UNIVERSITÉ DU LUXEMBOURG

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Quantum Thermodynamics

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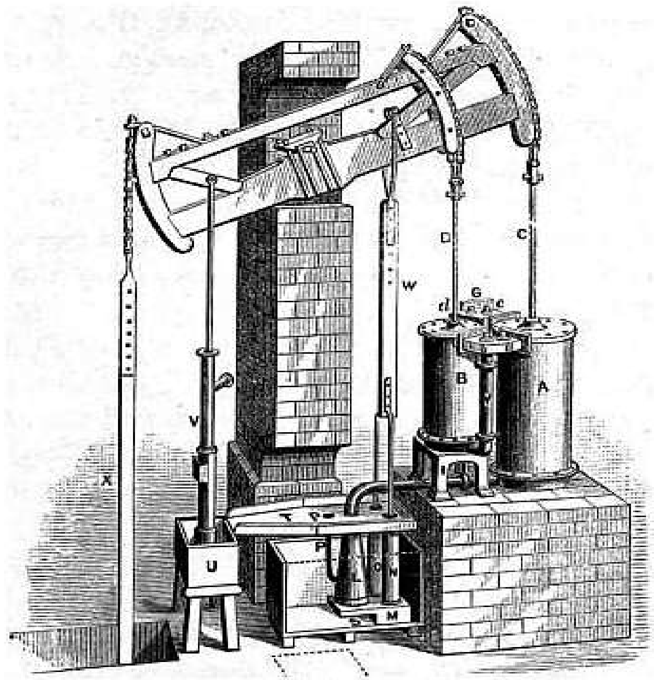
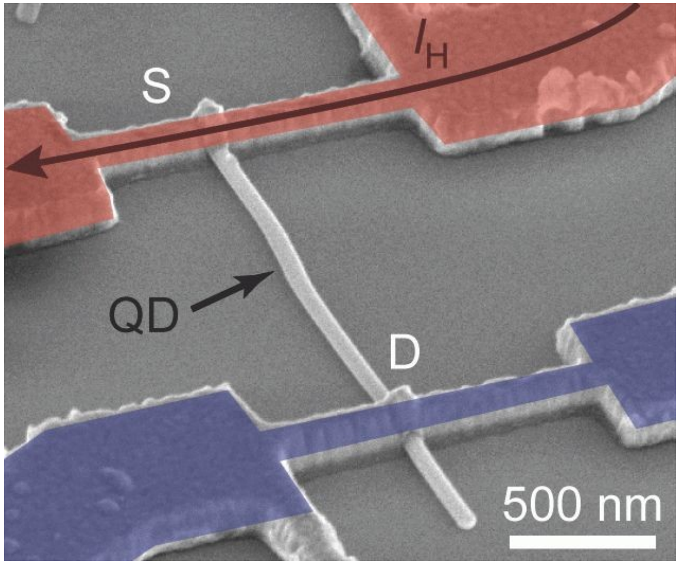
KITP – May 31, 2018

Introduction

Heat engine in the 19th century:



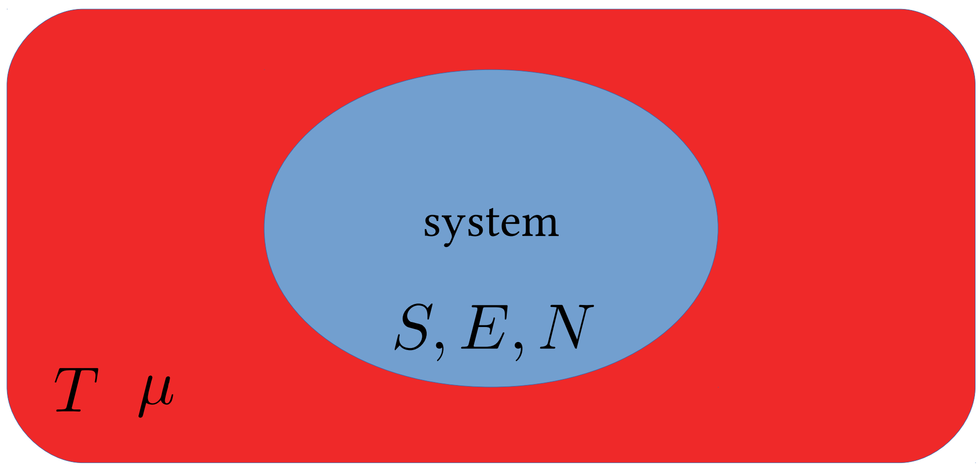
Heat engine in the 21th century:



Outline

1) Phenomenological thermodynamics
2) Stochastic Thermodynamics
3) A Hamiltonian formulation
4) Born-Markov-Secular Quantum Master Equation (QME)
5) Landau-Zener QME
6) Repeated interactions
7) Strong coupling
8) Conclusions

1) Phenomenological Nonequilibrium Thermodynamics



Zeroth law: System dynamics with an equilibrium

1st law:
$$d_t E = \dot{W} + \dot{Q}$$

2nd law:
$$\dot{\Sigma} = d_t S - \frac{\dot{Q}}{T} \geq 0$$

Slow transformation
$$d_t S \approx \frac{\dot{Q}}{T}$$

Entropy production
(dissipation)

2) Stochastic Thermodynamics

Esposito and Van den Broeck, Phys. Rev. E **82**, 011143 (2010)

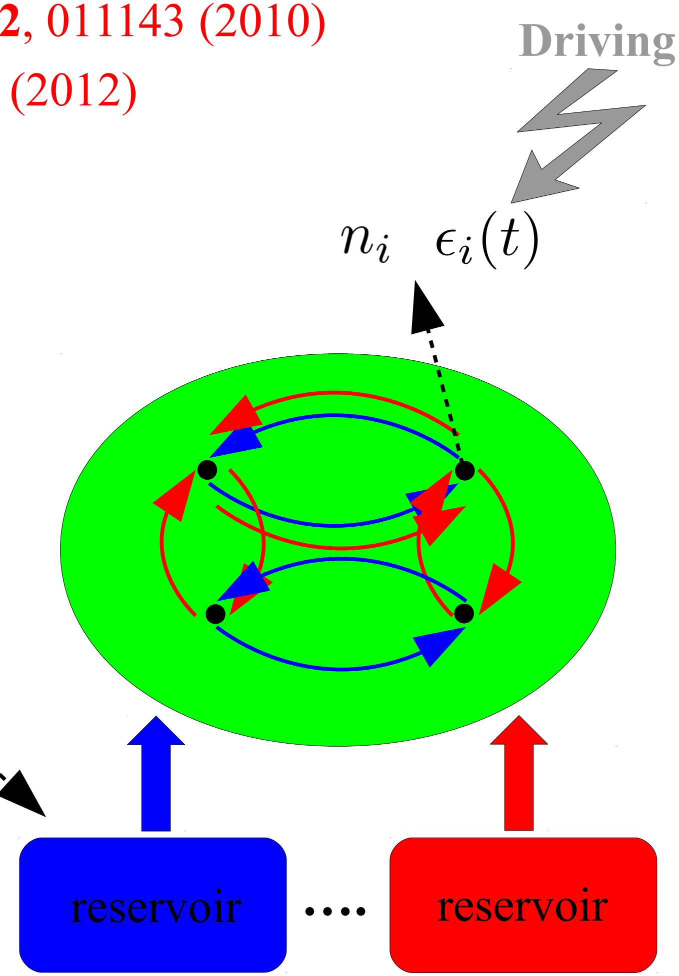
Esposito, Phys. Rev. E **85**, 041125 (2012)

Dynamics

Markovian master equation:

$$d_t p_i = \sum_j W_{ij} p_j = \sum_j \left(W_{ij} p_j - W_{ji} p_i \right)$$

$$W_{ij} = \sum_\nu W_{ij}^{(\nu)}$$



Different reservoirs $T^{(\nu)}$ $\mu^{(\nu)}$

Energy and Matter currents

$$I_E^{(\nu)} = \sum_{i,j} W_{ij}^{(\nu)} p_j (\epsilon_i - \epsilon_j)$$

$$I_M^{(\nu)} = \sum_{i,j} W_{ij}^{(\nu)} p_j (n_i - n_j)$$

Local detailed balance:

$$\frac{W_{ij}^{(\nu)}}{W_{ji}^{(\nu)}} = \exp\left(- \frac{(\epsilon_i - \epsilon_j) - \mu^{(\nu)}(n_i - n_j)}{k_b T^{(\nu)}} \right)$$

