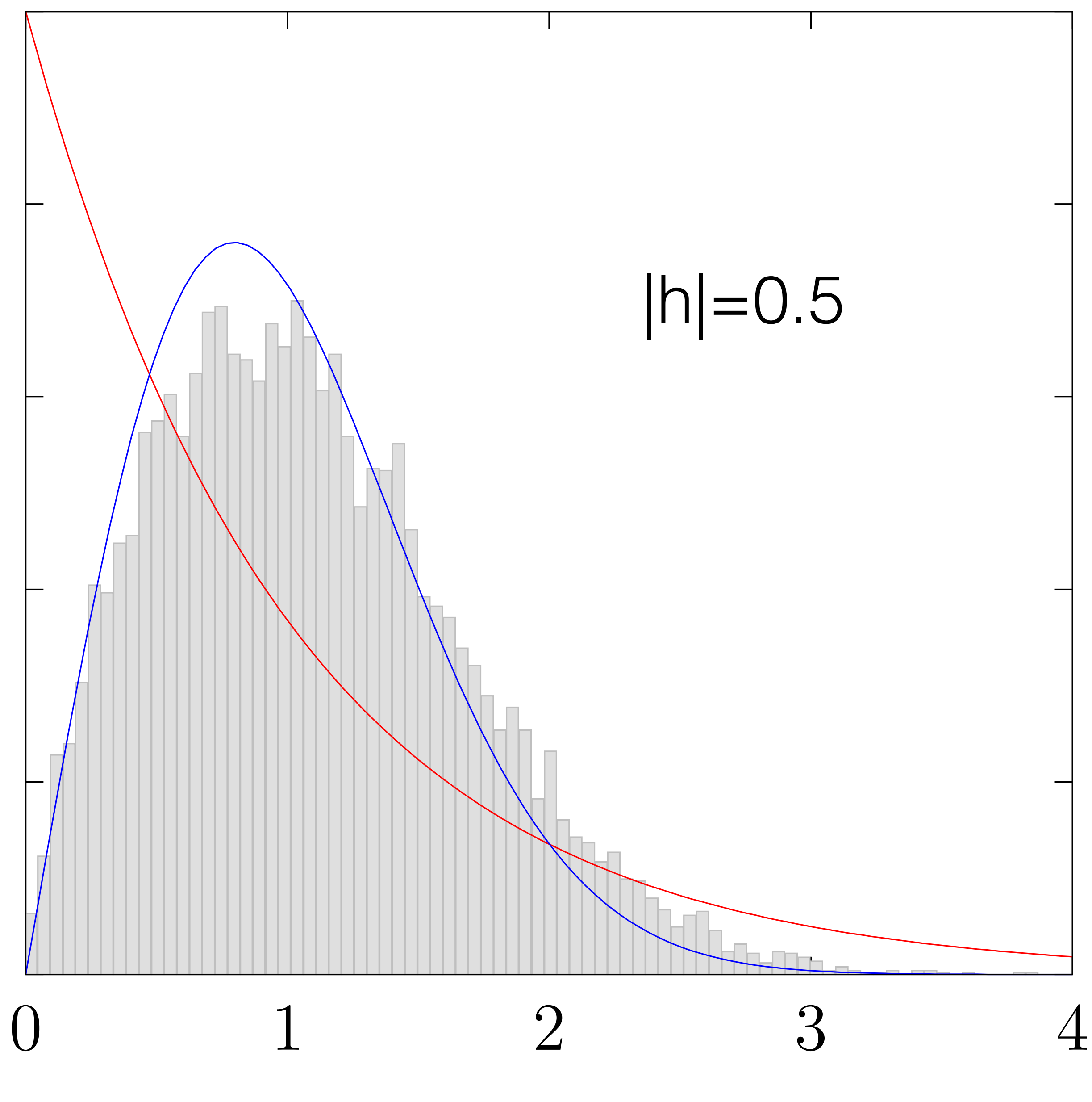
**Hallmark of integrable systems is that levels are not correlated and not prohibited from crossing**

$$s_n = (E_{n+1} - E_n)/\Omega, \ \Omega : \text{Average level spacing}$$

# XXZ +DISORDER

$$H_{XXZ} = \sum_{i=1}^{N-1} \left[\alpha\left(\sigma_i^x\sigma_{i+1}^x + \sigma_i^y\sigma_{i+1}^y\right) + \Delta\sigma_i^z\sigma_{i+1}^z\right] + \sum_i h_i\sigma_i^z$$

*weak disorder* $h_i \in [-h\ h]$




$$P(s) = \frac{\pi s}{2} e^{\frac{-\pi s^2}{4}}$$

Wigner dyson

Level repulsion!

$$s_n = (E_{n+1} - E_n)/\Omega,\ \Omega : \text{Average level spacing}$$

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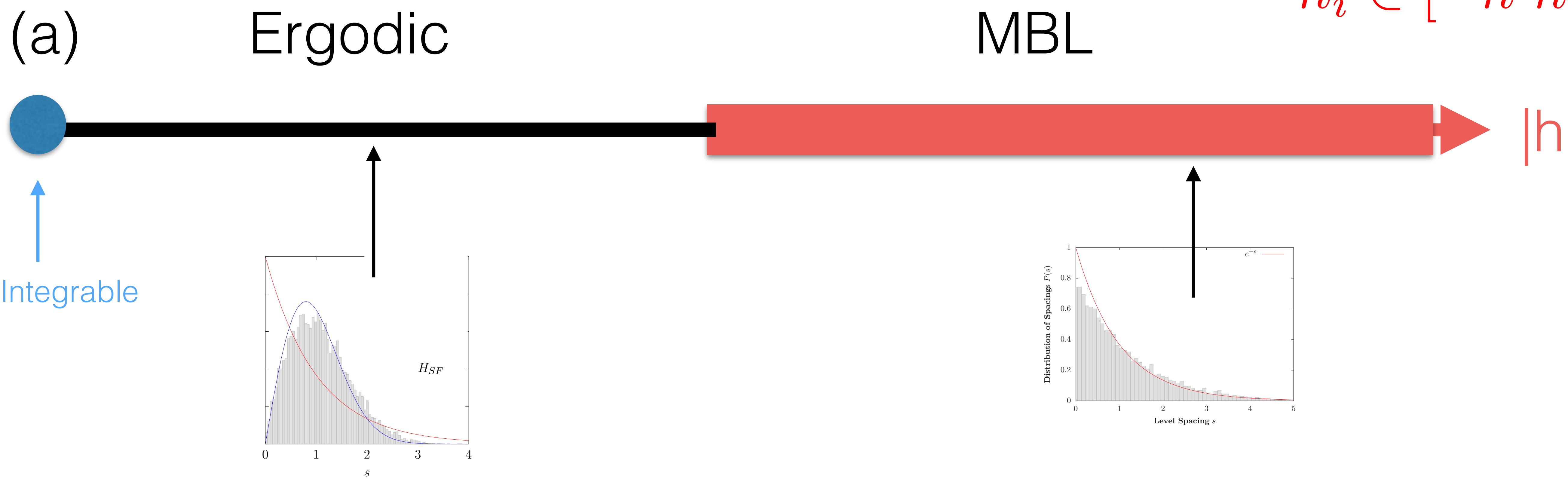

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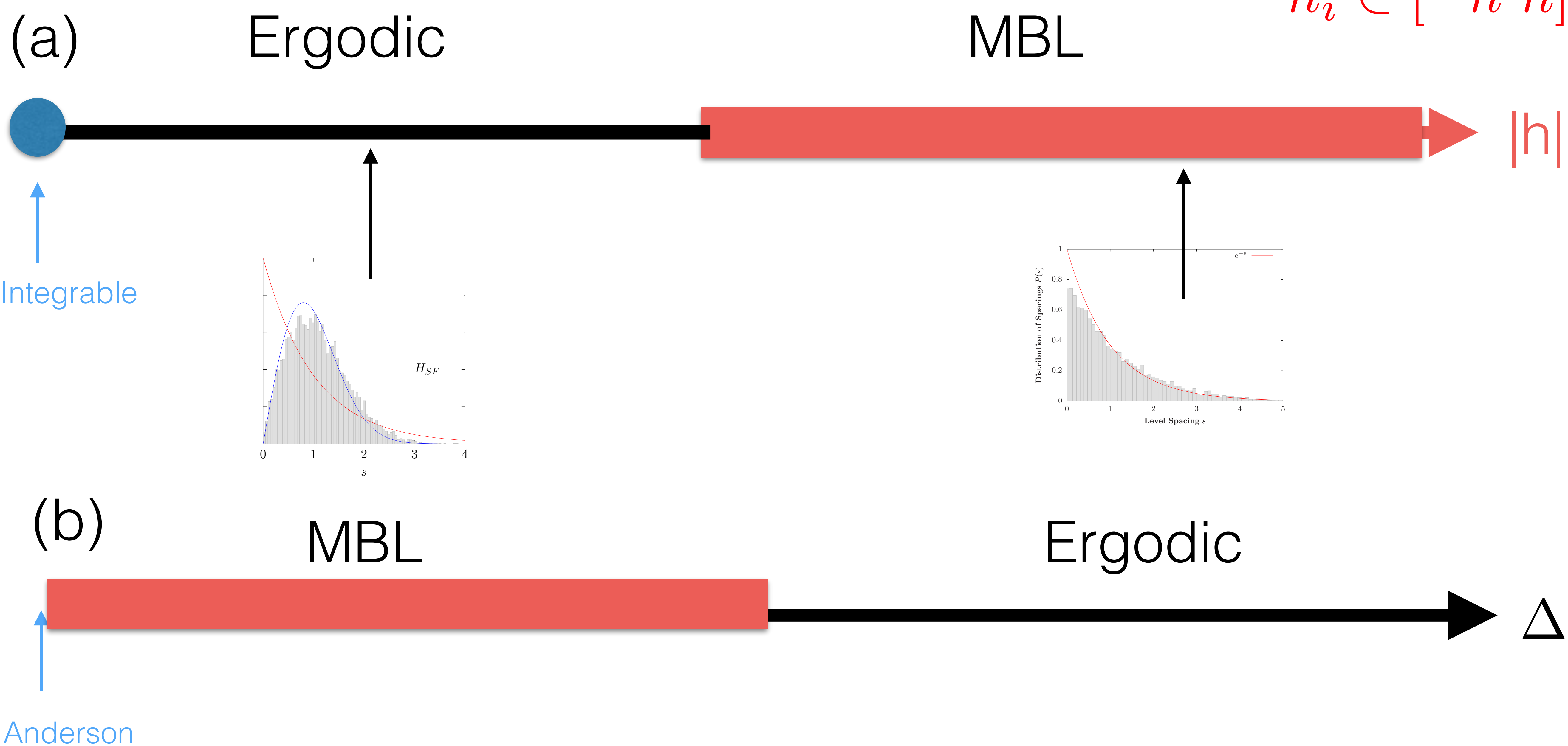

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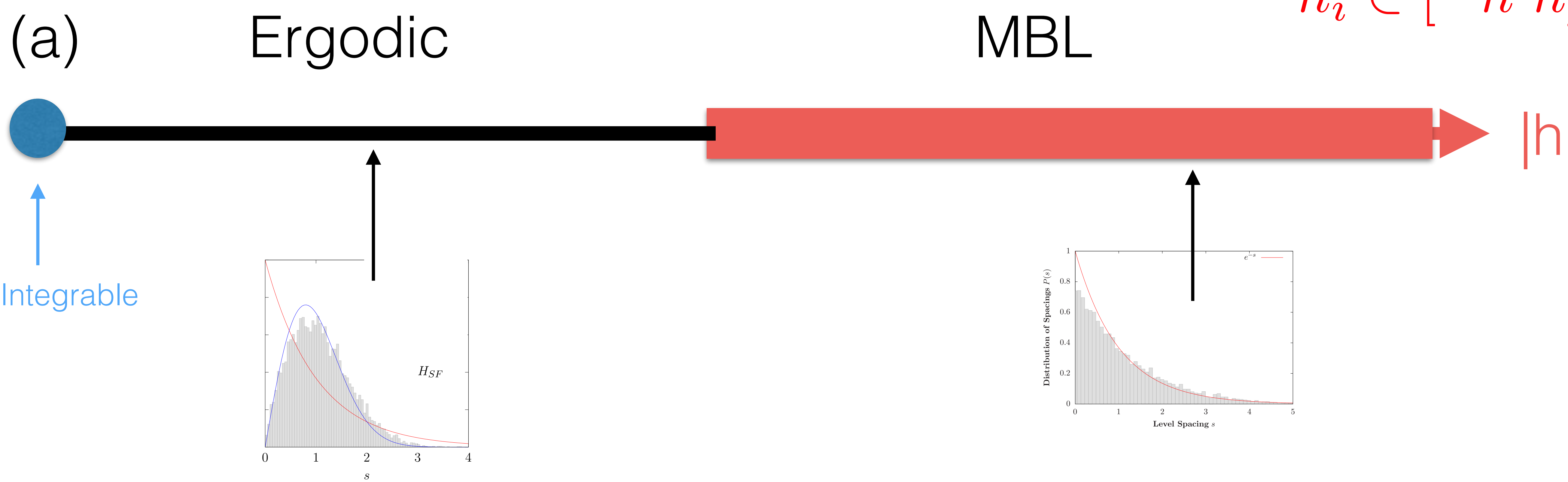

# Entanglement Entropy and ETH

$$H = \sum_i \sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \sigma_i^z \sigma_{i+1}^z + \sum_i h_i \sigma_i^z$$

$$H(|h|) = \sum E_n(h) |E_n(h)\rangle\langle E_n(h)|$$

$$\rho_A(h) = Tr_B(|E_n\rangle\langle E_n|)$$


Compute entanglement entropy on eigenstates !

Systems fulfilling eigenstate thermalisation hypothesis would generically have volume law scaling of entanglement entropy

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$S \propto L^d$ Ergodic Volume law

$S \propto L^{d-1}$ MBL area law

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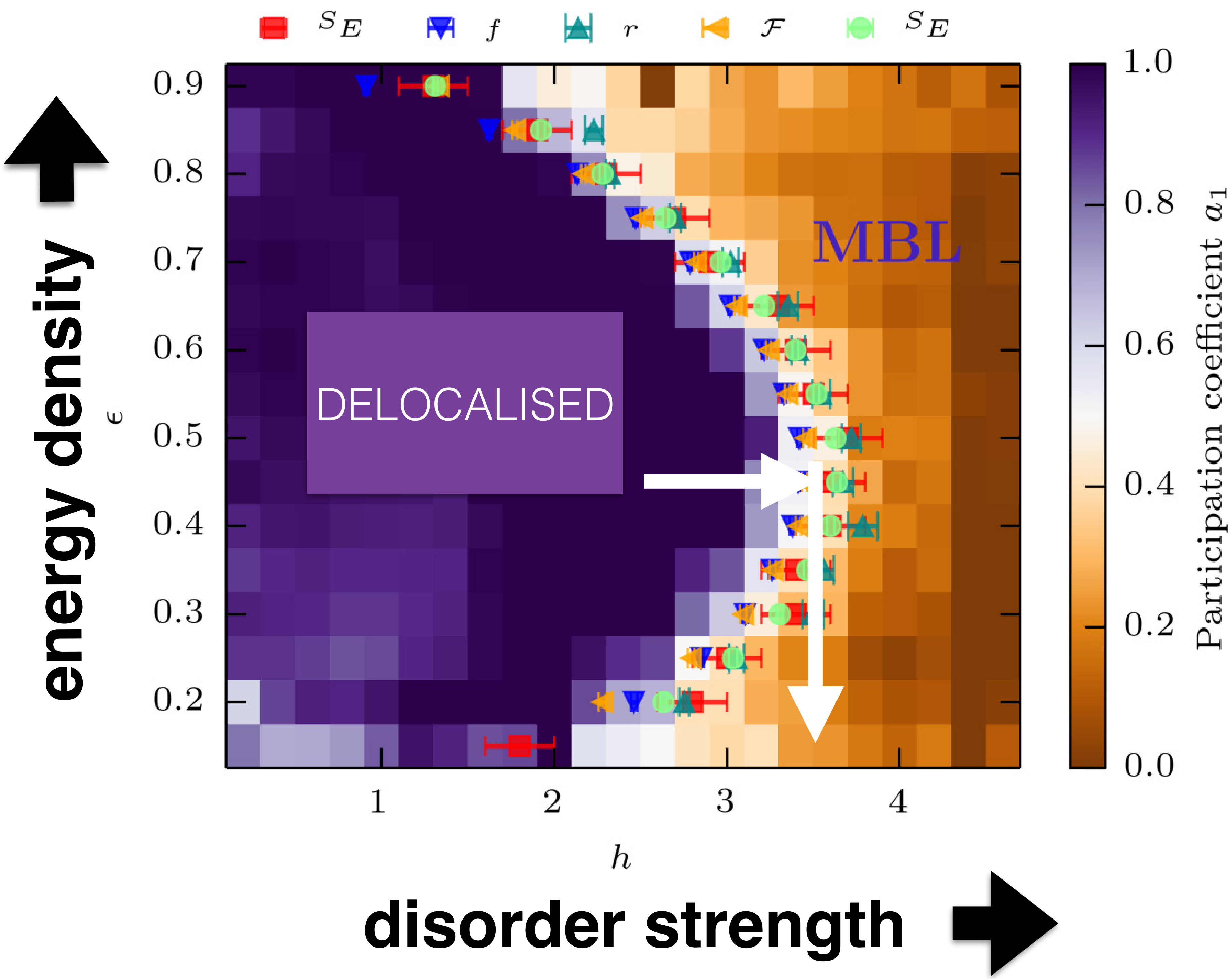


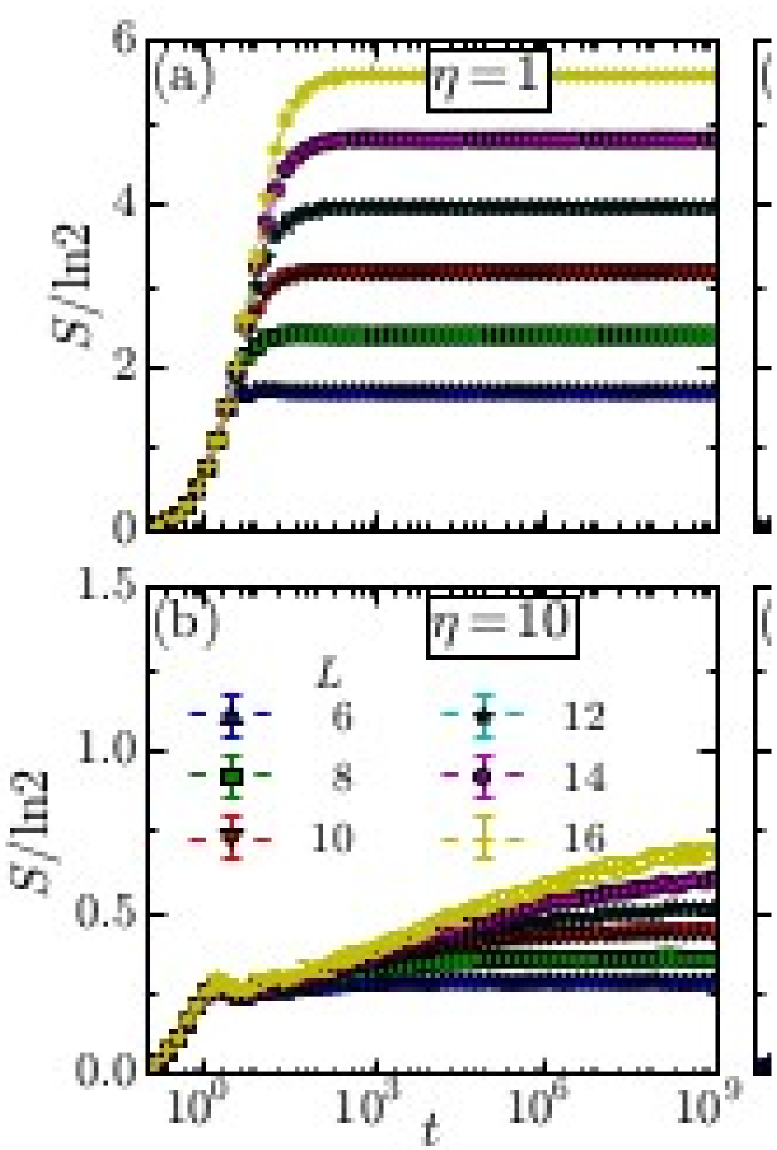

David J. Luitz, Nicolas Laflorencie, and Fabien Alet Phys. Rev. B 91, 081103(R) (2015)

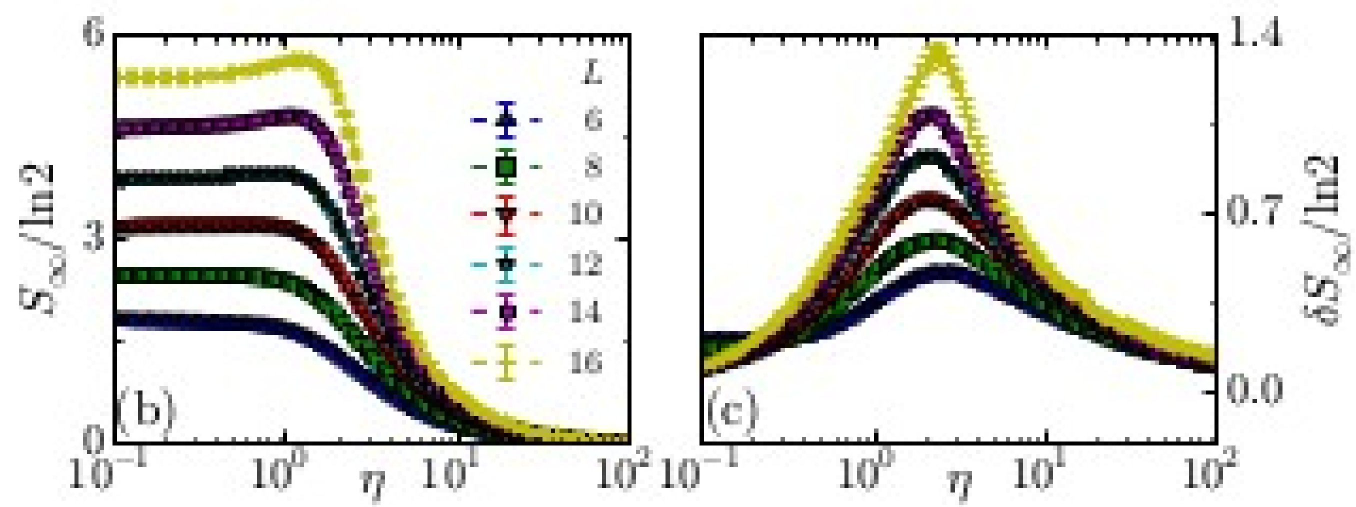
## Entanglement Growth

Starting from random initial product state

Ballistic growth on the ergodic side, logarithmic saturation on the MBL

In MBL far away regions of our system cannot exchange energy and get entangled only very slowly




Singh et al arXiv:1505.05045 (2015)

J. H. Bardarson et al Phys.Rev. Lett. 109, 017202 (2012).
M. Serbyn et al Phys. Rev. Lett. 110, 260601 (2013).
R. Vosk et al, Phys. Rev. Lett. 110, 06720 (2013)

+many others

# Entanglement Growth

Starting fr

$\propto t$ delocalised

$\propto log(t)$ MBL

Ballistic gr tion on the MBL

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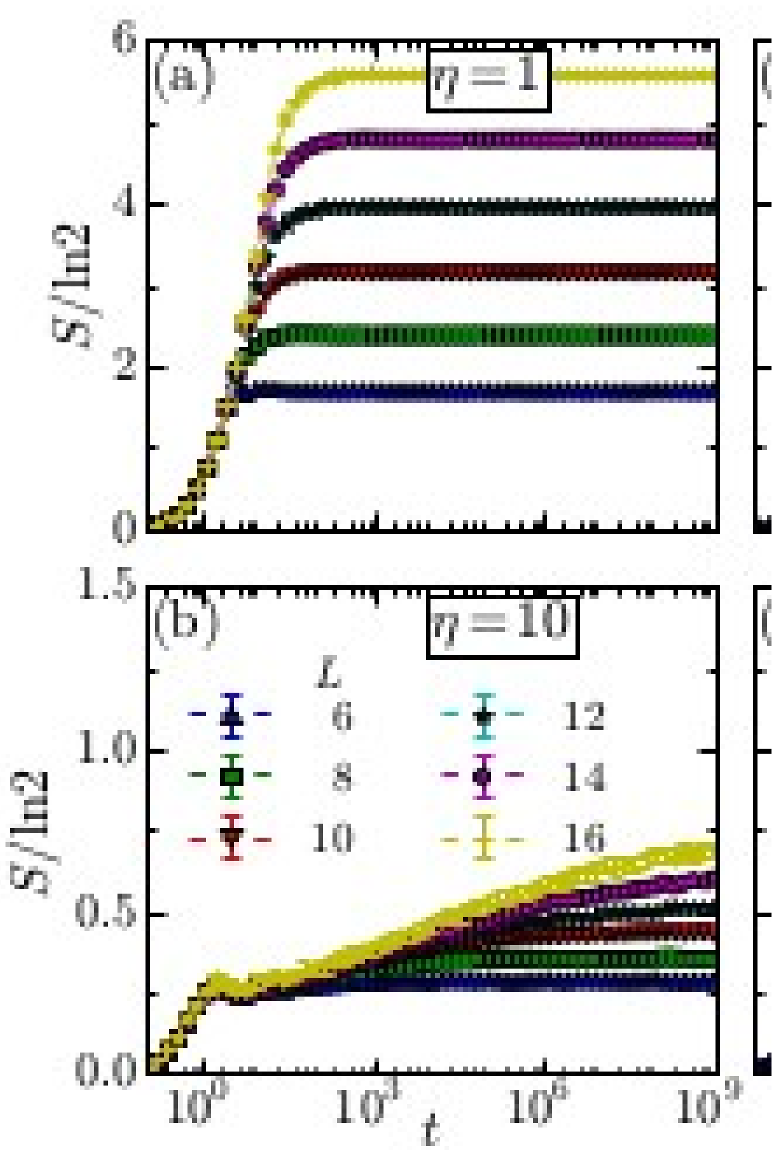


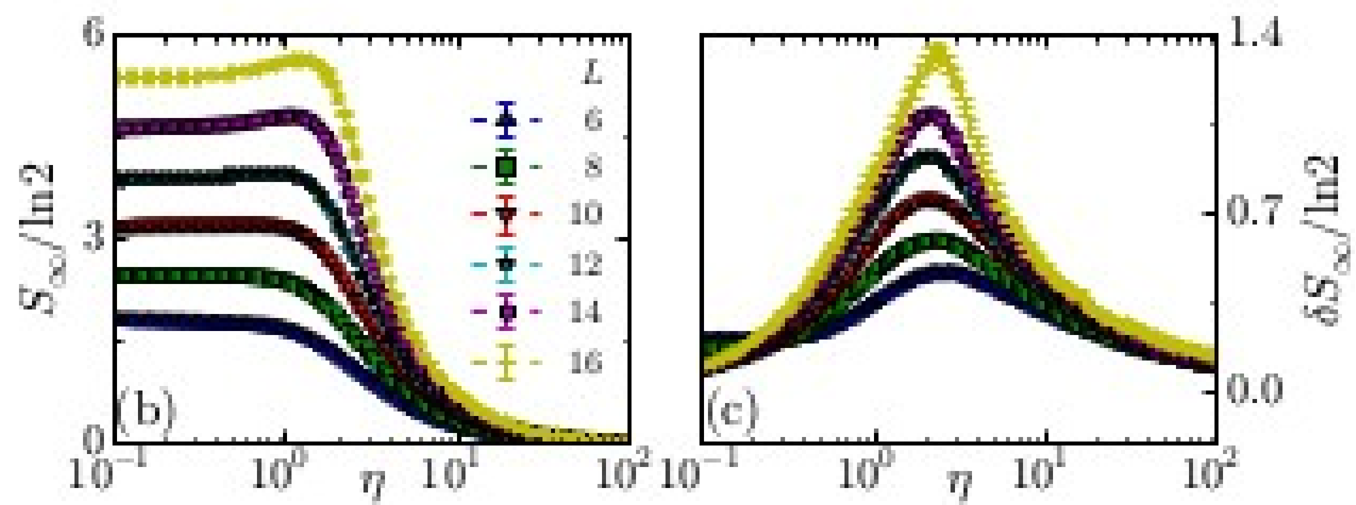


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# Dynamics

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$$\omega := \lim_{\tau \to \infty} \frac{1}{\tau} \int_0^\infty dt e^{-itH} \rho_i e^{itH}$$

$$= \sum_n |E_n\rangle\langle E_n|\rho_i|E_n\rangle\langle E_n|$$

# Experiments in cold atoms

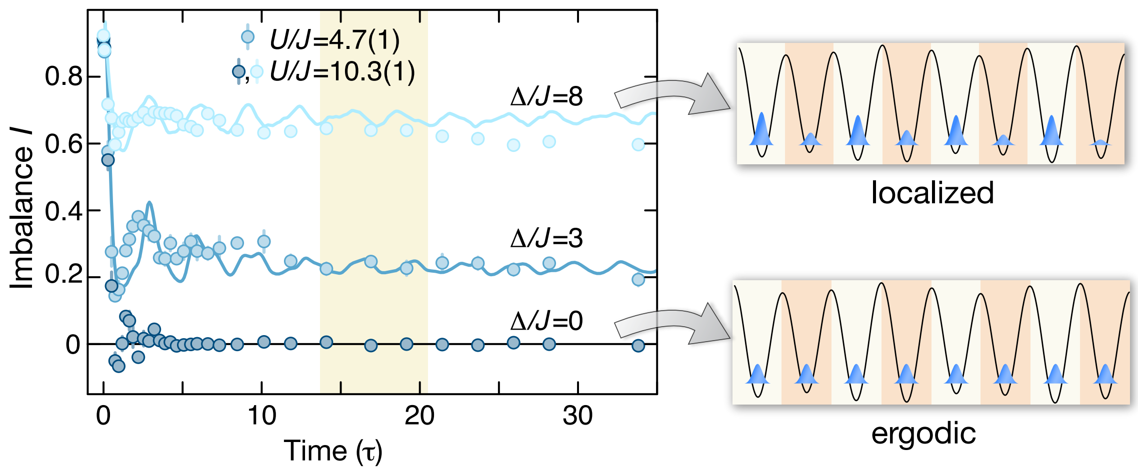



FIG. 7 Non-thermalizing out-of-equilibrium evolution of an initial density wave in the presence of a quasiperiodic detuning potential in the interacting Aubry- André model (see Eq. 17). Time traces of the imbalance $I$ for various strengths of the detuning potential $\Delta$. Points are experimental measurements, averaged over six different phases $\phi$ of the quasiperiodic detuning lattice. Lines denote DMRG simulations that take into account the trapping potential and the averaging over neighboring tubes, which are present in the experiment (Schreiber *et al.*, 2015).

