QSIM19

SCHWARZSCHILD BLACK HOLES AS MACROSCOPIC QUANTUM SYSTEMS

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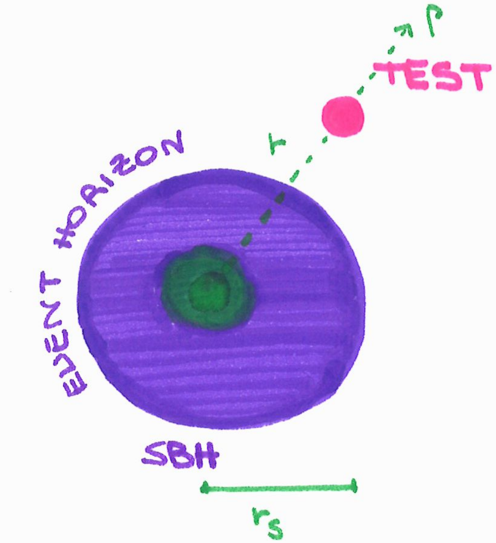
FIRENZE

THE SCHWARZSCHILD BLACK HOLE

$$h_{SBH}^{TEST}(p,r) = \frac{p^2}{2} + \frac{1}{2}\left(\frac{L^2}{r^2} + \mu^2\right)\left(1 - \frac{r_s}{r}\right)$$



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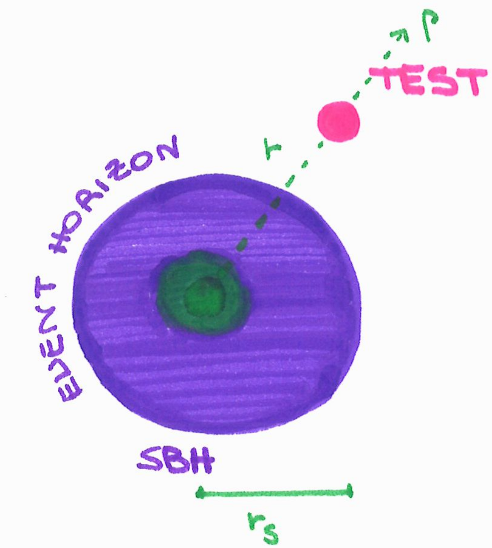

r_s Schwarzschild radius

L modulus of the conserved angular momentum

p, r radial momentum and position $\{p,r\}_{PB} = 1$

$\mu^2 = 1, 0$ mass^2

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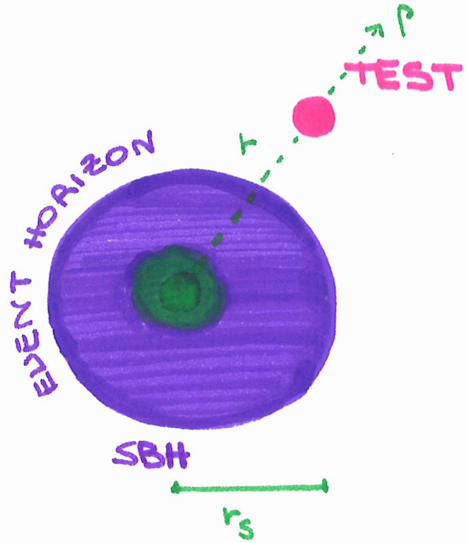

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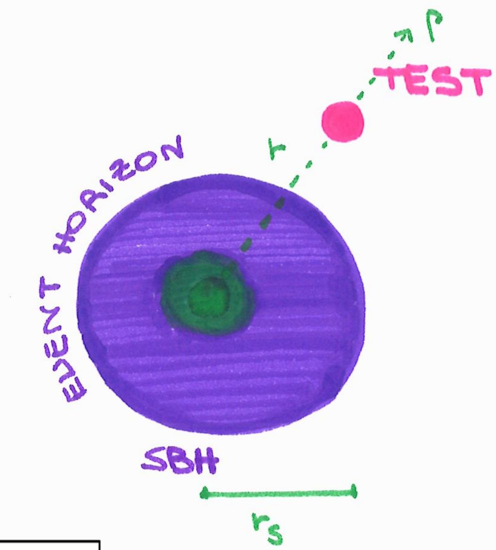

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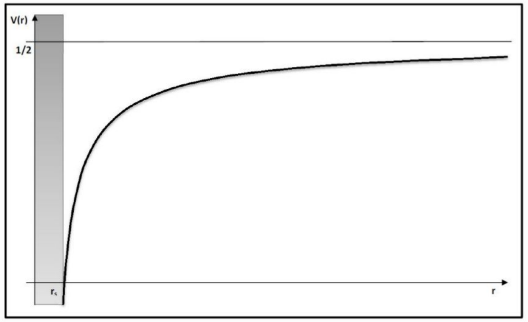
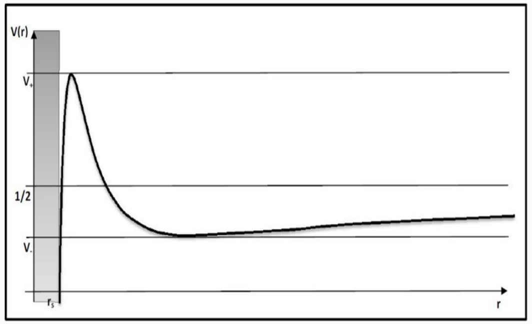