Average Thermodynamics

Energy, Particle number, Shannon entropy

$$E = \sum_i \epsilon_i p_i \quad , \quad N = \sum_i n_i p_i \quad , \quad S = \sum_i \underbrace{[-k_b \ln p_i]}_{s_i} p_i$$

1st law: Energy balance

$$\boxed{d_t E = \dot{W}_m + \dot{W}_c + \sum_\nu \dot{Q}^{(\nu)}}$$

Mechanical work

$$\dot{W}_m = \sum_i d_t \epsilon_i \, p_i$$

Particle balance

$$\boxed{d_t N = \sum_\nu I_M^{(\nu)}}$$

Chemical work

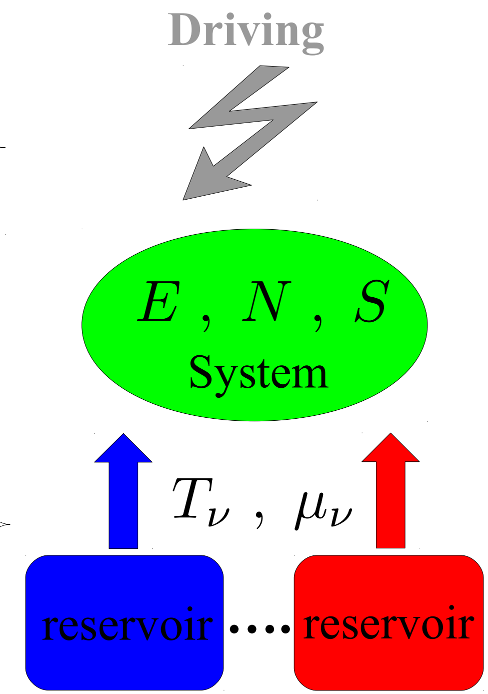
$$\dot{W}_c = \sum_\nu \mu^{(\nu)} I_M^{(\nu)}$$

2nd law: Entropy balance

$$\boxed{\dot{\Sigma} = d_t S - \sum_\nu \frac{\dot{Q}^{(\nu)}}{T_\nu} \geq 0}$$

Heat flow

$$\dot{Q}^{(\nu)} = I_E^{(\nu)} - \mu^{(\nu)} I_M^{(\nu)}$$



Entropy production

Entropy change in the reservoirs

Slow driving with 1 reservoir: $\dot{\Sigma} \approx 0$

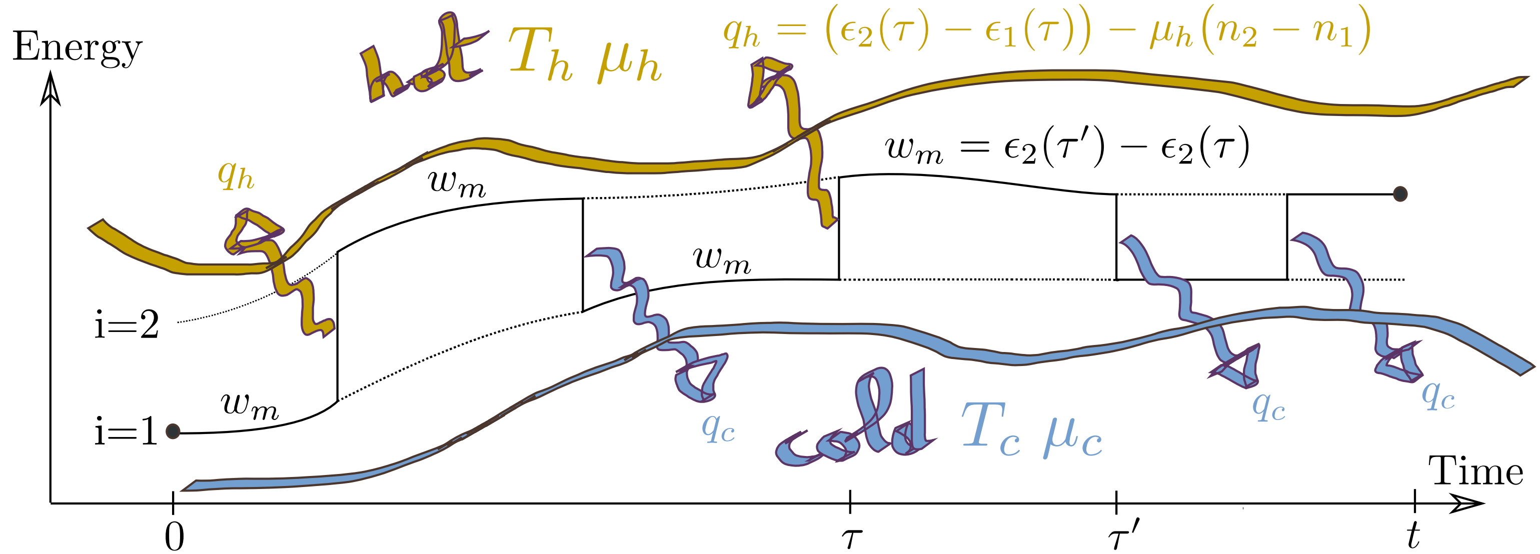
equilibrium thermo

$$T d_t S = d_t E - \dot{W}_m - \mu d_t N$$

$$\dot{\Sigma} = k_b \sum_{\nu,i,j} \left(W_{ij}^{(\nu)} p_j - W_{ji}^{(\nu)} p_i\right) \ln \frac{W_{ij}^{(\nu)} p_j}{W_{ji}^{(\nu)} p_i} \geq 0$$

$$\dot{\Sigma} = 0 \ \text{ iff } \ W_{ij}^{(\nu)} p_j = W_{ji}^{(\nu)} p_i \ \text{(detailed balance)}$$

Fluctuating Thermodynamics



Energy balance: $\epsilon_{i_t}(\lambda_t) - \epsilon_{i_0}(\lambda_0) = w_m\,[\Gamma|\lambda] + \underline{w_c\,[\Gamma|\lambda]} + \sum_{\nu=1}^{N} \underbrace{\left(\Delta\epsilon_\nu\,[\Gamma|\lambda] \underline{- \mu_\nu \Delta n_\nu\,[\Gamma|\lambda]}\right)}_{q_\nu\,[\Gamma|\lambda]}$

Particle balance: $n_{i_t} - n_{i_0} = \sum_{\nu=1}^{N} \Delta n_\nu\,[\Gamma|\lambda]$

Entropy balance: $\sigma\,[\Gamma|\lambda] = \ln \dfrac{P[\Gamma,\lambda]}{\tilde{P}[\bar{\Gamma},\lambda]} = \underbrace{\ln p_{i_0}(0) - \ln p_{i_t}(t)}_{\Delta s\,[\Gamma|\lambda]} - \sum_{\nu=1}^{N} \beta_\nu q_\nu\,[\Gamma|\lambda]$

Time-reversed driving
Time-reversed trajectory

not a physical observable

Integral fluctuation theorem: $\boxed{\langle e^{-\sigma} \rangle = 1}$ $\Rightarrow$ $\Sigma = \langle \sigma \rangle \geq 0$

Detailed FT
for entropy production

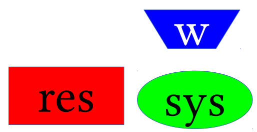
$$\boxed{\ln \frac{P(\sigma)}{\tilde{P}(-\sigma)} = \sigma}$$

if

Involution
$$\sigma\left[\Gamma|\lambda\right] = -\tilde{\sigma}\left[\bar{\Gamma}|\lambda\right]$$

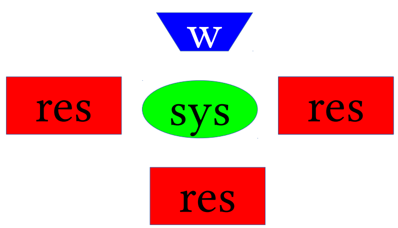
Seifert, PRL **95** 040602 (2005)

Fluctuation theorem for physical observable?