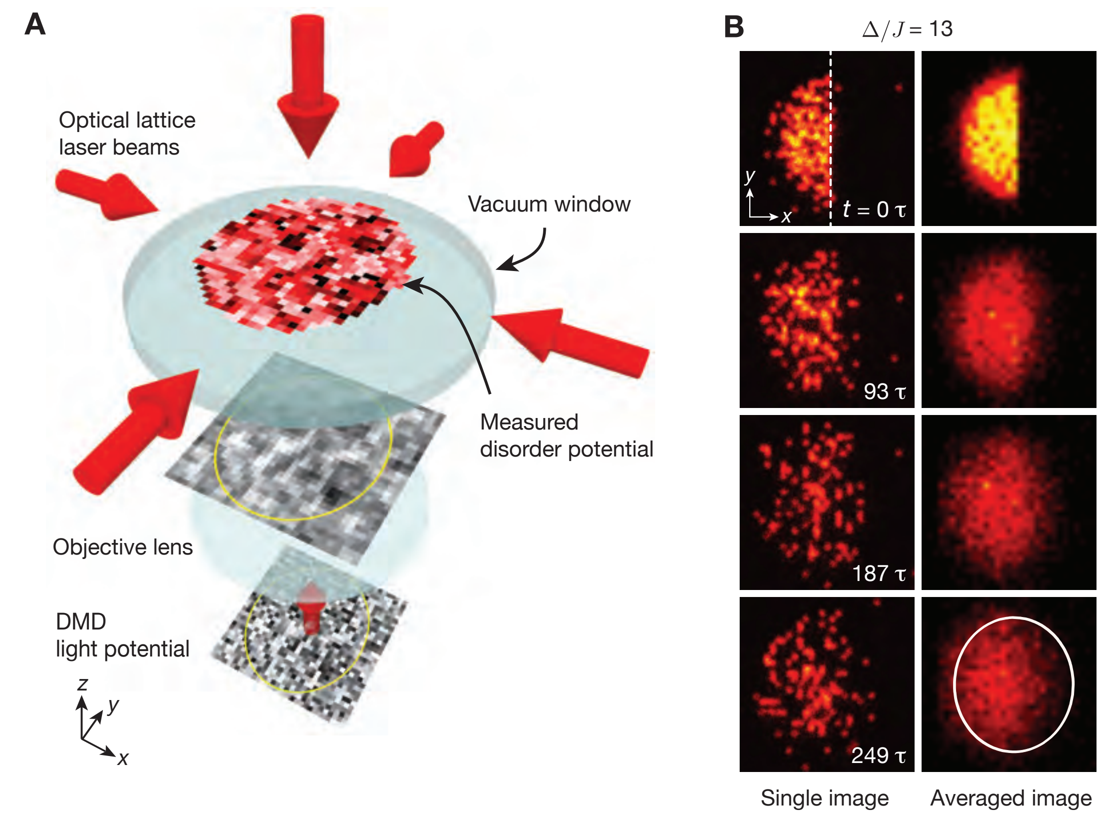



FIG. 8 Probing many-body localization in two dimensions. **(A)** Almost arbitrary disorder potentials of light are projected onto an ultracold bosonic atom cloud. The subsequent quantum evolution of an initial non-equilibrium state can then be tracked in the experiment. **(B)** In the experiment an initial domain wall of a bosonic Mott insulator is prepared ("half circle" in images). Even for long evolution times of $\simeq 250$ tunneling times, the system fails to thermalize, indicated by the remnant domain wall still visible in the experiment. In contrast, a thermalized state would not carry any information about the initial state of the system (Choi *et al.*, 2016).

**Bloch group**

*taken from*

D. A. Abanin, E. Altman, I. Bloch, M. Serbyn https://arxiv.org/abs/1804.11065 (2018)

# Trapped ions

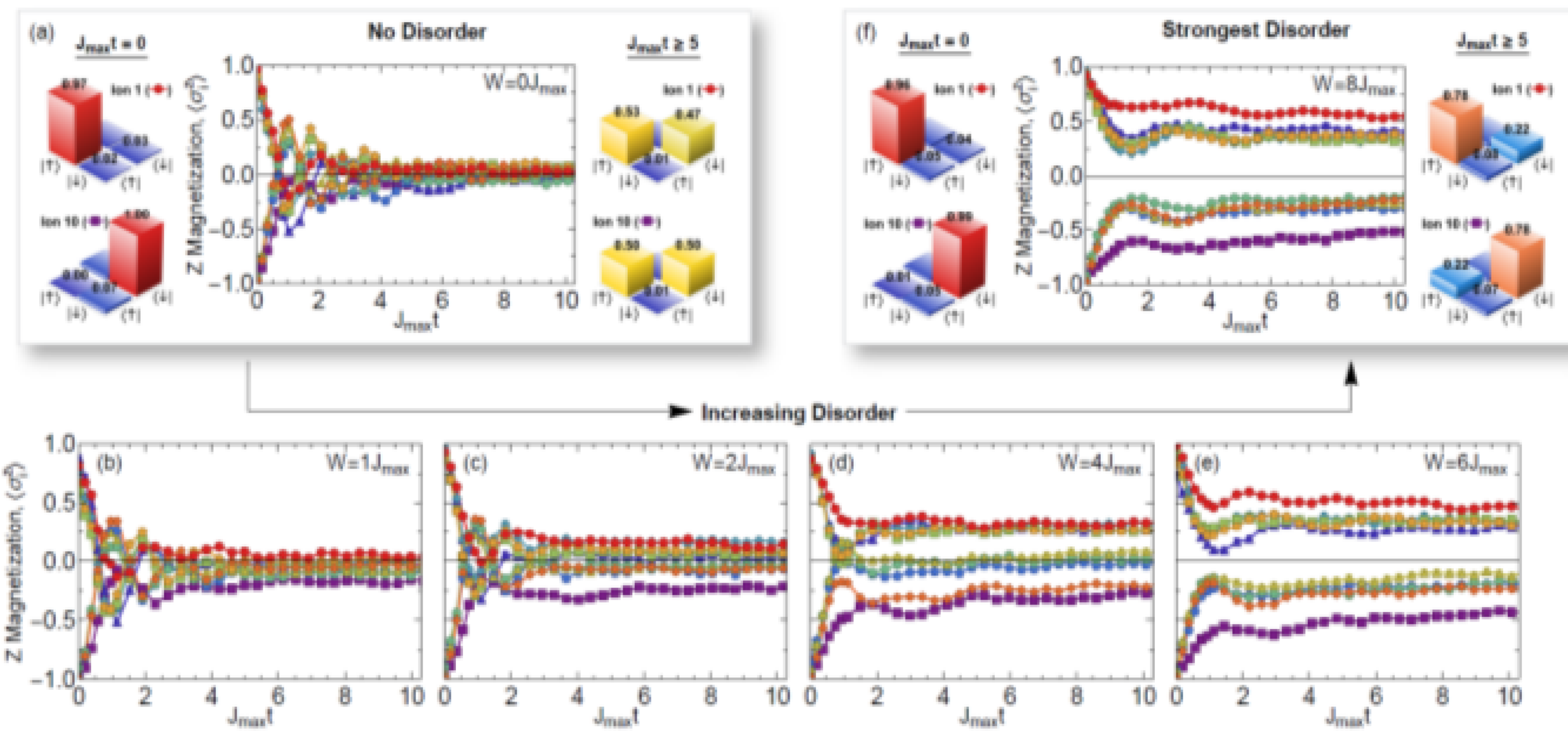
**Many-body localization in a quantum simulator with programmable random disorder**

J. Smith,[1] A. Lee,[1] P. Richerme,[2] B. Neyenhuis,[1] P. W. Hess,[1] P. Hauke,[3,4] M. Heyl,[3,4] D. Huse,[5] and C. Monroe[1]

$$H = \sum_{i<j} \sigma_x^i \sigma_x^j + \frac{B}{2} \sum_i \sigma_z^i + \sum_i \frac{h_i}{2} \sigma_x^i$$

**Nature Physics (2016)**

Prepare Neel state: $|\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\rangle$




Measure dynamics of local magnetisation

# Diagonal ensemble

$$H = H_i + H_f$$

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$$= \sum_n |E_n\rangle\langle E_n|\rho_i|E_n\rangle\langle E_n|$$

**We want to look at the multi partite correlations in the diagonal ensemble**

Natural since operationally it is the state connected to ergodic properties !!!

# Diagonal ensemble

$$\omega := \lim_{\tau\to\infty} \frac{1}{\tau}\int_0^\infty dt e^{-itH}\rho_i e^{itH}$$

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RAPID COMMUNICATIONS

PHYSICAL REVIEW B **92**, 180202(R) (2015)

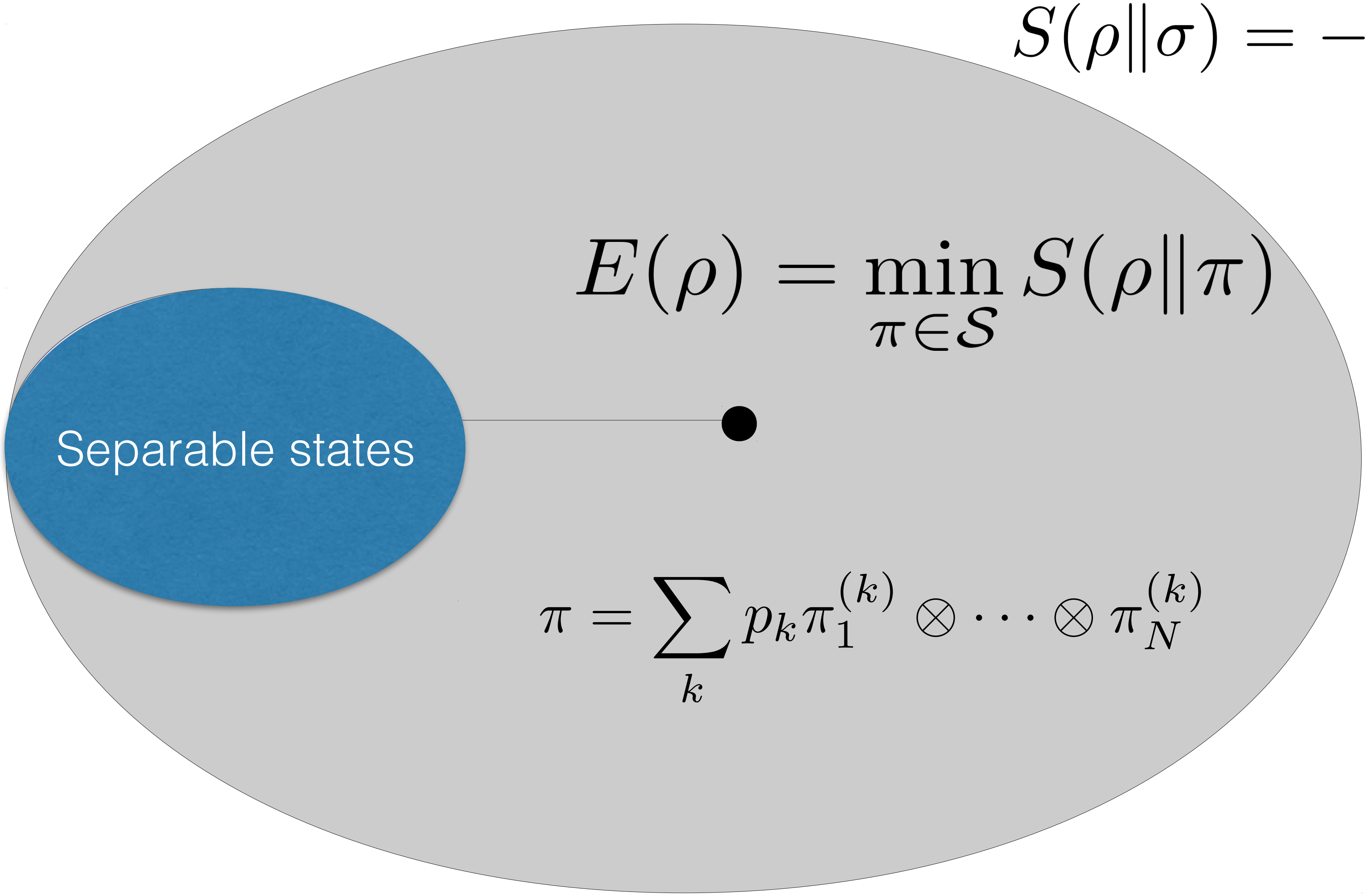
**Total correlations of the diagonal ensemble herald the many-body localization transition**

J. Goold,[1,*] C. Gogolin,[2,3,†] S. R. Clark,[4,5,6,‡] J. Eisert,[7,§] A. Scardicchio,[1,8,‖] and A. Silva[1,9,¶]

# Relative entropy and correlations

Multi-partite entanglement through relative entropy

$$S(\rho\|\sigma) = -(\rho \log_2 \sigma) - S(\rho)$$




$$E(\rho) = \min_{\pi \in \mathcal{S}} S(\rho\|\pi)$$

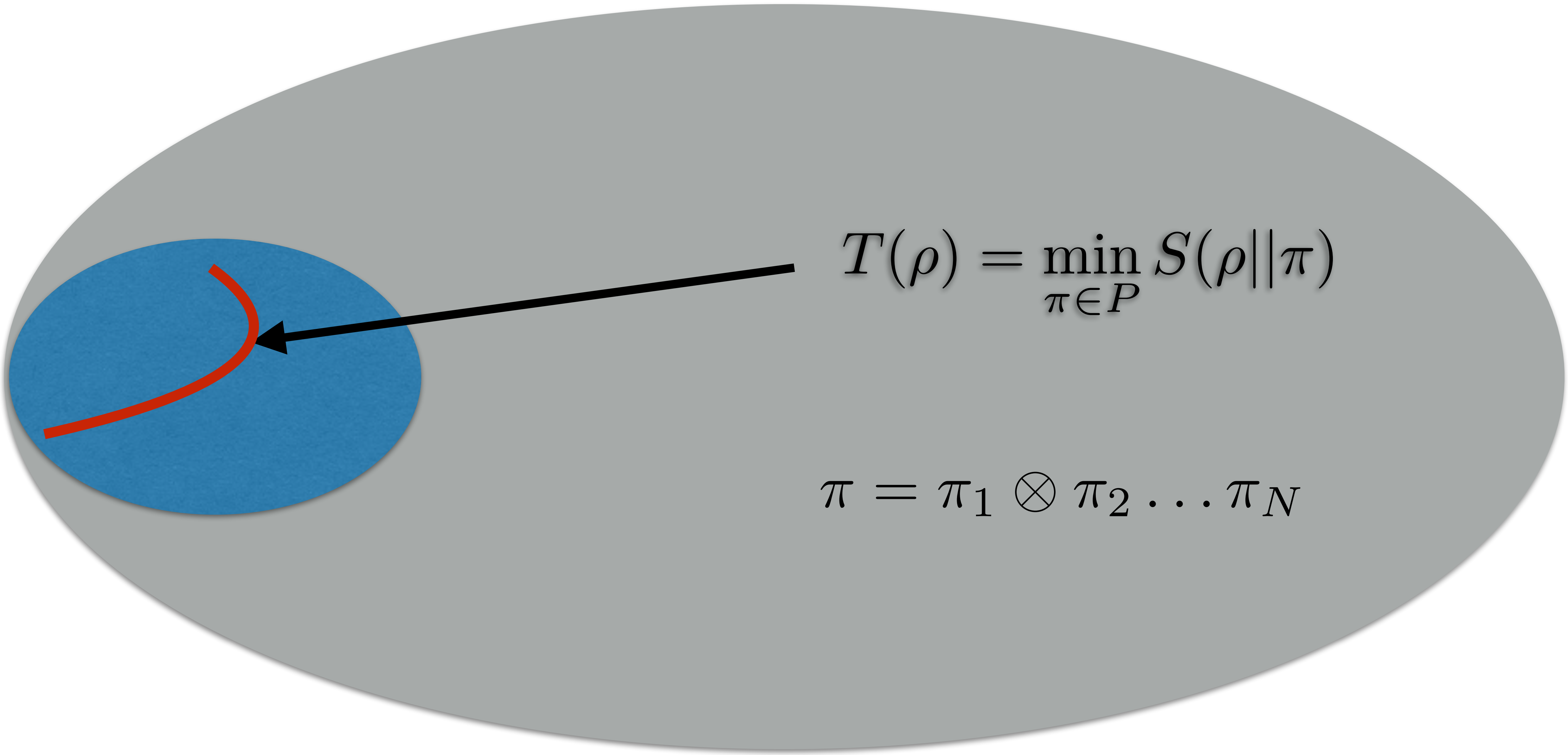
$$\pi = \sum_k p_k \pi_1^{(k)} \otimes \cdots \otimes \pi_N^{(k)}$$

V. Vedral, Rev. Mod. Phys. 74, 197, (2002)

Not a distance but upperbounds the trace distance

$$S(\rho\|\sigma) \geq \|\rho - \sigma\|_1^2/2$$

# Total correlations




Minimisation is easy

$$T(\rho) = S(\rho||\rho_1 \otimes \rho_2 \ldots \rho_N) = \sum_n S(\rho_n) - S(\rho)$$

For N=2 is mutual information

$$T(\rho) \geq E(\rho)$$

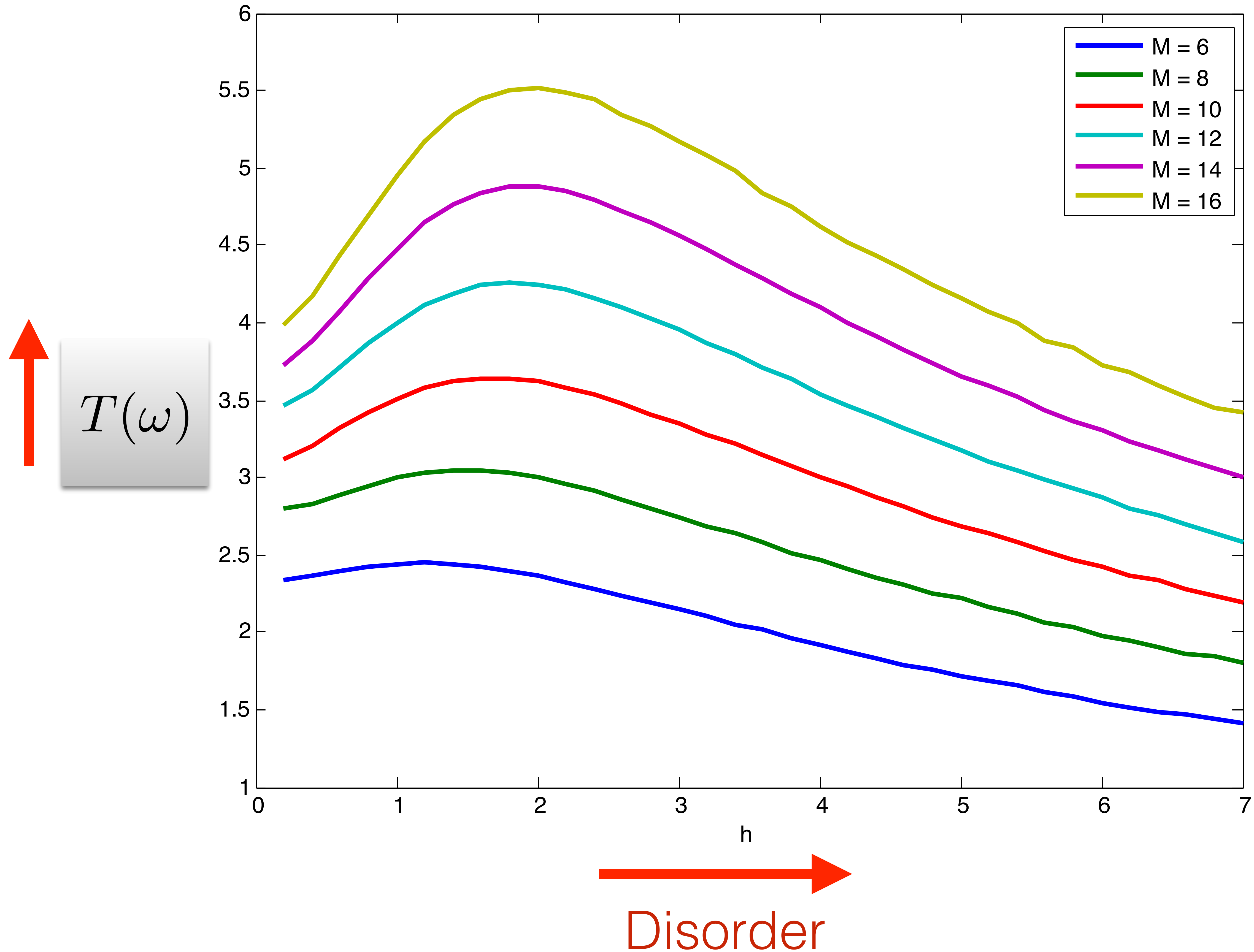
# Diagonal ensemble

$$\omega := \lim_{\tau \to \infty} \frac{1}{\tau} \int_0^\infty dt e^{-itH} \rho_i e^{itH}$$
$$= \sum_n |E_n\rangle\langle E_n|\rho_i|E_n\rangle\langle E_n|$$

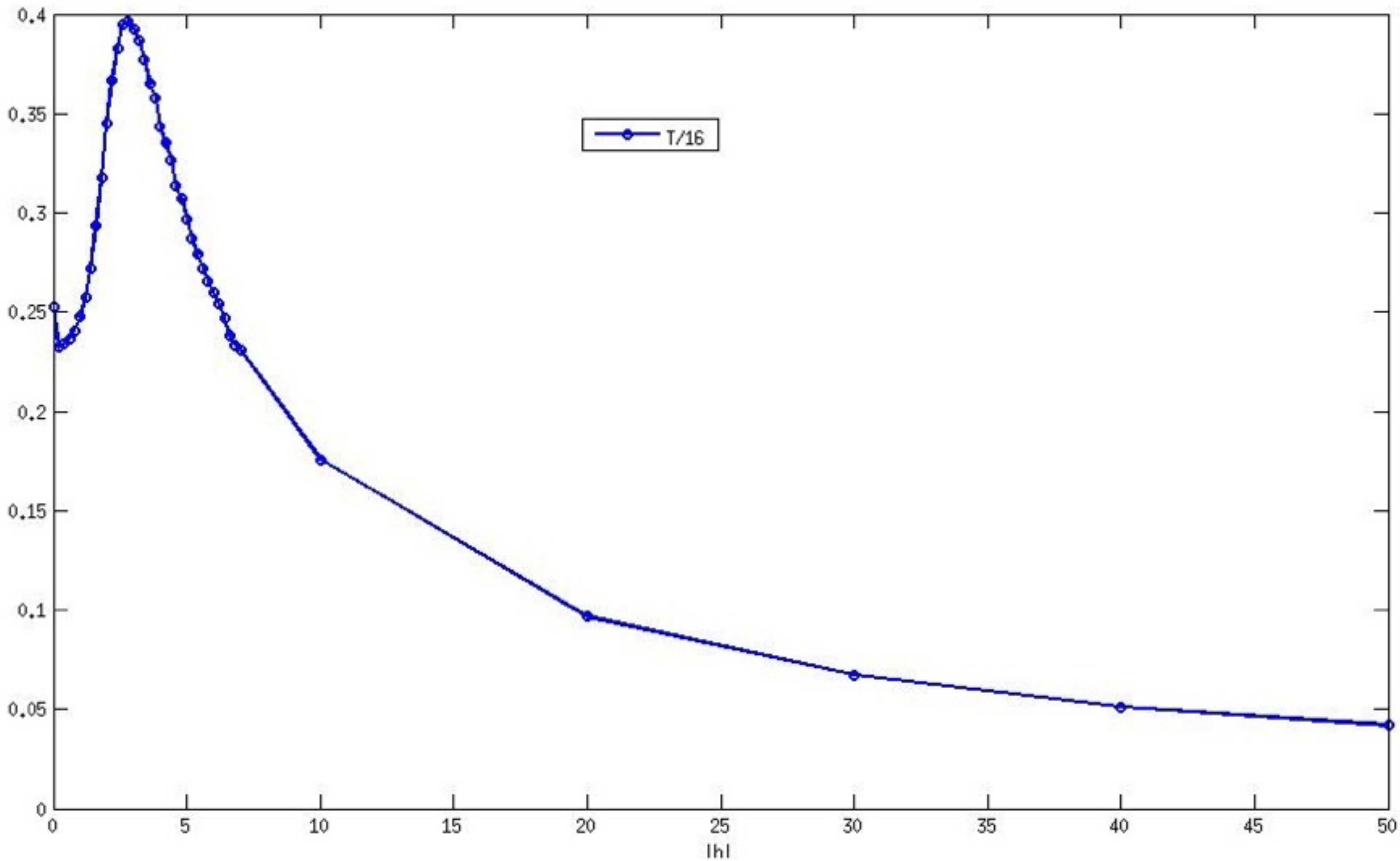
**We want to look at the correlations in the diagonal ensemble**

Initial states are computational basis states !

Average over disorder and
all initial computational
basis states !



M = 6
M = 8
M = 10
M = 12
M = 14
M = 16
h
$T(\omega)$
Disorder




Disorder

# Scaling with system size

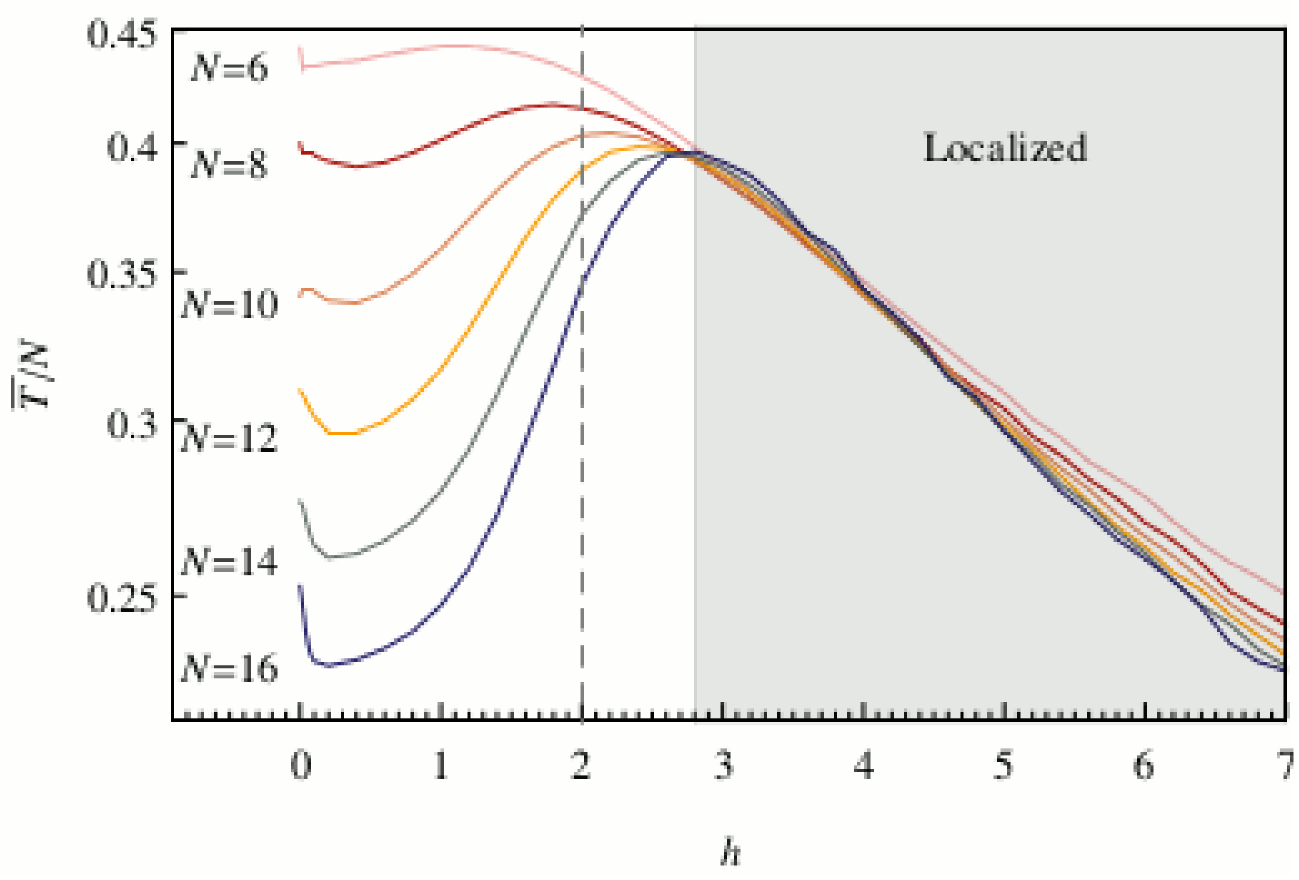
$$T(\omega) = \sum_m S(\omega_m) - S(\omega)$$

## Scaling with system size

$$T(\omega) = \sum_m S(\omega_m) - S(\omega)$$

MBL systems – easy should scale proportional to system size

Extensive due to sum of local terms




$$\omega := \sum_n |E_n\rangle\langle E_n|\rho|E_n\rangle\langle E_n| = \lim_{\mathcal{T}\to\infty}\int_0^{\mathcal{T}} dt e^{-itH}\rho e^{itH}$$

Fixed Hamiltonian H and randomly drawn states:

$$\Pr\left(S(\omega) \leq \log_2(d/2)\right) \leq 4\,\exp(-C\,d/\log_2(d)^2).$$

**Randomizing quantum states: Constructions and applications**
P. Hayden, D. Leung, P. W. Shor, A. Winter, Commun. Math. Phys. 250 371 (2004)

typically: $S(\omega) \geq \log_2(d/2)$

**We are not typical - have product states !**

Demand less - ergodic states explore a constant fraction of the available Hilbert space

$$S(\omega) \geq \log_2(\lambda d), \lambda > 0$$

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in subspace $d = \binom{N}{N/2} = N!/(\frac{N}{2}!)^2 \geq \sqrt{8\pi}\, e^{-2}\, 2^N/\sqrt{N}$

and $S(\omega_m) \leq \log_2 2 = 1$

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and $S(\omega_m) \leq \log_2 2 = 1$

$$T(\omega) \leq \log_2(N)/2 + const$$

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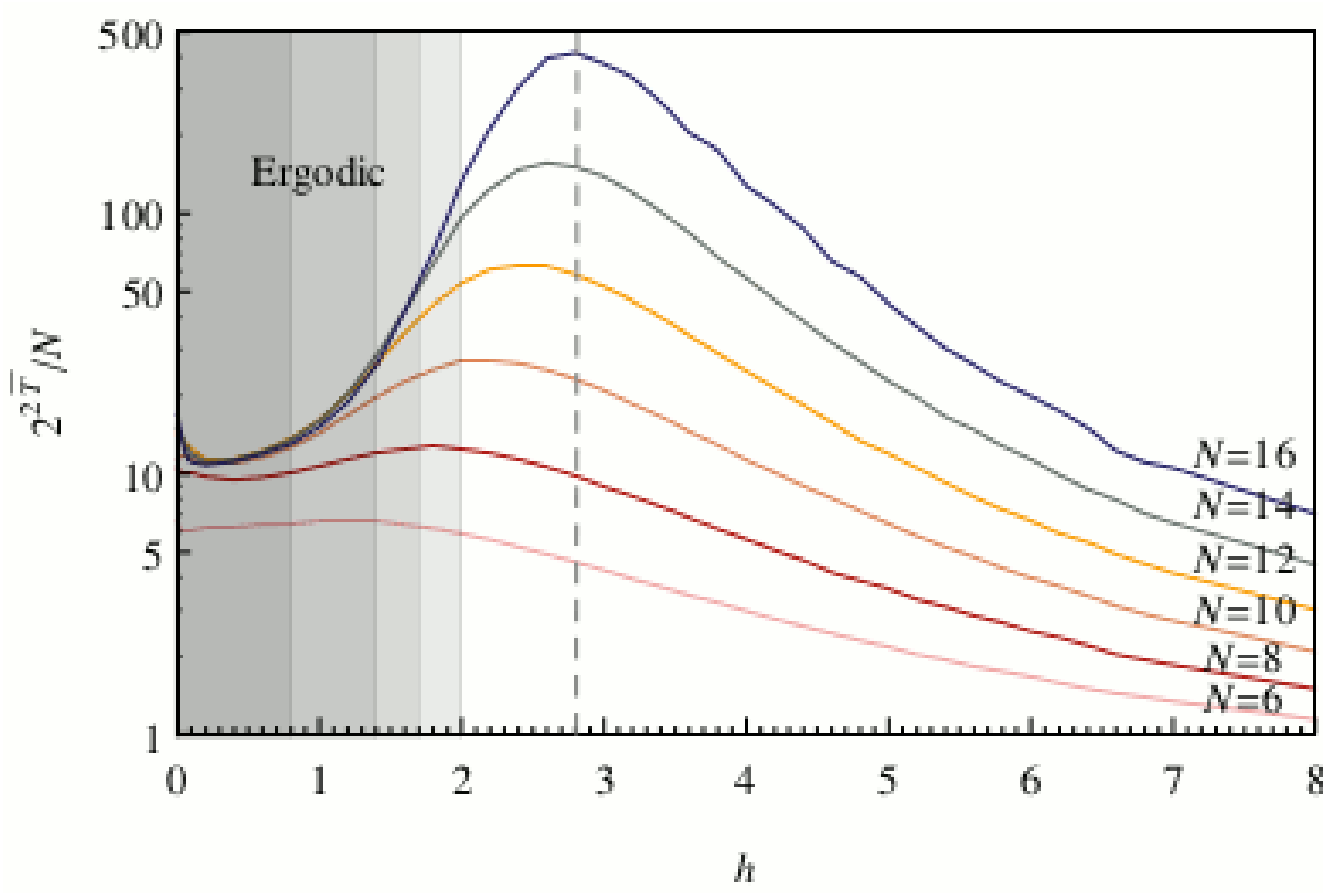
$$T(\omega) = \sum_m S(\omega_m) - S(\omega)$$