SBH potential for matter if $L^2 < 3r_s^2$



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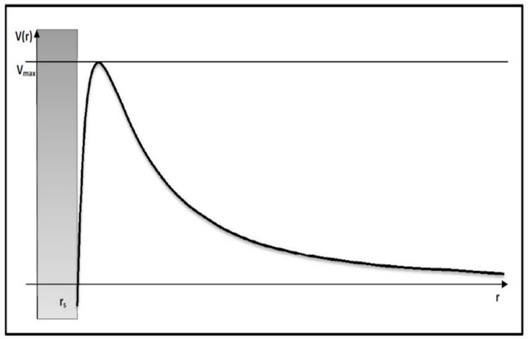

SBH potential for light

QUANTUM MECHANICS vs GENERAL RELATIVITY

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ALGEBRA vs GEOMETRY

MICRO vs MACRO

EVOLUTION IN TIME vs SYMMETRY TRANSFORMATION

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generalized coherent states

GCS

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generalized coherent states — GCS

large-N quantum (field) theory — large-N Q(F)T

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generalized coherent states

GENERALIZED COHERENT STATES

GCS

group-theoretic construction

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$Q \rightarrow \mathfrak{g}\ \&\ \mathcal{H} \longrightarrow G$ group (Lie)

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group-theoretic construction

$Q \rightarrow \mathfrak{g}\ \&\ \mathcal{H} \longrightarrow \mathcal{G}$ "dynamical" group (Lie)

propagators of Q are elements of a unitary irreducible representation of $\mathcal{G}$ obtained from $\mathfrak{g}$ via a Lie exponential map

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$\mathcal{M}$!! manifold

GCS

$$\hat{\Omega} \in \mathcal{G}/\mathcal{H}$$

$$\hat{\Omega} = e^{\sum_\beta (\Omega_\beta \hat{E}_\beta - \Omega^*_\beta \hat{E}_{-\beta})}$$

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$$\Updownarrow \qquad\qquad\qquad\qquad \Updownarrow$$

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manifold with a symplectic structure

$\Omega \to \zeta \to \tau$ complex projective coordinates

$$\{f, g\} = i \sum_{\alpha\beta} \left(\frac{\partial f}{\partial \tau_\alpha} \frac{\partial g}{\partial \tau^*_\beta} - \frac{\partial f}{\partial \tau^*_\beta} \frac{\partial g}{\partial \tau_\alpha} \right)$$

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$$d\mu(\Omega) \; : \; \int_{\mathcal{M}} d\mu(\Omega) |\Omega\rangle\langle\Omega| = \hat{\mathbb{1}}_{\mathcal{H}}$$

invariant measure

PSEUDO-SPIN COHERENT STATES SU(1,1)

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$su(1,1)$ algebra : $\mathrm{span}\{\hat{K}_0, \hat{K}_1, \hat{K}_2\}$ $\quad [\hat{K}_\alpha, \hat{K}_\beta] = i\varepsilon_{\alpha\beta\gamma}\hat{K}^\gamma$

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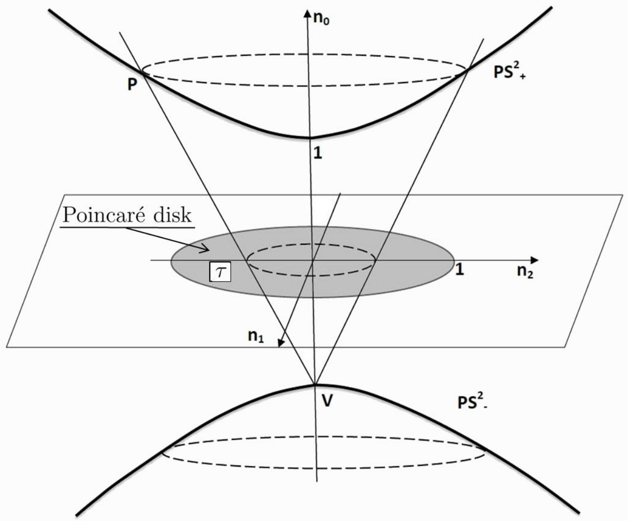
? $\mathcal{H}$?

THE MANIFOLD PS^2 (PSEUDO - SPHERE)



two-sheets hyperboloid

$\vec{X} = R\vec{n} \qquad R^2 = \frac{k}{2}$

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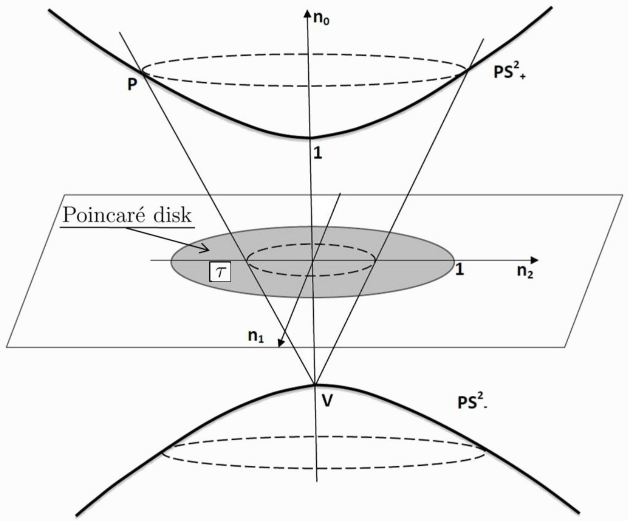