Driving + 1 reservoir + start at equilibrium: Work FT (Crooks FT)

No driving + multiple reservoirs + longtime limit $t \to \infty$: Current FT



Driving + multiple reservoirs + start at equilibrium vs reservoir $\nu = 1$:

$$\ln \frac{P(w_m, \{\Delta\epsilon_\nu\}, \{\Delta n_\nu\})}{\tilde{P}(-w_m, \{-\Delta\epsilon_\nu\}, \{-\Delta n_\nu\})} = \overbrace{\beta_1 \left(w_m - \Delta\Phi_1^{eq}\right)}^{\text{Work FT}} + \overbrace{\sum_{\nu=2}^{N} \left(A_\nu^\epsilon \Delta\epsilon_\nu + A_\nu^n \Delta n_\nu\right)}^{\text{Current FT}}$$

Bulnes Cuetara, Esposito, Imparato, PRE **89**, 052119 (2014)

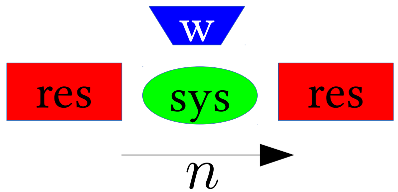
$$= \beta_1 - \beta_\nu \qquad = \beta_\nu \mu_\nu - \beta_1 \mu_1$$

Most general formulation: Rao & Esposito, NJP **20**, 023007 (2018)

Isothermal example: driven junction

Bulnes Cuetara, Esposito, Imparato, PRE **89**, 052119 (2014)

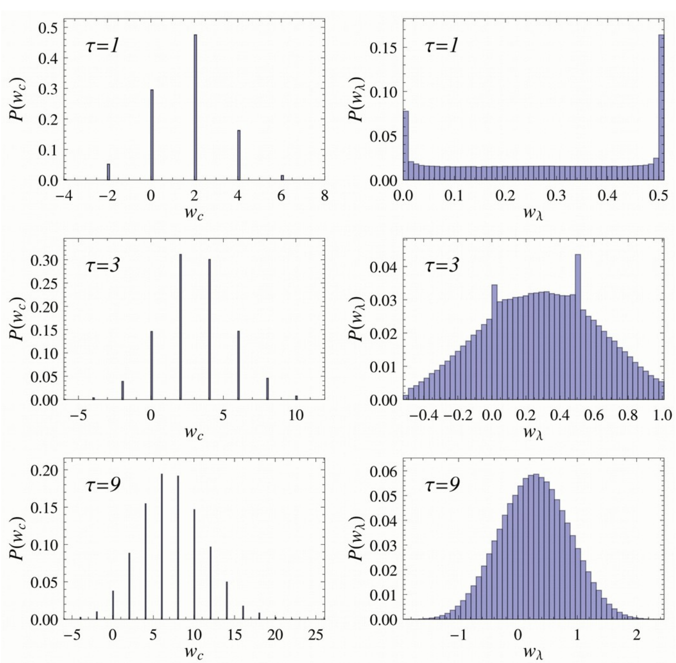
Setup:

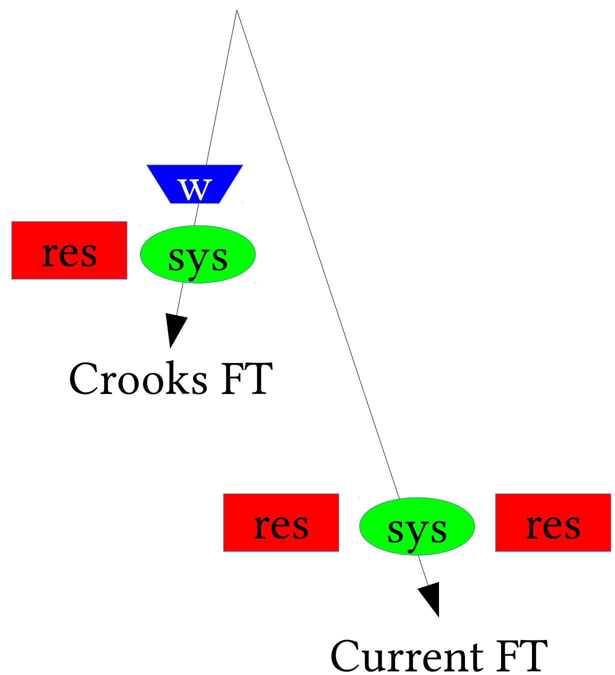


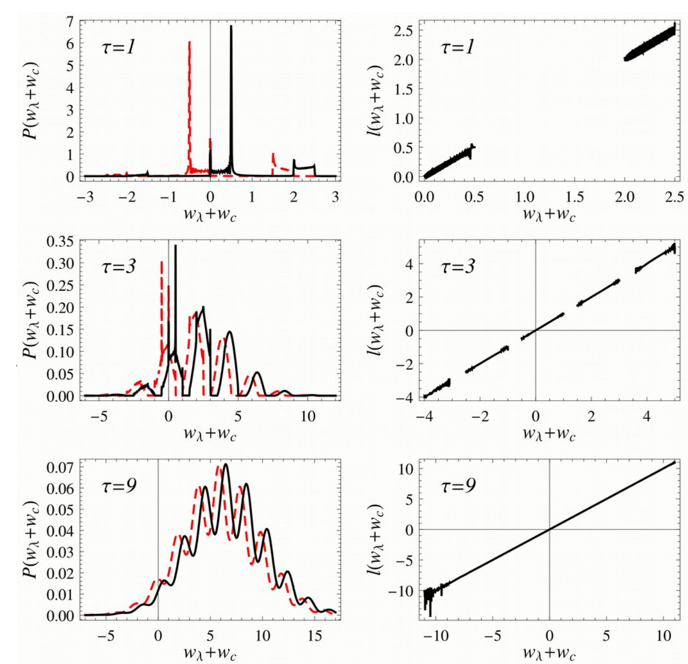
Initial condition: equilibrium vs a reference reservoir

Mechanical work: w_m Chemical work: $w_c = n\Delta\mu$

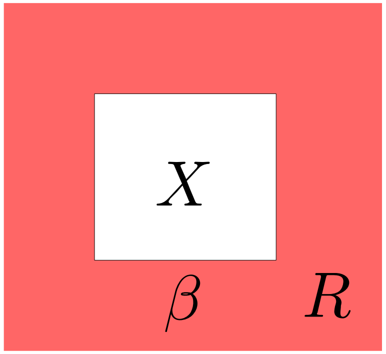
$$\frac{P_F(w_m + w_c)}{P_B(-w_m - w_c)} = \frac{P_F(w_m, w_c)}{P_B(-w_m, -w_c)} = \exp\left\{\frac{w_m + w_c - \Delta\Phi_1}{k_B T}\right\}$$







3) Hamiltonian formulation



$$H_{\text{tot}}(t) = H_X(t) + H_R + H_{XR}(t)$$

Assumption 1: $\rho_{XR}(0) = \rho_X(0)\rho_\beta^R \qquad \rho_\beta^R \equiv \dfrac{e^{-\beta H_R}}{Z_R}$

$$E_X(t) \equiv \text{tr}_{XR}\{[H_X(t) + H_{XR}(t)]\rho_{XR}(t)\}$$

1st law: $d_t E_X(t) = \dot{W}(t) + \dot{Q}(t)$ $\quad \begin{cases} \dot{W}(t) = d_t E_{XY}(t) \\ \dot{Q}(t) \equiv -\text{tr}_R\{H_R d_t \rho_R(t)\} \end{cases}$

2nd law: $\Sigma(t) \equiv \Delta S_X(t) - \beta Q(t) = D[\rho_{XR}(t) || \rho_X(t)\rho_\beta^R]$

$$= D[\rho_R(t)||\rho_\beta^R] + I_{X:R}(t) \geq 0$$

[Esposito, Lindenberg, & Van den Broeck, NJP **12**, 013013 (2010)]