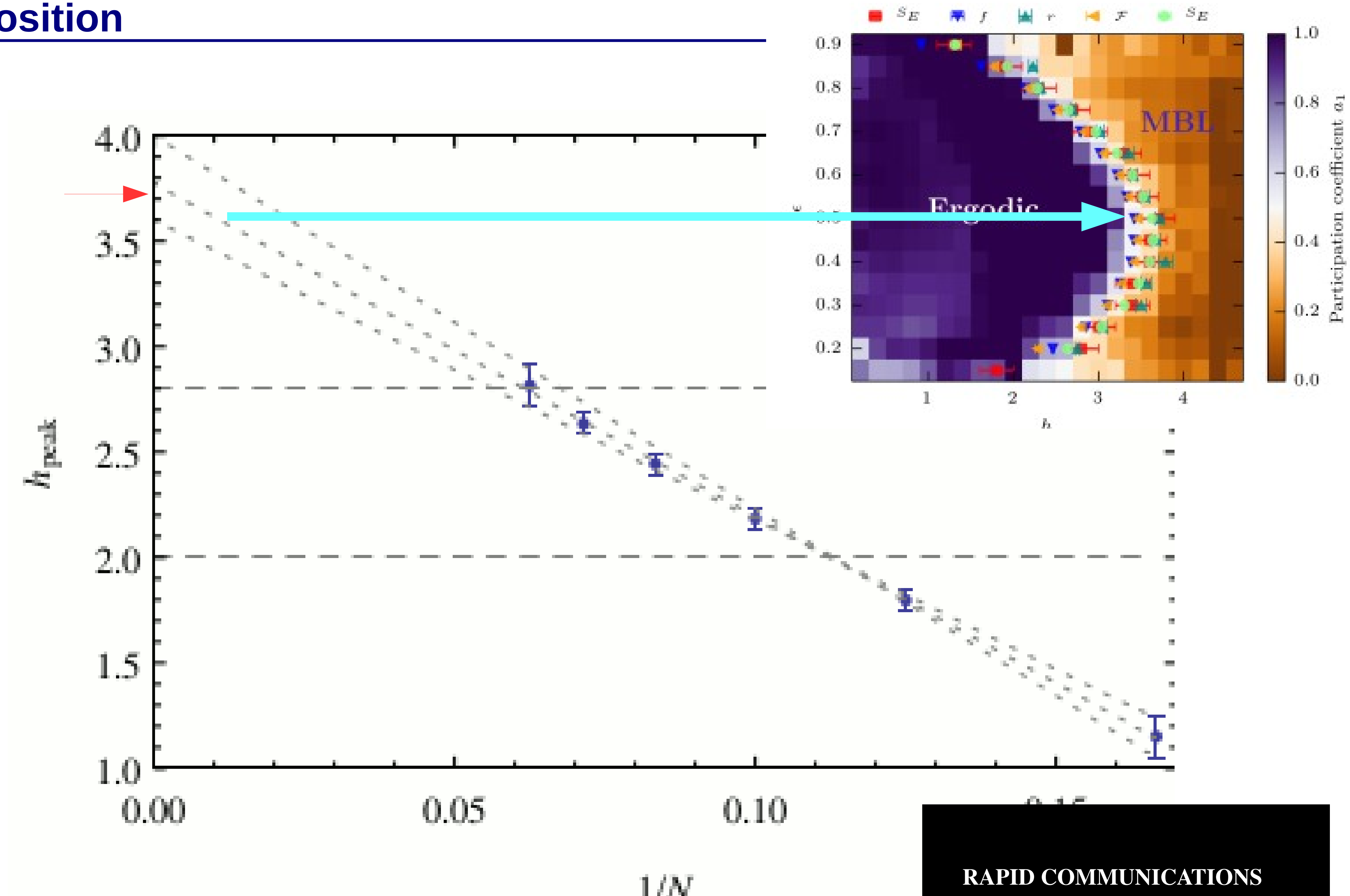
On Ergodic side we show subextensive scaling of the total correlations of the diagonal ensemble with system size! (Proof in paper)

In the total Sz=0 subspace it is logarithmic with system size



## Peak position

In good agreement with best numerics




RAPID COMMUNICATIONS

PHYSICAL REVIEW B **92**, 180202(R) (2015)

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[1]The Abdus Salam International Centre for Theoretical Physics (ICTP), Strada Costiera 11, 34151 Trieste, Italy

## Our point

Ergodicity breaking involves changes in the time average ensemble

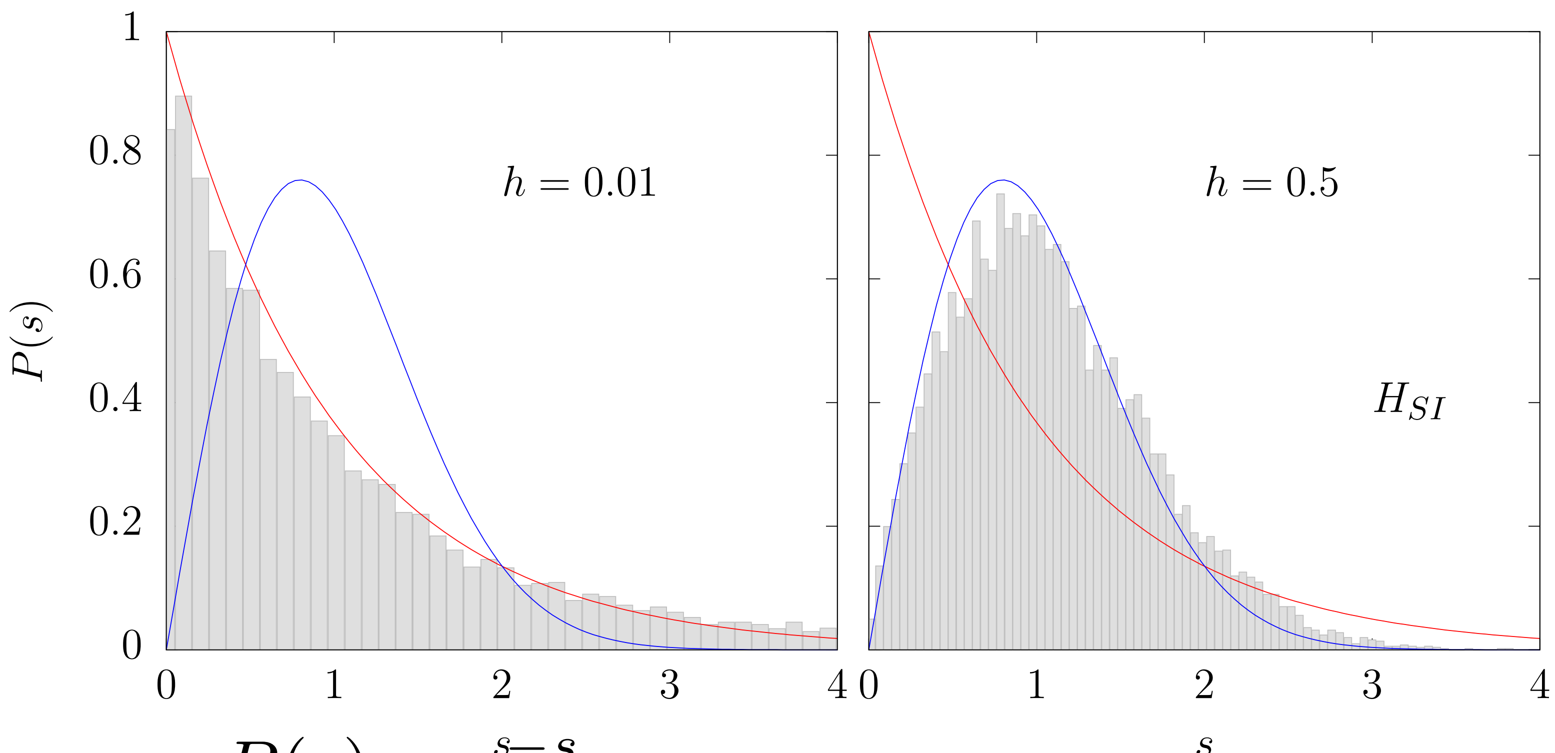
MBL is a novel phase which arises from ergodicity breaking in an abrupt fashion

When you want to understand it from perspective of correlations - look at the correlations in the ensemble which is operationally relevant for ergodicity breaking !

It is more general than MBL transition

# Single Magnetic Defect

$$H = t\sum_i \sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + V\sum_i \sigma_z^i \sigma_z^{i+1} + h\sigma_{L/2}^z$$




$$H_{SI} = H_{XXZ} + H_{IB}$$

$$\Delta = 0.5$$

$$P(s) = e^{-s}$$

$$P(s) = \frac{\pi s}{2} e^{\frac{-\pi s^2}{4}}$$

$$s_n = (E_{n+1} - E_n)/\Omega,\ \Omega : \text{Average level spacing}$$

**Papers by L. Santos, Xotos etc**

# Integrability breaking

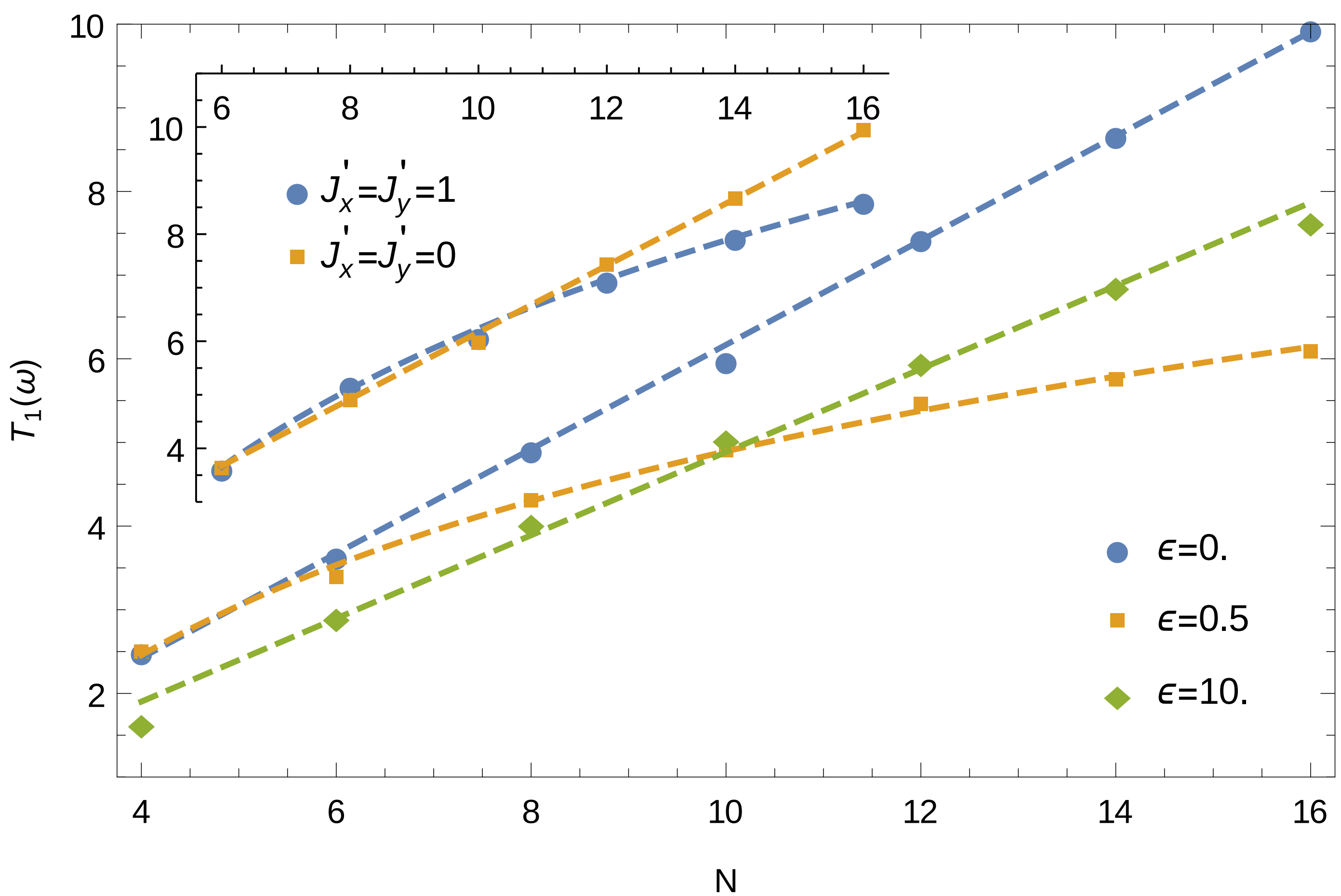



FIG. 1. (Color online) The Von Neumann total correlations of the diagonal ensemble starting with the Neel state for an XXZ chain with defect of strength $\epsilon$ placed at centre of the chain (Eq. (9) with parameters $J_x = J_y = 1$ and $J_z = 0.5$). When the defect strength is zero or very strong the model is integrable, which is reflected in a linear scaling of the total correlations, and when it is comparable with the interaction energy it shows a logarithmic growth indicative of ergodic dynamics. *Inset.* Total correlations for an XXZ chain with next-nearest-neighbour interaction (Eq. (10) with parameters $J_x = J_y = 1$, $J_z = 0.5$ and $J'_x = J'_y = 1$, compared to the same model with $J'_x = J'_y = 0$). The model is non-integrable and thus the scaling of the total correlations is logarithmic in the system size.

Inset shows quench with NNN terms!

$$|\psi_0\rangle = |\uparrow\downarrow\uparrow\ldots\downarrow\uparrow\rangle$$

Total correlations of the diagonal ensemble as a generic indicator for ergodicity breaking in quantum systems
F. Pietracaprina, C. Gogolin, and J. Goold
Phys. Rev. B **95**, 125118 (2017)

# Transport of energy and spin on the delocalised side?


?

No transport of either particle or energy

Ann. Phys. (Berlin) 529, No. 7, 1600350 (2017) / DOI 10.1002/andp.201600350

annalen der physik

Review Article

## The ergodic side of the many-body localization transition

*David J. Luitz[1,]* and Yevgeny Bar Lev[2,1,]***

Received 1 November 2016, revised 20 February 2017, accepted 24 March 2017
Published online 16 May 2017

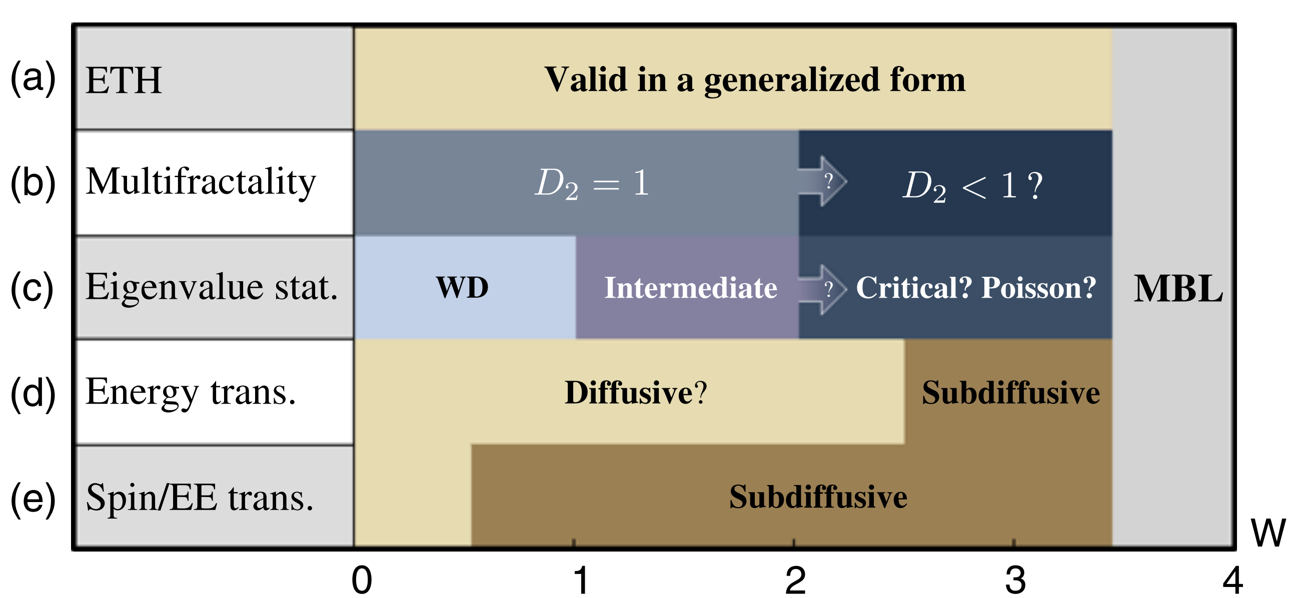



Table 1 Comparison of numerical methods for time evolution. Here $m$ is the number of Krylov vectors, $N_t$ is the number of time steps, $\chi$ is the bond dimension, $\mathcal{N}$ is the Hilbert space dimension and $L$ is the system size.

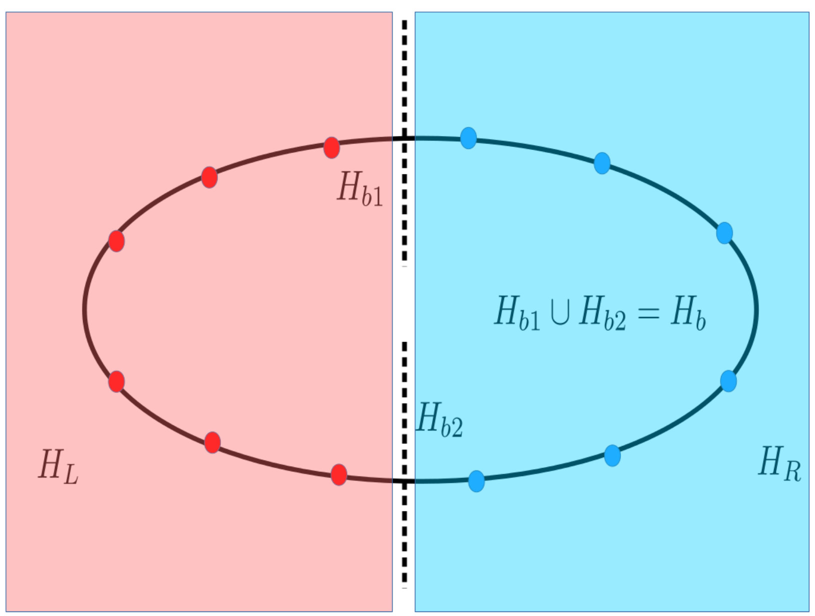
| Time evolution | memory | CPU | $L$ | time |
|---|---|---|---|---|
| ED | $\mathcal{O}(\mathcal{N}^2)$ | $\mathcal{O}(\mathcal{N}^3)$ | $\approx 18$ | $\infty$ |
| Krylov | $\mathcal{O}(m\mathcal{N})$ | $\mathcal{O}(LN_t\mathcal{N})$ | $\approx 30$ | $t_{\max}$ |
| tDMRG | $\mathcal{O}(L\chi^2)$ | $\mathcal{O}(LN_t\chi^3)$ | $> 100$ | $\approx \mathcal{O}(\ln \chi)$ |

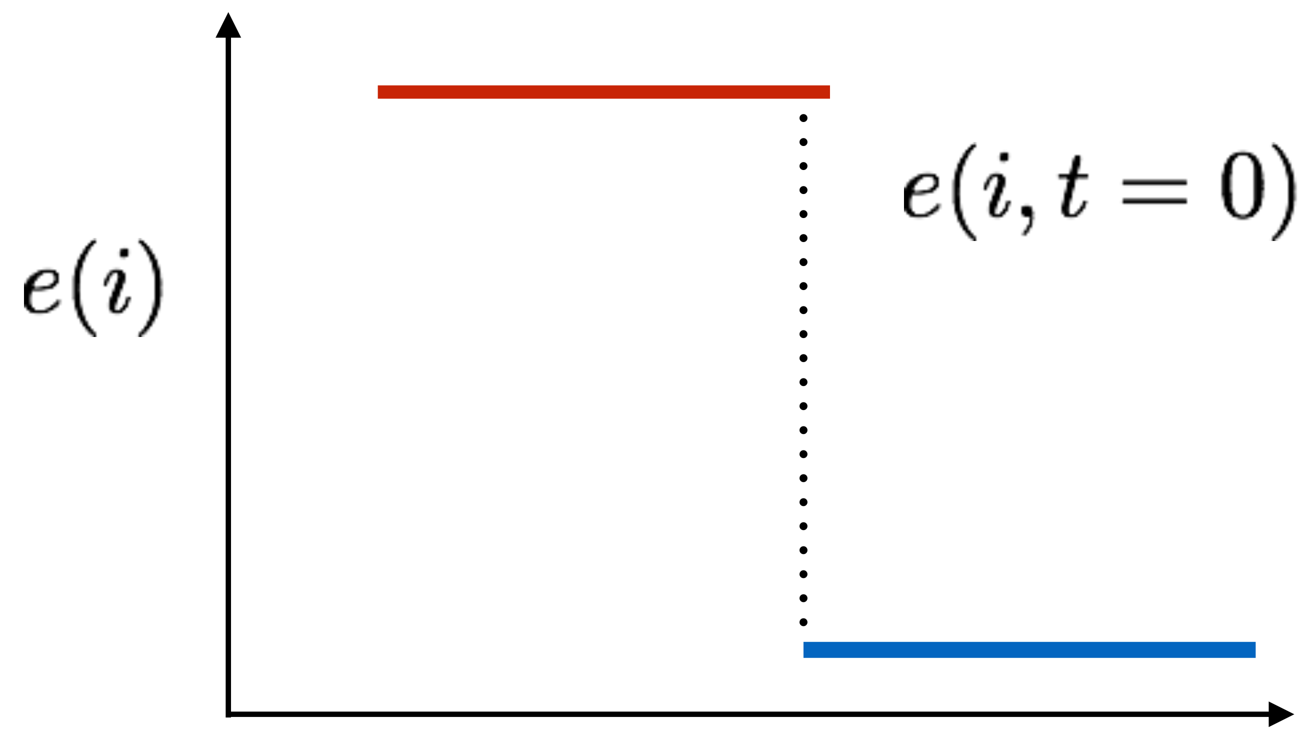
# Energy Imbalance

$$H = \sum_i \sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \sigma_i^z \sigma_{i+1}^z + \sum_i h_i \sigma_i^z$$

$$H_i = H_L \otimes \mathbb{I} + \mathbb{I} \otimes H_R$$

$$|\Psi_{in}\rangle = |\Psi_L^{es}\rangle \otimes |\Psi_R^{gs}\rangle$$




$$|\Psi(t)\rangle = \exp(-iHt)|\Psi_{in}\rangle \quad ?$$

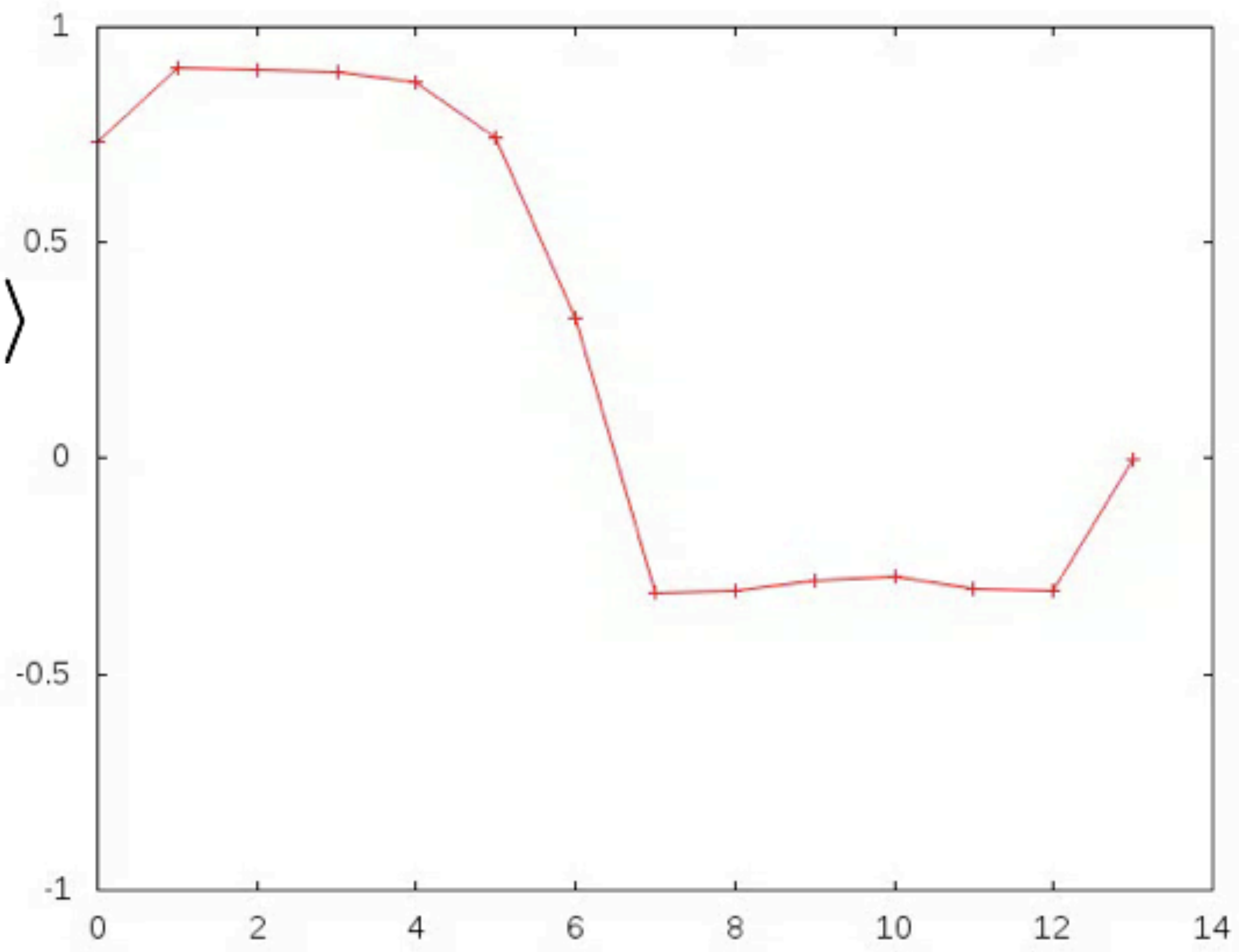
$$H_i = J\vec{s}_i.\vec{s}_{i+1} + h_i s_i^z$$

$$e(i,t) = \langle H_i(t)\rangle$$

use Krylov subspace techniques

# Delocalised side

$$e(i,t) = \langle H_i(t) \rangle$$



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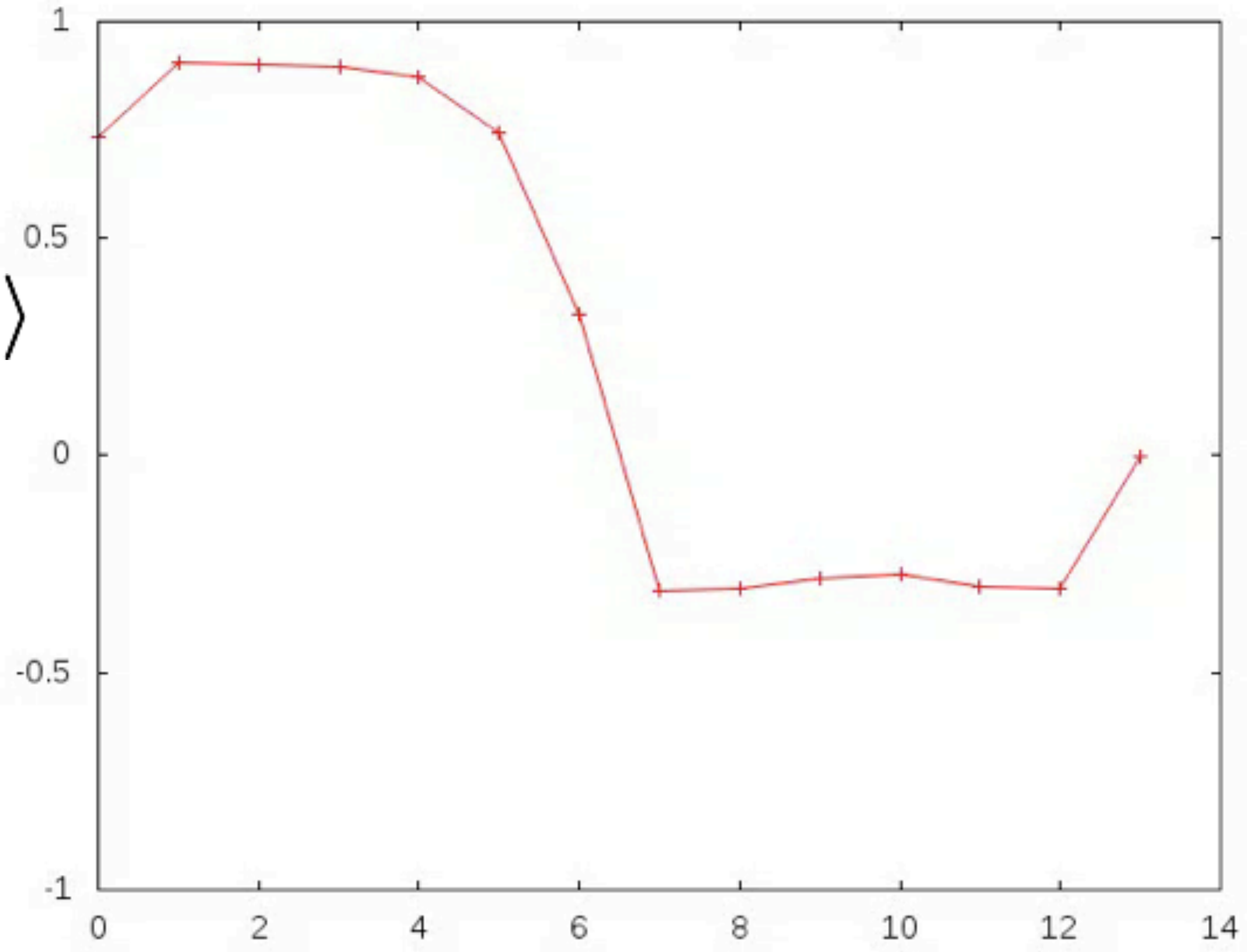