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$$\Omega \in PS^2 \longleftrightarrow i\Omega = \frac{\rho}{2} e^{i\varphi} \longleftrightarrow \vec{n} = (\cosh\rho,\ \sinh\rho\cos\varphi,\ \cosh\rho\sin\varphi)$$

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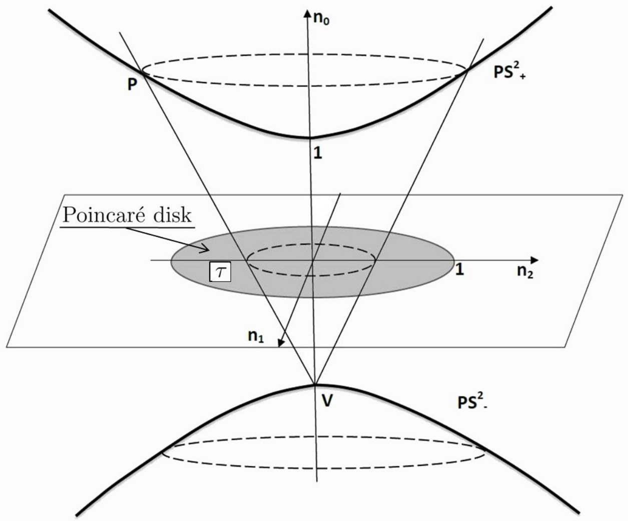

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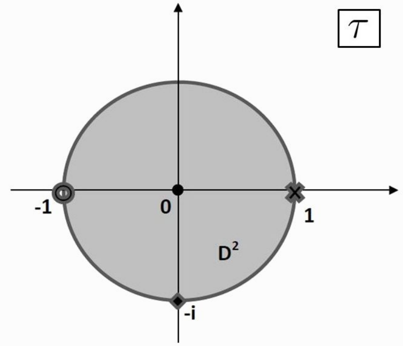

Poincaré disk D^2

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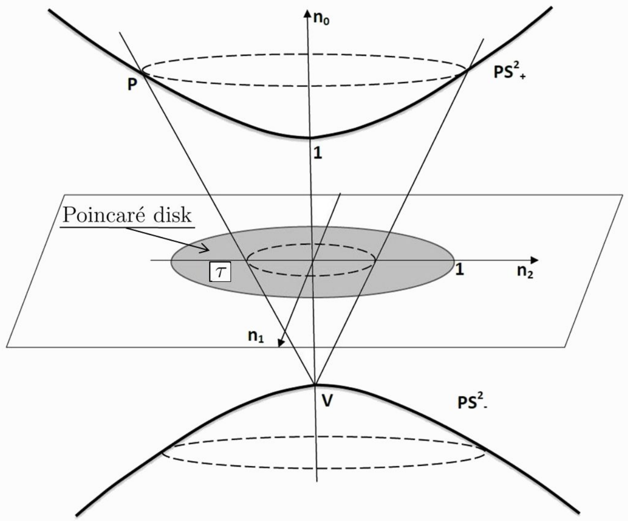

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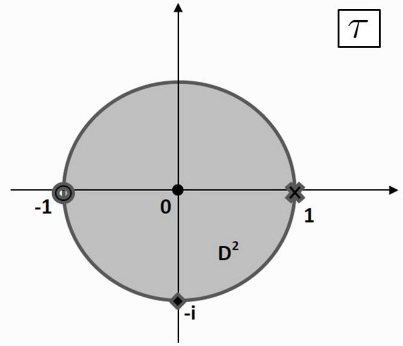

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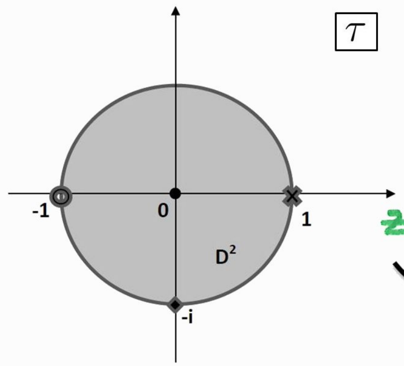
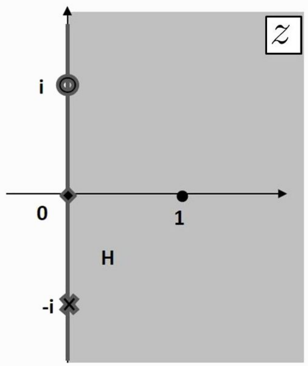


$z = \frac{i+\tau}{i-\tau}$



Poincaré half-plane $\mathbb{H}$

QUANTUM MECHANICS vs GENERAL RELATIVITY

MICRO vs MACRO

large-N quantum (field) theory

QUANTUM theory Q

Hilbert space $\mathcal{H}$

Lie algebra $\mathfrak{g}$

Hamiltonian operator $\hat{H} \in \mathfrak{g}$

QUANTUM theory $\mathcal{Q}$ | CLASSICAL theory $\mathcal{C}$

- Hilbert space $\mathcal{H}$
- Lie algebra $\mathfrak{g}$
- Hamiltonian operator $\hat{H} \in \mathfrak{g}$

* manifold $\mathcal{M}$
* symplectic form on $\mathcal{M}$ that defines Poisson Brackets $\{\cdot\}_{PB}$
* Hamiltonian function $h_{cl} : \mathcal{M} \to \mathbb{R}$

QUANTUM theory $\mathcal{Q}$ $\longleftrightarrow$ CLASSICAL theory $\mathcal{C}$

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$\lim_{k \to 0}$

WHEN $\lim_{k \to 0} Q_k = C$?