4) Born-Markov-Secular QME

$$H_{\rm tot}(t) = H_X(t) + H_R + \sum_k A_k \otimes B_k$$

slow weak

Closed description for the **dynamics** of the system

$$d_t \rho_X(t) = -i[H_X(t), \rho_X(t)] + \mathcal{L}_\beta(t)\rho_X(t) \equiv \mathcal{L}_X(t)\rho_X(t),$$

$$\mathcal{L}_\beta(t)\rho(t) = \sum_\omega \sum_{k,\ell} \gamma_{k\ell}(\omega) \left(A_\ell(\omega)\rho(t)A_k^\dagger(\omega) - \frac{1}{2}\{A_k^\dagger(\omega)A_\ell(\omega), \rho(t)\} \right)$$

$$A_k(\omega) \equiv \sum_{\epsilon-\epsilon'=\omega} \Pi_\epsilon A_k \Pi_{\epsilon'} \qquad \gamma_{k\ell}(\omega) = \int_{-\infty}^{\infty} dt e^{i\omega t} \mathrm{tr}_R\{B_k(t)B_\ell(0)\rho_\beta^R\}$$

Local detailed balance

$$\gamma_{k\ell}(-\omega) = e^{-\beta\omega}\gamma_{\ell k}(\omega)$$

$$\mathcal{L}_\beta(t)\rho_\beta^X(t) = 0, \quad \rho_\beta^X(t) = \frac{e^{-\beta H_X(t)}}{Z_X(t)}$$

Validity: $\tau_{th} = \hbar\beta < \tau_{rel} \sim \gamma < \tau_H = \hbar d$

[Redfield 65, Agarwal 69, Lindblad 76, Gorini-Frigerio-Verri-Kossakowki-Sudarshan 78]

Closed description for the **thermodynamics** of the system

Energy: $E_X(t) = -\mathrm{tr}_X\{H_X(t)\rho_X(t)\}$

Entropy: $S_X(t) = -\mathrm{tr}_X\{\rho_X(t)\ln\rho_X(t)\}$

1st law: $d_t E_X(t) = \dot{W}(t) + \dot{Q}(t)$

$$\dot{W}(t) = \mathrm{tr}_X\left\{\rho_X(t) d_t H_X(t)\right\}$$

$$\dot{Q}(t) = \mathrm{tr}_X\left\{H_X(t) d_t \rho_X(t)\right\} = \mathrm{tr}_X\{H_X(t)\mathcal{L}_X(t)\rho_X(t)\}$$

2nd law: $\dot{\Sigma}(t) = d_t S_X(t) - \beta\dot{Q}(t)$

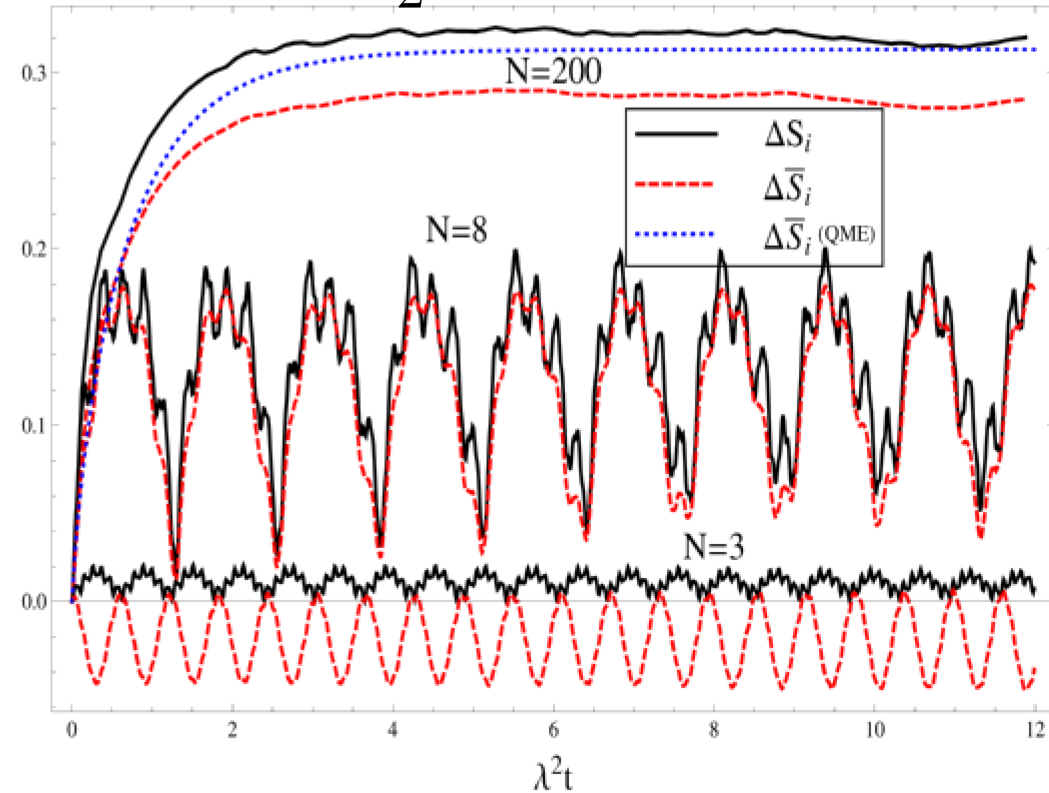
$$= -\mathrm{tr}\{[\mathcal{L}_X(t)\rho_X(t)][\ln\rho_X(t) - \ln\rho_\beta^X(t)]\} \geq 0$$

[Spohn 77, Spohn-Lebowitz 78, Alicki 79]

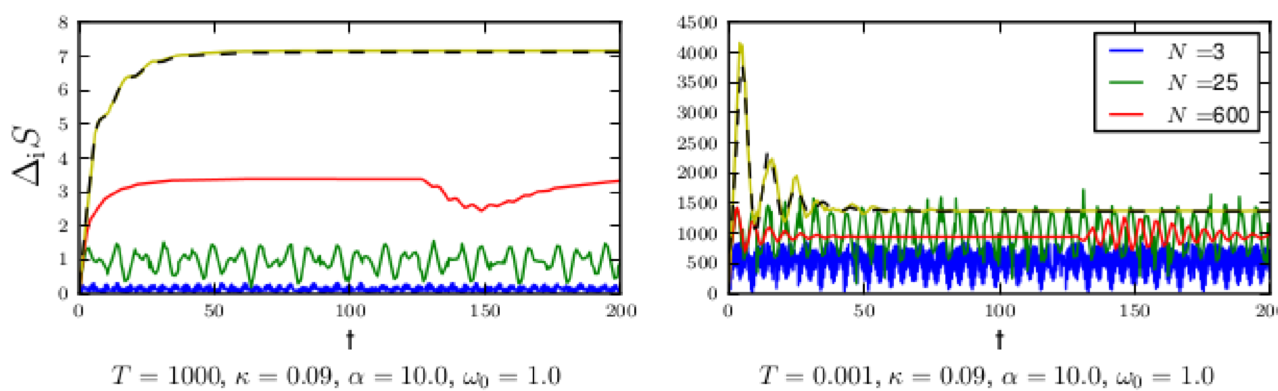
Reviews:[Kosloff, Entropy **15**, 2100 (2013)] [Gelbwaser &co, Adv.At.Mol.Opt.Phys. **64**, 329 (2015)]

With fluctuations:
- Non-degenerate states: Stochastic thermodynamics
- Degenerate states: [Bulnes-Cuetara, Esposito & Schaller, Entropy **18**, 447 (2016)]
- Fast periodic driving (Floquet):[Bulnes-Cuetara, Engel & Esposito, NJP **18**, 447 (2016)]

$$H = \frac{\Delta}{2}\sigma_z + H_r + \lambda\sigma_x R$$



[Esposito, Lindenberg, & Van den Broeck, NJP **12**, 013013 (2010)]