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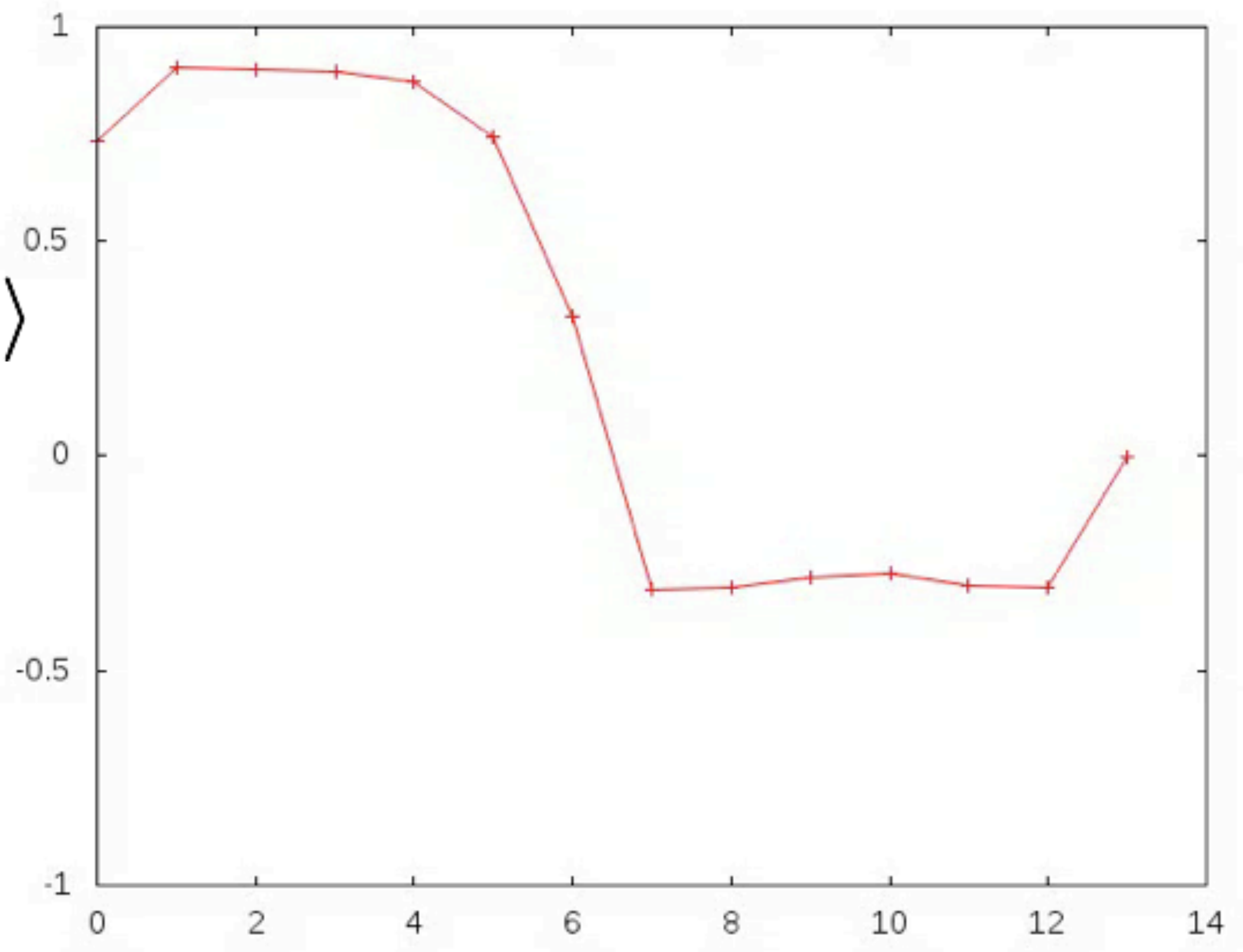
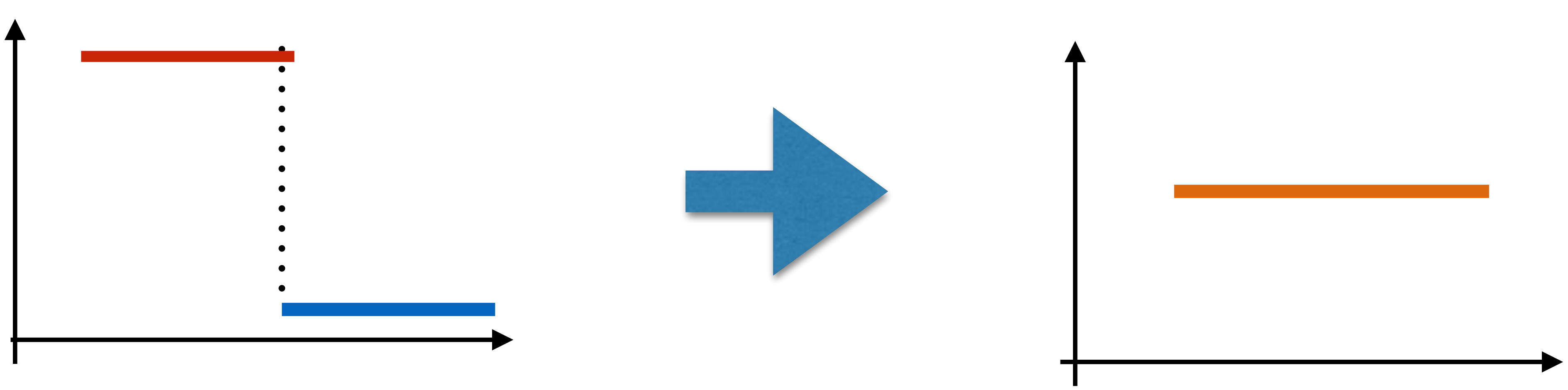
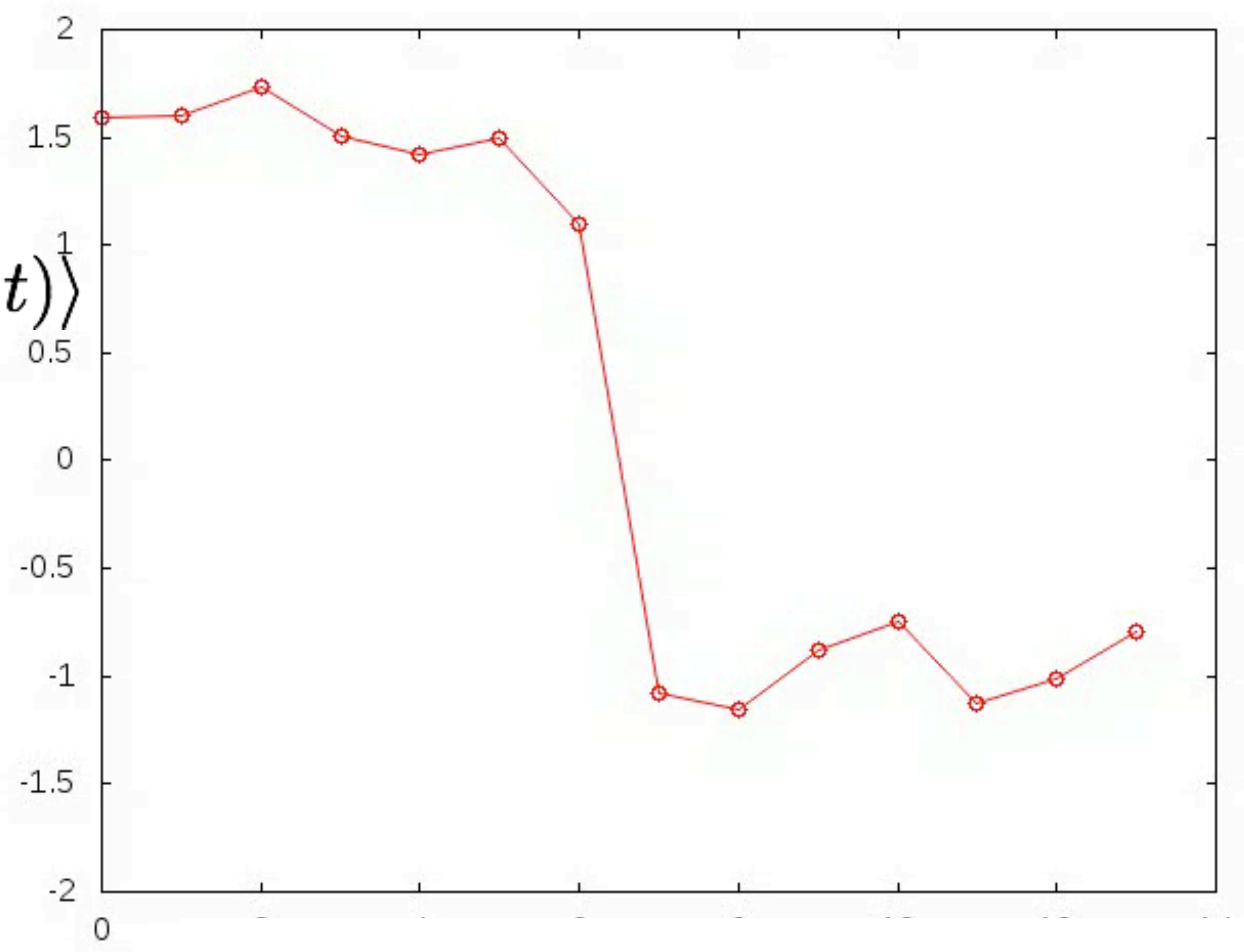


# MBL dynamics

$$e(i,t) = \langle H_i(t) \rangle$$



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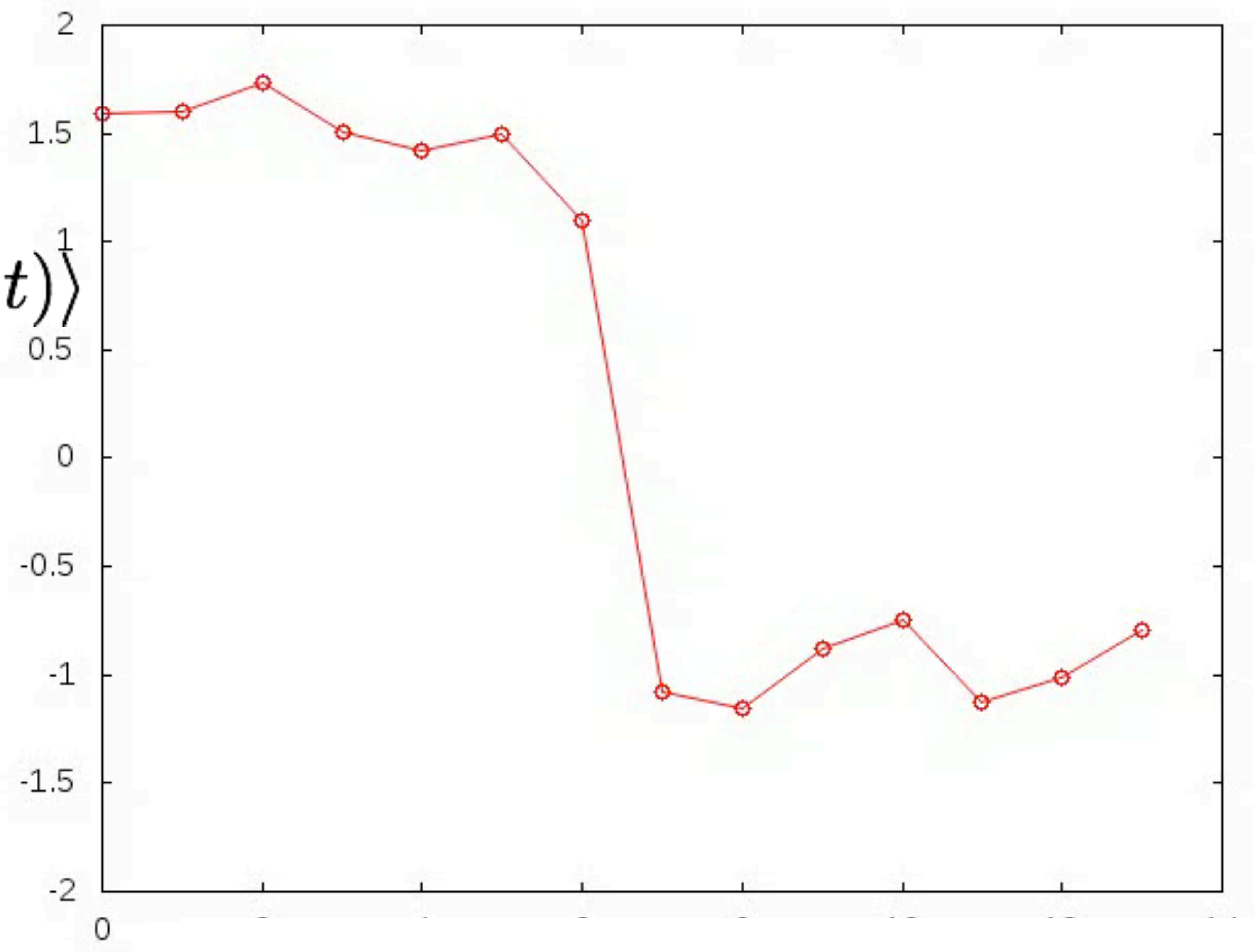

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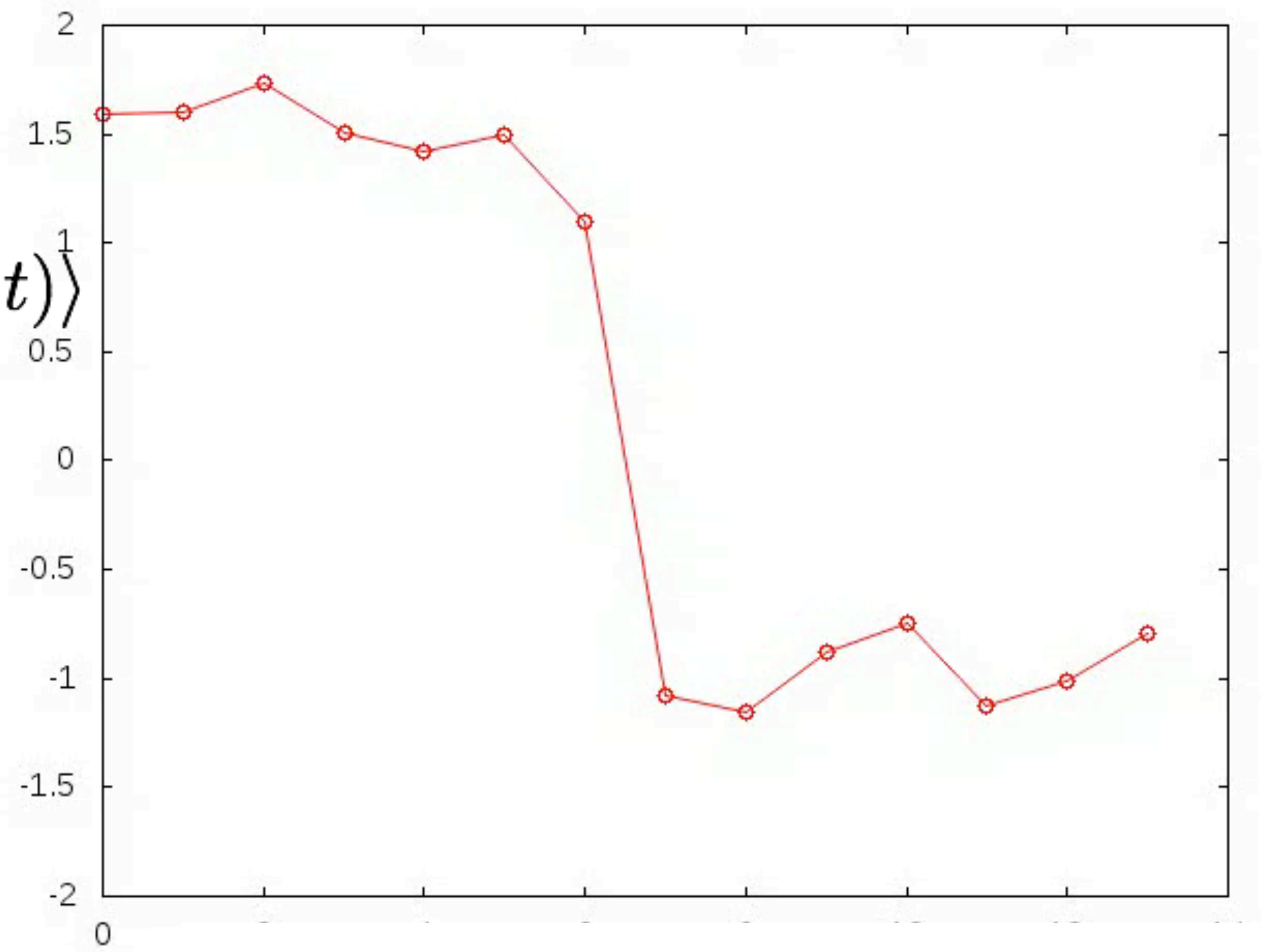


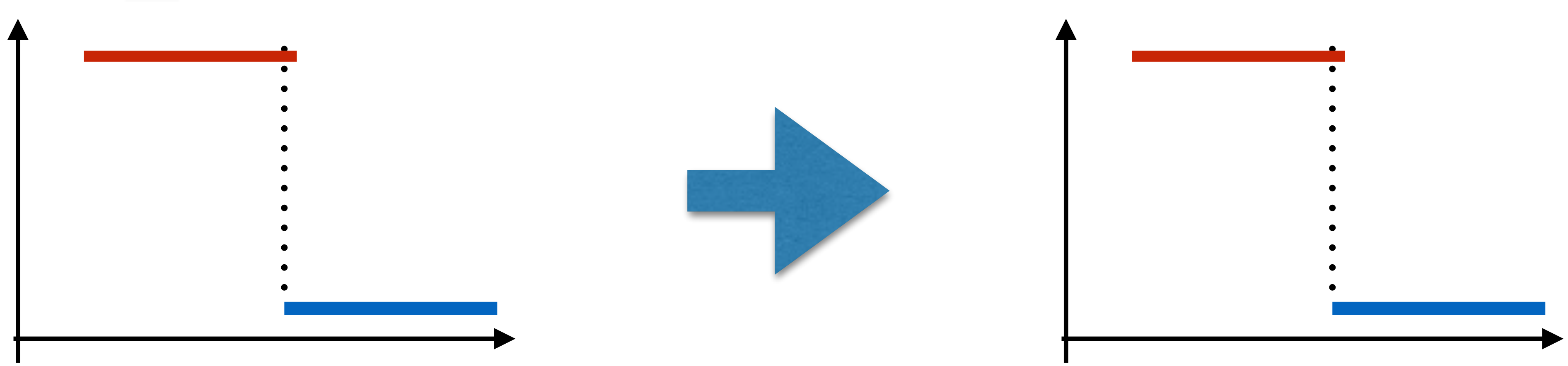

$$\Delta E(t) = \sum_{i=1}^{\frac{L}{2}} e(i,t) - \sum_{i=\frac{L}{2}+1}^{L} e(i,t)$$

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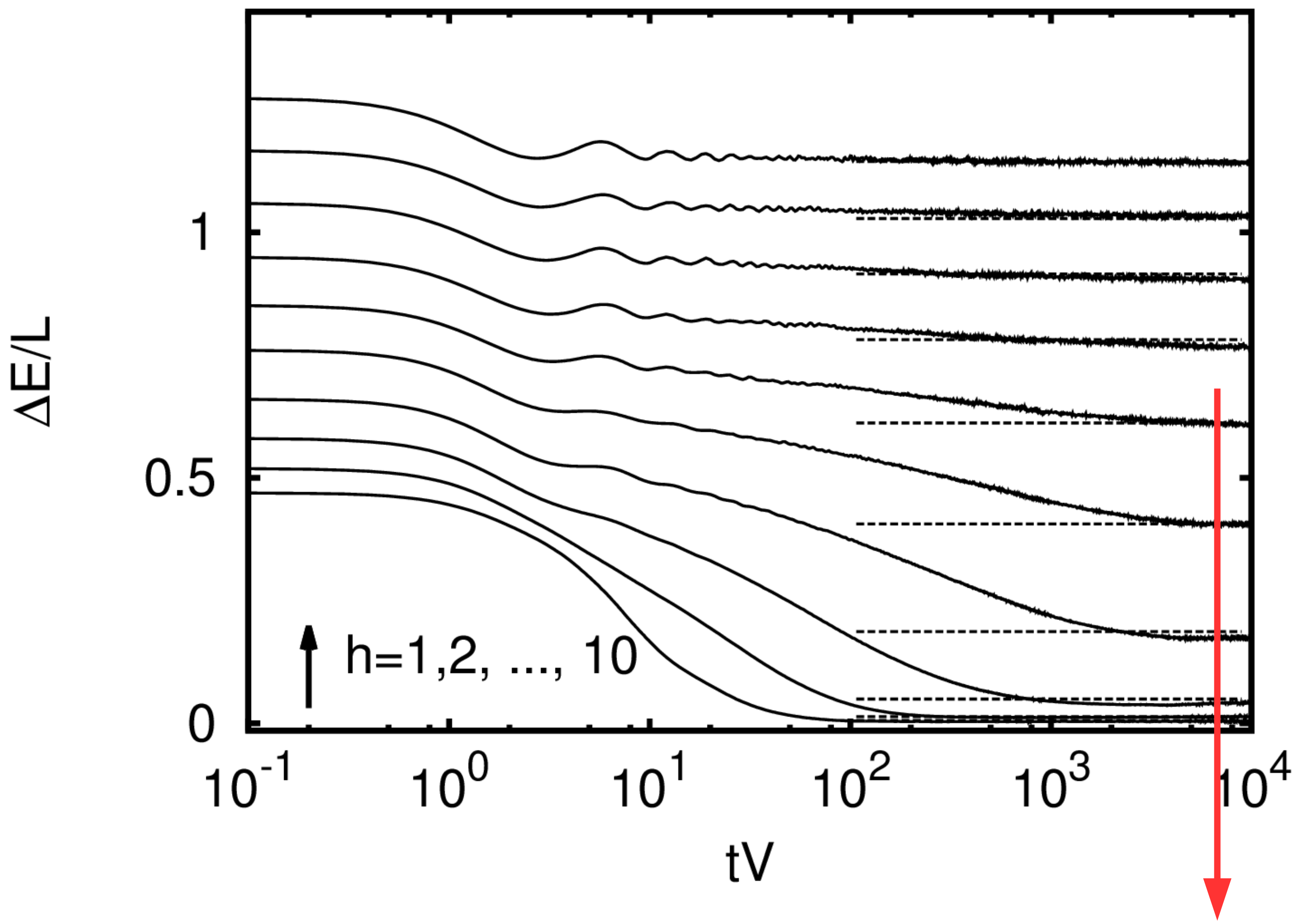
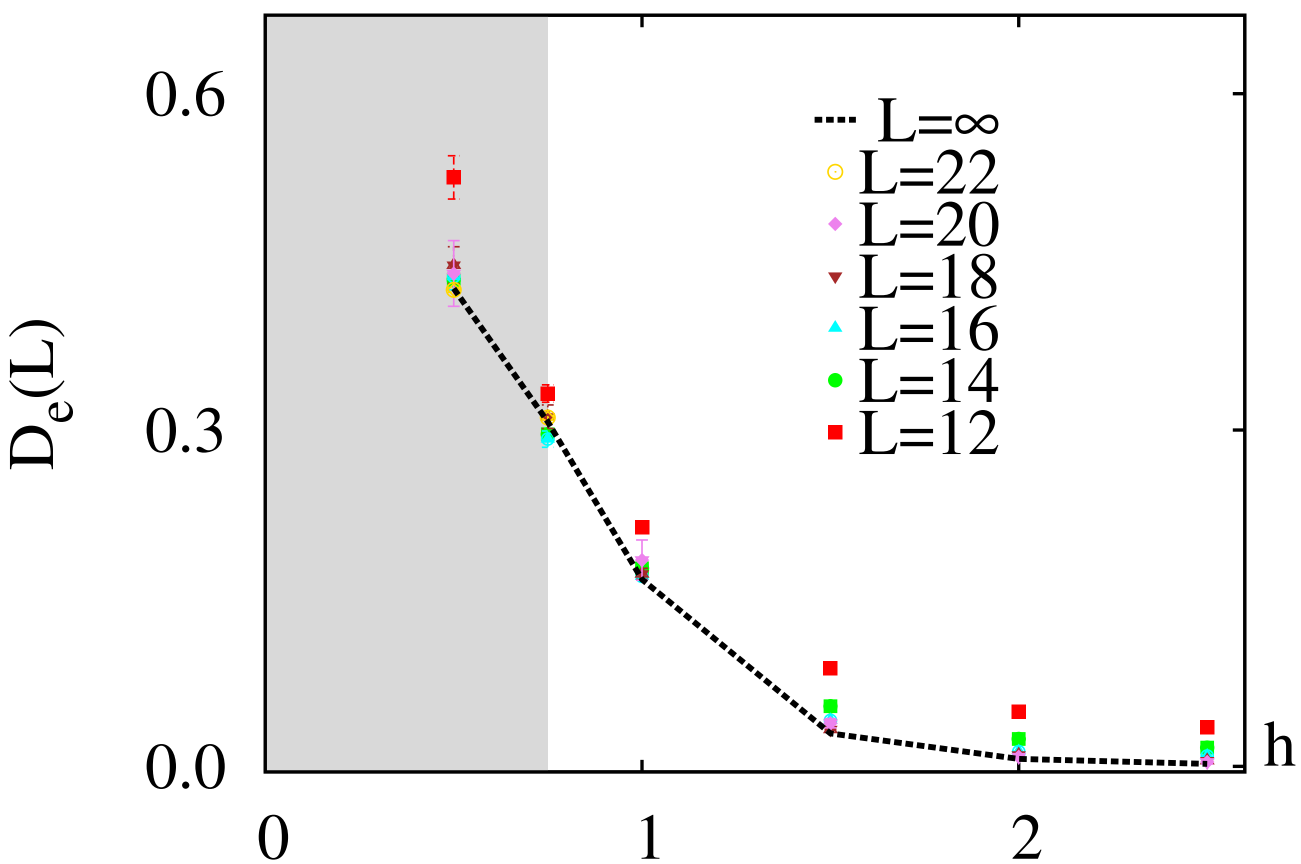
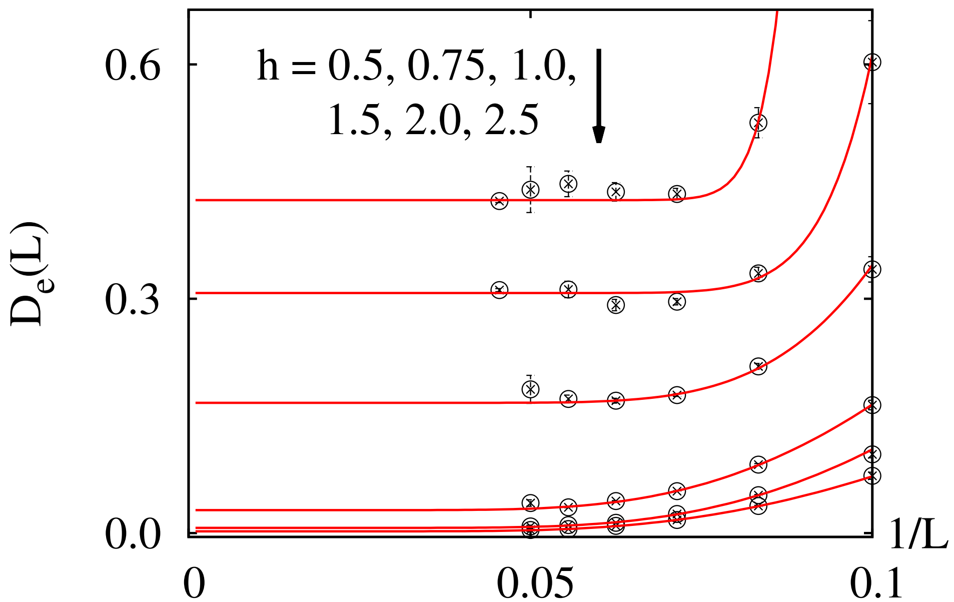



$$\omega := \sum_n |E_n\rangle\langle E_n|\rho|E_n\rangle\langle E_n| = \lim_{\tau\to\infty}\int_0^\tau dt e^{-itH}\rho e^{itH}$$

at least a small region of ergodic phase where energy transport appears to be diffusive, signatures of anomalous diffusion appears in large portion of ergodic phase

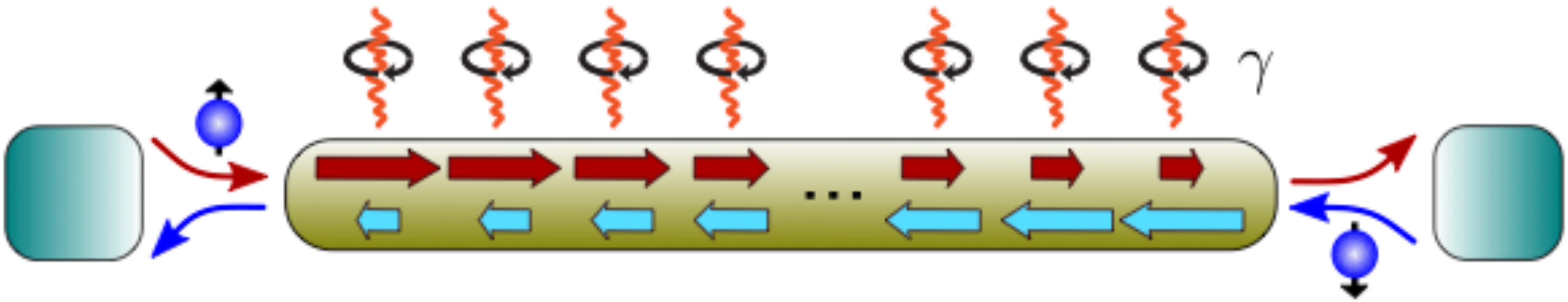



V. K. Varma, A. Lerose, F. Pietracaprina, J. Goold and A. Scardicchio, Journal of Statistical Mechanics (2017)

# Steady state transport

# Motivation (Long Term)




(Quasi)-Disordered and interacting bulk

How does the interplay of disorder, interactions and dephasing in a quantum systems give rise to emergent hydrodynamic behaviour at different length scales ?