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four conditions must hold

1 irreducibility of $\mathcal{G}_k$ $\rightarrow$ $c_k \int_{\mathcal{M}} d\mu(\Omega)\, |\Omega\rangle_k\langle\Omega| = \mathbb{1}$

2 uniqueness of the "zero" operator

3 exponentially decrease of different coherent states overlap

4 classical limit of the Hamiltonian $\lim_{k \to 0} k \dfrac{\langle\Omega|\hat{H}|\Omega\rangle_k}{\langle\Omega|\Omega\rangle_k} < \infty$

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$$\lim_{k \to 0} k H_k(\Omega) = h_{cl}(v,w)$$!!

LARGE-N LIMIT OF VECTOR MODELS

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Q_N describes a system of N spinless particles $i[\hat{q}_i, \hat{p}_j] = \frac{1}{N}\delta_{ij}\hat{\mathbb{1}}$

with a global symmetry whose invariants are

$$\hat{A} = \frac{1}{2}\sum_i \hat{q}_i^2 \qquad \hat{B} = \frac{1}{2}\sum_i (\hat{q}_i\hat{p}_i - \hat{p}_i\hat{q}_i) \qquad \hat{C} = \frac{1}{2}\sum_i \hat{p}_i^2$$

with Hamiltonian

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h an arbitrary polynomial

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set $\hbar = \frac{1}{N}$

LARGE-N LIMIT OF VECTOR MODELS

Q_N describes a system of N spinless particles $i[\hat{q}_i, \hat{p}_j] = \frac{1}{N}\delta_{ij}\hat{\mathbb{1}}$

with a global symmetry whose invariants are

$$\hat{A} = \frac{1}{2}\sum_i \hat{q}_i^2 \qquad \hat{B} = \frac{1}{2}\sum_i (\hat{q}_i\hat{p}_i - \hat{p}_i\hat{q}_i) \qquad \hat{C} = \frac{1}{2}\sum_i \hat{p}_i^2$$

with Hamiltonian

$$\hat{H}_N = N h[\hat{A}, \hat{B}, \hat{C}]$$

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$$A(v,w) = w \qquad B(v,w) = 2vw \qquad C(v,w) = w\left(\frac{k^2}{w^2} + v^2\right)$$

notice that $K^2(v,w) = k^2$ constant in $\mathcal{M}$

conserved angular momentum of the classical theory !!

EXAMPLE : FREE PARTICLES

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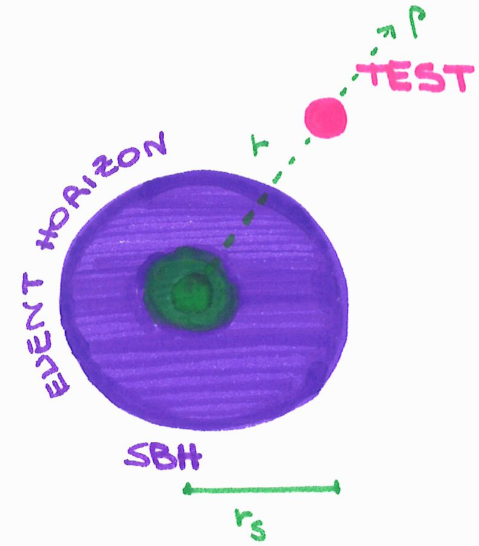
with $4k^2 = L^2$ angular momentum

CONSERVED

!

THE SCHWARZSCHILD BLACK HOLE

$$h_{SBH}^{TEST}(p,r) = \frac{p^2}{2} + \frac{1}{2}\left(\frac{L^2}{r^2} + \mu^2\right)\left(1 - \frac{r_s}{r}\right)$$



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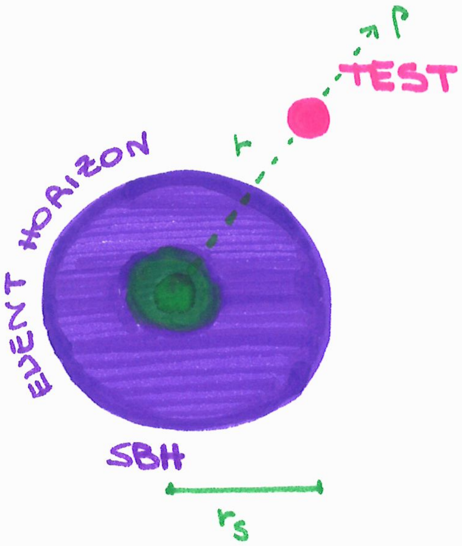
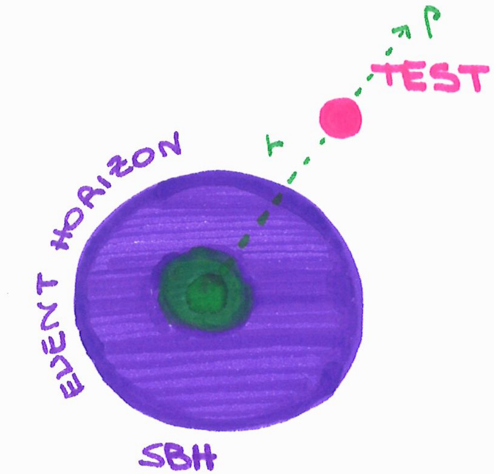
WHAT IS WHAT ?