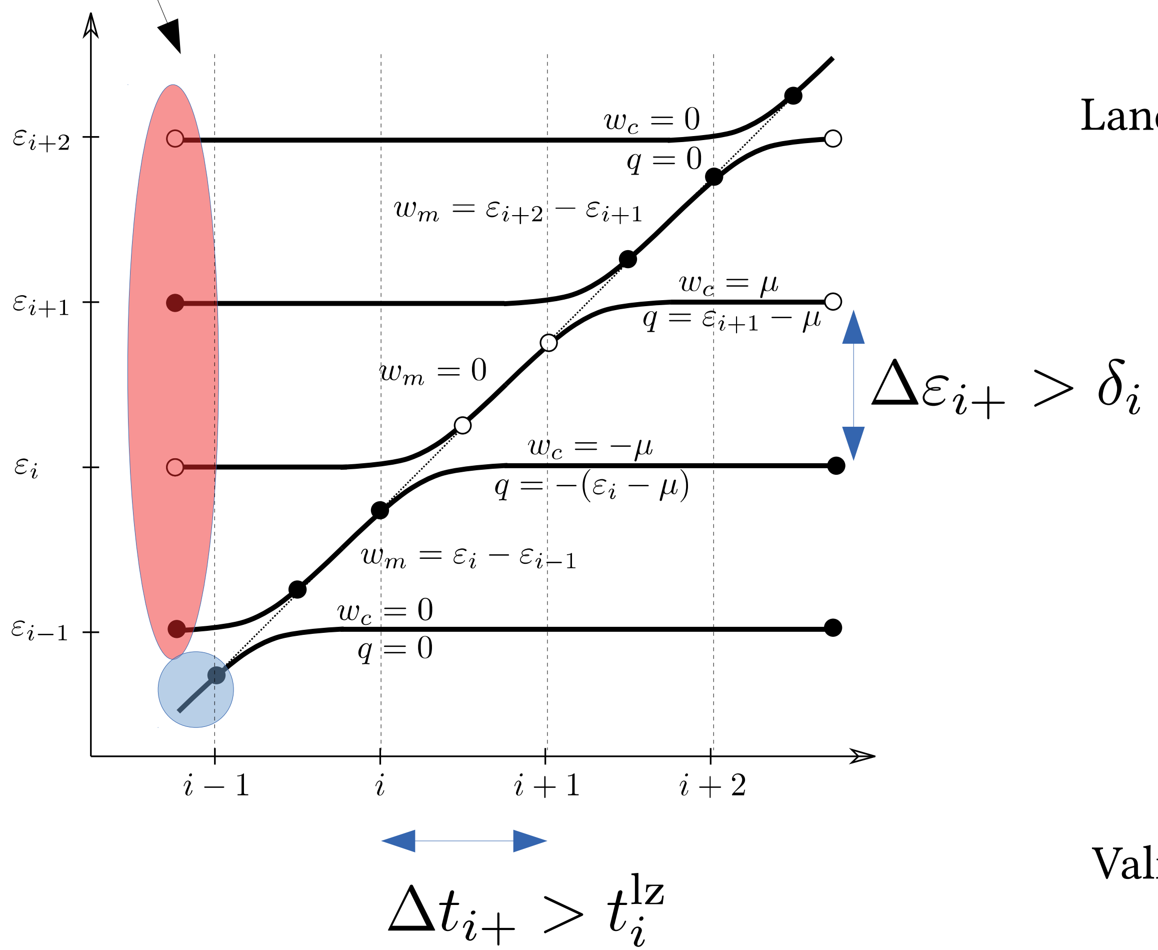


[Pucci, Esposito & Peliti, J. Stat. Mech. (2013) P04005]

5) A Landau-Zener approach

Thermal reservoir

$$H(t) = \epsilon_t c^\dagger c + \sum_{i=1}^{L} \varepsilon_i c_i^\dagger c_i + \frac{\delta}{2} \sum_{i=1}^{L} (c^\dagger c_i + c_i^\dagger c)$$



Landau-Zener prob. for a diabatic transition:

$$R_i = \exp\{-\pi\delta_i^2/(2\hbar\dot{\epsilon}_i)\}$$

valid after

$$t_i^{\mathrm{lz}} = \sqrt{\hbar/\dot{\epsilon}_i}\ \max[1, \sqrt{\delta_i^2/(\hbar\dot{\epsilon}_i)}]$$

Validity: $\Delta\varepsilon_{i+} > \sqrt{\hbar\dot{\epsilon}_i}, \delta_i$

[Barra & Esposito, PRE **93**, 062118 (2016)]

Effective dynamics

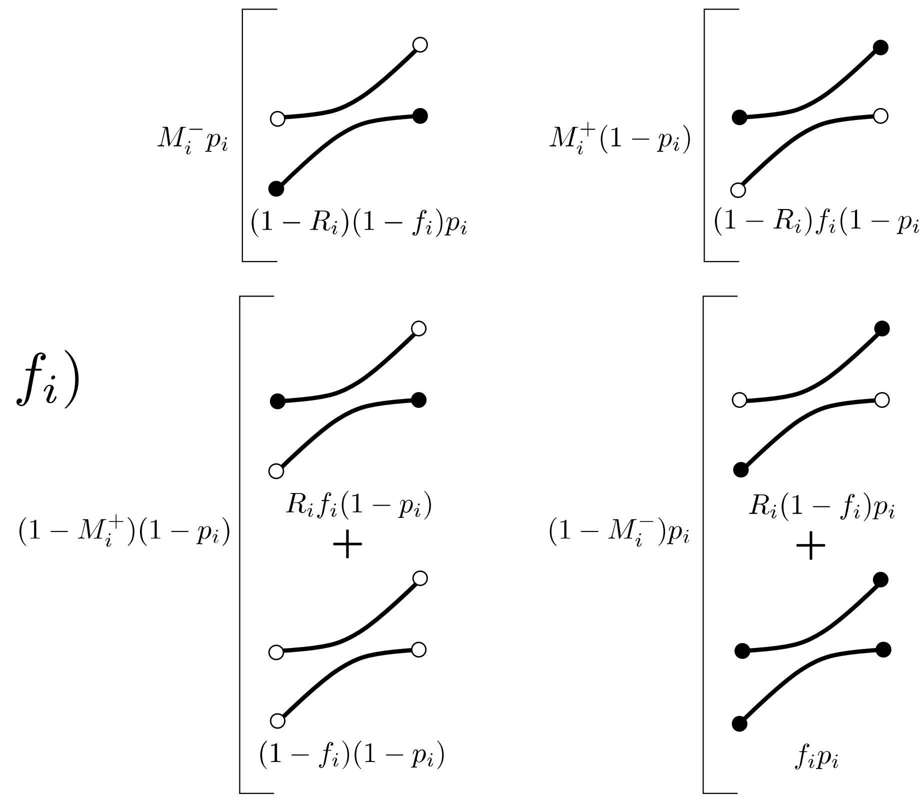
Master equation

$$p_{i+1} = (1 - M_i^-)p_i + M_i^+(1 - p_i)$$

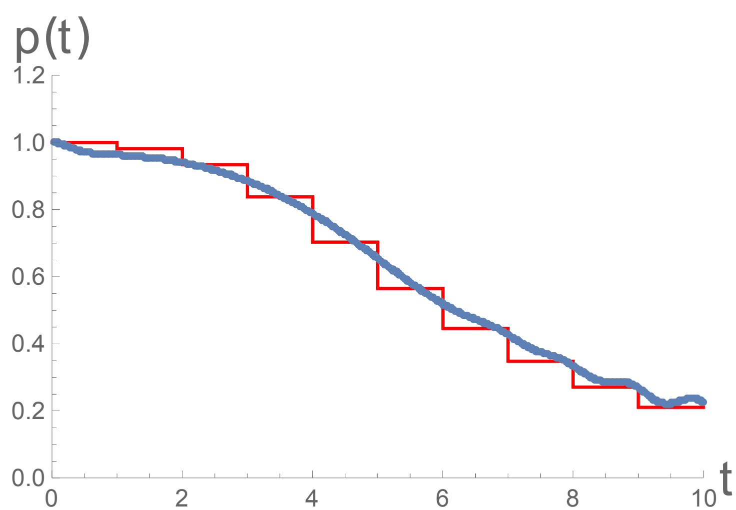
$$M_i^+ = (1 - R_i)f_i \qquad M_i^- = (1 - R_i)(1 - f_i)$$