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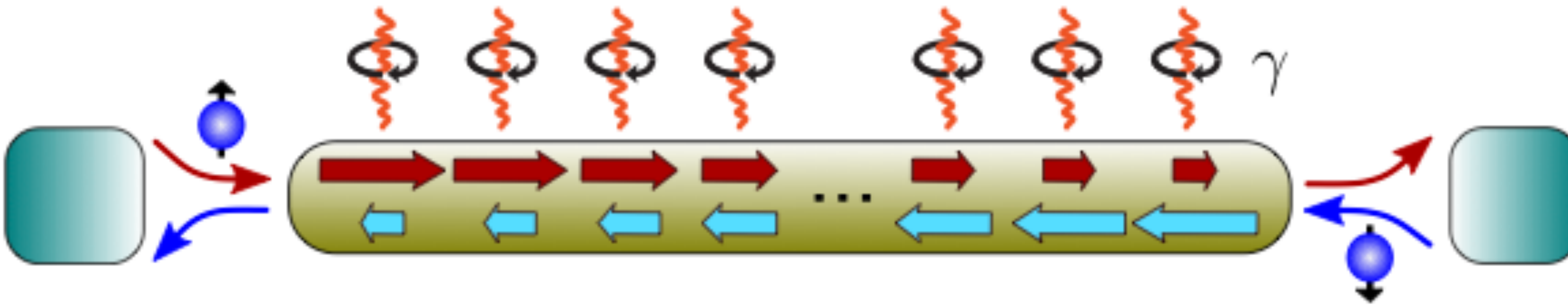


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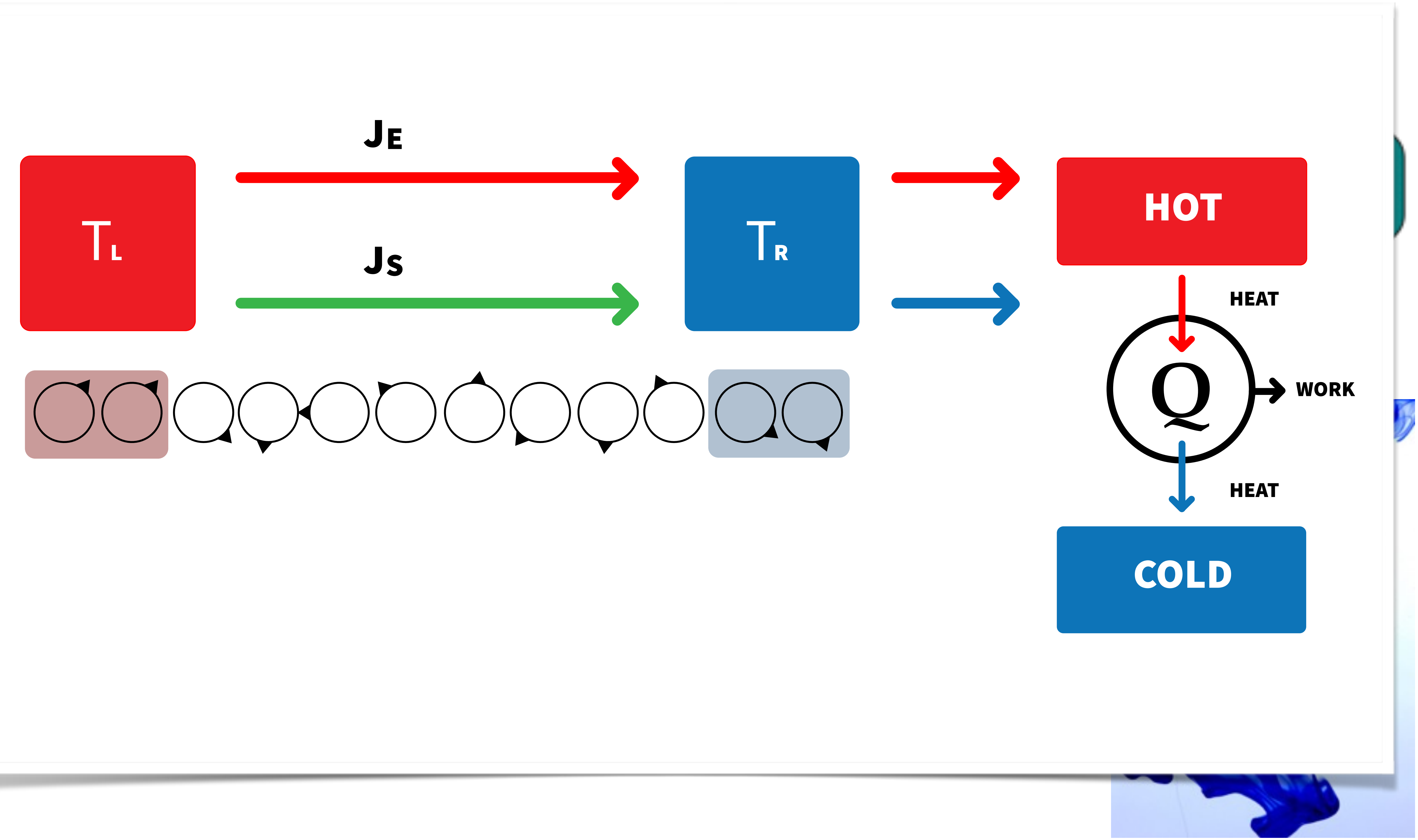
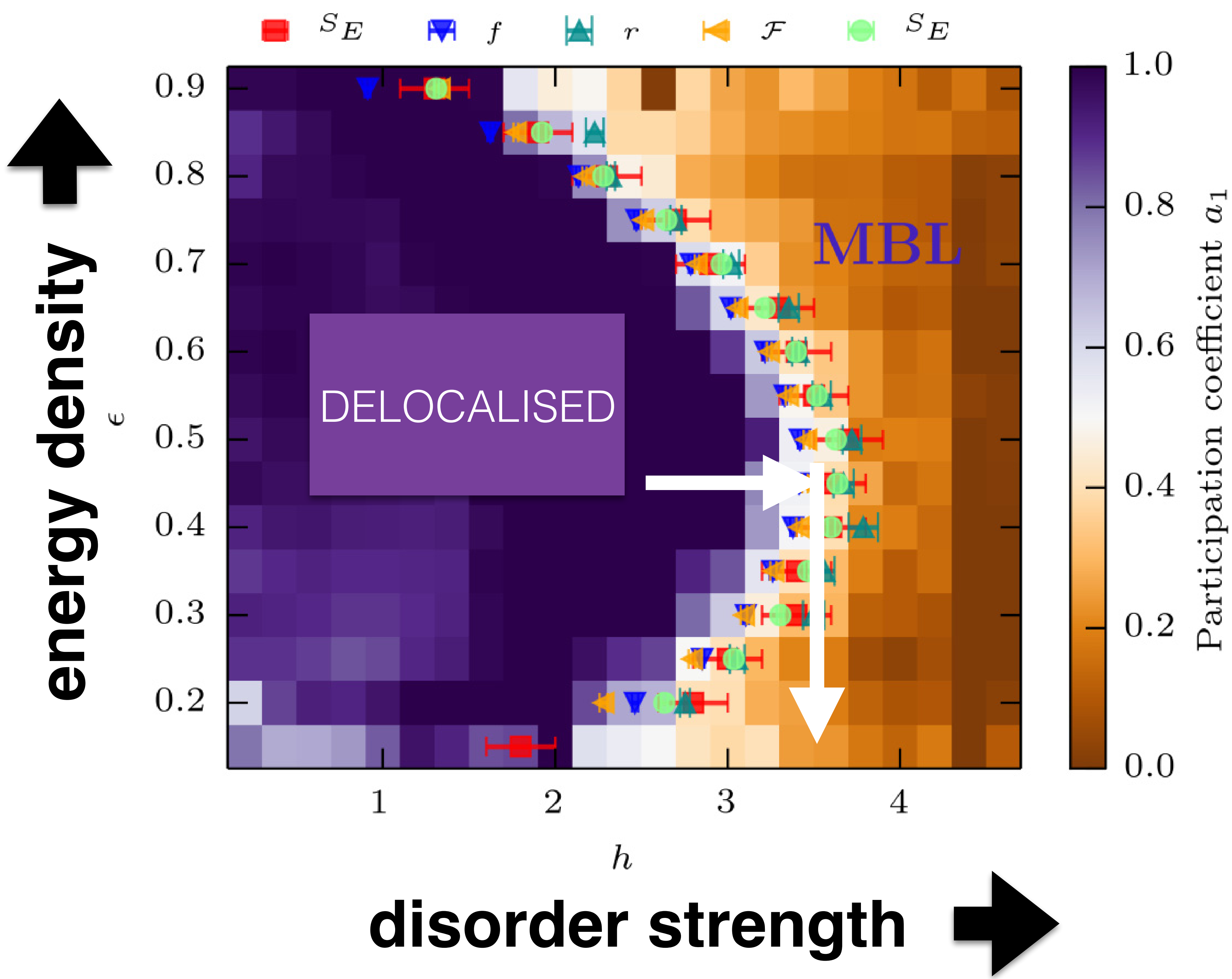





David J. Luitz, Nicolas Laflorencie, and Fabien Alet Phys. Rev. B 91, 081103(R) (2015)

PHYSICAL REVIEW B VOLUME 33, NUMBER 1 1 JANUARY 1986

# Multichannel Landauer formula for thermoelectric transport with application to thermopower near the mobility edge

U. Sivan and Y. Imry
*School of Physics and Astronomy, Tel Aviv University, 69978 Tel Aviv, Israel*
(Received 24 May 1985)

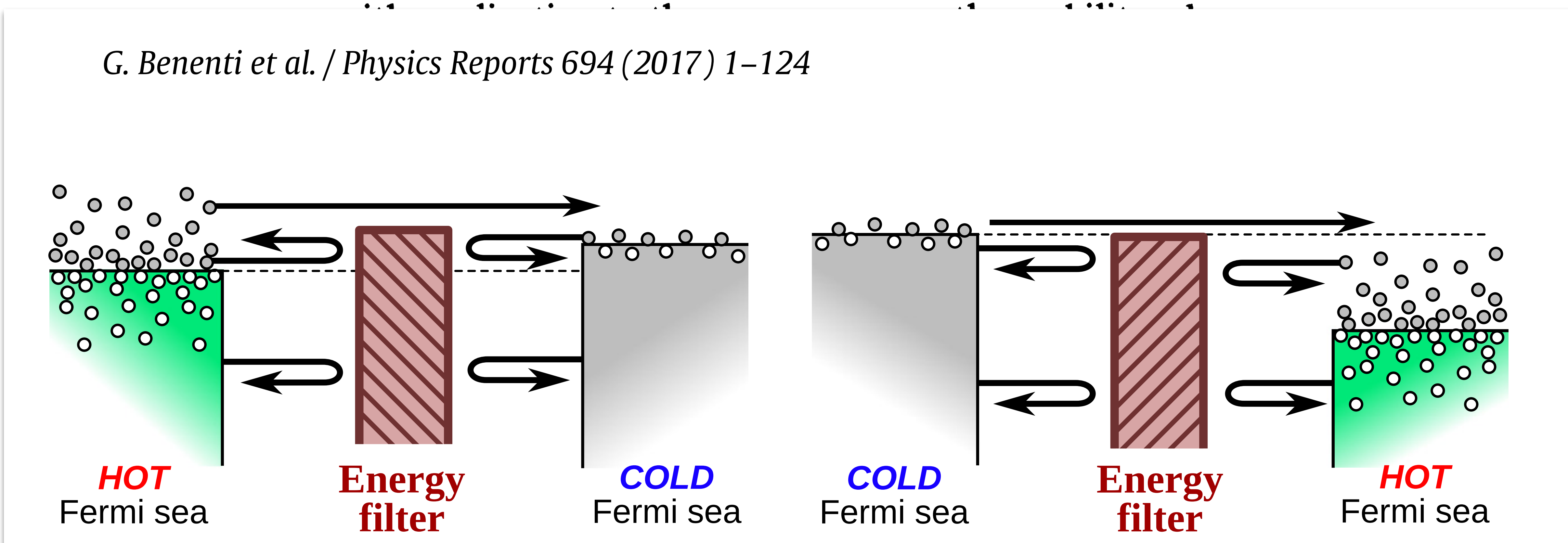
Various thermoelectric linear transport coefficients are defined and calculated for two reservoirs connected with ideal multichannel leads and a segment of an arbitrary disordered system. The reservoirs have different temperatures and chemical potentials. All of the inelastic scattering (and, thus, the dissipation) is assumed to occur only in the reservoirs. The definitions of the chemical potentials and temperature differences across the sample itself (mostly due to elastic scattering) are presented. Subtleties of the thermoelectric effects across the sample are discussed. The associated transport coefficients display deviations from the Onsager relations and from the Cutler-Mott formula for the thermopower (although the deviations vanish for a large number of channels and/or high resistance). The expression obtained is used to predict the critical behavior of the electronic thermopower near the mobility edge. It is shown to satisfy a scaling form in the temperature and separation from the mobility edge.

$h$

**disorder strength**

PHYSICAL REVIEW B VOLUME 33, NUMBER 1 1 JANUARY 1986

# Multichannel Landauer formula for thermoelectric transport

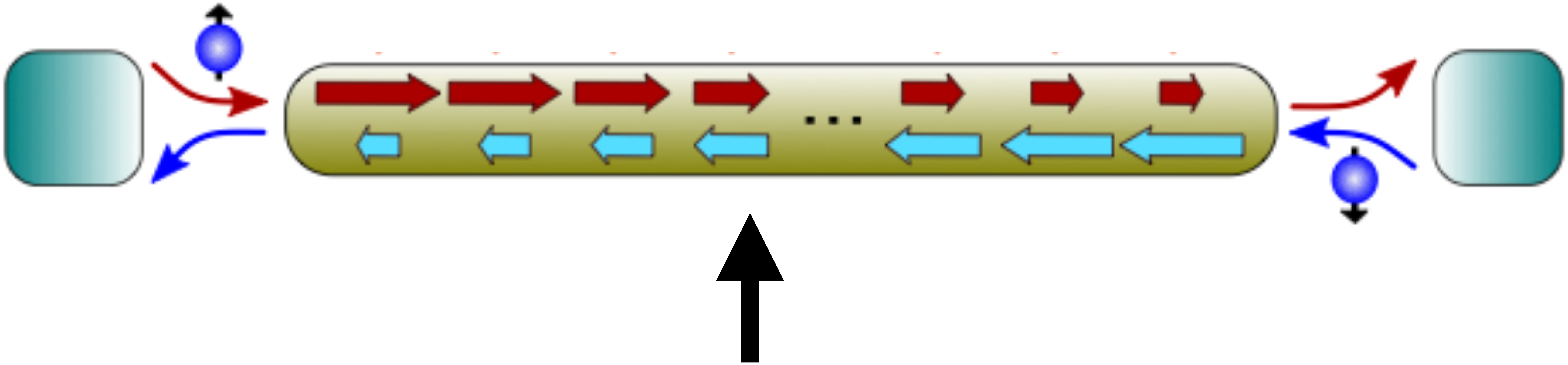



(b) Energy-filter as heat-engine. (c) Energy-filter as refrigerator.

$h$

**disorder strength**

# Spin Transport at high energy densities



$$H_{XXZ} = \sum_{i=1}^{N-1} \left[\alpha \left(\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y\right) + \Delta \sigma_i^z \sigma_{i+1}^z\right] + \sum_i h_i \sigma_i^z$$

# Boundary driving

$$\frac{d}{dt}\rho = i[\rho, H] + \mathcal{L}^{dis}(\rho)$$

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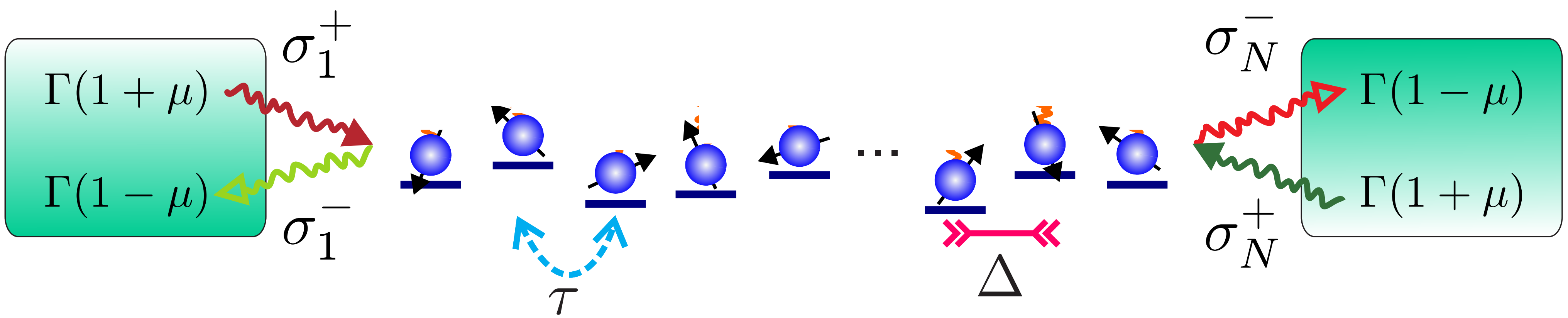
$$\mathcal{L}^{dis}(\rho) = \sum_k ([L_k\rho, L_k^\dagger] + [L_k, \rho L_k^\dagger])$$

$$\mathcal{L}^{boundary} = L_+^L + L_-^L + L_+^R + L_-^R$$

$$L_+^L = \sqrt{\Gamma(1+\mu)}\sigma_1^+ \qquad L_-^L = \sqrt{\Gamma(1-\mu)}\sigma_1^- \qquad L_+^R = \sqrt{\Gamma(1-\mu)}\sigma_L^+ \qquad L_-^R = \sqrt{\Gamma(1+\mu)}\sigma_L^-$$



# Non equilibrium steady state

System has unique non equilibrium steady state (NESS)

$$\lim_{t \to \infty} \hat{\rho}(t) = \hat{\rho}_\infty$$

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Think about spin current between sites

$$\frac{\mathrm{d}\hat{\sigma}_\ell^z}{\mathrm{d}t} = \hat{j}_\ell - \hat{j}_{\ell-1}$$

$$\hat{j}_\ell = \mathrm{i}[\hat{\sigma}_\ell^z, \hat{H}] = 2(\hat{\sigma}_\ell^x \hat{\sigma}_{\ell+1}^x + \hat{\sigma}_\ell^y \hat{\sigma}_{\ell+1}^y)$$

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Now try to compute expectation value in steady state

$$\langle \hat{j}_\ell \rangle = tr(\hat{\rho}_\infty \hat{j}_\ell)$$

# Scaling theory

Basic microscopic transport theory says variance of local inhomogeneity grows like:

$$\langle \Delta x^2 \rangle = 2Dt^{2\alpha}$$

$$0 < \alpha \leq 1$$

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Exponents are related via:

$$\alpha = \frac{1}{1+\nu}$$

B. Li and J. Wang, Phys. Rev. Lett. 91, 044301 (2003)

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$$\nu \geq 0$$

Diffusion $\langle J \rangle \propto \frac{1}{L}$ $\nu = 1$

$$< j_l > = -D\nabla < \sigma_l^z >$$

$$\alpha = \frac{1}{1+\nu}$$

# Scaling theory

Basic microscopic transport theory says variance of local inhomogeneity grows like:

$$\langle \Delta x^2 \rangle = 2Dt^{2\alpha}$$

$$0 < \alpha \leq 1$$

Ballistic transport

$$D \propto L \qquad \nu = 0$$

$$\langle j \rangle \propto \frac{1}{L^\nu}$$

$$\nu \geq 0$$

$L$

$$\alpha = \frac{1}{1+\nu}$$

$$< j_l >= -D\nabla < \sigma_l^z >$$

# Scaling theory

Basic microscopic transport theory says variance of local

$$\langle \Delta x^2 \rangle = 2Dt^{2\alpha}$$

$$0 < \alpha \le 1$$

| | |
|---|---|
| Anomalous diffusion | $D \propto L^{1-\nu}$ |
| Sub-diffusion | $\nu > 1$ |
| Super-diffusion | $\nu < 1$ |

$$\langle j \rangle \propto \frac{1}{L^{\nu}}$$

$$\nu \ge 0$$

$$\alpha = \frac{1}{1+\nu}$$

$$< j_l > = -D\nabla < \sigma_l^z >$$

# Scaling theory

Basic microscopic transport

th

$$\langle \Delta x^2 \rangle = 2Dt^{2\alpha}$$

$$\alpha \leq 1$$

Localization

$$< j > \propto \exp(-L) \qquad \nu \to \infty$$

$$\propto \frac{1}{L^{\nu}}$$

Super-diffusion $\nu < 1$

$$\nu \geq 0$$

Diffusion $\langle j \rangle \propto \frac{1}{L}$ $\nu = 1$

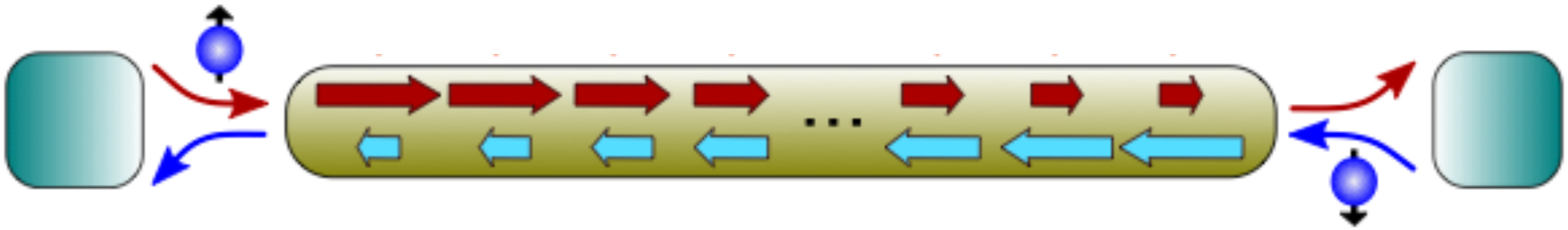
$$\alpha = \frac{1}{1+\nu}$$

$$< j_l > = -D\nabla < \sigma_l^z >$$

# Clean case

## Marko Znidaric PRL 2011

$$H_{XXZ} = \sum_{i=1}^{N-1} \left[ \alpha \left( \sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y \right) + \Delta \sigma_i^z \sigma_{i+1}^z \right]$$



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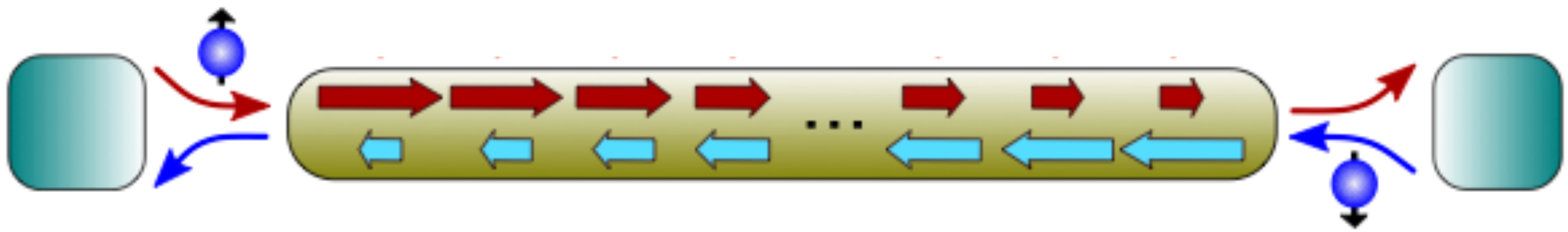
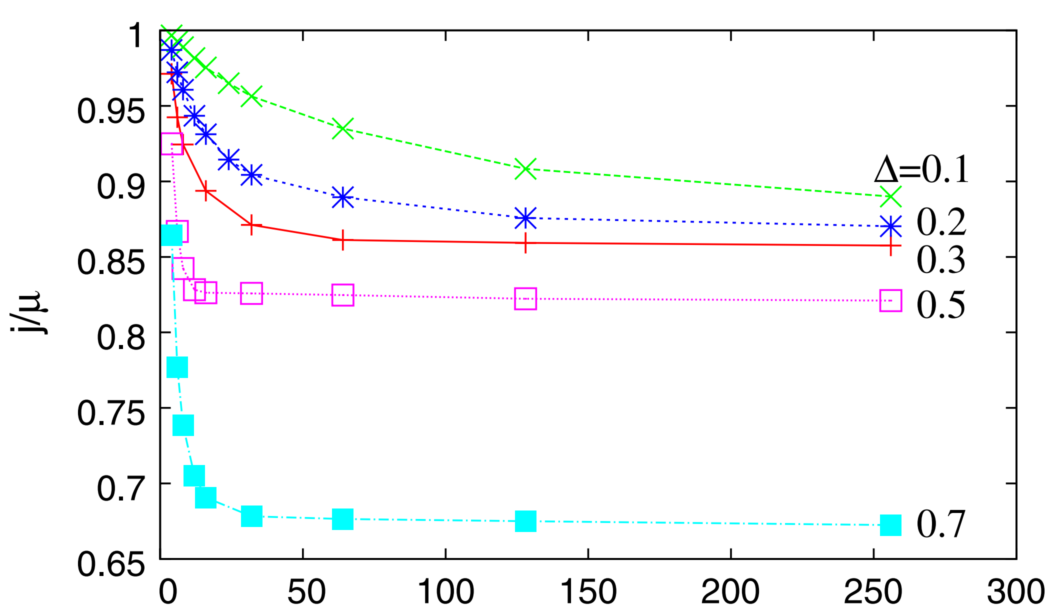




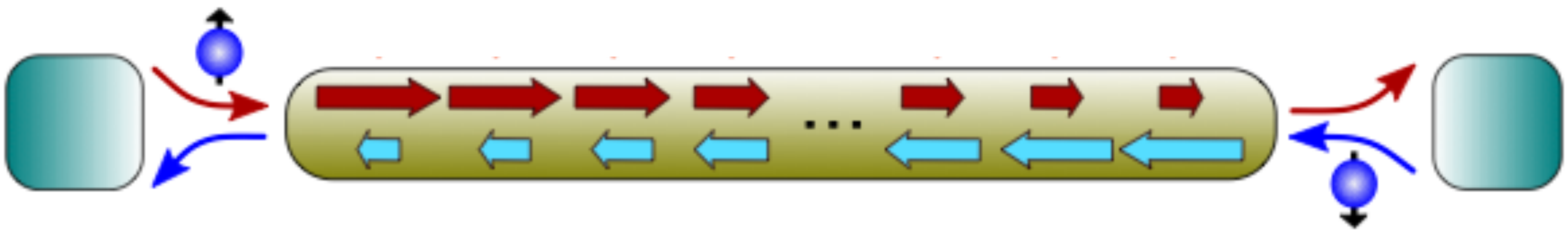

$$\Delta < 1 \quad \nu = 1$$

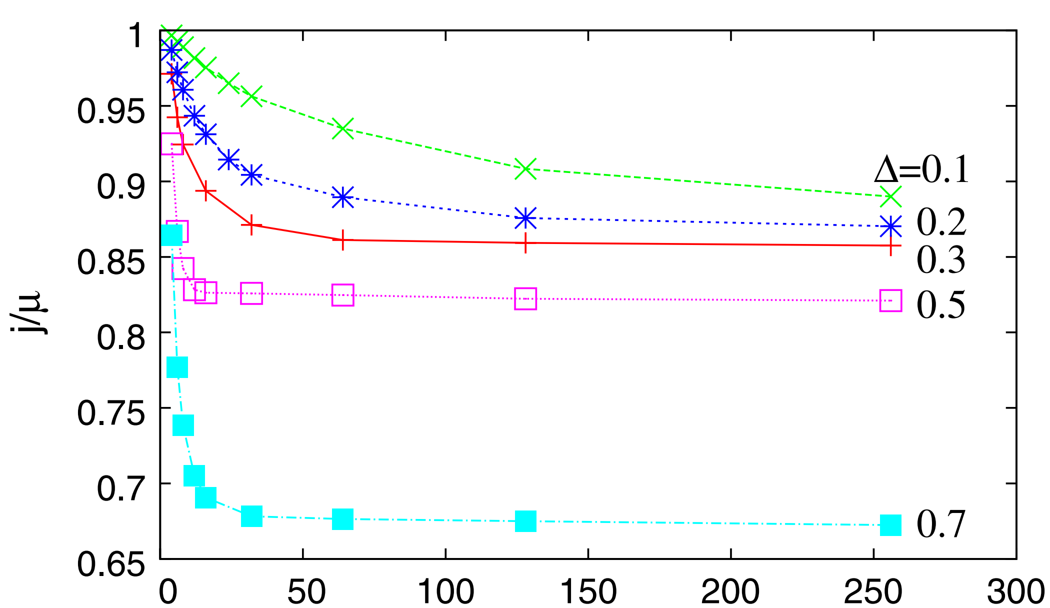
Ballistic

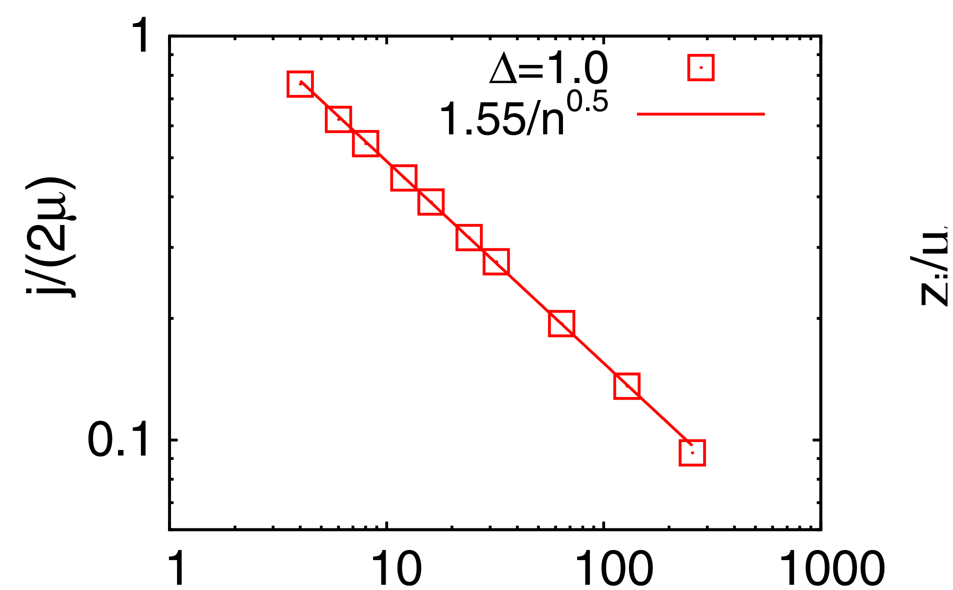
# Clean case

## Marko Znidaric PRL 2011

$$H_{XXZ} = \sum_{i=1}^{N-1} \left[ \alpha \left( \sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y \right) + \Delta \sigma_i^z \sigma_{i+1}^z \right]$$






$\Delta < 1 \quad \nu = 1$

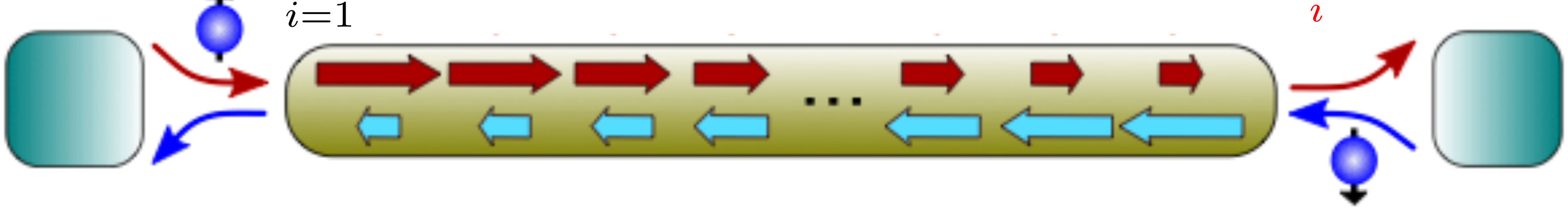
Ballistic

$\Delta = 1 \quad \nu = 0.5$

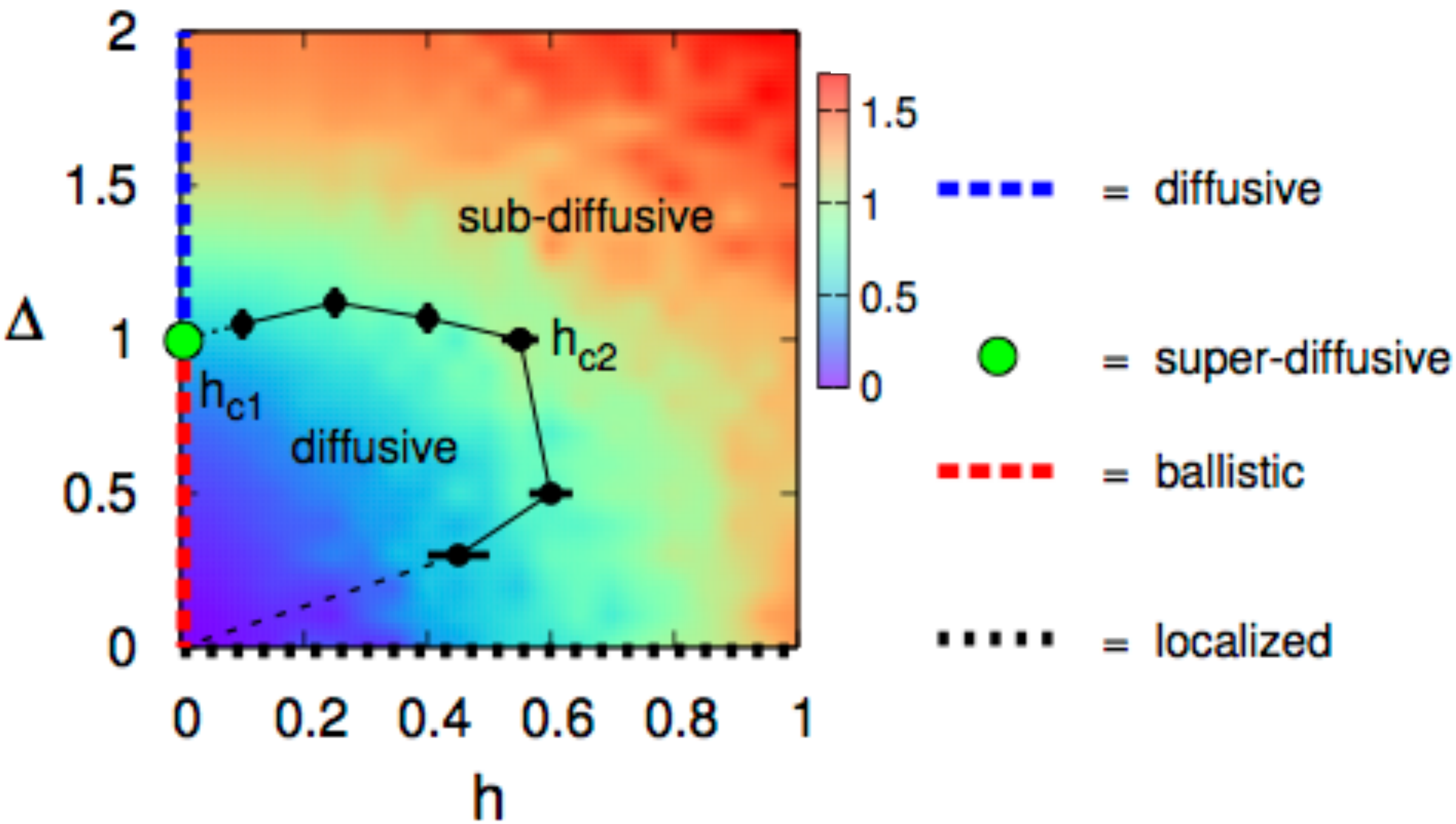
Super-diffusive

# with disorder

$$H_{XXZ} = \sum_{i=1}^{N-1} \left[\alpha\left(\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y\right) + \Delta \sigma_i^z \sigma_{i+1}^z\right] + \sum_i h_i \sigma_i^z$$