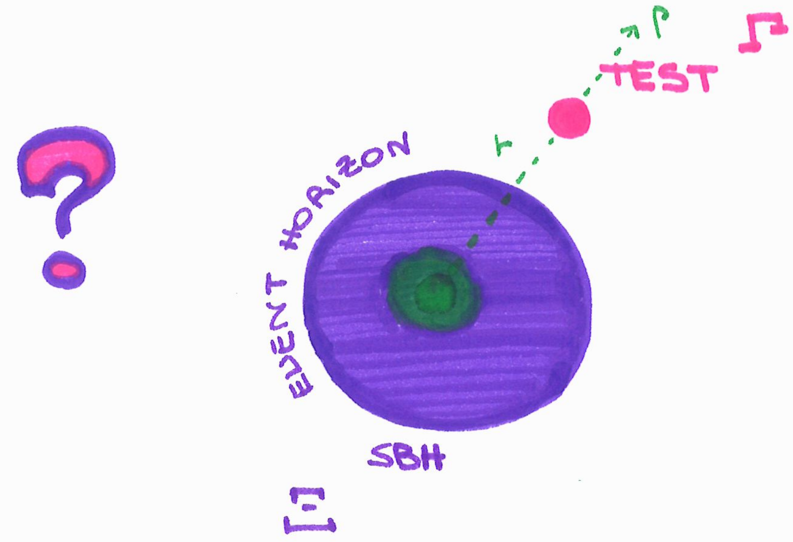


COMPOSITE SYSTEM



COMPOSITE SYSTEM

$$\Psi = \Gamma + \Xi$$



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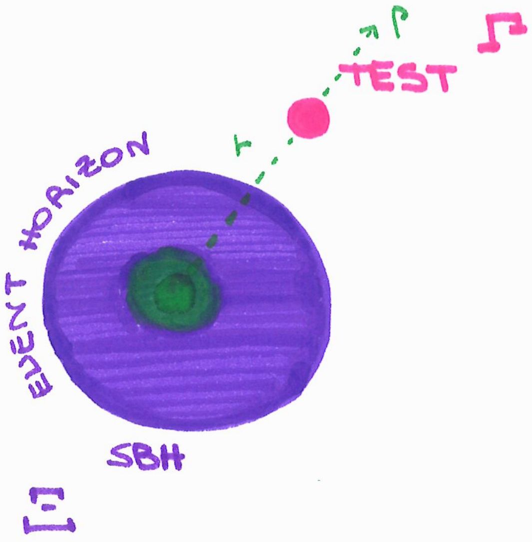

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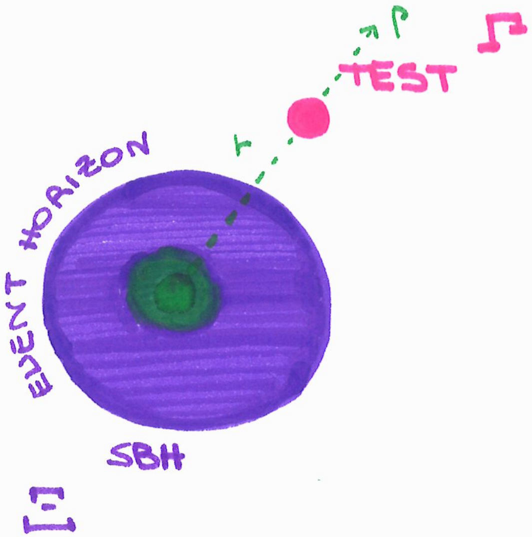

PARAMETRIC REPRESENTATION (WITH $\underbrace{\text{COHERENT STATES FOR } \Xi}_{\text{ECS}}$)

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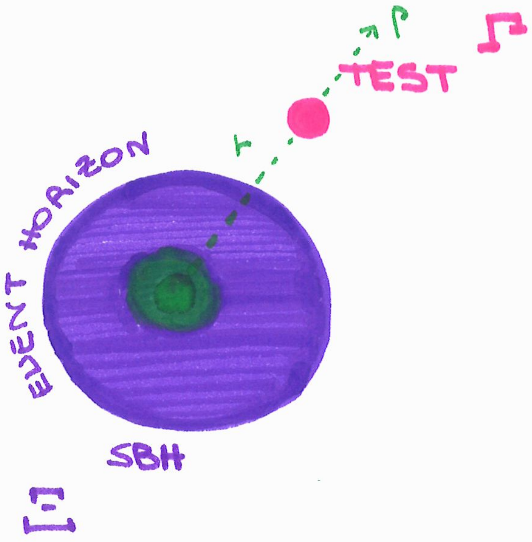

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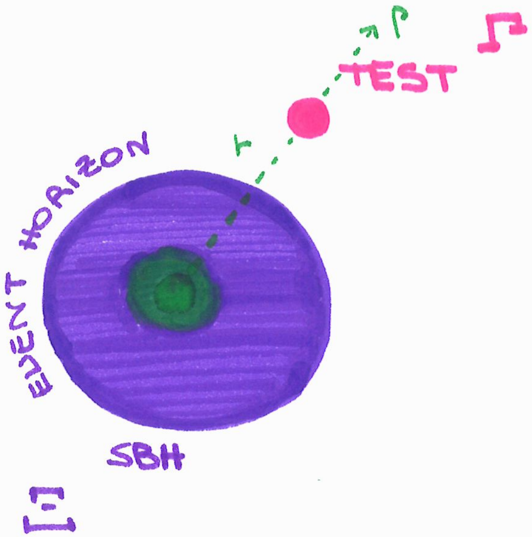