Local detailed balance

$$M_i^+/M_i^- = e^{-\beta(\varepsilon_i - \mu)}$$

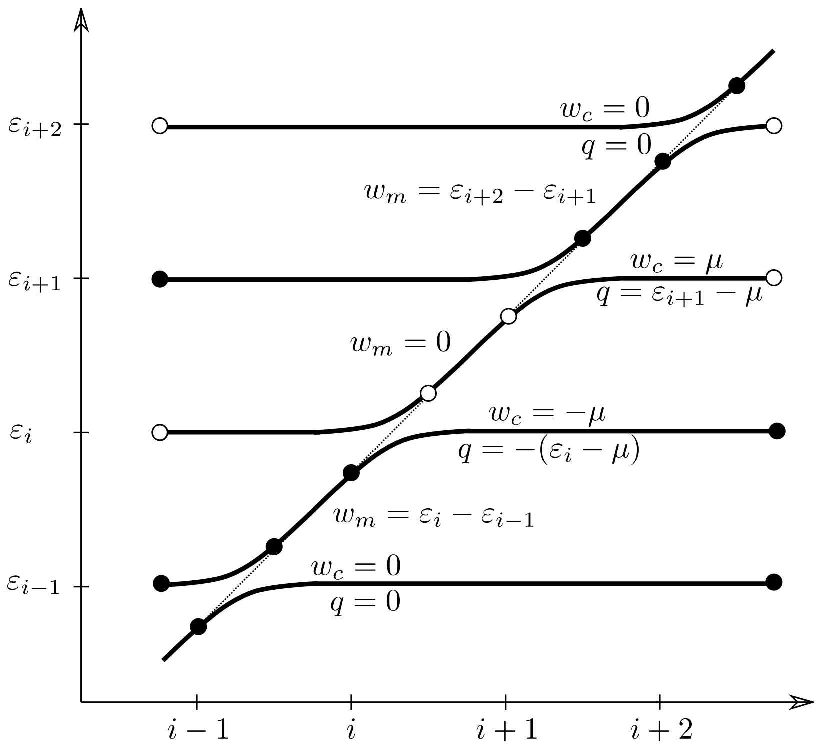


Exact vs stochastic dynamics



[Barra & Esposito, PRE **93**, 062118 (2016)]

Thermodynamics



1st law $\Delta E_{i+} = E_{i+1} - E_i = W_{i+} + Q_{i+}$

$$N_i = p_i \qquad E_i = \varepsilon_i p_i \qquad Q_{i+} = (\varepsilon_i - \mu)(p_{i+1} - p_i)$$

$$W_{i+} = W^{\mathrm{m}}_{i+} + W^{\mathrm{c}}_{i+} \quad \begin{cases} W^{\mathrm{m}}_{i+} = (\varepsilon_{i+1} - \varepsilon_i)p_{i+1} \\ W^{\mathrm{c}}_{i+} = \mu(p_{i+1} - p_i) \end{cases}$$

2nd law $\Delta S_{i+} = S_{i+1} - S_i = \Sigma_{i+} + Q_{i+}/T$

$$S_i = -k_B p_i \ln p_i - k_B (1 - p_i) \ln(1 - p_i)$$

$$\Sigma_{i+} = k_B \left(M_i^+(1 - p_i) - M_i^- p_i\right) \ln \frac{M_i^+(1 - p_i)}{M_i^- p_i} - k_B D(p_{i+1}|p_i) \geq 0$$

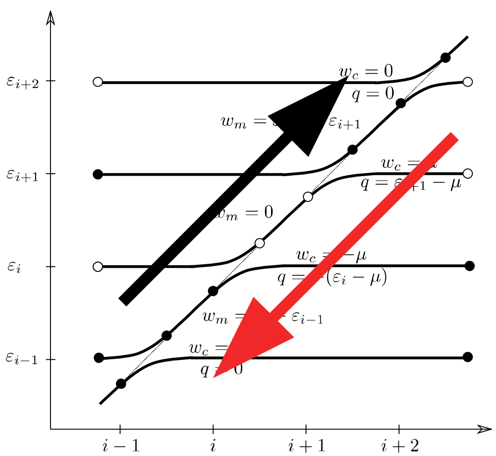
[Barra & Esposito, PRE **93**, 062118 (2016)]

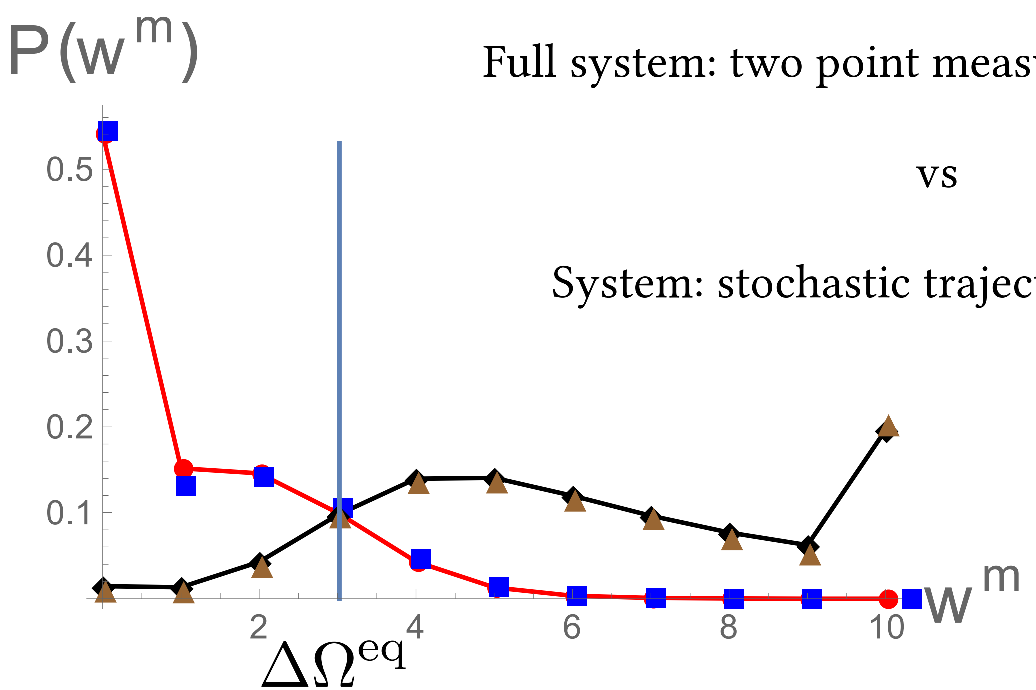
Work fluctuations

Jarzynski and Crooks fluctuation relation

System initially
at equilibrium

$$\frac{P(w^{\mathrm{m}})}{\tilde{P}(-w^{\mathrm{m}})} = \exp\left\{\beta(w^{\mathrm{m}} - \Delta\Omega^{\mathrm{eq}})\right\}$$





Full system: two point measurement approach

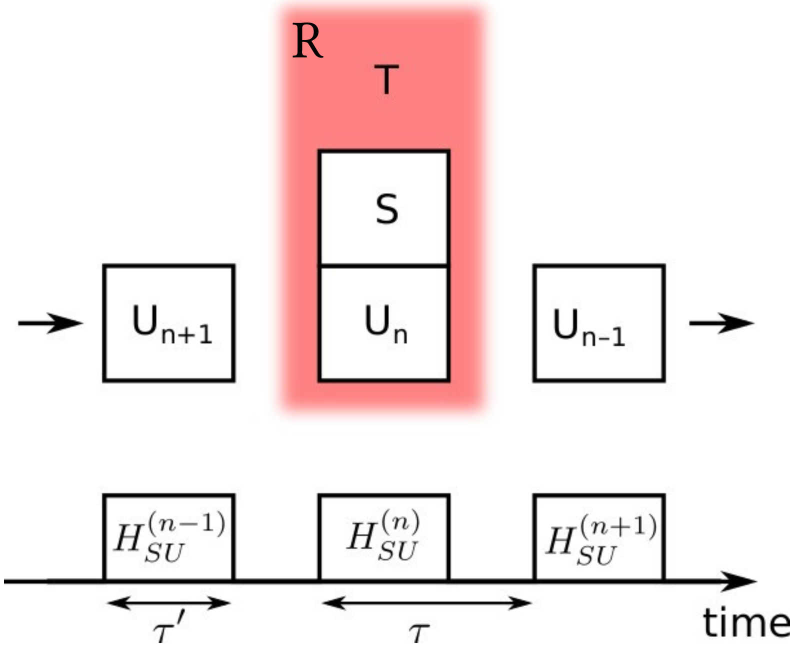
vs

System: stochastic trajectory approach

[Barra & Esposito, PRE **93**, 062118 (2016)]