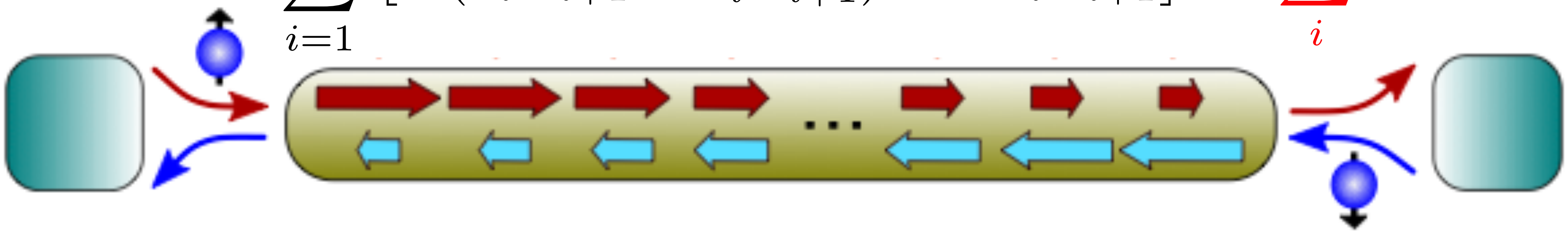


**Marko Znidaric et al PRL 2016**



# with disorder

$$H_{XXZ} = \sum_{i=1}^{N-1} \left[\alpha\left(\sigma_i^x\sigma_{i+1}^x + \sigma_i^y\sigma_{i+1}^y\right) + \Delta\sigma_i^z\sigma_{i+1}^z\right] + \sum_i h_i\sigma_i^z$$



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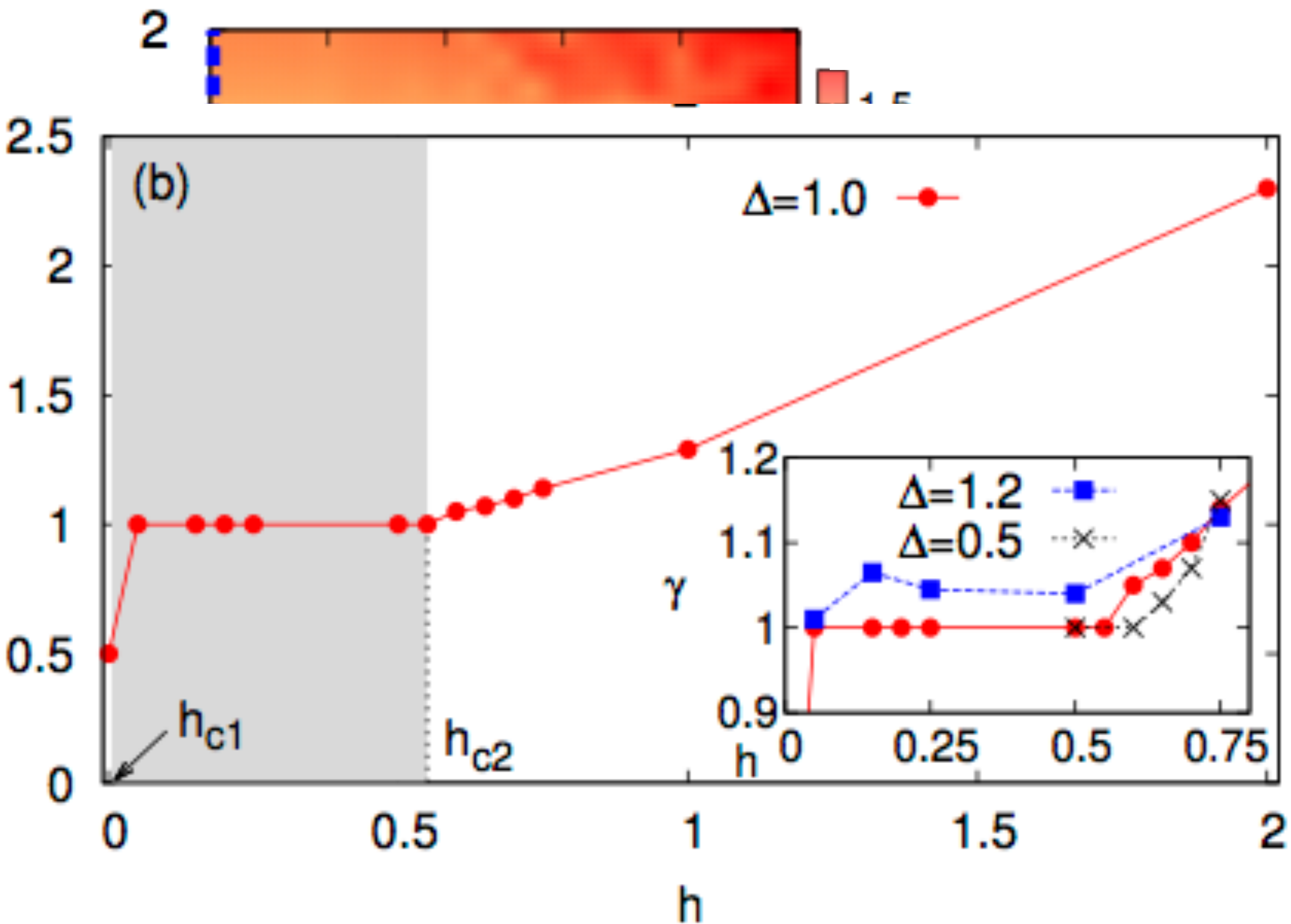
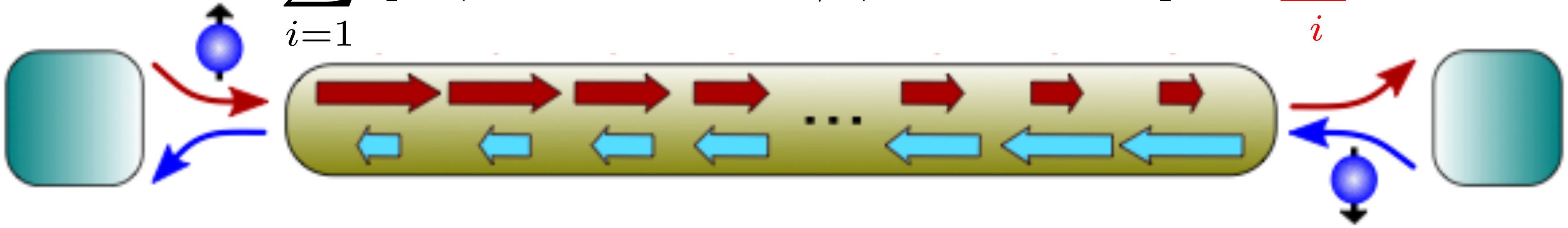


# with disorder

$$H_{XXZ} = \sum_{i=1}^{N-1} \left[\alpha\left(\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y\right) + \Delta \sigma_i^z \sigma_{i+1}^z\right] + \sum_i h_i \sigma_i^z$$



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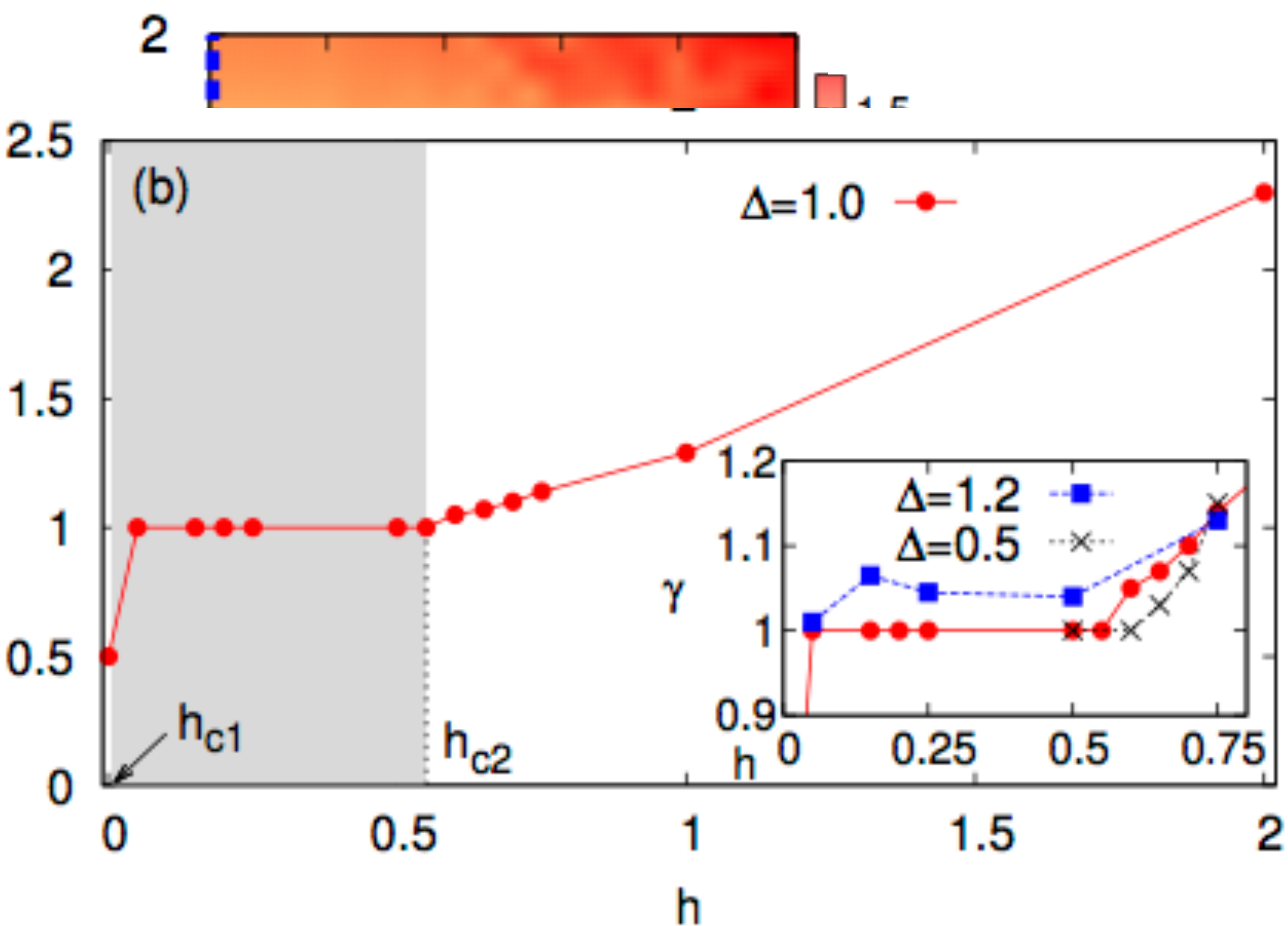
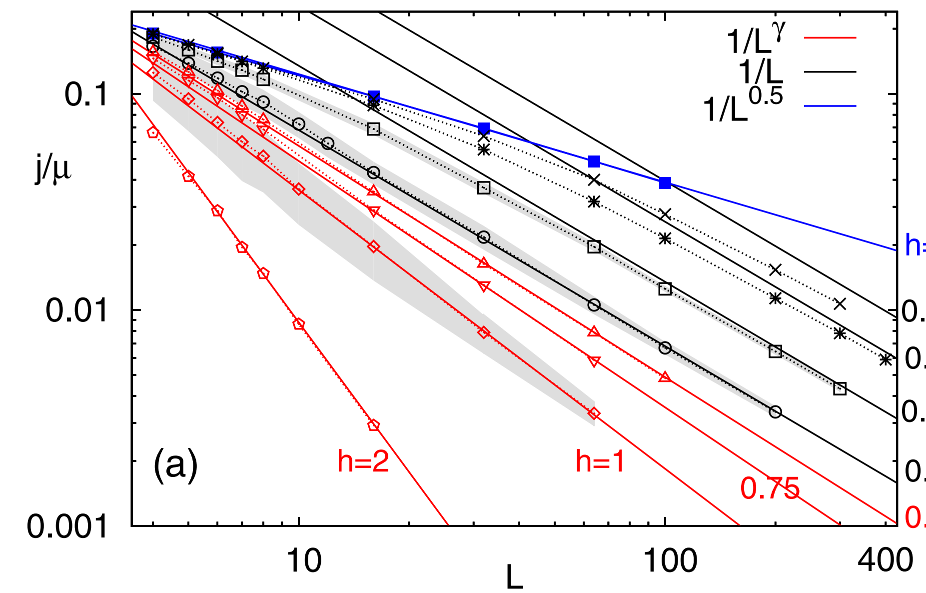
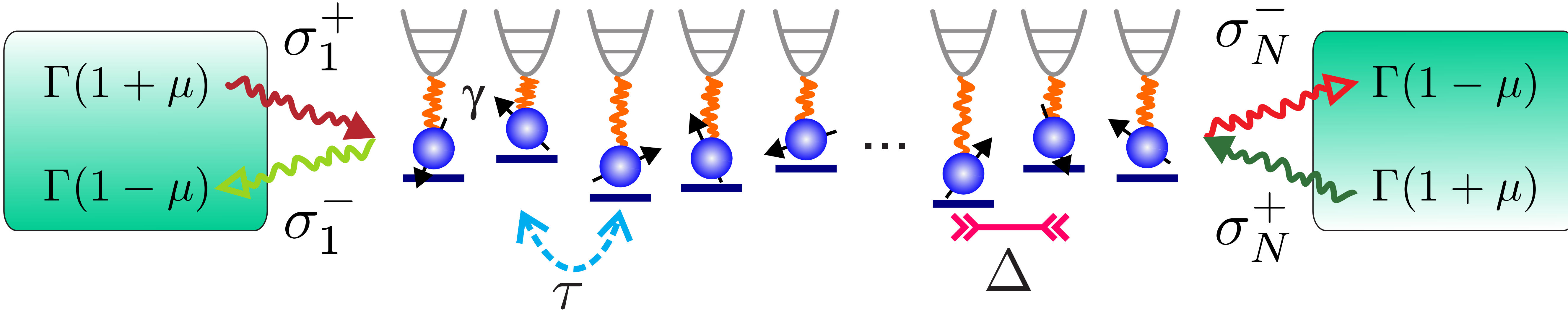



$$\eta = 1.33$$

$$L^* \propto h^{-\eta}$$

# With Dephasing




Marko Žnidarič, Juan Jose Mendoza-Arenas, Stephen R. Clark and John Goold

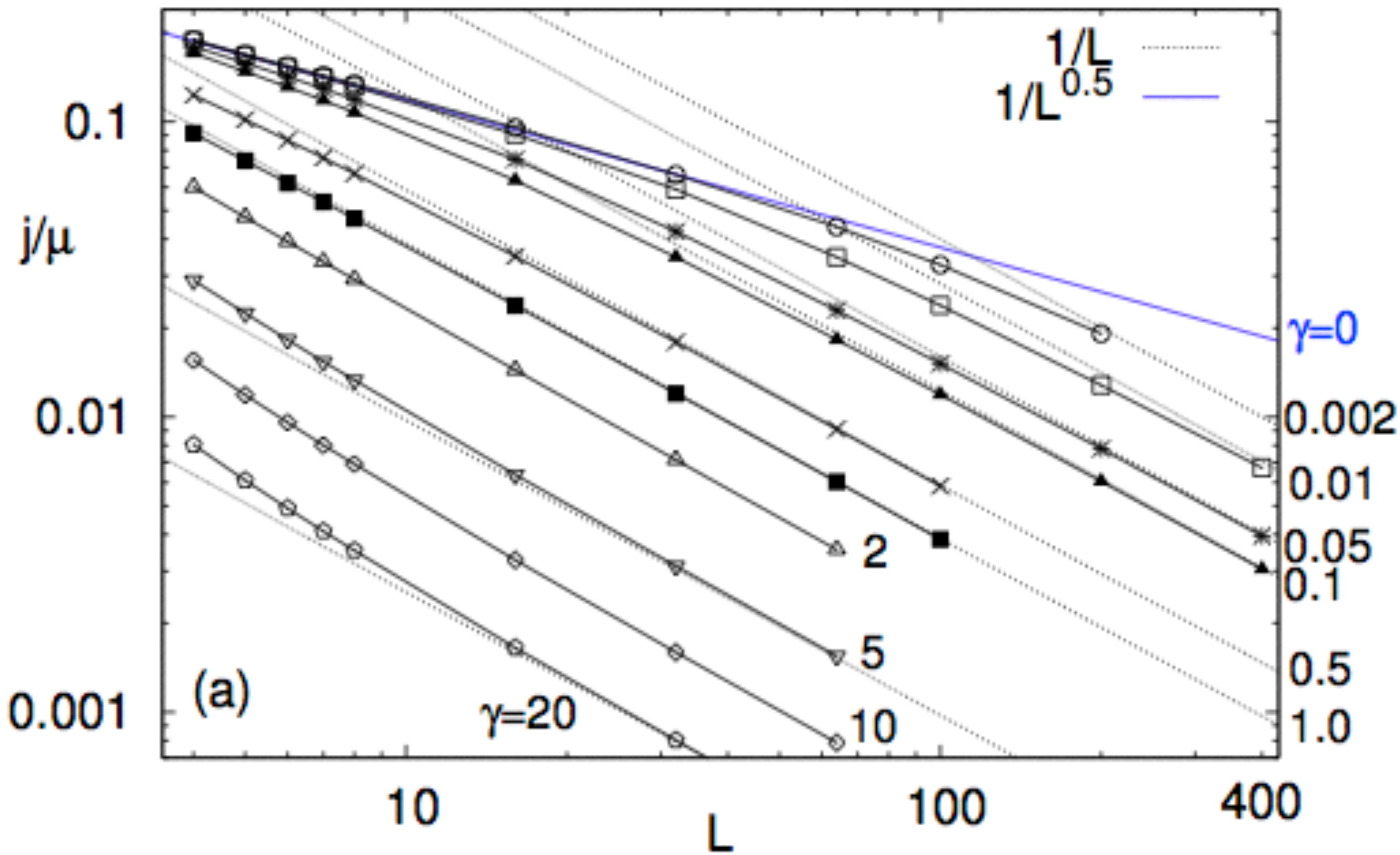
Annalen Der Physik 529, 1600298 (2017)

$$\mathcal{L}^{dis} = \mathcal{L}^{boundary} + \mathcal{L}^{dephasing}$$

$$\mathcal{L}^{dephasing} = \sum_{j=1}^{L} \sqrt{\frac{\gamma}{2}} \sigma_j^z$$

# Dephasing but no disorder

At the isotropic point: $\Delta = 1$ $\nu = 1/2$



# emergent length scale

timescale associated to dephasing: $\tau \propto \frac{1}{\gamma}$

inhomgeniety spreads length: $\sqrt{\langle \Delta x^2 \rangle} \propto t^{\alpha}$ $\quad 0 < \alpha \leq 1$

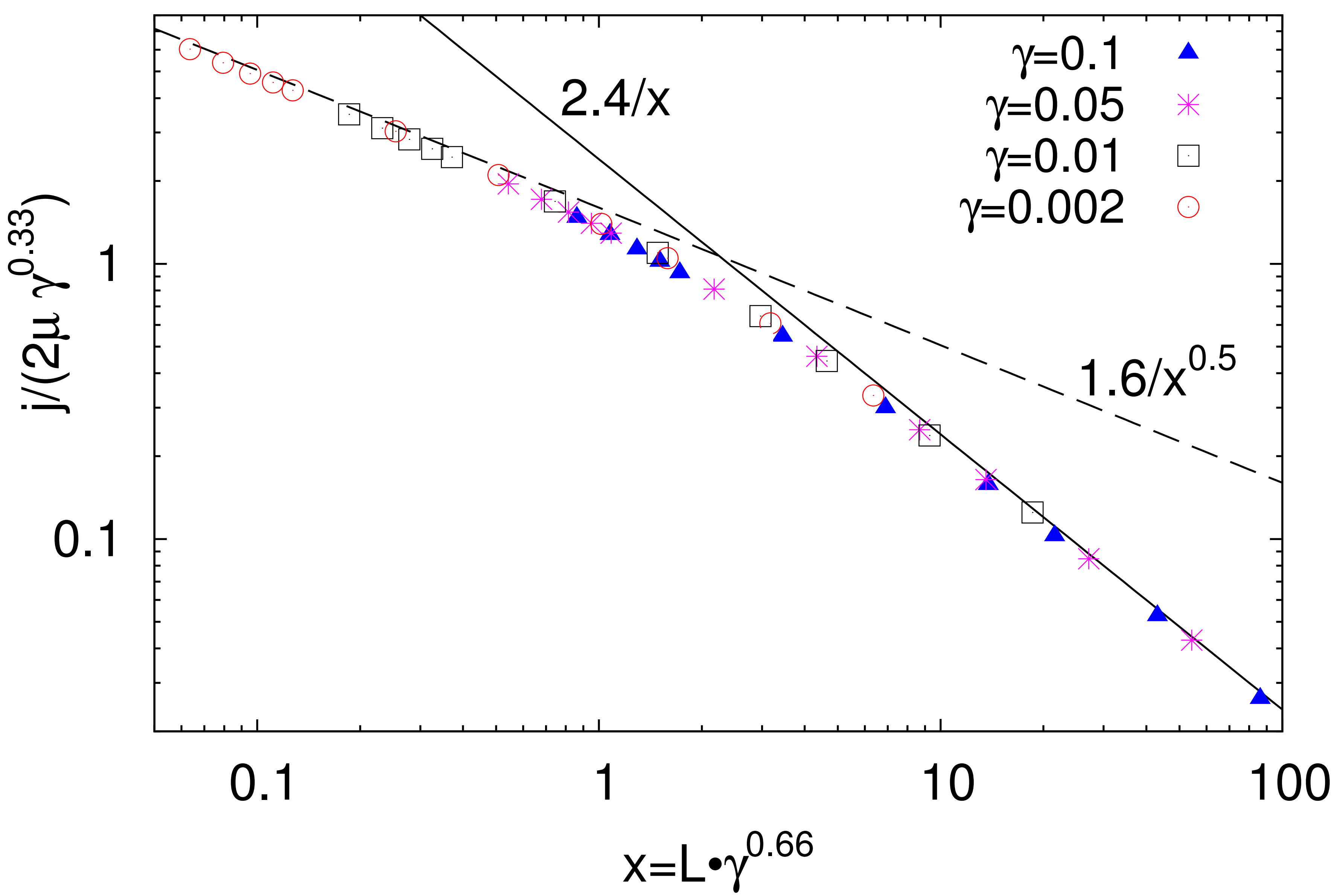
recall $\alpha = \frac{1}{1+\nu}$

exponent for clean case

so: $$L_\gamma \propto \gamma^{-\frac{1}{\nu+1}}$$

Length scale after which disorder dominates

# emergent length scale - weak dephasing




axis rescaled

$$x = L/L_\gamma$$

$$j' = jL_\gamma$$

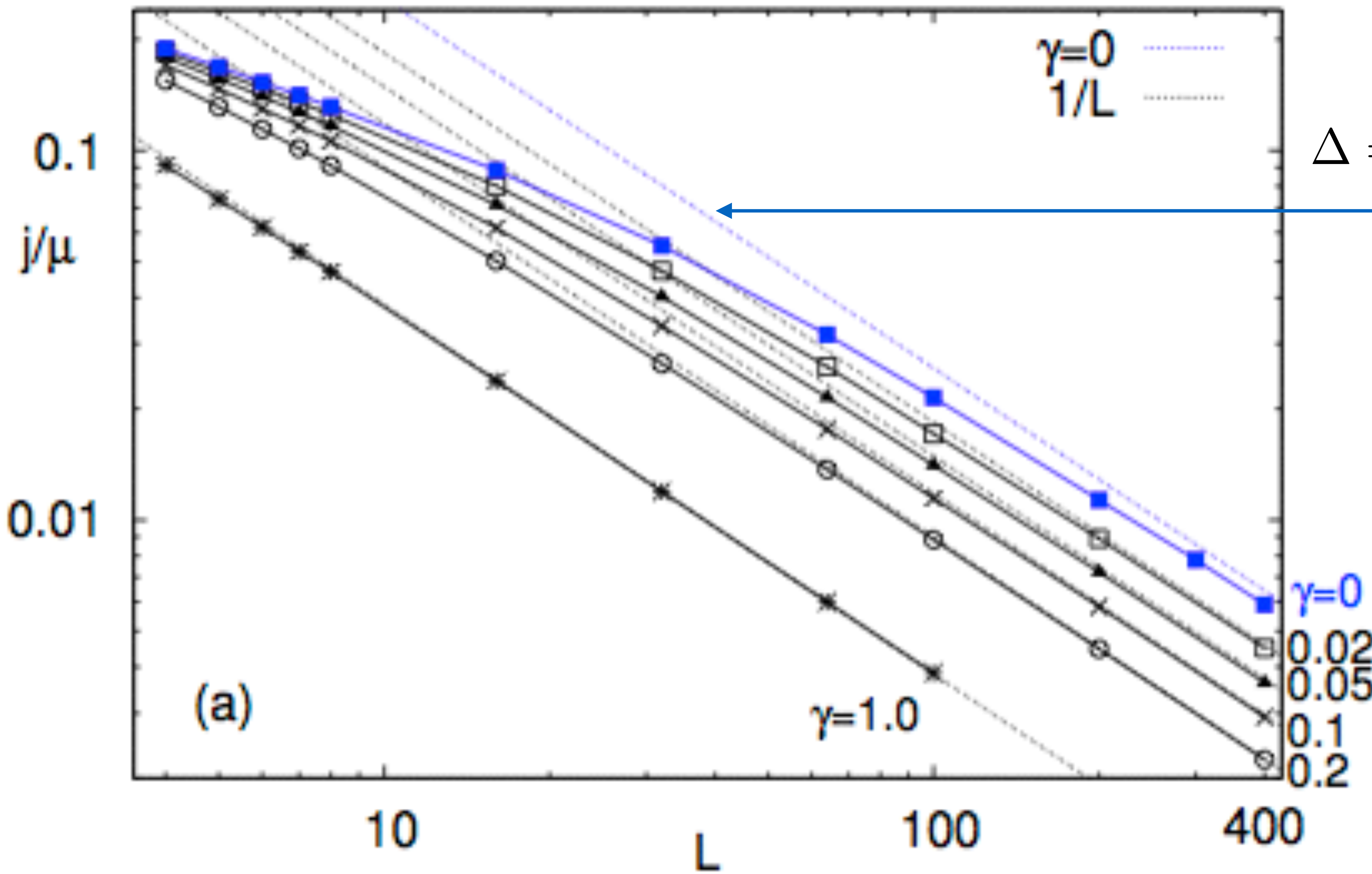
Universal form

$$j' \propto x^{-\nu}, x \leq 1$$

$$j' \propto x^{-1}, x \geq 1$$

# Dephasing low disorder

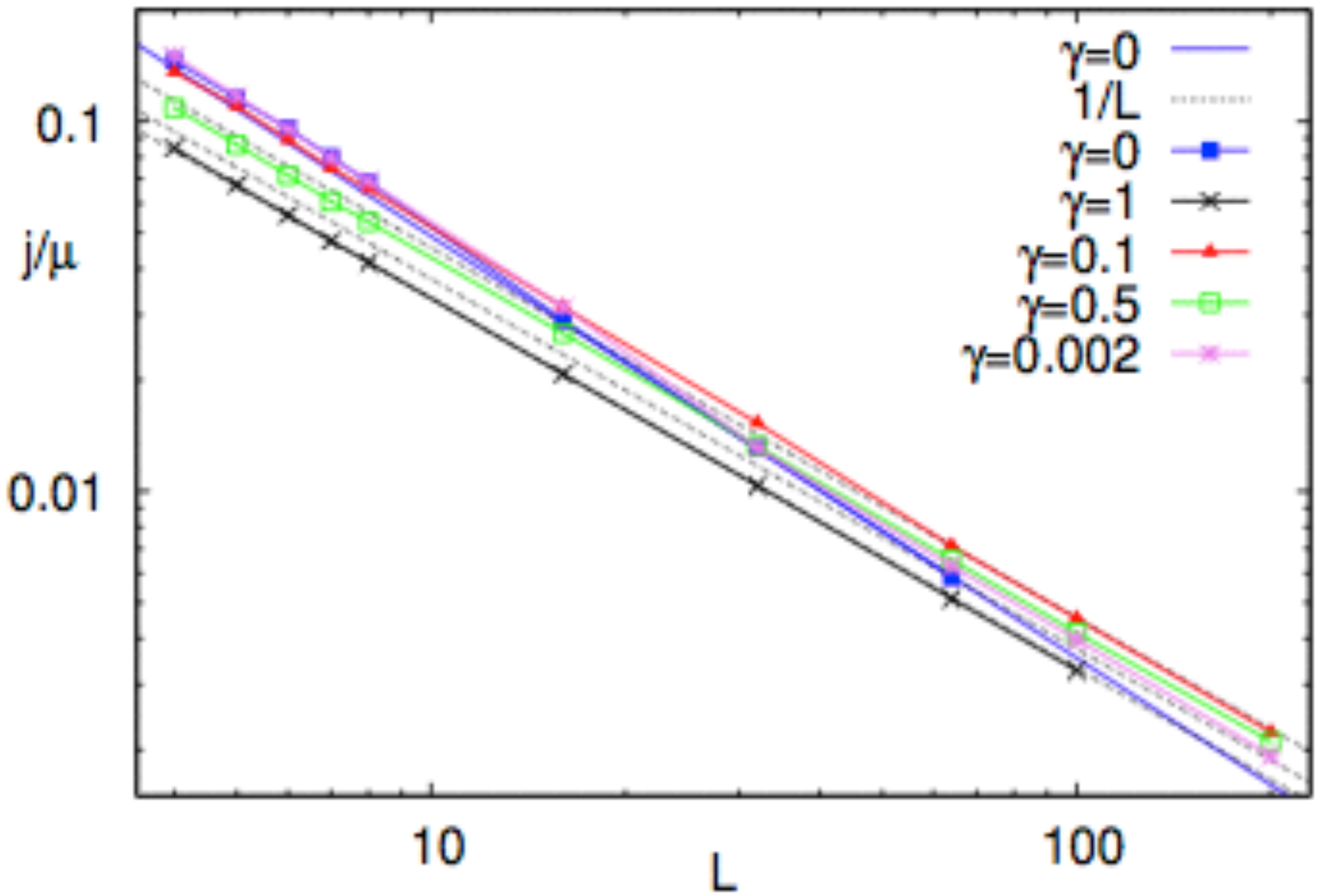
System is diffusive in absence of dephasing $|h| = 0.2$