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$$|\Psi\rangle = \sum_{\gamma\xi} \underbrace{c_{\gamma\xi} |\xi\rangle \otimes |\gamma\rangle}_{\int_{\mathcal{M}} d\mu(\Omega) |\Omega\rangle\langle\Omega|} = \int_{\mathcal{M}} d\mu(\Omega)\, \chi(\Omega)\, |\Omega\rangle \otimes |\phi(\Omega)\rangle$$

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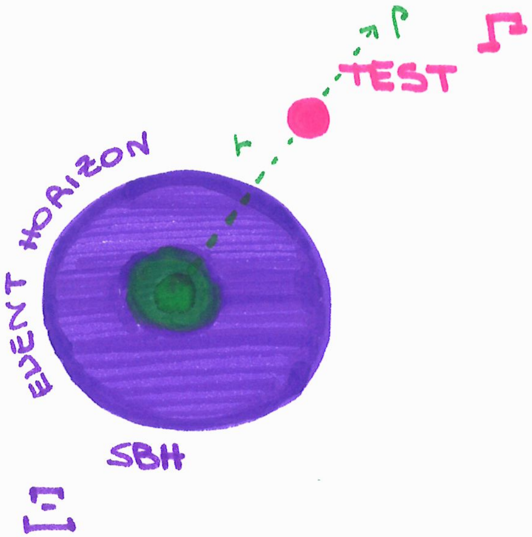

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$$\int_{\mathcal{M}} d\mu(\Omega)\, \chi^2(\Omega) = 1 \qquad \langle\phi(\Omega)|\phi(\Omega)\rangle = 1 \quad \forall\, \Omega \in \mathcal{M}$$

HOW CAN AN ENVIRONMENT

SHAPE THE SPECTRUM OF

A QUANTUM PARTICLE ?

SBH

TEST

HOW CAN AN ENVIRONMENT Ξ SHAPE THE SPECTRUM OF A QUANTUM PARTICLE Ω

SBH

TEST

- TIMELESSNESS $\qquad \hat{H}_{\Psi} |\Psi\rangle = 0$

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THESE ASSUMPTIONS ARE THE SAME UPON WHICH THE

PaW MECHANISM

and recent updates are based

with Ξ the clock for Ω

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LARGE-N Ξ

$N \longrightarrow \infty$

$$\int d\mu(\Omega)\, \chi(\Omega)\, \delta(\Omega - \bar{\Omega}) \left[H_\Xi(\Omega, \bar{\Omega}) |\phi(\Omega)\rangle - \hat{H}_\Omega |\phi(\Omega)\rangle \right] = 0$$

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! $\quad h_\Xi^\Omega(\Omega) = h_{SBH}^{TEST}(p, r) \quad$?

SBH AS MACROSCOPIC QUANTUM SYSTEMS

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$$g = SU(1,1)$$

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! $\mathfrak{g} = SU(1,1)$ + HAWKING RADIATION

TWO-MODE REALIZATION OF SU(1,1) ALGEBRA !!

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$$\hat{a} = \sum_i \hat{a}_i \qquad \hat{b} = \sum_i \hat{b}_i$$

$$[\hat{a}_i, \hat{a}_j^+] = \frac{1}{N^2}\delta_{ij}\hat{\mathbb{1}} = [\hat{b}_i, \hat{b}_j^+] \qquad [\hat{a}_i, \hat{b}_j^{(+)}] = 0 \qquad i,j: 1,2 \dots N$$

$$[\hat{a}, \hat{a}^+] = [\hat{b}, \hat{b}^+] = \frac{1}{N}\hat{\mathbb{1}} \qquad [\hat{a}, \hat{b}^{(+)}] = 0$$

$$SU(1,1) \quad \begin{cases} \hat{K}_+ = \hat{a}^+\hat{b}^+ \\ \hat{K}_- = \hat{a}\hat{b} \\ \hat{K}_0 = \frac{1}{2}(\hat{a}^+\hat{a} + \hat{b}\hat{b}^+) \end{cases} \qquad \hat{K}_\pm = \hat{K}_1 \pm i\hat{K}_2$$

$$2\hat{A} = \hat{N}_a + \hat{N}_b - i(\hat{a}^+\hat{b}^+ - \hat{a}\hat{b}) \qquad 2\hat{C} = \hat{N}_a + \hat{N}_b + i(\hat{a}^+\hat{b}^+ - \hat{a}\hat{b})$$

$$\hat{B} = \hat{a}^+\hat{b}^+ + \hat{a}\hat{b}$$

$$\hat{H}_{\Xi} = N\left[\hat{C} + \alpha\hat{A} + \beta\hat{B} + \varepsilon\hat{\mathbb{1}}\right] \quad ?$$

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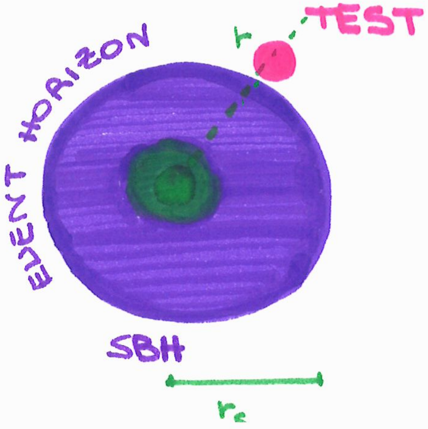