6) Repeated interactions

Exact identities

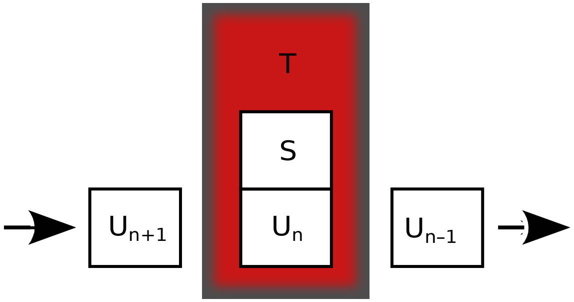


$$\Sigma = \underbrace{\Delta S_S + \Delta S_U - I_{S:U}(\tau)}_{\Delta S_X} - \beta Q \geq 0$$

$$\Delta E_X = \Delta E_U + \Delta E_S = W + Q$$

$$W = W_X + W_{\rm sw} \quad \begin{cases} W_X = \lim_{\epsilon \searrow 0} \int_{\epsilon}^{\tau' - \epsilon} dt' \mathrm{tr}_X \left\{ \rho_X(t) d_t \big(H_S(t) + V_{SU}(t) \big) \right\} \\ W_{\rm sw} = \mathrm{tr}_X \{ V_{SU}(0) \rho_X(0) - V_{SU}(\tau') \rho_X(\tau') \} \end{cases}$$

[Strasberg, Schaller, Brandes & Esposito, PRX **7**, 021003 (2017)]



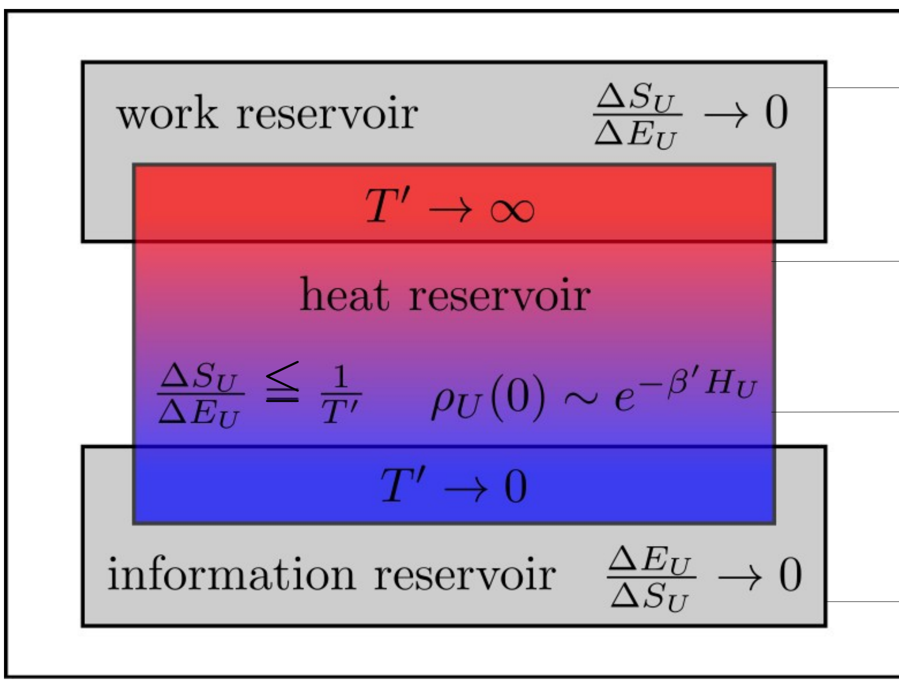
1$^{\text{st}}$ law $\quad \Delta E_S = W + Q \boxed{- \Delta E_U}$

2$^{\text{nd}}$ law $\quad \Sigma_S \equiv \Delta S_S \boxed{+ \Delta S_U} - \beta Q \geq I_{S:U}(\tau) \geq 0$

Steady state: $\quad \rho_S(0) = \rho_S(\tau)$

$\Delta E_S = \Delta S_S = 0$

$$\Sigma_S \overset{SS}{=} \beta(W \underbrace{\boxed{- \Delta F_U}}_{\text{resource}}) \geq 0$$



$\longrightarrow \Delta F_U = \Delta E_U$

$\xrightarrow{T = T'} \Delta F_U \geq 0 \quad W \geq 0$

$\xrightarrow[\text{Heat machine}]{T < T'} \eta = -\dfrac{W}{\Delta E_U} \leq 1 - \dfrac{T}{T'}$

$\longrightarrow \Delta F_U = -T \Delta S_U$

[Strasberg, Schaller, Brandes & Esposito, PRX **7**, 021003 (2017)]
[Deffner & Jarzynski, PRX **3**, 041003 (2013)]

“Classical” vs “Quantum” Resource

$$-W \leq -\Delta F_U = F_U(0) - F_U(\tau)$$

max min

$$H_U = \sum_n E_n |n\rangle\langle n|$$

$$\rho_U(0) = |N\rangle\langle N|$$

Assume a quantum state ρ_{QM} minimizing $F_U(\tau)$

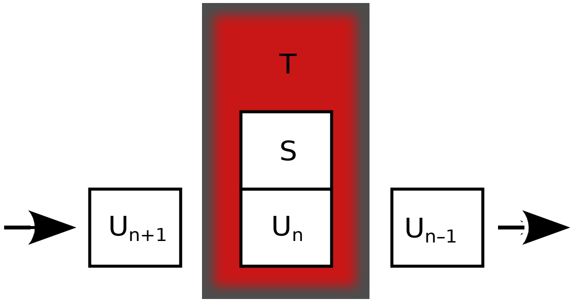
Build the corresponding classical state $\rho_{\mathrm{cl}} = \sum_n \langle n|\rho_{\mathrm{QM}}|n\rangle|n\rangle\langle n|$

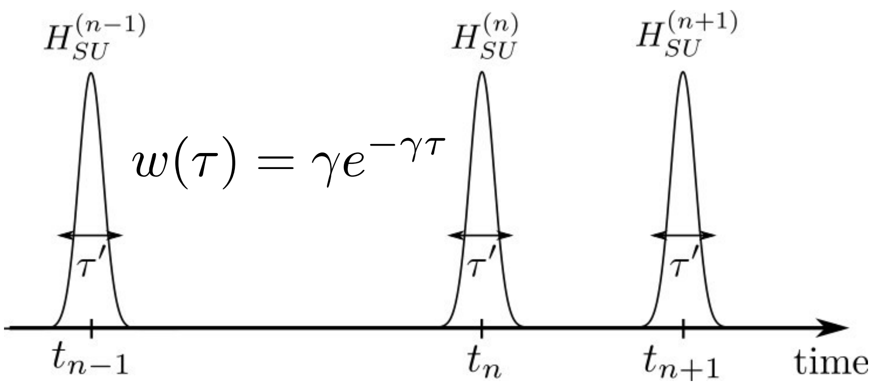
$E(\rho_{\mathrm{QM}}) = E(\rho_{\mathrm{cl}})$ $\quad S(\rho_{\mathrm{QM}}) < S(\rho_{\mathrm{cl}})$ $\Longrightarrow$ $F(\rho_{\mathrm{cl}}) < F(\rho_{\mathrm{QM}})$

Classical resource wins!