$\Delta = 1, \nu = 1/2$

# Dephasing “higher” disorder

System is sub-diffusive in absence of dephasing $|h| = 1.5$



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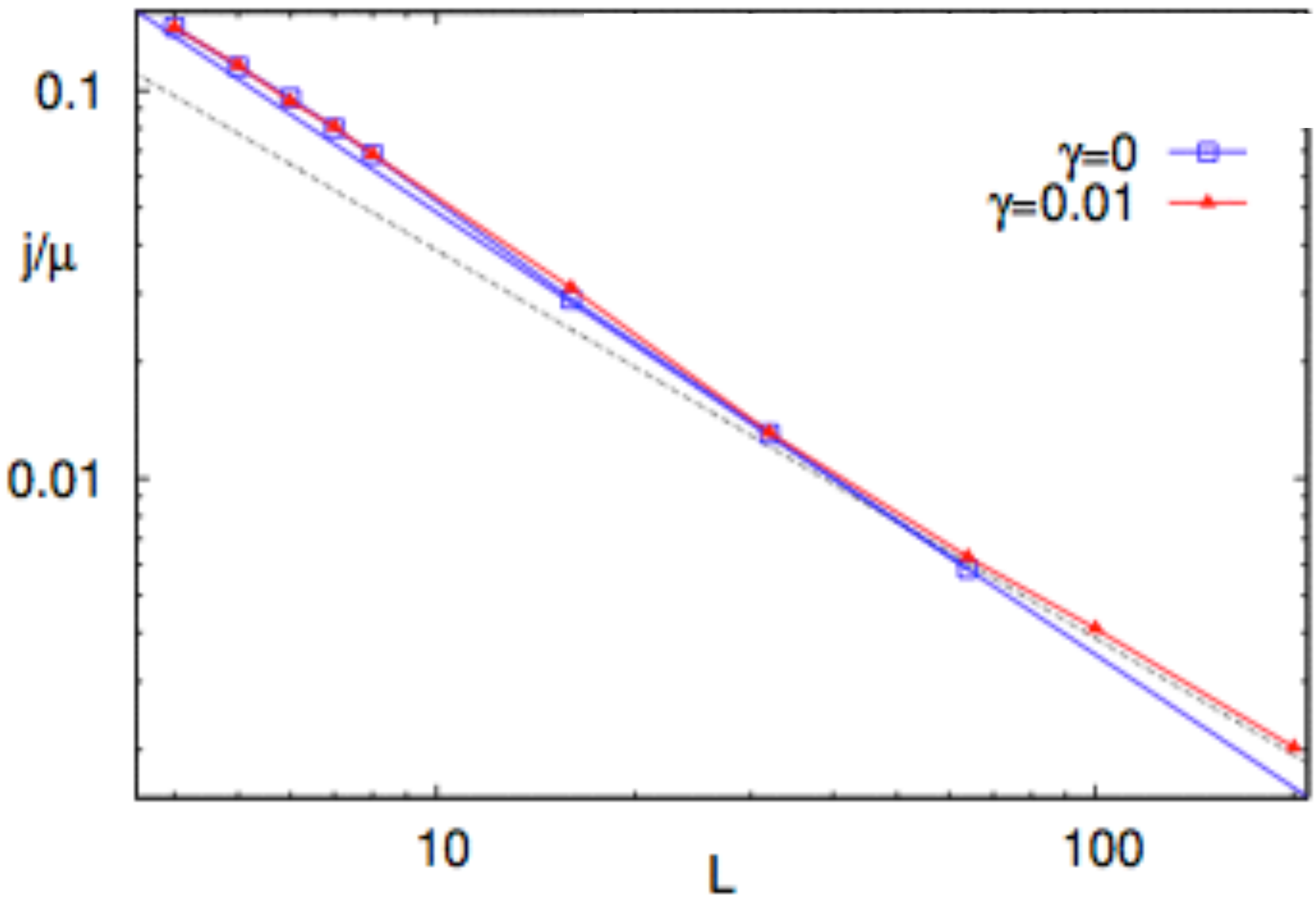
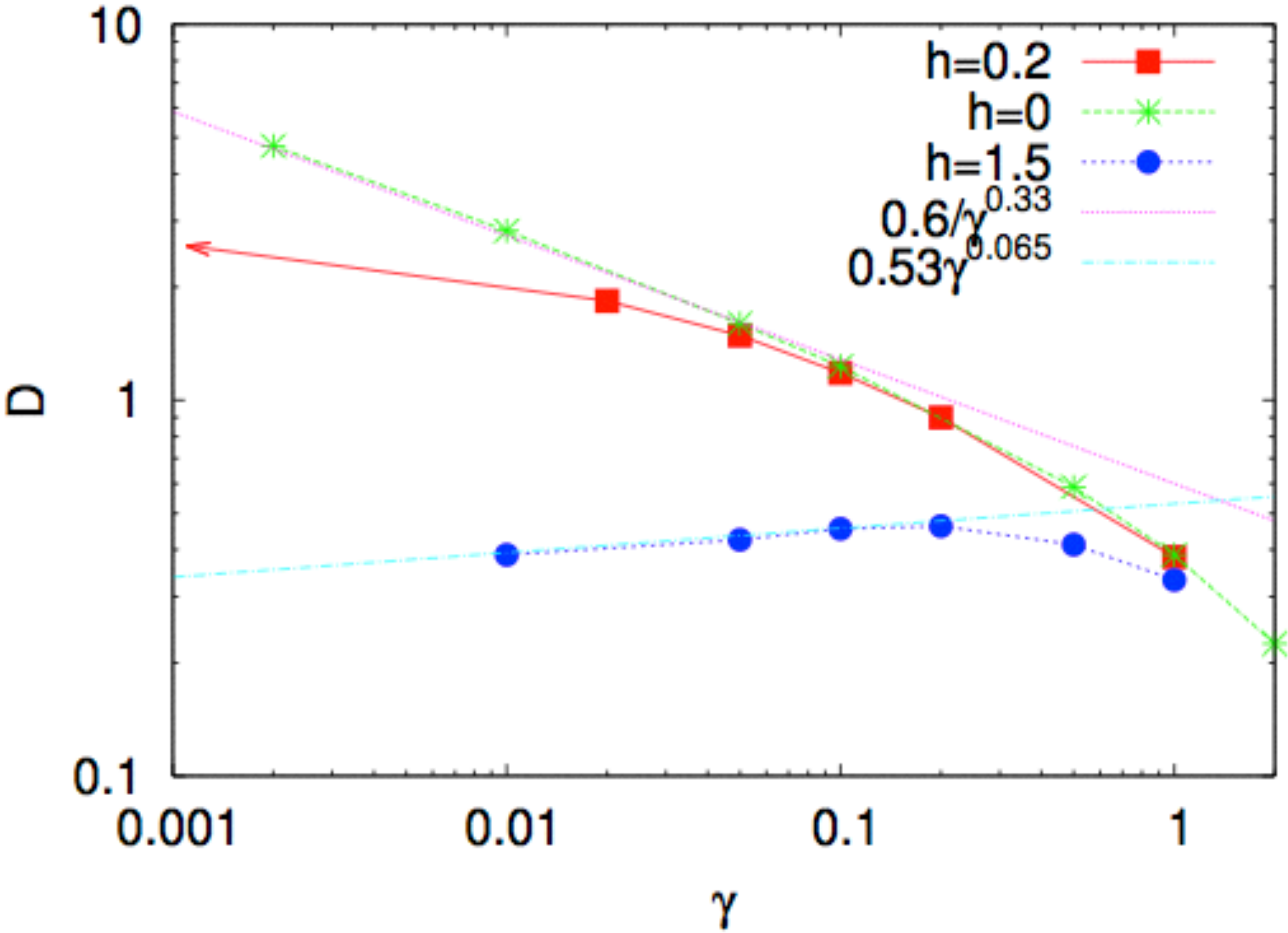


# Extraction of diffusion coefficient

System is sub-diffusive in absence of dephasing $|h| = 1.5$




Identified length scale after which disorder becomes important

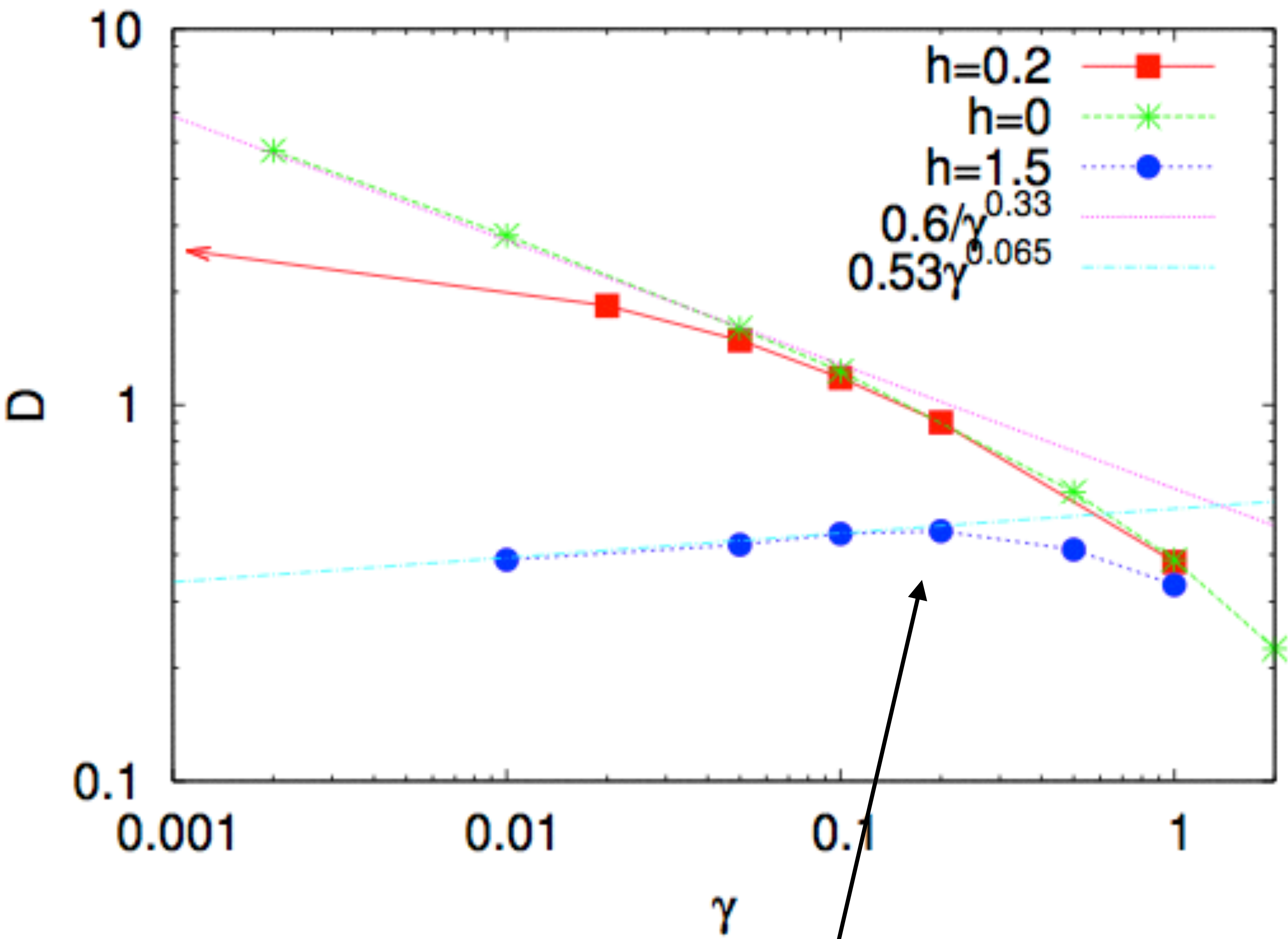
$$L_\gamma = \gamma^{-\frac{1}{\nu+1}}$$

Can compete with

$$L^* \propto h^{-\eta}, \eta = 1.33$$

# Extraction of diffusion coefficient

System is sub-diffusive in absence of dephasing $|h| = 1.5$




Identified length scale after which disorder becomes important

$$L_\gamma = \gamma^{-\frac{1}{\nu+1}}$$

Can compete with

$$L^* \propto h^{-\eta}, \eta = 1.33$$

**Dephasing enhanced transport !!!**

# Main Points for steady state transport

- Boundary driving can be a useful tool to understand asymptotic transport at high energy densities

- Large L scaling of xxz with disorder is highly non trivial

Our work on disorder + dephasing shows that dephasing enhanced transport must exist due to competition of lenghtscales

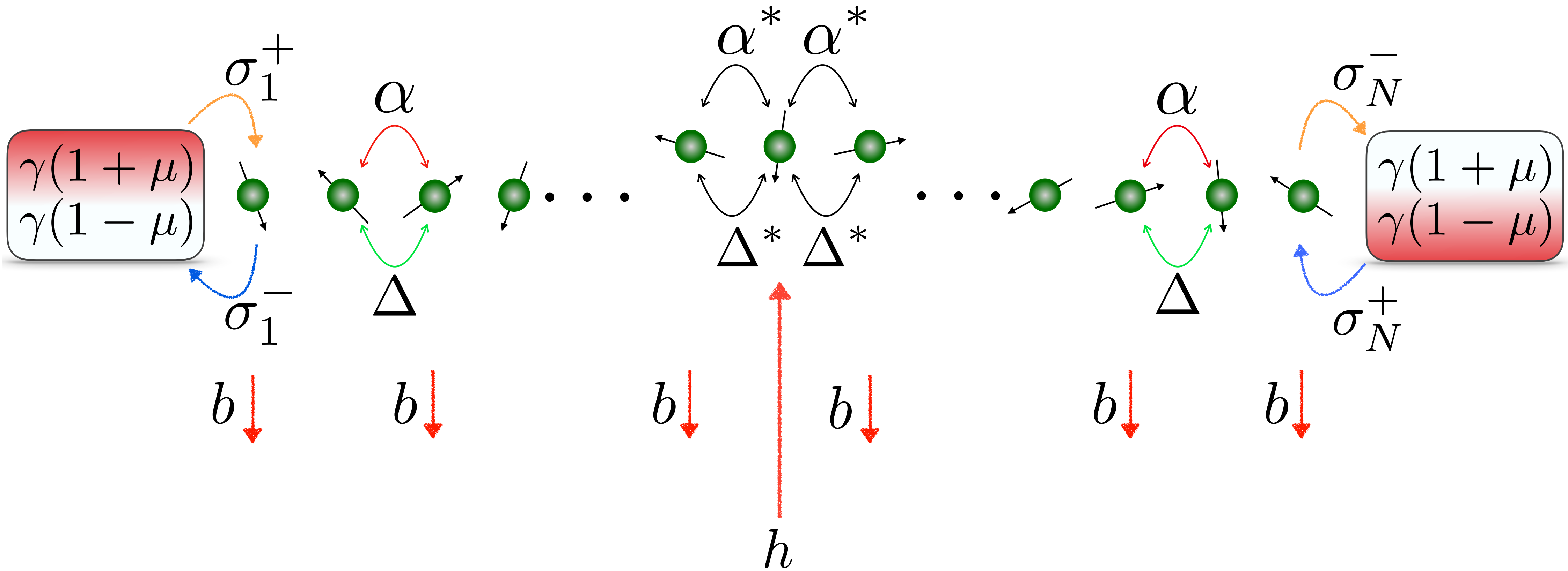
Generically, the weak disorder limit physics is dominated by the clean system physics

## Work in progress

THE ROYAL SOCIETY

With Marlon Brenes

See poster at conference




$$H = H_{XXZ} + H_{IB}$$

where

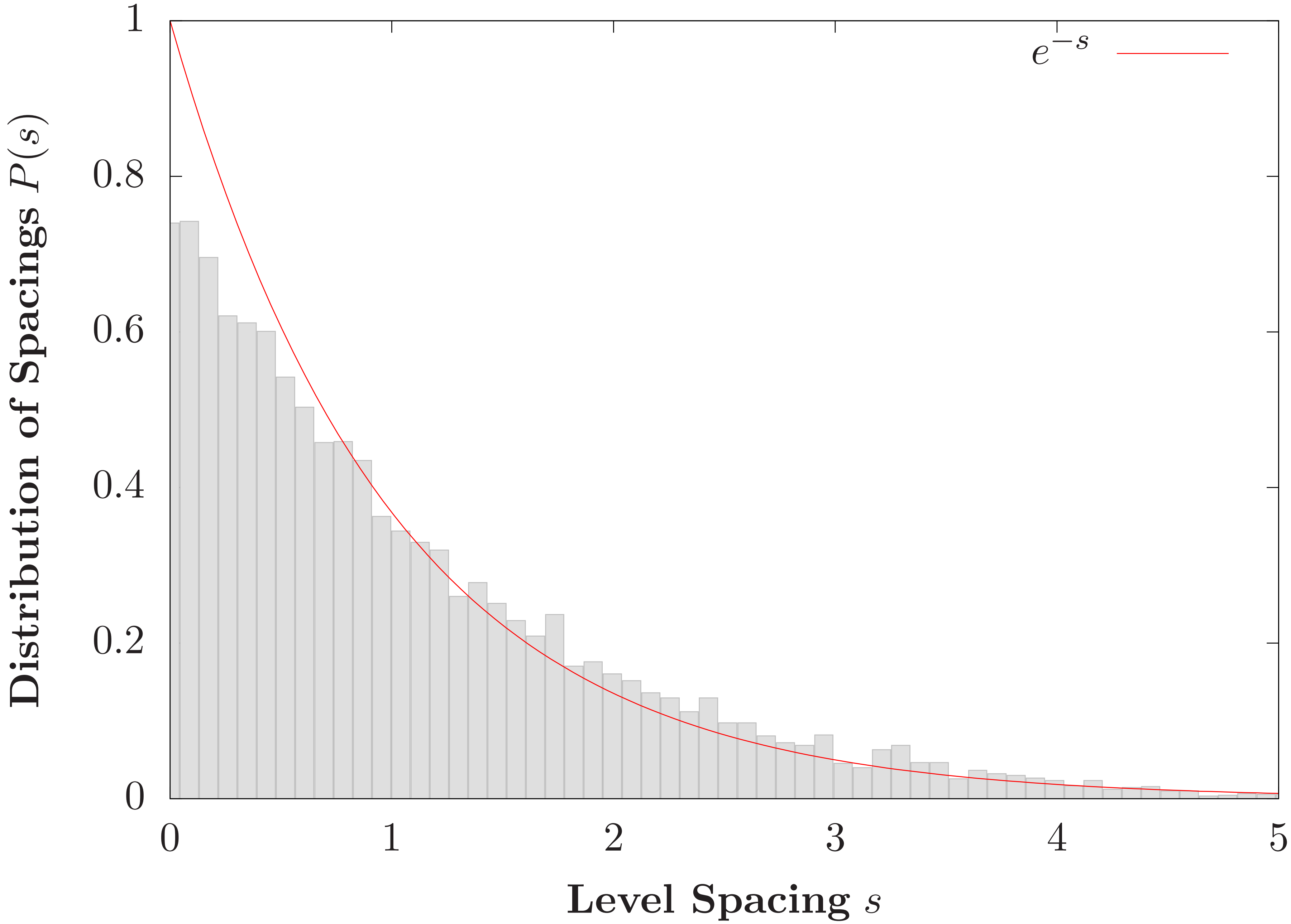
$$H_{XXZ} = \sum_{i=1}^{N-1} \left[\alpha \left(\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y\right) + \Delta \sigma_i^z \sigma_{i+1}^z\right]$$

$$H_{IB} = \begin{cases} \sum_{i=odd}^{N} b_i \sigma_i^z & \rightarrow \text{Staggered magnetic field} \\ h\sigma_{N/2}^z & \rightarrow \text{Single magnetic impurity} \end{cases}$$

[........h 0 h 0 h 0 .....]

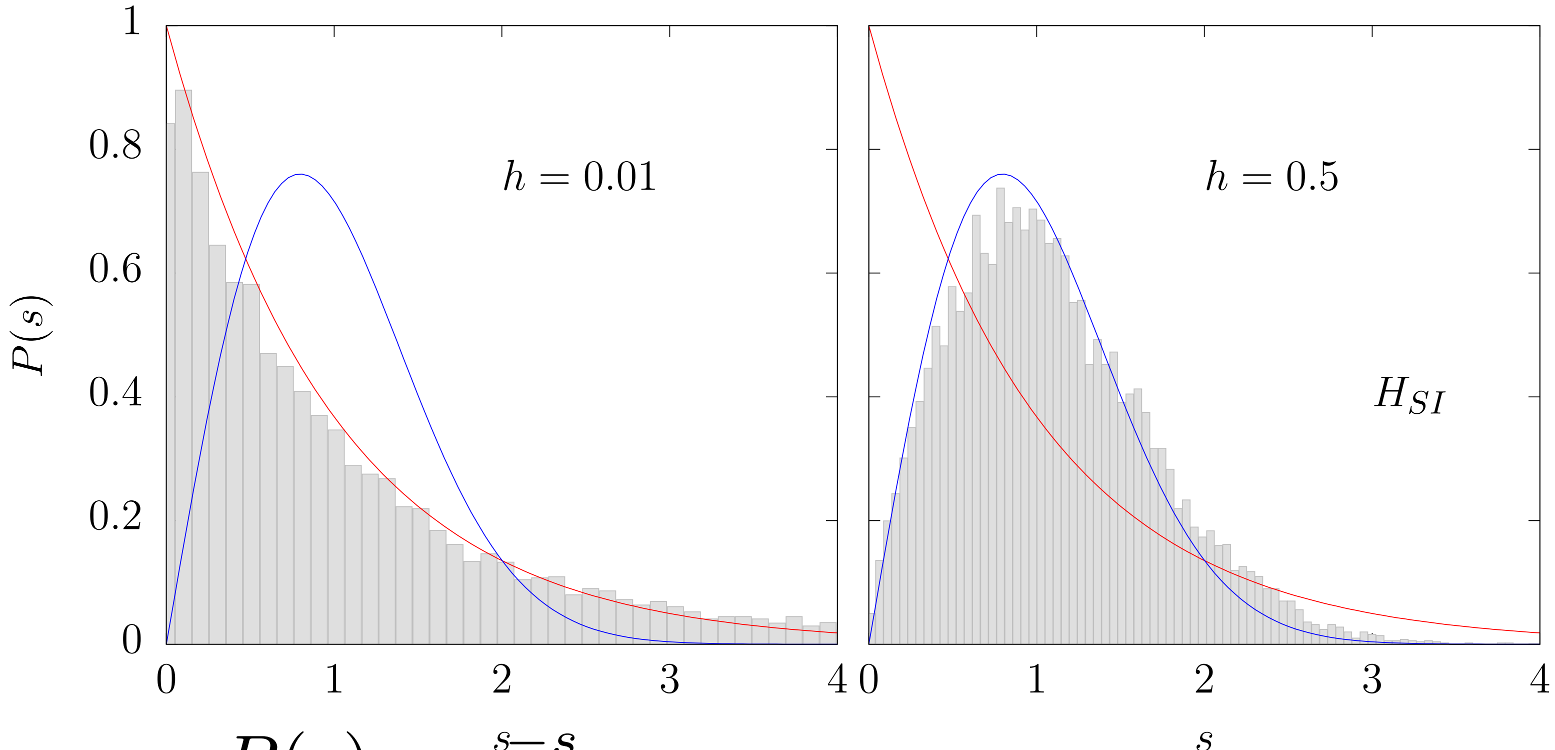
# Level spacing distribution

$$H = t\sum_i \sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y \quad + V\sum_i \sigma_z^i \sigma_z^{i+1}$$



# Single Magnetic Defect

$$s_n = (E_{n+1} - E_n)/\Omega, \ \Omega : \text{Average level spacing}$$




$$P(s) = e^{-s}$$

$$P(s) = \frac{\pi s}{2} e^{\frac{-\pi s^2}{4}}$$

$$H_{SI} = H_{XXZ} + H_{IB}$$

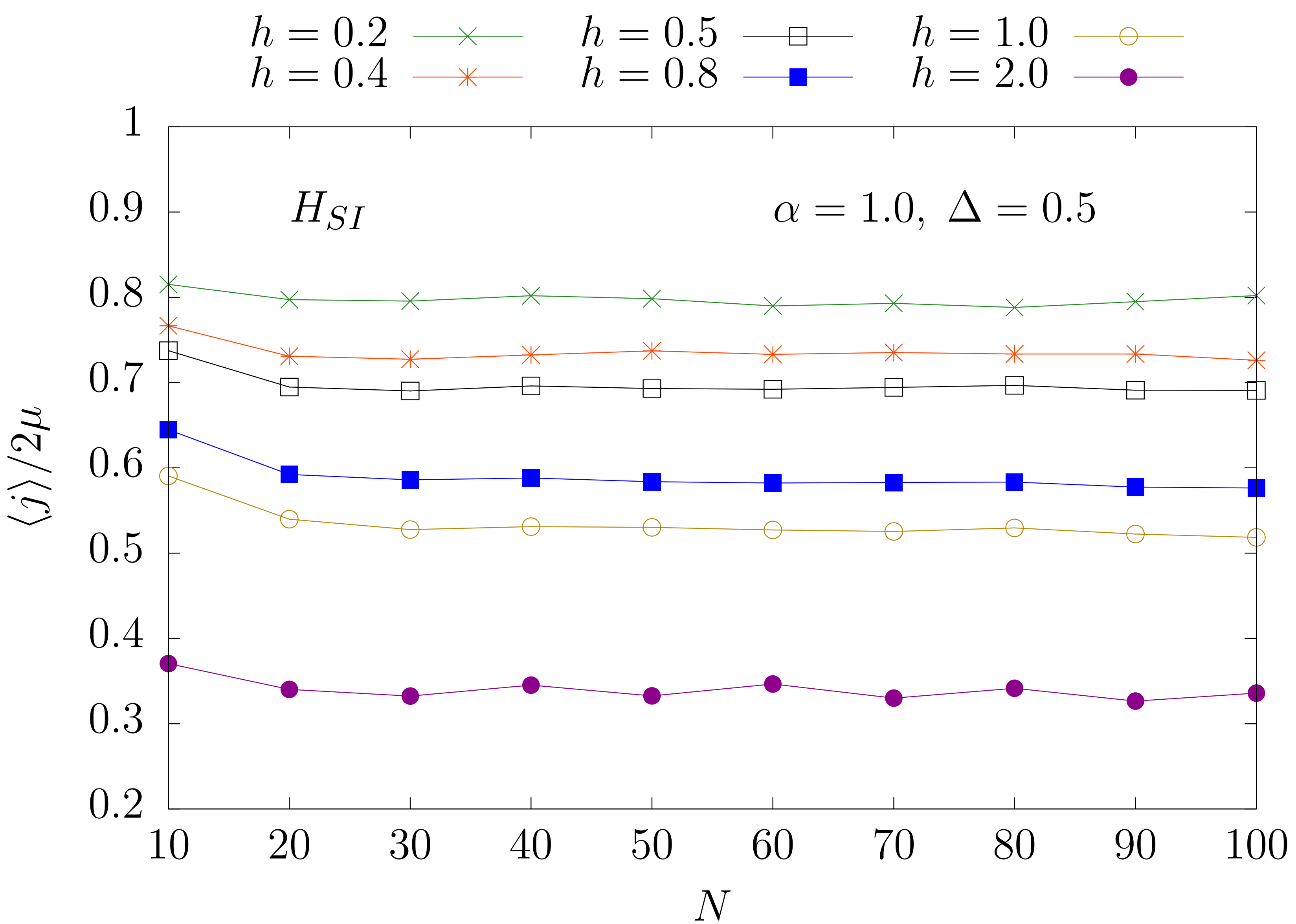
$$\Delta = 0.5$$

$$H = t \sum_i \sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + V \sum_i \sigma_z^i \sigma_z^{i+1} + h\sigma_{L/2}^z$$

# Single Magnetic Defect

- Transport across the system remains ballistic (same as the unperturbed transport regime)

$$\langle j \rangle \propto constant$$



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- Transport across the system remains ballistic (same as the unperturbed transport regime)

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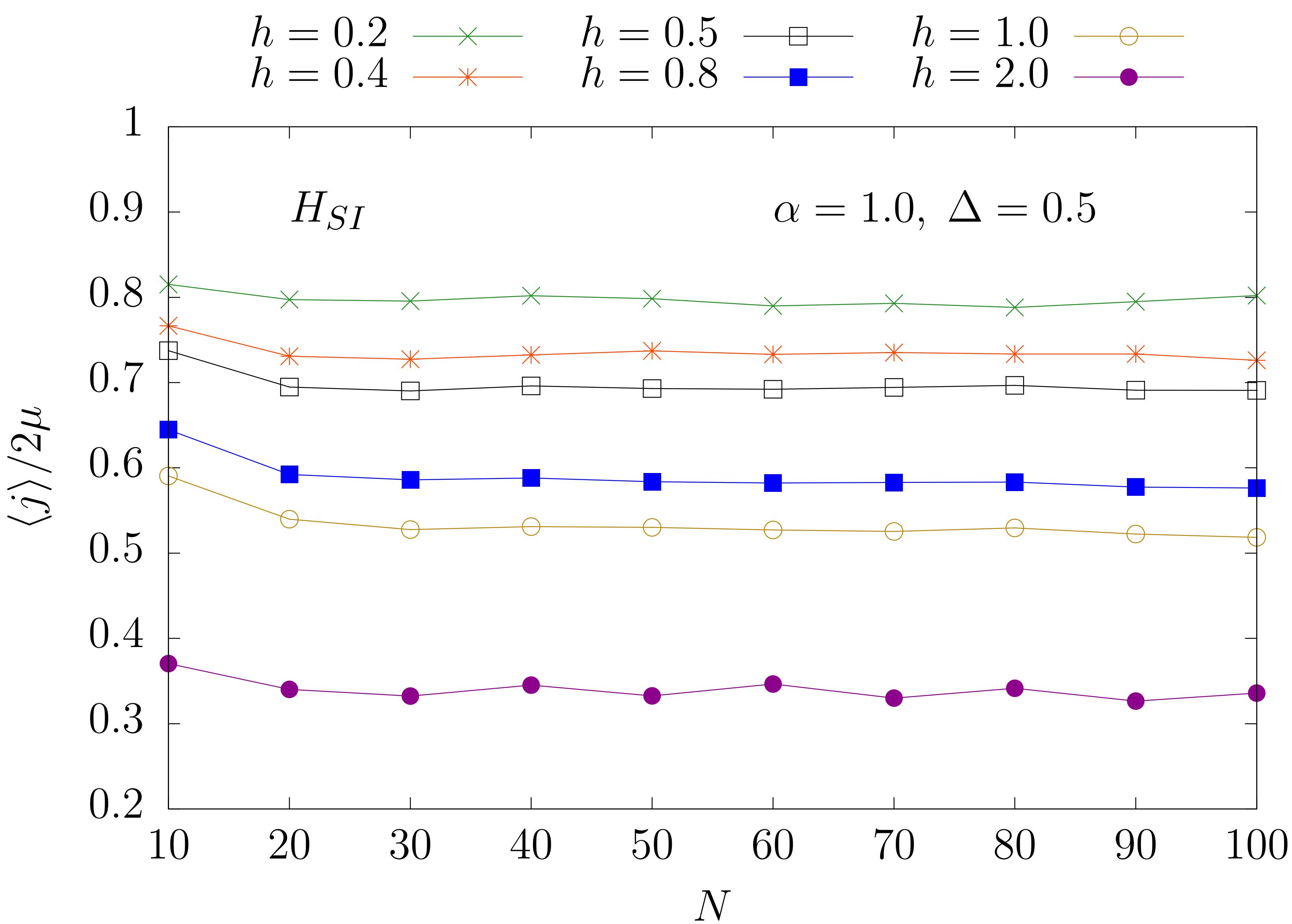
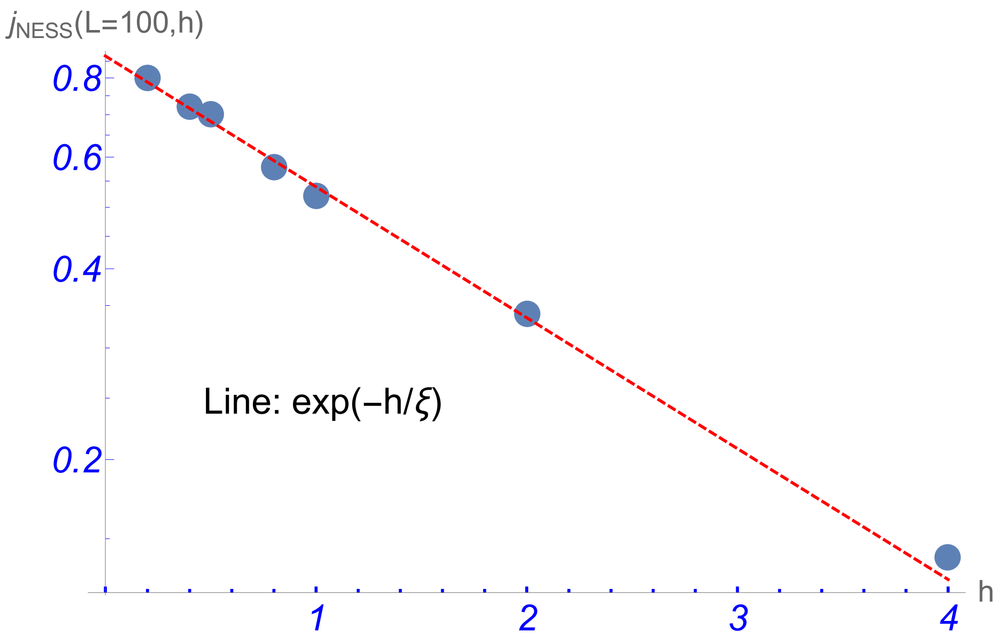