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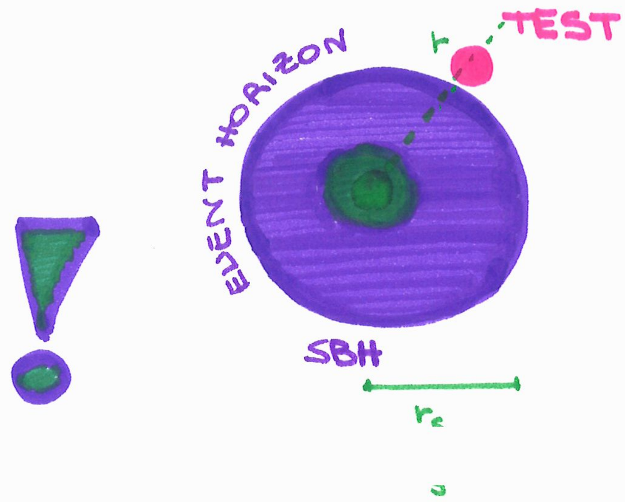
$$= h^{TEST}_{SBH}(p, r - r_s \ll r_s) \sim \frac{p^2}{2} + \frac{1}{2r_s^2}\left(\frac{L^2}{r_s^2} + \mu^2\right)\delta + o\left(\frac{\delta^2}{r_s^2}\right)$$

NEAR-HORIZON $\delta := r - r_s \ll r_s$

NEAR HORIZON DYNAMICS

$$r - r_s \ll r_s$$



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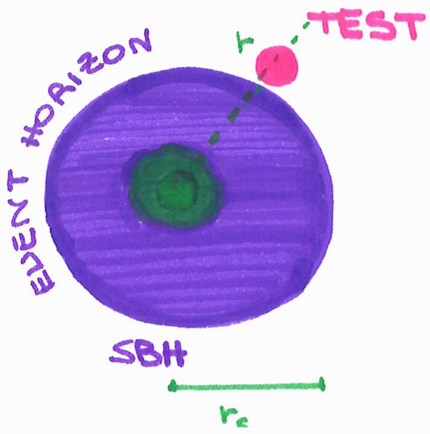

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free quadratic term

pairs creation-annihilation term
(Hawking ?)

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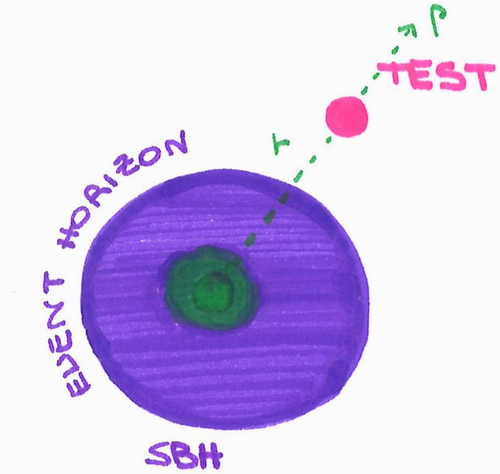

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THE MACROSCOPIC Ξ

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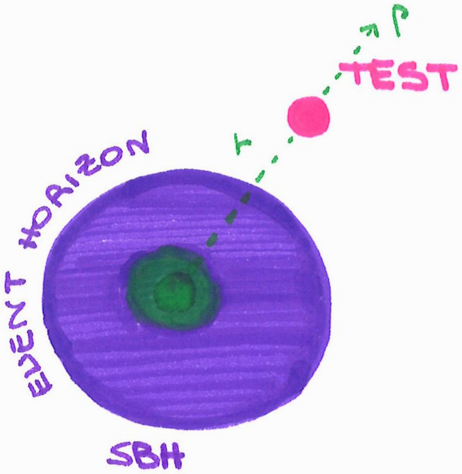
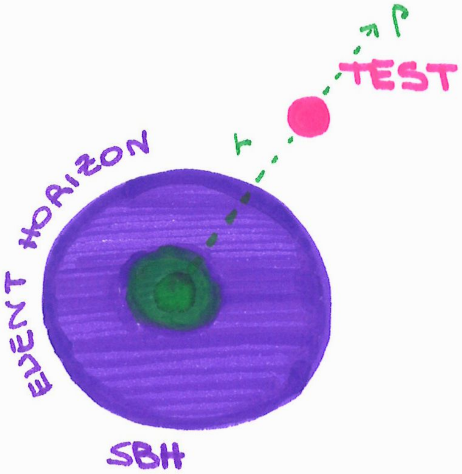
TEST PARTICLE Ω



$$\hat{H}_{\Omega} |\phi(\Omega)\rangle = H_{\Xi}(\Omega)|\phi(\Omega)\rangle$$



THE MACROSCOPIC Ξ IS THE SBH FOR THE TEST PARTICLE Ω

Its presence is reflected into the behaviour test particle Ω via the parametric dependence of its spectrum on the couple (p,r) provided by Ξ

(remember that $(p,r) \to \Omega \in \mathcal{M} \to$ coherent states for Ξ)