[Strasberg, Schaller, Brandes & Esposito, PRX **7**, 021003 (2017)]

Poisson interaction QME





$$H_{SU}(t) = \sum_n \delta(t - t_n) V_{SU}$$

Closed description for the **dynamics** of the system

Effect of a kick: $U = e^{-iV_{SU}}$

$$\mathcal{J}_S \rho_S(t) \equiv \mathrm{tr}_U\{U\rho_S(t) \otimes \rho_U U^\dagger\}$$

$$\mathcal{J}_U \rho_U \equiv \mathrm{tr}_S\{U\rho_S(t) \otimes \rho_U U^\dagger\}$$

$$d_t\rho_S(t) = -i[H_S(t), \rho_S(t)] + \mathcal{L}_\beta \rho_S(t) + \mathcal{L}_{\mathrm{new}}\rho_S(t)$$

$$\mathcal{L}_{\mathrm{new}}\rho_S(t) \equiv \gamma(\mathcal{J}_S - 1)\rho_S(t)$$

[Strasberg, Schaller, Brandes & Esposito, PRX **7**, 021003 (2017)]

Closed description for the **thermodynamics** of the system

1st law
$$d_t E_S(t) = \dot{W}_S(t) + \dot{W}_{\mathrm{sw}}(t) + \dot{Q}(t) - d_t E_U(t)$$

$$\dot{W}_S = \mathrm{tr}_S\{\rho_S(t) d_t H_S(t)\}$$

$$\begin{aligned}\dot{W}_{\mathrm{sw}} &= \gamma \mathrm{tr}_{SU}\{[H_S(t) + H_U][U\rho_S(t)\rho_U U^\dagger - \rho_S(t)\rho_U]\} \\ &= \gamma \mathrm{tr}_S\{H_S(t)(\mathcal{J}_S - 1)\rho_S(t)\} + \gamma \mathrm{tr}_U\{H_U(\mathcal{J}_U - 1)\rho_U\}\end{aligned}$$

$$\dot{Q}(t) = \mathrm{tr}_S\{H_S(t)\mathcal{L}_\beta \rho_S(t)\}$$

$$d_t E_U(t) = \gamma \mathrm{tr}_U\{H_U(\mathcal{J}_U - 1)\rho_U\}$$

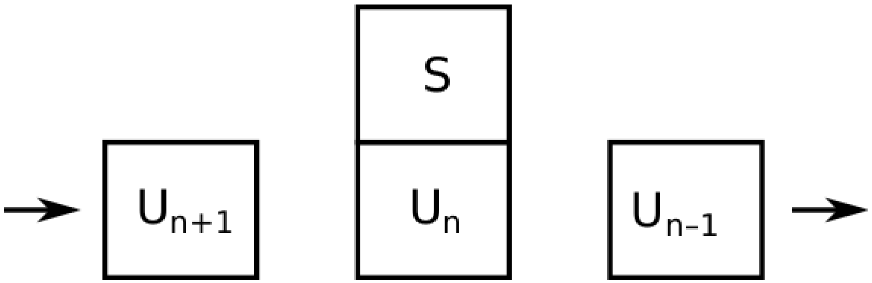
2nd law
$$\dot{\Sigma}_S(t) = d_t S_S(t) + d_t S_U(t) - \beta\dot{Q} \geq 0$$

$$\neq -\mathrm{tr}\{[\mathcal{L}_0\rho_S(t)][\ln\rho_S(t) - \ln\rho_\beta^S(t)]\} - \mathrm{tr}\{[\mathcal{L}_{\mathrm{new}}\rho_S(t)][\ln\rho_S(t) - \ln\bar{\rho}_{\mathrm{new}}]\}$$

thermodynamics cannot always be deduced from dynamics alone !!!

[Strasberg, Schaller, Brandes & Esposito, PRX **7**, 021003 (2017)]

Repeated interaction QME



- No reservoir
- $\rho_U \sim e^{-\beta H_U}$

Sequential interactions: $$H_{SU}(t) = \sum_n \Theta(t - n\delta t)\Theta(n\delta t + \delta t - t)\frac{\tilde{V}}{\sqrt{\delta t}}$$

Frequent Poisson interactions $$H_{SU}(t) = \sum_n \delta(t - t_n)\sqrt{\epsilon}\tilde{V}$$

$$w(\tau) = \gamma e^{-\gamma\tau} \qquad 1/\gamma = \epsilon \to 0$$

[Strasberg, Schaller, Brandes & Esposito, PRX **7**, 021003 (2017)]

Closed description for the **dynamics** of the system $\tilde{V} = \sum_k A_k \otimes B_k$

$$d_t\rho_S(t) = -i[H_S(t), \rho_S(t)] + \sum_{k,l} \langle B_l B_k \rangle_U \left(A_k \rho_S(t) A_l - \frac{1}{2}\{A_l A_k, \rho_S(t)\} \right)$$
$$= \mathcal{L}_S \rho_S(t)$$

Closed description for the **thermodynamics** of the system

1st law $\quad d_t E_S(t) = \dot{W}_{\mathrm{sw}}(t) + d_t E_U(t)$

heat

2nd law $\quad \dot{\Sigma}_S(t) = d_t I_{S:U}(t) = d_t S_S(t) + \beta' d_t E_U(t) - \gamma D(\mathcal{J}_U \rho_U \| \rho_U) \geq 0$

Is it like a normal reservoir? No $\mathcal{L}_S e^{-\beta H_U} \neq 0$ due to switching work

[Barra, Scientific Reports **5**, 14873 (2015)]
[Strasberg, Schaller, Brandes & Esposito, PRX **7**, 021003 (2017)]

Units entropy changes

Approach 1: $$\rho_U(t) = \mathcal{J}_U \rho_U$$

$$d_t S_U(t) = \gamma(-\mathrm{tr}_U\{(\mathcal{J}_U \rho_U)\ln(\mathcal{J}_U \rho_U)\} + \mathrm{tr}_U\{\rho_U \ln \rho_U\})$$

Approach 2:

Fraction of units which interacted $d_t n_t = \gamma N$

$$\bar{\rho}_U(t) = \frac{n_t}{N}\mathcal{J}_U \rho_U + \frac{N - n_t}{N}\rho_U$$

$$d_t \bar{S}_U(t) = -d_t \mathrm{tr}_U\{\bar{\rho}_U(t)\ln \bar{\rho}_U(t)\} = -\gamma \mathrm{tr}_U\{[(\mathcal{J}_U - 1)\rho_U]\ln \rho_U\}$$

Different by $d_t \bar{S}_U(t) - d_t S_U(t) = \gamma D(\mathcal{J}_U \rho_U \| \rho_U)$ "mixing" contribution

Thermal units $\rho_U = \rho^U_{\beta'}$
$$\begin{cases} d_t S_U(t) = \beta' d_t E_U(t) - \gamma D(\mathcal{J}_U \rho^U_{\beta'} \| \rho^U_{\beta'}) \\ d_t \bar{S}_U(t) = \beta' d_t \bar{E}_U(t) \quad \text{ideal reservoir} \end{cases}$$

[Strasberg, Schaller, Brandes & Esposito, PRX **7**, 021003 (2017)]

7) Strong coupling

Strong coupling using Nonequilibrium Green's functions (not for fluctuations!):

[Esposito, Ochoa & Galperin, PRL **114**, 080602 (2015)]
[Esposito, Ochoa & Galperin, PRB **92**, 235440 (2015)]
[Haughian, Esposito & Schmidt, PRB **97**, 085435 (2018)]

Classical strong coupling using time scale separation (including fluctuations):

[Strasberg & Esposito, Phys. Rev. E **95**, 062101 (2017)]

Quantum version is an open problem!!!

Only works in some cases: polaron transformation and QME (including fluctuations):

[Krause, Brandes, Esposito & Schaller, JCP **142**, 134106 (2015)]
[Schaller, Krause, Brandes & Esposito, NJP **15**, 033032 (2013)]

8) Conclusions

Stochastic thermodynamics is the canonical formulation of nonequilibrium thermodynamics

Exact Hamiltonian identity: Entropy production as system-reservoir correlations

For practical use and EP rate > 0, we use it to build thermodynamics on top of system's kinetics

- Born-Markov-Secular QME: (with or without Floquet) — Weak coupling, dense reservoir

- Landau-Zener QME: — Weak coupling, sparse reservoir

- Repeated interactions: — Importance of the switching work
Thermo can't always be deduced from dynamics
Fluctuations not explored yet

Strong coupling, quite clear now for classical systems, but not yet for quantum systems!

Thank you