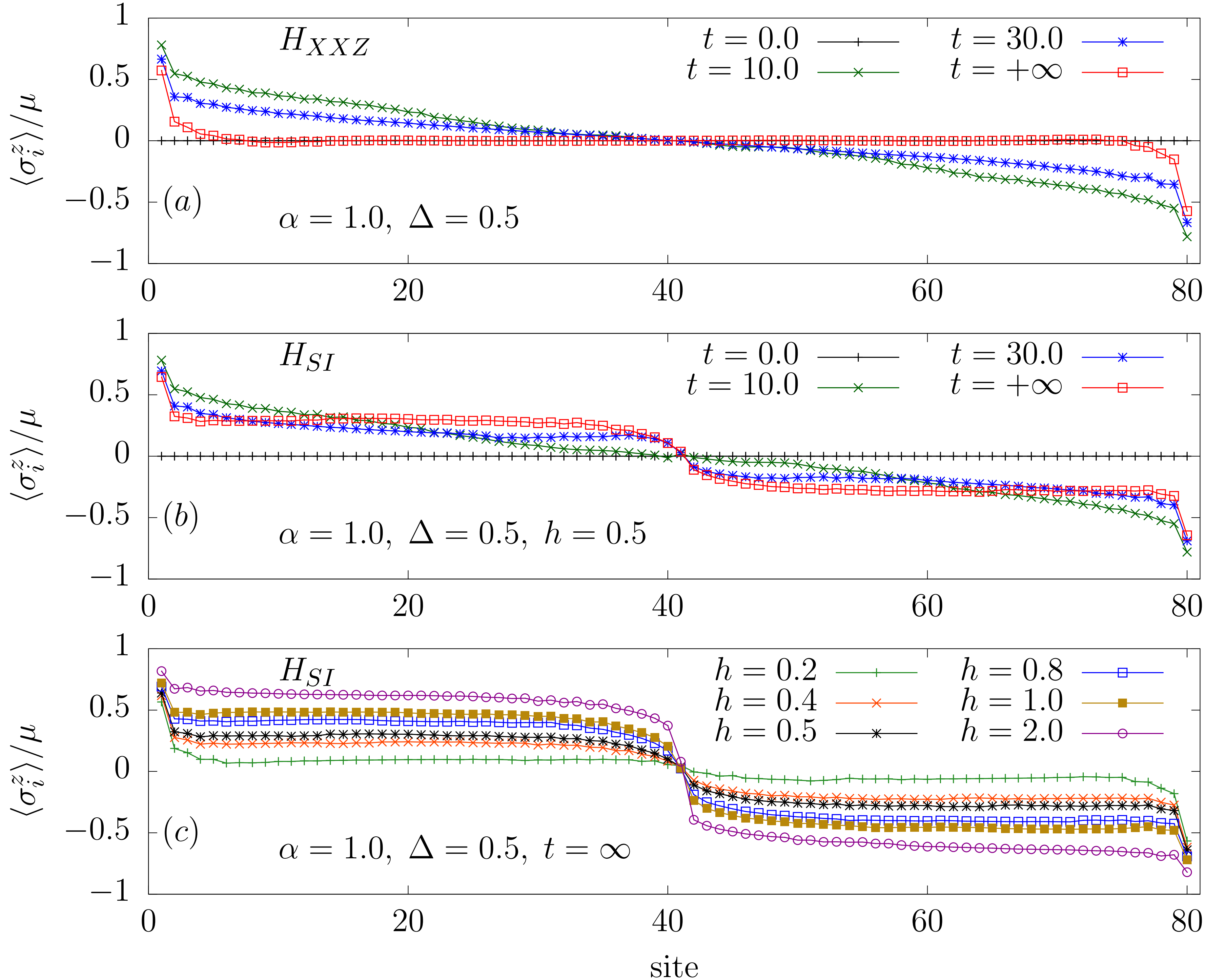



# Magnetisation profiles in the steady state

Clean case

Impurity - time ev

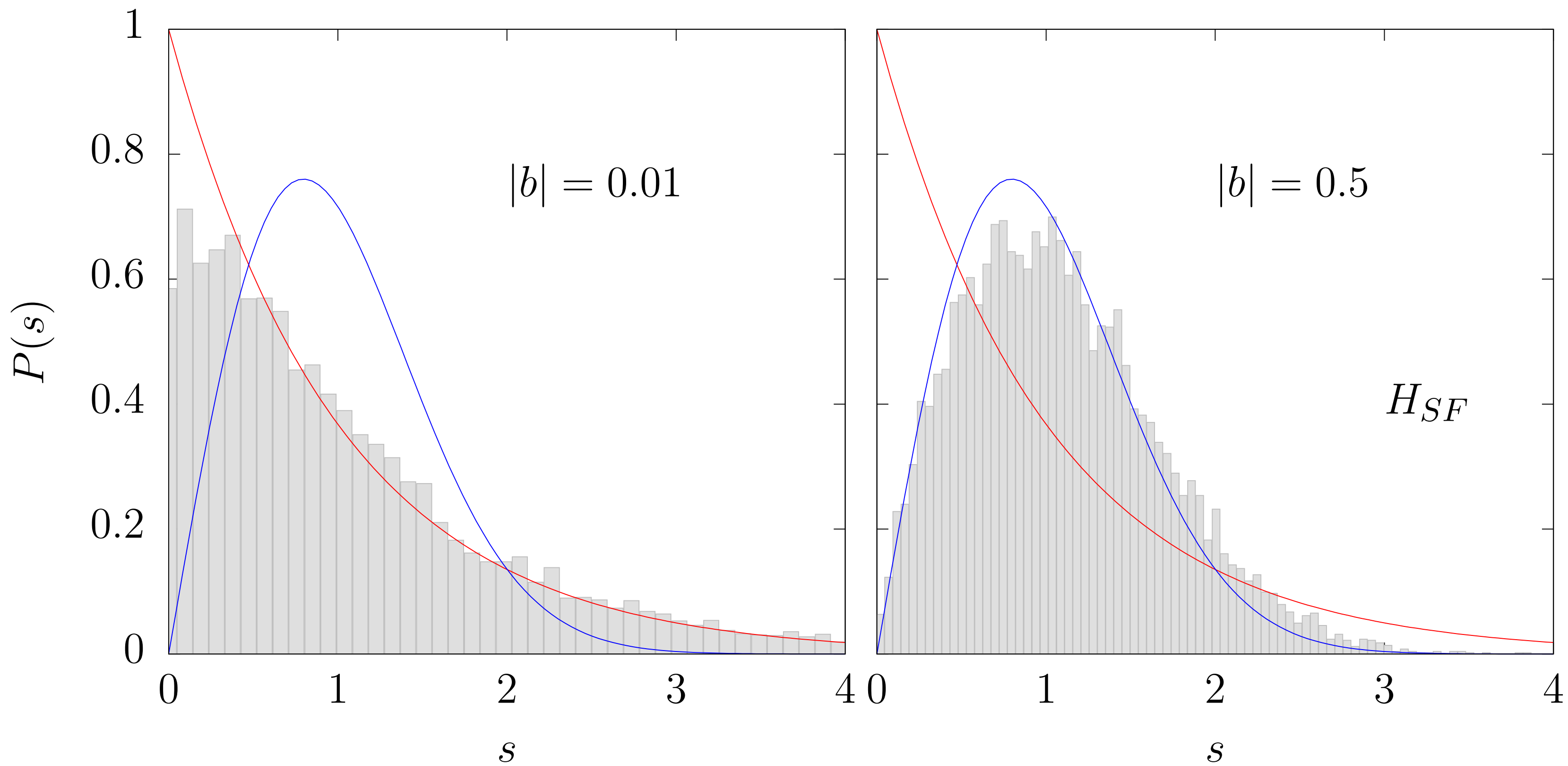
Impurity - steady state



# Spectral properties and level spacing statistics

## ***Staggered Magnetic Field***

$$s_n = (E_{n+1} - E_n)/\Omega,\ \Omega : \text{Average level spacing}$$

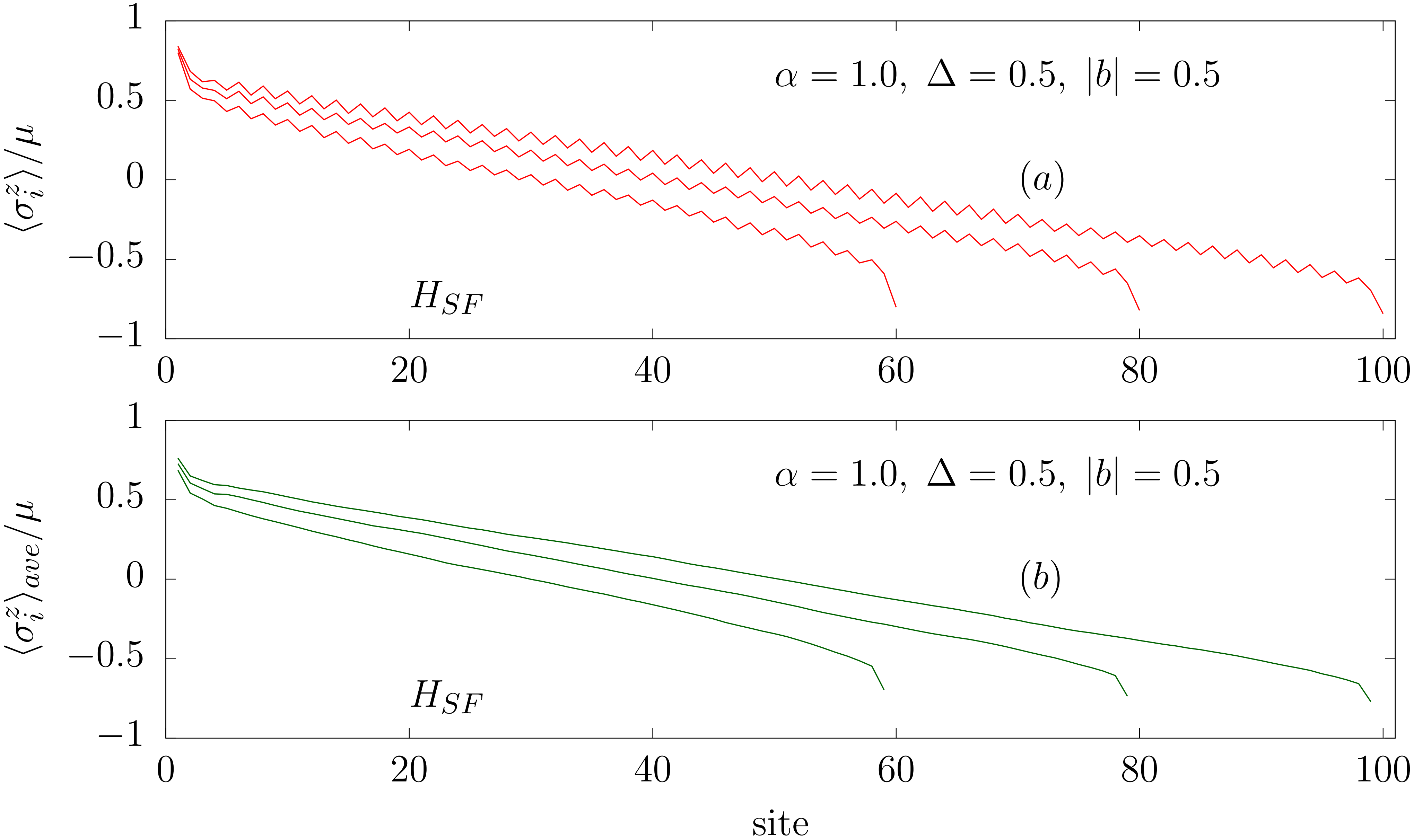



$$P(s) = e^{-s} \qquad P(s) = \frac{\pi s}{2} e^{\frac{-\pi s^2}{4}}$$

$$H_{SF} = H_{XXZ} + H_{IB}$$

$$\Delta = 0.5$$

# Magnetisation profiles in the steady state



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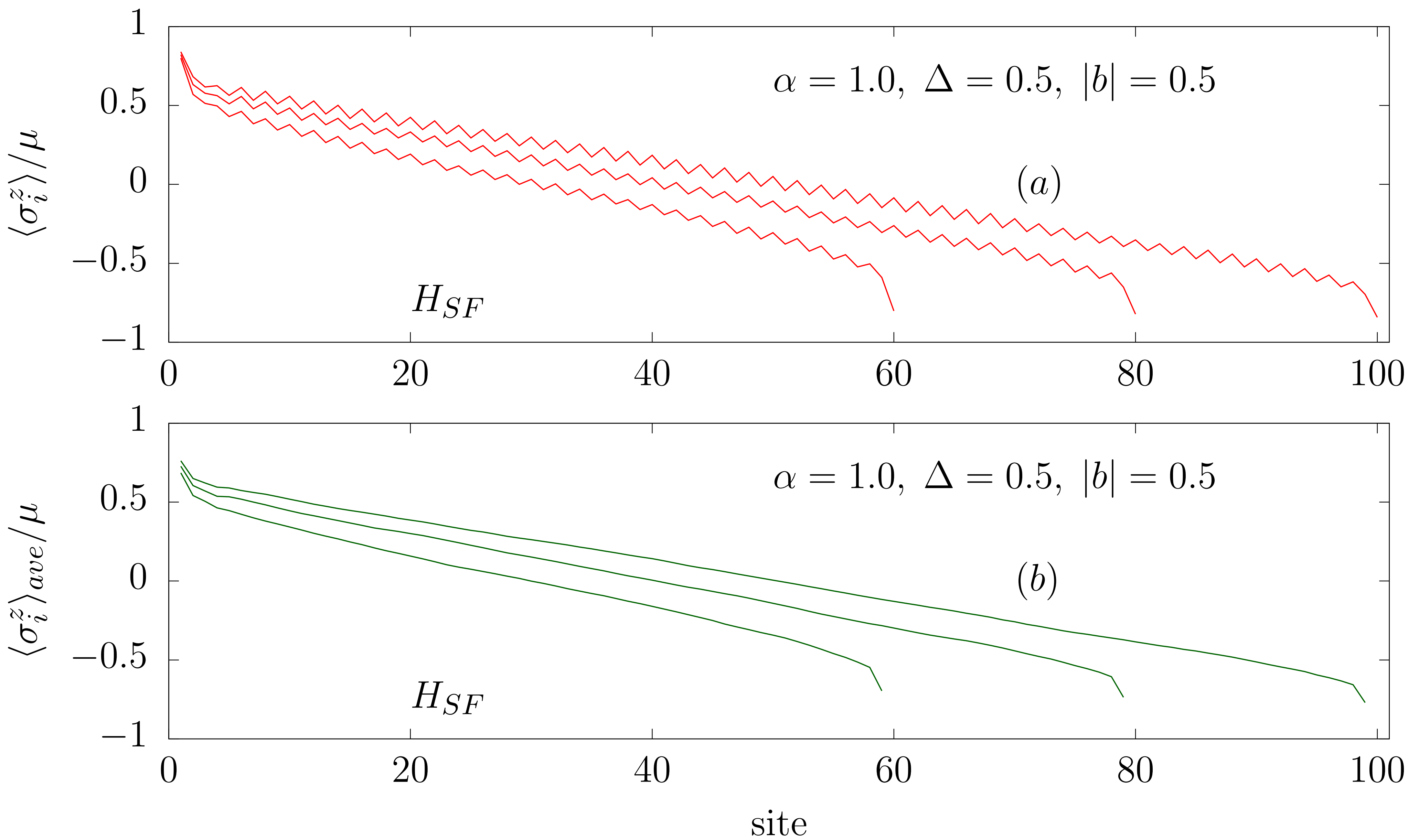
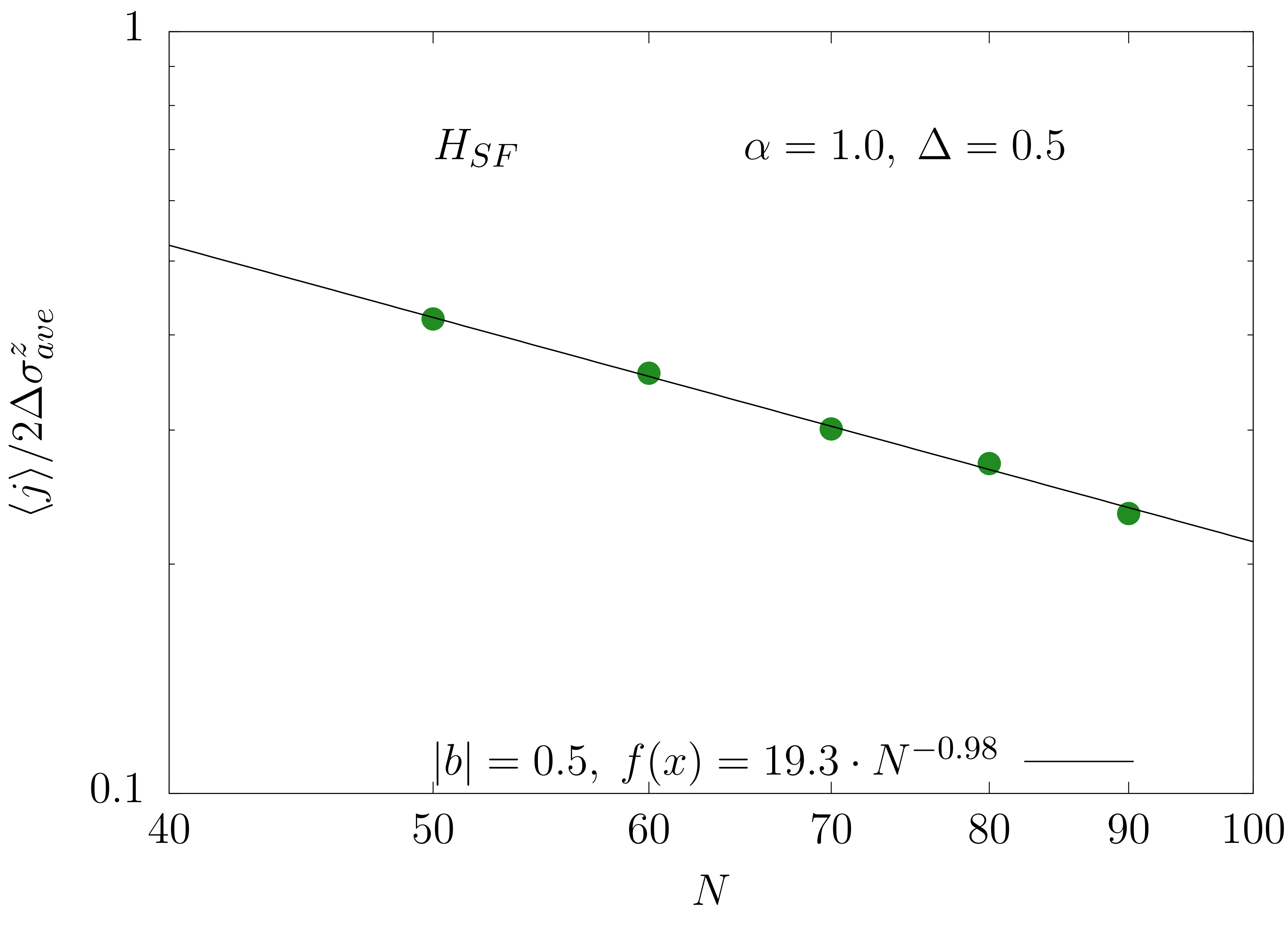



Diffusion $\langle J \rangle \propto \frac{1}{L}$ $\quad \nu = 1$

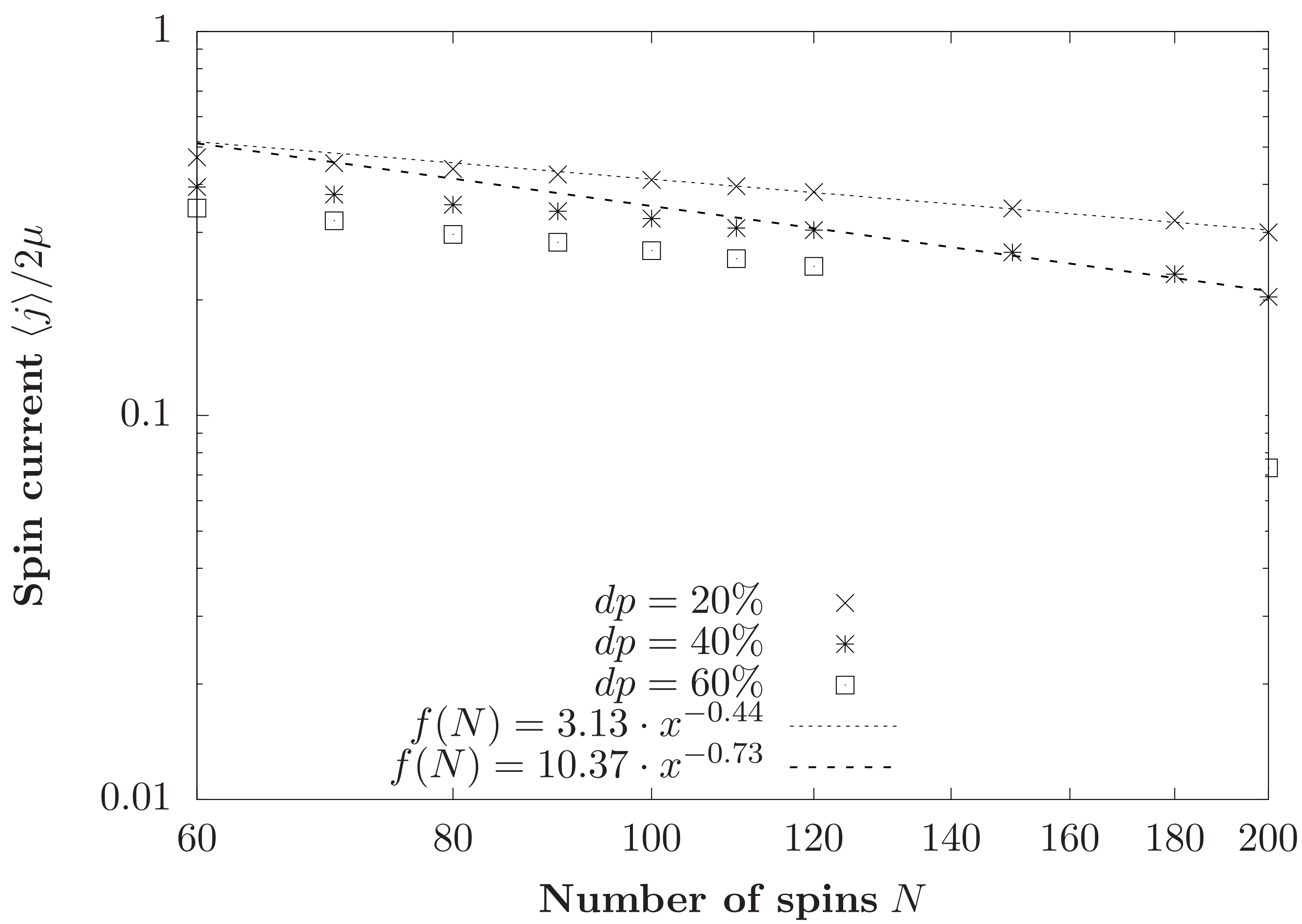
$$< j_l > = -D\nabla < \sigma_l^z >$$

# Transport is diffusive



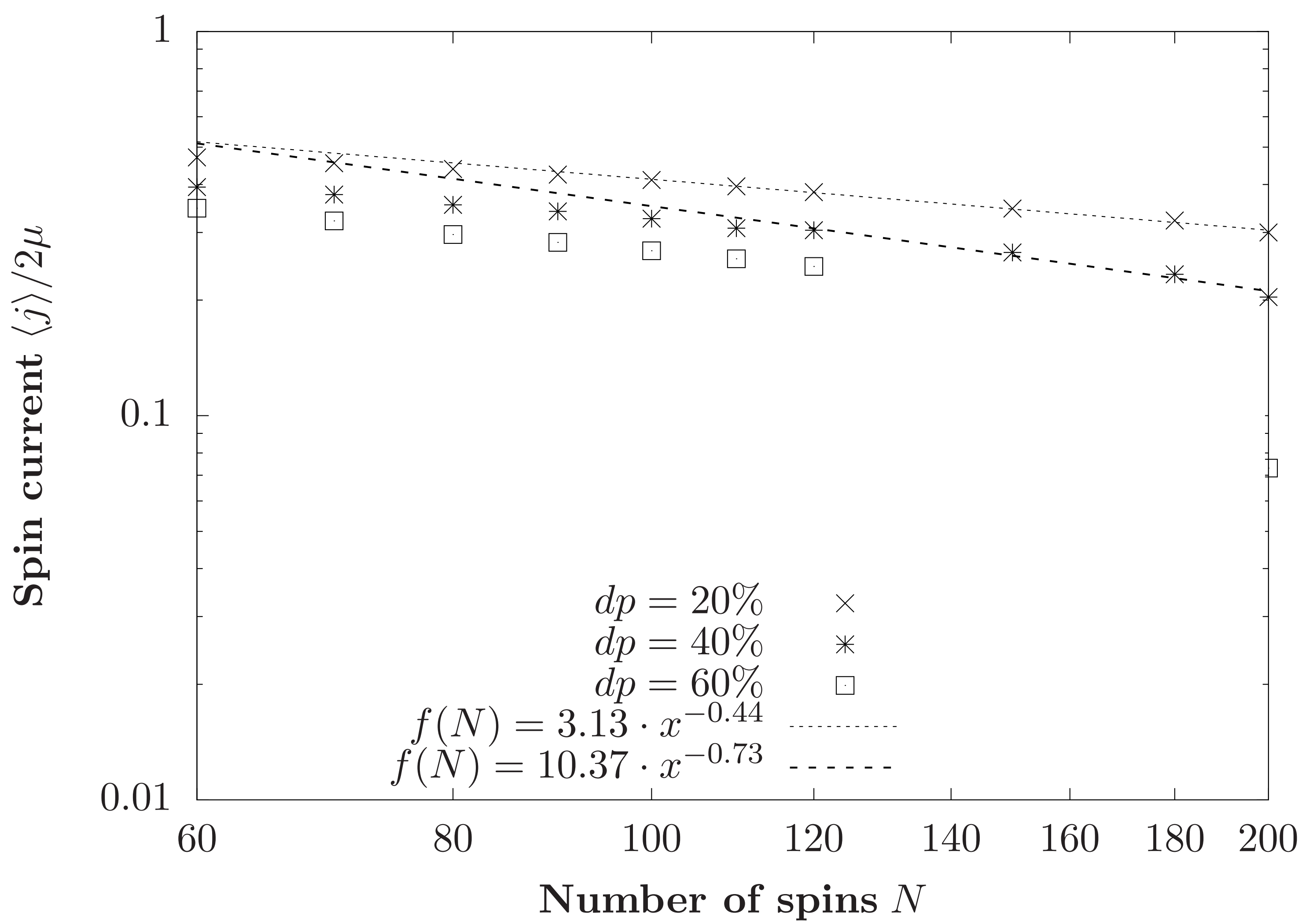

$$\langle j \rangle \propto 1/L$$

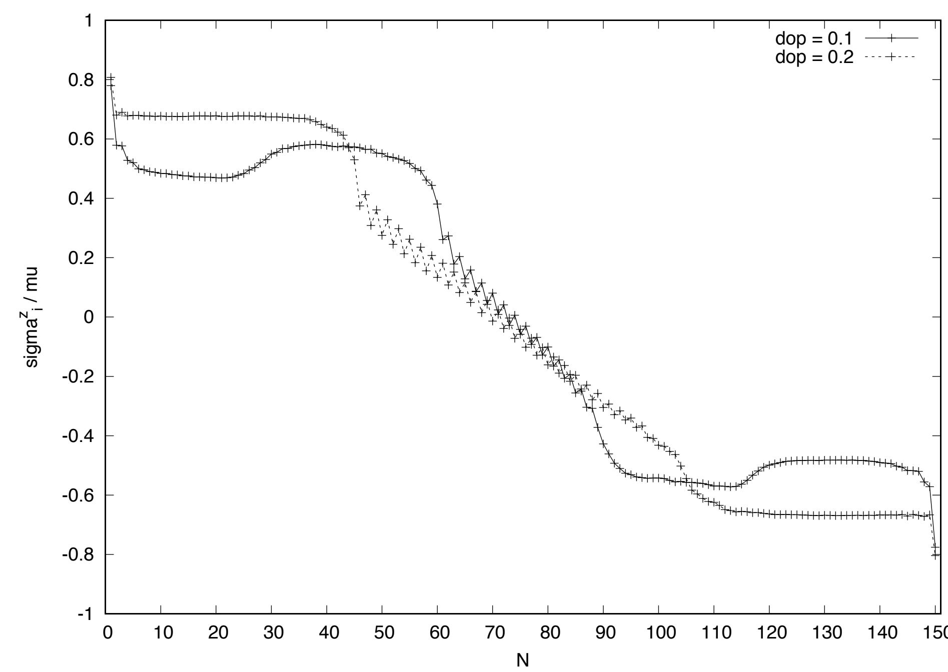
# Doping (super-undercooked)




Looks like anomalous **super-diffusion** with exponent depending on doping density

# Doping (super-undercooked)



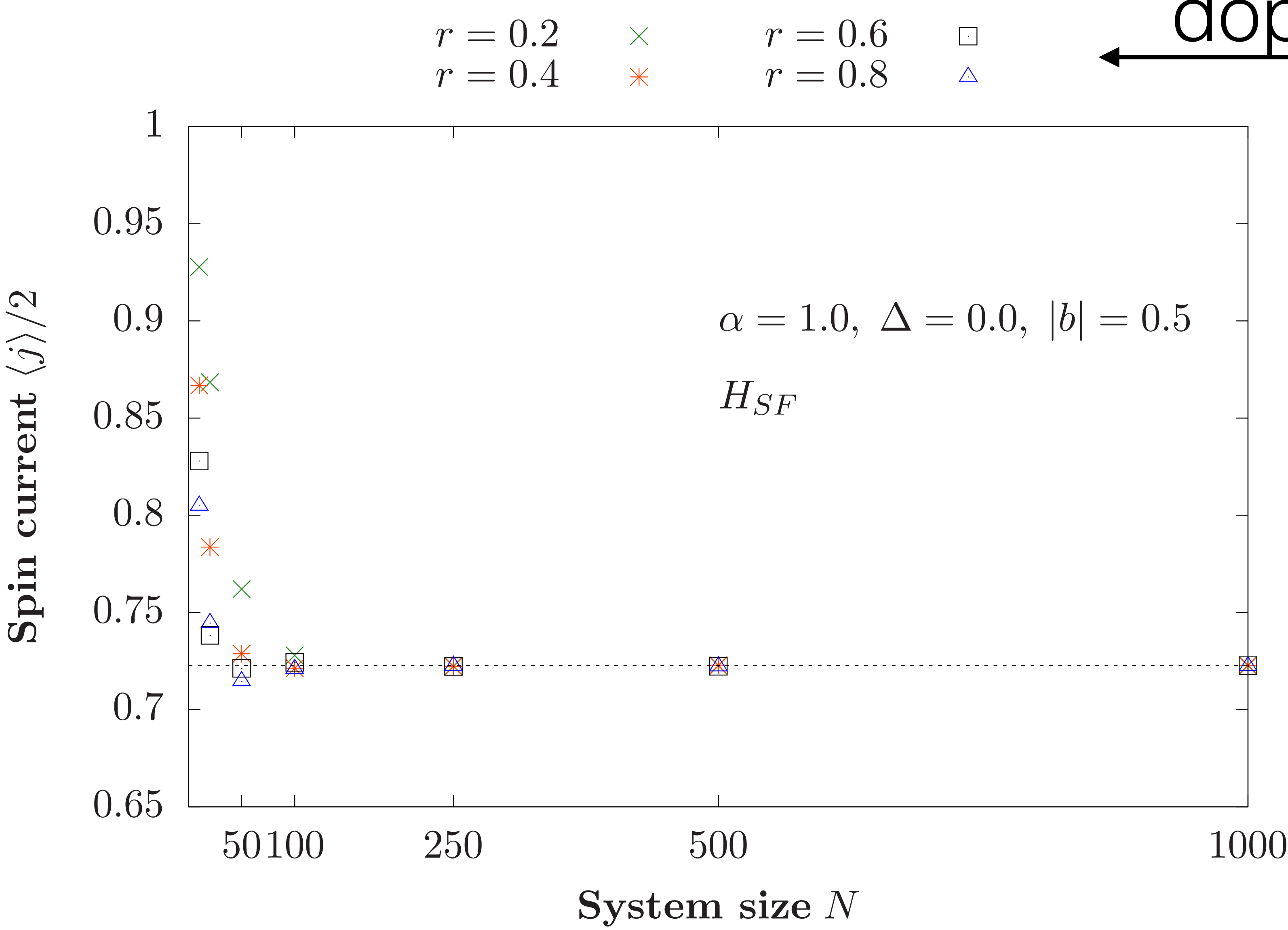




Looks like anomalous **super-diffusion** with exponent depending on doping density

# Role of interactions?

$$H_{XX} = \sum_{i=1}^{N-1} \left[\alpha\left(\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y\right) + \Delta \sigma_i^z \sigma_{i+1}^z\right] + \sum_i h_i \sigma_i^z$$

doping strength




[........h 0 h 0 h 0 .....]

# Welcome !

QuSys

THE ROYAL SOCIETY

Trinity College Dublin
Coláiste na Tríonóide, Baile Átha Cliath
The University of Dublin

erc European Research Council

Thank you for your time