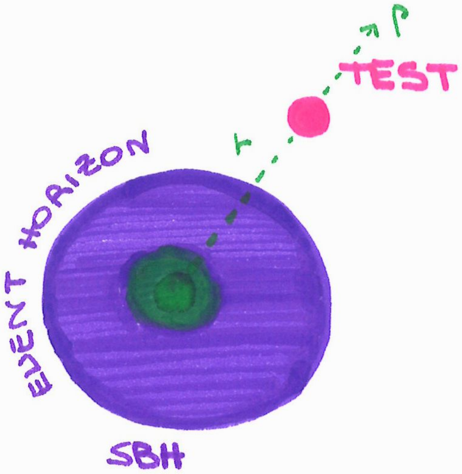
QUANTUM MECHANICS vs GENERAL RELATIVITY

EVOLUTION IN TIME vs SYMMETRY TRANSFORMATION

$\Downarrow$

Page & Wootters mechanism

$$\hat{H}_{\Omega} |\phi(\Omega)\rangle = H_{\Xi}(\Omega) |\phi(\Omega)\rangle$$



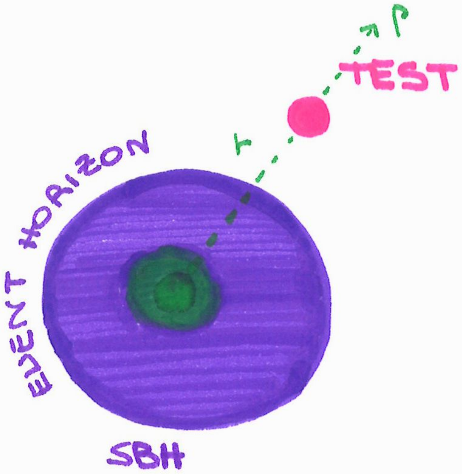
THE MACROSCOPIC Ξ

IS THE SBH FOR THE

TEST PARTICLE Ω

- TIMELESSNESS
- NON-INTERACTING PARTITION
- ENTANGLEMENT

$$\hat{H}_\Omega |\phi(\Omega)\rangle = H_\Xi(\Omega)|\phi(\Omega)\rangle$$



THE MACROSCOPIC Ξ

IS THE SBH FOR THE

TEST PARTICLE Ω

- TIMELESSNESS
- NON-INTERACTING PARTITION
- ENTANGLEMENT

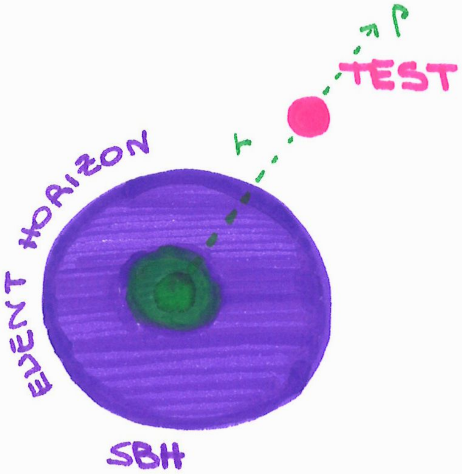
THESE ASSUMPTIONS ARE THE SAME UPON WHICH THE

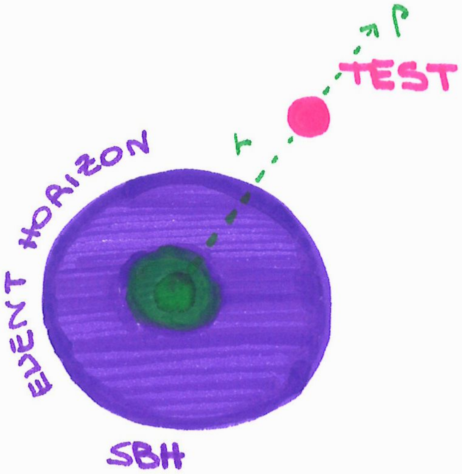
PaW MECHANISM

and recent updates are based

with Ξ the clock for Ω

Ξ IS THE SBH

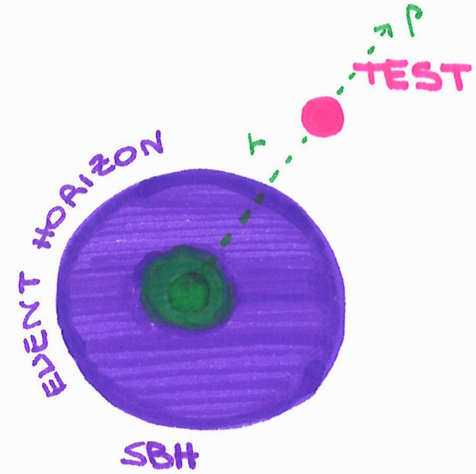




Ξ IS THE SBH

According to the PAW mechanism

Ξ IS A CLOCK FOR Ω



Ξ IS THE SBH

According to the PAW mechanism

Ξ IS A CLOCK FOR Ω

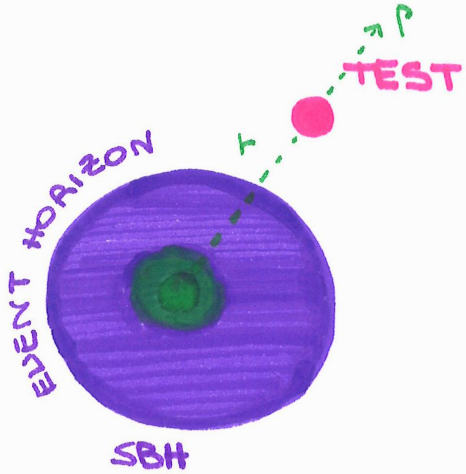
→

THE SBH IS THE

CLOCK

FOR WHAT GOES AROUND IT

THANKS



Ξ IS THE SBH

According to the PAW mechanism

Ξ IS A CLOCK FOR Ω

$\longrightarrow$

THE SBH IS THE

CLOCK

FOR WHAT GOES AROUND IT

!!